THE DETERMINATION OF THE AVERAGE LEAST DIMENSION OF SURFACING AGGREGATES BY COMPUTATION

B. DUMAS

Provincial Administration: Western Cape,
Department of Economic Affairs, Agriculture & Tourism,
31 Chiappini Street, Cape Town, 8001

ABSTRACT
The Average Least Dimension (ALD) of surfacing aggregates is required for the determination of the quantity of a bituminous product to be sprayed in the case of normal road surface treatments. The ALD is also needed for the calculation of the spread rate of surfacing aggregates. Method TMH1-B18(a), which is based on measuring the least dimension of an aggregate is the reference method for the determination of the ALD in South Africa. The accuracy of the measured ALD method depends on the sampling process, the sample size, measuring equipment, the interpretation of the least dimension, and the fraction of the sample not measured. The computational method TMH1-B18T(b), currently in use, is inaccurate and also does not take full account of the nature of the particle size distribution which has an effect on the estimated measured ALD. The computational method developed, as discussed in this paper, was designed to reduce the error between ALD measured and ALD computed by introducing more factors which will have a significant decreasing effect on the estimated measured ALD error.

Introduction

A reliable computational method for the determination of the ALD must comply to the following: (1) the variables must be obtainable from the normal tests required for bituminous surfacing and (2) the sample size must be of such an order that the sampling effects on the accuracy are reduced to the minimum.

The computational ALD method, currently in use, is not powerful enough to give a reliable estimate of the measured ALD. This method is over-simplified regarding the description of the distribution of the aggregates. Besides the inadequate formulation of the relationship, the nomogram given in TMH1, is awfully distorted (badly constructed).

A nomogram is a handy method when it comes to the application side of a relationship but it can be very difficult to handle when more variables are introduced or when the numeric scales are no longer linear. In the designing process of the new computational method, as discussed in this paper, the factors and principles involved in the determination of the ALD received all the attention. The construction or designing of a nomogram was not considered important seeing that computers and calculators are readily available nowadays.

Measured ALD method

The measured ALD method is described in TMH1-B18(a).

Definition
The least dimension of an aggregate particle is the smallest perpendicular distance between two parallel plates through which the particle will just pass. The average least dimension is the arithmetic mean of all the measured least dimensions of the aggregate particles measured.

There is a very important instruction mentioned in this method which has a significant effect on the measured ALD if not adhered to, and states, that the fraction passing the sieve aperture size half the nominal size (the smallest sieve aperture size through which 85% of the aggregates will pass) must not be measured. According to some historic data it would seem that this instruction was not always consistently adhered to.

Shortcomings
The measured ALD method is not free of errors. There are several areas where the reliability of the measured ALD can be significantly effected. The following exercises highlight three of the major areas where errors occur daily.

- The incautious sampling and preparation procedures have a diminishing effect on the reliability of the measured ALD. Even under good controlled sampling and preparation actions the variation can still be high. The following results were obtained from samples (30 kg) taken at nine different randomly selected sample sites from a stockpile which was constructed very carefully during its construction. From each sample a gradation as well as a measured ALD was carried out. The test data is tabulated as follows:

Sample	< 19,0 mm	< 13,2 mm	< 9,5 mm	< 6,7 mm	< 4,75 mm	Flakiness Index (%)	ALD measured (mm)
Site 1	100	99	59	15	5	21,3	5,74
Site 2	100	99	55	11	3	19,2	5,99
Site 3	100	99	39	9	4	14,6	6,42
Site 4	100	99	57	9	2	19,5	5,57
Site 5	100	99	57	16	5	19,8	5,91
Site 6		100	63	13	3	23,5	5,48
Site 7		100	47	8	3	15,7	6,14
Site 8		100	56	11	3	24,8	5,39
Site 9		100	62	18	6	22,7	5,52
						Mean	5,83
						Error	1,73*

* At a confidence level of 95%.

According to the analysis of the stockpile site samples the measured ALD error is of the order of 1,7 mm. It means that the actual measured ALD of a fairly controlled constructed stockpile can be any value within the range of the measured ALD ± 1,7 mm.

