Causality Between Per Capita Real GDP and Income Inequality in the U.S.: Evidence from a Wavelet Analysis

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Abstract This study applies wavelet coherency analysis to examine the relationship between the U.S. per capita real GDP and six income inequality measures over the period 1917 to 2012. Wavelet analysis allows the simultaneous examination of correlation and causality between the two series in both the time and frequency domains. Our findings provide robust evidence of positive correlation between the growth and inequality across frequencies. Yet, directions of causality vary across frequencies and evolve with time. Evidence that per capita real GDP leads inequality at both high- and low-frequencies exists for the Top 1 and 10% measures of inequality with little evidence that inequality leads real GDP per capita. In the time-domain, the time-varying nature of long-run causalities implies structural changes in the two series. These findings provide a more thorough picture of the relationship between the U.S. per capita real GDP and inequality measures over time and frequency, suggesting important implications for policy makers.

Keywords Income · Inequality · Wavelet analysis · U.S.

JEL Classification C49 · D31

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1 Introduction

Kuznets (1955) and Kaldor (1955) posed the issue of the relationship, if any, between income inequality and economic growth. Since then, researchers explore whether a country's inequality in the distribution of income increases or decreases in concert with its economic growth. Studies provide evidence that supports the view that inequality slows growth over the medium and long terms (Alesina and Perotti 1996; Alesina and Rodrik 1994; Persson and Tabellini 1992; Birdsall et al. 1995; Clarke 1995; Deininger and Squire 1996; Easterly 2007; Wilkinson and Pickett 2007; Berg et al. 2012). These researchers suggest several channels for a negative influence, such as inequality prevents the poor from accumulating human capital by delaying the timing of investment in human capital (Galor and Zeira 1993; Perotti 1996; Galor and Moav 2004; Aghion el al. 1999), and/or inequality generates political and economic instability that reduces investment (Persson and Tabellini 1992, 1994; Alesina and Perotti 1996) and obstructs the social consensus required to mitigate shocks and maintain growth (Rodrik 1999; Woo 2005). In contrast, a number of studies provide evidence of a positive relationship between inequality and growth. According to these researchers, inequality affects growth positively by providing incentives for entrepreneurship (Lazear and Rosen 1981; Hassler and Mora 2000), and/or by boosting saving and investment (Kaldor 1955; Bourguignon 1981), by developing human capital (Saint-Paul and Verdier 1993; Barro 2000).

In addition to the studies that consider the long-term relationship between inequality and growth, other studies focus on the ambiguous short-term relationship (Stiglitz 1969; Loury 1981; Tamura 1991; Perotti 1993; Benabou 1996; Galor and Tsiddon 1996, 1997; Aghion and Bolton 1997; Li and Zou 1998; Aghion et al. 1999; Maoz and Moav 1999; Fishman and Simhon 2002; Zilcha 2003; Galor el al. 2009; Forbes 2000; Banerjee and Duflo 2003; Halter el al. 2014). This literature uncovers a complex set of interactions, which depends on the specific research method and sample, between inequality and economic growth and highlights the difficulty of capturing a definitive causal relationship. Inequality either promotes, retards, or does not affect growth.

Most existing studies that examine the inequality growth nexus exclusively utilize time-domain methods. Few studies consider the frequency-domain relationships. The time- and frequency-varying relationships can provide significant implications for macroeconomic policymakers. The time-varying relationships indicate that the variables influence each other differently at different points in the business cycle (time) (Li et al. 2015). Frequency-varying relationships reveal short- versus long-term linkages between variables. Forbes (2000) emphasizes that a temporary relationship between inequality and growth does not directly contradict a permanent relationship and suggests a careful re-examination of the numerous linkages between inequality and growth.

Our paper explores these short- and long-term relationships between inequality and growth from the perspective of macroeconomic policy makers who undertake policies that could simultaneously improve growth and equality. We employ wavelet coherency analysis to examine the relationships between the U.S. per capita real GDP and inequality measures in the time and frequency domains. Wavelet coherency and phase differences simultaneously evaluate how causalities between U.S. per capita real GDP and the inequality measures fluctuate across frequencies and vary over time. This allows us to obtain short-term (high-frequency) and long-term (low-frequency) relationships between the two series–per capita real GDP and each of our income inequality measures–as well as potential structural breaks and time-varying relationships.

Wavelet analysis allows the extraction of time- and frequency-localized information, which permits deeper investigation of the causality between variables (Roueff and Sachs 2011). Economic processes emerge as outcomes of the actions of numerous agents at different frequencies, which implies that a macroeconomic time series incorporates information that operates at different time domains. Wavelet analysis separates the time series into several sub-series, which may associate with a particular time domain and which narrows the focus to provide fruitful insights on economic phenomena (Ramsey and Zhang 1996, 1997). Moreover, we can apply wavelet analysis to non-stationary and locally stationary as well as series with structural breaks (Roueff and Sachs 2011). By considering time series at different frequencies, we may obtain new insights about the series, which may allow isolation of interesting aspects of economic time series not observable in the time-domain.

2 Methodology: Wavelet Coherency and Phase Difference

Hudgins et al. (1993) and Torrence and Compo (1998) develop methodologies of the cross-wavelet power, the cross-wavelet coherency, and the phase difference. While wavelet analysis closely relates to Fourier analysis, wavelet analysis, however, possesses certain advantages. Wavelet analysis conserves information in both time and frequency domains by conducting the estimation of spectral characteristics of a time series as a function of time (Aguiar-Conraria et al. 2008). Also, wavelet analysis applies for non-stationary or locally stationary series (Roueff and Sachs 2011). Wavelet coherency allows for a three-dimensional analysis, which considers the time and frequency elements at the same time, as well as the strength of the correlation between the time-series elements (Loh 2013). In this way, we can observe both the time- and frequency-variations of the correlation between two series in a time-frequency domain. Consequently, wavelet coherency provides a much better measure of co-movement between variables, U.S. per capita real GDP and our various income inequality measures, in comparison to conventional causality and correlation analysis. Following the approach of Li et al. (2015), we estimate wavelet coherency by using the cross-wavelet and auto-wavelet power spectrums as follow:

$$R_{xy}^2(\tau,s) = \frac{|S(s^{-1}W_{xy}(\tau,s))|^2}{S(s^{-1}|W_x(\tau,s)|^2)S(s^{-1}|W_y(\tau,x)|^2},$$

where complex argument $\arg W_{xy}(\tau,s)$ represents the local relative phase between x_t and y_t, $|W_x(\tau,s)|^2$ is the wavelet power, $\arg W_x(\tau,s)$ represents local phase, and S is a smoothing operator.[1] The ratio of the cross-wavelet spectrum to the product of the spectrum of each series equals the local correlation of the two series. This formula gives a quantity between 0 and 1 in a time-frequency window. Zero coherency indicates no co-movement between per capita real GDP and an income inequality measure, while the highest coherency implies the strongest co-movement between the two series. On the wavelet coherency plots, red colours correspond to strong co-movement whereas blue colours correspond to weak co-movement.

We cannot easily distinguish between positive and negative co-movements as the wavelet coherency is squared. Thus, we use the phase difference to provide information on positive and negative co-movements as well as the leading relationships between the two series.[2] Bloomfield et al. (2004) characterizes the phase difference relationship between $x(t)$ and $y(t)$ such that:

$$\phi_{xy} = tan^{-1}\left(\frac{\mathcal{J}\{S(s^{-1}W_{xy}(\tau,s))\}}{\Re\{S(s^{-1}W_{xy}(\tau,s))\}}\right), with\ \phi_{xy} \in [-\Pi,\Pi],$$

where $\mathcal{J}$ and $\Re$ equal the imaginary and real parts of the smoothed cross-wavelet transform, respectively.

A phase difference of zero reveals that the two underlying series move together, while a phase difference of $\pi(-\pi)$ indicates that two series move in the opposite directions. If $\phi_{xy} \in (0,\pi/2)$, then the series move in phase (positively co-move) with $y(t)$ preceding $x(t)$. If $\phi_{xy} \in (\pi/2,\pi)$, then the series move out of phase (negatively co-move) with $x(t)$

[1] Without smoothing, the squared wavelet coherency is always equal to 1 at any frequency and time. Torrence and Compo (1998) show that smoothing in time or frequency increases the degrees of freedom of each point and increases the confidence of the wavelet spectrum.

[2] The term phase means the position in the pseudo-cycle of the series as a function of frequency.

preceding $y(t)$. If $\phi_{xy} \in (-\pi, -\pi/2)$, then the series move out of phase with $y(t)$ preceding $x(t)$. Finally, if $\phi_{xy} \in (-\pi/2, 0)$, then the series move in phase with $x(t)$ preceding $y(t)$. Also, the phase difference can imply causality between $x(t)$ and $y(t)$ in both the time and frequency domains. In sum, wavelet analysis permits deeper understanding than the conventional Granger causality test, which assumes that a single causal link holds for the whole sample period as well as at each frequency (Grinsted et al. 2004; Tiwari et al. 2013). For example, in wavelet analysis, if $x(t)$ precedes $y(t)$, then a causal relationship runs from $x(t)$ to $y(t)$ at a particular time and frequency (Li et al. 2015).

3 Data

Our analysis relies on the natural logarithm of U.S. per capita real GDP and the four income inequality measures[3] - Atkinson Index, Gini Coefficient, the Relative Mean Deviation, Theil's entropy Index,– as well as Top 10%, and Top 1% income shares as useful proxies for inequality across the income distribution (Leigh 2007) over the period 1917 – 2012. Income inequality measures as well as income share measures come from the online data segment of Professor Mark W. Frank's website.[4] Real GDP (at constant 2009 prices) comes from the Global Financial Database, which we divide by population from the data segment of Shiller website[5], to derive the real per capita GDP. We conduct the analysis considering two frequency cycles. The 1-2-year cycle associates with the short-term (high-frequency) and the 2-4-year cycle associates with the long-term analysis (low-frequency).

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[3] We take natural logarithms to correct for potential heteroskedasticity and dimensional differences between the series. Also, taking natural logarithms is standard practice, since it implies that we can interpret the coefficients as elasticities.

[4] See http://www.shsu.edu/eco_mwf/inequality.html. Professor Frank constructed dataset based on the Internal Revenue Service (IRS) information, which has a limitation of omission of some individual earning less than a threshold level of gross income. For this reason, we focus more on top income shares as primary indicators of inequality measures. We examine six inequality measures as each offers a different insight as to the inequality of income.

[5] See http://www.econ.yale.edu/~shiller/data.htm.

4 Preliminary Analysis

Though our focus considers wavelets, we initially do a preliminary analysis, involving standard causality tests. To start, we first test the data series for unit roots, using the standard augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests (see Dickey and Fuller 1979; Phillips and Perron 1988). Table 1 shows that these tests fail to reject the null hypothesis of non-stationarity for the six income inequality measures as well as per capita real GDP at the 5-percent level. These tests further indicate that the first differences of the series do reject the null of a unit root. Therefore, the unit-root tests indicate that the data conform to I(1) processes.

The presence of unit roots makes the traditional asymptotic inference invalid by violating asymptotic normality. Toda and Yamamoto (1995) propose an interesting, yet simple, procedure requiring the estimation of an augmented VAR that guarantees the asymptotic distribution of the Wald statistics (an asymptotic Chi square distribution), since the testing procedure proves robust to the integration and cointegration[6] properties of the processes. In other words, the result holds no matter whether series are I(0) or I(1) and/or whether cointegration does or does not exist. Table 2 shows that the Toda-Yamamoto causality tests indicate that one-way causality exists from the inequality measures to per capita real GDP for Atkin05, Rmeandev and Theil, whereas one-way causality exists from per capita real GDP to the Top 10%. Also, it shows two-way causality exists between the Gini coefficient and per capita real GDP and no causality between the Top 1% and per capita real GDP. The Toda-Yamamoto test, however, cannot distinguish between short- and long-run causality. Thus, we should test for cointegration and causality jointly across the frequency domain.

