The implications of forcing beta from one down towards beta neutrality on key risk and return and other measures in long only mean variance efficient equity portfolios.

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Abstract

Hedge fund strategies such as the equity market neutral have provided significant risk adjusted returns in the form of alpha, but their short selling and debt has made them generally costly and prone to failure under changing market conditions. There is a need to isolate the benefits of long short equity hedging without the added costs and dangers associated with short selling and leverage.

Isolating the set of lowest possible market beta long equity portfolios that can mimick long short equity hedging can provide investors cost effective hedge fund replication. A systematic procedure involving mean variance optimisation and quantitative analytical techniques was used to characterise the behaviour of targeted beta portfolios on key risk and return metrics and variables as a beta constraint was applied to optimisation on a finely calibrated scale of one down to zero.

This research was able to isolate a sample from the JSE/FTSE Top 40 Index into a solution set (P) of low beta portfolio alternatives extending from a target beta value of 0.475 to a beta value of 0.600 which was identified, characterised and disaggregated into definitive solution tuples P1 (beta 0.600, beta 0.575, beta 0.550) and P2 (beta 0.525, beta 0.500, beta 0.475).

Key words

Hedge, replication, beta, equity, short

Declaration

I declare that this research project is my own work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the Gordon Institute of Business Science, University of Pretoria. It has not been submitted before for any degree or examination in any other University. I further declare that I have obtained the necessary authorisation and consent to carry out this research.

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1 INTRODUCTION

1.1 Research Problem

The vulnerabilities of hedge fund strategies were starkly revealed particularly during the depths of the 2008 financial crisis when the majority of hedge funds registered huge negative returns exposing hitherto unknown strong positive correlations (Kaiser and Haberfelner, 2011) to equity and fixed income asset classes (Roumpis & Syriopoulos, 2014). The unfavourable asset correlations negatively impacted the performance of alpha seeking hedge funds employing strategies of selling short overpriced stocks and buying cheaper ones in order to capitalize on market mispricing anomalies as stocks move together more in response to macro-economic signals and news (Authers, 2012; Sabbaghi, 2011).

1.1.1 Management Fees

The problem has arguably been further compounded by one of the main distinguishing characteristics of the hedge fund industry: high management fees comprising typically a fixed component of about two percent of assets under management and a performance related 20 percent share of profits not usually subject to watermark restrictions (Ibbotson, Chen, & Zhu, 2011).

As Cochrane (2013) points out, the high active management fees that hedge funds are known to charge have been consistent over long periods of time but the returns to most of their investors have been disappointing and mediocre at best. French (2008) argues that the fees charged by hedge funds, comprising the management fees which are fixed and the performance fees, tend to be one sided and accrue even when the fund does not perform. As a result, the net return available to investors laden with fees for active management. In fact the French (2008) study reveals the average annual hedge fund fee for the 1996 to 2007 period was 4.26 percent of assets because of the double fee structure, whereas mutual funds actually reduced fees from 2.08 percent of assets to 0.95 percent in 2006. The sheer extent of the cost disparity is put into perspective by the author's observation that fees paid by hedge fund clients to invest US$458.6

billion dollars in 2006, were 36 percent higher than fees that non-hedge fund clients were charged for their US$6.18 trillion in investments.

The perspective drawn from the study is that active investing in most hedge funds is not worth it for investors compared to well diversified passive investments with less fees, less risk and better returns. Jones and Wermers (2011) find that managers are unlikely to deliver active returns of more than zero if all expenses are considered because market have become highly efficient and ultra-competitive.

1.1.2 Frequent Rebalancing and associated transaction costs

Furthermore, hedge fund strategies such as equity market neutral or low beta , particularly those related to statistical arbitraging techniques are known to rebalance very frequently (L'habitant, 2002; Muhtaseb, 2012), sometimes as much as every 15 minutes. Frequent rebalancing results in very high turnover and the associated transaction costs which tend to increase very significantly. High transaction costs are almost always eventually charged to clients, which is undesirable. Cakici, Tessitore and Usmen (2002) suggest that rebalancing should be very infrequent in markets with high costs so that the effects of wrong bets are not worsened.

1.1.3 Short selling

The viability of hedge fund strategies involving shorting of stocks is now under question amongst professional investors (Authers, 2012). Hedge fund practices such as short selling been under increasing regulatory scrutiny with certain types of short selling being curtailed by regulatory authorities in various capital markets around the world (Beber & Pagano, 2013).

The unsavoury reputation of some of the proponents of short selling have done much to undermine the strategy, as Karpoff and Lou (2010) point out in their study , some short sellers have been known to spread false rumours about those firms in which they would have taken short positions in order to force a price decline and take the ensuing profits, ruining other investors in the process. Although the authors note that short sellers at times do help bringing to light financial shenanigans for the benefit of the investing public they still sometimes muddy the waters with regard to price discovery with white noise trades.

Short selling is an exercise which can be fraught with danger. Whereas in a long sale the maximum loss to the investor is limited to the initial investment in made to buy a certain share, in the case of a short sale, the potential loss can be unlimited and dependent on the level to which the share price can rise against the negative growth bet taken in shorting. Bankruptcies such as those of the Lehman Brothers in 2008, served to underline the risks associated with careless use of financial strategies such as short selling resulting in restrictions and even prohibitions to short selling (Saffi & Sigurdsson, 2011).

It is also to be borne in mind that in short selling involves borrowing and associated fees which can easily run above 10 percent annually (Diether, Lee & Werner, 2009). Within a hedge fund context, the investors are generally disadvantaged by high fees and obtain a lowered return (French, 2008).

1.1.4 Leverage

In addition to leverage from borrowing shares, hedge funds tend to borrow heavily and thus have leverage which is relatively higher than that found in mutual funds. (Brunel, 2007). In order for leverage to succeed in yielding superior returns it needs to be very dynamically managed by highly skilled managers. Hedge funds may create risks to other financial institutions owing to excessive borrowing, particularly in instances of sharp declines in asset pricing and markets become illiquid as was the case with the Long Term Capital Fund (Stulz, 2007). The availability of cheap debt in good times is exploited by hedge funds, in tough economic conditions however that high leverage increases vulnerability to liquidity issues and can complicate the path to optimal de-leveraging , often leading to increased systemic risks (Acharya & Viswanathan, 2011).

Accessibility of traditional hedge fund structures is somewhat limited, arguably by the inherently high structural costs that come with the need to implement sophisticated strategies that require constant monitoring by highly skilled staff.

1.1.5 Opacity

The general absence of regulatory requirements to report and the use of only favorable past results contributes to biased return calculations found in many hedge fund databases (Malkiel & Saha, 2005). Most hedged fund tend to operate in a black-box approach with information not made readily available to investors.

1.1.6 Mutual fund industry challenges

The mutual fund industry on the other hand has generally tended to struggle to effectively deliver impressive risk adjusted returns, leading to the rise of the closet indexing phenomenon, whereby a pretense to active management is masked in a minimal churning of stocks around a core passive index (Cremers & Petajisto, 2009). It would appear that regulatory hurdles and possibly skill issues conspire to lower the level of risk management sophistication and ability to explore alpha, relative to the hedge fund counterparts as alluded to by Agarwal, Boyson, and Naik (2009). Management fees on the other hand tend to be generally lower in comparison to hedge funds (Ibbotson et al., 2011). Given that investors are now more than ever highly concerned about the level of risk exposure to low return investments, low cost alternatives to main stream hedge fund strategies or clones have begun to gain traction (Tuchschmid, Wallerstein, & Zaker, 2011). Alternatives in the mutual fund space can play an increasingly important role in providing access to emulated hedge-fund strategies but at lower fees with the added benefits of greater transparency, liquidity, and the relatively more reassuring regulatory protections of mutual funds (Agarwal, Boyson, & Naik, 2009). However there are still challenges around determining how to best clone hedge fund strategies cost effectively or to generate more viable alternatives (Roumpis & Syriopoulos, 2014).

1.1.7 Going forward

As institutional investors become more important players, the hedge fund management skills may become redundant, to the need for other services provided to investors such as risk management, transparency, reporting, liquidity and balance sheet capability needed to take on large new investments (Stulz, 2007). Moreover, many of these services can be obtained by institutions without paying a large performance fee to a hedge fund. Perhaps most strikingly, there is increasing evidence that the performance of hedge fund indices can largely be replicated by machines (Kat and Palaro, 2006), which is opening up opportunities for investors that require hedge fund exposure without having to overpay expensive hedge fund managers.

Mutual funds have been trying hard to come up with hedge fund strategies that can bypass structural hurdles. A sizeable growth in what are termed hedge fund lite strategies is noted by Agarwal, Boyson, and Naik (2006). Performance relative to hedge funds have been dismal though, with key weakness in skill, incentive structures and limited flexibility. A clear trend towards commoditisation of hedge fund products is apparent, with investors yearning for access to hedge fund strategies at mutual fund level fees. Other players at the institutional level including endowment funds and the pensions are busy trying to devise new strategies to gain access to the hedge fund prize promising lower fees.

The deterioration of average hedge fund performances and the need for liquidity and transparency amongst other issues undoubtedly provides the impetus and sufficient room for the revival of hedge fund replication and financial innovation that can deliver effective and cheaper means for investors seeking to obtain hedge fund like returns (Kat, 2007).

1.2 Purpose of the Research

There is therefore a need to focus on "best of both worlds" quantitative approaches to semi active management of equities, lying between passive management and fully active management in the mutual fund space, that can potentially emulate the hedge fund strategies particularly those involving short selling such as market neutral and long-short which explicitly promise alpha while controlling exposure to systematic risk.

The idea behind this study therefore has been to explore the possibilities of creating an alternative to equity market neutral and low beta hedge funds, that does not require the use of costly and risky traditional methods such as shorting and leverage and the inherent need for frequent portfolio rebalancing. Essentially a no shorting, no leverage, low rebalancing low beta alternative hedge fund portfolio structure, which would be cost effective.

To the best of the author's knowledge no definitive studies have been done that characterize the performance of a beta constrained initially well diversified portfolio of equity stocks in a mean variance optimised setting through the forcing of beta from a market identical value of one down to market neutral zero, and studying the implications on the interactions and behavior of key risk and return metrics and variables that include active share (Cremers & Petajisto, 2009), tracking error, industry concentration index, the Sharpe ratio (Sharpe, 1964) and the Treynor ratio (Treynor & Black, 1973) in general and also particularly in an emerging market context.

The aim of the research was therefore to ascertain if a beta constrained mean variance optimised equity portfolio may be able to offer a cost effective alternative to long short and especially market neutral portfolios for investors seeking controlled market exposure to systematic risk without the added risks and costs that come with hedging products. Harry Markowitz's (1952) mean variance analysis as well as insights from work done by Charpin and Lacaze (2006) on equity constrained portfolios amongst others were be used in the construction, simulation and evaluation of beta targeted constrained equity portfolios. Furthermore the insights from the work by Clarke, De Silva, & Thorley, (2011) on minimum variance portfolios as they relate to potential impact of threshold betas on portfolio composition in mean variance portfolios were drawn upon in characterising portfolio behaviour.

The rest of this document is structured as follows: chapter 2) a theory and literature review focusing on semi active management, hedge fund strategies, replication and mean variance optimisation; chapter 3) research questions and objectives; while chapter 4) covers the methodology and design which is of a quantitative nature. Chapter 5) presents the empirical results, chapter 6) contains the analysis and interpretation of results and Chapter 7 concludes with recommendations.

2 THEORY AND LITERATURE REVIEW

2.1 Semi Active Portfolio Management

Some investors are interested in the ability to capture risk adjusted returns in a manner that efficiently controls for systematic risk (Sharpe, 1964) and creates room for alpha, a skill based return as described by Grinold and Kahn (2000), Clarke et al. (2002) in their empirical law of active management as well as more recently by Cremers and Petajisto (2009) in introducing the concept of active share. The focus of this research is on quantitative approaches to semi active management of equities that can be implemented algorithmically, lying between passive management and fully active management.

In the mutual fund industry semi active strategies are generally designed for investors wanting to outperform the benchmark while carefully managing portfolio risk exposures. The two main forms of semi active strategies are derivatives based and stock based (Maginn, Tuttle, McLeavey, & Pinto, 2007) where the former aim to provide beta exposure through a derivative and the enhanced return/alpha through non-equity instruments such as fixed income and the latter involves a degree of active management wherein investment insights would be used to delicately deviate portfolio weights from benchmark figures with a high degree of risk control.

Although Grinold and Kahn (2000) mathematically showed that discipline and skill may result in high information ratios the introduction of constraints was shown by Clarke, De Silva and Thorley (2002) to complicate matters if the information ratio were decomposed into the more practical transfer coefficient and other noise measures involved in the realisation of active returns. It would therefore not be unreasonable to believe that the majority of closet indexers (Cremers and Petajisto, 2009) could be spawned from enhanced indexing.

2.2 Capital Asset Pricing Model

The capital asset pricing model (CAPM) by Markowitz, (1952) remains relevant despite the criticisms and the concept of beta as measure of systematic risk and for application in the estimation of expected returns remains arguably sound (Modigliani & Pogue, 1974) (Black, 1995) and in practical use (Leote de Carvalho, Lu, & Moulin, 2012).

The CAPM market model can be used in estimating expected returns $E(R_i)$ as inputs into mean variance optimization for instance with the following simplified relationship.

$$E(R_i) = R_f + \beta_i(R_M - R_f)$$

Where R_f is the risk free rate and β_i is the sensitivity of the systematic risk and $R_M - R_f$ represents the equity risk premium.

One of the limitations that have to be considered in using CAPM estimates is the inability of the model to properly account for multiple sources of systematic risk which would not necessarily be adequately reflected in the underpinning one risk factor assumption. Furthermore some of the more strict assumptions of CAPM such as those that relate to homogeneity of return expectations among different investors and liquidity (De Fusco, McLeavey, Pinto, & Runkle, 2011) have been shown to be a marked departure from the heterogeneity and thin trading that is generally observed in reality. Limitations not withstanding CAPM has facilitated the context of portfolio management and the development of underpinning constructs such as beta and alpha in passive, active and semi active management of investment securities.

2.3 The search for Alpha

Ibbotson et al. (2011) described alpha as returns that are added value as a result of investment skills and are generally hard to obtain in the broader mutual fund industry context. Alternatively according Jaeger and Wagner (2005) alpha is that component of returns that are not accessible by taking on systematic risk in the broad markets whereas the returns from beta arise from direct investment in a diversified portfolio of stocks and bonds with no application of special investment nous needed.

The pursuit of risk adjusted returns has been very forcefully enacted particularly in the hedge fund industry (van Dyk, van Vuuren, & Heymans, 2014) albeit with mixed success as evidenced in literature on the subject matter including Fung, Hsieh, Naik, and Ramadorai (2008), Fung and Hsieh (2011), Ibbotson et al. (2011), Roumpis and Syriopoulos (2014) and others through a variety of strategies including equity market neutral and long short which attempt to control the level of exposure to systematic risk factors, whilst allowing for access to returns from non-market factors including alternative beta, under the umbrella of alpha. In the case of hedge funds Jaeger and Wagner (2005) argued that a large component of the returns (as much as 80 percent or more) actually arose from systematic factors either as beta or alternative beta. This argument obtained further support from Fung et al. (2008) who noted that to a large extent hedge fund fees generally charged on total returns were actually compensating managers for taking on systematic risk rather as opposed to skill based alpha. The authors warning that the hedge industry itself may be headed towards a zero alpha equilibrium as it would get harder to generate alpha in the crowded capital markets deserves attention given that it is the search for alpha in the first place that has attracted vast sums of money into the hedge fund sector.

The mutual fund sector however has generally tended to struggle to effectively deliver impressive risk adjusted returns in comparison to a well-diversified portfolio, leading to the rise of the closet indexing phenomenon (Cremers & Petajisto, 2009). It would appear that regulatory hurdles and possibly skill issues have conspired to lower the level of risk management sophistication and ability to explore alpha, relative to the hedge fund counterparts as alluded to by Agarwal et al, (2009). Management fees on the other hand have tended to be generally lower than those charged by hedge funds, comprising an annual management fee of up to 2 % and typically without the performance related 20% carry component as has been the case in the hedge fund sector (Ibbotson et al., 2011).

2.4 Hedge Fund strategies

There are numerous categories of hedge fund strategies including fixed-income arbitrage, event driven, convertible arbitrage, emerging market, long-short, equity managed futures and equity market neutral (Jaeger & Wagner, 2005). Fung and Hsieh (2002) noted the difficulties in categorizing hedge fund strategies owing to information opacity and proliferation of new funds. For the purposes of this research the focus was on the equity market neutral and long-short equity strategies.

2.4.1 Equity Market Neutral

Equity market neutral funds would typically target a beta exposure in equity markets of zero (Jaeger & Wagner, 2005) but may on occasion be exposed to a beta of ±0.20 (Black, 2009). For beta neutrality to be achieved, the money value and beta of the long positions are matched to equivalent short positions (Patton, 2009). Although beta risks are minimized in this strategy, portfolios may be exposed to alternative beta: substantial risks in other areas of the equity markets, such as market capitalization, value, growth, or industry. A key challenge would be skillful and effective access to alternative beta and alpha.

2.4.2 Long Short

The long short strategy is a combination of long and short positions on equity that are not money or beta matched as such funds do not target zero beta exposure. Instead long short funds may target a net positive or negative exposure to equity markets. Long–short equity funds have been the largest hedge fund strategy, earning close to 40 percent of all investor dollars allocated to hedge funds (Black, 2009). Black, (2009) estimated that long short funds would typically average a net long beta of 0.3 to 0.6 over extended periods unless bear markets were expected, in which case exposure would be reduced to negative to the equity market. One of the weaknesses in the implementation of the long short strategy particularly in 1x0/x0 portfolios in which long portfolio is independently derived is the inefficient use of correlations between long and short assets for integrated reduction of portfolio risk (Charpin & Lacaze, 2006). It would be reasonable to assume that such inefficiencies are probably magnified during market crises when correlations converge as was the case during 2008.

2.5 Shorting of stocks

In theory, unconstrained portfolios in which shorting is allowed enable the investor to make more efficient use of information compared to long only portfolios owing to the ability to use insights on both expected positive performing stocks which can be fully over weighted and poor performing stocks which can be negatively weighted according to Grinold and Kahn (2000). However they did note that despite there also being other advantages in terms of the ability to port alpha in certain applications, at low numbers of stocks and low active risks, shorting strategies fail to beat returns of long only strategies. Furthermore the actual advantages of shorting have not always translated into relatively higher returns, as Sorensen, Hua, and Qian (2007) found that financing and transactional costs involved in shorting can overwhelm the informational benefits as a result of increasing turnover, the added monitoring associated with the potential for unrealized losses to short position price spikes and the research costs. The implementation costs of long only strategies are therefore generally lower than those involving the use of shorting. Sorensen et al. (2007) also found that relaxing the long only constraint leads to higher ex-post tracking error and the emergence of sizeable differences between decay of the information ratio relative to the transfer coefficient. It is therefore reasonable to begin to question the long term vability of hedging strategies that use shorting (Authers, 2012).

2.6 Hedge Fund Replication - the search for cost effective alternatives

With uncertainties in the global capital markets, underlined by the 2008, sub-prime fueled credit crisis and now the Eurozone fragilities as exemplified by the Greek debt crisis, there may be need to enhance the development of potentially more cost effective and less risky semi active portfolio replicas of desirable hedge strategies (Tancar, Poddig, & Ballis-Papanastasiou, 2012), such as market neutral and long short in a mutual fund setting employing transparent assets and prudent constraints.

2.6.1 Financial Crisis of 2008 and hedge funds

Hedge funds built up a reputation as alternative investments able to deliver returns during good and bad times even weathering the technology bubble (Ibbotson et al., 2011). Global investment in hedge funds according to van Dyk et al. (2014) increased from US$50 billion in 1990 to US$2.2 trillion just after 2006, on the back of some growth surges in which some funds achieved absolute returns at both upward and downward market phases. However, the reputation of the industry was arguably bruised in during 2008 global financial crisis as hedge funds experienced severe losses and restructuring with more than 600 funds thought to have liquidated in 2009 alone, a figure more than double the then historical ten year average (Roumpis, & Syriopoulos, 2014). According to Kaiser and Haberfelner (2011) post the 2008 financial crisis, the correlations between asset classes and equity have become very high, raising questions as to the continued viability of the industry as a whole.

Tancar et al. (2012) in fact argued that the financial crisis and subsequent consolidation of hedge fund industry justified a search for more cost effective replication which had stagnated.

The authors pointed to the high positive correlation with equity and credit markets as well as an absence of alpha as convincing evidence of the existence of systematic risks inherent in the hedge fund industry.

2.6.2 Regulatory issues

Hedge funds have also begun to catch the attention of uneasy regulators. Apart from the opacity of information issues highlighted by the demise of Long Term Capital Management (van Dyk et al., 2014). The negative attention has also been focused on typical hedge fund practices such as short selling to the extent that certain types of short selling have been curtailed by regulatory authorities in various capital markets around the world (Beber & Pagano, 2013).

2.6.3 Fees and incentives

A strong combination of fixed fees for management and generous performance incentives set the fee structure for hedge funds apart from average mutual equity funds. The typical hedge fund fee structure has been highly skewed in favour of the manager with management fees comprising typically a fixed component of about two percent of assets under management and a performance related 20 percent share of profits not usually subject to watermark restrictions (Ibbotson et al., 2011). Despite some similarities in terms of the fixed component of management fees which can be on par with hedge funds, mutual funds do not generally charge share of profits fees according to Ibbotson et al. (2011) and in the few instances when they do charge, such fees would carry high-water mark restrictions - meaning that such fees would be earned only after the recovery of past losses.

2.6.4 Types of replication

Significant research work has been done in terms of modeling the returns of mainstream hedge fund strategies including Tuchschmid et al. (2011) and Kooli and Sharma (2011) whereby hedge fund strategies are typically compared to multifactor strategy clones. However there appears to be limited consensus on return constituents owing to the multiplicity of factors affecting returns such as market timing (Tudor and Cao, 2012).

Replication involves imitation of the hedge fund risk return profile using portfolio constituents that are liquid, transparent, tradeable, scaleable and cost efficient covering a range of asset classes (Tancar et al., 2012). According to Kooli and Sharma (2011), the techniques that can be used to model hedge fund strategies including rules-based strategies, factor-based approaches and the distributional pay-off approach and for replicating hedge funds are all still far from perfect. The research to date, does tentatively point towards the view that hedge funds earn risk factor returns that can be achieved more cost effectively through alternatives (Tuchschmid et al., 2011).

Tancaret et al. (2012) identified two objectives for replication, firstly the precise capture of target beta in hedging, benchmarking or as the passive element in a core satellite approach applications with strict tracking error mandates. The authors believed that top down risk factor and to a lesser extent the distributional model were best suited for such replication. Secondly replication could be used to generate reasonable hedge fund mimicking risk return profile with relatively more relaxed risk constraints, which would still exploit systematic risks and corresponding risk premia but focused more on performance and diversification benefits. Products arising from the second objective would be suitable for diversifying other portfolios or as add-ins in specialized hedge fund core satellite portfolios. Although Tancar et al. (2012) suggested a modified top down factor model as more suitable for enabling investors to forego some tracking error precision in exchange for more risk adjusted performance, it can be argued that rules based approaches are likely more efficient for such implementations.

2.7 Performance measurement and evaluation tools

2.7.1 Mean Variance Optimization

Mean variance analysis as pioneered by Harry Markowitz (1952) has contributed greatly to the understanding of how to construct optimal portfolios that make efficient use of risk on the understanding that the risk and return relationship in a portfolio is based on expected asset returns and variances as well as the correlations of returns. In the equation below the estimation of expected returns $E(R_j)$, variances σ_i^2 and covariances $Cov(R_i, R_j$ between a set of n assets as inputs enables a solution to be found for the optimal set of portfolio weights that create a minimum variance portfolio, which can trace through an efficient frontier by minimizing

$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(R_i, R_j)$ By choice of weights and subject to

$E(R_p) = \sum_{j=1}^{n} w_j E(R_j)$ = z and subject to $\sum_{j=1}^{n} w_j = 1$ z, rmin < z > rmax.

Where z lies between the minimum and maximum expected returns for the individual assets. The above can be constrained against short sales by adding $w_j \geq 0$.

Although there are some limitations to mean variance optimization in respect of factors such as instability of the efficient frontiers, particularly due to optimization input noise and serial correlations, the technique remains objective and fairly adaptable for narrowing the unlimited set of choices in selecting an asset allocation (De Fusco et al., 2011).

Charpin and Lacaze (2006) showed how mean variance optimization can be used to construct efficient market neutral and more generally long short portfolios, taking into account practical restrictions on leverage and short selling.

2.7.2 Law of Active Management

Building on Grinold and Kahn's (2000) fundamental law of active management which relates the information ratio to the product of the information coefficient and breadth, Clarke et al. (2002) attempted to operationalize the law through the introduction of a performance coefficient as product of information coefficient and the transfer coefficient. As per their generalized average expected active return equation:

$$E(R_a) = TC\ x\ IC\ \sqrt{N}\sigma_A$$

Where transfer coefficient TC measures correlation between residual returns forecasted and the active weights, with IC as the information coefficient measuring the correlation between residual returns forecasted and realised active returns. The term $\sqrt{N}\,\sigma_A$ measures aggressiveness in the management of the portfolio.

In an empirical investigation Clarke et al. (2002) found that the realised information coefficient explained only part of the performance if a portfolio was subjected to certain constraints such as restrictions on the use of leverage and short sales. The transfer coefficient was useful in measuring hitherto unexplained performance losses. Clarke et al. (2002) argued therefore that a practical diagnostic technique measuring the impacts and correlations between constraints was required to link performance to portfolio managerial skill as per equation below.

$$R_A = \left(TC\rho_{\alpha,r} + \sqrt{(1-TC^2)}\,\rho_{c,r}\right) \; x \; \sigma_A \sqrt{N}\, std(\frac{r_i}{\sigma_i})$$

$\rho_{\alpha,r}$ is the realised information coefficient

$\rho_{c,r}$ is the realised cross sectional correlation coefficient measuring the constraint related noise

$\sigma_A \sqrt{N}$ is a measure of portfolio aggressiveness.

$std(\frac{r_i}{\sigma_i})$ represents cross sectional standard deviation of risk adjusted realised residual returns or return dispersion

The proper evaluation and attribution of investment performance would undoubtedly be an important element to understanding the effectiveness of constrained hedge fund replicas in semi active portfolio management therefore the extent to which a diagnostic tool such as that developed by Clarke et al. (2002) could be applied to this research would be of interest.

2.7.3 Active Share and Tracking Error

Cremers and Petajisto (2009) contended that active managers could only beat their benchmark index either through stock selection or market timing or both. They argued that tracking error, a measure of standard deviation of active returns of a portfolio to its target benchmark, while useful as a proxy for determining market timing ability could not be relied upon to provide a clear picture of stock selection ability. According to Cremers & Petajisto, (2009) any portfolio managed against a benchmark could be broken down into a 100% long portion and an active zero net investment long and short component which would account for the overweighting and underweighting of individual stocks relative to the benchmark. The size of the active long short component relative to the 100% long component is the Active Share, essentially a proportion of the portfolio whose weightings $w_{fund,i}$ different from the benchmark index $w_{index,i}$ as per equation below:

$$Active\ Share = \frac{1}{2}\sum_{i=1}^{N}\left|w_{fund,i} - w_{index,i}\right|$$

The authors argued that active share together with tracking error could be used to provide a comprehensive picture of the level of active management in a portfolio relative to its benchmark. As tracking error was measured using the covariance matrix of returns, they pointed out, it was biased towards market risk factor based stock choices or asset allocation, whilst active share would equally weight such factor based allocation, therefore Cremers and Petajisto (2009) argued that tracking error could be used to measure factor bets whereas active share would assess the active stock picking.

What is also of interest is that Cremers and Petajisto (2009) modified the standard tracking error equation by a linear regression of excess fund returns on excess index returns:

$$R_{fund,t} - R_{f,t} = \alpha_{fund} - \beta_{fund}(R_{index,\ t} - R_{f,t}) + \varepsilon_{fund,t}$$

$$Tracking\ error = Stdev\ [\varepsilon_{fund,t}]$$

This modification they argued, would remove noise from frivolous deviations ensuring that recurrent allocation to high or low beta shares would not distort measured tracking error. Another of the key benefits of using tracking error together with active share to analyse portfolios highlighted by Cremers and Petajisto, (2009) was that factor compositions of portfolio do not have to be constructed which greatly simplifies matters in constrained equity portfolio construction and optimization analysis.

2.7.4 Industry Concentration Index

A measure related to active share that would be also useful in evaluating a constrained equity portfolio is industry concentration as developed by Kacperczyk, Sialm, and Zheng (2005) and shown in the equation below:

$$Industry\ Concentration\ Index = \sum_{i=1}^{I} (w_{fund,i} - w_{index,i})^2$$

Where $w_{fund,i}$ and $w_{index,i}$ are industry weights in the fund and in the index respectively across *I* industry portfolios. Although Cremers and Petajisto (2009) considered the industry concentration index to be a less useful hybrid of tracking error and active share because of its use of squared weights, it can be argued that it is a measure that can provide information that is of use to understanding the composition of a portfolio from the perspective of exposure to different industries as different constraints are applied.

2.8 Opportunities for Enhanced Indexation with constrained Equity Portfolios

2.8.1 Mean Variance Optimisation

Charpin and Lacaze (2006) show how mean variance optimization can be used to construct efficient market neutral and more generally long short portfolios, taking into account practical restrictions on leverage and short selling. They suggest a process involving 1) choice of investable stocks 2) selection of stocks and 3) portfolio construction in which multifactor models are used to rank the universe of investable stocks according to return expectations and then an optimization program employed to create optimised portfolios.

Charpin and Lacaze (2006) recommended a very systematic process involving no use of qualitative information in selection of stocks and regular rebalancing irrespective of performance. They noted the practices in equity market neutral funds of generally more frequent rebalancing and the use of qualitative complementary information in estimating returns and risks. In order reduce optimisation errors Chan, Karcesk, and Lakonishok (1999) suggested the use of shrinkage estimators in constructing the covariance matrix based on a single index factor model such as the Standard and Poor's (S&P) 500, rather than the historical sample covariance matrix.

2.8.2 Minimum Variance Optimisation

Minimum variance portfolios can be problematic in that they can lower risk to suboptimal levels at the expense of the returns and also that the optimisation process typically results in portfolios that do not properly exploit correlation and are highly over-weighted with low volatility stocks (Stoyanov, 2011). However despite the problems minimum variance portfolios can have useful applications in controlling risk. Clarke et al. (2011) provided an interesting analytic solution to a typically intractable problem of optimising security weights in both long only and long short using minimum variance portfolios under a single factor risk model assumption. An interesting finding of the authors was that minimum variance portfolios were comprised of stocks with beta values that had to fall below a specific threshold even if non market security correlations were factored.

The authors were able to show that the ratio of the portfolio beta β_P to the specified long only portfolio threshold beta β_L specifies the level of ex ante portfolio variance σ_{LMV}^2 to market risk $\beta_P^2\sigma_M^2$ according to the equation:

$$\frac{\beta_P^2\sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_P}{\beta_L}$$

Where β_L - although not of closed mathematical form can be determined by a ranking procedure - is specified according to the following equation: $\beta_L = \frac{\frac{1}{\sigma_M^2}+\sum_{\beta_i<\beta_L}\frac{\beta_i^2}{\sigma_{\varepsilon i}^2}}{\sum_{\beta_i<\beta_L}\frac{\beta_i}{\sigma_{\varepsilon i}^2}}$

σ_M^2 is the variance of market index returns whilst σ_i^2 and β_i are the ex-ante variances and betas for individual portfolio securities.

According to Clarke et al. (2011) β_L could also be roughly equated to the mean portfolio beta plus the cross-sectional variance in betas if it were assumed there was no cross-sectional correlation between σ_i and $\beta_i$and number of securities was large.

Clarke et al. (2011) derive the long only constrained equation for portfolio weights as follows:

$$w_i = \frac{\sigma_{LMV}^2}{\sigma_{\varepsilon i}^2}\left(1-\frac{\beta_i}{\beta_L}\right)\ for\ \beta_i < \beta_L \ \ else\ w_i = 0$$

Where σ_{LMV}^2 is ex-ante variance of the long only minimum variance portfolio, $\sigma_{\varepsilon i}^2$ is the variance of returns that is not systematic for i , β_i is the ex ante market beta for security i and β_L is the long only threshold beta.

Optimal security weights are driven at the portfolio level by σ_{LMV}^2 and β_L whilst the cross sectional variation in individual positive weights depends on beta β_i and variance $\sigma_{\varepsilon i}^2$. These relationships would be important for this research in understanding constrained equity portfolios that attempt to mimic hedge fund risk return profiles.

Clarke et al. (2011) analysed the portfolio β_P and threshold beta β_L ratio over a multi-year period, and found 80 to 90% of the risk in minimum variance portfolios with a long only constraint to be related to beta under single factor model assumptions. They also found and attributed relatively strong returns from some of the minimum variance portfolios to the low beta high return anomaly and criticism of CAPM.

Furthermore and of even more significance particularly to this research is that Clarke et al. (2011) proposed that some of the minimum variance portfolio optimization findings could be generalised to mean variance portfolio optimisation as follows:

- The variance minimisation aspects of general mean variance objective functions play a very dominant role in determining the small subset of stocks that make it into the optimal portfolio. Issues such as complexity, constraint correlations, noise, trading costs and the interplay of variables such as return expectations play a relatively very minimal role. Clarke et al. (2011) essentially suggest therefore that the optimal mean variance portfolios will be very similar to minimum variances portfolios including having small solution sets with similar candidates.

- For long-only mean-variance optimisation, the focus should be on a small set of risk estimates in covariance matrix construction because many of the constituents of the matrix and their associated influence do not make it into the solution set. It is generally the lowest total risk quintile of stocks; say *n* percent that likely end up in the solution set, meaning that the values of only n^2 percent of members of the matrix would actually impact portfolio weights. The findings discourage the development of the traditional full sample covariance matrix.

The literature review arguably supports the existence of the need and opportunity to conduct research that further increases the universe of risk management products available to investors that can effectively draw on the risk return profiles of hedge fund strategies in a more cost effective mutual fund setting with the associated lower fees, increased transparency and regulatory benefits.