In the following exercise one of the above mentioned 30 kg samples (Site 9) was divided into one kilogram portions. Ten of these portions were selected randomly. After the gradations were carried out, the portions were further divided into four sub-portions, large enough for a measured ALD (number of aggregate particles varied between 250 and 350). The results are as follows:

Sample	< 19,0 mm	< 13,2 mm	< 9,5 mm	< 6,7 mm	< 4,75 mm	Flakiness Index (%)	Mean ALD measured (mm)
Portion 1		100	64	18	6	25,4	5,41
Portion 2	100	99	63	17	6	23,0	5,36
Portion 3		100	62	17	6	22,1	5,41
Portion 4		100	61	17	6	29,0	5,56
Portion 5	100	99	60	17	6	23,3	6,07
Portion 6		100	65	20	6	19,3	5,41
Portion 7	100	99	63	18	6	22,9	5,42
Portion 8	100	99	62	17	6	18,9	5,37
Portion 9	100	99	58	17	6	20,7	5,64
Portion 10		100	65	18	6	22,1	5,52
						Mean	5,52
						Error	1,01

The ALD variation within a sample according to this exercise varied between 5,36 and 6,07 mm (a range of 0,71 mm). Regarding sample preparation under good controlled conditions the measured ALD error is about 1,01 mm.

- Secondly, the instruction mentioned earlier, regarding the discarding of the fraction of aggregates not to be measured, has a significant effect on the reliability of the measured ALD, if not applied correctly. Take for example the following case where the difference in gradation of the two samples is insignificant. In the case of sample A and according to the above mentioned rule the aggregates passing the 6,7 mm sieve must not be measured while in the case of sample B the aggregates passing the 6,7 mm sieve and retained on the 4,75 mm sieve must be measured.

Sample	< 19,0 mm	< 13,2 mm	< 9,5 mm	< 6,7 mm	< 4,75 mm	Flakiness Index (%)	Mean ALD measured (mm)
A	100	85	55	10	0	20.0	6,502
B	100	86	55	10	0	20.0	5,722

According to the results the difference between the two ALDs is about 0,8 mm (14%) which is a significant difference.

- The instruction, viz. “ *by means of a riffler, divide out a representative sample of such a size as to give at least 200 aggregate particles of each of the fractions to be tested* “ is sometimes wrongly applied. Some operators measure only the first 200 aggregate particles and discard the rest. For obvious reasons the error in the measured ALD due to this negligence is unpredictable and therefore renders the ALD completely unreliable.

The computational ALD method currently used

This method is a computational method where the ALD of the aggregates is calculated by using the gradation and flakiness index data. The method is intended to be used as a quick assessment or controlling method to find the ALD.

In this computational method the median is the only variable which describes the particle size distribution of the aggregates. The median on its own cannot fully reflects the characteristics of the particle size distribution. Hence more information is required besides the median.

The nomogram, given in the TMH1-B18(b)T method, is a metricated and reconstructed version of the original imperial scaled nomogram which appeared in a document published by Shell (Surface Dressing – 1963). The interval markings of the scales of the nomogram are not so good. In Figure 1a, which is a copy of the above mentioned nomogram, one can clearly see how distorted the interval markings of the scales of the nomogram are. Figure 1b is an example of how badly the Median-scale intervals are marked (14,8 to 16,6). An example of the ALD scale (8,1 to 10,2) is given in Figure 1c and an example of the Flakiness Index scale (43 to 52) is given in Figure 1d.

In practice this method became very popular and attractive due to the speed of determination of the ALD. This method is no longer seen as a controlling method and is used more and more as the basis to determine the spray rates of bituminous products. For this reason it is imperative to find a new computational method which will produce a more accurate estimated value for the measured ALD.

New computational ALD method

It will be desirable to have a computational method where the ALD can be calculated fairly accurate and that the variables, required in the relationship, is obtainable from the normal test methods used for surfacing aggregate.

ALD measured

As stated before the measured ALD is the average least dimension and it is obtained by determining the arithmetic mean of the least dimension of all the aggregates, measured with a calliper (comparator table). The ALD is therefore a certain type of mean which describes the central tendency of the distribution of least dimension of the aggregates. It depends mainly on the particle size distribution of the aggregates as well as the shape of the particles.

Prelude

The median and the arithmetic mean of a symmetrical normal distribution are equal while in the case of asymmetrical distributions they are different. Therefore if the median and the arithmetic mean (measured ALD) of the least dimension distribution is plotted out graphically then all the points on a straight line (mean equals median) will represent a symmetrical distri-bution. Any other point deviating from the straight line will reflect asymmetrical distributions. Asymmetrical distributions consisting of more fine than coarse particles are pointed out by the points lying below the straight line. In the cases where the coarse particles are dominating the finer particles the corresponding points will lie above the straight line.