[6] Cointegration is the long-term, or equilibrium, relationship between two series. To ascertain long-run stability of the parameters, we perform the Johansen (1988, 1991) cointegration tests to determine whether the per capita real GDP and each of six income inequality measures cointegrate with each other. The test results show that no cointegration exists between per capita real GDP and each inequality measure, implying that per capita real GDP and the income inequality measures do not maintain a long-term relationship.

To examine the short- and long-run stability of the coefficients of the VAR model formed by each one of the six income inequality measures and per capita real GDP, we apply the Lc tests of Nyblom (1989) and Hansen (1990), which test the null hypothesis of constant parameters against the alternative hypothesis that the parameters follow a random-walk process (Gardner 1969). When the series are I(1), the Lc test can also serve as a test of cointegration, which indicates stability of the implied long-run relationship. According to Andrew (1993) and Andrew and Ploberger (1994), the F-statistics test the null hypothesis of no structural break against the alternative hypothesis of a single shift of unknown change point. We also apply these tests for stability of the short-run parameters, using the three different test statistics: Sup-F, Ave-F, and Exp-F. Contrary to the Lc test, the F-tests require trimming from the ends of the sample. The *p* values and critical values for all stability tests come from parametric bootstrapping, which avoids the use of asymptotic distribution.

Tables 3 and 4, report the results of the parameter stability tests for the per capita real GDP and the six income inequality measures. Andrew and Ploberger (1994) suggest that the use of the Sup-F, Mean-F, and Exp-F tests, which test the same null hypothesis but differ in the alternative hypotheses, depends on the purpose of the test. The Sup-F statistic tests parameter constancy against a one-time sharp shift in parameters, so that the alternative hypothesis for the Sup-F test is an immediate shift in the regime. If the system shift gradually, however, then the Mean-F and Exp-F statistics, which assume that parameters follow a martingale process, are suitable. Both statistics test the global constancy of the parameters, implying that the Mean-F and Exp-F tests are appropriate to investigate whether the underlying relationship among the variables stays stable over time. Table 3A, B, D, F show that the Sup-F, Mean-F, and Exp-F tests reject the null hypothesis of parameter constancy, implying parameter non-constancy in the per capita real GDP equations as well as Aktin 05, Gini, and Theil index equations. Table 3C reports significant evidence of parameter non-constancy in the per capita real GDP equation but not in the null of overall stability of the VAR (2) model. Table 3E reports significant evidence of parameter non-constancy in the Top 10% equation but not in the null of overall stability of the VAR (2) model.

Investigating the causal relationship between the variables, using short-run parameters of the differenced or cointegrated VAR can lead to meaningless results with biased inference and inaccurate forecasts and Granger causality tests will show sensitivity to changes in the sample period. Overall, the parameter stability test show that the cointegrated VAR model possesses unstable short- and long-run parameters, suggesting the existence of structural changes.[7]

To check for the robustness of long-run stability of the parameters, we also estimate the cointegration equation between the variables based on the FM-OLS estimator.

Table 4 reports the results of the Lc tests. For all six FM-OLS estimators, the Nyblom-Hansen Lc test rejects the null hypothesis of cointegration at the 5-percent level. Thus, we observe both short- and long-run instability, motivating wavelet coherency analysis. When the frequency components exhibit nonstationarity, the traditional approach may miss such frequency components. Wavelet analysis provides localised information to deal with the time-varying characteristics found in most economic time series. Thus, we can avoid the assumption of stationarity (Fan and Gençay 2010). Furthermore, wavelet analysis allows us to examine the time- and frequency-localized information with structural breaks.[8]

5 Main analysis

[7] We examine the existence of structural breaks in VAR. Results of the Bai and Perron (1998) tests show that there are structural breaks. Also, given the existence of structural breaks in the series, we test for unit roots with one or two structural breaks using methodologies for endogenising dates, including Zivot and Andrews (2012), Lumsdaine and Papell (1997) and Lee and Strazicich (2003). These results are available upon request.

[8] The results of the cointegration test motivate our focus on a time-varying approach. One way to implement time-varying cointegration uses a rolling causality analysis. We choose not to follow this method for the following reasons. First, the results may depend on the optimal window length. Second, rolling causality analysis only works in the time domain.

From 1983 to 2012, the U.S. per capita real GDP and Atkin05 show a statistically significant high coherency across 1-2 year frequency band in Fig. 1. Figure 1 also shows positive correlations between the U.S. per capita real GDP and Atkin05 over the short and long term.

Across the 2-4 year frequency band in Table 5, U.S. per capita real GDP leads the Atkin05 inequality measure in 1917-1948 and 1977-2012, while the Atkin05 inequality measure leads U.S. per capita real GDP in 1949-1976. The change of direction of the causality, from per capita real GDP leads to inequality leads in the late 1940s probably relates to a democratization of wealth in the post-war period. Also, stagnating real wages for the majority of the population despite increasing productivity. Across the 1-2 year frequency band, we see the causal link running from per capita real GDP to the Atkin05 inequality measure for several periods – 1965-1973, 1978-1987, and 2011-2012 (see Table 5). The 1970s saw couple of oil price spikes as OPEC began affecting prices. After the 1973 oil shocks, productivity growth suddenly slowed and the oil price shocks led to higher unemployment and inflation.

The Gini coefficient exhibits a positive and statistically significant correlation with U.S. per capita real GDP from 1917 to 1930 and from 1970 to 2012 in Fig. 2. Figure 2 also shows causality between U.S. per capita real GDP and the Gini coefficient. Over the short and long term, the two series show positive correlation.

U.S. per capita real GDP leads the Gini coefficient from 1967-1972 at high frequency in Table 6, while the Gini coefficient leads per capita real GDP from 1917-1970 to 1983-2012 at low frequency. The Vietnam War covered the 1967-1972 period which in turn productivity growth slowed. Also, as a consequence of fiscal and monetary policies during this War, the U.S. experienced rising inflation and unemployment during most of the 1960s into the early 1980s. Moreover, OPEC oil price shocks also occurred during the 1970s, as noted above. We can see the temporary causality does not determine long-run causality (see Table 6).