3 RESEARCH QUESTIONS

To the best of the author's knowledge no definitive studies have been undertaken that specifically characterize the performance of a beta constrained initially well diversified portfolio of equity stocks under mean variance optimised conditions through forcing beta from a market identical value of one down towards market neutral zero, and studying the implications on the interactions and behavior on key risk and return metrics and variables that include active share, tracking error, the industry concentration index, the Sharpe ratio, the information ratio and the Treynor ratio in general and also particularly in an emerging market context such as South Africa. The aim of the research was therefore to ascertain if a beta constrained mean variance optimised equity portfolio may be able to offer a cost effective alternative to long short and market neutral portfolios for investors seeking controlled market exposure to systematic risk without the added risks and costs that come with hedging products. The topic area is not well researched and the literature does not appear to provide probable solutions.

Harry Markowitz's (1952) mean variance analysis and associated techniques were used to conduct mean variance optimization of a long only equity portfolio with a varying beta constraint from one down to zero. An attempt was made to derive efficient frontiers and investigate the performance of the equity long only strategy with varying beta constraints. The research drew on insights from Charpin and Lacaze (2006) and others on optimising constrained equity portfolios. Furthermore the insights from the work by Clarke et al. (2011) on minimum variance portfolios as they relate to the impact of threshold betas on portfolio composition were used where applicable in studying the mean variance case.

Accordingly this research sought to find answers to the following specific questions in respect of the optimization of beta constrained equity portfolios:

3.1 Research Question One

Active share: Does active share behave in a linear manner as portfolio beta is progressively reduced from 1 down to zero and how does it relate to ex-post portfolio returns?

3.2 Research Question Two

Tracking error: How does the tracking error respond to change in the beta constraint?

3.3 Research Question Three

Industry concentration Index: Does the industry concentration index exhibit any interesting patterns?

3.4 Research Question Four

Number of stocks: How does the number of stocks in the solution set portfolio vary with the changing target beta?

3.5 Research Question Five

Returns: What are the patterns and effects on realized average returns, Sharpe ratios, information ratios, Treynor ratios and related measures of the varying constraint beta portfolios and how do they compare with a market neutral long short strategy?

3.6 Research Question Six

Portfolio weights: Do the optimal portfolio weights of securities approximate the simplified analytic equation by Clarke et al. (2011)?

3.7 Research Question Seven

Does the beta constraint force the long only portfolio to behave similar to a minimum variance portfolio rather than a mean variance efficient portfolio?

3.7.1 Research Question Seven (a)

Systematic risk: Do any of the portfolios conform to the 80 to 90% range of portfolio risk being systematic that Clarke et al. (2011) find for minimum variance portfolios?

3.7.2 Research Question Seven (b)

Cross sectional variation of portfolio weights: Does the cross sectional variation in weights of stocks in the portfolio depend on the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$ and market beta β_i parameters as in the Clarke et al. (2011) case?

3.8 Research Question Eight

The answers to the preceding questions are envisaged to provide insights as to the interactions of variables in the generated beta constrained portfolios that can help to provide a tentative answer to the following overarching long /short hedge fund replication question:

Is there an optimal level set of low beta constrained long only equity portfolios that can generate a risk return profile to satisfy investors interested in a controlled range of exposure to exploitable broad systematic risk from unit beta down to beta neutrality without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral, but focused more on performance and diversification benefits?

4 METHODOLOGY

4.1 Choice of methodology

A quantitative, causal, experimental investigation (Saunders & Lewis, 2012) of a hedging strategy replication – market neutral with constraints - using secondary data was undertaken. Specifically, historical stock market financial data comprising mainly the prices of stocks and indices as well as broad capital market data including historical treasury bill rates was used to test the performance and an attempt was made to understand the behavior of long only constrained equity mean variance optimised portfolios over a controlled range of exposure to systematic risk defined as beta in comparison to standard market neutral from a risk return perspective. A longitudinal study perspective was taken in respect of the comparison of performance between the various alternative portfolios, in order to allow for a more judicious use of the rich historical data set that was available. Although a considerable amount of research is undertaken involving the comparative analysis of performance of different strategies it typically uses cross-sectional analysis techniques that rely on point estimates which can be problematic when the quantities being compared are of parameters that cannot be directly measured but must instead be estimated from historical data such as for instance the sharpe ratio (Ledoit & Wolf, 2008). As contended by Laurini, Monteiro and Sanvicente (2011) the dearth in robust statistical methodology, has stood in the way of appropriate performance analysis and attribution. This research was designed to make use of enabling statistical and qualitative techniques, including the longitudinal perspective to understand differences in performance particularly in dealing with portfolio returns that were not independently and identically distributed in the multivariate normal form. Given that the share price returns from hedge fund portfolios would be of a time series nature with abnormal skewness and kurtosis measures and exhibit features such as heteroscedasticity (De Miguel, Garlappi, Nogales & Uppal, 2009) an investigation of a time series nature in respect of elements of some of the research questions was considered to be appropriate.

4.2 Population

According to Saunders and Lewis (2012) the population is the complete set of group members. The relevant population would be all Johannesburg Stock Exchange listed company shares on the main board as it is the exposure to broad systematic risks as represented through securities on the stock exchange that this research seeks to better understand.

4.3 Unit of analysis

The unit of analysis would be a single share on the Johannesburg Stock Exchange (JSE) main board. As the aim of the research was to explore potentially optimal low beta alternative portfolios to typical hedge fund portfolios, the fundamental constituent of such long only alternatives would be the individual stocks or shares , listed on a stock exchange such as the JSE. Our primary motivation was to compare risk adjusted return performance of the individual shares as part of unique portfolios calibrated on the different values of market beta. Although a number of variables were being measured, analysed and compared, such as active ratio, tracking error volatility, they are all fundamental drawn from the performance of a share on a stock exchange.

4.4 Sampling method and size

The sample comprised of share price data of the top 160 companies on the JSE in existence for the past 10 years, in order to allow for a considerable interval of time for analyzing returns prior to, during and after the financial crisis of 2008. The research focused on analyzing those stocks that were part of the FTSE/JSE All Share Index (ALSI) in respect of the estimation of market betas and for purposes of portfolio construction and performance analysis, a smaller subset comprising the FTSE/JSE All Africa Top 40 shares were selected. The reduced sample size in respect of portfolios was justified by the need to deal with issues such as thin trading, illiquidity and a lack of sufficient historical data. The FTSE/JSE Top 40 shares are those that are highest ranked on the JSE in terms of market capitalization. Accordingly the level of available free float, which are those shares available for trading, tends to be relatively higher in comparison to those counters that are outside of the Top 40 cohort. As prices of stocks are affected by trading and liquidity , the sample had to consist of those stocks that would have pricing series that would be amenable to the development of expected returns, variance of returns, and covariance inputs to mean variance optimization. This a measure of convenient and judgemental sampling was employed (Saunders & Lewis, 2012). The data was manipulated in order to account for changes potentially affecting share price and return calculations such as listings and delistings, share splits and buy backs and dividend history.

Some of the shares even in the Top 40 did not have pricing history for the desired 10 year interval, and were excluded from consideration. The final subset of shares used in the Top 40 set was trimmed down to 30 and provided a nine year interval of sample data.

4.5 Data gathering process

Historical financial data was publicly available and was collected from various databases. Share price data and associated information was sourced from Google Finance, I-Net BFA database and the JSE. Information on risk free rates relating to treasury bills was sourced from the South African Reserve Bank website. The data sourced was also complemented by other public sources of information available through search engines such as Google. For instance detailed information on the index calculation methodology for the JSE/FTSE Top 40 index was sourced from the FTSE website.

4.6 Measurement instrument

The research was based on the use of mean variance optimisation techniques as formulated by the likes of Markowitz (1954) and Sharpe (1994) and more recently DeMiguel et al (2011), Charpin and Lacaze (2006) amongst many others to build efficient constrained long only equity portfolios therefore an optimisation programming package called Solver in Microsoft Excel was used to manipulate share price data, including share price expected returns, variances, covariances and beta in developing mean variance optimised portfolios and then testing the portfolios over in sample and out of sample data. The algorithms that were employed to effect the optimizations were the Generalized Reduced Gradient (GRG2) Algorithm in respect of nonlinear aspects as developed by Leon Lasdon, of the University of Texas at Austin, and Allan Waren, of Cleveland State University. Whilst the linear and integer problems were addressed using the simplex method with bounds on the variables and the branch and bound technique as implemented by John Watson and Dan Fylstra, of Frontline Systems, Inc.

Mean variance optimization with constraints including target beta, is similar to a non-convex quadratic programming problem, which introduces issues relating to the feasibility of computing a solution (Konno & Yamazaki, 1991;Lobo, Fazel & Boyd, 2007) particularly were large dense covariance matrices are involved. There are, however, several types of constraints that are convex but cannot be expressed as linear constraints (Branke, Scheckenbach, Stein, Deb, & Schmeck, 2009). One such constraint would be an upper bound on the variance of the portfolio. One of the constraints that was introduced into the optimization was an upper bound of the beta of the portfolio. The relationship of the beta to the variance of the portfolio therefore by extension arguably introduces non –convexity to the optimization search space, particularly as the value of the beta constraint is reduced towards zero.

4.7 Analysis approach

The basic approach to analysis was governed by an iterative process involving the use of mean variance optimisation tools, testing and analysis (Clarke et al., 2002; Charpin & Lacaze, 2006). The process involved the following:

4.7.1 Optimisation

The sample data extended from January 2006 to December 2014. A 60 month period from January 2006 to December 2010 was used to calculate the expected returns, construct the covariance matrix and individual security level beta. This inputs were then used to calculate estimated expected return and standard deviation. Together with an estimated risk free rate, Sharpe ratio would be calculated, which were then used as the objective function in a non-convex quadratic optimization program based on the Markowitz (1957) mean variance model.

The constraints applied to the optimisation model were, lower (0 %) and upper bounds (10 %) of portfolio weights for long only portfolios and lower (-10 %) and upper (10%) for the long short portfolio in order to minimize error maximisation, and a target beta ranging from one to zero for the long only portfolios, and zero for the long / short portfolio. The objective function for maximization was the Sharpe ratio.

The initial optimization output was used to make a portfolio allocation from January 2011 for a 24 month holding period. The holding period represented an out of sample testing period within which buy and hold portfolio returns were calculated based on the optimized weights. Monthly return data was computed.

4.7.1.1 Inputs – estimation of returns

Mean variance analysis inputs to the optimiser in respect of expected returns and covariance were developed using both historical estimates and the market model (Charpin & Lacaze, 2006). Expected returns estimates were derived from the historical price and dividend data series.

4.7.1.2 Estimation of Variance - Covariance matrices

Variance and covariance estimates were derived from monthly returns in a two stage process involving first the construction of the sample variance –covariance matrix from historical data and then the use of statistical techniques to enhance the stability and robustness of the inputs. A 60 month estimation period for variances has been used in many mean variance optimization settings (Clarke et al, 2011). Forecasts of covariances and variances of monthly excess returns (over the monthly Treasury bill rate) were generated from different models, using the prior 60 months of data for each qualifying stock in the JSE/FTSE Top 40.

A shrinkage estimation procedure was applied for constructing a simplified covariance matrix using a single index model (Chan et al., 1999). This robustification of variance covariance inputs originally draws from Stein's estimation techniques (Chopra, Hensel & Turner, 1993) enhanced by Ledoit and Wolf (2003) through which the historical sample covariance matrix is combined or shrunk with a single factor variance matrix by applying an appropriate shrinkage intensity factor. Ledoit and Wolf (2003) found that the optimal shrinkage intensity for any sample is between 0 and 1 and is particularly stable through time. Although the intensity is derivable through a loss function it is a view on estimation error in the sample covariance matrix relative to the single index model matrix. The more conservative the view , the higher the intensity, accordingly a heuristic view is taken to employ a shrinkage intensity of 0.8 in favour of the single index model, implying that the level of estimation error in the sample covariance matrix is four times higher than the level of bias in the single index model matrix. The conservative shrinkage intensity arguably contributes to a more efficient estimator of the covariance matrix which would be invertible.

The matrix dimensions of the sample under consideration for this research at P = 30 was considerable relative to the number of observations N available of 120 , thefore although the ratio of P/N was less than one , it was not negligible, and would have resulted in sample covariance matrices that would be invertible, but not numerically well conditioned, leading to a high amplification of estimation error Accordingly shrinkage to a well-conditioned estimator for large-dimensional covariance matrices was considered to be appropriate for this research. Furthermore in comparing the relative out of sample performance of the various types covariance matrix inputs including, constant correlation , principal components and shrinkage to identity matrix ,Ledoit and Wolf (2003) found the shrinkage to single index model. able to generate the lowest standard deviation. Shrinkage allows extra market covariance to be factored in a non-arbitrary manner to the sample covariance matrix which is not well structured.

Ledoit and Wolf (2003) note that the single-index covariance matrix coming is highly biased but is less error prone compared to the sample covariance matrix

4.7.1.3 Beta

The betas for the individual stocks was calculated using ordinary least squares regression (OLS). Regression analysis using time-series data on returns on the market was undertaken using an appropriate index such as the JSE All Share Index (Strugnell, Gilbert, & Kruger, 2011) and returns to each asset would be used to estimate alpha and beta which would be used to then derive expected returns and covariances. A period of 60 months ending December 2010 was used in the estimation of beta, which can be considered to be an ideal time frame (Ward & Muller, 2012; Chan et al,1999).

4.7.1.4 Initial optimisation and Rebalancing

The optimisation process was repeated whilst varying the beta constraint value, decrementing the values by 0.025 until it is reduced to zero, to create a set of 41 long only portfolios with associated risk and return characteristics for comparison with a 42nd long /short portfolio created using the same data generating process.

After 24 months, the expected returns were resampled from December 2012, to the prior 60 months. The covariance matrix was then reconstructed over the last 60 months. The OLS betas were also calculated over the 60 months from January 2008 to December 2012. The new set of inputs were used to rerun the optimization program and develop new portfolio weights that were used to rebalance the portfolio and recalculate portfolio returns and standard deviations from January 2012 for a 24 month period up to December 2014. At the December 2012 data point, the optimisation was done for the 42 portfolios (including the long/short) varying by the level of the target portfolio beta. The out of sample data for the optimized portfolios was then used to calculate the attributes that were used to compare the performance of the different beta portfolios.

A fairly long rebalancing period of 24 months was used primarily because part of the aim of the study was to develop low beta alternatives that would be stable over short to medium term holding periods, not have high turnover and the high transaction costs that are invariably associated with frequent portfolio rebalancing. As one of the major disadvantages of market neutral investing relative to other strategies is frequent rebalancing to maintain a hedged position (L'habitant, 2002; Muhtaseb, 2012) , this study sought to isolate alternatives to frequent rebalancing. Furthermore it is known that the use of stein estimation techniques for the covariance matrix construction results in fairly stable portfolios that do not change in composition from month to month therefore high rebalancing frequency is not considered to be pertinent when Stein estimation is used to compute the inputs. (Chopra, Hensel & Turner, 1993). Also considering Jagannathan and Ma (2003) who found that the no-short sales constraint was able to enhance the performance of even the sample covariance matrix, a combination , in this study of Ledoit and Wolf (2003; 2004) shrinkage together with the short constraint was intended to confer significant stability and performance to the resultant matrices.

4.7.2 Testing

A nine year timeline was used for conducting in sample portfolio construction and then the out of sample testing which time frame was considered sufficiently long enough to account for regime changes in risk, return and correlations. Upon completion of in-sample testing to create expected returns, variance and covariance matrices and betas for input into the optimization program, optimal portfolio weights were produced, with out of sample data then being used within the out-of sample testing period to determine realized risk and return metrics for each of those portfolios.

Therefore a trace of optimal target beta portfolios and their associated realized returns and risks was developed across the beta range from one down to zero. Testing metrics or variables such as active share, industry concentration index, tracking error, Sharpe and Treynor ratios were computed for each portfolio. The different beta portfolios that were optimized were subjected to performance measurement and testing on a number of dimensions. The mains categories of measurement were returns risk, activity and portfolio composition.

4.7.2.1 Return Measures

The monthly realised returns on the optimized portfolios were calculated on the holding period out of sample return data, to determine how well the optimized portfolios would have performed. Furthermore using the ALSI and the FTSE/JESE Top 40 indices as benchmarks ,excess to benchmark returns during the holding periods , were also computed. In order to facilitate further analysis the cumulative index value of each portfolio over the testing timeframe was plotted in line with Ward and Muller (2012).

4.7.2.2 Risk

Risks in respect of the generated out of sample returns was measured in the form the average standard deviation of returns or portfolio volatility. The standard deviation of excess returns relative to benchmark ALSI and FTSE/JSE Top 40 returns was measured. This second measure is known as the tracking error volatility. Although beta may be considered to be a measure of systematic risk, in the study it was used more as an independent variable that was expressed in the optimization constraints. The standard deviation of return in excess to the risk free was measured on the basis of the monthly 92 day South Africa Treasury bill rates as proxy for the risk free rate.

4.7.2.3 Risk adjusted return measures

In order to account for risk in comparing performance between the different portfolios, the Sharpe, Information ratio and Treynor ratios were calculated. The Sharpe ratio was calculated as the ratio of the returns in excess to the risk free rate to the standard deviation of excess returns. The excess return was calculated as the difference between the monthly return of the relevant portfolio and the return of the 92 day South African Treasury bill. In order to summarise the mean variance properties of the optimized target beta portfolios the Information ratio was calculated as the ratio of the average excess return to benchmark and the standard deviation of those returns. Essentially the ratio of excess returns and tracking error volatility. The Treynor ratio, which measures systematic risk adjusted return was calculated as the ratio of the excess returns to risk free divided by the portfolio beta

4.7.2.4 Portfolio composition and activity

Portfolio composition and activity was measured using a number of techniques. The extent of implied active factor bets in the portfolios was measured by the Active Share. Composition of the competing portfolios was measured first simply through the number of shares, then through the Industry Concentration Index. The benchmark used in computing the industry concentration index was the JSE ALSI and the JSE/FTSE Top 40, using the standard industry classification.

Thus upon completion of in-sample testing, and optimal portfolio weights were produced, simulations would be run on out of sample data within the testing period to determine realised risk and return metrics for each of those portfolios. Therefore a trace of optimal target beta portfolios and their associated realized returns and risks was developed across the beta range from one down to zero. Testing metrics or variables such as active share, industry concentration index, tracking error and Sharpe ratios was computed for each portfolio. Statistical and qualitative analyses including time series regression and trend analysis was undertaken in order to characterize the different target beta portfolio variants relative to each other as well as to a reference long/short market neutral portfolio. Specific tests for differences between the categorized portfolios included robust analysis of variance and Sharpe ratios incorporated Ledoit and Wolf (2008) and Ledoit and Wolf (2011) amongst other appropriate measures.

4.7.3 Analytical Tests

On the basis of the information gathered, the following categories of test were conducted:

4.7.3.1 Visual inspection of graphically represented measures

For return measures this included cumulative portfolio returns index value data over the test period (Ward & Muller, 2012). Composition and activity measures specifically active share and the industry concentration index and the number of shares were graphically plotted to facilitate comparison between the different portfolios. Visual inspection can be useful particularly in some instances as an aid to robust statistical tests, particularly in respect of time series data, displaying serial correlation of returns and heteroskedacity, therefore it was employed in this research.

4.7.3.2 Ranking

Composition and activity measures especially the active share and the industry concentration index were used to rank the different portfolios. Cremers and Petajisto (2009) contend that ranking is an appropriate analytical method for comparing portfolios on the basis of exposure to specific industry sectors relative to a benchmark. cumulative portfolio returns index values were also used to compare and rank portfolios.

4.7.3.3 Robust time series statistical tests

a. Sharpe ratio

The out-of-sample Sharpe ratios and variances of the different target beta portfolios were used to compare performance. The statistical significance of the differences between the Sharpe ratios of any two portfolios was measured using bootstrapping methods known to be suitable when portfolio returns are not independently and identically distributed normally (Ledoit & Wolf, 2008; DeMiguel et al, 2009). Since hedge fund like returns abnormal skewness and kurtosis and were of a time series nature, Ledoit and Wolf's (2008) bootstrapping methodology was employed in pairwise tests of the target beta portfolios ranked by the respective Sharpe ratios. To test the difference in Sharpe ratios between two adjacent portfolios *i* and *j*, $H_0: \frac{\mu_i}{\sigma_i} - \frac{\mu_j}{\sigma_j} = 0$ a two sided *p*- value employing the studentized circular block bootstrap in Ledoit and Wolf (2008) was used. For each test 5000 bootstrap resamples and a data dependent block size of b = 6 were used. Additional tests including block sizes of 1 and 4 were done as a check. This testing procedure was implemented in the R statistical programming language and included some of the modules of developed by Ledoit and Wolf (2008).

b. Variances

Unlike deMeguel et al (2009) for the test for variances, we do not use the non-studentized stationary bootstrap of Politis and Romano (1994) in combination with Ledoit and Wolf (2008) bootstrap method to generate *p*-values, and instead opt for the arguably more integrated robust inference method of Ledoit and Wolf (2011) involving a studentized version of the Politis and Romano (1994) stationary time series bootstrap. A major problem with non-studentized bootstraps is that they are unable to improve inference accuracy compared to tests for normal data (Ledoit & Wolf, 2011). For all ranked target beta portfolios, the pair wise test involved testing the hypothesis that the variance of the returns of any two respective portfolios $H_0: \sigma_i^2 - \sigma_n^2 = 0$. Ledoit and Wolf (2011) studentized bootstrap using M = 5000 bootstrap resamples and a data dependent block size of B = 1 was used to generate p-values to the 5 percent significance level. Different block sizes, including 4 and 6 were also tested in order to enhance the reliability of the resultant findings.

c. Family wise error correction

In order to account for familywise error rates in multiple tests of significance between different portfolio pairs, the author considered the nature of the research which required a balance between the control of Type I errors and the ability to detect effects when they are present. Therefore as in Kesselman, Miller and Holland (2011) the design and choice of pairs to test was undertaken in a manner that reduced the need to use inappropriate and overly conservative tests with limited power such as Bonferroni methods. Care was taken to avoid stringent Type I error control which could result in a loss of statistical power and consequently reducing the detection of important differences. Accordingly the error rate is set per hypothesis/test at a 5 percent level of significance. A false discovery rate correction was undertaken using the Benjamini and Yekutieli (2001) correction. Furthermore expectations were managed in a way similar to a multiple comparisons correction but less strictly tied to familywise error rates by way of informal calibration (Gelman, Hill, & Yajima, 2012). Although closed test procedures typically involve the computation of all possible combinations of a hypothesis, a limited set of pertinent combinations is generated in line with the reasoning of Wright (1992).

4.7.4 Minimum Variance comparisons

In order to understand the minimum variance versus mean variance relationship and generalizations with respect to the role of systematic risks in determining the optimised solution set, mathematical and qualitative analysis was conducted. Imputed threshold betas for the optimised portfolios were derived and compared in terms of the associated equations from Clarke et al. (2011) including :

$$\frac{\beta_P^2 \sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_P}{\beta_L}$$

The approach essentially involved testing for whether the ratio of the portfolio beta β_P to the specified long only portfolio threshold beta β_L (imputed for the solution set of targeted beta mean variance portfolios, given that threshold beta β_L is a minimum variance concept) determined the level of portfolio's variance attributable to market risk in accordance with the above equation.

To calculate $\beta_L = \frac{\frac{1}{\sigma_M^2}+\sum_{\beta_i<\beta_L}\frac{\beta_i^2}{\sigma_{\varepsilon i}^2}}{\sum_{\beta_i<\beta_L}\frac{\beta_i}{\sigma_{\varepsilon i}^2}}$ a recursive procedure was employed.

A graphical assessment was undertaken to establish whether the ratios calculated in respect of the targeted beta mean variance portfolios, were falling within the Clarke et al (2011) range of 80 to 90%. Secondly for each target beta portfolio the measure of systematic / total portfolio risk $\frac{\beta_P^2\sigma_M^2}{\sigma_{LMV}^2}$ was calculated and compared to the portfolio beta/threshold beta $\frac{\beta_P}{\beta_L}$. One of the intentions of this study was to understand the risk return characteristics of the target portfolio in relation to the extent to which the proposed Clarke et al (2011) equality would hold.

With regard cross sectional variation optimal weights, the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$ was computed on the basis of the following equation for each stock in the sample set:

Total variance (σ_{LMV}^2) = Systematic variance $(\beta_P^2\sigma_M^2)$+ Unsystematic variance $(\sigma_{\varepsilon i}^2)$.

The individual optimal weights w_i implied by Clarke et al (2011) were then computed using the equation

$$w_i = \frac{\sigma_{LMV}^2}{\sigma_{\varepsilon i}^2}\left(1-\frac{\beta_i}{\beta_L}\right)\ for\ \beta_i < \beta_L\ \ else\ w_i = 0$$

The resultant Clarke et al (2011) portfolios were then compared with the target beta optimised portfolios for similarities in respect of the number of stocks included and the weights.

Furthermore for each target beta the optimal security weights are regressed against the unsystematic variance of returns and against the individual stock market beta at the 5 percent significance level to determine the extent to which the cross sectional variation in weights of stocks in the portfolio depend on the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$ and market beta β_i parameters as in the Clarke et al. (2011) case.

4.7.5 Integrative analysis

An integrated assessment of all quantitative and qualitative data, statistical test results, visual inspection, mathematical and investment analysis through triangulation was then employed to conclusively answer the overarching research question.

4.8 Limitations

1. The study relies heavily on Harry Markowitz's mean variance analysis and associated tools include the CAPM. Inherent weaknesses of CAPM would affect the findings including:

    a. The homogeneity assumption in respect of investors views on risky assets, expected mean returns, variances and correlations. In practice views may be heterogeneous and depart from equilibrium views

    b. The assumption that investors can buy and sell in any quantity without affecting price does not hold in reality as liquidity would vary for the different shares traded on the JSE

    c. A single risk factor assumption under CAPM fails to capture various other sources of systematic risk in the manner enabled and enriched by multifactor models (De Fusco et al., 2011)

2. The low-beta high-return phenomenon (Campbell, & Vuolteenaho, 2004; Clarke et al., 2011) may not be explicitly adjusted for in the optimisation and analysis of results. The findings of this study may therefore be impacted to the extent that the low-beta high return anomalies were to be present in the samples

3. Mean variance optimisation is highly sensitive to inputs therefore the efficient frontiers that are constructed may be unstable and prone to errors originating from the inputs as well as from the over-fitting of data by the optimisation algorithms

4. Hard assumptions on the limited use of qualitative information in the formulation expected returns and their variances together with fixed rebalancing frequency for portfolios may not match practice in terms of using complementary information and rebalancing discipline which is conditioned by market information

5. This study is based on a static model of trading whereas hedge funds, especially the high frequency ones trade dynamically (Kat, 2008)

6. Standard deviation, one of the measures of risk used in this research generally significantly underestimates skewness to the left-tail risk in hedging strategies. (Liang & Park, 2010; Agarwal & Naik, 2004).

5 RESULTS

The broad aim of the research was to attempt to reduce the level of portfolio beta, in a long only mean variance optimized setting, to the lowest extent possible towards zero. Such low beta portfolio may have significant risk and cost benefits and possible alternatives for long /short portfolios. Of interest was to observe and characterise the interactions of variables in the generated beta constrained portfolios in seeking to tentatively answer the question of whether or not there exists optimal level beta constrained equity portfolio that can generate a risk return profile to satisfy investors interested in controlled range of exposure to exploitable broad systematic risk from unit beta down to beta neutrality without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral, but focused more on performance and diversification benefits. A key step in this research was to perform mean variance optimisation to produce the desired portfolio weights from which return out data could be manipulated and analysed. The research questions one through to eight make use of essentially the same sample data derived from the output of the mean variance optimisation process, therefore descriptions on sample summary statistics and data reliability and validity are not necessarily repeated in the proceeding results sections.

5.1 Data reliability and validity

According to Saunders and Lewis (2012) reliability relates to the level to which consistent findings can be produced as a result of the data collection and analysis procedures and validity is concerned with intended accurate measurement and the delivery of findings that are consistent with what they are meant achieve.

This research is by design meant to reduce observation and measurement errors that threating reliability by the use of systematic algorithm procedures in the development and processing of optimisation inputs. The optimisation procedure used itself algorithmic, repeatable and not subject to subject bias as it the rebalancing process. Observer bias (Saunders & Lewis, 2012) in the subsequent analysis of results was reduced through the use of robust statistical methodologies augmented with triangulation with other measures and analytical techniques to ensure cogent and impersonal interpretation of results. Validity was also enhanced through the use of logical and algorithmic processes and mathematics, accompanied by triangulation of results. The presence of a clear overarching reason question to which all the other questions contribute input answers enhance the internal consistency of the project.

5.2 Mean Variance Optimisation

The optimization of the portfolio was a computationally resource demanding activity. The mean variance optimization procedure undertaken was able to provide long only portfolios that were optimized up to a target portfolio beta of 0.450.

Table 1: Initial mean variance optimisation portfolio 31 Dec 2012

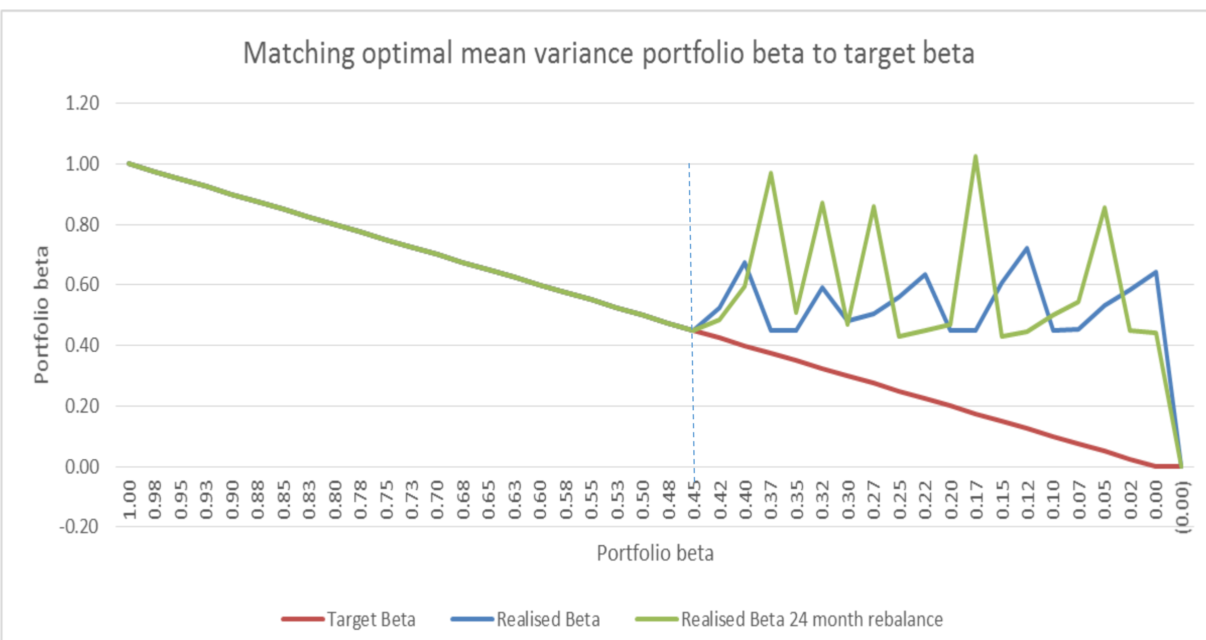
	number of counters	18	16	14	14	14	15	15	14	14	13	13	22	29	17	14	13	29	30
	target beta	1.000	0.900	0.800	0.700	0.600	0.575	0.550	0.525	0.500	0.475	0.450	0.425	0.400	0.300	0.200	0.100	0.000	(0.000)
AGL	Anglo American plc	6.1%	9.1%	6.3%	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.2%	-3.5%
AMS	Anglo American Plat Ltd	10.0%	8.1%	5.8%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	1.2%	-4.3%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.8%	9.8%	7.6%	2.4%	10.0%	10.0%	10.0%	0.1%	0.1%
APN	Aspen Pharmacare Hldgs Ltd	4.1%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	1.6%	10.0%	10.0%	10.0%	4.7%	10.0%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	7.4%	3.8%	0.2%	0.0%	0.0%	0.0%	0.0%	0.3%	1.8%	0.0%	0.0%	0.0%	1.8%	-3.1%
BVT	Bidvest Ltd	0.1%	2.2%	0.0%	0.0%	0.1%	0.1%	0.1%	0.1%	0.2%	3.1%	2.9%	10.0%	5.8%	9.2%	10.0%	10.0%	2.1%	4.7%
DSY	Discovery Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.3%	0.4%	2.0%	7.0%	10.0%	10.0%	10.0%	6.8%	10.0%	10.0%	10.0%	6.8%	10.0%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.9%	0.0%	0.0%	0.0%	3.6%	10.0%
GRT	Growthpoint Prop Ltd	0.0%	0.1%	5.8%	9.9%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	5.1%	5.2%	10.0%	10.0%	10.0%	10.0%	10.0%
IMP	Impala Platinum Hlgs Ltd	0.3%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	0.0%	-9.3%
INL	Investec Ltd	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	3.8%	0.0%	0.0%	0.0%	3.4%	-10.0%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.6%	1.3%	0.0%	0.0%	0.0%	0.8%	0.2%
IPL	Imperial Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	1.7%	0.0%	0.0%	0.0%	2.6%	0.1%
MDC	Mediclinic Internat Ltd	0.0%	0.0%	0.0%	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	8.0%	4.5%	0.1%	10.0%	10.0%	5.1%	10.0%
MPC	Mr Price Group Ltd	8.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	6.7%	2.2%	10.0%	10.0%	10.0%	9.5%	10.0%
MTN	MTN Group Ltd	10.0%	5.1%	8.1%	7.8%	3.0%	2.6%	2.8%	0.3%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	3.4%	10.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.7%	0.5%	1.1%	0.0%	0.0%	1.6%	2.9%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	7.9%	3.6%	0.0%	0.0%	0.0%	2.4%	0.0%	0.0%	0.0%	2.4%	-1.1%
NTC	Netcare Limited	7.4%	4.3%	5.2%	5.3%	3.5%	4.4%	4.7%	3.5%	2.9%	0.6%	0.0%	6.2%	4.1%	0.0%	0.0%	0.0%	1.7%	2.7%
OML	Old Mutual plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	1.0%	0.0%	0.0%	0.0%	1.8%	-5.2%
REM	Remgro Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	0.0%	0.0%	0.0%	1.5%	-2.6%
RMH	RMB Holdings Ltd	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.0%	4.0%	0.0%	0.0%	0.0%	4.2%	10.0%
SAB	SABMiller plc	5.2%	4.2%	6.7%	9.5%	10.0%	10.0%	10.0%	10.0%	10.0%	7.8%	0.0%	1.1%	10.0%	0.0%	0.0%	0.0%	3.7%	10.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%	0.0%	0.0%	3.9%	10.0%
SHF	Steinhoff Int Hldgs Ltd	8.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.0%	0.0%	0.0%	1.9%	-1.2%
SHP	Shoprite Holdings Ltd	0.5%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	3.3%	10.0%	10.0%	10.0%	8.0%	10.0%
SLM	Sanlam Limited	0.0%	1.4%	4.1%	7.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	0.3%	10.0%	9.9%	10.0%	10.0%	1.3%	3.1%
SOL	Sasol Limited	10.0%	5.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.2%	0.0%	0.0%	0.0%	2.2%	-3.4%
TBS	Tiger Brands Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	2.0%	4.7%	9.5%	10.0%	10.0%	10.0%	8.4%	10.0%	10.0%	10.0%	10.0%	2.9%	10.0%
WHL	Woolworths Holdings Ltd	7.5%	10.0%	7.9%	6.5%	6.1%	6.9%	7.2%	6.8%	6.3%	6.7%	7.3%	6.0%	3.3%	9.6%	0.0%	0.0%	6.7%	10.0%
		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Risk free rate	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%	0.7%
	Portfolio mean return μ	1.5%	1.8%	1.9%	1.9%	1.9%	1.9%	1.8%	1.8%	1.7%	1.7%	1.6%	1.4%	1.1%	1.6%	1.5%	1.5%	1.4%	2.2%
	Portfolio standard dev σ	5.9%	5.4%	4.9%	4.4%	4.0%	4.0%	3.9%	3.8%	3.7%	3.6%	3.6%	3.8%	4.3%	3.8%	3.5%	3.5%	4.2%	16.7%
	Sharpe ratio	0.14	0.21	0.24	0.28	0.30	0.30	0.29	0.29	0.28	0.27	0.25	0.20	0.10	0.24	0.24	0.24	0.16	0.09
	Portfolio beta achieved β	1.000	0.900	0.800	0.700	0.600	0.575	0.550	0.525	0.500	0.475	0.450	0.522	0.674	0.482	0.449	0.449	0.644	0.000