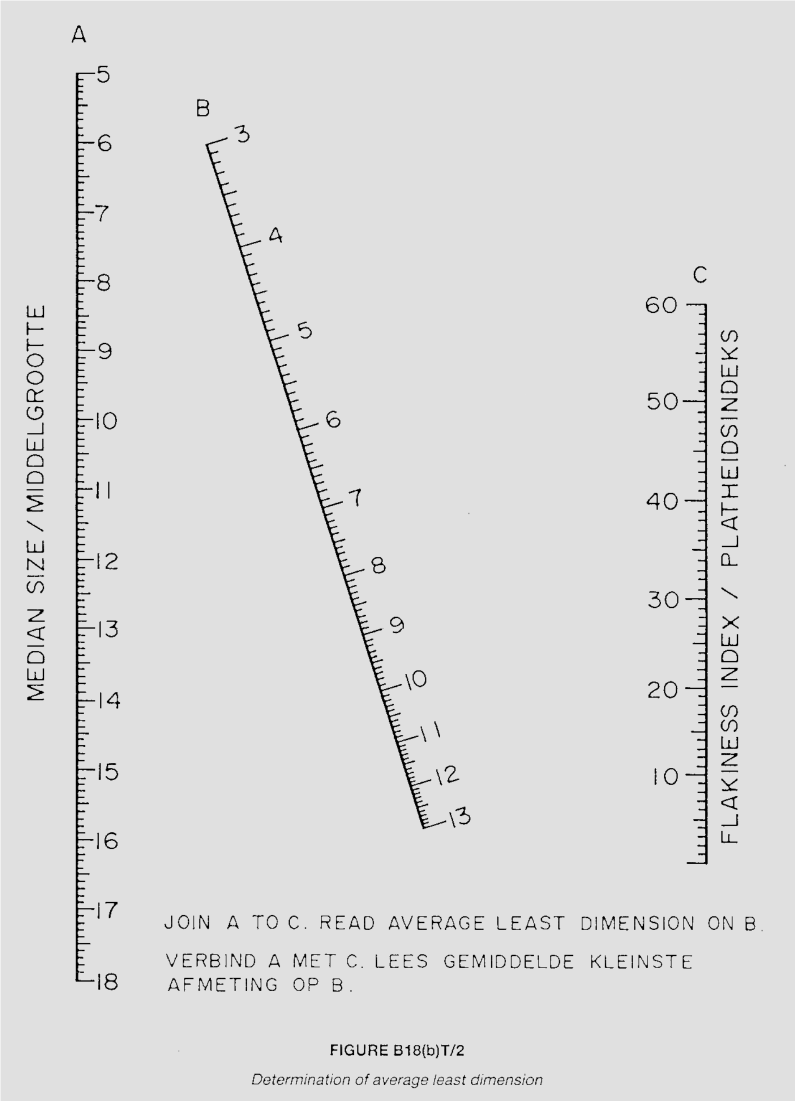

Figure 1a

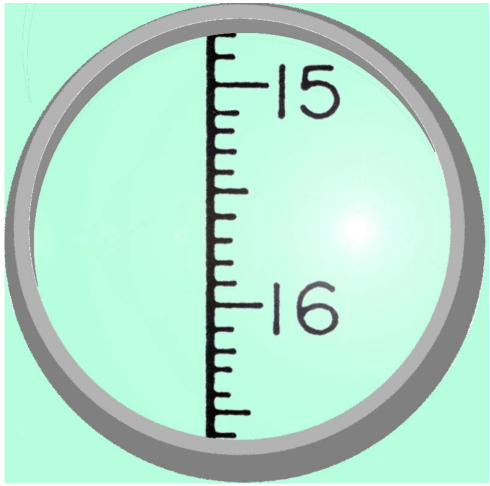

Figure 1b

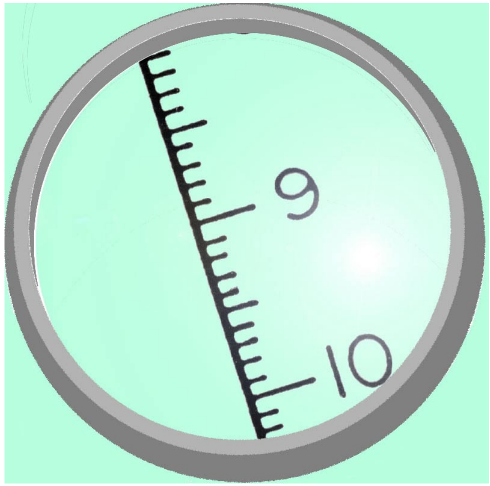

Figure 1c

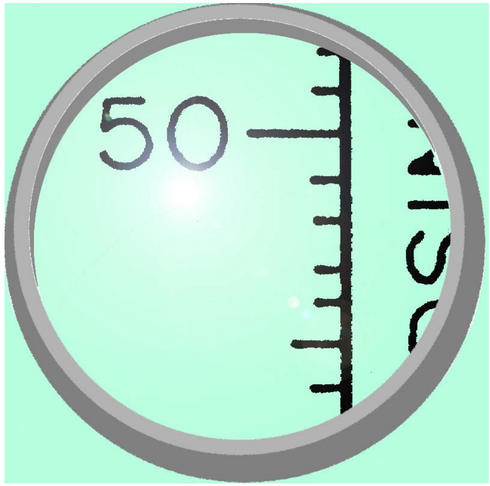

Figure 1d

The graph on figure 2 illustrates the relationship between the median and the mean of the least dimension distributions obtained from 186 different samples. According to the graph it is clear that the degree of symmetry of the particle size ditribution has an effect on the measured ALD because all the points do not lie on the straight line. The points are slightly dispersed which means that the particles of all the samples are not singular in size. Besides the fact that asymmetrical distributions do exist the correlation between the median and the mean (ALD) least dimension is nevertheless very good (0,996). For all practical purposes the mean of the least dimension measurements can be replaced by the median of the least dimensions.

Before a relationship can be formulated it is important to establish whether there exist a relation between the median of the least dimension distribution and the median obtained from the gradation. The graph of figure 3 demonstrates the degree of correlation between the median of the least dimensions and the median of the gradation.

The correlation is fairly good seeing that the median of the least dimensions is obtained by means of a comparator table (one dimensional measurement) while the median of the gradation is obtained by means of a set of sieves (square aperture). Hence the two median determination methods measure dimensional properties of the particles differently and this is the reason for the scattering of the points. To reduce this dispersion significantly it is important to bring in more variables which are related to the characteristic of the particle size distribution to the formulation process. The basic relation between the median of