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LINEAR STABILITY BOUNDARY FOR SIMULATED OSCILLATORY FLOW PHENOMENA IN VERTICAL CONDENSING FLOWS

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ABSTRACT

In an earlier paper the results of a simulation of unstable flow phenomena in vertical condensing flows were presented. The horizontal condensing flow model was extended to include gravitational effects. The simulation of the non-linear model revealed existence of limit-cycle type of oscillations of large amplitude, including possibilities of flow reversals, as were observed on horizontal flows. The simulations also revealed that the gravitational effects have an attenuating effect on the oscillatory behavior in downward flows, and an enhancing effect on the upward flows. Matlab/Simulink tools were used for the system simulations. In the present paper the non-linear model was linearized, leading to an identification of the stable and unstable domains of operation for vertical condensing flows in both upward and downward flow directions. The stability boundary for the horizontal condensing flows lies in between the stability boundaries for vertical flows. The downward condensing flow is more stable and has a narrower unstable region than the horizontal flow. By comparison the vertical condensing upward flow is more unstable than both horizontal and downward flows. These observations are in correspondence with those of the non-linear model. A linear stability criterion including effects of gravity for vertical flows, that is an extension of such a criterion for horizontal flows, is presented.

INTRODUCTION

The two-phase region in condensing flow, undergoing complete condensation inside a tube, acts as an amplifier of any small internal or external disturbances. The time dependent characteristics of condensing flows are important in a variety of applications like reheat and re-boiler systems in conventional or nuclear power plants, reactor cooling systems, space power generation, refrigeration systems, moisture separators, and chemical processing. A sufficiently large excursion or oscillation could affect the performance of the processes taking place, cause damage to the mechanical equipment and endanger the safety of such systems. Large oscillations in the sub-cooled liquid region at the condenser outlet, including the possibilities of flow reversals, are likely associated with large impulse loads that may cause substantial damage to the piping and other components of the overall system.

The dynamic characteristics of condensing flows may be categorized into two classes. The first is the result of externally forced changes in a particular input variable, and the second is a phenomenon of internally induced self-sustained oscillations of large amplitude. These studies have been done on horizontal, single or multi tube, condensing flows [1-4] as well as vertical condensing flows [5-7].

The self-sustained condensing flow oscillations are internally induced solely by the processes taking place within the flow system and aided or abated by the coupling that exists between the condenser and other systems components. These represent a particular unstable mode of operation when the steady state parameters of the condensing flow assume some specific values. Once the condenser is left to itself at this operating level, one may see an initiation, a growth, and finally a limit cycle type of sustained oscillations of very large amplitude, including the possibility of flow reversal. Bhatt and Wedekind [1] made a systemic study of this oscillatory condensing flow instability from a fundamental perspective. This study was done on a horizontal condensing flow. Both the experimentally observed linear stability boundary and the non-linear limit cycle behavior were predicted using the system mean void fraction model.

Since many condensing flows involve vertical flows, including substantial nuclear reactor safety issues involving such systems, it is necessary to study such flows under the influence of gravity in both down flow as well as up flow configurations. A simulation of such flows using system mean void fraction model with non-linear elements was reported in [7]. Fig 1 and 2 show such a simulation for both a horizontal as well as vertical condensing flows. This was done using Matlab/Simulink tools in solving non- linear system of equations.