Table 2 : 24 month rebalanced portfolio 31 Dec 2012

	number of counters	15	14	16	13	15	15	15	13	20	21	18	21	29	14	21	20	18	30
	target beta	1.000	0.900	0.800	0.700	0.600	0.575	0.550	0.525	0.500	0.475	0.450	0.425	0.400	0.300	0.200	0.100	0.000	(0.000)
AGL	Anglo American plc	4.6%	1.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%	-10.0%
AMS	Anglo American Plat Ltd	5.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	0.0%	0.0%	-10.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	0.5%	5.7%	7.0%	10.0%	0.9%	10.0%	3.3%	10.0%	9.8%
APN	Aspen Pharmacare Hldgs Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	5.8%	1.4%	10.0%	10.0%	0.9%	10.0%	10.0%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	4.8%	2.3%	1.3%	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%	0.0%	0.0%	0.0%	0.0%	-8.7%
BVT	Bidvest Ltd	0.0%	0.0%	3.8%	0.0%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	0.1%	10.0%	0.0%	10.0%	5.1%	4.5%	0.0%	10.0%
DSY	Discovery Ltd	0.0%	0.0%	0.4%	0.9%	7.6%	8.6%	6.7%	9.2%	5.3%	7.4%	10.0%	10.0%	3.2%	9.2%	10.0%	7.3%	10.0%	10.0%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	0.0%	3.2%	10.0%	0.0%	0.0%	4.6%	0.5%	1.0%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	3.8%	4.7%	7.6%	9.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	9.6%	10.0%	10.0%
IMP	Impala Platinum Hlgs Ltd	0.0%	9.5%	4.3%	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.9%	0.0%	0.0%	0.0%	0.0%	-10.0%
INL	Investec Ltd	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	0.0%	0.0%	0.0%	0.1%	0.1%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.0%	0.0%	0.2%	0.0%	-8.1%
IPL	Imperial Holdings Ltd	2.8%	10.0%	5.2%	4.8%	1.2%	1.2%	1.2%	0.0%	0.0%	0.3%	0.0%	0.5%	5.5%	0.0%	0.0%	0.0%	0.0%	3.1%
MDC	Mediclinic Internat Ltd	0.0%	0.1%	1.1%	9.9%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	9.5%	9.6%	10.0%	10.0%	10.0%
MPC	Mr Price Group Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	0.8%	10.0%	3.8%	10.0%	7.6%	10.0%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	4.8%	0.0%	1.0%	1.9%	0.0%	10.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.0%	1.0%	3.4%	3.4%	2.9%	9.1%	3.7%	10.0%	10.0%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	8.7%	10.0%	5.2%	0.2%	0.0%	0.0%	1.9%	0.0%	0.3%	0.0%	0.0%	10.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.7%	10.0%	9.3%	0.2%	5.2%	10.0%	0.3%	10.0%	8.4%	10.0%
OML	Old Mutual plc	10.0%	5.4%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%	-9.7%
REM	Remgro Ltd	10.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.0%	0.0%	-6.2%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.7%	0.0%	4.0%	7.9%	8.4%	0.5%	1.3%	1.0%	2.9%
SAB	SABMiller plc	5.8%	3.4%	10.0%	10.0%	8.7%	8.2%	7.5%	3.3%	3.2%	2.6%	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	10.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.0%	2.4%	0.6%	3.6%	3.6%	0.5%	1.0%
SHF	Steinhoff Int Hldgs Ltd	10.0%	10.0%	9.3%	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.2%	0.0%	0.2%	0.1%	0.0%	-2.0%
SHP	Shoprite Holdings Ltd	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	2.7%	3.3%	10.0%	10.0%	4.4%	10.0%	10.0%
SLM	Sanlam Limited	0.0%	0.1%	1.1%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	3.2%	3.8%	10.0%	4.2%	0.0%	1.8%	10.0%	1.1%	9.9%
SOL	Sasol Limited	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.0%	0.0%	0.0%	0.0%	4.1%
TBS	Tiger Brands Ltd	0.0%	0.0%	1.8%	0.0%	3.8%	4.7%	6.8%	8.3%	10.0%	10.0%	10.0%	10.0%	6.1%	8.4%	8.9%	10.0%	10.0%	10.0%
WHL	Woolworths Holdings Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	4.7%	10.0%	0.0%	1.1%	0.0%	5.1%	4.5%	1.0%	2.9%
		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
	Risk free rate	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%	0.60%
	Portfolio mean return μ	1.39%	1.78%	1.92%	2.08%	2.15%	2.17%	2.17%	2.20%	2.14%	1.96%	2.01%	1.64%	1.21%	1.84%	1.68%	1.70%	1.69%	2.67%
	Portfolio standard dev σ	5.88%	5.39%	4.84%	4.37%	3.93%	3.85%	3.76%	3.70%	3.58%	3.47%	3.42%	3.35%	3.83%	3.46%	3.38%	3.41%	3.33%	3.77%
	Sharpe ratio	0.13	0.22	0.27	0.34	0.40	0.41	0.42	0.43	0.43	0.39	0.41	0.31	0.16	0.36	0.32	0.32	0.33	0.55
	Portfolio beta achieved β	1.000	0.900	0.800	0.700	0.600	0.575	0.550	0.525	0.500	0.475	0.450	0.485	0.597	0.468	0.469	0.499	0.441	0.000

Below a level of 0.45, the optimizer was unable to find feasible solutions in the long only form, the chart below indicates the extent of the optimization.

Figure 1: Extent to which long only beta portfolios optimisation was possible with decreasing target beta



A period of 60 months of data was used to construct the variance –covariance matrix and return estimates used to produce the optimized portfolio estimates. 60 months data is considered to be a reasonable period of time (Ward & Muller, 2012; Chan et al, 1999). Furthermore, shrinkage estimation was applied on the historical matrices to increase the stability and reliability of the optimization inputs (Chopra, Hensel & Turner, 1993)

5.2.1 Summary statistics for optimised portfolios returns.

On the basis of the optimisation and rebalancing, the summary statistics for the continuous total return series for the total portfolio set of 41 long only ,and 1 long short market neutral constructed for the period January 2011 to December 2014 is indicated below. The returns were monthly and not annualized as the comparison are within a common group of portfolios with differing beta constraints and the long / short.

Table 3: Summary statistics of share return series for optimised portfolio. This data spans 48 months beginning 1 January 2010 to 31 December 2014

Portfolio by target beta		n	mean	sd	median	trimmed mean	median abs dev	min	max	range	skew	kurtosis	standard error	AR(1)	AR(N)	sharpe ratio
beta	1.000	48	0.0153	0.0344	0.0180	0.0152	0.0366	-0.0552	0.0824	0.1377	-0.0470	-0.6094	0.0050	0.0262	0.2008	0.4434
beta	0.975	48	0.0138	0.0343	0.0170	0.0145	0.0344	-0.0593	0.0845	0.1439	-0.1986	-0.5571	0.0050	0.0584	0.3794	0.4024
beta	0.950	48	0.0142	0.0342	0.0185	0.0151	0.0370	-0.0638	0.0820	0.1458	-0.2886	-0.5381	0.0049	0.0644	0.3911	0.4145
beta	0.925	48	0.0141	0.0351	0.0193	0.0149	0.0401	-0.0679	0.0842	0.1521	-0.2622	-0.6012	0.0051	0.0830	0.4572	0.4009
beta	0.900	48	0.0142	0.0351	0.0200	0.0152	0.0371	-0.0712	0.0843	0.1555	-0.3098	-0.5513	0.0051	0.1055	0.5305	0.4054
beta	0.875	48	0.0153	0.0350	0.0196	0.0164	0.0377	-0.0723	0.0845	0.1568	-0.3875	-0.4035	0.0050	0.0803	0.4680	0.4367
beta	0.850	48	0.0157	0.0348	0.0189	0.0169	0.0365	-0.0747	0.0829	0.1575	-0.4050	-0.3214	0.0050	0.0692	0.4187	0.4510
beta	0.825	48	0.0161	0.0342	0.0189	0.0173	0.0356	-0.0753	0.0801	0.1554	-0.4145	-0.2489	0.0049	0.0722	0.4235	0.4703
beta	0.800	48	0.0172	0.0345	0.0199	0.0185	0.0371	-0.0742	0.0830	0.1572	-0.4493	-0.2071	0.0050	0.0515	0.3452	0.4994
beta	0.775	48	0.0179	0.0345	0.0194	0.0192	0.0353	-0.0753	0.0830	0.1583	-0.4601	-0.1060	0.0050	0.0533	0.3554	0.5204
beta	0.750	48	0.0182	0.0346	0.0232	0.0197	0.0366	-0.0805	0.0831	0.1635	-0.5425	0.0892	0.0050	0.0562	0.3652	0.5267
beta	0.725	48	0.0190	0.0348	0.0249	0.0206	0.0385	-0.0823	0.0832	0.1655	-0.5733	0.2172	0.0050	0.0532	0.3499	0.5480
beta	0.700	48	0.0197	0.0351	0.0266	0.0215	0.0395	-0.0839	0.0833	0.1672	-0.6071	0.2268	0.0051	0.0491	0.3442	0.5598
beta	0.675	48	0.0200	0.0351	0.0242	0.0219	0.0378	-0.0841	0.0849	0.1690	-0.6232	0.2977	0.0051	0.0619	0.4023	0.5710
beta	0.650	48	0.0208	0.0352	0.0255	0.0226	0.0384	-0.0868	0.0850	0.1718	-0.6577	0.4018	0.0051	0.0685	0.4270	0.5922
beta	0.625	48	0.0211	0.0351	0.0261	0.0229	0.0374	-0.0878	0.0832	0.1710	-0.6824	0.4584	0.0051	0.0867	0.4991	0.6017
beta	0.600	48	0.0220	0.0349	0.0279	0.0240	0.0345	-0.0926	0.0800	0.1726	-0.8227	0.8888	0.0050	0.1028	0.5460	0.6291
beta	0.575	48	0.0225	0.0351	0.0287	0.0248	0.0324	-0.0948	0.0775	0.1723	-0.9139	1.1712	0.0051	0.1500	0.6232	0.6423
beta	0.550	48	0.0225	0.0353	0.0293	0.0249	0.0308	-0.0956	0.0795	0.1750	-0.9479	1.2555	0.0051	0.2487	0.6861	0.6358
beta	0.525	48	0.0230	0.0359	0.0304	0.0256	0.0323	-0.0959	0.0838	0.1797	-0.9536	1.1585	0.0052	0.2460	0.6510	0.6401
beta	0.500	48	0.0224	0.0364	0.0296	0.0252	0.0312	-0.1006	0.0812	0.1818	-1.0006	1.3145	0.0053	0.4302	0.6714	0.6147
beta	0.475	48	0.0223	0.0359	0.0310	0.0253	0.0319	-0.1052	0.0782	0.1834	-1.1556	1.9179	0.0052	0.7245	0.6714	0.6207
beta	0.450	48	0.0193	0.0347	0.0289	0.0226	0.0352	-0.0904	0.0717	0.1622	-0.9921	0.9290	0.0050	0.5439	0.7599	0.5553
beta	0.425	48	0.0184	0.0304	0.0236	0.0204	0.0249	-0.0700	0.0695	0.1395	-0.7991	0.7592	0.0044	0.3620	0.7483	0.6051
beta	0.400	48	0.0155	0.0304	0.0203	0.0171	0.0279	-0.0547	0.0717	0.1264	-0.4843	-0.4780	0.0044	0.4814	0.7947	0.5110
beta	0.375	48	0.0206	0.0355	0.0308	0.0240	0.0303	-0.0955	0.0763	0.1718	-1.0643	1.2141	0.0051	0.0623	0.3052	0.5797
beta	0.350	48	0.0215	0.0352	0.0277	0.0246	0.0312	-0.1098	0.0772	0.1870	-1.2706	2.5664	0.0051	0.3085	0.7223	0.6097
beta	0.325	48	0.0201	0.0362	0.0274	0.0223	0.0337	-0.1023	0.0766	0.1789	-0.8542	1.1427	0.0052	0.0798	0.4309	0.5540
beta	0.300	48	0.0208	0.0343	0.0288	0.0239	0.0273	-0.0991	0.0742	0.1733	-1.2101	2.1105	0.0050	0.5006	0.7490	0.6054
beta	0.275	48	0.0202	0.0343	0.0261	0.0224	0.0340	-0.0830	0.0771	0.1602	-0.6716	0.3227	0.0050	0.1008	0.3264	0.5888
beta	0.250	48	0.0180	0.0317	0.0252	0.0205	0.0355	-0.0654	0.0632	0.1286	-0.6660	-0.2134	0.0046	0.6993	0.6385	0.5672
beta	0.225	48	0.0167	0.0321	0.0227	0.0188	0.0339	-0.0663	0.0644	0.1307	-0.5855	-0.2591	0.0046	0.6136	0.6508	0.5200
beta	0.200	48	0.0164	0.0319	0.0215	0.0181	0.0276	-0.0613	0.0701	0.1314	-0.5616	-0.3018	0.0046	0.6089	0.7629	0.5130
beta	0.175	48	0.0227	0.0392	0.0349	0.0259	0.0421	-0.0940	0.0878	0.1818	-0.8236	0.4395	0.0057	0.0792	0.2996	0.5792
beta	0.150	48	0.0150	0.0295	0.0200	0.0168	0.0311	-0.0618	0.0608	0.1226	-0.5434	-0.3852	0.0043	0.6621	0.7665	0.5087
beta	0.125	48	0.0163	0.0315	0.0219	0.0180	0.0306	-0.0681	0.0719	0.1400	-0.5568	-0.2431	0.0045	0.6894	0.7433	0.5163
beta	0.100	48	0.0190	0.0331	0.0247	0.0215	0.0278	-0.0952	0.0770	0.1723	-1.0287	1.5908	0.0048	0.3865	0.7707	0.5756
beta	0.075	48	0.0177	0.0322	0.0232	0.0198	0.0266	-0.0676	0.0685	0.1361	-0.7123	0.1286	0.0047	0.4014	0.7406	0.5484
beta	0.050	48	0.0179	0.0354	0.0207	0.0196	0.0344	-0.0782	0.0821	0.1602	-0.5158	-0.0492	0.0051	0.2786	0.4314	0.5045
beta	0.025	48	0.0172	0.0314	0.0257	0.0193	0.0350	-0.0647	0.0697	0.1344	-0.5988	-0.0641	0.0045	0.5978	0.7322	0.5477
beta	0.000	48	0.0172	0.0322	0.0262	0.0192	0.0337	-0.0608	0.0672	0.1280	-0.5605	-0.4263	0.0046	0.5628	0.7029	0.5337
L/S beta	0.000	48	0.0285	0.0424	0.0379	0.0328	0.0320	-0.1114	0.1017	0.2130	-1.1290	1.2666	0.0061	0.9866	0.8212	0.6724

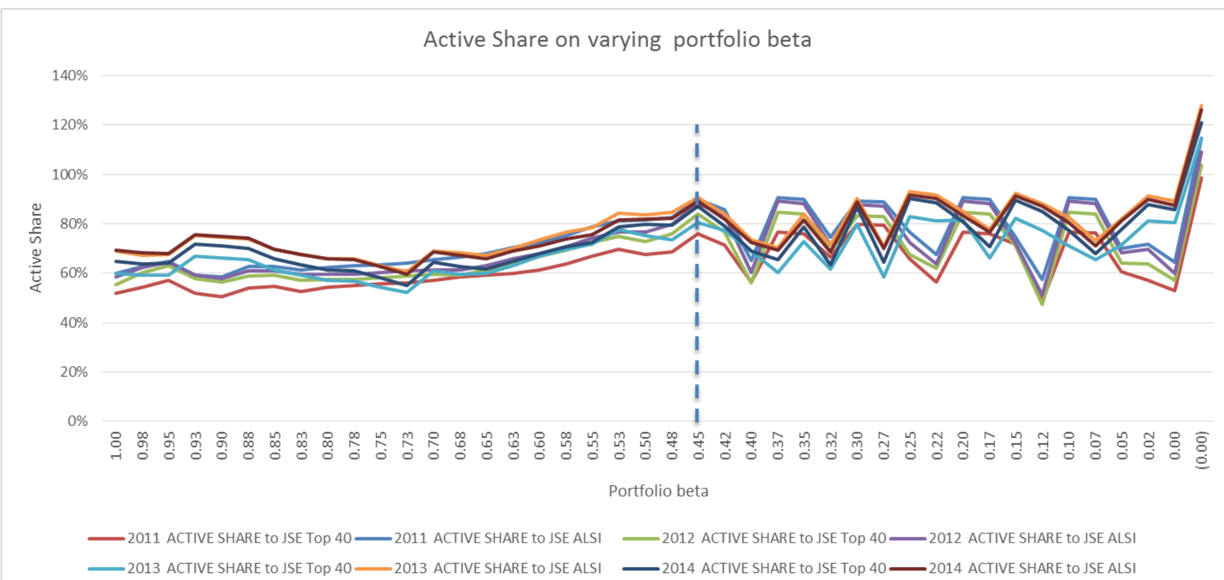
From the statistical summary, non-normal behaviour is exhibited by the transformed return series particularly in respect of the skewness and kurtosis. The distributions for the different series are not symmetrical around the respective means as would be the case in normal distributions. Kurtosis is less than 3 in all 42 cases, indicating platykurtic distributions with a wider spread around the mean. We would expect a kurtosis value of 3 for a normal distribution..

5.3 Research Question One – Active share

Active share: Does active share behave in a linear manner as portfolio beta is progressively reduced from 1 down to zero and how does it relate to ex-post portfolio returns?

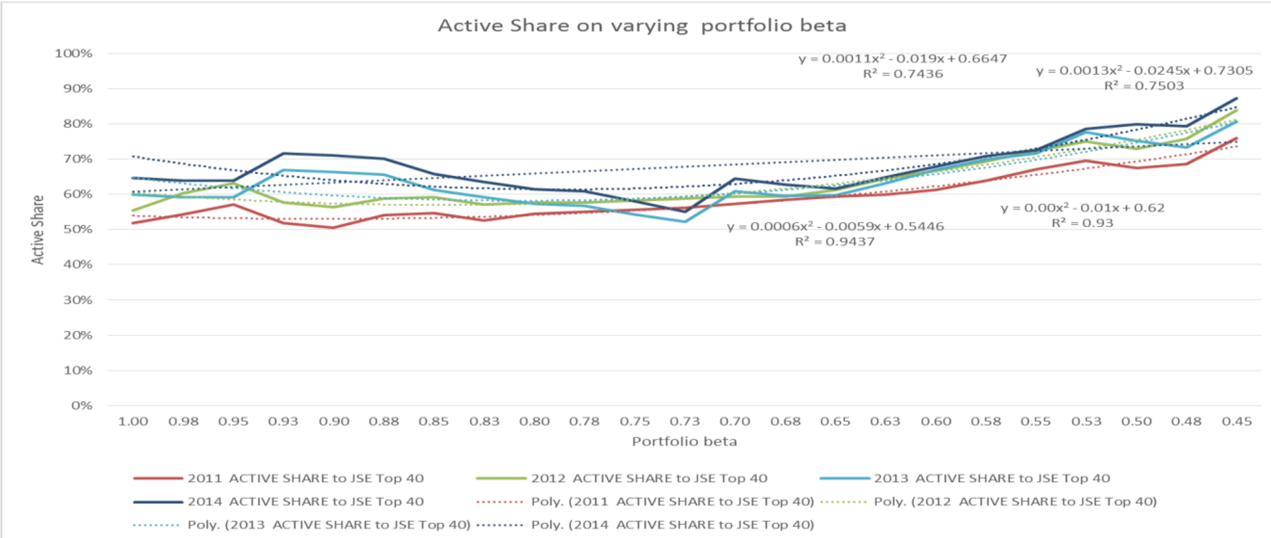
The mean variance optimization procedure undertaken was able to provide portfolios that were optimized up to a target portfolio beta of 0.45 optimized portfolio returns. Active share was calculated on the basis of such returns against the ALSI and the Top 40 indeces. The level of active share in the optimizable region of 1 to 0.45 shows a steady increase , but is mostly similar for either benchmark used.

Figure 2: Active share on JSE Top 40 and JSE ALSI for the optimised target beta portfolios over the 2011 to 2014 testing period



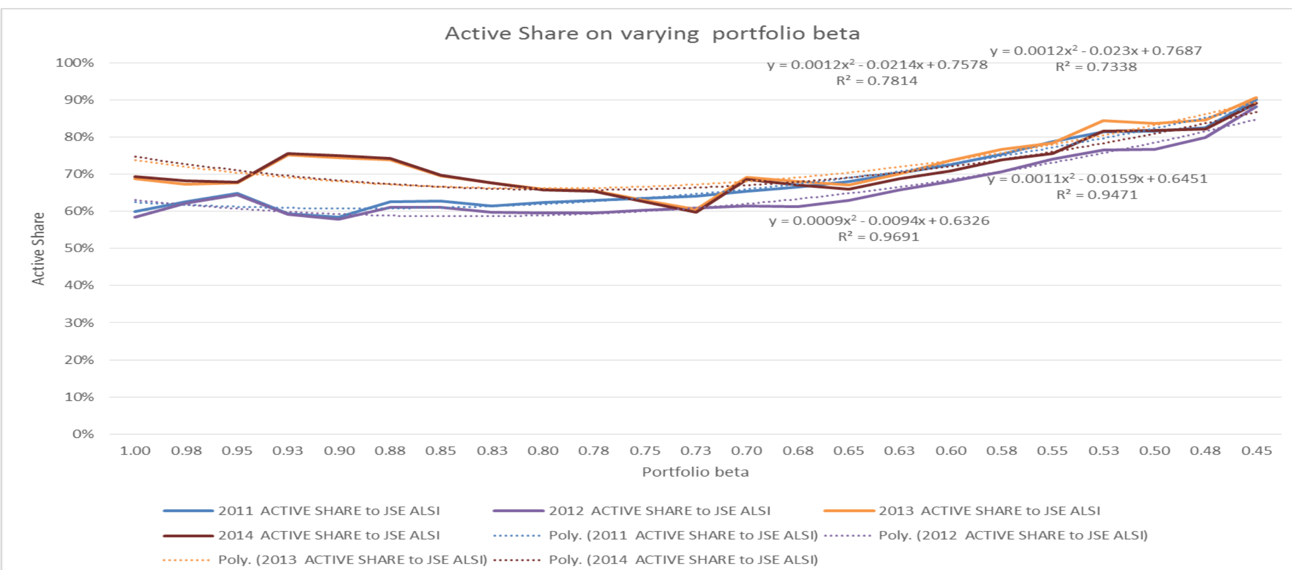
Overall active share ranges from a minimum of 51 % to a maximum of 91 % over the 1 to 0.45 beta range. The trendline indicate a cross sectional relationship between the active share that is is characterized as second order polynomial/ quadratic rather than first order linear as shown in the graph below of the Top 40 benchmarked portfolios.

Figure 3: Active share JSE Top 40 cross-sectional variation to portfolio beta



The same relationship is indicated for the ALSI benchmarked portfolios as well.

Figure 4: Active share JSE ALSI cross-sectional variation to portfolio beta



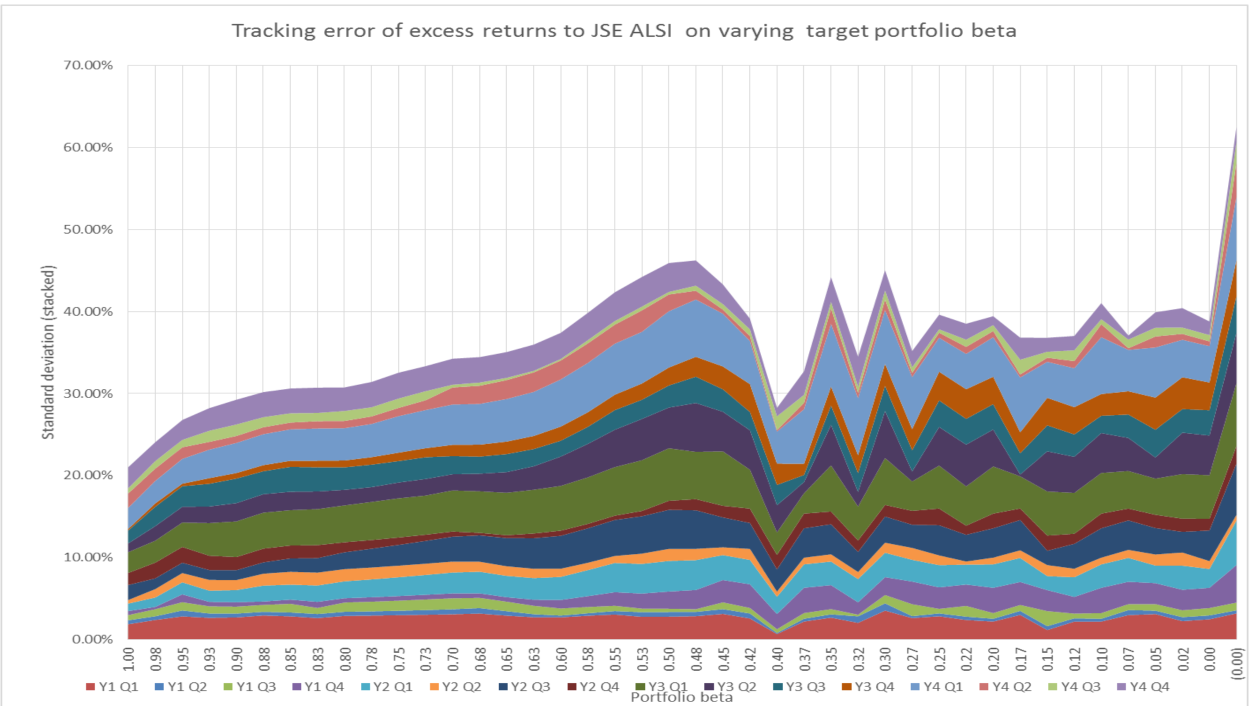
The active share values are calculated on first optimisation for years 2011 and 2012, then second rebalancing for years 2013 and 2014. The relationship persists through the rebalancing. A period of 48 months is used to calculate the active share figures. The reliability (Saunders & Lewis, 2012) is therefore seen through the rebalancing. Stability is also been conferred through the use of shrinkage estimation to produce the optimisation variance –covariance matrices.

5.4 Research Question Two – Tracking error

Tracking error: How does the tracking error respond to change in the beta constraint?

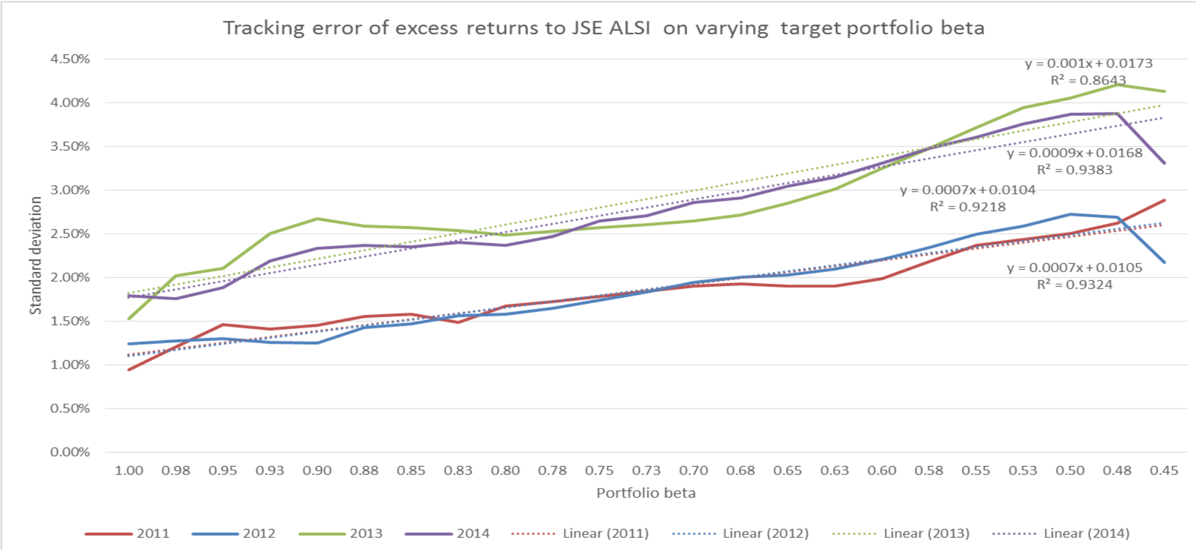
The standard deviations of the returns series of the optimized portfolios were computed against the JSE ALSI and the JSE/FTSE Top 40 benchmarks. The results are indicated in the tables below

Figure 5: Tracking error to JSE ALSI for 2011 to 2014 calculated quarterly and presented in a stacked form to show the time based profile for each target beta portfolio, including the long/short equity neutral indicated by (0.000) on the x axis



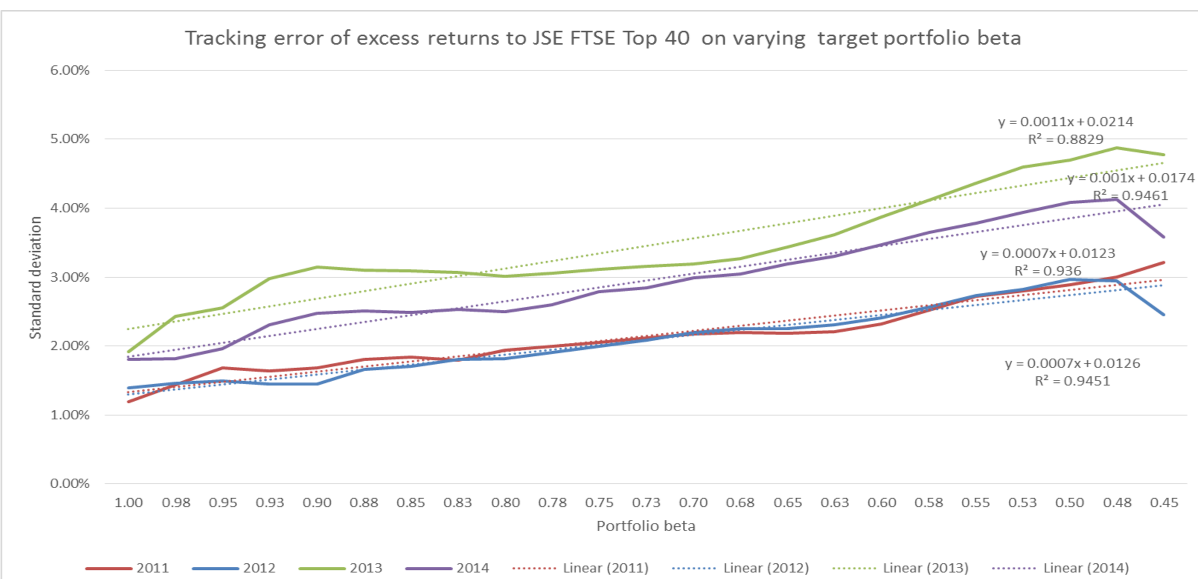
In the case of the JSE ALSI benchmark tracking error for optimized portfolios increases linearly within a range of 0.95 % to 4.25 % as target portfolio beta is reduced from 1 down to 0.45.

Figure 6: Tracking error to JSE ALSI calculated annually and related to the different target beta portfolios up to the minimum beta threshold of 0.45



The graphs indicate a highly linear cross sectional relationship between tracking error and the targeted portfolio beta. Thus in respect of the JSE Top 40 benchmark, the tracking error increases linearly as the beta is reduced, staying within a 1 to 5% range.

