Evolutionary Game Theoretic Demand-Side Management and Control for a Class of Networked Smart Grid☆

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Abstract

In this paper, a new demand-side management problem of networked smart grid is formulated and solved based on evolutionary game theory. The objective is to minimize the overall cost of the smart grid, where individual communities can switch between grid power and local power according to strategies of their neighbors. The distinctive feature of the proposed formulation is that, a small portion of the communities are cooperative, while others pursue their own benefits. This formulation can be categorized as control networked evolutionary game, which can be solved systematically by using semi-tensor product. A new binary optimal control algorithm is applied to optimize transient performances of the networked evolutionary game.

Keywords: Demand-side management, game theory, semi-tensor product, optimal control, binary decision systems.

1. Introduction

Demand-side management of energy systems becomes increasingly popular, because of its great potential in improving energy efficiency in industries. Smart grid is a typical platform where demand-side management strategies can be applied. A core issue in smart grid is that, dynamic user behaviors should be addressed in designing demand-side management strategies. Widely-used techniques for demand-side management of smart grid include game theoretic approach (Mohsenian-Rad *et al.*, 2014), multi-objective optimization (Nwulu and Xia, 2015; Malatji *et al.*, 2013), distributed energy consumption control (Ma *et al.*, 2014), and model predictive control (Zhang and Xia, 2011), etc.

Smart grids can be analyzed in the perspective of network systems, since there usually exist multiple interactive users consuming powers from grids. In networked smart grid systems, stability and optimization are two main issues. Stability of the networked smart grid system indicates that interactive users reach an equilibrium. Some methodologies, i.e. game theory (Mohsenian-Rad *et al.*, 2014), can be applied to prove the existence of equilibria in networked smart grid system. Optimization of the network smart grid system implies that, in the transient process to reach the equilibrium, some indexes can be optimized. The grid provider is capable of influencing decisions of users in the network by presenting dynamic pricing strategies (Liang *et al.*, 2013; Jiang *et al.*, 2014). It is possible that the smart grid provider and some of the users cooperate to affect decisions of other users, such that the common benefit can be improved.

Game theory has been widely applied to energy systems (Du *et al.*, 2015; Hong *et al.*, 2014). In previous researches on game theoretic policy for energy systems, fundamental games are usually played between two individual users (Xiao *et al.*, 2015), or between the power company and users (Fadlullah *et al.*, 2014). Pay-off functions and strategies are usually defined such that existence of Nash Equilibrium (NE) can be proved. Optimization (or model predictive control (Stephens *et al.*, 2015)) can be employed to search for NE. Sometimes the fundamental game is played repeatedly, and strategies of users are updated in real-time. In this situation, it is named evolutionary game (Cheng *et al.*, 2015). Networked evolutionary game indicates that, the repeated game is played among networked users, and updating laws relate to topological structure of the network (Cheng, 2009). In some networked evolutionary games, actions of some users can be actively assigned, such that other users are induced to improve common benefit. The users with actively assigned actions can be defined as controllers; and the networked evolutionary game with controllers can be defined as control networked evolutionary game (Zhao *et al.*, 2011).

During recent years, a new semi-tensor product (Cheng *et al.*, 2007) is developed to solve the problem of networked evolutionary game. The semi-tensor product is an extension of ordinary matrix product. By using semi-tensor product, dynamics of evolutionary games can be formulated into an algebraic form (Cheng, 2009), and the existence of NE can be proved systematically (Cheng *et al.*, 2015). For the control networked evolutionary game, control strategies can be designed to reach the NE by using semi-tensor product. Moreover, classical control methods can be introduced and extended in the framework of semi-tensor product to attain the NE of the networked evolutionary game.

In this paper, demand-side management of a class of smart grid is studied within the framework of control networked evolutionary game. The smart grid is built among interactive communities using either grid power or local generated power. It is assumed that a small portion of the communities are subsidized, thus cooperative with the grid provider. However, other communities are un-subsidized and pursuing individual benefits. We aim to design actions for the cooperative communities (controllers), such that the com-

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mon benefits can be improved even if other communities are noncooperative. The main *contributions* of this paper include that: 1) the demand-side management of a smart grid is modeled into a control networked evolutionary game; 2) the networked evolutionary game is composed by fundamental games played simultaneously among several players instead of 2-player games; 3) semi-tensor product is applied to solve the demand-side management problem; and 4) a new binary optimal control is introduced to optimize the transient performance of the control networked evolutionary game.