From 1980 to 2012, U.S. per capita real GDP and the Rmeandev inequality measure show a statistically significant high coherency across the 1-2 year frequency band (see Fig. 3) with an in-phase relation (see Table 7).

We observe across the 1-2 year frequency band in Table 7 an in-phase relationship in 1966-1975 with per capita real GDP leading. At low frequencies, we see the causal link running from per capita real GDP to Rmeandev from 1917 to 1948 and Rmeandev leads per capita real GDP from 1949 to 2012, which relates to compression in wages during the 1940s.

Theil index exhibits a strong positive correlation with U.S. per capita real GDP from 1980 to 2012 across the 1-2 year frequency band in Fig. 4.

The phase difference shows causality between the U.S. per capita real GDP and the Theil index in Table 8. Throughout the period from 1917 to 2012, per capita real GDP leads the Theil index at low frequency. This indicates that per capita real GDP positively affects income inequality (Theil). At high frequencies, per capita real GDP leads Theil index repeatedly from 1963 to 1972 (see Table 8), which also corresponds to the Vietnam War period.

Across the 1-2 years frequency band, two significant islands exist of high coherency between U.S. per capita real GDP and the Top 10% around 1955 and from 1985 to 2012 in Fig. 5. Across the 2-3 years frequency band, we observe a significant island from 1945 to 1957 (see Fig. 5), which is related to the World War II as the Top 10% income share fell substantially during the World War II (Goldin and Margo 1992). We observe the consistent strong positive correlation between U.S. per capita real GDP and inequality measures at the 1-2 years frequency at the recent sample years (see Fig. 5). This may relate to the Tax Reform Act of 1986, which lowered the top tax rate and raised the bottom tax rate. As a result, income inequality leads U.S. per capita real GDP in the recent sample years.

Table 9 shows causality between the U.S. per capita real GDP and the Top 10%. At high frequency, per capita real GDP leads the Top 10% from 1917 to 1988. At low frequency, per capita real GDP leads the Top 10% from 1917 to 1973 to 1979-1984 (see Table 9).

In Fig. 6, we observe a statistically positive correlation from the 1926 to the 1949 between per capita real GDP and the Top 1 % across the 2-3 year frequency band as during the Great Depression the top 1% declined extensively.

At high frequency, per capita real GDP leads the Top 1% from 1917-1993 to 2003-2012 in Table 10. At low frequency, per capita real GDP leads the Top 1% from 1917-1983 to 1986-2012 (see Table 10).

Overall, we observe a positive correlation between per capita real GDP and income inequality. Also, we observe that the directions of short- and long-term causality vary.[9] If we restrict our analysis to classical time series, we cannot find any information about frequency differences. To develop a deeper understanding of the relationships between U.S. per capita real GDP and our measures of income inequality requires wavelet analysis.

6 Conclusion

Policy makers attempt to reduce inequality and to sustain and/or boost economic growth. The relationship between inequality and growth received much analysis in the existing literature. Unfortunately, numerous variables affect these variables simultaneously or at different points of time, rendering net causality and correlation results difficult to document. This paper investigates the causal relationship between U.S. per capita real GDP and six measures of income inequality. We use wavelet coherency analysis, which allows the causal relationship between the two series to vary over time and frequency. Wavelet analysis is robust to lag length, stationarity, cointegration, and model specification. Furthermore, it permits the

[9] The wavelet coherency and phase difference for the levels of per capita real GDP and income inequality measures still show very similar correlation and causalities.

consideration of cointegration and causality. We use annual time-series data from 1917 to 2012 from the U.S., which covers numerous economic expansions and recessions.

This paper addresses the possible presence of structural breaks. We employ tests for parameter constancy to examine the stability of the estimated VAR model and to test for both short- and long-term instability. Also, we test the existence of structural breaks in each series. Observed instability and structural change, therefore, make the traditional Granger causality test inappropriate. We apply the time- and frequency-varying wavelet coherency analysis to assess the causal relationship between the U.S. per capita real GDP and our six income inequality measures.

Results show that the periods and directions of short- and long-term causality vary. Also, short-term relationships do not necessarily coincide with long-term relationships. Causality changes direction – from inequality leading to per capita real GDP leading. We find different directions of causality for our six income inequality measures – especially during periods of volatility such as World War II (1939-1945), the OPEC oil shocks (1973-1979), the early 1980s recession, the transitory recession in the 1990s, and the recent financial crisis and Great Recession. An exception is that per capita real GDP mainly leads the Top 1 and 10% inequality measures at both high- and low-frequencies.

This paper began with a mass of mutually conflicting findings on how inequality affects growth. Our findings support the view that inequality and growth are positively correlated in the short and long term, which implies that the benefits of economic growth do not trickle down across society. In addition, we find not only inequality matters for growth but also growth matters for inequality, especially the Top 1 and 10% income shares.

The most used and direct policy to reduce inequality redistributes income through government spending, taxes, and transfer payments. Yet, rapid and forced redistribution from rich to poor may not provide the best solution. In particular, significant adjustments to fiscal policy to achieve a lower level of income inequality may cause slower economic growth. For example, higher transfer payments to low income families may lead to a higher budget deficit, absent other fiscal actions. A higher budget deficit, then, may lead to higher interest rates and, thus, to reduced investment, net exports, and consumption, leading to reduced growth in real GDP. In this case, policies that help to reduce inequality may undermine growth.

As another strategy, policy makers use taxes to redistribute income from the rich to the poor. Such tax induced redistribution may not work because it takes away incentives and may produce rent-seeking (Lazear and Rosen 1981; Hassler and Mora 2000). This paper finds that inequality and growth are positively correlated. While the literature on this topic remains contentious, the view of a trade-off between inequality and growth seems embedded in policy makers' choice. In this example, we see once again that policies that help to reduce inequality may undermine growth.

Future research could consider the relationship between fiscal policy adjustments and income inequality. That is, how do changes in different fiscal controls to address income inequality affect economic growth and how significant a change in fiscal policy can occur without impinging on economic growth?