Figure 7: Tracking error to JSE Top 40 calculated annually and related to the different target beta portfolios up to the minimum beta threshold of 0.45



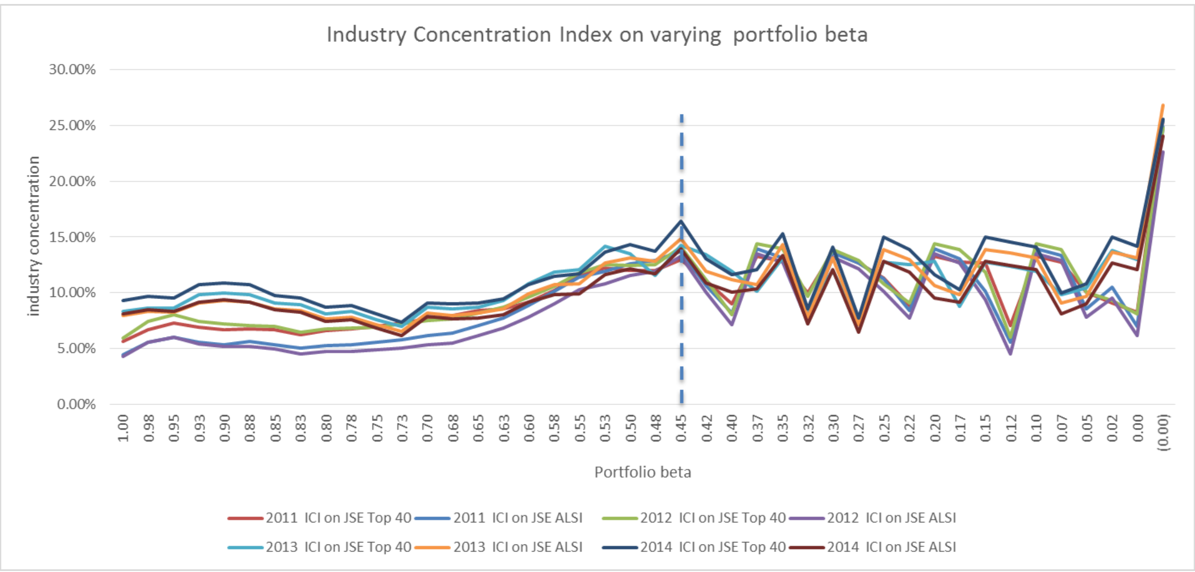
Under the out of sample calculations, the high R^2 values are exhibited for all years Y1 -2011 (0.945), Y2- 2012 (0.936), Y3-2013 (0.946) and Y4-2014 (0.883).

5.5 Research Question Three – Industry concentration

Industry concentration Index: Does the industry concentration index exhibit any interesting patterns?

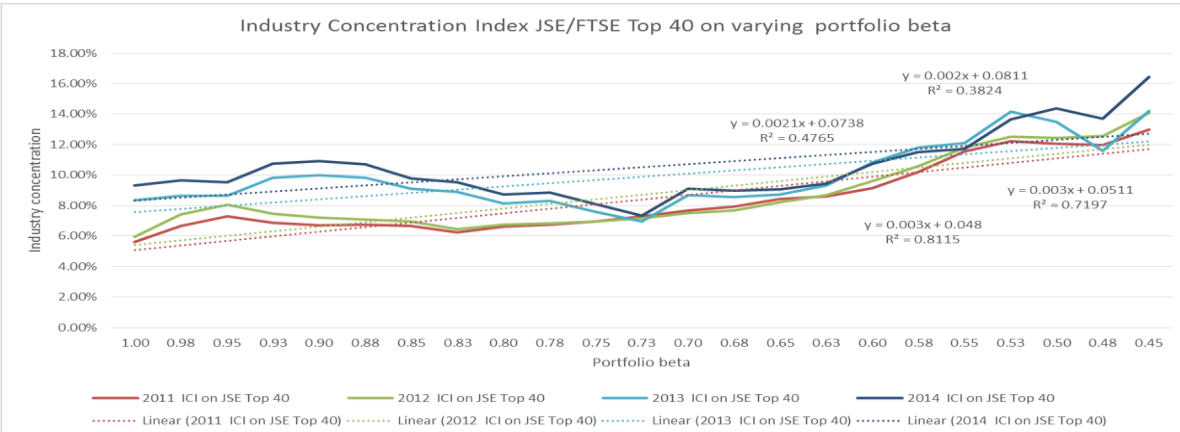
Industry concentration was calculated on the basis of the two benchmarks JSE ALSI and the JSE/FTSE Top 40 indices. Overall the concentration increases with decreasing beta, for either benchmark as indicated in the graph below.

Figure 8: Industry Concentration Index on JSE Top 40 and JSE ALSI for the optimised target beta portfolios over the 2011 to 2014 testing



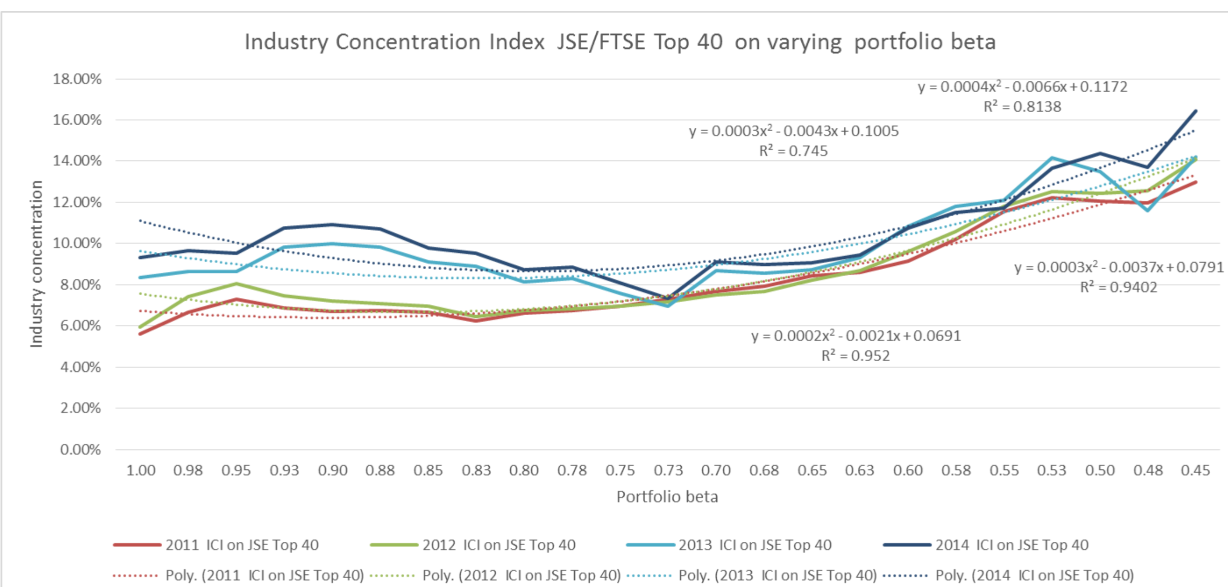
A linear relationship appears to hold initially as per graph below for the JSE/FTSE Top 40

Figure 9: Industry Concentration Index JSE Top 40 cross-sectional variation to portfolio beta first order polynomial relationship



However on closer inspection a second order polynomial relationship better characterizes the relationship of the industry concentration to the portfolio beta as indicated in the same graph below with polynomial trend lines.

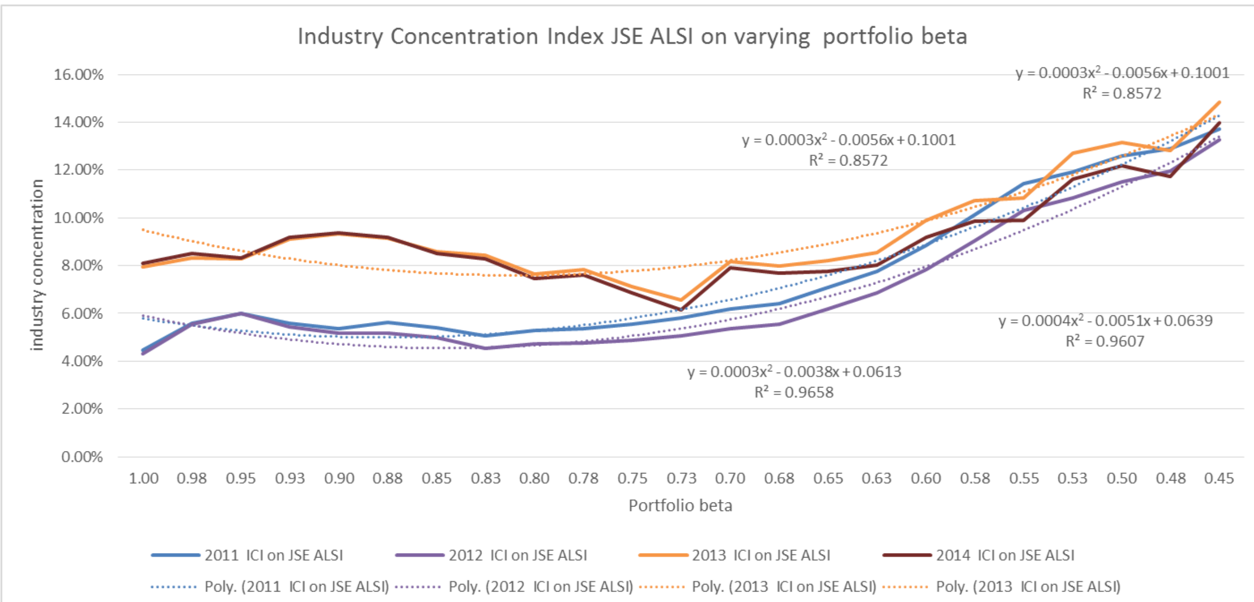
Figure 10: Industry Concentration Index JSE Top 40 cross-sectional variation to portfolio beta second order polynomial relationship



The relationship is denoted by very high R^2 values exhibited for all the years considered: Y1-2011 (0.952), Y2- 2012 (0.940), Y3-2013 (0.745) and Y4-2014 (0.814).

A similar relationship is obtained in respect of the JSE ALSI calculations for the optimized portfolios where the curvilinear fluctuations are evident. The R^2 values are above 0.85 for all the four years indicating upwardly trending but fluctuating industry concentration as portfolio beta is decremented.

Figure 11: Industry Concentration Index JSE ALSI cross-sectional variation to portfolio beta second order polynomial relationship

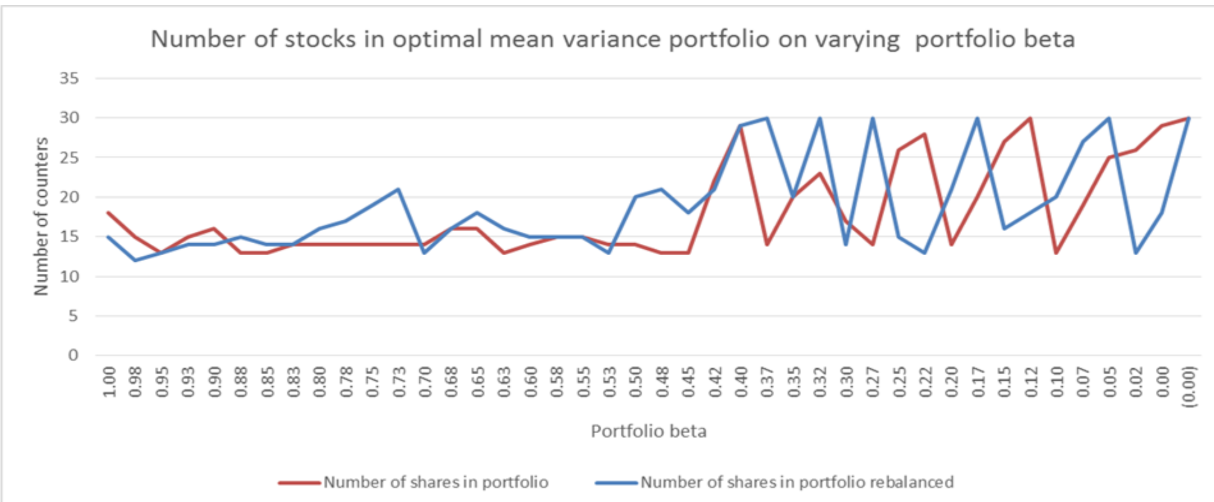


5.6 Research Question Four – Size

Number of stocks: How does the number of stocks in the solution set portfolio vary with the changing target beta?

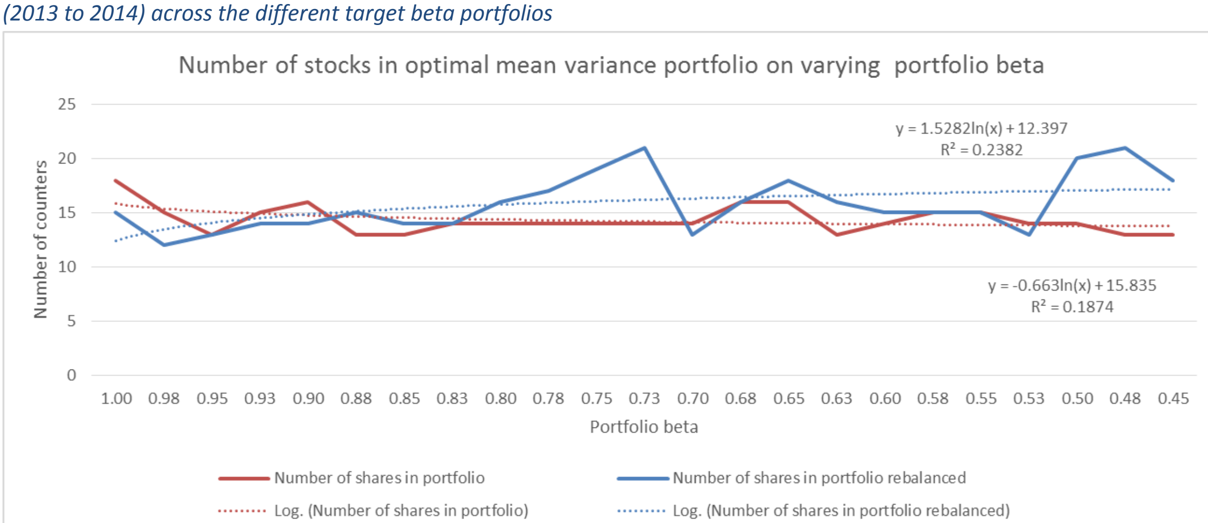
As the target beta is reduced , the number of stocks in the solution set is seen to decrease.

Figure 12: Number of stocks in optimised initial portfolio (period 2011 to 2012) and rebalanced portfolio (2013 to 2014) across the different target beta portfolios



The relationships is weakly log linear as indicated in the chart below.

Figure 13: Cross sectional variation in number of stocks in optimised initial portfolio (period 2011 to 2012) and rebalanced portfolio (2013 to 2014) across the different target beta portfolios



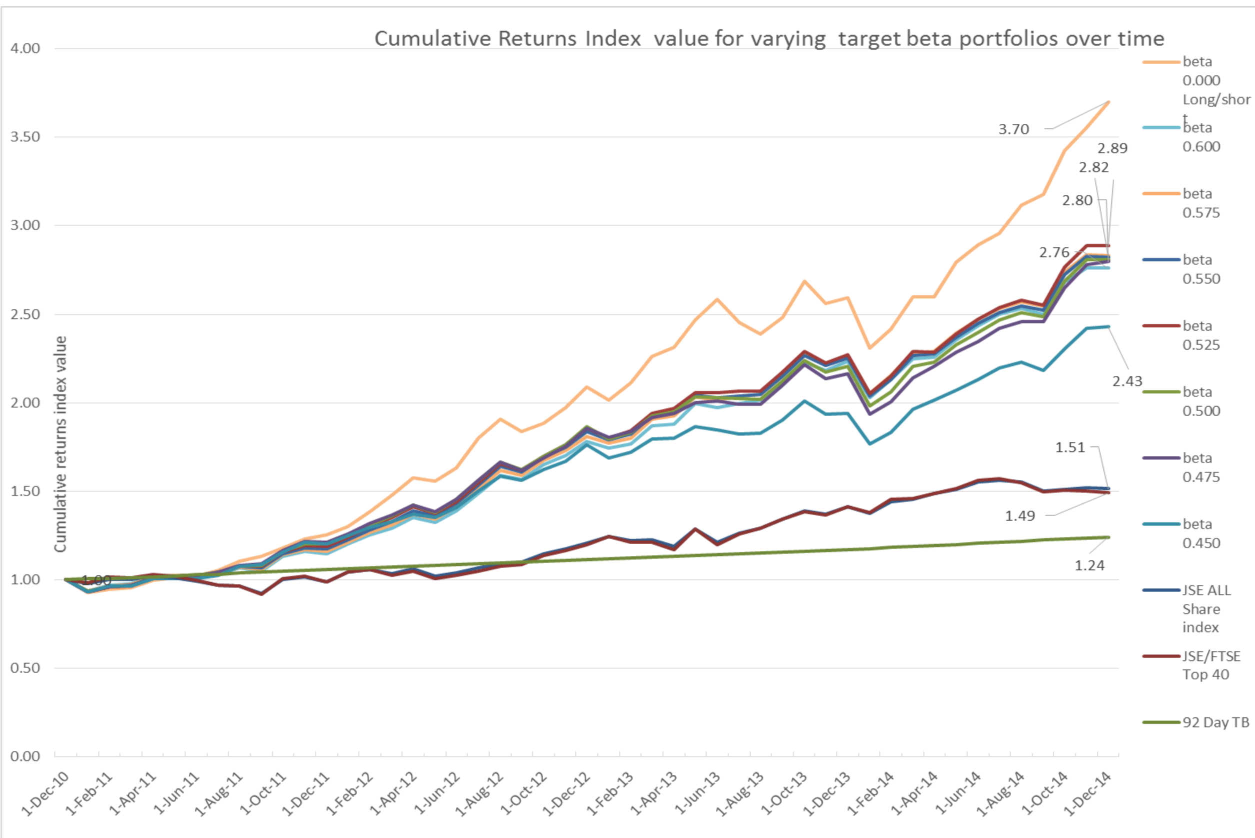
5.7 Research Question Five - Returns

Returns: What are the patterns and effects on realized average returns, Sharpe ratios, and the treynor ratios measures of the varying constraint beta portfolios and how do they compare with a market neutral long short strategy?

5.7.1 Cumulative index returns value

Results of the cumulative index value of returns for the portfolios plus the relevant rankings are presented. The cumulative index value, based on monthly returns, is calculated in a similar manner to that of Ward and Muller (2012) but was based on monthly returns as opposed to daily turns.

Figure 14: Cumulative returns index value time series plot for selected target beta portfolios for 2011 to 2014, showing growth in index values from a 31 Dec 2010 base value of 1.00



The cumulative returns indicate that certain portfolios have a clear lead in creating value. The table below provides the rankings highlighting the top ten by value. The long short portfolio is clearly leading however there is a cluster of relatively high value portfolios in the 0.475 to 0.600 beta range.

Table 4: Cumulative returns index values for the full target beta portfolio set showing rankings by value

Target Portfolio Beta	Cum Return Index Apr 2015	Ranking	Cum Return Index Dec 2014	Ranking	Cum Return Index Dec 2013	Ranking	CR Index Dec 2012	Ranking
1	2.22169	36	2.01415	36	1.73860	37	1.32554	42
0.975	2.04159	42	1.88075	42	1.72886	39	1.38376	41
0.95	2.07716	39	1.91199	40	1.75539	35	1.40637	40
0.925	2.07119	41	1.90184	41	1.75220	36	1.44554	39
0.9	2.07562	40	1.91513	39	1.76728	34	1.47187	38
0.875	2.19560	37	2.01221	37	1.82714	29	1.50100	36
0.85	2.23196	35	2.05252	34	1.85320	28	1.50458	34
0.825	2.29163	32	2.09515	33	1.86272	27	1.49553	37
0.8	2.40116	27	2.20715	29	1.93257	22	1.54922	33
0.775	2.49716	25	2.28456	24	1.97667	20	1.57166	32
0.75	2.52269	23	2.31623	22	1.99055	19	1.59527	31
0.725	2.61863	19	2.40601	20	2.03367	17	1.61955	29
0.7	2.73760	17	2.47578	17	2.08582	14	1.64447	27
0.675	2.76022	15	2.51919	15	2.10855	12	1.68193	21
0.65	2.85705	12	2.61572	11	2.15576	10	1.71871	13
0.625	2.87322	11	2.64832	10	2.17316	8	1.74900	9
0.6	3.01015	6	2.75942	8	2.23371	6	1.78011	7
0.575	3.07932	4	2.83341	3	2.26664	3	1.81061	6
0.55	3.03311	5	2.82274	5	2.25268	5	1.83705	5
0.525	3.10273	3	2.88873	2	2.27151	2	1.85483	4
0.5	2.97927	7	2.80875	6	2.20540	7	1.86496	2
0.475	2.94740	8	2.79772	7	2.16257	9	1.85779	3
0.45	2.59317	20	2.43010	18	1.94178	21	1.76490	8
0.425	2.54193	21	2.35136	21	1.87322	26	1.68280	20
0.4	2.28179	34	2.04927	35	1.71255	41	1.59590	30
0.375	2.89856	9	2.58499	13	2.10975	11	1.68393	17
0.35	2.89809	10	2.69208	9	2.06048	16	1.71851	14
0.325	2.78978	14	2.51641	16	2.06640	15	1.64975	26
0.3	2.75466	16	2.61114	12	2.02393	18	1.72246	12
0.275	2.83432	13	2.54383	14	2.08779	13	1.70490	15
0.25	2.46445	26	2.30067	23	1.81893	30	1.69946	16
0.225	2.31331	31	2.16024	30	1.73457	38	1.65666	25
0.2	2.28688	33	2.12861	31	1.77717	32	1.68393	19
0.175	3.32986	2	2.83290	4	2.25850	4	1.74575	10
0.15	2.12859	38	2.00231	38	1.60087	42	1.50341	35
0.125	2.31696	30	2.11980	32	1.72375	40	1.65898	24
0.1	2.62892	18	2.41013	19	1.90010	25	1.68393	18
0.075	2.50513	24	2.26410	26	1.90442	24	1.74486	11
0.05	2.52495	22	2.27358	25	1.91016	23	1.62767	28
0.025	2.35012	29	2.21710	27	1.76978	33	1.67209	23
0.000	2.35462	28	2.21007	28	1.77833	31	1.67494	22
0.000	4.09927	1	3.70155	1	2.59580	1	2.09049	1

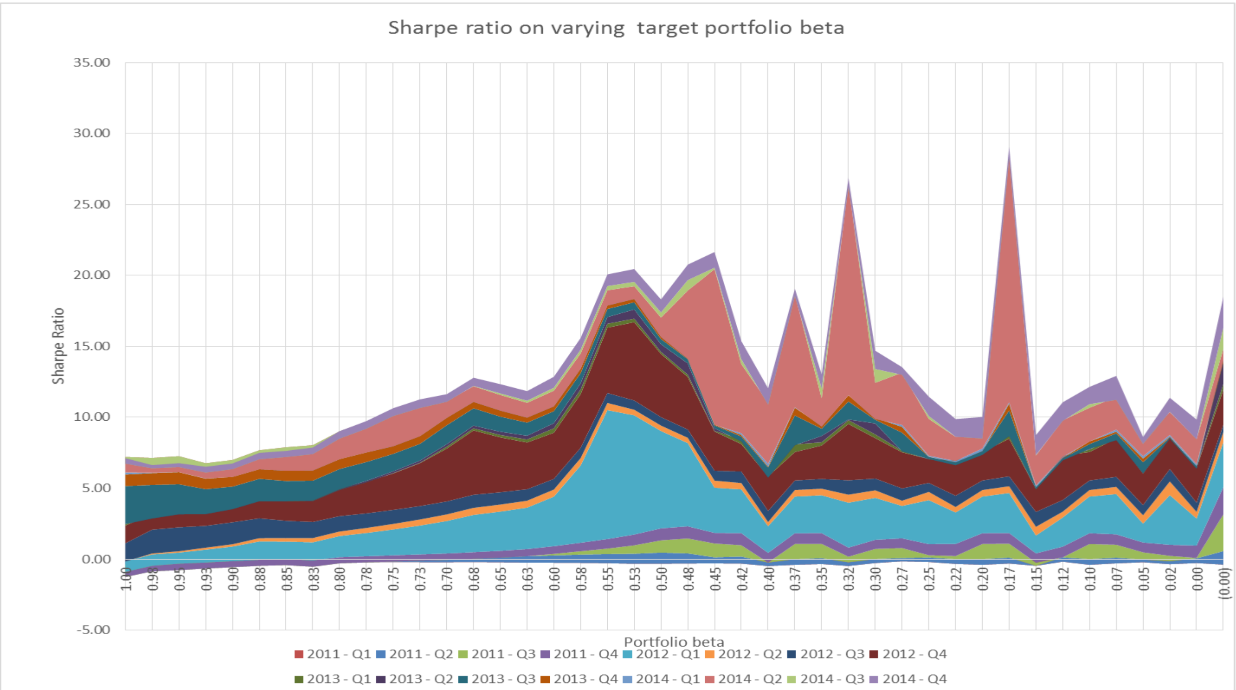
5.7.1.1 Reliability and validity of the data

The index is created on the basis of actual historical return data. The rankings from the cumulative returns index are generally consistent with the ex-ante sharpe ratio calculation indicated in the summary statistics. The portfolio with the highest sharpe ratio, the long/short beta 0 , is ranked first. Portfolios such as those with betas in the 1 to 0.8 range for instance reflected relatively lower cumulative return values as shown in the chart above. The index is created on the basis of actual historical return data.

5.7.2 Sharpe ratios

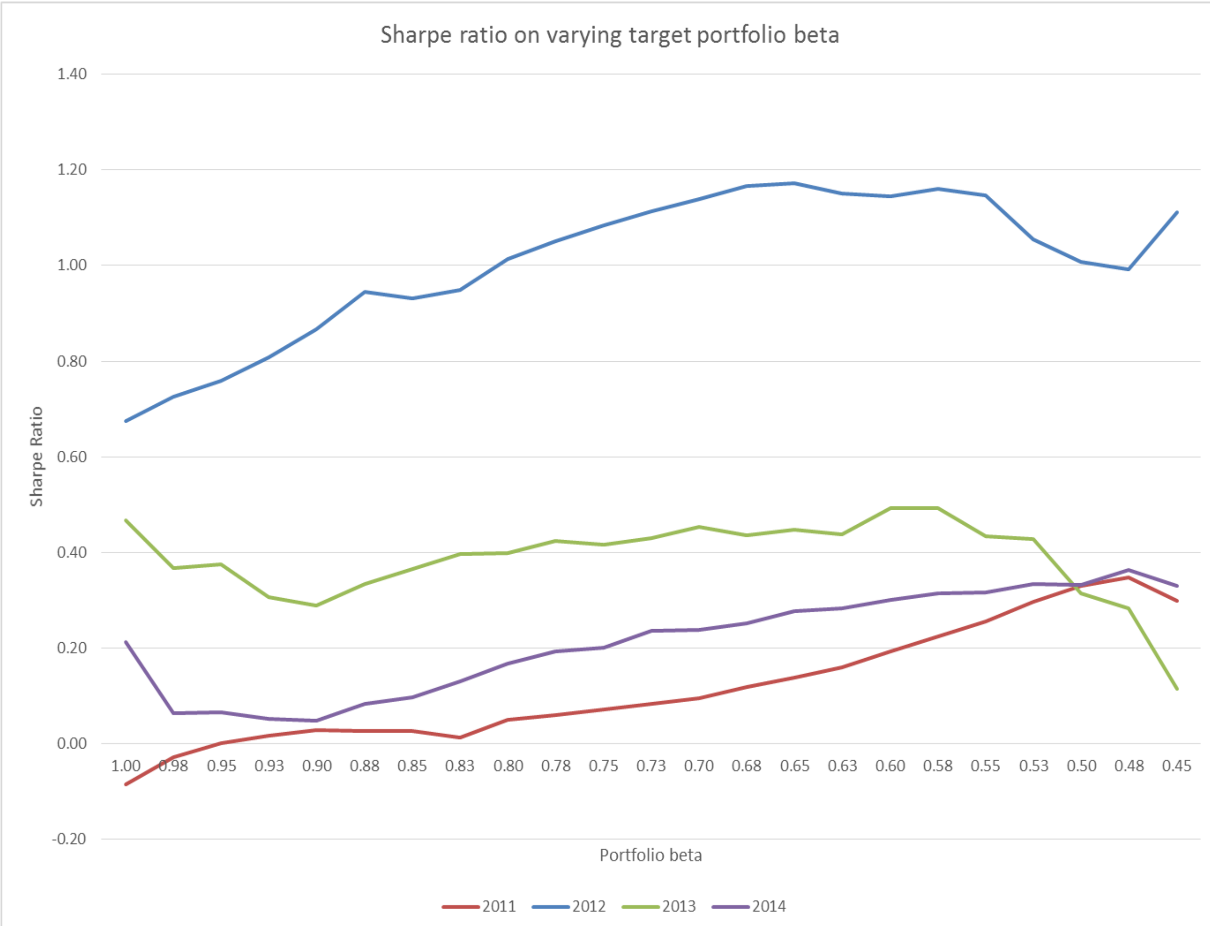
The graph below shows Sharpe ratios calculated on a quarterly basis and stacked to give a sense of magnitude of the profile of risk adjusted returns over the 2011 to 2014 testing period. Sharpe ratios were computed on the optimized portfolio return series.

Figure 15: Sharpe ratios at quarterly intervals over the full set of target beta portfolios



The same Sharpe ratios figures taken on yearly intervals provide a sense that the ratio increases albeit modestly as the target beta is lowered. The relationship is evident in the graph below.

Figure 16: Sharpe ratios at yearly intervals over the 1 down to 0.45 target beta portfolio subset



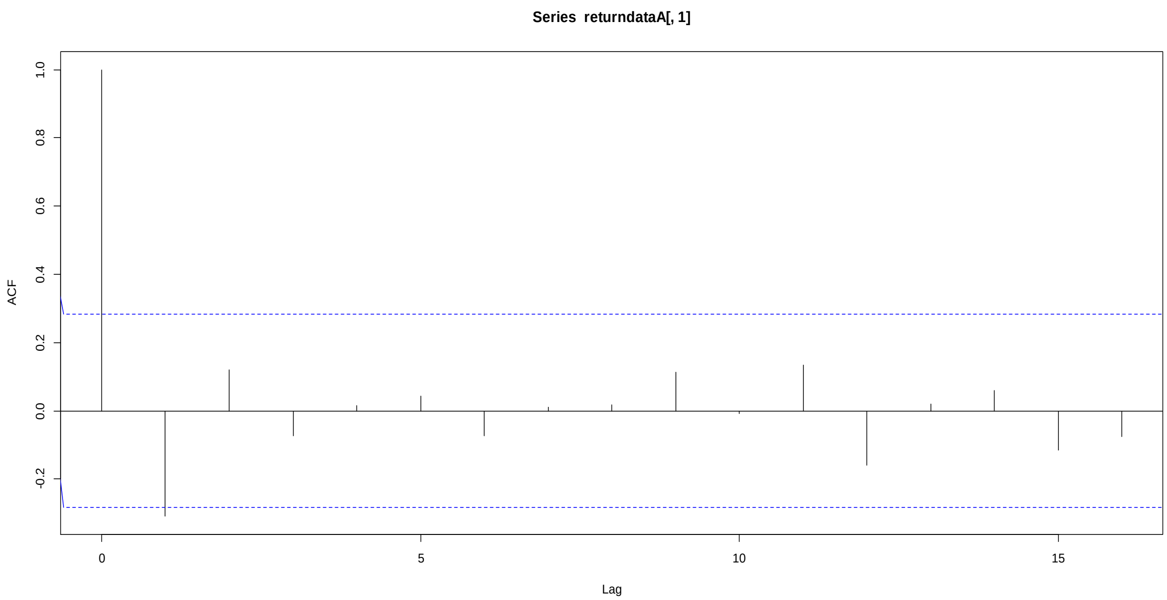
5.7.3 Summary statistics of returns in excess of risk free rate

Table 5: Summary statistics of share returns in excess of the month risk free rate of return series for optimised target beta portfolios. This data spans 48 months beginning 1 January 2010 to 31 December 2014