The layout of this paper is arranged as following. In Section 2, mathematical preliminaries are introduced. In Section 3, the demand-side management of a simple smart grid is formulated within the framework of control networked evolutionary game. In Section 4, the proposed control evolutionary game is analyzed and solved by using semi-tensor product, and a new optimal control approach is proposed to improve transient performance. In Section 5, a simulation example is presented to illustrate the proposed demand-side management approach. This paper is concluded in the final section.

2. Mathematical preliminaries

2.1. Control networked evolutionary game

Information interchange within networked system can be described by a directed graph $\mathscr{G} = \{\mathscr{V},\mathscr{E}\}$, where $\mathscr{V} = \{\pi_1,\pi_2,\dots,\pi_n\}$ is a set of nodes, and $\mathscr{E} \subseteq \mathscr{V}\times\mathscr{V}$ is a set of edges that depict information flow between nodes. An edge (π_i,π_j) in $\mathscr{G}$ denotes that the information of node π_i is available to π_j, and π_i is defined as a neighbor of π_j. The index set of all neighbors of node π_j is denoted by $\mathscr{N}_j = \{i : (\pi_i,\pi_j) \in \mathscr{E}\}$. In an undirected graph, $(\pi_i,\pi_j) \in \mathscr{E} \Leftrightarrow (\pi_j,\pi_i) \in \mathscr{E}$. The adjacent matrix $\mathscr{A} \triangleq [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} = 1$ if $(\pi_j,\pi_i) \in \mathscr{E}$, and $a_{ij} = 0$ otherwise. It is assumed that $a_{ii} = 0$. More details on network system can be found in Ren (2010).

Definition 1. A normal finite game $\mathscr{H}$ can be formulated by 1) the set of players: $\mathscr{V} = \{\pi_1,\pi_2,\dots,\pi_n\}$; 2) the strategy set for each player: $\mathscr{X}_i = \{x_{i1}, x_{i2},\dots,x_{ik}\}$, where $i = 1,\dots,n$; and 3) the cost function: $c_i(x_i, x_{-i})$, where $x_i \in \mathscr{X}_i$ denotes the strategy selected by player i, and x_{-i} denotes strategies of other players excluding player i.

Definition 2. Nash equilibrium (NE), denoted by $(x_1^*, x_2^*,\dots,x_n^*)$, is a local optimal response for a normal finite game, where no individual would gain by unilaterally changing its own strategy: $c_i(x_i^*, x_{-i}^*) \le c_i(x_i, x_{-i}^*)$.

If a game can be played repeatedly with an updating law: $\Pi: \quad x_i(t+1) = f(x_i(t), x_{-i}(t), c_i(t))$, where $t \ge 0$ denotes the discrete sampling time, then it is named evolutionary game.

In an evolutionary game played by multiple players, a typical updating law can be given by Unconditional Imitation with fixed priority (Cheng *et al.*, 2015):

$$x_i(t+1) = x_{j^*}(t), \;\; j^* = \arg\min_{j\in\mathscr{N}_i} c_j(x_j(t), x_{-j}(t)). \tag{1}$$

If j^* is non-unique, then select the minimal j^* as priority.

Definition 3. The networked evolutionary game is composed by 1) a networked graph $\mathscr{G}$; 2) a normal finite game $\mathscr{H}$ that can be played repeatedly; and 3) an updating law Π.

Remark 1. The above definition of the networked evolutionary game is slightly different from that of Cheng *et al.* (2015), where fundamental networked game (FNG) is required. In this paper, the normal finite game is used in Definition 3.

Definition 4. The control networked evolutionary game is composed by 1) a normal finite game $\mathscr{H}$ that is played repeatedly; 2) a networked graph $\mathscr{G}_c = (\mathscr{X}\cup\mathscr{U},\mathscr{E})$, where $\{\mathscr{X},\mathscr{U}\}$ is a partition of $\mathscr{V}$ ($\mathscr{X}\cup\mathscr{U} = \mathscr{V}$ and $\mathscr{X}\cap\mathscr{U} = \emptyset$), and strategies of $\mathscr{U}$ can be actively assigned; and 3) an updating law Π.

2.2. Semi-tensor product

Definition 5. The semi-tensor product of two matrix $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$ can be defined by

$$A \ltimes B \triangleq (A\otimes I_{o/n})(B\otimes I_{o/p}) \in \mathbb{R}^{(mo/n)\times(qo/p)}, \tag{2}$$

where $o = \mathrm{lcm}(n, p)$ denotes the least common multiple of n and p; and $\otimes$ denotes the Kronecker product.

Definition 6. The fundamental vector $\delta_n^i \in \mathscr{D}_n$ is defined as the ith column of the identity matrix $I_{n\times n}$. It can be further defined that $\delta_n[i, j,\dots,k] \triangleq [\delta_n^i,\delta_n^j,\dots,\delta_n^k]$.