the least dimension measurements and the median of the gradation will therefore forms the basis of the formulation of the computational ALD model. New variables must be incorporated in the formulation process to reduce the dispersion of the points which is mainly caused by the nature of the particle size distribution and the shape of the aggregate particles.

It is not possible to eliminate the above mentioned scattering completely. The reason being that there are still other factors which have an effect on the relationship but are considered unimportant. For example the flakiness index on its own is not strong enough to describe the shape of the aggregate in full. The aim of this formulation is to strive for an error of the order that is obtainable for the measured ALD.

Theoretical model

According to the definitions of the measured ALD and the median of the gradation we can start off by stating that the ALD is directly proportional to the central tendency of the particle size distribution, shape and type of distribution of the particles.

Axiomatically the statement is formulated as follows:

$$\text{ALD} \propto \mathbf{M} \circ \mathbf{P} \qquad \text{eq. 1}$$

The Euclidean space **M** describes the central tendency of the particle size distribution of the particles. It contains the optimal median value for a singular and symmetrical particle size distribution and does not take into account the shape and the nature of the size distribution of the particles. On the other hand the space **P** contains the elements that describes the shape of the particles and the nature of the size distribution of the particles. The mathematical operator between the two spaces transform the space **P** functionally into a scalar.

The next step is to convert eq. 1 into a real relationship. To do that the important elements of each set must first be identified and defined before the conversion can be made.