Portfolio by target beta		n	mean	sd	median	trimmed mean	median abs dev	min	max	range	skew	kurtosis	standard error	AR(1)	AR(N)	sharpe ratio
beta	1.000	48	0.0107	0.0345	0.0135	0.0106	0.0367	-0.0596	0.0778	0.1374	-0.0471	-0.6115	0.0050	0.0272	0.2033	0.3115
beta	0.975	48	0.0093	0.0344	0.0127	0.0100	0.0332	-0.0642	0.0799	0.1441	-0.1968	-0.5574	0.0050	0.0608	0.3870	0.2702
beta	0.950	48	0.0096	0.0342	0.0136	0.0106	0.0366	-0.0687	0.0779	0.1465	-0.2869	-0.5353	0.0049	0.0670	0.3994	0.2815
beta	0.925	48	0.0096	0.0352	0.0145	0.0104	0.0405	-0.0728	0.0795	0.1523	-0.2620	-0.5972	0.0051	0.0855	0.4651	0.2717
beta	0.900	48	0.0097	0.0351	0.0152	0.0106	0.0377	-0.0760	0.0797	0.1557	-0.3101	-0.5456	0.0051	0.1083	0.5394	0.2761
beta	0.875	48	0.0107	0.0350	0.0149	0.0119	0.0374	-0.0772	0.0799	0.1570	-0.3878	-0.3972	0.0051	0.0827	0.4775	0.3068
beta	0.850	48	0.0112	0.0348	0.0140	0.0124	0.0361	-0.0795	0.0782	0.1578	-0.4052	-0.3138	0.0050	0.0714	0.4276	0.3204
beta	0.825	48	0.0116	0.0343	0.0141	0.0128	0.0365	-0.0802	0.0754	0.1556	-0.4149	-0.2402	0.0049	0.0744	0.4322	0.3376
beta	0.800	48	0.0127	0.0345	0.0155	0.0140	0.0372	-0.0791	0.0783	0.1574	-0.4496	-0.1992	0.0050	0.0530	0.3522	0.3676
beta	0.775	48	0.0134	0.0345	0.0150	0.0147	0.0357	-0.0801	0.0784	0.1585	-0.4611	-0.0970	0.0050	0.0547	0.3626	0.3887
beta	0.750	48	0.0137	0.0347	0.0186	0.0152	0.0362	-0.0853	0.0784	0.1638	-0.5441	0.1007	0.0050	0.0575	0.3719	0.3956
beta	0.725	48	0.0145	0.0348	0.0203	0.0161	0.0382	-0.0871	0.0786	0.1657	-0.5749	0.2296	0.0050	0.0542	0.3552	0.4174
beta	0.700	48	0.0151	0.0352	0.0220	0.0170	0.0393	-0.0888	0.0787	0.1675	-0.6090	0.2411	0.0051	0.0502	0.3499	0.4306
beta	0.675	48	0.0155	0.0351	0.0197	0.0174	0.0376	-0.0890	0.0803	0.1692	-0.6254	0.3135	0.0051	0.0630	0.4077	0.4416
beta	0.650	48	0.0163	0.0352	0.0210	0.0181	0.0380	-0.0916	0.0804	0.1720	-0.6604	0.4194	0.0051	0.0694	0.4316	0.4633
beta	0.625	48	0.0166	0.0351	0.0212	0.0184	0.0370	-0.0927	0.0785	0.1713	-0.6852	0.4774	0.0051	0.0877	0.5038	0.4723
beta	0.600	48	0.0174	0.0349	0.0229	0.0195	0.0345	-0.0974	0.0754	0.1728	-0.8263	0.9113	0.0050	0.1036	0.5501	0.4992
beta	0.575	48	0.0180	0.0351	0.0241	0.0202	0.0328	-0.0997	0.0729	0.1726	-0.9183	1.1956	0.0051	0.1505	0.6261	0.5130
beta	0.550	48	0.0179	0.0353	0.0248	0.0204	0.0312	-0.1004	0.0743	0.1748	-0.9526	1.2803	0.0051	0.2483	0.6878	0.5075
beta	0.525	48	0.0185	0.0359	0.0258	0.0210	0.0328	-0.1008	0.0786	0.1794	-0.9589	1.1817	0.0052	0.2446	0.6515	0.5138
beta	0.500	48	0.0179	0.0364	0.0246	0.0207	0.0318	-0.1054	0.0761	0.1815	-1.0053	1.3400	0.0053	0.4263	0.6700	0.4904
beta	0.475	48	0.0178	0.0359	0.0261	0.0208	0.0322	-0.1101	0.0731	0.1831	-1.1606	1.9503	0.0052	0.7169	0.6681	0.4947
beta	0.450	48	0.0147	0.0347	0.0238	0.0180	0.0346	-0.0953	0.0669	0.1622	-0.9962	0.9557	0.0050	0.5376	0.7556	0.4248
beta	0.425	48	0.0139	0.0305	0.0191	0.0159	0.0248	-0.0748	0.0655	0.1403	-0.7999	0.7845	0.0044	0.3578	0.7419	0.4563
beta	0.400	48	0.0110	0.0304	0.0158	0.0126	0.0274	-0.0596	0.0670	0.1266	-0.4828	-0.4693	0.0044	0.4818	0.7919	0.3616
beta	0.375	48	0.0161	0.0356	0.0259	0.0195	0.0296	-0.1004	0.0719	0.1723	-1.0656	1.2324	0.0051	0.0625	0.3048	0.4520
beta	0.350	48	0.0169	0.0352	0.0231	0.0200	0.0304	-0.1147	0.0723	0.1870	-1.2796	2.6079	0.0051	0.3025	0.7142	0.4811
beta	0.325	48	0.0155	0.0362	0.0230	0.0178	0.0343	-0.1072	0.0717	0.1789	-0.8601	1.1627	0.0052	0.0800	0.4317	0.4287
beta	0.300	48	0.0163	0.0343	0.0244	0.0194	0.0279	-0.1040	0.0691	0.1730	-1.2176	2.1433	0.0050	0.4930	0.7425	0.4735
beta	0.275	48	0.0157	0.0344	0.0215	0.0179	0.0340	-0.0879	0.0723	0.1602	-0.6731	0.3363	0.0050	0.1014	0.3267	0.4566
beta	0.250	48	0.0135	0.0317	0.0208	0.0160	0.0353	-0.0703	0.0592	0.1294	-0.6654	-0.1928	0.0046	0.6905	0.6313	0.4246
beta	0.225	48	0.0122	0.0321	0.0184	0.0142	0.0337	-0.0712	0.0600	0.1312	-0.5874	-0.2396	0.0046	0.6062	0.6432	0.3788
beta	0.200	48	0.0118	0.0319	0.0168	0.0136	0.0282	-0.0660	0.0660	0.1319	-0.5597	-0.2905	0.0046	0.6077	0.7627	0.3709
beta	0.175	48	0.0182	0.0392	0.0298	0.0214	0.0417	-0.0988	0.0835	0.1823	-0.8245	0.4540	0.0057	0.0793	0.2992	0.4633
beta	0.150	48	0.0105	0.0295	0.0152	0.0123	0.0316	-0.0667	0.0565	0.1231	-0.5487	-0.3682	0.0043	0.6522	0.7568	0.3552
beta	0.125	48	0.0117	0.0315	0.0171	0.0135	0.0303	-0.0729	0.0673	0.1402	-0.5585	-0.2251	0.0045	0.6830	0.7368	0.3725
beta	0.100	48	0.0145	0.0331	0.0200	0.0170	0.0274	-0.1001	0.0722	0.1723	-1.0352	1.6276	0.0048	0.3797	0.7621	0.4387
beta	0.075	48	0.0132	0.0323	0.0184	0.0153	0.0264	-0.0723	0.0641	0.1364	-0.7114	0.1427	0.0047	0.4018	0.7393	0.4076
beta	0.050	48	0.0133	0.0354	0.0161	0.0150	0.0341	-0.0830	0.0777	0.1607	-0.5178	-0.0362	0.0051	0.2792	0.4310	0.3765
beta	0.025	48	0.0127	0.0314	0.0212	0.0148	0.0350	-0.0696	0.0653	0.1349	-0.6004	-0.0457	0.0045	0.5920	0.7278	0.4036
beta	0.000	48	0.0126	0.0322	0.0215	0.0147	0.0337	-0.0657	0.0626	0.1282	-0.5606	-0.4116	0.0046	0.5583	0.6976	0.3929
L/S beta	0.000	48	0.0240	0.0424	0.0334	0.0282	0.0322	-0.1163	0.0976	0.2139	-1.1311	1.2898	0.0061	0.9766	0.8191	0.5659

A Box-Ljung autocorrelation test (Monti, 1994) was applied. Some level of serial correlation is seen in some of the portfolios, for instance the correlogram (Venables & Ripley, 2002) for the beta 1 portfolio below shows AR (1) autocorrelation. Excess return data essentially has sample risk and return measures that are more or less similar to those of the full return data in table (3).

Figure 17: Box-Ljung test Correlogram for the target beta 1 portfolio's returns in excess to risk free rate over the four year test period



5.7.4 Ledoit and Wolf (2008) test for differences in Sharpe ratios

The Ledoit and Wolf (2008) robust test for differences in Sharpe ratios was conducted on 1) the ranked 41 ranked adjacent pairs and 2) long short zero beta against each of the 41 long target beta portfolios. Ranked adjacent pair comparison test result outputs are presented in the table below.

Table 6: Results for Ledoit and Wolf (2008) test for differences in Sharpe ratios undertaken on adjacent target beta portfolio pairs

		Block size = 4		Block size = 6			
target beta portfolio	Portfolio Pair	Difference	p.Value	Difference	p.Value	achieved beta	achieved beta on rebalance
1.000	1-2	0.0414	0.2142	0.0414	0.2266	1.000	1.000
0.975	2-3	-0.0114	0.4031	-0.0114	0.4227	0.975	0.975
0.950	3-4	0.0099	0.7129	0.0099	0.7113	0.950	0.950
0.925	4-5	-0.0044	0.6885	-0.0044	0.7005	0.925	0.925
0.900	5-6	-0.0308	0.6771	-0.0308	0.6875	0.900	0.900
0.875	6-7	-0.0135	0.2286	-0.0135	0.2681	0.875	0.875
0.850	7-8	-0.0172	0.2210	-0.0172	0.2358	0.850	0.850
0.825	8-9	-0.0300	0.0172	-0.0300	0.0252	0.825	0.825
0.800	9-10	-0.0211	0.0824	-0.0211	0.0954	0.800	0.800
0.775	10-11	-0.0069	0.7071	-0.0069	0.6989	0.775	0.775
0.750	11-12	-0.0218	0.1286	-0.0218	0.1474	0.750	0.750
0.725	12-13	-0.0132	0.3787	-0.0132	0.4097	0.725	0.725
0.700	13-14	-0.0110	0.2887	-0.0110	0.3317	0.700	0.700
0.675	14-15	-0.0217	0.1514	-0.0217	0.1718	0.675	0.675
0.650	15-16	-0.0091	0.4941	-0.0091	0.5259	0.650	0.650
0.625	16-17	-0.0269	0.2440	-0.0269	0.2731	0.625	0.625
0.600	17-18	-0.0139	0.3111	-0.0139	0.3137	0.600	0.600
0.575	18-19	0.0055	0.6419	0.0055	0.6477	0.575	0.575
0.550	19-20	-0.0063	0.6673	-0.0063	0.7069	0.550	0.550
0.525	20-21	0.0234	0.2290	0.0234	0.2386	0.525	0.525
0.500	21-22	-0.0043	0.8706	-0.0043	0.8726	0.500	0.500
0.475	22-23	0.0699	0.1632	0.0699	0.2276	0.475	0.475
0.450	23-24	-0.0315	0.5231	-0.0315	0.5287	0.450	0.450
0.425	24-25	0.0946	0.2563	0.0946	0.2707	0.522	0.485
0.400	25-26	-0.0904	0.2883	-0.0904	0.2931	0.674	0.597
0.375	26-27	-0.0290	0.7612	-0.0290	0.7744	0.449	0.969
0.350	27-28	0.0524	0.5259	0.0524	0.5333	0.451	0.510
0.325	28-29	-0.0448	0.5893	-0.0448	0.5883	0.592	0.870
0.300	29-30	0.0168	0.8654	0.0168	0.8734	0.482	0.468
0.275	30-31	0.0320	0.7157	0.0320	0.7131	0.503	0.858
0.250	31-32	0.0458	0.1790	0.0458	0.1978	0.559	0.431
0.225	32-33	0.0079	0.8198	0.0079	0.8352	0.633	0.450
0.200	33-34	-0.0924	0.1988	-0.0924	0.2450	0.449	0.469
0.175	34-35	0.1081	0.2218	0.1081	0.2681	0.451	1.023
0.150	35-36	-0.0173	0.7223	-0.0173	0.7375	0.608	0.432
0.125	36-37	-0.0662	0.3631	-0.0662	0.3635	0.722	0.447
0.100	37-38	0.0311	0.6941	0.0311	0.7041	0.449	0.499
0.075	38-39	0.0312	0.4709	0.0312	0.4897	0.452	0.543
0.050	39-40	-0.0271	0.6921	-0.0271	0.7081	0.534	0.856
0.025	40-41	0.0106	0.7337	0.0106	0.7451	0.583	0.448
0.000	41-42	-0.1730	0.1882	-0.1730	0.2288	0.644	0.441

The first pair (1-2) comprises the beta 1 and the beta 0.975 portfolios whilst last pair (41-42) comprises the long only beta 0 and the long/short beta 0 portfolios. At a significance level α= 5%, the null hypothesis of equal Sharpe ratios cannot be rejected as seen from the respective p values which are all above the 0.05 threshold expect for the (8–9). Although the average data dependent block size for the studentised boot strap on Sharpe ratios was 6, block sizes of 4 as well as 1 were also used in order to enhance reliability.

Long/short global pair comparison test results are summarized below.

Table 7: Results for Ledoit and Wolf (2008) test for differences in Sharpe ratios undertaken on long/short to target beta portfolio pairs

		Block size = 4		Block size = 6			
target beta portfolio	Portfolio pair	Difference	p.Value	Difference	p.Value	achieved beta	achieved beta on rebalance
1.000	42-1	-0.2544	0.2679	-0.2544	0.3107	1.000	1.000
0.975	42-2	-0.2957	0.2076	-0.2957	0.2396	0.975	0.975
0.950	42-3	-0.2844	0.2036	-0.2844	0.2492	0.950	0.950
0.925	42-4	-0.2942	0.1806	-0.2942	0.2190	0.925	0.925
0.900	42-5	-0.2898	0.1882	-0.2898	0.2176	0.900	0.900
0.875	42-6	-0.2590	0.2150	-0.2590	0.2444	0.875	0.875
0.850	42-7	-0.2455	0.2244	-0.2455	0.2464	0.850	0.850
0.825	42-8	-0.2283	0.2294	-0.2283	0.2733	0.825	0.825
0.800	42-9	-0.1983	0.2819	-0.1983	0.3043	0.800	0.800
0.775	42-10	-0.1772	0.3155	-0.1772	0.3447	0.775	0.775
0.750	42-11	-0.1703	0.2829	-0.1703	0.3249	0.750	0.750
0.725	42-12	-0.1485	0.3289	-0.1485	0.3633	0.725	0.725
0.700	42-13	-0.1353	0.3551	-0.1353	0.4015	0.700	0.700
0.675	42-14	-0.1243	0.3909	-0.1243	0.3907	0.675	0.675
0.650	42-15	-0.1026	0.4391	-0.1026	0.4477	0.650	0.650
0.625	42-16	-0.0936	0.4293	-0.0936	0.4621	0.625	0.625
0.600	42-17	-0.0667	0.5463	-0.0667	0.5563	0.600	0.600
0.575	42-18	-0.0528	0.5805	-0.0528	0.5977	0.575	0.575
0.550	42-19	-0.0584	0.5205	-0.0584	0.5503	0.550	0.550
0.525	42-20	-0.0521	0.5471	-0.0521	0.5663	0.525	0.525
0.500	42-21	-0.0755	0.3527	-0.0755	0.3799	0.500	0.500
0.475	42-22	-0.0712	0.3753	-0.0712	0.4013	0.475	0.475
0.450	42-23	-0.1411	0.1368	-0.1411	0.1820	0.450	0.450
0.425	42-24	-0.1096	0.3243	-0.1096	0.3383	0.522	0.485
0.400	42-25	-0.2043	0.2424	-0.2043	0.2649	0.674	0.597
0.375	42-26	-0.1138	0.4227	-0.1138	0.4693	0.449	0.969
0.350	42-27	-0.0848	0.3087	-0.0848	0.3389	0.451	0.510
0.325	42-28	-0.1372	0.3103	-0.1372	0.3483	0.592	0.870
0.300	42-29	-0.0924	0.2625	-0.0924	0.2879	0.482	0.468
0.275	42-30	-0.1092	0.4679	-0.1092	0.5025	0.503	0.858
0.250	42-31	-0.1413	0.2078	-0.1413	0.2428	0.559	0.431
0.225	42-32	-0.1871	0.1798	-0.1871	0.2164	0.633	0.450
0.200	42-33	-0.1950	0.2240	-0.1950	0.2537	0.449	0.469
0.175	42-34	-0.1026	0.5401	-0.1026	0.5665	0.451	1.023
0.150	42-35	-0.2107	0.1084	-0.2107	0.1364	0.608	0.432
0.125	42-36	-0.1933	0.1434	-0.1933	0.1748	0.722	0.447
0.100	42-37	-0.1272	0.2056	-0.1272	0.2368	0.449	0.499
0.075	42-38	-0.1583	0.2777	-0.1583	0.3101	0.452	0.543
0.050	42-39	-0.1894	0.2294	-0.1894	0.2635	0.534	0.856
0.025	42-40	-0.1623	0.1940	-0.1623	0.2270	0.583	0.448
0.000	42-41	-0.1730	0.1996	-0.1730	0.2302	0.644	0.441

The first pair (42-1) comprises the long/short beta 0 and the beta 1 portfolios whilst last pair (42-41) comprises the long/short beta 0 portfolios and the long only beta 0. At a significance level α= 5%, the null hypothesis of equal Sharpe ratios cannot be rejected for any of the pairs tested as seen from the respective p values which are all above the 0.05 threshold. This applies for the block size 4 or 6 tests.

5.7.5 Ledoit and Wolf (2011) robust test for differences in variances

In order to provide a more complete picture the Ledoit and Wolf (2011) robust test for differences in variances was also applied and the results are as follows for ranked adjacent pair comparisons of portfolio variances:

Table 8:Results for Ledoit and Wolf (2011) test for differences in variances undertaken on adjacent target beta portfolio pairs

		Block size = 1		Block size = 4		Block size = 6			
target beta portfolio	Portfolio Pair	Difference	p.Value	Difference	p.Value	Difference	p.Value	achieved beta	achieved beta on rebalance
1.000	1-2	0.0050	0.9410	0.0050	0.9430	0.0050	0.9480	1.000	1.000
0.975	2-3	0.0100	0.7130	0.0100	0.6960	0.0100	0.7080	0.975	0.975
0.950	3-4	-0.0550	0.1880	-0.0550	0.2250	-0.0550	0.2450	0.950	0.950
0.925	4-5	0.0020	0.9790	0.0020	0.9760	0.0020	0.9840	0.925	0.925
0.900	5-6	0.0080	0.9230	0.0080	0.9270	0.0080	0.9270	0.900	0.900
0.875	6-7	0.0110	0.5550	0.0110	0.5810	0.0110	0.5890	0.875	0.875
0.850	7-8	0.0320	0.0940	0.0320	0.1250	0.0320	0.1350	0.850	0.850
0.825	8-9	-0.0130	0.5020	-0.0130	0.5150	-0.0130	0.5360	0.825	0.825
0.800	9-10	-0.0010	0.9730	-0.0010	0.9770	-0.0010	0.9720	0.800	0.800
0.775	10-11	-0.0090	0.7330	-0.0090	0.7380	-0.0090	0.7560	0.775	0.775
0.750	11-12	-0.0080	0.7900	-0.0080	0.8070	-0.0080	0.8160	0.750	0.750
0.725	12-13	-0.0210	0.3830	-0.0210	0.4040	-0.0210	0.4330	0.725	0.725
0.700	13-14	0.0030	0.8300	0.0030	0.8470	0.0030	0.8550	0.700	0.700
0.675	14-15	-0.0050	0.8600	-0.0050	0.8600	-0.0050	0.8660	0.675	0.675
0.650	15-16	0.0070	0.7290	0.0070	0.7500	0.0070	0.7540	0.650	0.650
0.625	16-17	0.0080	0.8500	0.0080	0.8580	0.0080	0.8650	0.625	0.625
0.600	17-18	-0.0100	0.6790	-0.0100	0.7060	-0.0100	0.6960	0.600	0.600
0.575	18-19	-0.0140	0.4830	-0.0140	0.5130	-0.0140	0.5190	0.575	0.575
0.550	19-20	-0.0320	0.1930	-0.0320	0.1860	-0.0320	0.1870	0.550	0.550
0.525	20-21	-0.0300	0.1950	-0.0300	0.2190	-0.0300	0.2260	0.525	0.525
0.500	21-22	0.0280	0.4800	0.0280	0.4850	0.0280	0.5270	0.500	0.500
0.475	22-23	0.0700	0.3970	0.0700	0.3820	0.0700	0.3800	0.475	0.475
0.450	23-24	0.2600	0.0170	0.2600	0.0130	0.2600	0.0300	0.450	0.450
0.425	24-25	0.0060	0.9640	0.0060	0.9610	0.0060	0.9620	0.522	0.485
0.400	25-26	-0.3160	0.0780	-0.3160	0.0910	-0.3160	0.0860	0.674	0.597
0.375	26-27	0.0210	0.9030	0.0210	0.9150	0.0210	0.9170	0.449	0.969
0.350	27-28	-0.0580	0.6060	-0.0580	0.6250	-0.0580	0.6310	0.451	0.510
0.325	28-29	0.1080	0.3150	0.1080	0.3560	0.1080	0.3720	0.592	0.870
0.300	29-30	-0.0020	0.9900	-0.0020	0.9890	-0.0020	0.9890	0.482	0.468
0.275	30-31	0.1590	0.3470	0.1590	0.3780	0.1590	0.3950	0.503	0.858
0.250	31-32	-0.0210	0.6050	-0.0210	0.6140	-0.0210	0.6230	0.559	0.431
0.225	32-33	0.0110	0.8760	0.0110	0.8800	0.0110	0.8850	0.633	0.450
0.200	33-34	-0.4110	0.0300	-0.4110	0.0460	-0.4110	0.0640	0.449	0.469
0.175	34-35	0.5700	0.0030	0.5700	0.0070	0.5700	0.0100	0.451	1.023
0.150	35-36	-0.1330	0.0440	-0.1330	0.0660	-0.1330	0.1080	0.608	0.432
0.125	36-37	-0.0980	0.4900	-0.0980	0.4930	-0.0980	0.5070	0.722	0.447
0.100	37-38	0.0490	0.7060	0.0490	0.7040	0.0490	0.6840	0.449	0.499
0.075	38-39	-0.1880	0.0310	-0.1880	0.0620	-0.1880	0.0900	0.452	0.543
0.050	39-40	0.2410	0.0170	0.2410	0.0260	0.2410	0.0460	0.534	0.856
0.025	40-41	-0.0460	0.5100	-0.0460	0.5300	-0.0460	0.5380	0.583	0.448
0.000	41-42	-0.5540	0.0030	-0.5540	0.0100	-0.5540	0.0210	0.644	0.441

At a significance level α= 5%, the null hypothesis of equal variances is rejected for the beta 0.45 and beta 0.425 pairs tested as seen from the respective p values which are all below the 0.05 threshold. This applies for the block size 4 or 6 tests as well. The null hypothesis is also rejected for portfolio pairs (33-34) (34-35), (35-36) , (38-39),(39-40) and (41-42). It should be noted that the average data dependent block size for the studentised boot strap on variances is 1, block sizes of 4 as well as 6 were also included in order to enhance reliability and for comparative purposes. Long/short global pair comparison of variances Ledoit and Wolf (2011) test results are summarized in the table below

Table 9:Results for Ledoit and Wolf (2011) test for differences in variances undertaken on long/short with target beta portfolio pairs

		Block size = 1		Block size = 4		Block size = 6			
target beta portfolio	Portfolio pair	Difference	p.Value	Difference	p.Value	Difference	p.Value	achieved beta	achieved beta on rebalance
1.000	42-1	-0.4140	0.1350	-0.4140	0.1770	-0.4140	0.1970	1.000	1.000
0.975	42-2	-0.4190	0.1060	-0.4190	0.1380	-0.4190	0.1550	0.975	0.975
0.950	42-3	-0.4290	0.0670	-0.4290	0.1050	-0.4290	0.1140	0.950	0.950
0.925	42-4	-0.3740	0.1010	-0.3740	0.1360	-0.3740	0.1530	0.925	0.925
0.900	42-5	-0.3760	0.0980	-0.3760	0.1410	-0.3760	0.1430	0.900	0.900
0.875	42-6	-0.3840	0.0860	-0.3840	0.1120	-0.3840	0.1420	0.875	0.875
0.850	42-7	-0.3950	0.0680	-0.3950	0.1030	-0.3950	0.1230	0.850	0.850
0.825	42-8	-0.4260	0.0460	-0.4260	0.0730	-0.4260	0.0900	0.825	0.825
0.800	42-9	-0.4130	0.0480	-0.4130	0.0850	-0.4130	0.1060	0.800	0.800
0.775	42-10	-0.4120	0.0510	-0.4120	0.0780	-0.4120	0.1170	0.775	0.775
0.750	42-11	-0.4040	0.0410	-0.4040	0.0800	-0.4040	0.1060	0.750	0.750
0.725	42-12	-0.3960	0.0340	-0.3960	0.0740	-0.3960	0.0920	0.725	0.725
0.700	42-13	-0.3750	0.0370	-0.3750	0.0920	-0.3750	0.1000	0.700	0.700
0.675	42-14	-0.3770	0.0330	-0.3770	0.0730	-0.3770	0.1010	0.675	0.675
0.650	42-15	-0.3720	0.0240	-0.3720	0.0690	-0.3720	0.0780	0.650	0.650
0.625	42-16	-0.3800	0.0150	-0.3800	0.0450	-0.3800	0.0680	0.625	0.625
0.600	42-17	-0.3880	0.0020	-0.3880	0.0060	-0.3880	0.0190	0.600	0.600
0.575	42-18	-0.3780	0.0020	-0.3780	0.0040	-0.3780	0.0130	0.575	0.575
0.550	42-19	-0.3640	0.0030	-0.3640	0.0080	-0.3640	0.0150	0.550	0.550
0.525	42-20	-0.3330	0.0060	-0.3330	0.0100	-0.3330	0.0310	0.525	0.525
0.500	42-21	-0.3030	0.0080	-0.3030	0.0220	-0.3030	0.0450	0.500	0.500
0.475	42-22	-0.3320	0.0080	-0.3320	0.0140	-0.3320	0.0210	0.475	0.475
0.450	42-23	-0.4020	0.0010	-0.4020	0.0050	-0.4020	0.0100	0.450	0.450
0.425	42-24	-0.6620	0.0000	-0.6620	0.0010	-0.6620	0.0060	0.522	0.485
0.400	42-25	-0.6680	0.0040	-0.6680	0.0210	-0.6680	0.0310	0.674	0.597
0.375	42-26	-0.3520	0.0270	-0.3520	0.0450	-0.3520	0.0630	0.449	0.969
0.350	42-27	-0.3730	0.0220	-0.3730	0.0280	-0.3730	0.0510	0.451	0.510
0.325	42-28	-0.3150	0.0300	-0.3150	0.0410	-0.3150	0.0690	0.592	0.870
0.300	42-29	-0.4230	0.0030	-0.4230	0.0040	-0.4230	0.0100	0.482	0.468
0.275	42-30	-0.4210	0.0070	-0.4210	0.0170	-0.4210	0.0130	0.503	0.858
0.250	42-31	-0.5800	0.0010	-0.5800	0.0030	-0.5800	0.0030	0.559	0.431
0.225	42-32	-0.5590	0.0010	-0.5590	0.0060	-0.5590	0.0150	0.633	0.450
0.200	42-33	-0.5690	0.0020	-0.5690	0.0090	-0.5690	0.0170	0.449	0.469
0.175	42-34	-0.1580	0.3500	-0.1580	0.3810	-0.1580	0.3970	0.451	1.023
0.150	42-35	-0.7290	0.0010	-0.7290	0.0010	-0.7290	0.0030	0.608	0.432
0.125	42-36	-0.5950	0.0000	-0.5950	0.0090	-0.5950	0.0190	0.722	0.447
0.100	42-37	-0.4980	0.0000	-0.4980	0.0010	-0.4980	0.0070	0.449	0.499
0.075	42-38	-0.5470	0.0020	-0.5470	0.0100	-0.5470	0.0290	0.452	0.543
0.050	42-39	-0.3590	0.0310	-0.3590	0.0320	-0.3590	0.0310	0.534	0.856
0.025	42-40	-0.6000	0.0010	-0.6000	0.0020	-0.6000	0.0050	0.583	0.448
0.000	42-41	-0.5540	0.0030	-0.5540	0.0110	-0.5540	0.0200	0.644	0.441

At a significance level α= 5%, the null hypothesis of equal variances is rejected for all portfolio pairs from the beta 0.825 and long/short beta 0 pair down to long beta 0 and long/short beta 0 with the exception of (42 -10) and (42-34) in application of the Ledoit and Wolf (2011) test as seen from the respective p values which are all below the 0.05 threshold as highlighted above.

Even for the block 4 and 6 tests, significant differences are evident. This applies for the block size 4 or 6 tests as well. The null hypothesis is also rejected for portfolio pairs (33-34) (34-35), (35-36) , (38-39),(39-40) and (41-42).

It should be noted that the average data dependent block size for the studentised boot strap on variances is 1, block sizes of 4 as well as 6 were also included in order to enhance reliability and for comparative purposes.

For purposes of this study Type II error rather than Type I error control would be of concern. Therefore in order to avoid the pitfall of conventional familywise error rate control levels that are too stringent (Keselman & Holland, 2011) to allow for the detection of departures from the null hypothesis techniques such as Bonferonni are not undertaken. However in order to robustify the results and address reliability and validity of data issues, some familywise error correction is applied exploratively to the block size 1 p -value set.

A Benjamini and Yekutieli (2001) family error rate correction based on the false discovery rate , applied on the pvalue, provides some reduction in the set of rejection nulls as highlighted in the table below.

Table 10: Benjamini and Yekutieli (2001)adjusted p-values in results for Ledoit and Wolf (2011) test for differences in variances undertaken on long/short with target beta portfolio pairs

		Block size = 1		
target beta portfolio	Portfolio pair	Difference	p.Value	Benjamini & Yekutieli p.Value
1.000	42-1	-0.4140	0.1350	0.5954
0.975	42-2	-0.4190	0.1060	0.4795
0.950	42-3	-0.4290	0.0670	0.3428
0.925	42-4	-0.3740	0.1010	0.4689
0.900	42-5	-0.3760	0.0980	0.4673
0.875	42-6	-0.3840	0.0860	0.4214
0.850	42-7	-0.3950	0.0680	0.3428
0.825	42-8	-0.4260	0.0460	0.2618
0.800	42-9	-0.4130	0.0480	0.2646
0.775	42-10	-0.4120	0.0510	0.2726
0.750	42-11	-0.4040	0.0410	0.2411
0.725	42-12	-0.3960	0.0340	0.2142
0.700	42-13	-0.3750	0.0370	0.2251
0.675	42-14	-0.3770	0.0330	0.2142
0.650	42-15	-0.3720	0.0240	0.1841
0.625	42-16	-0.3800	0.0150	0.1260
0.600	42-17	-0.3880	0.0020	0.0294
0.575	42-18	-0.3780	0.0020	0.0294
0.550	42-19	-0.3640	0.0030	0.0353
0.525	42-20	-0.3330	0.0060	0.0623
0.500	42-21	-0.3030	0.0080	0.0706
0.475	42-22	-0.3320	0.0080	0.0706
0.450	42-23	-0.4020	0.0010	0.0221
0.425	42-24	-0.6620	0.0000	0.0000
0.400	42-25	-0.6680	0.0040	0.0441
0.375	42-26	-0.3520	0.0270	0.1985
0.350	42-27	-0.3730	0.0220	0.1764
0.325	42-28	-0.3150	0.0300	0.2103
0.300	42-29	-0.4230	0.0030	0.0353
0.275	42-30	-0.4210	0.0070	0.0686
0.250	42-31	-0.5800	0.0010	0.0221
0.225	42-32	-0.5590	0.0010	0.0221
0.200	42-33	-0.5690	0.0020	0.0294
0.175	42-34	-0.1580	0.3500	1.0000
0.150	42-35	-0.7290	0.0010	0.0221
0.125	42-36	-0.5950	0.0000	0.0000
0.100	42-37	-0.4980	0.0000	0.0000
0.075	42-38	-0.5470	0.0020	0.0294
0.050	42-39	-0.3590	0.0310	0.2103
0.025	42-40	-0.6000	0.0010	0.0221
0.000	42-41	-0.5540	0.0030	0.0353

Despite the fairly conservative nature of the test, portfolios such as (42-17), (42-18) and (42-19) amongst others remain in the solution set as highlighted in the table above.

5.7.6 Information Ratio

Information ratios in respect of the portfolios have been calculated in the same manner as the Sharpe ratio, with the key difference being that the denominator in the Sharpe ratio equation, the standard deviation of returns in excess to the risk free rate was replaced by the standard deviation of returns in excess of a relevant benchmark return. In simpler terms the information ratio is the return in excess to the benchmark divided by the tracking error. In this study, the benchmark returns used were those of the JSE All Share Index (ALSI) and the JSE/FTSE Top 40. The graphs below show how the portfolios performed over the test period.

Figure 18:Information ratios on JSE ALSI at yearly intervals over the full set of target beta portfolios

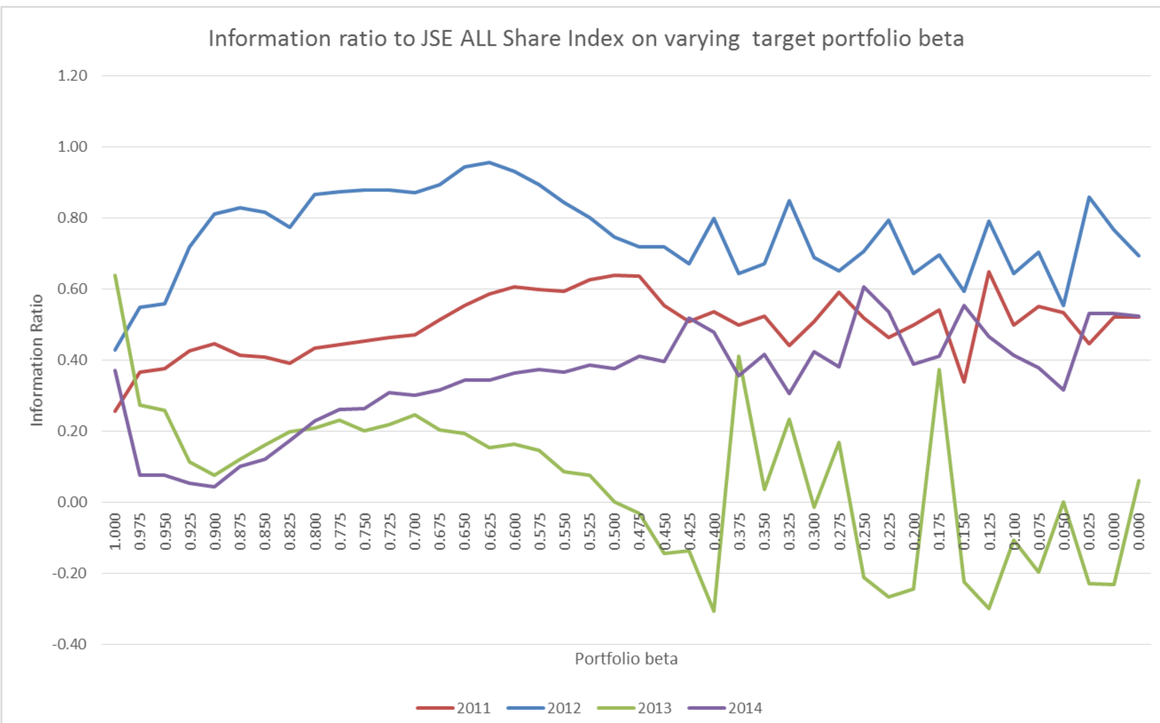
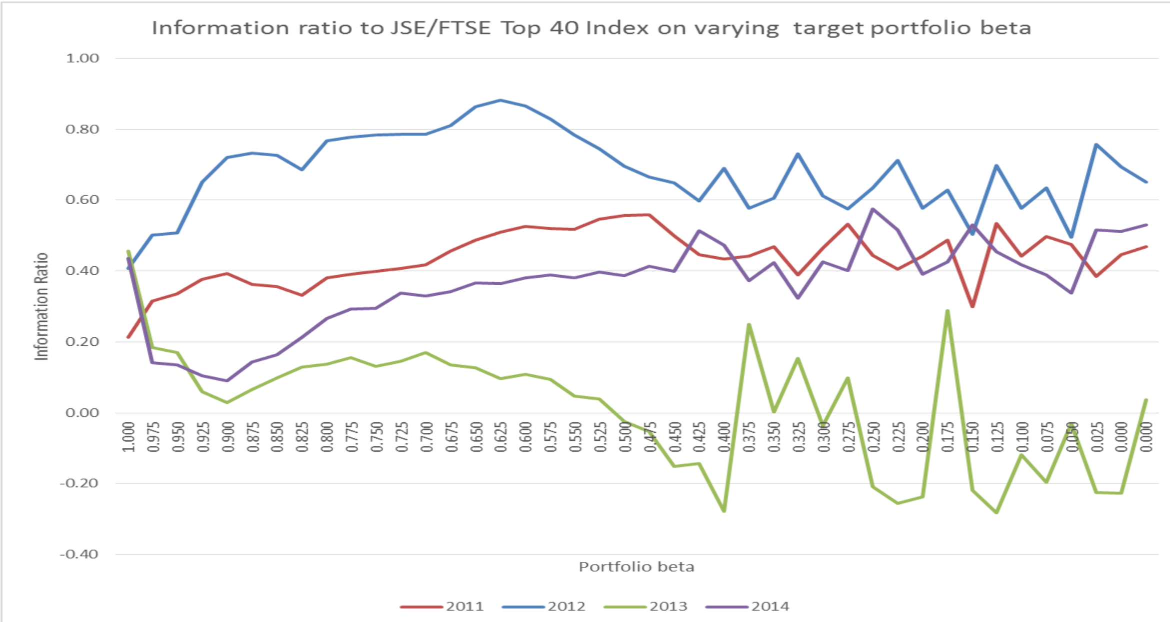


Figure 19:Information ratios on JSE Top 40 at yearly intervals over the full set of target beta portfolios



The results show that in general the information ratio increased with the lowering of the target beta. Year 2013 saw significant sharp decreases in the information ratio into negative territory even before the threshold of beta 0.45 is reached. In the following year a very strong recovery is observed as the profile returns to its previous character. The performance of the portfolios as indicated by the information ratio is similar on both benchmarks and the stacked graphs reveal more of the performance profile.