Theorem 1. *(Cheng* et al.*, 2015) With equivalence $i \sim \delta_n^i$, $i = 1,2,\dots,n$, a logic function $f : \mathscr{D}_n^k \to \mathscr{D}_n$ can be rewritten by $f(x_1, x_2,\dots,x_k) = M_f \ltimes_{i=1}^k x_i$, where M_f is the structure matrix of logic function f.*

Theorem 2. *(Cheng* et al.*, 2015) For a logic dynamic system $x_i(t+1) = f_i(x_i(t), x_{-i}(t)) = M_{fi} \ltimes_{i=1}^n x_i$, $i = 1,\dots,n$, it can be rewritten in the form of*

$$x(t+1) = M_f x(t), \tag{3}$$

where $x(t) \triangleq \ltimes_{i=1}^n x_i$, and $M_f \triangleq M_{f1} * M_{f2} * \cdots * M_{fn}$. Here, $*$ denotes the Khatri-Rao product: $M * N \triangleq [\mathrm{col}_1(M)\ltimes \mathrm{col}_1(N),\cdots,\mathrm{col}_s(M)\ltimes\mathrm{col}_s(N)]$, where $M\in\mathbb{R}^{p\times s}$ and $N\in\mathbb{R}^{q\times s}$; and $\mathrm{col}_i(M)$ denotes the ith column of matrix M.

Theorem 3. *(Cheng* et al.*, 2015) For a logic dynamic system given by* (3)*, δ_n^i is its fixed point, if and only if the diagonal element m_{ii} of M_f equals* 1*.*

3. Problem formulation

In this paper, the evolutionary game is played among some remote rural communities, where a networked power grid is newly constructed. Before the construction of the power grid, the communities were using power generated by local facilities, e.g., diesel generators. To cover the cost, the price of grid power is high when there are less users. As the number of users grows, the price of grid power would decrease. However, if the number of users grows excessively large, the price would increase again due to supply shortage.

The main problem of this price policy is that, no individual community would like to become the first user of the grid, since its price would be high at the initial stage. Another problem is that, even if an optimal common benefit is reached, it might be unstable.

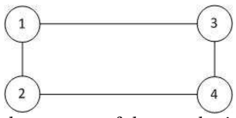

Figure 1: Topological structure of the graph given in Cases 1 and 2

Table 1: Prices of diesel power and grid power

user number	0	1	2	3	4
grid power price	8	7	7	6.5	7.5
diesel power price	7.2	7.2	7.2	7.2	7.2

Note: values in this table are not absolute prices; they are assigned to reflect differences of prices in various scenarios.

Case 1. Consider a grid connecting 4 communities. Each community has the choices of either local diesel power or grid power. The diesel power price p_d is constant, and the grid power price $p_g(t)$ varies with the numbers of users, as is displayed in Table 1. Denote the strategy space of community i by $\mathscr{X}_i = \{1,2\}$, where 1 indicates using grid power, and 2 indicates using of local diesel power. The topological structure of the network is given by Fig. 1, where edges are all undirected. It follows that the adjacent matrix is given by

$$\mathscr{A} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}.$$

Communities have no direct knowledge of real-time prices, but they fully know costs of their neighbors. Define p_i as the price paid by community i. The cost function is defined by

$$c_i(x_i(t), x_{-i}(t)) = p_i(t) + \alpha\left(p_i(t) - \min_{j\in\mathscr{N}_i} p_j(t)\right), \tag{4}$$

where $\alpha > 0$ is a constant weight coefficient. The cost function (4) indicates that each community, while pursuing the lowest price, feels uncomfortable if paying higher price than its neighbors. The updating law Π is given by (1), implying one community would change its strategy to that of the neighbor with the lowest cost. The common benefit at time t is defined as $C(t) = \sum_{i=1}^{4} p_i(t)$. It should be noted that, in this case, the optimal common benefit happens when three communities use grid power, and one uses diesel power.

Consider the scenario that the states are already in one of the cases of the optimal common benefit:

$$\{x_1(t), x_2(t), x_3(t), x_4(t)\} = \{2,1,1,1\}. \tag{5}$$

It follows that, according to the updating law (1), $\{x_1(t+1), x_2(t+1), x_3(t+1), x_4(t+1)\} = \{1,1,1,1\}$, deviating from the optimal common benefit.

In this case, $\{2,2,2,2\}$ indicates a fixed point; however, it is not an NE. If any community changes from 2 to 1, its cost would become smaller; however, no community would like to change unilaterally, because others might also make the same choice, and the states come to $\{1,1,1,1\}$.

