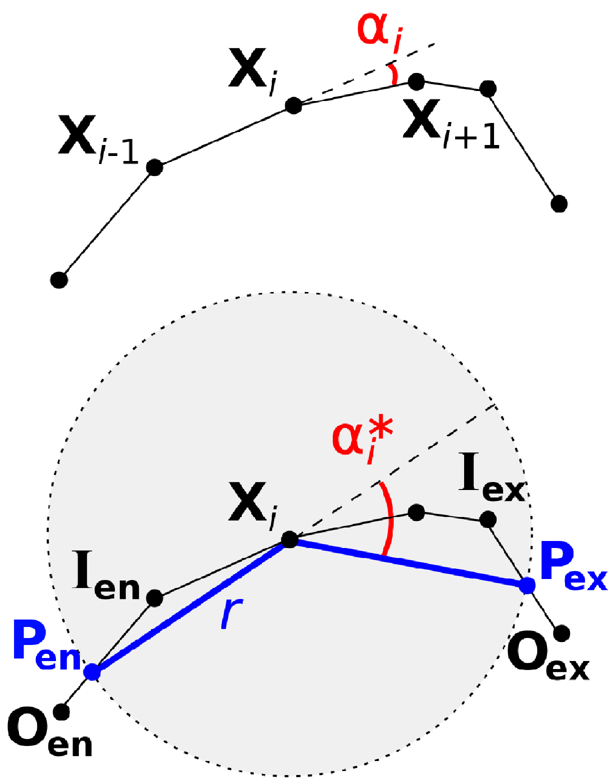
Supporting Information 2

Interpolating entrance and exit points of a circle



Given a series of locations $\mathbf{X}_i = (X_i, Y_i)$ recorded at constant time intervals Δt. Whereas the turning angle at constant time interval α_i (**top**) corresponds to the change in direction between vectors $\mathbf{X}_{i\text{-}1}\rightarrow\mathbf{X}_i$ (with length L_i) and $\mathbf{X}_i\rightarrow\mathbf{X}_{i+1}$ (with length L_{i+1}) and therefore acts as a proxy for angular speed ($\alpha_i/\Delta t$), the turning angle at constant length interval α_i^* (**bottom**) corresponds to the change in direction between vectors $\mathbf{P}_{\mathbf{en}}\rightarrow\mathbf{X}_i$ and $\mathbf{X}_i\rightarrow\mathbf{P}_{\mathbf{ex}}$, where $\mathbf{P}_{\mathbf{en}}$ and $\mathbf{P}_{\mathbf{ex}}$ are the last entrance and first exit locations, respectively, of a virtual circle with radius r centred on current location $\mathbf{X}_i$. Let $\mathbf{I} = (X_{\text{in}}, Y_{\text{in}})$ and $\mathbf{O} = (X_{\text{out}}, Y_{\text{out}})$ be the last inside and first outside recorded locations, respectively, of the first passage at the circle perimeter, either backwards ($\mathbf{I} = \mathbf{I}_{\mathbf{en}}$ and $\mathbf{O} = \mathbf{O}_{\mathbf{en}}$; $\mathbf{I}_{\mathbf{en}} = \mathbf{X}_i$ if $L_i > r$) to determine $\mathbf{P}_{\mathbf{en}}$, or forwards ($\mathbf{I} = \mathbf{I}_{\mathbf{ex}}$ and $\mathbf{O} = \mathbf{O}_{\mathbf{ex}}$; $\mathbf{I}_{\mathbf{ex}} = \mathbf{X}_i$ if $L_{i+1} > r$) to determine $\mathbf{P}_{\mathbf{ex}}$. The location $\mathbf{P}$ (either $\mathbf{P}_{\mathbf{en}}$ or $\mathbf{P}_{\mathbf{ex}}$) corresponds to the point where the vector $\mathbf{I}\rightarrow\mathbf{O}$ intersects the circle perimeter. The length of this vector is $d_{\mathbf{IO}} = (d_x^2 + d_y^2)^{0.5}$, with $d_x = X_{\text{out}} - X_{\text{in}}$ and $d_y = Y_{\text{out}} - Y_{\text{in}}$, and its orientation is θ, with $\cos(\theta) = d_x/d_{\mathbf{IO}}$ and $\sin(\theta) = d_y/d_{\mathbf{IO}}$. In a new orthonormal frame of reference (U, V) originating at $\mathbf{I}$ and with U axis running through $\mathbf{O}$, the coordinates of current location $\mathbf{X}_i$ become $U_i = (X_i - X_{\text{in}}) \cos(\theta) + (Y_i - Y_{\text{in}}) \sin(\theta)$ and $V_i = (Y_i - Y_{\text{in}}) \cos(\theta) - (X_i - X_{\text{in}}) \sin(\theta)$. By applying Pythagoras' theorem, one gets $r^2 = (d_{\mathbf{IP}} - U_i)^2 + V_i^2$, where $d_{\mathbf{IP}}$ corresponds to the distance between $\mathbf{I}$ and $\mathbf{P}$, with $d_{\mathbf{IP}} > U_i$. Entrance or exit location can therefore be linearly interpolated as $\mathbf{P} = \mathbf{I} + (\mathbf{O} - \mathbf{I})\, d_{\mathbf{IP}}/d_{\mathbf{IO}}$ (i.e. $X_{\text{P}} = X_{\text{in}} + \cos(\theta)\, d_{\mathbf{IP}}$ and $Y_{\text{P}} = Y_{\text{in}} + \sin(\theta)\, d_{\mathbf{IP}}$), with $d_{\mathbf{IP}} = U_i + (r^2 - V_i^2)^{0.5}$.