Figure 20: Stacked representation of information ratios on JSE ALSI at yearly intervals over the full set of target beta portfolios

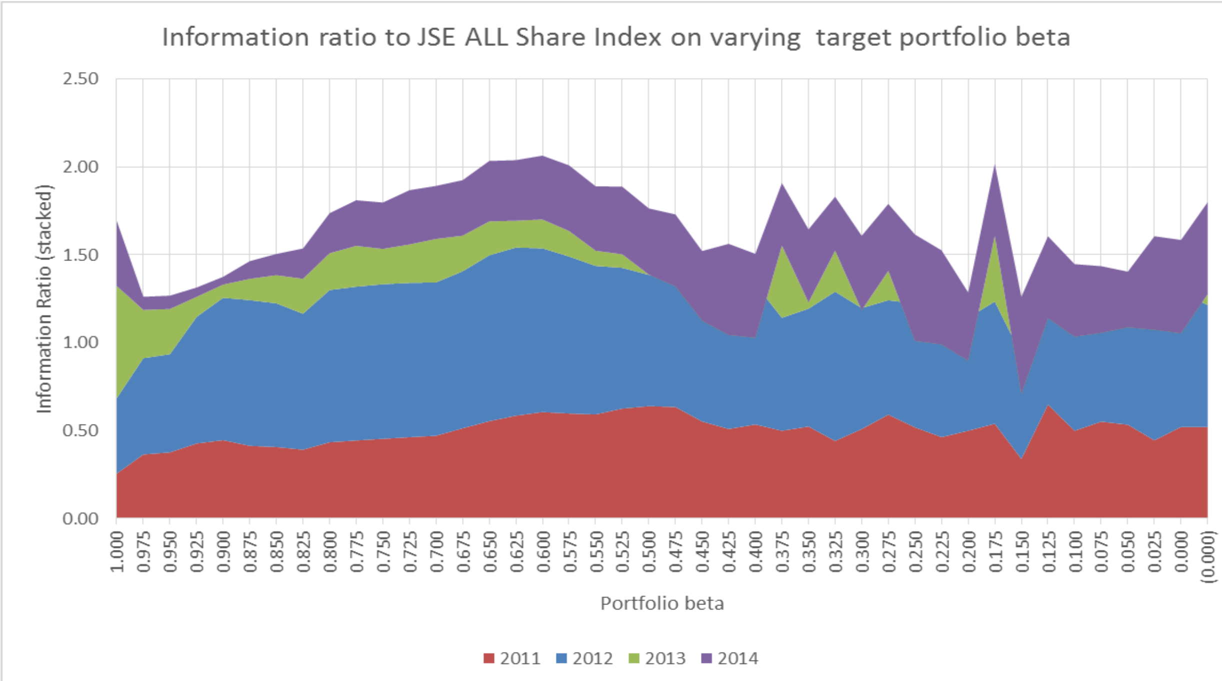
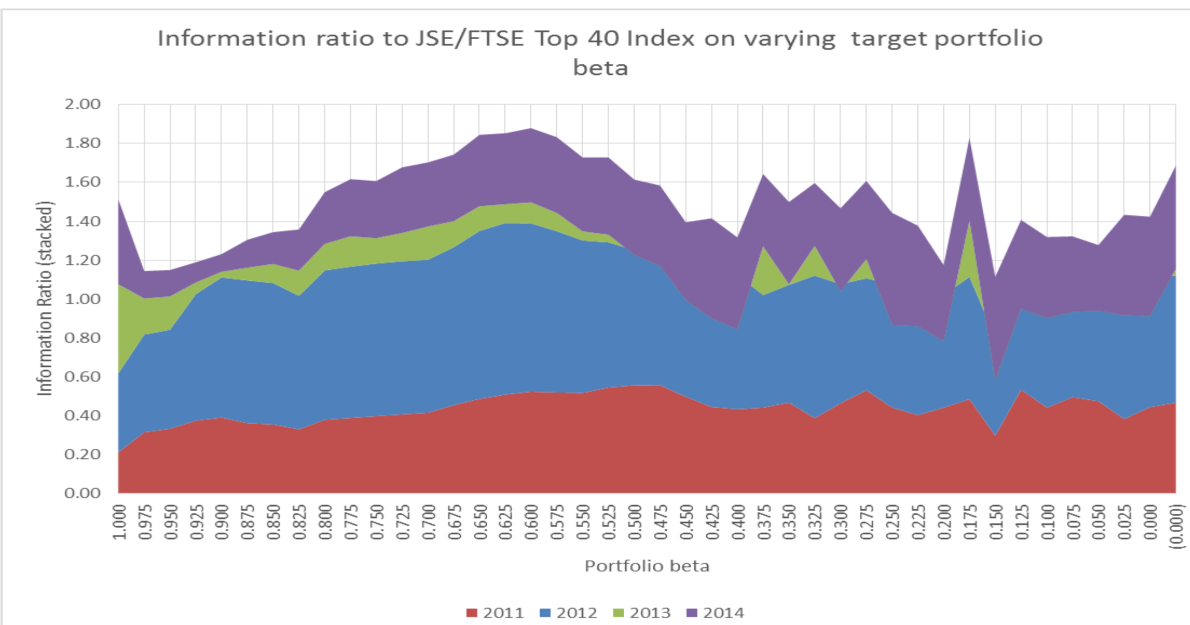


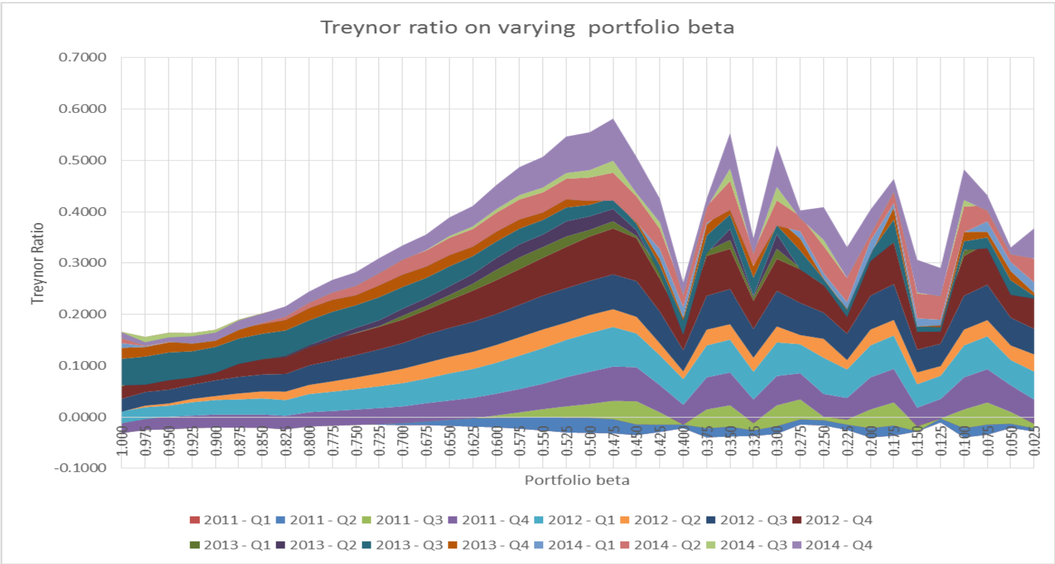
Figure 21:Stacked representation of information ratios on JSE Top 40 at yearly intervals over the full set of target beta portfolios



5.7.7 Treynor ratio.

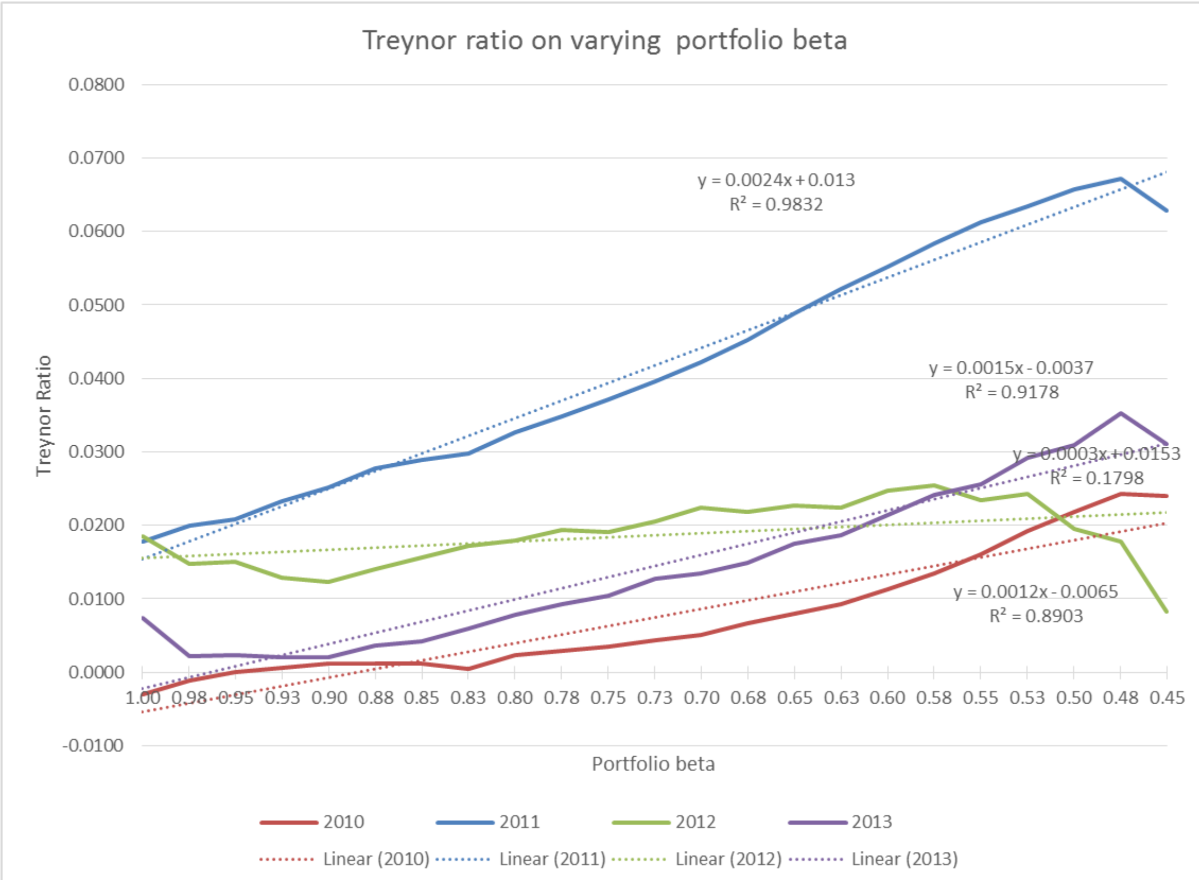
Whereas with the Sharpe and the information ratios, excess returns are adjusted in respect of total portfolio risk, the Treynor ratio adjusts excess returns by the systematic component of portfolio risk, being the beta. As the research assess the performance of the different portfolios as beta is changed, the Treynor ratio allows insight into the extent to which systematic risk influences returns.

Figure 22: Treynor ratios calculated quarterly across target beta 1.00 down to 0.025



Stacked returns, on a quarterly basis show a pronounced increase in the ratio as target beta is decreased. The yearly graph of Treynor ratios shows a strong linear relationship between the excess returns to risk free rate and the target beta.

Figure 23: Treynor ratios cross sectional variation to target beta portfolios from 1.000 down to 0.450



5.8 Research Question Six - Portfolio weights

Portfolio weights: Do the optimal portfolio weights of securities approximate the simplified analytic equation by Clarke et al. (2011)?

5.8.1 A description of the sample input processing

Clarke et al (2011)'s work on minimum variance portfolios involved the use of ex-ante inputs for beta and variances to derived simplified analytic equations describing some of the characteristics of such portfolios. In aiming to calculated imputed minimum variance equivalent portfolios for this research, the ex-anted individual betas β_i used were estimated through ordinary least squares regression. In the first optimization for a period of 60 months up to 31 December 2010 and for the subsequent rebalancing a period of 60 months up to 31 December 2012. The individual ex ante variances σ_i^2 and the market variance σ_M^2were calculated for respectively similar 60 month periods.

The derivation of the Clarke et al equivalent portfolio depended on the estimated of an imputed threshold beta, based on the equation $\frac{\beta_P^2\sigma_M^2}{\sigma_P^2} = \left(\frac{\beta_P}{\beta_L}\right)$ where calculate $\beta_L = \frac{\frac{1}{\sigma_M^2}+\sum_{\beta_i<\beta_L}\frac{\beta_i^2}{\sigma_i^2}}{\sum_{\beta_i<\beta_L}\frac{\beta_i}{\sigma_i^2}}$ is calculated by a recursive procedure. From a reliability perspective it therefore has to be borne in mind that the simplified analytic equations are not of closed form and different methods in calculating beta may impact the result of the estimated threshold beta obtained and associated derivation of imputed weights.

Portfolio weights are presented again below with the imputed Clarke et al (2011) minimum equivalents for the initial optimization.

Table 11:Initial beta constrained optimised portfolios and the Clarke et al (2011) minimum variance equivalents

Initial Optimisation 31 Dec 2010

	number of counters	18	16	14	14	14	14	29	17	14	13	29
	target beta	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	6.1%	9.1%	6.3%	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.2%
AMS	Anglo American Plat Ltd	10.0%	8.1%	5.8%	2.9%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	1.2%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	10.0%	10.0%	10.0%	0.1%
APN	Aspen Pharmacare Hldgs Ltd	4.1%	10.0%	10.0%	10.0%	10.0%	10.0%	1.6%	10.0%	10.0%	10.0%	4.7%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	7.4%	0.0%	1.8%	0.0%	0.0%	0.0%	1.8%
BVT	Bidvest Ltd	0.1%	2.2%	0.0%	0.0%	0.1%	0.2%	5.8%	9.2%	10.0%	10.0%	2.1%
DSY	Discovery Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	7.0%	6.8%	10.0%	10.0%	10.0%	6.8%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.9%	0.0%	0.0%	0.0%	3.6%
GRT	Growthpoint Prop Ltd	0.0%	0.1%	5.8%	9.9%	10.0%	10.0%	5.2%	10.0%	10.0%	10.0%	10.0%
IMP	Impala Platinum Hlgs Ltd	0.3%	0.2%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	3.8%	0.0%	0.0%	0.0%	3.4%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	0.0%	0.0%	0.0%	0.8%
IPL	Imperial Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.7%	0.0%	0.0%	0.0%	2.6%
MDC	Mediclinic Internat Ltd	0.0%	0.0%	0.0%	0.0%	10.0%	10.0%	4.5%	0.1%	10.0%	10.0%	5.1%
MPC	Mr Price Group Ltd	8.0%	10.0%	10.0%	10.0%	10.0%	10.0%	2.2%	10.0%	10.0%	10.0%	9.5%
MTN	MTN Group Ltd	10.0%	5.1%	8.1%	7.8%	3.0%	0.0%	0.4%	0.0%	0.0%	0.0%	3.4%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	1.1%	0.0%	0.0%	1.6%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	3.6%	2.4%	0.0%	0.0%	0.0%	2.4%
NTC	Netcare Limited	7.4%	4.3%	5.2%	5.3%	3.5%	2.9%	4.1%	0.0%	0.0%	0.0%	1.7%
OML	Old Mutual plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.0%	0.0%	0.0%	0.0%	1.8%
REM	Remgro Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	0.0%	0.0%	0.0%	1.5%
RMH	RMB Holdings Ltd	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	4.0%	0.0%	0.0%	0.0%	4.2%
SAB	SABMiller plc	5.2%	4.2%	6.7%	9.5%	10.0%	10.0%	10.0%	0.0%	0.0%	0.0%	3.7%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%	0.0%	0.0%	3.9%
SHF	Steinhoff Int Hldgs Ltd	8.2%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.0%	0.0%	0.0%	1.9%
SHP	Shoprite Holdings Ltd	0.5%	10.0%	10.0%	10.0%	10.0%	10.0%	3.3%	10.0%	10.0%	10.0%	8.0%
SLM	Sanlam Limited	0.0%	1.4%	4.1%	7.0%	10.0%	10.0%	10.0%	9.9%	10.0%	10.0%	1.3%
SOL	Sasol Limited	10.0%	5.2%	0.0%	0.0%	0.0%	0.0%	0.2%	0.0%	0.0%	0.0%	2.2%
TBS	Tiger Brands Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	2.9%
WHL	Woolworths Holdings Ltd	7.5%	10.0%	7.9%	6.5%	6.1%	6.3%	3.3%	9.6%	0.0%	0.0%	6.7%
		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Minimum Variance Clarke et al (2011) equivalent

	6	8	7	7	9	11	13	12	11	11	13
	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AMS	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
ANG	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.3%	6.2%	5.5%	5.5%	7.9%
APN	12.3%	10.3%	8.5%	7.1%	5.8%	4.8%	6.7%	5.0%	4.5%	4.5%	6.4%
BIL	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
BVT	11.7%	9.8%	0.0%	0.0%	5.5%	4.6%	6.4%	4.8%	4.2%	4.2%	6.1%
DSY	0.0%	0.0%	0.0%	0.0%	0.0%	12.2%	17.0%	12.7%	11.2%	11.2%	16.1%
FSR	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
GRT	0.0%	44.3%	36.6%	30.3%	25.0%	20.8%	28.9%	21.6%	19.1%	19.1%	27.5%
IMP	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INL	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INP	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
IPL	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
MDC	0.0%	0.0%	0.0%	0.0%	32.0%	26.7%	37.1%	27.6%	24.5%	24.5%	35.2%
MPC	37.0%	31.2%	25.8%	21.3%	17.6%	14.6%	20.4%	15.2%	13.4%	13.4%	19.3%
MTN	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NED	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	1.8%	0.0%	0.0%	2.2%
NPN	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NTC	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
OML	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
REM	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
RMH	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SAB	4.5%	3.8%	3.1%	2.6%	2.1%	1.8%	2.5%	0.0%	0.0%	0.0%	2.4%
SBK	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SHF	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SHP	37.4%	31.5%	26.0%	21.5%	17.8%	14.8%	20.6%	15.3%	13.6%	13.6%	19.5%
SLM	0.0%	20.2%	16.7%	13.8%	11.4%	9.5%	13.2%	9.8%	8.7%	8.7%	12.5%
SOL	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
TBS	0.0%	0.0%	0.0%	0.0%	0.0%	16.6%	23.1%	17.2%	15.2%	15.2%	21.9%
WHL	4.9%	4.2%	3.4%	2.8%	2.3%	2.0%	2.7%	2.0%	1.8%	1.8%	2.6%
	107.8%	155.5%	120.1%	99.5%	119.6%	128.4%	189.3%	139.2%	121.8%	121.8%	179.7%

Differences between the mean variance portfolio and the Clarke et al (2011) equivalent are indicated in the split table below with 1) numerical differences in weights weight in mean variance less weight in imputed minimum variance portfolio and 2) a common size statement in which the weight in the mean variance is 100 % and the weight in the imputed minimum variance portfolio is calculated as a percentage of the former i.e. weight minimum variance / weight mean variance.

Table 12:Weight differences and common size weight comparisons between Initial beta constrained optimised portfolios and the Clarke et al (2011) minimum variance equivalents

Differences in weight

	d	18	16	14	14	14	14	29	17	14	13	29
	target beta	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	6.1%	9.1%	6.3%	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.2%
AMS	Anglo American Plat Ltd	10.0%	8.1%	5.8%	2.9%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	1.2%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-5.9%	3.8%	4.5%	4.5%	-7.8%
APN	Aspen Pharmacare Hldgs Ltc	-8.2%	-0.3%	1.5%	2.9%	4.2%	5.2%	-5.2%	5.0%	5.5%	5.5%	-1.7%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	7.4%	0.0%	1.8%	0.0%	0.0%	0.0%	1.8%
BVT	Bidvest Ltd	-11.6%	-7.6%	0.0%	0.0%	-5.5%	-4.5%	-0.6%	4.4%	5.8%	5.8%	-4.0%
DSY	Discovery Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	-5.2%	-10.2%	-2.7%	-1.2%	-1.2%	-9.4%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.9%	0.0%	0.0%	0.0%	3.6%
GRT	Growthpoint Prop Ltd	0.0%	-44.2%	-30.8%	-20.4%	-15.0%	-10.8%	-23.8%	-11.6%	-9.1%	-9.1%	-17.5%
IMP	Impala Platinum Hlgs Ltd	0.3%	0.2%	0.0%	0.0%	0.0%	0.0%	0.4%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	3.8%	0.0%	0.0%	0.0%	3.4%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.3%	0.0%	0.0%	0.0%	0.8%
IPL	Imperial Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.7%	0.0%	0.0%	0.0%	2.6%
MDC	Mediclinic Internat Ltd	0.0%	0.0%	0.0%	0.0%	-22.0%	-16.7%	-32.6%	-27.5%	-14.5%	-14.5%	-30.1%
MPC	Mr Price Group Ltd	-29.0%	-21.2%	-15.8%	-11.3%	-7.6%	-4.6%	-18.2%	-5.2%	-3.4%	-3.4%	-9.9%
MTN	MTN Group Ltd	10.0%	5.1%	8.1%	7.8%	3.0%	0.0%	0.4%	0.0%	0.0%	0.0%	3.4%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-1.9%	-0.7%	0.0%	0.0%	-0.6%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	3.6%	2.4%	0.0%	0.0%	0.0%	2.4%
NTC	Netcare Limited	7.4%	4.3%	5.2%	5.3%	3.5%	2.9%	4.1%	0.0%	0.0%	0.0%	1.7%
OML	Old Mutual plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.0%	0.0%	0.0%	0.0%	1.8%
REM	Remgro Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.1%	0.0%	0.0%	0.0%	1.5%
RMH	RMB Holdings Ltd	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	4.0%	0.0%	0.0%	0.0%	4.2%
SAB	SABMiller plc	0.7%	0.4%	3.5%	6.9%	7.9%	8.2%	7.5%	0.0%	0.0%	0.0%	1.3%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.4%	0.0%	0.0%	0.0%	3.9%
SHF	Steinhoff Int Hldgs Ltd	8.2%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.0%	0.0%	0.0%	1.9%
SHP	Shoprite Holdings Ltd	-36.9%	-21.5%	-16.0%	-11.5%	-7.8%	-4.8%	-17.3%	-5.3%	-3.6%	-3.6%	-11.5%
SLM	Sanlam Limited	0.0%	-18.9%	-12.6%	-6.8%	-1.4%	0.5%	-3.2%	0.1%	1.3%	1.3%	-11.2%
SOL	Sasol Limited	10.0%	5.2%	0.0%	0.0%	0.0%	0.0%	0.2%	0.0%	0.0%	0.0%	2.2%
TBS	Tiger Brands Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	-6.6%	-13.1%	-7.2%	-5.2%	-5.2%	-19.0%
WHL	Woolworths Holdings Ltd	2.5%	5.8%	4.5%	3.6%	3.7%	4.4%	0.6%	7.6%	-1.8%	-1.8%	4.1%
		-7.8%	-55.5%	-20.1%	0.5%	-19.6%	-28.4%	-89.3%	-39.2%	-21.8%	-21.8%	-79.7%

Common size weight comparison

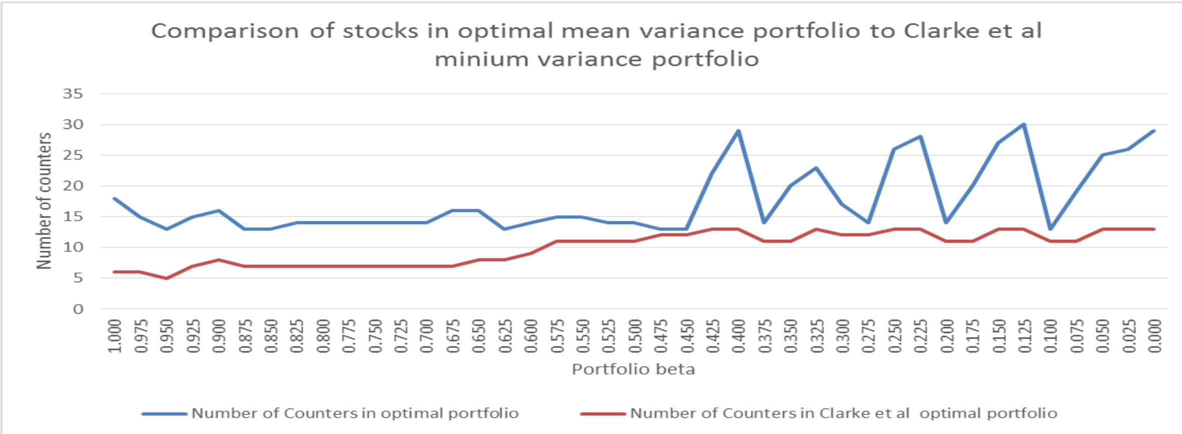
		18	22	23	23	25	27	14	25	27	28	14
		1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	0%	0%	0%	0%	100%	100%	100%	100%	100%	0%	0%
AMS	Anglo American Plat Ltd	0%	0%	0%	0%	100%	0%	0%	100%	0%	100%	0%
ANG	Anglogold Ashanti Ltd	100%	100%	100%	100%	100%	100%	347%	62%	55%	55%	8247%
APN	Aspen Pharmacare Hldgs Ltc	301%	103%	85%	71%	58%	48%	428%	50%	45%	45%	135%
BIL	BHP Billiton plc	0%	0%	0%	0%	0%	100%	0%	100%	100%	100%	0%
BVT	Bidvest Ltd	11385%	439%	100%	100%	10601%	2938%	111%	52%	42%	42%	295%
DSY	Discovery Ltd	100%	100%	100%	100%	100%	175%	249%	127%	112%	112%	238%
FSR	Firstrand Ltd	100%	100%	100%	100%	100%	100%	0%	0%	100%	100%	0%
GRT	Growthpoint Prop Ltd	100%	32142%	628%	307%	250%	208%	559%	216%	191%	191%	275%
IMP	Impala Platinum Hlgs Ltd	0%	0%	0%	0%	0%	100%	0%	100%	100%	100%	100%
INL	Investec Ltd	0%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
INP	Investec plc	100%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
IPL	Imperial Holdings Ltd	100%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
MDC	Mediclinic Internat Ltd	100%	100%	100%	100%	320%	267%	820%	19312%	245%	245%	695%
MPC	Mr Price Group Ltd	463%	312%	258%	213%	176%	146%	915%	152%	134%	134%	204%
MTN	MTN Group Ltd	0%	0%	0%	0%	0%	100%	0%	100%	100%	100%	0%
NED	Nedbank Group Ltd	100%	100%	100%	100%	100%	100%	519%	167%	100%	100%	136%
NPN	Naspers Ltd -N-	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%	0%
NTC	Netcare Limited	0%	0%	0%	0%	0%	0%	0%	100%	100%	100%	0%
OML	Old Mutual plc	100%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
REM	Remgro Ltd	0%	100%	100%	100%	100%	100%	0%	0%	100%	100%	0%
RMH	RMB Holdings Ltd	0%	100%	100%	100%	100%	100%	0%	0%	0%	100%	0%
SAB	SABMiller plc	87%	90%	47%	27%	21%	18%	25%	0%	100%	0%	64%
SBK	Standard Bank Group Ltd	100%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
SHF	Steinhoff Int Hldgs Ltd	0%	100%	100%	100%	100%	100%	0%	100%	100%	100%	0%
SHP	Shoprite Holdings Ltd	7850%	315%	260%	215%	178%	148%	620%	153%	136%	136%	244%
SLM	Sanlam Limited	100%	1492%	404%	197%	114%	95%	132%	99%	87%	87%	942%
SOL	Sasol Limited	0%	0%	100%	100%	100%	100%	0%	100%	100%	100%	0%
TBS	Tiger Brands Ltd	100%	100%	100%	100%	100%	166%	231%	172%	152%	152%	752%
WHL	Woolworths Holdings Ltd	66%	42%	43%	44%	39%	31%	83%	21%	4514877%	2032976%	39%
		108%	155%	120%	100%	120%	128%	189%	139%	122%	122%	180%

The results show that the weights in the optimized portfolios are generally less than those that might be computed with the Clarke et al (2011) weights equation.

$$w_i = \frac{\sigma_{LMV}^2}{\sigma_{\varepsilon i}^2}\left(1 - \frac{\beta_i}{\beta_L}\right) \; for \; \beta_i < \beta_L \quad else \; w_i = 0$$

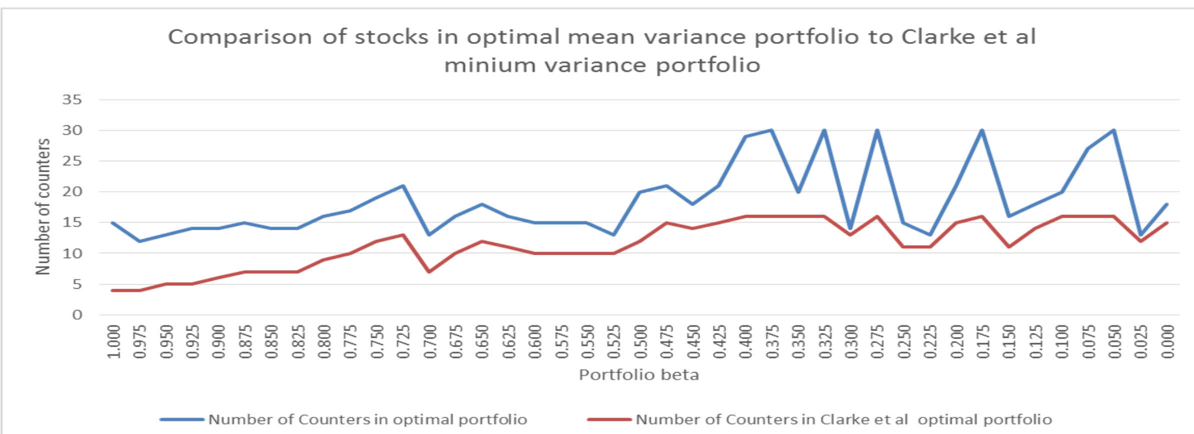
Furthermore the number of counters represented in the Clarke et al equivalent portfolio is seen to be considerably less as indicated in the graphs below for initial optimization and subsequent rebalancing.

Figure 24:Number of stocks in initial optimal portfolio compared with that of the Clarke et al (2011) derived equivalent



On rebalancing:

Figure 25: Number of stocks in rebalanced optimal portfolio compared with that of the Clarke et al (2011) derived equivalent



The results hold for the rebalanced portfolios as well as seen in the proceeding table of respective mean variance and Clarke et al derived minimum variance portfolios.