Remark 2. The weight coefficient $\alpha > 0$ is necessary in (4), because the direct subtraction would be inappropriate if the two terms are with different physical implications. Another physical meaning of α is users' priorities between actual costs and psychological comforts. Without loss of generality, $\alpha = 1$ is assumed in this paper. Uncertain/time-varying α will be investigated in the future research.

Case 2. Suppose that all conditions in Case 1 are satisfied, except that the real-time price can be fully accessible, and the updating law is given by

$$x_i(t+1) = g(x_i(t), x_{-i}(t)) = \begin{cases} 2, & \text{if } p_g(t) > p_d(t); \\ 1, & \text{if } p_g(t) \le p_d(t), \end{cases} \tag{6}$$

indicating that the less expensive choice is preferred.

Consider one of the cases of optimal common benefit given by (5). It follows from (6) that $\{x_1(t+1), x_2(t+1), x_3(t+1), x_4(t+1)\} = \{1,1,1,1\}$, and $\{x_1(t+2), x_2(t+2), x_3(t+2), x_4(t+2)\} = \{2,2,2,2\}$, and the situation will remain in the future.

Suppose that the game is a controlled network evolutionary game defined by Definition 3. The *objective* of this paper is to design strategies for $\mathscr{U}$, such that optimal common benefit is achieved and maintained.

Case 3. Consider node 4 as the controller; and suppose the updating law is given by (1). The objective is to design $u = x_4 \in \mathscr{X}_4$, such that the total cost $\sum_{i=1}^{4} p_i(x_i, x_{-i})$ is minimized.

Case 4. Consider node 4 as the controller; and suppose the updating law is given by (6). The objective is to design $u = x_4 \in \mathscr{X}_4$, such that the total cost $\sum_{i=1}^{4} p_i(x_i, x_{-i})$ is minimized.

4. Control design

4.1. Algorithm for calculating the algebraic form

Suppose that there are n communities, and their communication topology can be given by adjacent matrix $\mathscr{A} = \{a_{ik}\}_{n\times n}$. The algorithm for calculating the algebraic form (3) can be designed as following.

i. Set $j = 1$.
ii. Set initial value $x_0 = \delta_{2^n}^{j}$. Based on the initial value, calculate the grid power price p_g and the price p_i.
iii. Use a_{ik} and p_i to calculate the cost function c_i.
iv. Based on a_{ik}, c_k and the updating law, the updated strategy can be obtained: $x_i(j) = f(x_0, a_{ik}, c_k)$.
v. Set $j = j+1$, and go to ii until $j = 2^n$.
vi. Calculate $M_f = M_{f1} * M_{f2} * \cdots * M_{fn}$, where $M_{fi} = [x_i(1), x_i(2), \cdots, x_i(2^n)]$. The algebraic form can be obtained by $x(k+1) = M_f x(k)$.

4.2. Analysis on Case 3

Based on the updating law given by (1), the true value diagram can be calculated with the algorithm in Section 4.1, and listed by Tables 2 and 3. Identify 1 with $\delta_2^1 \triangleq [1,0]^T$, and 2 with $\delta_2^2 \triangleq [0,1]^T$. The cost function can be calculated by (4). According to semi-tensor product theory, the controlled network evolutionary game can be described by

$$x_i(t+1) = f(x_i(t), x_{-i}(t), c_i(t)) = M_{fi}\, x(t), \tag{7}$$

Table 2: True value diagram of Case 3 when $u=1$

profile	111	112	121	122	211	212	221	222
c_1	7.5	6.5	6.5	7	7.9	7.2	7.4	7.2
c_2	7.5	6.5	7.9	7.4	6.5	7	7.4	7.4
c_3	7.5	7.9	6.5	7.4	6.5	7.4	7	7.4
c_4	7.5	6.5	6.5	7	6.5	7	7	7
$x_1(t+1)$	1	1	1	1	1	1	1	2
$x_2(t+1)$	1	1	1	1	1	1	1	1
$x_3(t+1)$	1	1	1	1	1	1	1	1