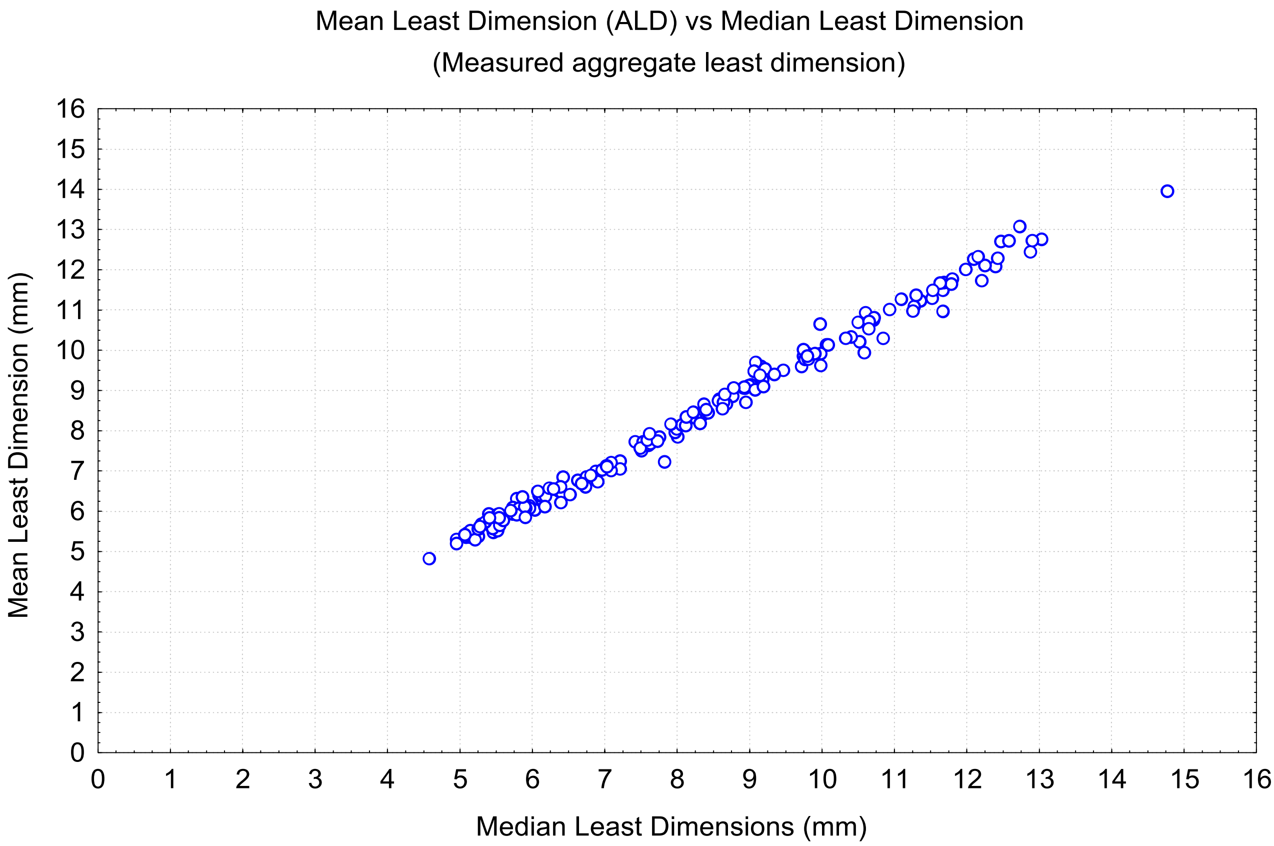

Figure 2

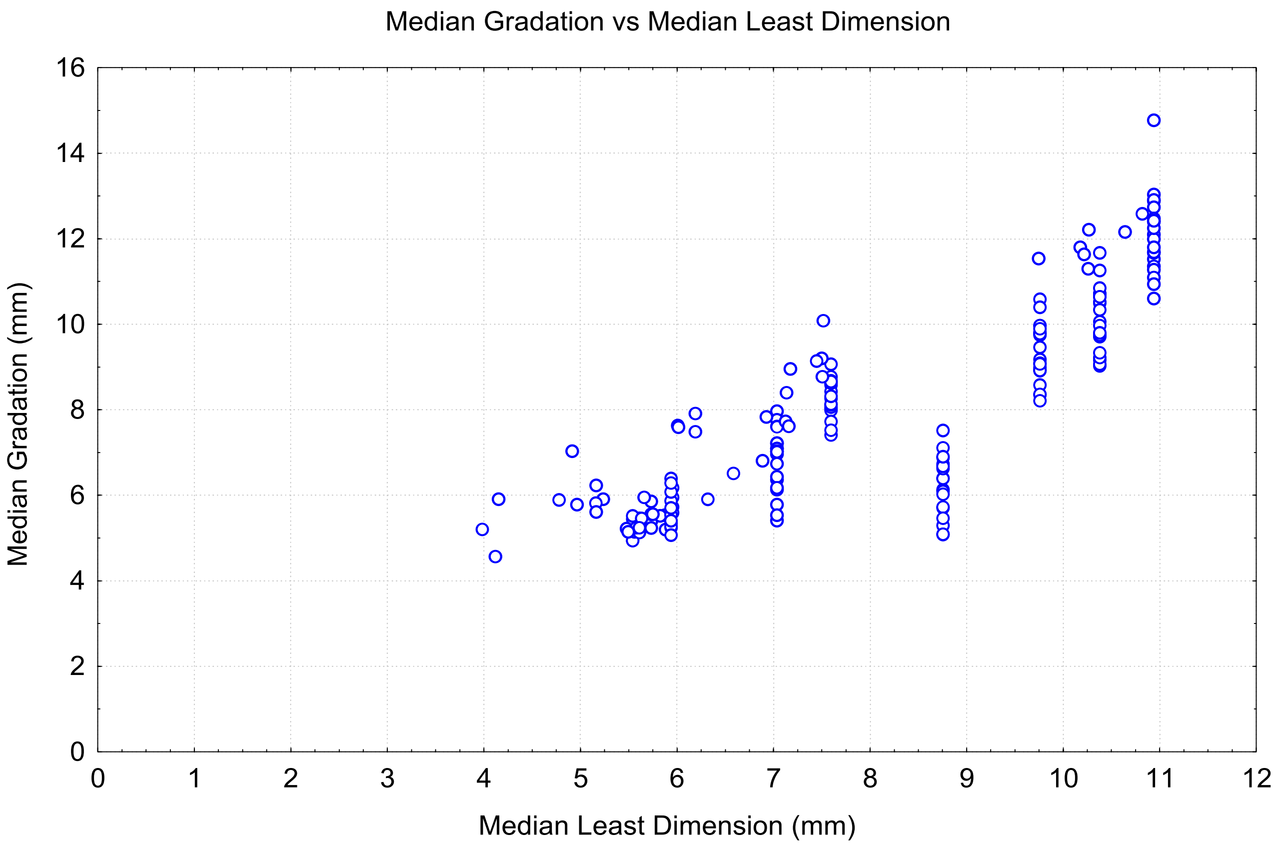

Figure 3

The two variables, viz. the *gradation median* value and the *fraction not measured* are the basic elements of the set **M**.

Median (M_e)
The median of the gradation is that sieve size through which 50% of the aggregates will pass. It is obtained by the accumulation of sieve fractions and with sieves having prescribed square apertures. This median value is also a certain type of mean. It is different from the median least dimension, because it describes the central tendency of the square aperture (sieve size) through which 50% of the particles will pass. While the median least dimension is the value where 50% of the least dimension of the particles is less than the median value. As in the case of the median least dimension, the median of the gradation also depends on the particle size distribution and the shape of the aggregates.

Fraction not measured (F_r)
According to TMH1-B18(a) the particles of the fraction passing the sieve, half the nominal size of the aggregate, must not be measured. This fraction does have a significant effect on the measured ALD if not applied correctly.

Flakiness Index (F_i)
The flakiness index , carried out according to TMH1-B3, is a rough guide to describe the shape of the aggregate. It stands to reason that the flakiness index cannot reflect the full shape characteristic of the aggregate which will have an effect on the measured ALD. For practical reasons and for the purpose in mind, we'll suffice for the time being with the flakiness index as a particle shape indicator.

The degree of peakiness and the degree of symmetry of the particle size distribution forms an integral part of the formulation and, therefore must also form part of the basic elements of set **P**.

Distribution Peakiness (K)
The more peaky the particle size distribution (leptokurtic) the more singular the aggregates are in size. In other words, the range in sizes will be small and the least dimension of the individual particle will be closer to the mean (ALD). When the particle size distribution is flatter than normal (platykurtic) the aggregates will not be singular anymore. The range between the smallest and largest particle size becomes wider as the distribution becomes flatter. The least dimension of the individual particle deviates quite significantly from the average least dimension. The degree of peakiness of a particle size distribution is indicated by the K-value which normally varies between 0,15 and 0,35 for surfacing aggregates. For a fairly singular aggregate this value is 0,263.