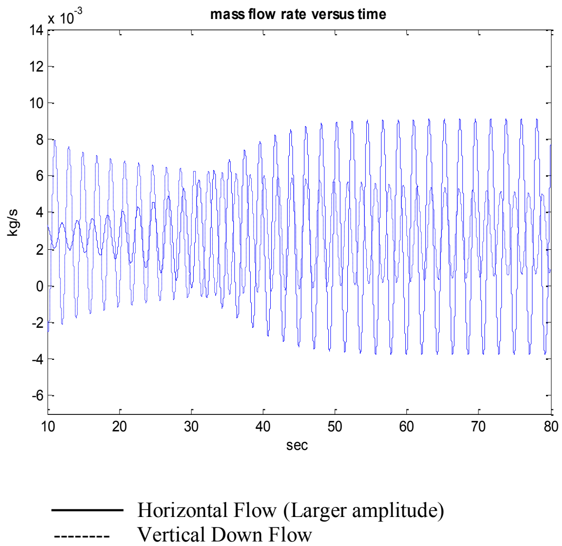

Figure 1 Comparison of Horizontal and Vertical Down Flow

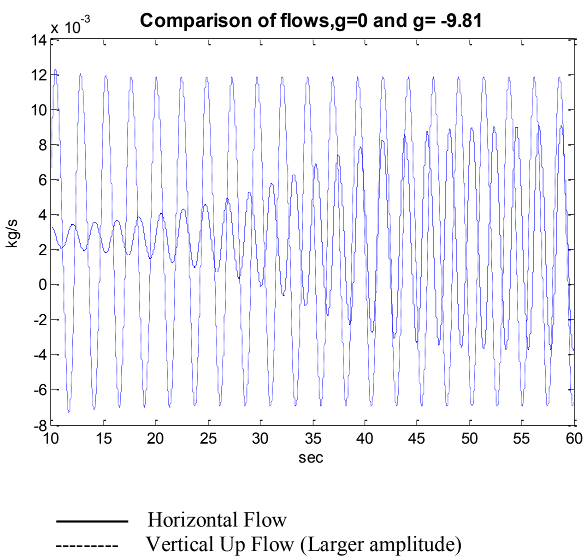

Figure 2 Comparison of Horizontal and Vertical Up-flow

However, from a practical perspective it is necessary to develop a criterion for stable and unstable operation. This is achieved by linearizing the governing set of equations. Therefore the objective of this paper is to capitalize on the success achieved earlier on horizontal flows [1] and extend that approach in developing a linear stability criterion for vertical condensing flows incorporating the gravitational effects.

NOMENCLATURE

A_t cross-sectional area of the tube; m^2

D inside diameter of tube; m

$\bar{f}_q$ heat flux; w/m^2

g acceleration due to gravity, m^2

h enthalpy of saturated liquid; J/kg

h' enthalpy of saturated vapor; J/kg

k_i inlet orifice coefficient

k_o outlet orifice coefficient

L total length of condenser; m

$\overline{m}$ steady state mean mass flow rate; kg/s

$m_o(t)$ vapor mass flow rate at Z=0; kg/s

$\overline{m}_L(t)$ sub-cooled liquid mass flow rate at exit; kg/s

$\overline{m}_s(t)$ inlet vapor mass flow rate; kg/s

P inside perimeter of tube; m

p_c condensing pressure in two-phase region;

p_i pressure upstream of inlet orifice, *Pa*

p_a average pressure across inlet orifice, *Pa*

p_o pressure downstream of outlet valve, *Pa*

t time; sec

V_s upstream vapor volume; m^3

Greek Symbols

$\overline{\alpha}_s$ system mean void fraction (smvf)

γ $d\rho'/dp$ s^2/m^2

$\bar{\eta}_m$ steady state position of the effective point of complete condensation; m

$\bar{\eta}(t)$ non-fluctuating effective point of complete condensation

ρ density of saturated liquid; kg/m^3

ρ' density of saturated vapor; kg/m^3

ρ'_a average vapor density across inlet valve; kg/m^3

τ_c condensing flow time constant; sec

N_c, N_i, N_o dimensionless numbers, equation (26)

k_i^* linearized valve resistances at inlet, $2(k_i/\rho_a' A_t^2)\overline{m}$; $KN.s/m^2.gm$

k_o^* linearized valve resistances at inlet, $2(k_o/\rho A_t^2)\overline{m}$; $KN.s/m^2.gm$

2. THEORETICAL ANALYSIS:

Only a brief overview of the analysis and its proposed extension to the vertical flows will be presented here. The backbone of this modeling technique is the incorporation of a system mean void fraction in integral formulation of the governing conservation equations. Consider a generalized schematic of a condensing flow system as shown in Fig 3. This schematic depicts a coupling between the two phase region of the condenser and upstream vapor volume, downstream liquid inertia, and the upstream and downstream flow resistances.