Table 3: True value diagram of Case 3 when $u=2$

profile	111	112	121	122	211	212	221	222
c_1	6.5	7	7	7	7.4	7.4	7.4	7.2
c_2	6.5	7	7.4	7.4	7	7	7.4	7.2
c_3	6.5	7.4	7	7.4	7	7.4	7	7.2
c_4	7.9	7.4	7.4	7.4	7.4	7.4	7.4	7.2
$x_1(t+1)$	1	1	1	1	1	1	1	2
$x_2(t+1)$	1	1	1	1	1	1	2	2
$x_3(t+1)$	1	1	1	1	1	2	1	2

where $x(t)=\ltimes_{i=1}^{3}x_i(t)$, and

$$M_{f1}=\begin{cases}\delta_2[1,1,1,1,1,1,1,2], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,1,1,2], & \text{for } u=2,\end{cases}$$

$$M_{f2}=\begin{cases}\delta_2[1,1,1,1,1,1,1,1], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,1,2,2], & \text{for } u=2,\end{cases}$$

$$M_{f3}=\begin{cases}\delta_2[1,1,1,1,1,1,1,1], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,2,1,2], & \text{for } u=2.\end{cases}$$

It follows from Theorem 2 that the overall controlled logic dynamics can be expressed by $x(t+1)=M_f(u(t))x(t)$, where

$$\begin{cases} M_f(\delta_2^1) & = M_{f1}(\delta_2^1)*M_{f2}(\delta_2^1)*M_{f3}(\delta_2^1)\\ & =\delta_8[1,1,1,1,1,1,1,5],\\ M_f(\delta_2^2) & = M_{f1}(\delta_2^2)*M_{f2}(\delta_2^2)*M_{f3}(\delta_2^2)\\ & =\delta_8[1,1,1,1,1,2,3,8].\end{cases} \tag{8}$$

By using Theorem 3, it can be claimed that

1) When the controller is $u=\delta_2^1$ (grid power), there is only one fixed point δ_8^1. Here, $(x_1,x_2,x_3,u)\sim(1,1,1,1)$ is an NE, but not the optimal NE.
2) When the controller is $u=\delta_2^2$ (diesel power), there are two fixed points, namely (1,1,1) and (2,2,2). Here, $(x_1,x_2,x_3,u)\sim(1,1,1,2)$ is an optimal NE, while $(x_1,x_2,x_3,u)\sim(2,2,2,2)$ is not an NE.

The following strategies of $u(t)$ is capable of reaching and maintaining the optimal NE point:

1) For $x(0)\in\{\delta_8^1,\delta_8^2,\delta_8^3,\delta_8^4,\delta_8^5\}$,select either $u(0)=\delta_2^1$ or $u(0)=\delta_2^2$, and it follows from Table 2 and 3 and (8) that $x(1)=\delta_8^1$. Then, set $u(t)=\delta_2^2$ for $t\geq 1$, such that the optimal NE $(1,1,1,2)$ will be maintained.
2) For $x(0)\in\{\delta_8^6,\delta_8^7\}\sim\{212,221\}$, select $u(0)=\delta_2^1$. and it follows that $x(1)=\delta_8^1$. Then, set $u(t)=\delta_2^2$ for $t\geq 1$, such that the optimal NE $(1,1,1,2)$ will be maintained.
3) For $x(0)\in\{\delta_8^8\}\sim\{222\}$, it can be calculated that $M_f(\delta_2^1)\ltimes M_f(\delta_2^1)=\delta_8[1,1,1,1,1,1,1,1]$. Consequently, select $u(0)=\delta_2^1$ and $u(1)=\delta_2^1$; it follows that $x(2)=\delta_8^1$. Then, set $u(t)=\delta_2^2$ for $t\geq 2$, such that the optimal NE $(1,1,1,2)$ will be maintained.

Table 4: True value diagram of Case 4 when $u=1$

profile	111	112	121	122	211	212	221	222
p_d	7.2	7.2	7.2	7.2	7.2	7.2	7.2	7.2
p_g	7.5	6.5	6.5	7	6.5	7	7	7
$x_1(t+1)$	2	1	1	1	1	1	1	1
$x_2(t+1)$	2	1	1	1	1	1	1	1
$x_3(t+1)$	2	1	1	1	1	1	1	1

Table 5: True value diagram of Case 4 when $u=2$

profile	111	112	121	122	211	212	221	222
p_d	7.2	7.2	7.2	7.2	7.2	7.2	7.2	7.2
p_g	6.5	7	7	7	7	7	7	8
$x_1(t+1)$	1	1	1	1	1	1	1	2
$x_2(t+1)$	1	1	1	1	1	1	1	2
$x_3(t+1)$	1	1	1	1	1	1	1	2