Table 1. Unit root Tests

	Level					
	ADF			PP		
	C	C+T	N	C	C+T	N
Per capita real GDP	-0.519	-2.885	2.129	-0.731	-2.665	3.653
Atkin05	-1.22	-2.037	-0.924	-1.495	-2.795	-0.494
Gini	-0.832	-2.578	-0.751	-0.943	-2.787	-0.733
Rmeandev	-0.26	-2.3	-1.032	-1.632	-3.183	-0.818
Theil	-0.884	-0.942	-1.005	-1.318	-2.098	-0.816
Top 10%	-0.694	-0.794	-0.698	-0.756	-0.788	-0.698
Top 1%	-1.141	-1.162	-0.451	-1.078	-1.022	-0.457
	First difference					
	ADF			PP		
	C	C+T	N	C	C+T	N
Per capita real GDP	-6.655***	-6.612***	-6.172***	-6.773***	-6.733***	-6.172***
Atkin05	-8.781***	-6.033***	-8.786***	-8.781***	-8.77***	-8.787***
Gini	-9.638***	-6.361***	-9.589***	-9.63***	-9.608***	-9.575***
Rmeandev	-6.578***	-6.72***	-6.502***	-9.165***	-9.125***	-9.169***
Theil	-8.392***	-5.736***	-8.412***	-8.381***	-8.491***	-8.402***
Top 10%	-8.788***	-8.894***	-8.801***	-8.747***	-8.856***	-8.761***
Top 1%	-9.748***	-9.882***	-9.787***	-9.809***	-10.14***	-9.848***

The Augmented Dickey-Fuller (ADF) test and Phillips-Perron (PP) test corresponds to Dickey and Fuller (1979) and Phillips and Perron (1988) unit root tests; *** indicates the rejection of the null hypothesis at 1% level of significance.

Table 2. Toda-Yamamoto Causality modified WALD Test

Null Hypothesis	Chi-sq	Prob.	Granger Causality
per capita real GDP does not granger cause Atkin05	3.345	0.188	One-way directional Causality
Atkin05 does not granger cause per capita real GDP	10.268	0.006	Aktin05 -> per capita real GDP
per capita real GDP does not granger cause Gini	8.04	0.045	Two-way directional Causality
Gini does not granger cause per capita real GDP	13.736	0.003	Gini <-> per capita real GDP
per capita real GDP does not granger cause Rmeandev	4.346	0.114	One-way directional Causality
Rmeandev does not granger cause per capita real GDP	6.291	0.043	Rmeandev -> per capita real GDP
per capita real GDP does not granger cause Theil	3.009	0.222	One-way directional Causality
Theil does not granger cause per capita real GDP	8.598	0.014	Theil -> per capita real GDP
per capita real GDP does not granger cause Top10 percent	10.705	0.005	One-way directional Causality
Top10 percent does not granger cause per capita real GDP	1.455	0.483	Per capita real GDP -> Top 10%
per capita real GDP does not granger cause Top1 percent	3.036	0.219	No causality
Top1 percent does not granger cause per capita real GDP	3.86	0.145	

Table 3. Parameter Stability tests in VAR(2) model

A	Per capita real GDP Equation		Atkin05 Equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	44.57	<0.01	31.8	<0.01	54.13	<0.01
Mean-F	6.69	0.03	12.11	<0.01	11.87	0.020
Exp-F	18.07	<0.01	12.3	<0.01	23.56	<0.01
B	Per capita real GDP Equation		Gini Equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	44.54	<0.01	16.27	0.020	50.05	<0.01
Mean-F	7.84	0.01	6.11	0.020	11.23	0.030
Exp-F	18.07	<0.01	4.71	0.030	20.98	<0.01
C	Per capita real GDP Equation		Rmeandev equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	37.87	<0.01	27.57	<0.01	51.62	<0.01
Mean-F	7.62	0.02	5.33	0.090	11.37	0.030
Exp-F	14.84	<0.01	9.59	<0.01	21.73	<0.01
D	Per capita real GDP Equation		Theil Equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	62.55	<0.01	54.57	<0.01	56.42	<0.01
Mean-F	11.11	<0.01	10.83	<0.01	13.87	0.010
Exp-F	27.35	0.01	23.07	<0.01	25.42	<0.01
E	Per capita real GDP Equation		Top 10 Equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	260.95	<0.01	21.33	<0.01	42.85	<0.01
Mean-F	11.65	<0.01	12.48	<0.01	17.45	<0.01
Exp-F	126.25	1	7.81	<0.01	17.62	<0.01
F	Per capita real GDP Equation		Top 1 Equation		VAR(2) System	
	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value	Statistics	Bootstrap p-value
Sup-F	45.64	<0.01	33.84	<0.01	46.69	<0.01
Mean-F	6.84	0.03	18.34	<0.01	18.94	<0.01
Exp-F	19.1	<0.01	13.51	<0.01	20.28	<0.01

The parameter stability tests exhibit non-standard asymptotic distributions. Using the parametric bootstrap procedure, Andrews (1993) and Andrews and Ploberger (1994) report the critical values and *p* values for the non-standard asymptotic distributions of these tests. We obtain the critical values and *p* values using asymptotic distribution constructed by means of Monte Carlo simulations using 10,000 samples generated from a VAR model with constant parameters. Besides, according to Andrews (1993), 15-percent trimming from both ends of the sample is required for the Sup-F, Mean-F and Exp-F. Hence, we apply the tests to the fraction of the sample in (0.15, 0.85).

Table 4 **Parameter stability tests in long-run relationship FM-OLS**

	Atkin05		Gini		Rmeandev		Theil		Top 10%		Top 1%	
	Stats	Bootstrap p-value	Stats	Bootstrap p-value	Stats	Bootstrap p-value	Stats	Bootstrap p-value	Stats	Bootstrap p-value	Stats	Bootstrap p-value
Lc	14.59	<0.01	11.48	<0.01	14.08	<0.01	16.92	<0.01	15.71	<0.01	15.47	<0.01

We apply the Lc test proposed by Nyblom (1989) and Hansen (1992) to investigate the long-run parameter stability with the long-run relationship estimated using the Fully Modified ordinary least squares (FM-OLS) estimator of Phillips and Hansen (1990). When the underlying series are I(1), it also serves as a test of cointegration. We calculate p-value using 10,000 bootstrap repetitions.