Distribution Symmetry (S)
Only when the aggregates are very singular, will the shape of the particle size distribution be symmetrical. As soon as the aggregates becomes non-singular the distribution becomes more asymmetrical. In the case of a symmetrical distribution the effect of the median least dimen-sion on the average least dimension will be zero, otherwise it will effect it negatively or positively. The shape of the particle size distribution is reflected by the S-value which normally varies between –0,4 and 0,5 . For a symmetrical particle size distribution the S-value is zero.

The relationship
The statement made above (eq. 1) is converted functionally as follows:

$$ALD_e = f(M_e, F_r, C_f) \otimes g(F_i, K, S, C_g) \qquad \text{eq. 2}$$

where ALD_e is the estimated measured ALD and C_f and C_g are constants which can only be determined from experimental data.

Function f()
When the *fraction not measured* (F_r) is greater than zero, the median will be additively effected. When F_r is equal to zero no adjustment is required. It means that when F_r is zero the median is unchanged but when F_r is greater than zero the median will be increased. The reason being that the particles smaller than half the nominal size are not measured in the measured ALD method in spite of the fact that it was included in the gradation used for the determination of the median.

Function g()
The higher the Flakiness Index, the more flaky the aggregate, which means that as F_i increases from zero, the median (M_e) will decrease proportionally. When F_i is zero (no flakes) the median will not be affected, hence the effect of F_i on M_e will therefore be additive.

The K-value (peakiness indicator) is also an additive term. Although the effect on the median (M_e) is significant, it is not so drastic, as in the case of the Flakiness Index.

Regarding the symmetry of the particle size distribution the S-value will have a greater effect on the median as the K-value. Due to the fact that the S-value varies from a negative value through zero to a positive value, the term must also be additive. The median will be decreased by the S-value in the case where a sample has more finer fractions than is normally expected. In the case where the sample has more coarser fractions the median will be increased by the S-value.