4.3. Analysis on Case 4

Based on the updating law given by (6), the true value diagram can be calculated through the algorithm proposed in Section 4.1, and listed by Table 4 and 5. It follows that, according to semi-tensor product, the controlled network evolutionary game can be described by $x_i(t+1)=g(x_i(t),x_{-i}(t))=M_{gi}x(t)$, where $x(t)=\ltimes_{i=1}^{3}x_i(t)$, and

$$M_{g1}=\begin{cases}\delta_2[2,1,1,1,1,1,1,1], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,1,1,2], & \text{for } u=2,\end{cases}$$

$$M_{g2}=\begin{cases}\delta_2[2,1,1,1,1,1,1,1], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,1,1,2], & \text{for } u=2,\end{cases}$$

$$M_{g3}=\begin{cases}\delta_2[2,1,1,1,1,1,1,1], & \text{for } u=1,\\ \delta_2[1,1,1,1,1,1,1,2], & \text{for } u=2.\end{cases}$$

It follows from Theorem 2 that the overall dynamics can be expressed by $x(t+1)=M_g(u(t))x(t)$, where the structure matrix can be calculated by

$$\begin{cases} M_g(\delta_2^1) & = M_{g1}(\delta_2^1)*M_{g2}(\delta_2^1)*M_{g3}(\delta_2^1)\\ & =\delta_8[8,1,1,1,1,1,1,1],\\ M_g(\delta_2^2) & = M_{g1}(\delta_2^2)*M_{g2}(\delta_2^2)*M_{g3}(\delta_2^2)\\ & =\delta_8[1,1,1,1,1,1,1,8].\end{cases}$$

It can be seen from structure matrix that, when $u=\delta_2^1$, there is no fixed point; when $u=\delta_2^2$, there are two fixed points $x=\delta_8^1$ and $x=\delta_8^8$. The fixed point $x=\delta_8^1$ is an optimal NE, while $x=\delta_8^8$ is not an optimal NE.

The optimal NE can be reached and maintained by using the following strategies.

1) For $x(0)=\delta_8^1$, select $u(t)=\delta_8^2$, such that the optimal NE can be maintained.
2) For $x(0)\in\{\delta_8^2,\delta_8^3,\delta_8^4,\delta_8^5,\delta_8^6,\delta_8^7,\}$, select either $u(0)=\delta_2^1$ or $u(0)=\delta_2^2$, such that $x(1)=\delta_8^1$. Then, select $u(t)=\delta_8^2$ for $t\geq 2$, such that the optimal NE can be maintained.
3) For $x(0)=\delta_8^8$, select $u(t)=\delta_8^1$, such that $x(1)=\delta_8^1$. Then, select $u(t)=\delta_8^2$ for $t\geq 2$, such that the optimal NE can be maintained.

Remark 3. Case 3 and 4 are controllable cases. It should be noted that controllability of networked evolutionary games depends on topological structure, strategy set, updating law, and selection of control variables. Detailed information of controllability can be found in Cheng *et al.* (2015).

4.4. Optimal control design

As can be seen from Section 4.2, to achieve the optimal NE, there may exist different strategies. For example, if the initial states are given by $x(0) = \delta_8^4 \sim (1,2,2)$, either $u(0) = \delta_2^1$ or $u(0) = \delta_2^2$ enables the state to become $x(1) = \delta_8^1$. If $u(0) = \delta_2^1$, the overall cost at $t = 0$ is $C(x(0), u(0)) = 28.4$; if $u(0) = \delta_2^2$, the overall cost at $t = 0$ is $C(x(0), u(0)) = 28.6$. Comparatively, for initial states $x(0) = \delta_8^4$, $u(0) = \delta_2^1$ is superior. In this section, we propose an optimal control to minimize the overall cost in transient process.

Suppose that the system is required to reach the optimal NE within T steps, and define $U = [u(0), u(1), \ldots, u(T)]^T$. The cost function for optimization can be designed by $J \triangleq \sum_{t=0}^{T} C(x(t), u(t))$. Dynamics of the system can be given in algebraic form (3). The terminal states should reach the optimal NE: $x(T) = x_{NE^*}$. Suppose that the initial states is denoted by $x(0) = x_0$. The optimization can be formulated by

$$U^* = \arg\min_U J, \tag{9}$$

$$\begin{aligned} \text{s.t.} \quad x(t+1) =& M_f(u(t))x(t), && (10)\\ u(t) =& \ltimes u_i(t), \quad u_i(t) \in [\delta_2^1, \delta_2^2], && (11)\\ x(0) =& x_0, && (12)\\ x(T) =& x_{NE^*}, && (13)\end{aligned}$$

where the optimal solution $U^* = [u^*(0), u^*(1), \ldots, u^*(T)]^T$ can be regarded as the optimal control sequence.