Table 5. Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Atkinson Index)

High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1964	$(0,\frac{\pi}{2})$, In-phase	+	Atkin05 -> U.S. per capita real GDP
	1965-1973	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP-> Atkin05
	1974-1977	$(0,\frac{\pi}{2})$, In-phase	+	Atkin05 -> U.S. per capita real GDP
	1978-1987	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP-> Atkin05
	1988-2010	$(0,\frac{\pi}{2})$, In-phase	+	Atkin05 -> U.S. per capita real GDP
	2011-2012	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP-> Atkin05
Low frequency	1917-1948	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP-> Atkin05
	1949-1976	$(0,\frac{\pi}{2})$, In-phase	+	Atkin05 -> U.S. per capita real GDP
	1977-2012	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP-> Atkin05

Table 6. Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Gini coefficient)

High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1966	$(0,\frac{\pi}{2})$, In-phase	+	Gini -> U.S. per capita real GDP
	1967-1972	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Gini
	1973-2012	$(0,\frac{\pi}{2})$, In-phase	+	Gini -> U.S. per capita real GDP
Low frequency	1917-1970	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Gini
	1971-1982	$(0,\frac{\pi}{2})$, In-phase	+	Gini -> U.S. per capita real GDP
	1983-2012	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Gini

Table 7. **Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Rmeandev)**

High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1965	$(0,\frac{\pi}{2})$, In-phase	+	Rmeandev -> U.S. per capita real GDP
	1966-1975	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Rmeandev
	1976-2012	$(0,\frac{\pi}{2})$, In-phase	+	Rmeandev -> U.S. per capita real GDP
Low frequency	1917-1948	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Rmeandev
	1949-2012	$(0,\frac{\pi}{2})$, In-phase	+	Rmeandev -> U.S. per capita real GDP

Table 8. **Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Theil Index)**

High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1962	$(0,\frac{\pi}{2})$, In-phase	+	Theil -> U.S. per capita real GDP
	1963-1972	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Theil
	1973-2012	$(0,\frac{\pi}{2})$, In-phase	+	Theil -> U.S. per capita real GDP
Low frequency	1917-2012	$(\frac{-\pi}{2},0)$, In-phase	+	U.S. per capita real GDP -> Theil

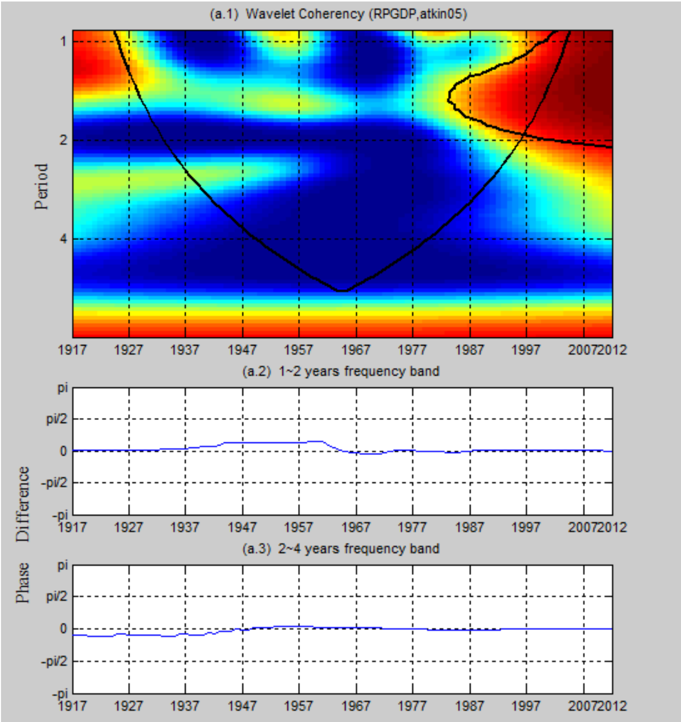
Table 9. Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Top 10%)

High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1988	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top10%
	1989-2012	$(0, \frac{\pi}{2})$, In-phase	+	Top10% -> U.S. per capita real GDP
Low frequency	1917-1973	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top10%
	1974-1978	$(0, \frac{\pi}{2})$, In-phase	+	Top10% -> U.S. per capita real GDP
	1979-1984	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top10%
	1985-2012	$(0, \frac{\pi}{2})$, In-phase	+	Top10% -> U.S. per capita real GDP

Table 10. Wavelet phase difference (logarithm of U.S. per capita real GDP, logarithm of Top 1%)

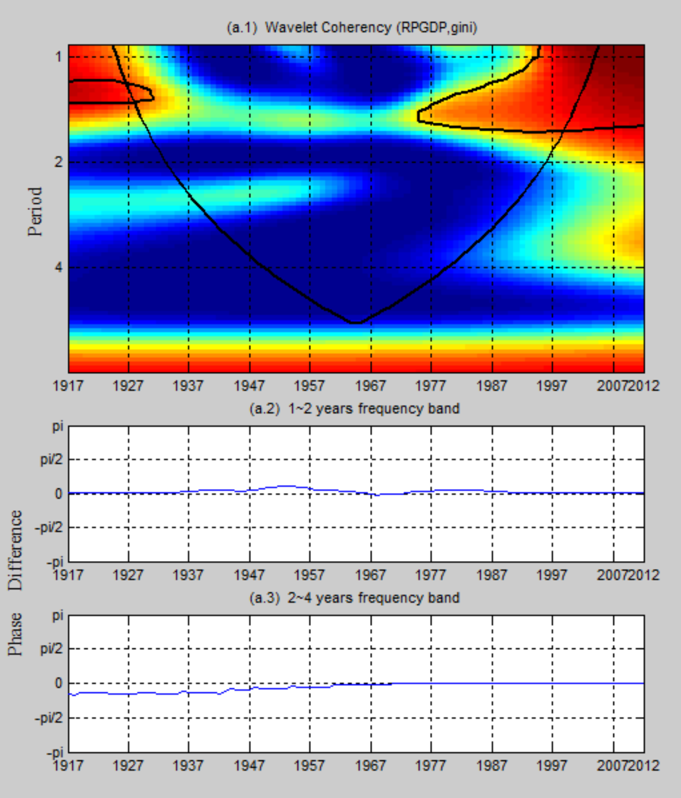
High frequency	Period	Phase	Sign of co-movement	Causality
	1917-1993	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top1%
	1994-2002	$(0, \frac{\pi}{2})$, In-phase	+	Top1% -> U.S. per capita real GDP
	2003-2012	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top1%
Low frequency	1917-1983	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top1%
	1984-1985	$(0, \frac{\pi}{2})$, In-phase	+	Top1% -> U.S. per capita real GDP
	1986-2012	$(\frac{-\pi}{2}, 0)$, In-phase	+	U.S. per capita real GDP -> Top1%