Table 13: Rebalance beta constrained optimised portfolios and the Clarke et al (2011) minimum variance equivalents

24 Month Rebalance 31 Dec 2012

	number of counters	15	14	16	13	15	20	29	14	21	20	18
	target beta	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	4.6%	1.5%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
AMS	Anglo American Plat Ltd	5.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	0.0%	0.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	10.0%	0.9%	10.0%	3.3%	10.0%
APN	Aspen Pharmacare Hldgs Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	1.4%	10.0%	10.0%	0.9%	10.0%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	4.8%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
BVT	Bidvest Ltd	0.0%	0.0%	3.8%	0.0%	0.1%	0.1%	0.0%	10.0%	5.1%	4.5%	0.0%
DSY	Discovery Ltd	0.0%	0.0%	0.4%	0.9%	7.6%	5.3%	3.2%	9.2%	10.0%	7.3%	10.0%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.0%	0.0%	0.0%	4.6%	0.5%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	3.8%	10.0%	10.0%	10.0%	10.0%	9.6%	10.0%
IMP	Impala Platinum Hlgs Ltd	0.0%	9.5%	4.3%	0.7%	0.0%	0.0%	0.9%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	0.0%	0.0%	0.0%	0.1%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.0%	0.0%	0.2%	0.0%
IPL	Imperial Holdings Ltd	2.8%	10.0%	5.2%	4.8%	1.2%	0.0%	5.5%	0.0%	0.0%	0.0%	0.0%
MDC	Mediclinic Internat Ltd	0.0%	0.1%	1.1%	9.9%	10.0%	10.0%	10.0%	9.5%	9.6%	10.0%	10.0%
MPC	Mr Price Group Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	0.8%	10.0%	3.8%	10.0%	7.6%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%	1.0%	1.9%	0.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	2.9%	9.1%	3.7%	10.0%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	5.2%	1.9%	0.0%	0.3%	0.0%	0.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	5.7%	5.2%	10.0%	0.3%	10.0%	8.4%
OML	Old Mutual plc	10.0%	5.4%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%
REM	Remgro Ltd	10.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.0%	0.0%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.9%	8.4%	0.5%	1.3%	1.0%
SAB	SABMiller plc	5.8%	3.4%	10.0%	10.0%	8.7%	3.2%	0.0%	0.0%	0.5%	0.0%	0.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.6%	3.6%	3.6%	0.5%
SHF	Steinhoff Int Hldgs Ltd	10.0%	10.0%	9.3%	3.7%	0.0%	0.0%	0.2%	0.0%	0.2%	0.1%	0.0%
SHP	Shoprite Holdings Ltd	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	3.3%	10.0%	10.0%	4.4%	10.0%
SLM	Sanlam Limited	0.0%	0.1%	1.1%	10.0%	10.0%	10.0%	4.2%	0.0%	1.8%	10.0%	1.1%
SOL	Sasol Limited	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.0%	0.0%	0.0%	0.0%
TBS	Tiger Brands Ltd	0.0%	0.0%	1.8%	0.0%	3.8%	10.0%	6.1%	8.4%	8.9%	10.0%	10.0%
WHL	Woolworths Holdings Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	1.1%	0.0%	5.1%	4.5%	1.0%
		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Minimum Variance Clarke et al (2011) equivalent

		4	6	9	7	10	12	16	13	15	16	15
		1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AMS	Anglo American Plat Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	7.7%	8.7%	7.1%	6.8%	6.9%	6.6%
APN	Aspen Pharmacare Hldgs Ltd	16.8%	14.1%	11.4%	9.3%	7.5%	6.3%	7.1%	5.8%	5.6%	5.7%	5.4%
BIL	BHP Billiton plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
BVT	Bidvest Ltd	0.0%	0.0%	9.8%	0.0%	6.5%	5.4%	6.1%	5.0%	4.8%	4.9%	0.0%
DSY	Discovery Ltd	0.0%	0.0%	24.0%	19.6%	15.9%	13.2%	15.0%	12.3%	11.8%	11.9%	11.4%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.9%	0.9%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	23.4%	19.4%	22.1%	18.1%	17.3%	17.6%	16.7%
IMP	Impala Platinum Hlgs Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
IPL	Imperial Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
MDC	Mediclinic Internat Ltd	0.0%	58.6%	47.2%	38.6%	31.2%	26.0%	29.6%	24.1%	23.1%	23.5%	22.3%
MPC	Mr Price Group Ltd	35.4%	29.7%	23.9%	19.6%	15.8%	13.2%	15.0%	12.2%	11.7%	11.9%	11.3%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	2.7%	2.6%	2.6%	2.5%
NPN	Naspers Ltd -N-	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	6.8%	7.7%	6.3%	6.0%	6.1%	5.8%
OML	Old Mutual plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
REM	Remgro Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	2.2%	2.1%	2.1%	2.0%
SAB	SABMiller plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.6%	2.1%	2.0%	2.0%	1.9%
SHF	Steinhoff Int Hldgs Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SHP	Shoprite Holdings Ltd	40.3%	33.9%	27.3%	22.3%	18.1%	15.0%	17.1%	14.0%	13.4%	13.6%	12.9%
SLM	Sanlam Limited	0.0%	12.4%	10.0%	8.2%	6.6%	5.5%	6.3%	0.0%	4.9%	5.0%	4.7%
SOL	Sasol Limited	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
TBS	Tiger Brands Ltd	0.0%	0.0%	32.7%	0.0%	21.6%	18.0%	20.5%	16.7%	16.0%	16.3%	15.5%
WHL	Woolworths Holdings Ltd	3.7%	3.1%	2.5%	2.0%	1.6%	1.4%	1.6%	0.0%	1.2%	1.2%	1.2%
		96.2%	151.9%	188.9%	119.6%	148.2%	137.6%	166.5%	128.6%	129.4%	132.4%	121.1%

A comparative tabulated analysis on the rebalanced portfolios in the diagram below shows the relatively underweighting in the mean variance portfolio as well as the relatively higher number of stocks in comparison to the imputed minimum variance equivalent.

Table 14: Weight differences and common size weight comparisons between rebalanced beta constrained optimised portfolios and the Clarke et al (2011) minimum variance equivalents

Differences in weight

	d	15	14	16	13	15	20	29	14	21	20	18
	target beta	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	4.6%	1.5%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
AMS	Anglo American Plat Ltd	5.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	0.0%	0.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	-7.2%	1.3%	-6.2%	3.2%	-3.6%	3.4%
APN	Aspen Pharmacare Hldgs Ltc	-6.8%	-4.1%	-1.4%	0.7%	2.5%	3.7%	-5.8%	4.2%	4.4%	-4.8%	4.6%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	4.8%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
BVT	Bidvest Ltd	0.0%	0.0%	-6.0%	0.0%	-6.3%	-5.3%	-6.1%	5.0%	0.3%	-0.4%	0.0%
DSY	Discovery Ltd	0.0%	0.0%	-23.6%	-18.7%	-8.3%	-7.9%	-11.9%	-3.1%	-1.8%	-4.7%	-1.4%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	8.9%	0.0%	0.0%	3.7%	-0.4%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	-19.6%	-9.4%	-12.1%	-8.1%	-7.3%	-8.0%	-6.7%
IMP	Impala Platinum Hlgs Ltd	0.0%	9.5%	4.3%	0.7%	0.0%	0.0%	0.9%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	0.0%	0.0%	0.0%	0.1%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.0%	0.0%	0.2%	0.0%
IPL	Imperial Holdings Ltd	2.8%	10.0%	5.2%	4.8%	1.2%	0.0%	5.5%	0.0%	0.0%	0.0%	0.0%
MDC	Mediclinic Internat Ltd	0.0%	-58.6%	-46.1%	-28.6%	-21.2%	-16.0%	-19.6%	-14.7%	-13.5%	-13.5%	-12.3%
MPC	Mr Price Group Ltd	-25.4%	-19.7%	-13.9%	-9.6%	-5.8%	-3.2%	-14.2%	-2.2%	-7.9%	-1.9%	-3.8%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%	1.0%	1.9%	0.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%	6.5%	1.1%	7.5%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	5.2%	1.9%	0.0%	0.3%	0.0%	0.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	-1.1%	-2.5%	3.7%	-5.7%	3.9%	2.5%
OML	Old Mutual plc	10.0%	5.4%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%
REM	Remgro Ltd	10.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.0%	0.0%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.2%	6.3%	-1.6%	-0.9%	-1.1%
SAB	SABMiller plc	5.8%	3.4%	10.0%	10.0%	8.7%	3.2%	0.0%	0.0%	0.5%	0.0%	0.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	-0.1%	-1.5%	1.6%	1.6%	-1.5%
SHF	Steinhoff Int Hldgs Ltd	10.0%	10.0%	9.3%	3.7%	0.0%	0.0%	0.2%	0.0%	0.2%	0.1%	0.0%
SHP	Shoprite Holdings Ltd	-40.3%	-23.9%	-17.3%	-12.3%	-8.1%	-5.0%	-13.9%	-4.0%	-3.4%	-9.2%	-2.9%
SLM	Sanlam Limited	0.0%	-12.3%	-8.9%	1.8%	3.4%	4.5%	-2.0%	0.0%	-3.1%	5.0%	-3.7%
SOL	Sasol Limited	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.0%	0.0%	0.0%	0.0%
TBS	Tiger Brands Ltd	0.0%	0.0%	-30.8%	0.0%	-17.8%	-8.0%	-14.3%	-8.3%	-7.1%	-6.3%	-5.5%
WHL	Woolworths Holdings Ltd	6.3%	6.9%	7.5%	8.0%	8.4%	8.6%	-0.4%	0.0%	3.9%	3.3%	-0.2%
		3.8%	-51.9%	-88.9%	-19.6%	-48.2%	-37.6%	-66.5%	-28.6%	-29.4%	-32.4%	-21.1%

Common size weight comparison

		19	22	23	24	25	22	17	29	24	26	27
		1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	0%	0%	100%	100%	100%	100%	0%	100%	100%	100%	100%
AMS	Anglo American Plat Ltd	0%	100%	100%	100%	0%	0%	0%	100%	100%	100%	100%
ANG	Anglogold Ashanti Ltd	100%	100%	100%	100%	100%	1652%	87%	791%	68%	209%	66%
APN	Aspen Pharmacare Hldgs Ltc	168%	141%	114%	93%	75%	63%	527%	58%	56%	652%	54%
BIL	BHP Billiton plc	0%	0%	0%	0%	0%	100%	0%	100%	100%	100%	100%
BVT	Bidvest Ltd	100%	100%	256%	100%	4821%	4014%	15356%	50%	94%	109%	0%
DSY	Discovery Ltd	100%	100%	6398%	2271%	209%	247%	477%	134%	118%	164%	114%
FSR	Firstrand Ltd	100%	100%	100%	100%	100%	100%	11%	100%	100%	19%	171%
GRT	Growthpoint Prop Ltd	100%	100%	100%	100%	617%	194%	221%	181%	173%	184%	167%
IMP	Impala Platinum Hlgs Ltd	100%	0%	0%	0%	100%	0%	0%	100%	100%	100%	100%
INL	Investec Ltd	0%	100%	100%	100%	100%	100%	0%	100%	0%	100%	0%
INP	Investec plc	100%	100%	100%	100%	100%	0%	0%	100%	0%	0%	0%
IPL	Imperial Holdings Ltd	0%	0%	0%	0%	0%	100%	0%	0%	100%	100%	100%
MDC	Mediclinic Internat Ltd	100%	92326%	4335%	388%	312%	260%	296%	254%	240%	235%	223%
MPC	Mr Price Group Ltd	354%	297%	239%	196%	158%	132%	1838%	122%	307%	119%	150%
MTN	MTN Group Ltd	100%	100%	100%	100%	100%	100%	0%	100%	0%	0%	100%
NED	Nedbank Group Ltd	100%	100%	100%	100%	100%	100%	97%	91%	28%	71%	25%
NPN	Naspers Ltd -N-	0%	0%	0%	0%	0%	0%	0%	100%	0%	100%	100%
NTC	Netcare Limited	100%	100%	100%	100%	100%	119%	149%	63%	1969%	61%	70%
OML	Old Mutual plc	0%	0%	0%	100%	100%	0%	0%	100%	100%	0%	100%
REM	Remgro Ltd	0%	100%	100%	100%	100%	100%	0%	100%	100%	100%	100%
RMH	RMB Holdings Ltd	100%	100%	100%	100%	100%	100%	34%	26%	386%	170%	211%
SAB	SABMiller plc	0%	0%	0%	0%	0%	0%	100%	100%	0%	100%	100%
SBK	Standard Bank Group Ltd	100%	100%	100%	100%	100%	100%	106%	342%	55%	56%	401%
SHF	Steinhoff Int Hldgs Ltd	0%	0%	0%	0%	100%	0%	0%	100%	0%	0%	100%
SHP	Shoprite Holdings Ltd	901698%	339%	273%	223%	181%	150%	526%	140%	134%	306%	129%
SLM	Sanlam Limited	100%	22336%	896%	82%	66%	55%	147%	100%	269%	50%	450%
SOL	Sasol Limited	0%	100%	100%	100%	100%	0%	0%	100%	100%	100%	100%
TBS	Tiger Brands Ltd	100%	100%	1766%	100%	574%	180%	335%	198%	181%	163%	155%
WHL	Woolworths Holdings Ltd	37%	31%	25%	20%	16%	14%	138%	100%	24%	27%	119%
		96%	152%	189%	120%	148%	138%	167%	129%	129%	132%	121%

5.9 Research Question Seven – Minimum variance behaviour

Minimum variance behaviour: Does the beta constraint force the long only portfolio to behave similar to a minimum variance portfolio rather than a mean variance efficient portfolio?

The inputs used to perform the calculations in respect of systematic risk for the mean variance portfolio and the minimum variance portfolio are the same as those used in calculating the imputed Clarke et al (2011) weights.

5.9.1 Research Question Seven (a)

Systematic risk: Do any of the portfolios conform to the 80 to 90% range of portfolio risk being systematic that Clarke et al. (2011) find for minimum variance portfolios?

The portfolio risk figures for the optimized portfolios are presented below.

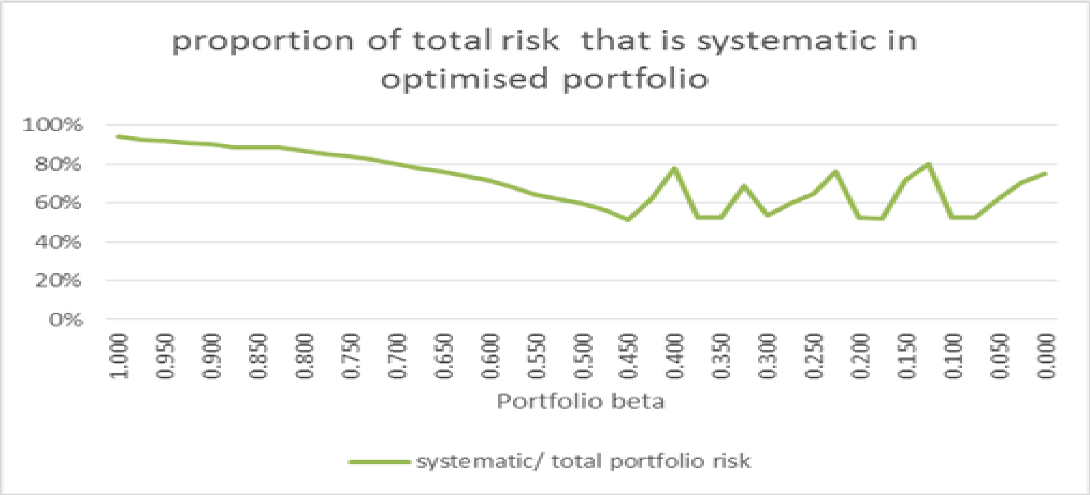
Table 15: Summarised risk measures for initial portfolios across target betas

Initial optimisation

Total Market Risk	σ_M^2	0.0032																			
Total Portfolio risk	σ_{LMV}^2	0.0034	0.0033	0.0032	0.0031	0.0029	0.0028	0.0026	0.0025	0.0024	0.0020	0.0019	0.0018	0.0017	0.0016	0.0014	0.0019	0.0014	0.0012	0.0012	0.0018
Portfolio Beta target	β_{pT}	1.0000	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7000	0.6750	0.6500	0.6250	0.6000	0.5000	0.4000	0.3000	0.2000	0.1000	0.0000
Achieved Beta	β_p	1.0000	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7000	0.6750	0.6500	0.6250	0.6000	0.5000	0.6736	0.4818	0.4488	0.4488	0.6444
	β_p^2	1.0000	0.9506	0.9025	0.8556	0.8100	0.7656	0.7225	0.6806	0.6400	0.4900	0.4556	0.4225	0.3906	0.3600	0.2500	0.4537	0.2322	0.2014	0.2014	0.4153
Threshold portfolio beta	β_L	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442	0.7442
Systematic Portfolio risk	$\beta_p^2\ \sigma_M^2$	0.0032	0.0031	0.0029	0.0028	0.0026	0.0025	0.0023	0.0022	0.0021	0.0016	0.0015	0.0014	0.0013	0.0012	0.0008	0.0015	0.0008	0.0007	0.0007	0.0013
systematic/ total portfolio risk	$\frac{\beta_p^2\ \sigma_M^2}{\sigma_{LMV}^2}$	94%	92%	92%	91%	91%	89%	88%	88%	87%	80%	78%	76%	74%	71%	60%	78%	53%	52%	52%	75%
portfolio beta/threshold beta	$\frac{\beta_p}{\beta_L}$	134%	131%	128%	124%	121%	118%	114%	111%	107%	94%	91%	87%	84%	81%	67%	91%	65%	60%	60%	87%

The relationship between systematic and total risk is shown in the graph below.

Figure 26: Relationship between the proportion of systematic risk to total portfolio risk across the different target beta portfolios from initial optimisation



The calculated values in respect of the 24 month rebalanced portfolio are similar to those of the initial portfolio.

Table 16: Summarised risk measures for rebalanced portfolios across target betas

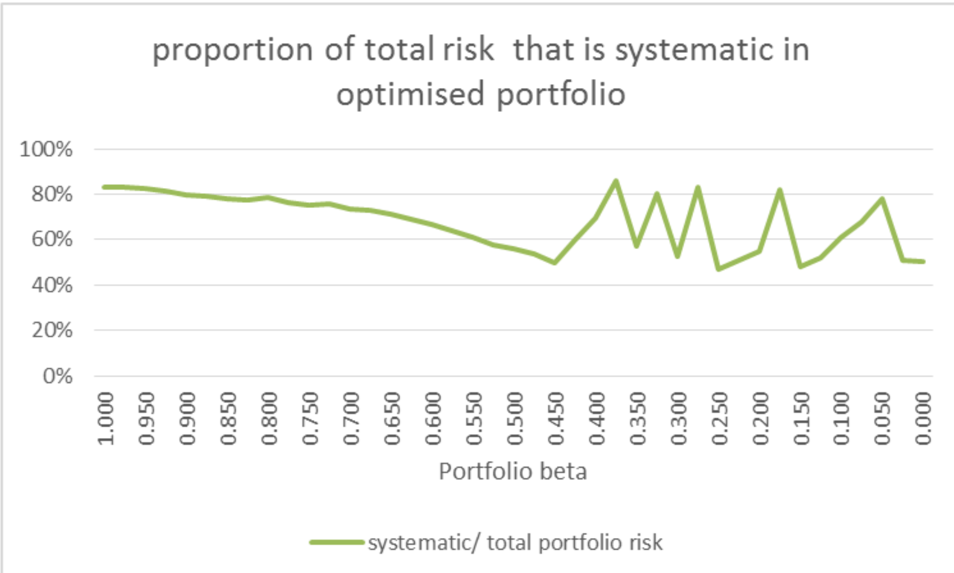
Rebalanced optimisation

Total Market Risk	σ_M^2	0.0029									
Total Portfolio risk	σ_{LMV}^2	0.0035	0.0033	0.0031	0.0030	0.0029	0.0028	0.0026	0.0025	0.0023	0.0019
Portfolio Beta target	β_{pT}	1.0000	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7000
Achieved Beta	β_p	1.0000	0.9750	0.9500	0.9250	0.9000	0.8750	0.8500	0.8250	0.8000	0.7000
	β_p^2	1.0000	0.9506	0.9025	0.8556	0.8100	0.7656	0.7225	0.6806	0.6400	0.4900
Threshold portfolio beta	β_L	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185
Systematic Portfolio risk	$\beta_p^2\ \sigma_M^2$	0.0029	0.0027	0.0026	0.0025	0.0023	0.0022	0.0021	0.0020	0.0018	0.0014
systematic/ total portfolio risk	$\frac{\beta_p^2\ \sigma_M^2}{\sigma_{LMV}^2}$	83%	83%	82%	82%	80%	79%	78%	77%	78%	73%
portfolio beta/threshold beta	$\frac{\beta_p}{\beta_L}$	139%	136%	132%	129%	125%	122%	118%	115%	111%	97%

Total Market Risk	σ_M^2											
Total Portfolio risk	σ_{LMV}^2	0.0018	0.0017	0.0016	0.0015	0.0013	0.0015	0.0012	0.0011	0.0012	0.0011	
Portfolio Beta target	β_{pT}	0.6750	0.6500	0.6250	0.6000	0.5000	0.4000	0.3000	0.2000	0.1000	0.0000	
Achieved Beta	β_p	0.6750	0.6500	0.6250	0.6000	0.5000	0.5966	0.4683	0.4688	0.4989	0.4414	
	β_p^2	0.4556	0.4225	0.3906	0.3600	0.2500	0.3559	0.2193	0.2197	0.2489	0.1949	
Threshold portfolio beta	β_L	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	0.7185	
Systematic Portfolio risk	$\beta_p^2\ \sigma_M^2$	0.0013	0.0012	0.0011	0.0010	0.0007	0.0010	0.0006	0.0006	0.0007	0.0006	
systematic/ total portfolio risk	$\frac{\beta_p^2\ \sigma_M^2}{\sigma_{LMV}^2}$	73%	71%	69%	67%	56%	70%	53%	55%	61%	50%	
portfolio beta/threshold beta	$\frac{\beta_p}{\beta_L}$	94%	90%	87%	84%	70%	83%	65%	65%	69%	61%	

The ratio of systematic to total risk is sustained through the rebalancing as the graph below indicates.

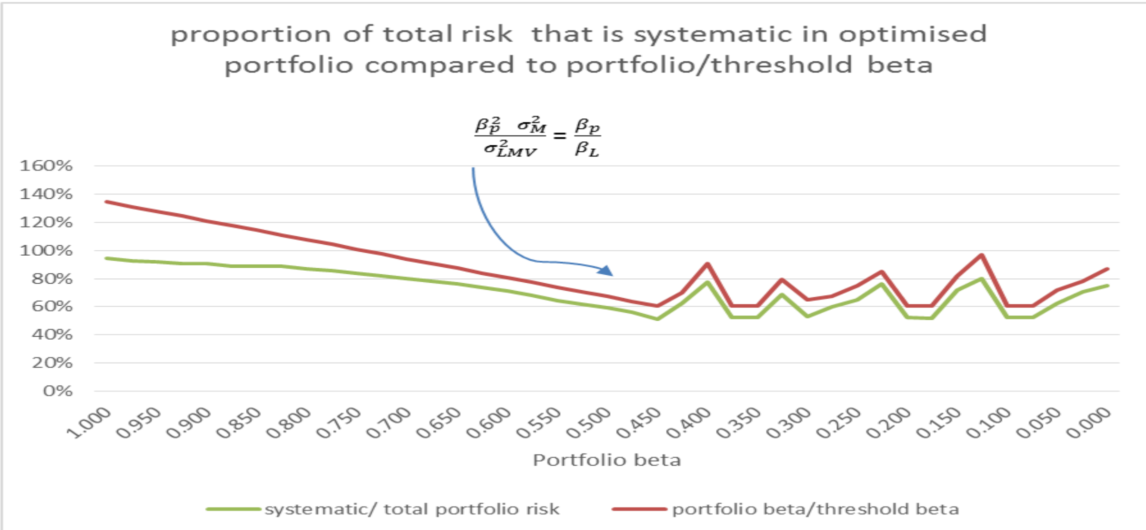
Figure 27:Relationship between the proportion of systematic risk to total portfolio risk across the different rebalanced target beta portfolios



As shown in the preceding calculation tables and graphs, target beta 0.975 to 0.65 in the initial optimization and target beta 0.1 to 0.725 portfolios after rebalancing approximately fall within the 80 to 90 % range of portfolio risk being systematic.

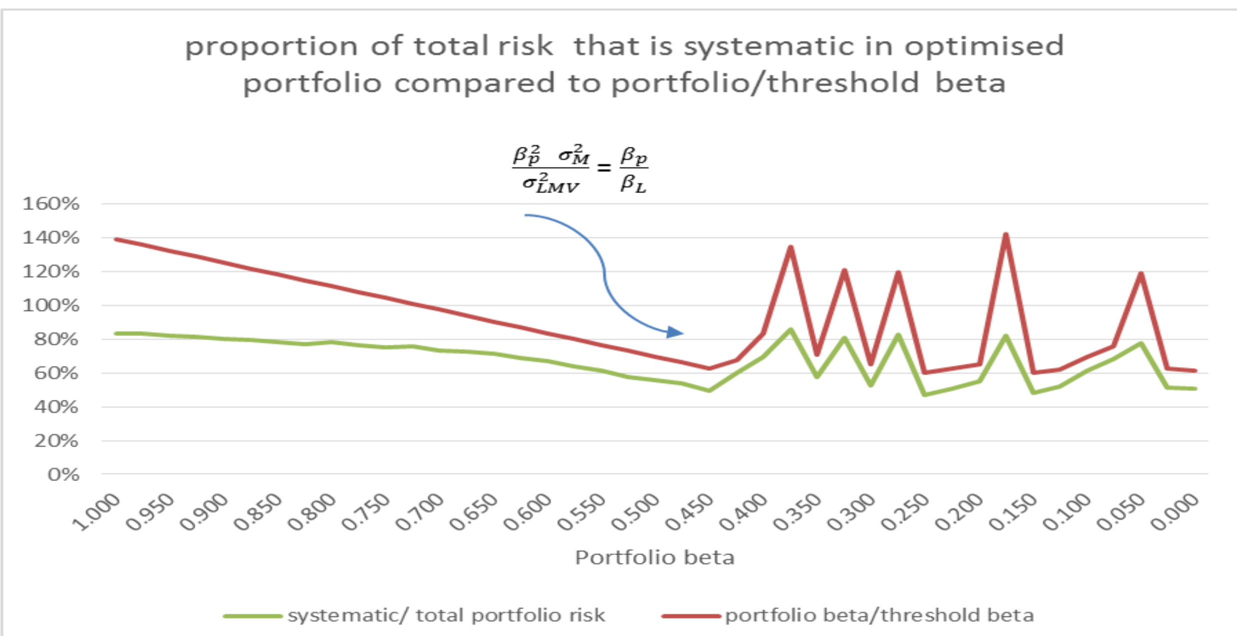
Furthermore it is clear that the equation $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ begins to be more characteristic of the optimized portfolios as the target beta is reduced. This is evident from the graphs below.

Figure 28: Relationship between the equality of proportion of total risk that is systematic to the portfolio beta/threshold beta ratio across initial optimisation portfolios to determine if Clarke et al analytic equation is valid for mean variance optimisation



The same relationship applies in the rebalanced portfolio as shown below

Figure 29: Relationship between the equality of proportion of total risk that is systematic to the portfolio beta/threshold beta ratio across rebalanced optimisation portfolios to determine if Clarke et al analytic equation is valid for mean variance optimisation

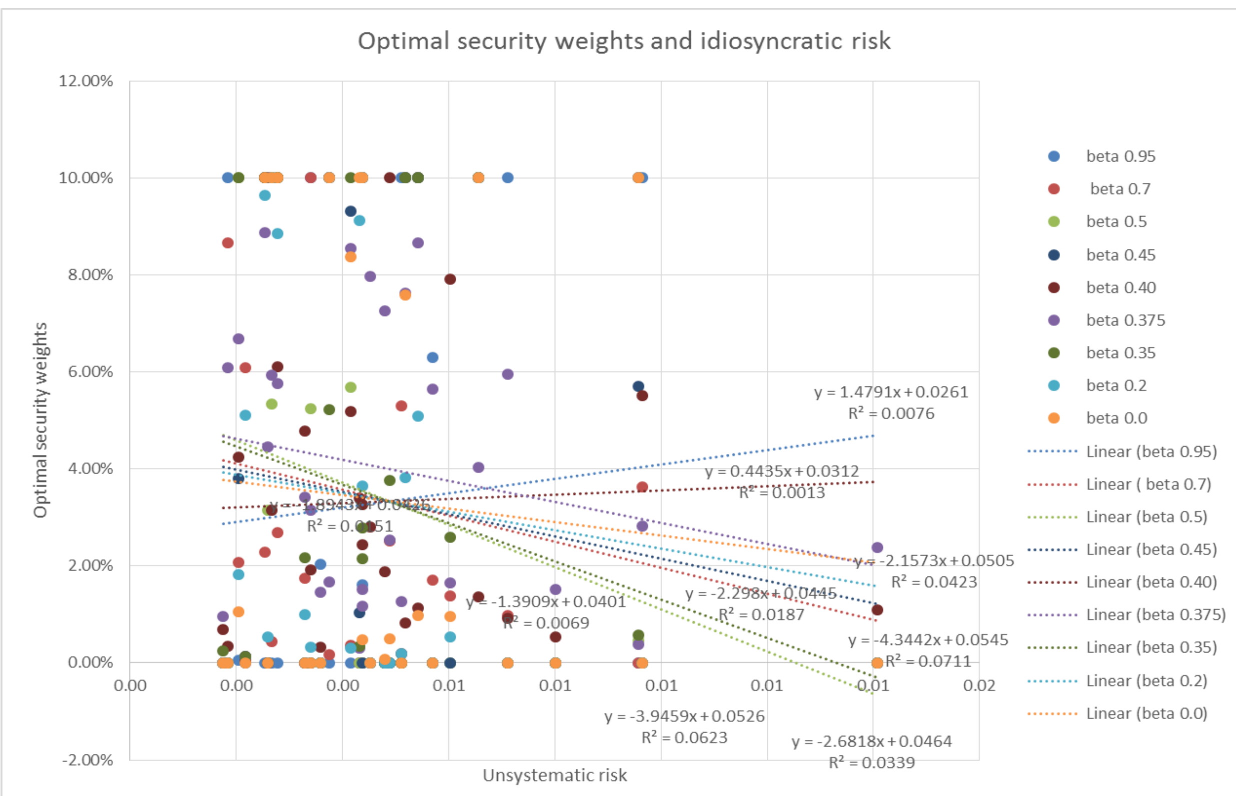


5.9.2 Research Question Seven (b)

Cross sectional variation of portfolio weights: does the cross sectional variation in weights of stocks in the portfolio depend on the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$ and individual market beta β_i parameters as in the Clarke et al. (2011) case?

Regression of weights of stocks in the mean variance optimized portfolios against the individual stock ex-ante unsystematic variances of returns $\sigma_{\varepsilon i}^2$ was undertaken. A cross sectional dependence relationship could not be established. The following compound chart illustrates the lack of a discernable relationships in the variation in weights against return variances.

Figure 30: Cross sectional variation in weights of stocks in the rebalanced portfolios to the ex-ante unsystematic variance of returns



The R^2 values in respect of the target beta rebalanced portfolios included in the above chart are all below 0.05 except for the beta 0.35 portfolio who R^2 value is 0.06. The low values indicate that the cross sectional relationship between weights in optimized target beta portfolios and unsystematic variance of returns $\sigma_{\varepsilon i}^2$ is not well pronounced at all.

Weights of stocks in the optimized portfolios were regressed against their individual market beta. The cross sectional variation of weights appears to depend more on the individual market beta in a more pronounced manner at the lower levels of target beta, within the confines of the optimizable range. As can be seen in the beta 0.45 case, for the initial optimization and subsequent rebalancing where the R^2 values of 0. 22 and 0.47 respectively are indicative of a strong relationship.

Figure 31:Cross sectional variation in individual weights to individual market beta in initial target beta 0.45 optimised portfolio

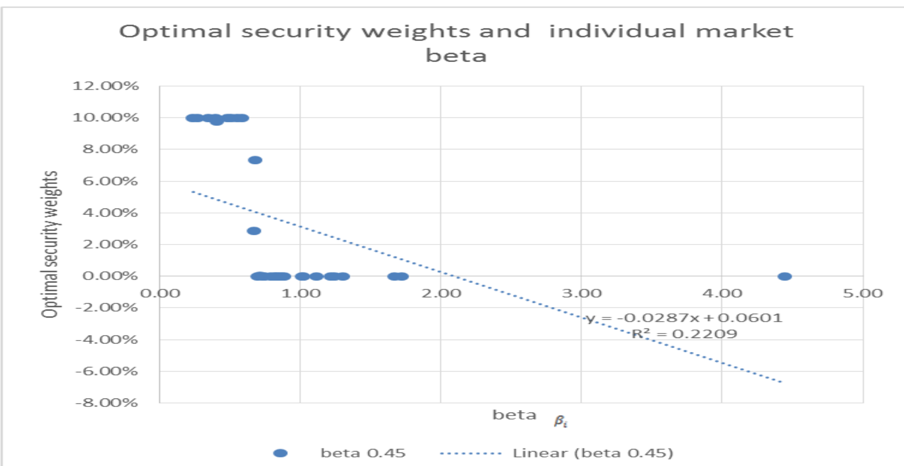
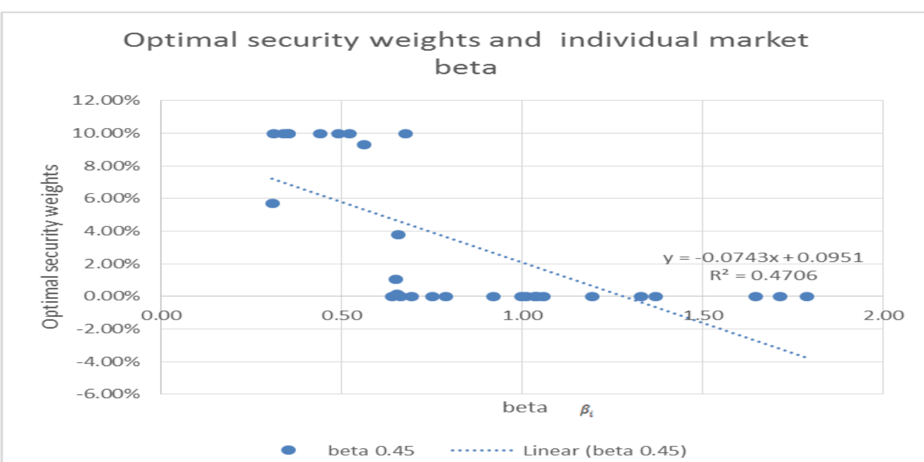


Figure 32:Cross sectional variation in individual weights to individual market beta in rebalanced target beta 0.45 optimised portfolio



Whereas at the higher levels of beta such as 0.7 the relationship is less pronounced as shown below with R^2 values of 0. 058 and 0.048 respectively .