As can be seen from the control constraint (11), the problem (9) is a binary optimization. The system constraint (10) seems linear; however, since the product is semi-tensor product, it is actually nonlinear. The nonlinear binary optimization can be solved by using a newly developed algorithm named Bounded Neighborhood Field Optimization (BNFO) (Wu and Chow, 2013a,b). The algorithm of BNFO is incapable of addressing the terminal constraint (13). Consequently, the optimization can be reformulated by (9)–(12), where no terminal constraint is included.

Remark 4. BNFO can be categorized as switching optimization. Typical results in switching optimization can be seen in Li *et al.* (2006) and literatures therein. General optimal control of boolean networks can be found in Zhao *et al.* (2011).

Theorem 4. *Suppose that the following conditions are satisfied: (1) the optimal NE x_{NE^*} is a global optimal point; and (2) with certain control series $[\hat{u}(0), \hat{u}(1), \cdots, \hat{u}(N)]$, the optimal NE x_{NE^*} can be reached within finite time $t = N < T$ from initial states $x(0) = x_0$. Then, with large enough control horizon T, the closed-loop system with the optimal control* (9)–(12) *is capable of reaching the optimal NE.*

Proof. The result can be proved by contradiction. Assume that, with control horizon T and the optimal control (9)–(12), the closed-loop system fails to reach x_{NE^*}. It follows that

$$\begin{aligned} J^* =& \sum_{t=0}^{T} C(x^*(t), u^*(t))\\ =& \sum_{t=0}^{N-1} C(x^*(t), u^*(t)) + \sum_{t=N}^{T} C(x^*(t), u^*(t)),\end{aligned}$$

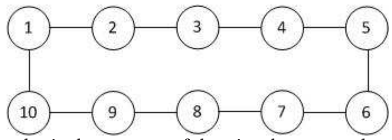

Figure 2: Topological structure of the circular network with 10 nodes

where x^*t is the corresponding optimal states under optimal control $u^*(t)$; and $x^*(t) \neq x_{NE^*}$.

In another aspect, according to conditions of the theorem, there exists at least another control series $[\hat{u}(0), \hat{u}(1), \cdots, \hat{u}(N), \cdots, \hat{u}(T)]$, such that x_{NE^*} is reached and maintained. It follows that

$$\begin{aligned} \hat{J} =& \sum_{t=0}^{T} C(\hat{x}(t), \hat{u}(t)) = \sum_{t=0}^{N-1} C(\hat{x}(t), \hat{u}(t)) + \sum_{t=N}^{T} C(\hat{x}(t), \hat{u}(t))\\ =& \sum_{t=0}^{N-1} C(\hat{x}(t), \hat{u}(t)) + (T-N)C(x_{NE^*}, u_{NE^*}),\end{aligned}$$

where u_{NE^*} is the corresponding control to maintain the optimal NE. It then follows that

$$\begin{aligned} \hat{J} - J^* =& \left(\sum_{t=0}^{N-1} C(\hat{x}(t), \hat{u}(t)) - \sum_{t=0}^{N-1} C(x^*(t), u^*(t))\right)\\ &+ \sum_{t=N}^{T} \big(C(x_{NE^*}, u_{NE^*}) - C(x^*(t), u^*(t))\big),\end{aligned}$$

where $\sum_{t=0}^{N-1} C(\hat{x}(t), \hat{u}(t)) - \sum_{t=0}^{N-1} C(x^*(t), u^*(t))$ is finite; and $\sum_{t=N}^{T} (C(x_{NE^*}, u_{NE^*}) - C(x^*(t), u^*(t)))$ is negative and decreases strictly as T increases. Consequently, it can be claimed that $\hat{J} - J^* < 0$ for large enough T, indicating that $\hat{u}(t)$ is superior over $u^*(t)$; hence $u^*(t)$ is not an optimal solution, which contradicts the assumption given at the beginning of this proof. Based on the contradiction, it can be proved that the optimal control (9)–(12) guarantees the convergence to the optimal NE x_{NE^*}. □

For Case 3, suppose the initial state is given by $x(0) = \delta_8^4$. Set $T = 3$, and $x(T) = \delta_8^1$. Solving the optimization formulated by (9)–(12) yields $U^* = [\delta_2^1, \delta_2^2, \delta_2^2]$.

5. A simulation example

In this section, an illustrative example is presented by considering a smart grid with more communities. Its topological structure is given by an undirected circular network with 10 nodes, as can be seen from Fig. 2. Based on concepts in Section 2.1, its adjacent matrix can be calculated accordingly. The diesel power price is given by $p_d = 7$, and the grid power price is given by $p_g(n) = \frac{(n-5)^2}{25} + 6.5$, where n is the number of communities using the grid power. In this simulation example, x_7 and x_{10} are selected as controls u_1 and u_2; their values can be assigned arbitrarily to δ_2^1 or δ_2^2. We suppose that the updating law is given by (1), and the cost function is given by (4) with the coefficient $\alpha = 0.8$.