The non-fluctuating system mean void fraction, [1, 2, 3], within the two-phase region is expressed as

$$\bar{\alpha}_s = \frac{1}{\bar{\eta}\,(t)} \int_{z=0}^{\bar{\eta}\,(t)} \bar{\alpha}_a \,(z,t)dz \qquad (1)$$

Where $\bar{\alpha}_a$ is the local area mean void fraction, at an arbitrary location, z, within the two-phase region and $\bar{\eta}(t)$ is the time dependent length of the two-phase region.

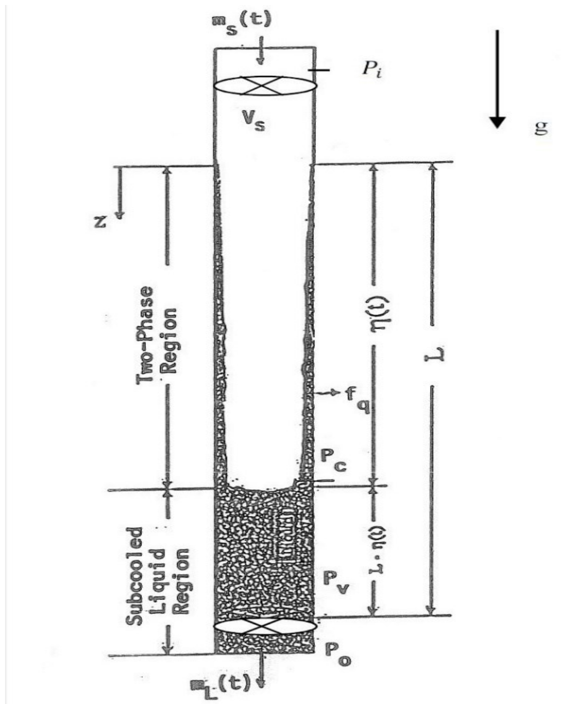

Figure 3 Schematic of a Vertical Two-Phase Condensing Flow

Assuming a time invariant system mean void fraction, [1,2], and incorporating it into the conservation equations, the following system of equations is obtained for the various regions of the condensing flow system irrespective of the flow direction.

Upstream Vapor Region

$$\frac{d\rho'(t)}{dt} = \frac{1}{V_s}[m_s(t) - m_o(t)] \qquad (2)$$

Where ρ' is the density of vapor

Pressure drop across the inlet flow resistance:

$$p_i \;-\; \mathrm{p}\;(\mathrm{t}) \;=\; \frac{k_i}{\rho' {A_t}^2}\;\bar{m}_s^2\;(t) \qquad (3)$$

Where k_i is the inlet flow coefficient and A_t is the cross-sectional area of the tube.

Compressibility of the vapor:

$$\frac{d\rho\prime(t)}{dt} \;=\; \gamma\;\frac{dp}{dt} \qquad (4)$$

Where, gamma, γ, is $\frac{d\rho\prime}{dp}$

Two-Phase region:

Conservation of mass:

$$[A_t\,\rho\,(1-\bar{\alpha}_s) +\, \rho'\bar{\alpha}_s]\frac{d\bar{\eta}\,(t)}{dt} + A_t\,\bar{\alpha}_s\bar{\eta}\,(t)\frac{d\rho\prime}{dt} =\; \bar{m}_t\,(z,t)_{z=0} - \bar{m}_t^* \qquad (5)$$

Conservation of mass and energy:

$$A_t\,\rho'\big(\acute{h} - h\big)\bar{\alpha}_s\frac{d\bar{\eta}\,(t)}{dt} + A_t\,\big(\acute{h} - h\big)\bar{\alpha}_s\bar{\eta}\,(t)\frac{d\acute{\rho}}{dt} \;= -\,\bar{f}_q\;\mathrm{P}\;\bar{\eta}\,(t) + \;\big(\acute{h} - h\big)\;\bar{m}_t\,(z,t)_{z=0} \qquad (6)$$

Where, f_q is the spatially averaged heat flux.