Figure 1. **Causal relationship between Per Capita Real GDP and Atkison Index**



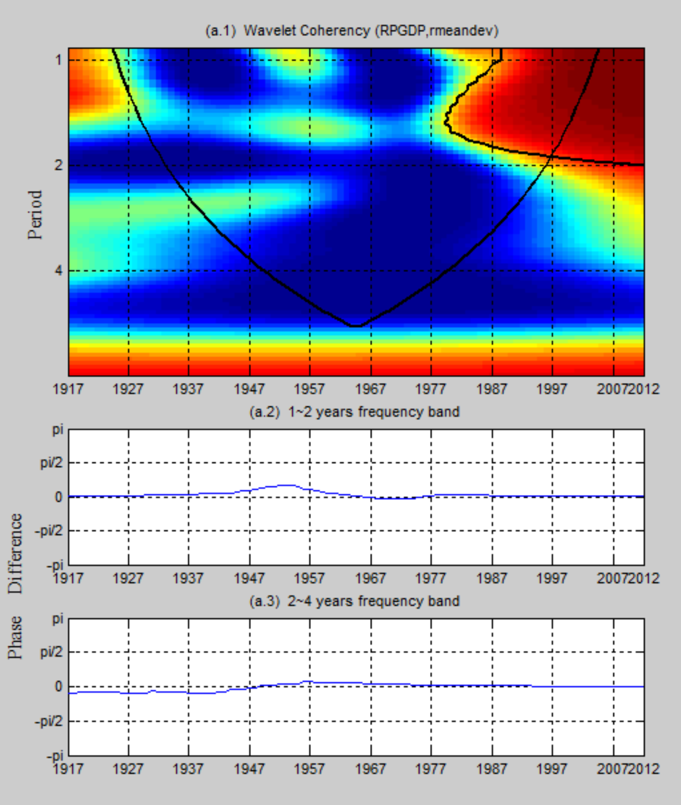
Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and Aktin05. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Figure 2. Causal relationship between Per Capita Real GDP and Gini coefficient



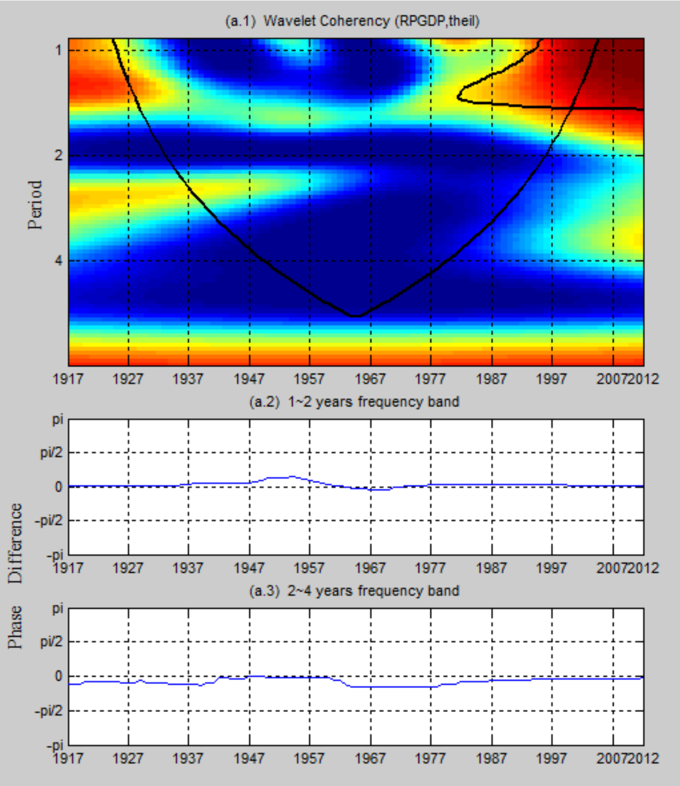
Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and Gini coefficient. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Figure 3. Causal relationship between Per Capita Real GDP and the Relative Mean Deviation



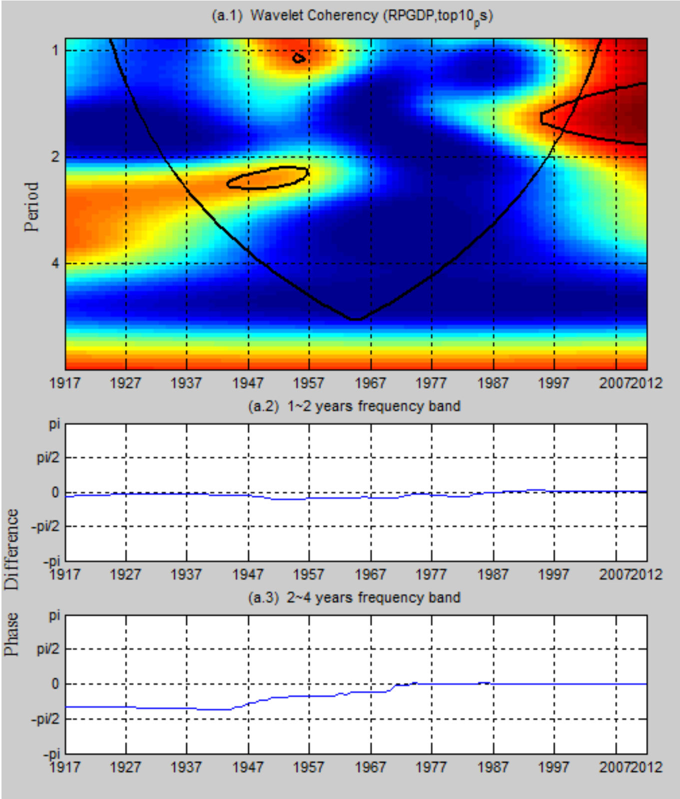
Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and the relative mean deviation. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Figure 4. **Causal relationship between Per Capita Real GDP and Theil's entropy Index**