The equation
The next step is to put eq. 2 into an equation form. The functional form of eq. 2 is as follows:

$$ALD_e = (A_0 + A_1 M_e + A_2 F_r) * (B_0 + B_1 F_i + B_2 K + B_3 S) \qquad \text{eq. 3}$$

where A_0 and B_0 are constants and A_1, A_2, B_1, B_2 and B_3 are term coefficients and must be obtained from either experimental or historic data.

Mathematical process
To find the constants and coefficients of eq. 3 from a inductive mathematical point of view is not so easy, if not impossible. The best way to overcome all the side effects of the influential factors, and those not considered in the model, and to establish the type and magnitude of the errors involved, is to make use of a stochastic process.

Confidence limits
The only way to find the true measured ALD is to measure quite a number of samples according to TMH1. It is impractical to measure a certain number of samples to establish a good measured ALD every time. In this paper a relationship is formulated to determine the ALD from certain gradation and shape properties. The ALD obtained from any computa-tional relationship is not the true measured ALD, it is an estimated measured ALD, or in short a computed ALD. It must be remembered that this estimated ALD is still a variate distributed in some way about the true measured ALD. However, an interval, within which the true measured ALD will lie, can be established with a certain amount of confidence. A confidence level of 95% was used in the formulation process, because of the many variations, deviations and uncertainties that had to be dealt with, it was unnecessary to aim for a higher confidence level.

Data
The data used for the determination of the required constants and term coefficients are not given here, but the number of samples involved in the analysis was 504. It consists of information obtained from historic data as well as from certain properly designed statistical experiments.

Constants and coefficients
According to the data used in the analysis the values of the constants and term coefficients of eq. 3 were calculated as follows:

A_0	=	0,064 402
A_1	=	0,245 081
A_2	=	-0,014 381
B_0	=	3,270 825
B_1	=	-0,019 573
B_2	=	-0,149 364
B_3	=	0,326 787

Reliability
According to the data used in the determination of the constants and term coefficients, 93,7% of the total variation is explained by eq. 3. Hence the variation that cannot be explained is about 6% which is due to certain minor factors which were not considered in the analysis or formulation process. This means that the variables selected and the type of mathematical model formulated provided the results aimed for.

Estimated measured ALD
The standard error of estimate is 0,522 which means that the confidence interval of the true measured ALD can be calculated.

Therefore the estimated measured ALD is as follows:

$$ALD_c = (A_0 + A_1 M_e + A_2 F_r) * (B_0 + B_1 F_i + B_2 K + B_3 S) + \textit{error} \qquad \text{eq. 4}$$

To avoid confusion with the actual ALD measured the estimated measured ALD is designated by the symbol ALD_c (computed ALD). The error depends on the critical values of the *confi-dence interval*, the *standard error of estimate,* and the value of the *estimated ALD*. The error does not change significantly whether the minimum estimated ALD (4,5 mm) or maximum estimated ALD (13,0 mm) is used to calculate the error. Hence, to find the error, the estimated ALD in the determination of the confidence interval can be replaced by the mean estimated ALD value (9,137 mm). The error is calculated as 1,027 mm. The final equation for the computed ALD is therefore

$$ALD_c = (A_0 + A_1 M_e + A_2 F_r) * (B_0 + B_1 F_i + B_2 K + B_3 S) \pm 1{,}027 \qquad \text{eq. 5}$$

Computed ALD versus measured ALD
Figure 4 reflects the distribution of the residuals (difference between the computed ALD and the measured ALD). From this distribution it is clear that the majority of the residuals are within the error range of the measured ALD due to sample preparation. It means that the relationship, as defined above, can be used to determine the estimated measured ALD and that the error is well within the accuracy limits aimed for.