By using the algorithm proposed in Section 4.1, the algebraic form of the game can be obtained by $x(t+1) = M_f(u_1(t), u_2(t))x(t)$, where $x(t) = \ltimes_{i=1, i\neq 7, i\neq 10}^{10} x_i$. All NEs can be calculated by using Theorem 3: $x_{NE}(u_1 = \delta_2^1, u_2 = \delta_2^1) = \delta_{256}[1]$, $x_{NE}(u_1 = \delta_2^1, u_2 = \delta_2^2) = \delta_{256}[256]$, $x_{NE}(u_1 = \delta_2^2, u_2 = \delta_2^1) = \delta_{256}[256]$, $x_{NE}(u_1 = \delta_2^2, u_2 = \delta_2^2) = \delta_{256}[1,4,253,256]$, where $x_{NE^*} = \delta_{256}^4$ is the optimal NE.

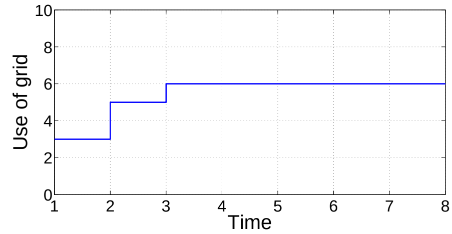

Figure 3: Number of communities using grid power, in case of $x(0) = \delta_{256}^{200}$

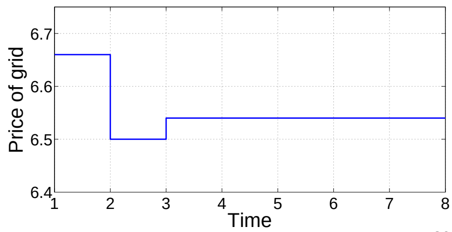

Figure 4: Price of grid power, in case of $x(0) = \delta_{256}^{200}$

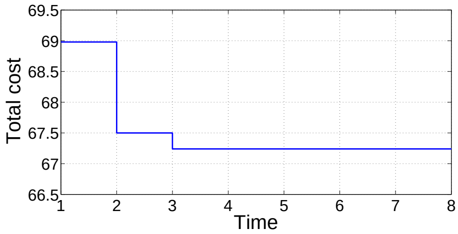

Figure 5: Total cost paid by communities, in case of $x(0) = \delta_{256}^{200}$

Suppose that the initial states are given by $x(1) = \delta_{256}^{200}$. Applying the optimal control (9)–(12) with control horizon $T = 8$ to the control networked evolutionary game yields $u(t) = u_1(t) \ltimes u_2(t) = \delta_4[4,4,4,4,4,4,4,4]$, and $x(t) = \delta_{256}[200,132,4,4,4,4,4,4]$, where $t = 1,2,\cdots,8$; and the closed-loop system reaches $x_{NE^*} = \delta_{256}^4$. Simulation results are illustrated by Figs. 3–5, where the number of communities using grid power, the real-time price of the grid power, and the overall cost paid by all communities are displayed. As can be seen from simulation results, with the proposed optimal control based on evolutionary game theory and semi-tensor products, the overall cost converges to the optimal NE.

Suppose that the initial states are given by $x(1) = \delta_{256}^{78}$, and the control horizon is set to $T = 6$. The result can be obtained by using the optimal control (9)–(12):

$$u(t) = u_1(t) \ltimes u_2(t) = \delta_4[3,4,4,4,4,4], \tag{14}$$

$$x(t) = \delta_{256}[78,5,1,1,1,1], \tag{15}$$

where $t = 1,2,3,4,5,6$. As can be seen from (15), the closed-loop system fails to reach $x_{NE^*} = \delta_{256}^4$; the reasons include that 1) x_{NE^*} may be un-reachable from the initial point $x(0) = \delta_{256}^{78}$, and 2) the control horizon is not large enough.

6. Conclusion

In this paper, control networked evolutionary game and semi-tensor product are applied to solve the demand-side management problem of a simple smart grid. By using the semi-tensor product to solve the control networked evolutionary game, NEs can be proved systematically, and control series can be designed to reach and maintain the optimal NE. The BNFO algorithm is introduced to optimize the transient performance of the control networked evolutionary game.

Some future works of this research include: 1) optimal control with dynamic price policies (instead of static ones in this paper), and 2) adaptive control in case of uncertain/time-varying weight coefficient in the cost function.

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