Sub-cooled Region

Conservation of mass:

$$-\rho\;A_t\,\frac{d\bar{\eta}\,(t)}{dt} \;=\; \bar{m}_t^* \;-\; \bar{m}_L(t) \qquad (7)$$

Where m_t^* is the flow rate of the liquid leaving the two-phase region relative to the moving boundary, $\eta(t)$.
Pressure drop in the sub-cooled liquid region is sum of the friction, inertia, and gravitational components:

$$p_c\,(t) - p_o = \frac{k_o}{\rho A_t^2}\;m_L^2(t) + \;\frac{[L-\eta(t)]}{A_t}\frac{dm_L\;(t)}{dt} + \Delta p_g(\mathrm{t}) \qquad (8)$$

3. LINEARIZATION OF GOVERNING EQUATIONS:

A digital simulation of the above set of non-linear equations was carried out using Matlab/Simulink. For example, the results of such non-linear simulations [7] are depicted in figures 1 and 2.

The characteristics of non-linear simulations are useful in understanding the physical mechanisms involved in the condensing flow systems. However, from a practical perspective it is more useful to delineate stable and unstable regions based on a linear stability boundary. Such an approach was followed for horizontal condensing flow [1]. That led to a determination of relevant non-dimensional groups encompassing the effects of physical parameters affecting the stability of condensing flow systems.

An extension of above approach, for vertical flows involving gravitational effects, was carried out by linearizing the governing equations. The following assumptions were made:

1. The coefficient of all differential terms were assumed to be represented by their mean values.
2. Non-linear flow resistances at the inlet and outlet were assumed to be piecewise linear around a given operating point.

Based on the above assumptions the governing equations of previous section lead to the following sets of linear equations.

Equation (3) and (8) become:

$$p_i - p(t) = k_i^* \, m_s(t) + c_i \tag{9}$$

$$p(t) - p_o = [k_o^* \, m_L(t) + c_o] + \frac{L_o}{A_t}\frac{dm_L(t)}{dt} - [L - \eta(t)]\rho g \tag{10}$$

$$p_i - p_o = \text{constant} \tag{11}$$

and is given by,

$$p_i - p_o = k_i^* \, m_s(t) + k_o^* \, m_L(t) + \frac{L_o}{A_t}\frac{dm_L(t)}{dt} - [L - \eta(t)]\rho g + c \tag{12}$$

Where,c $= c_i + c_o$, are constants.

Re-arranging equation (12),

$$m_s(t) = \frac{1}{k_i^*}\left[\begin{matrix}(p_i - p_o) - k_o^* \, m_L(t) - \\ \left(\frac{L_o}{A_t}\right)\frac{dm_L(t)}{dt} + [L - \eta(t)]\rho g - c\end{matrix}\right] \tag{13}$$

Equation (4) and (10) lead to

$$\frac{d\rho'}{dt} = \gamma\left[k_o^* \frac{dm_L(t)}{dt} + \left(\frac{L_o}{A_t}\right)\frac{d^2 m_L(t)}{dt^2} + \rho g \frac{d\eta(t)}{dt}\right] \tag{14}$$

Combined conservation of mass gives,

$$-A_t(\rho - \rho')\alpha_s \frac{d\eta(t)}{dt} + (A_t\alpha_s\eta(t) + V_s)\frac{d\rho'}{dt} = m_s(t) - m_L(t) \tag{15}$$

Conservation of energy, equation (5) along with (2) gives,

$$\frac{d\eta(t)}{dt} + \frac{1}{\tau_c}\eta(t) = \frac{m_s(t)}{A_t\rho'\alpha_s} - \left(\frac{1}{A_t\rho' V_s}\right)(V_s + A_t\alpha_s\eta_m)\frac{d\rho'}{dt} \tag{16}$$

Where, $\tau_c = \frac{A_t\rho'\alpha_s(h' - h)}{f_q P}$

An alternate form of equation (16) is

$$\frac{d\eta(t)}{dt} + \left(\frac{\rho'}{\rho}\right)\frac{1}{\tau_c}\eta(t) = \frac{1}{A_t\rho'\alpha_s} m_L(t) \tag{17}$$