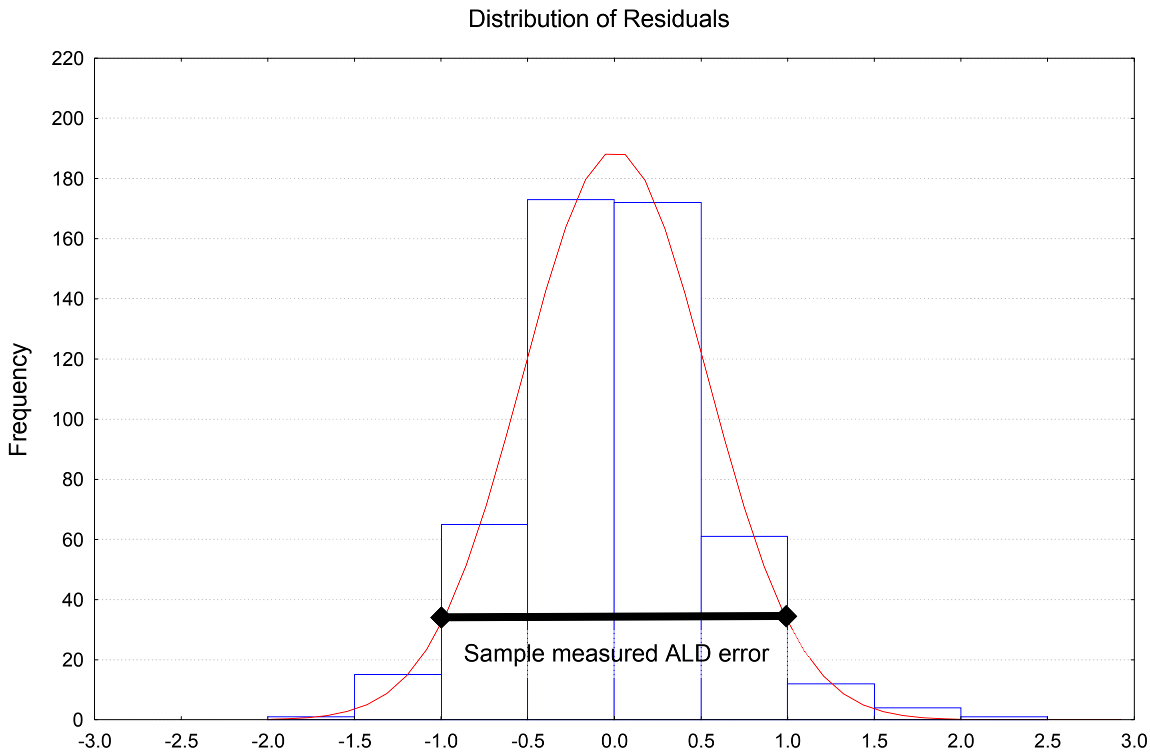

Figure 4

Calculation

From the gradation information, calculate the median value to an accuracy of 0,001 mm as follows:

$$A = \mathrm{Log}_{10}(S_L) \qquad B = \frac{P_r - P_L}{P_U - P_L} \qquad C = \mathrm{Log}_{10}(S_U) - \mathrm{Log}_{10}(S_L) \qquad Q_r = 10^{A + B \cdot C}$$

For each r calculate Q_r

r	1	2	3	4	5
P_r	10	25	50	75	90

where

S_L : The first sieve where the material passing is less than P_r %.

S_U : The first sieve where the material passing is greater than P_r %.

P_L : Percentage passing S_L

P_U : Percentage passing S_U

Calculate the particle size distribution parameters

$$M_e = Q_3$$

$$K = (Q_4 - Q_2) / 2(Q_5 - Q_1)$$

$$S = (Q_5 - 2Q_3 + Q_1) / (Q_5 - Q_1)$$

Calculate the ALD_c to an accuracy of 0,001 mm as follows:

$$f(M_e, F_r) = A_0 + A_1M_e + A_2F_r$$

$$g(F_i, K, S) = B_0 + B_1F_i + B_2K + B_3S$$

$$ALD_c = f(M_e, F_r) \cdot g(F_i, K, S)$$

Conclusions

1. The estimated measured Average Least Dimension (ALD_c) as calculated by means of eq. 3 has an error of approximately 1 mm which is of the same order as that obtained for the ALD measured under close controlled conditions.
2. The sample properties used in the computational method is obtained from a sample size approximately four times larger than the sample used for the measured ALD.
3. The variables required are obtained from normal testing of surfacing aggregates.

References

1. Surface Dressing by G.P. Jackson, published by the Bitumen Division, Shell International Petroleum Co. Ltd. London, 1963.
2. Tentative Method for the Determination of the Average Least Dimension of Aggre-gates by computation, TMH1 (1986), Technical Methods for Highways.

Acknowledgement

The author wish to acknowledge the support provided by the Provincial Administration of Western Cape and the staff of the Pavement Technology component of the Department of Economic Affairs, Agriculture and Tourism.

THE DETERMINATION OF THE AVERAGE LEAST DIMENSION OF SURFACING AGGREGATES BY COMPUTATION

B. DUMAS

Provincial Administration: Western Cape, Department of Economic Affairs, Agriculture & Tourism, 31 Chiappini Street, Cape Town, 8001

BARRY DUMAS

On the 9th March 1962, Barry started at the Provincial Administration of the Western Cape (previously CPA). His interest is mainly in the field of pavement technology. In l966 he obtained his National Diploma in Chemistry (Materials) at the Pretoria Technikon. With his B.Sc. knowledge in Mathematics and Statistics he assisted Dr Semmelink (CSIR) in l986 with the development of the statistical model documented in TRH 5. Barry likes hiking and astronomy.