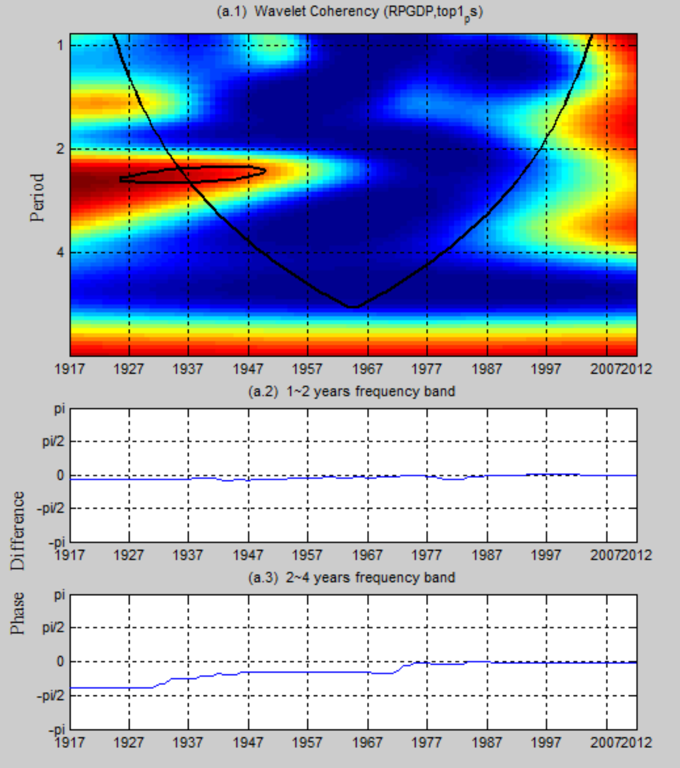
Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and Theil's entropy index. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Figure 5. Causal relationship between Per Capita Real GDP and Top 10% income share



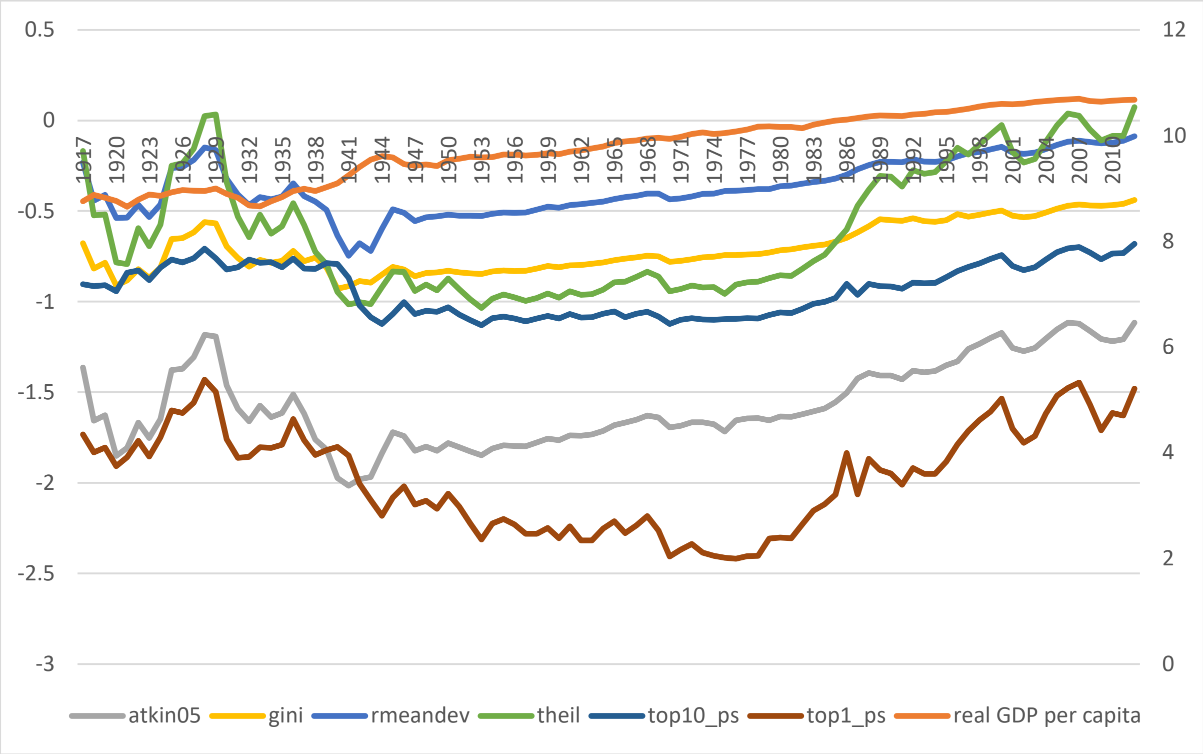
Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and Top 10% income share. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Figure 6. Causal relationship between Per Capita Real GDP and Top 1% income share



Note: The wavelet coherency (a.1) and phase difference (a.2 , a.3) between the per capita real GDP and Top 1% income share. The y-axis refers to the frequencies (measured in years); the x-axis refers to the time period over the period 1917-2012.

Appendix 1. Graph of Per Capita Real GDP and Income inequality measures



Note: Variables are in natural logarithms.

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