Equations (12), (15) and (16) are solved simultaneously for $\eta(t)$. When $\eta(t)$ is substituted in equation (17) this leads, after considerable analysis, to the following differential equation that has been non-dimensionalized as indicated below.

$$a_o^* \frac{d^3 m_L^*(t)}{dt^{3*}} + a_1^* \frac{d^2 m_L^*(t)}{dt^{2*}} + a_2^* \frac{dm_L^*(t)}{dt^*} + a_3^* m_L^*(t) = b_o^* \tag{18}$$

Where, $\quad m_L^* = \frac{m_L}{\bar{m}}, \quad t^* = \frac{t}{T_1}\left(\frac{\rho'}{\rho}\right),$

$$1/\tau_c' = (1/\tau_c + \rho g/\rho' k_i^* \alpha_s A_t)$$

$$\frac{1}{T_1} = \left[\frac{1}{\psi\,\tau_c'} + \frac{\rho g}{\phi\, k_i^*}\right]$$

$$\psi = 1 + w\gamma\rho g, \quad w = \left[\frac{V_s + A_t\alpha_s\eta_m}{A_t\rho'\alpha_s}\right],$$

$$\phi = (\rho - \rho')A_t\alpha_s - V'\gamma\rho g,$$

$$V' = A_t\alpha_s\eta_m + V_s$$

To investigate the condition for marginal stability [1] a Laplace transform of equation (18), the characteristics equation in the Laplace variable, is given by

$$a_o^* s^3 + a_1^* s^2 + a_2^* s + a_3^* = 0 \tag{19}$$

$$a_1^* a_2^* - a_o^* a_3^* \geq 0 \text{ for stability} \tag{20}$$

The equality sign is for marginal stability and associated dimensionless frequency is given by:

$$\omega_n^{*\,2} = \frac{a_3^*}{a_1^*} = \frac{a_2^*}{a_o^*} \tag{21}$$

Where,

$$a_o^* = \left(\frac{\rho'}{\rho}\right)^3 \frac{1}{T_1^{\,2}}\left(\frac{\gamma w L_o\phi}{A_t} + \frac{\gamma V' L_o}{A_t}\right) \tag{22}$$

$$a_1^* = \left(\frac{\rho'}{\rho}\right)^2 \frac{1}{T_1}\left(\frac{L_o\phi}{\psi\rho'\alpha_s A_t^{\,2}\, k_i^*} + \gamma w k_o^*\phi + \gamma V' k_o^* + \frac{L_o}{A_t k_i^*}\right) + \left(\frac{\rho'}{\rho}\right)^3 \frac{1}{T_1\tau_c}\left(\frac{\gamma w L_o\phi}{A_t} + \frac{\gamma V' L_o}{A_t}\right) \tag{23}$$

$$a_2^* = \left(\frac{\rho'}{\rho}\right)\left(\frac{k_o^*\phi}{k_i^*\psi\rho'\alpha_s A_t} + \frac{k_o^*}{k_i^*} + 1\right) + \left(\frac{\rho'}{\rho}\right)^2 \frac{1}{\tau_c}\left(\frac{L_o\phi}{\psi\rho'\alpha_s A_t^{\,2}\, k_i^*} + \gamma w k_o^*\phi + \gamma V' k_o^* + \frac{L_o}{A_t k_i^*}\right) \tag{24}$$

$$a_3^* = \frac{\phi}{\rho\alpha_s A_t} + \left(\frac{\rho'}{\rho}\right)\left(\frac{T_1}{\tau_c}\right)\left(\frac{k_o^*\phi}{k_i^*\psi\rho'\alpha_s A_t} + \frac{k_o^*}{k_i^*} + 1\right) \tag{25}$$

An order of magnitude analysis was done on coefficients a_o^*, a_1^*, a_2^* and a_3^* of equation (22-25), assuming:

$\phi \approx (\rho - \rho')A_t\alpha_s$
$1/\tau_c \approx 1/\tau_c'$

$T_1 \approx \psi\tau_c$

Therefore neglecting insignificant terms, equation (20) reduces to:

$$[N_c + N_i] \geq N_o \quad (26)$$

$$N_c = \frac{\tau_c}{\gamma V' k_i^*} \quad (27)$$

$$N_i = \frac{A_t \tau_c k_o^*}{L_o} \quad (28)$$

$$N_o = \frac{1+\left({\rho'}/{\rho}\right)(\psi-1)}{\left({\rho}/{\rho'}\right)\left({k_o^*}/{k_i^*}\right)+\psi} - \frac{1}{\psi}\left({\rho'}/{\rho}\right) \quad (29)$$

The gravitational effects are encompassed in equation (29). For horizontal flows, when g=0, equation (29) reduces to equation (32) of [1]. A plot of equation (26) delineates stable and unstable regions as shown in Figure [4].

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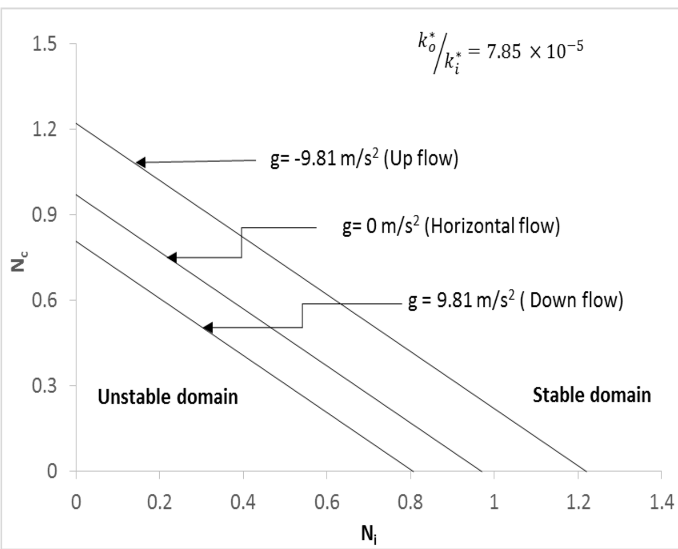

Figure 4 Stability boundaries for horizontal and vertical condensing flows

Common Data for Figure 4

Fluid: Freon-12

D=0.00762 m, ρ_a'= 58.278kg/ m^3, $\bar{m}$ = 2.72 kg/s, p_i= 1388.73 kpa, p_o= 643.46 kpa, ρ= 1245.9 kg/m^3 k_o= 44.905
k_i= 12217, ρ'=37.0694 kg/m^3, α_s= 0.83, V_s= 3.45*$10^{-4}m^3$,
γ =5.7*$10^{-5}s^2/m^2$, $\bar{\eta}$= 2.0703m, f_q=7.587810^{3}j/s m^2
τ_c=1.0684 s, L=8.595 m.

It is interesting to note that the stability boundaries for the horizontal condensing flow is in between the condensing down flow and the condensing up flow. The condensing down flow appears to be more stable than up flow, with a narrower unstable domain in comparison to both horizontal and vertical up flow. This is also suggested by the non-linear simulations presented in figs 1 and 2.

CONCLUSION

This paper was primarily concerned with linearization of the non-linear model that was developed earlier to simulate self-sustained oscillatory flow phenomena in vertical two-phase condensing flows both in the downward as well as upward flow directions. The non-linearized model simulations were done using Matlab/Simulink tools. However, from a practical stand point, a linearized model is more useful. It identifies the boundary between the stable and unstable domains, through a set of non- dimensional parameters, which were developed as an extension of the analysis for horizontal flows. The stability boundary for horizontal condensing flow is in between the stability boundaries for the vertical down flow and vertical upward condensing flows. This observation is in correspondence with simulations of the non-linear model. The vertical down flow is more stable than the horizontal condensing flow and has a narrower unstable region. By comparison the vertical up flow appears to be more unstable and has a wider unstable region in comparison to both the horizontal and vertical downward condensing flow. However, the reflux flow that can occur in vertical upward condensing flows was not investigated in the present study

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