Figure 33: Cross sectional variation in individual weights to individual market beta in initial target beta 0.70 optimised portfolio

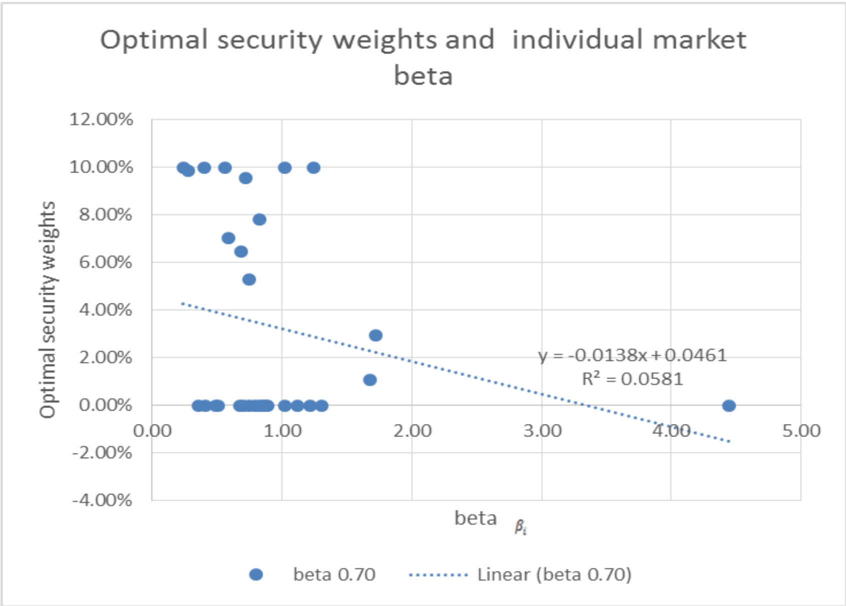
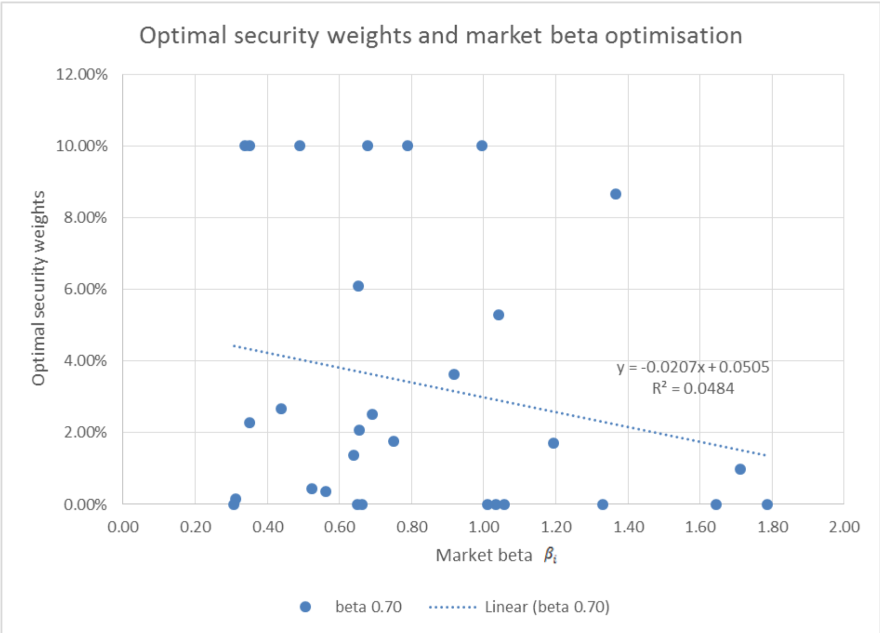
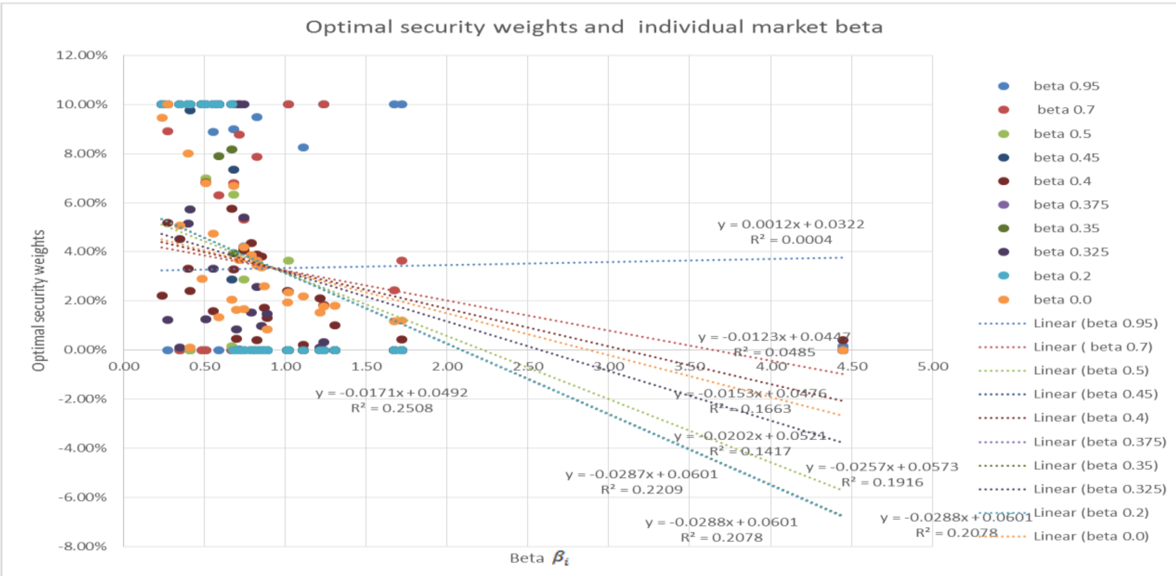


Figure 34:Cross sectional variation in individual weights to individual market beta in rebalanced target beta 0.70 optimised portfolio



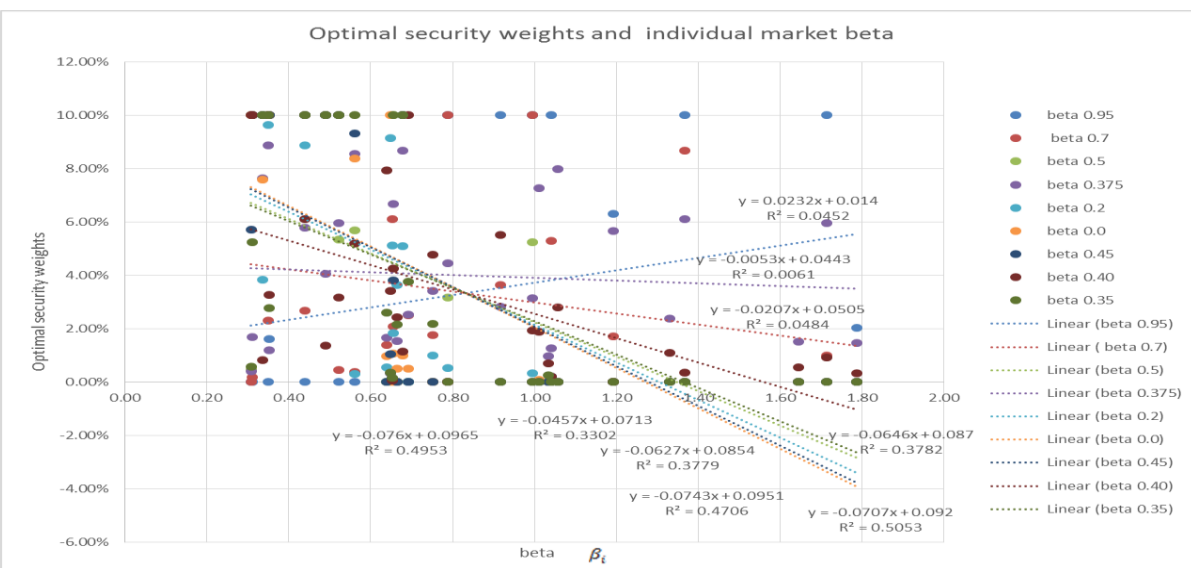
A compound graph of regressions of different target beta portfolios, illustrates the relationship further, as the R^2 values of the high target beta portfolios are markedly lower than those of the low target beta portfolios down to the 0.45 beta threshold in the initial optimisation. A similar relationship is sustained through the rebalancing.

Figure 35: Cross sectional variation in weights of stocks to the individual market betas in selected initial optimised portfolios



Rebalanced portfolios:

Figure 36 Cross sectional variation in weights of stocks to the individual market betas in selected in rebalanced portfolios



The results clearly show that cross sectional variation of weights in stocks in the mean variance optimized portfolio is dependent on individual market betas of those stocks particularly as the portfolio target beta is decremented towards zero, within the optimizable threshold. However the relationship between weights in stocks with respect to the ex-ante unsystematic stock variances does not appear to be present at all.

5.10 Research Question Eight –Long /Short Hedge Fund Replication

Is there an optimal level set of low beta constrained long only equity portfolios that can generate a risk return profile to satisfy investors interested in a controlled range of exposure to exploitable broad systematic risk from unit beta down to beta neutrality without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral, but focused more on performance and diversification benefits?

The results in respect of mean variance optimization show that the research was able to controllably constrain a long only portfolio to mimick a long/short market neutral down to the level of a target beta 0.45 without having to employ shorting or leveraging techniques. Important characterising constructs such and active share and the industry concentration index were shown to exhibit strongly quadratic / second order polynomial relationship to the target beta constraint.

Whereas a very strong first order linear relationship between tracking error volatility and the target portfolio beta has been established, the relationship between the number of shares and target beta has proved to be somewhat tenuous. A weakly log-linear relationship was what could be characterised based on the results and data produced by this study.

Risk and risk adjusted returns measures including the variances, the Sharpe ratio, the information ratio and Treynor ratio provided valuable insights into the return behaviour of low beta optimized portfolios. Robust testing for the equality of Sharpe ratios (Ledoit & Wolf, 2008) in respect of various optimized portfolios pairs at the 5 percent significance level did not yield conclusively positive results. However the testing of differences between variances using another robust test from Ledoit and Wolf (2011) yielded strong positive results. At the alpha level of 5 percent, the null hypothesis of equal variances was able to be rejected and accordingly relevant target beta portfolios were shown to be different from each other. These results helped to further characterize the optimization solution set.

Cumulative Index value results were able to provide sharp focus both quantitatively and qualitatively through visual inspection (Ward and Muller, 2012) as to the real world value creation potential of the optimized portfolios with the low beta cohort performing well. A ranking of the portfolios indicated that the low beta threshold portfolios ranging from beta target 0. 525 down to 0.475 would have significantly outperformed the JSE All share index and JSE/FTSE Top 40. In the solution set the beta target 0.525 portfolio would have produced a cumulative index value of 2.89 against a value of 3.8 for the long /short portfolio and 1.51 for the JSE All share index.

Table 17:summary of selected risk / return measures and variables across the target portfolios

Target beta	1.0000	0.9000	0.8000	0.7000	0.6000	0.5000	0.4750	0.4500	0.4250	0.4000	0.3000	0.2000	0.1000	0.0000
Achieved beta initial	1.0000	0.9000	0.8000	0.7000	0.6000	0.5000	0.4750	0.4500	0.5225	0.6736	0.4818	0.4488	0.4488	0.6444
Achieved beta rebalance	1.0000	0.9000	0.8000	0.7000	0.6000	0.5000	0.4750	0.4500	0.4853	0.5966	0.4683	0.4688	0.4989	0.4414
Active share 2014 JSE Top 40	65%	71%	61%	64%	68%	80%	79%	87%	79%	69%	87%	81%	77%	86%
Active share 2014 JSE ALSI	69%	75%	66%	69%	71%	82%	82%	89%	82%	72%	89%	83%	81%	88%
Tracking error JSE/FTSE Top 40	0.0157	0.0225	0.0232	0.0263	0.0305	0.0371	0.0380	0.0362	0.0320	0.0251	0.0368	0.0331	0.0341	0.0319
Tracking error JSE ALSI	0.0140	0.0198	0.0208	0.0234	0.0272	0.0334	0.0341	0.0323	0.0279	0.0210	0.0329	0.0292	0.0302	0.0278
Industry conc. index 2014 JSE Top 40	9%	11%	9%	9%	11%	14%	14%	16%	13%	12%	14%	12%	14%	14%
Industry conc. index 2014 JSE ALSI	8%	9%	7%	8%	9%	12%	12%	14%	11%	10%	12%	10%	12%	12%
Number of stocks 2011-2012	18	16	14	14	14	14	13	13	22	29	17	14	13	29
Number of stocks 2013-2014	15	14	16	13	15	20	21	18	21	29	14	21	20	18
Average monthly return	0.0107	0.0097	0.0127	0.0151	0.0174	0.0179	0.0178	0.0147	0.0139	0.0110	0.0163	0.0118	0.0145	0.0126
Cumulative returns index value	2.0141	1.9151	2.2072	2.4758	2.7594	2.8087	2.7977	2.4301	2.3514	2.0493	2.6111	2.1286	2.4101	2.2101
Standard deviation	0.0345	0.0351	0.0345	0.0352	0.0349	0.0364	0.0359	0.0347	0.0305	0.0304	0.0343	0.0319	0.0331	0.0322
Sharpe ratio	0.3115	0.2761	0.3676	0.4306	0.4992	0.4904	0.4947	0.4248	0.4563	0.3616	0.4735	0.3709	0.4387	0.3929
Information ratio JSE Top 40	0.4402	0.2703	0.3910	0.4419	0.4562	0.3867	0.3708	0.3176	0.3250	0.2983	0.3425	0.2626	0.3179	0.2964
Information ratio JSE ALSI	0.4398	0.2572	0.3982	0.4515	0.4724	0.3979	0.3864	0.3139	0.3333	0.3040	0.3545	0.2480	0.3284	0.2887

As the table above summarizing some of the risk / return measures shows, there is are characterisable patterns useful to the optimisation and development of low beta long alternatives.

The results from the study indicate some scope within which optimal target beta portfolio weights of securities are shown to approximate the simplified analytic equation by Clarke et al. (2011). The common size comparison show some similarities in magnitude of weights at a global level. However the number of stocks included in the optimal portfolio is markedly less than in the Clarke et al (2011) imputed portfolio but appears to converge as the target beta is lowered. There is thus a convergence towards $\frac{\beta_P^2\sigma_M^2}{\sigma_P^2} = \left(\frac{\beta_P}{\beta_L}\right)$ as the target beta is lowered.

Futhermore the some portfolios in the optimized portfolio, in particular from beta 0.9 down to 0.65 appear to fall with the minimum variance range of systematic/total risk of between 90 % down to 80 % that Clarke et al (2011) suggest to generalizable for minimum variance portfolios.

However with respect to the cross sectional variation of portfolio weights in the target portfolios with ex-ante unsystematic variances, this research study could not find a significant relationship to at play in the case of mean optimized portfolios relative to minimum variance portfolios. Where a strong and significant relationship was exhibited was in the cross sectional variation of portfolio weights to individual market beta. The relationship was found to be increasingly pronounced particularly in the low target beta portfolios.

The sum total of the results summarized above provide a solid grounds with which to answer in the affirmative the broader research question of whether or not there exists an optimal level beta constrained equity portfolio set that can generate a risk return profile to satisfy investors interested in controlled range of exposure to exploitable broad systematic risk from unit beta down to beta neutrality without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral.

6 ANALYSIS

The proceeding discussion of results aims to show the extent to which the research questions posed have been addressed, the extent to which objectives of the research, particularly in the characterization of low beta alternatives to the long/short market neutral or long /short low beta hedge funds have been met. An understanding of the empirically achievable limits to mean variance constrained optimisation is considered critical from a contextual perspective, to the ability to pose viable long only alternatives to long/short equity hedge portfolios. The manner in which the behavior of the possible long only low beta alternatives can be effectively characterized in terms of clear theoretical constructs and frameworks such as mean variance and minimum variance as well as mathematically sound descriptions of relationship between relevant risk / return variables and other sample features such as number of shares and portfolio weights is extremely important not only from a perspective of gaining adequate understanding of expected behaviour of alternative solutions but also as foundation for building other innovative risk and investment management products in a manner that assures reliability and validity. In this regard therefore the results of this study are discussed and analysed per each of the research question posed.

6.1 Research Question One – Active share

Active share: Does active share behave in a linear manner as portfolio beta is progressively reduced from 1 down to zero and how does it relate to ex-post portfolio returns?

The results of the study indicate that active share in the optimized target beta portfolios behaves in a linear manner that is quadratic in nature as the target beta is reduced from 1.0 down to 0, within the scope of the optimizable threshold on 0.45.

The trendline of active share against target beta for optimised portfolios shows a relationship of the form.

$$y = ax^2 - bx + c$$

where y is active share , a and b are polynomial co-efficients and c is a constant . For the optimisation test period of four years , the value of c was observed to range between 0.54 and 0.73 with respect to active share calculated on the JSE/FTSE Top 40 benchmark and between 0.63 and 0.77 on the JSE ALSI.

The quadratic nature of the behaviour underlies the marked increase in returns that is observed as the target beta is reduced towards zero. The range of active share registered of between 51 to 91 percent on either JSE/FTSE Top 40 and JSE ALSI indicates that the optimised portfolios are in the upper half of the 0 to 100 percent active share range that Cremers and Petajisto (2009) observe to be available to long only portfolios. The findings of this study in which returns in excess of benchmarks, including the risk free rate are seen to increase significantly as the level of active share increases is theoretically consistent with the Cremers and Petajisto (2009) contention that active share significantly explains fund outperformance of benchmarks.

Although Muller and Ward (2011) did not find a relationship between level of active share and a fund's risk adjusted performance on the JSE, the algorithmic implementation of stock picking and the inherent systematic market timing in the case of this study's active share portfolios may account for the different results. The benchmarks are significantly outperformed by the portfolios with the higher active shares as indicated by the cumulative index values for example the beta target 0.525 portfolio produces a cumulative index value of 2.89 against a value of 1.51 for the JSE All share index and of 3.8 for the long /short portfolio with the highest active share. It is important to note and view the active share dimensions of stock selection and market timing that are observed and measured in this research, as being implemented algorithmically by means of the mean variance optimisation under constraints rather than from active human management.

6.2 Research Question Two – Tracking error

Tracking error: How does the tracking error respond to change in the beta constraint?

Cremers and Petajisto (2009) find tracking error to be a poor predictor of fund performance, this study it to be increasing significantly with decreasing beta. If the theory is correct then from a logical transitive perspective the relationship between tracking error and a declining beta is expected to be less pronounced relative to that shared between active share and declining beta. The relationship between tracking error and beta as shown in figure (6) for the JSE ALSI is highly linear.

But what is of interested is that it stays within a 0.5 to 5 percent range indicating that the volatility of excess returns to the benchmark increases in a less volatile manner to reductions in exposure to systematic risk, than is observed in the case with active share. So a first order polynomial relationship is characterized between tacking error and the target portfolio beta, whereas a second order polynomial relationship is apparent in the case of active share to target beta.

Furthermore in the case of professional investment manager settings in which tracking error control is a significant requirement Chan et al (1999), some of the optimised portfolios that are the subject of this research may not be useful for strict tracking error minimization, but may have application in respect of those investment requirements that tracking error be kept within a specified range.

6.3 Research Question Three – Industry concentration

> **Industry concentration Index:** Does the industry concentration index exhibit any interesting patterns?

The Industry Concentration Index is shown to behave in a manner similar to that of Active share. As per the results section in figures (9 and 10) a strong quadratic relationship is observed between the industry concentration index and the target portfolio beta.

The relation is of the form $y = ax^2 - bx + c$ where y : industry concentration index , a and b are polynomial co-efficients and c is a constant . For the test period from years 2010 to 20-15 , the value of c was observed to range between 0.069 and 0.12 with respect to active share calculated on the JSE/FTSE Top 40 benchmark and 0.061 and 0.1 on the JSE ALSI. The b coefficient had values of 0.0002 and 0.0004 for JSE/FTSE active share and 0.0003 to 0.0004 for that of JSE ALSI.

According to Kacperczyk et al (2005) funds with higher industry concentration as measured by the industry concentration ratio exhibit a strong relationship to outperformance on a risk adjusted basis, compared to more diversified funds with lower ratios. Despite their contention that the industry concentration index is a less useful square weights burdened hybrid measure falling between active share and tracking error, Cremers and Petajisto (2009) further corroborated its positive relationship to fund performance in their study which found active share and the industry concentration index to be both significant in the prediction of superior performance on benchmark adjusted returns.

The findings of this study that the optimised target beta portfolios show a generally decreasing number of stocks with decreasing beta and a quadratic increase in the industry concentration index, together with a concomitant increase in the benchmark adjusted returns resonates with the literature in respect of the behavior of industry concentration.

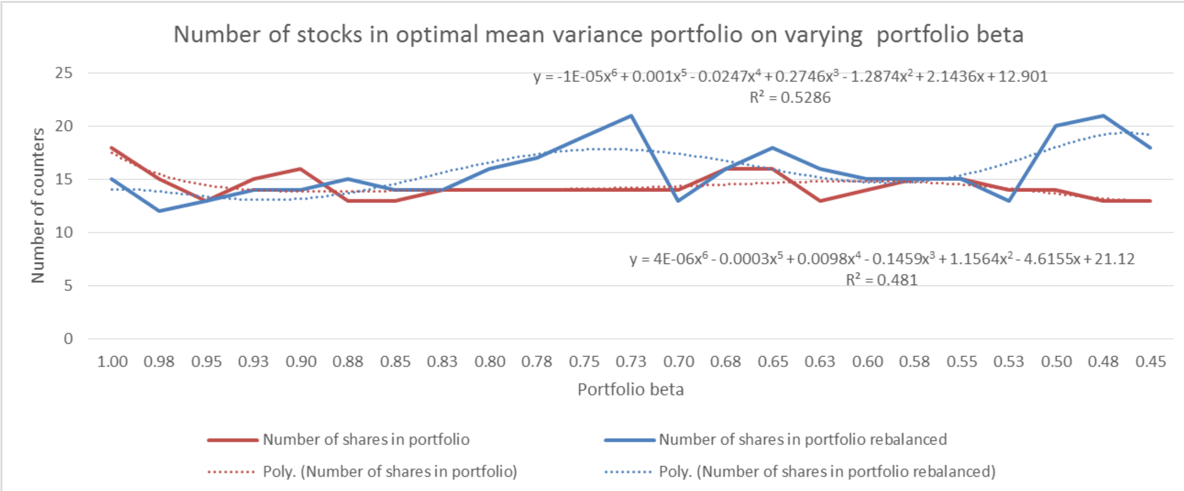
6.4 Research Question Four - Size

Number of stocks: How does the number of stocks in the solution set portfolio vary with the changing target beta?

As the target beta is reduced a reduction in the number of stocks can seen albeit with a decidedly fluctuating pattern as indicated in figure (13). The modest R^2 values of 0.18 and 0.24 for the initial optimisation and the subsequent rebalance are indicative of the reality that a considerable amount of the fluctuations in the number of stocks cannot be explained in a log linear manner.

A view worth considering is that because the optimisation constraints such as the limits in the portfolio weights and target beta potentially lead to a non-convex optimisation setting as suggested by Lobo et al (2007) or even the alternative view drawn from Branke et al (2009) whereby the target beta can be considered as a special type of constraint that is convex in nature and mimicking an upper bound on the variance of the portfolio. Either perspective introduces significant complexity to the optimisation on the basis of which it can therefore be argued that the number of stocks becomes a complex function of the ability of the optimisation algorithm to find relevant global or local minima, if not at least viable Karush–Kuhn–Tucker conditions (Boyd & Vandenberghe, 2004). The relationship may therefore be better explained by a relatively more complex higher order polynomial equation as the following sixth order polynomial approximations tentatively suggest with higher R^2 values of 0.48 and 0.52 respectively.

Figure 37: Cross sectional variation in number of stocks in optimised initial portfolio (period 2011 to 2012) and rebalanced portfolio (2013 to 2014) across the different target beta portfolios with sixth order polynomial trend approximation



Thus the Clarke et al (2011) generalization that the mean variance portfolios would hold a relatively small of the set of the universe of stocks available for optimisation despite the existence of constraints and complexity, citing the dominant role of the variance minimization aspects of general mean variance objective functions does not appear to hold as strongly in the terms of the results of this study. The type of constraints introduced, particularly in respect of the minimum and maximum weights and the declining target beta clearly increase the level of complexity to the extent that the number of stocks in the solution set is generally more than would be expected in a typical minimum variance optimisation.

6.5 Research Question Five - Returns

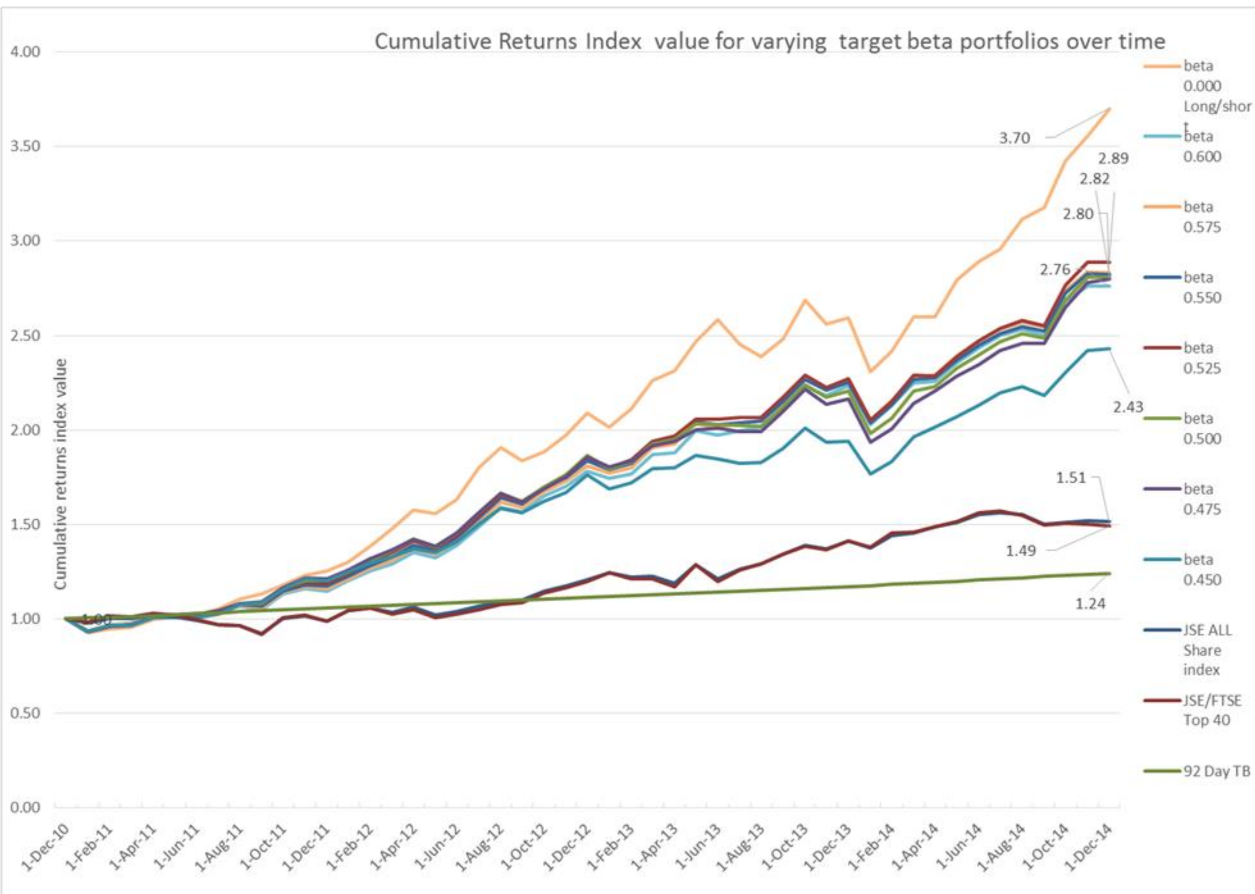
Returns: What are the patterns and effects on realised average returns, Sharpe ratios, information ratios, the treynor ratios as well as related measures of the varying constraint beta portfolios and how do they compare with a market neutral long short strategy?

6.5.1 Returns

Ward and Muller (2012) contend that the use of average return figures and applying statistical tests is methodologically weak owing to the time series nature of the data which invalidates the power of typical tests. Indeed Ledoit and Wolf (2008), deMiguel et al (2009), Laurini et al (2012) amongst others point out some of the challenges with time series data, in particular the strict requirements for data to be independent and identically distributed as well as homoscedastic and mesokurtic in order for tests of differences to be useful.

Visual inspection of the cumulative returns index value of the optimised portfolios offers compelling evidence of the performance differentials in the absence of robust statistical tests or as a complement to such tests.

Figure 38: Figure 14: Cumulative returns index value time series plot for selected target beta portfolios for 2011 to 2014, showing growth in index values from a 31 Dec 2010 base value of 1.00



On the basis of that inspection of the chart above, the beta range 0.45 to 0.60 portfolios show the high cumulative return index values of 2.43 up to 2.89. The long / short beta 0.00 has the highest index value of 3.7. The ALSI and Top 40 lag behind with values of 1.51 and 1.49 respectively.

6.5.2 Sharpe ratio

The visual inspection is qualitatively useful in interrogating the results from robust statistical testing. The results from testing for differences in the Sharpe ratios using Ledoit and Wolf's (2008) bootstrapping techniques were that none of the portfolio showed significant risk adjusted differences. However from the visual inspection significant differences in risk adjusted returns by way of the cumulative index value can be readily observed. Chopra, Hensel and Turner (1993) note high volatility of returns and relatively short testing time horizons as being key possible contributors to instances were a lack of statistical significance arises in risk adjusted performance testing. The out of sample testing period in this research of 48 months is considerably shorter than the 72 months cited in the Chopra et al (1993) study as being relatively short. Furthermore the relative size of the standard deviation in comparison to the mean return is large and in some instances more than double the size as shown in the summary table (17). Accordingly it may require a greater period of time for differences to be found significant by the test used.

6.5.3 Information and Treynor ratios

The similarity between the information ratio and the Sharpe ratio save for the choice of benchmark (Goodwin, 1998) is evident in the information ratio profile that has been in line with the Sharpe ratio characteristic in which it increases with the lowering of the target beta to the extent that the underlying benchmarks move in tandem with the risk free rate for the study's set of results. The impact of the different benchmarks is seen in year 2013 which saw a significant sharp decrease in the information ratio into negative territory even before the threshold of beta 0.45 was reached. The Treynor ratio profiles wherein the underlying denominator is systematic risk exhibit a very strong linear relationship with respect to the decreasing target of portfolio beta. This signifies an increase in systematic risk adjusted returns as the level of portfolio beta is reduced.

6.5.4 Integrated analysis of results from risk return metrics and measures and statistical test results

Given the preceding analysis on the Sharpe and information ratios therefore it was not unexpected that the Ledoit and Wolf (2011) robust test for differences in variances was more powerful in distinguishing between the different portfolios. The results of the Ledoit and Wolf (2011) test prove to be interesting from a triangulation perspective. It is shown through the different risk return metrics, active share, industry concentration index, tracking error, cumulative index return value that the range of interest for a potential solution set is from beta 0.475 to beta 0.600 – referred to from here onwards as set (P). The highest Sharpe ratios are all within this set and the top rankings by cumulative index return value are heavily represented in this set.

Table 18 Summary of key results and measures including Ledoit and Wolf (2011) variances test , cumulative return index values , industry concentration index values and Sharpe ratios

Test of variances - Ledoit and Wolf (2011)					Comparative meausures								
target beta portfolio	Portfolio pair	Difference	p.Value	Benjamini & Yekutieli p.Value	Sharpe ratio	Excess Returns	Standard deviation	Variance	Active share ALSI 2014	Ind. Conc.Ind ALSI 2014	Tracking error ALSI	Cum Ret Index Value	Value Ranking
1.000	42-1	-0.4140	0.1350	0.5954	0.3115	0.0107	0.0345	0.0011885	69.4%	8.1%	1.4%	2.014	36
0.975	42-2	-0.4190	0.1060	0.4795	0.2702	0.0093	0.0344	0.0011830	68.2%	8.5%	1.6%	1.881	42
0.950	42-3	-0.4290	0.0670	0.3428	0.2815	0.0096	0.0342	0.0011713	67.9%	8.3%	1.7%	1.912	40
0.925	42-4	-0.3740	0.1010	0.4689	0.2717	0.0096	0.0352	0.0012369	75.5%	9.2%	1.9%	1.902	41
0.900	42-5	-0.3760	0.0980	0.4673	0.2761	0.0097	0.0351	0.0012348	74.9%	9.4%	2.0%	1.915	39
0.875	42-6	-0.3840	0.0860	0.4214	0.3068	0.0107	0.0350	0.0012255	74.2%	9.2%	2.0%	2.012	37
0.850	42-7	-0.3950	0.0680	0.3428	0.3204	0.0112	0.0348	0.0012123	69.8%	8.5%	2.0%	2.053	34
0.825	42-8	-0.4260	0.0460	0.2618	0.3376	0.0116	0.0343	0.0011747	67.7%	8.3%	2.0%	2.095	33
0.800	42-9	-0.4130	0.0480	0.2646	0.3676	0.0127	0.0345	0.0011902	65.8%	7.4%	2.0%	2.207	29
0.775	42-10	-0.4120	0.0510	0.2726	0.3887	0.0134	0.0345	0.0011908	65.4%	7.6%	2.1%	2.285	24
0.750	42-11	-0.4040	0.0410	0.2411	0.3956	0.0137	0.0347	0.0012012	62.5%	6.9%	2.2%	2.316	22
0.725	42-12	-0.3960	0.0340	0.2142	0.4174	0.0145	0.0348	0.0012105	59.7%	6.2%	2.2%	2.406	20
0.700	42-13	-0.3750	0.0370	0.2251	0.4306	0.0151	0.0352	0.0012366	68.6%	7.9%	2.3%	2.476	17
0.675	42-14	-0.3770	0.0330	0.2142	0.4416	0.0155	0.0351	0.0012333	67.1%	7.7%	2.4%	2.519	15
0.650	42-15	-0.3720	0.0240	0.1841	0.4633	0.0163	0.0352	0.0012396	66.0%	7.7%	2.5%	2.616	11
0.625	42-16	-0.3800	0.0150	0.1260	0.4723	0.0166	0.0351	0.0012307	68.9%	8.0%	2.6%	2.648	10
0.600	42-17	-0.3880	0.0020	0.0294	0.4992	0.0174	0.0349	0.0012207	70.9%	9.2%	2.7%	2.759	8
0.575	42-18	-0.3780	0.0020	0.0294	0.5130	0.0180	0.0351	0.0012324	73.9%	9.9%	2.9%	2.833	3
0.550	42-19	-0.3640	0.0030	0.0353	0.5075	0.0179	0.0353	0.0012494	75.7%	9.9%	3.1%	2.823	5
0.525	42-20	-0.3330	0.0060	0.0623	0.5138	0.0185	0.0359	0.0012894	81.6%	11.6%	3.2%	2.889	2
0.500	42-21	-0.3030	0.0080	0.0706	0.4904	0.0179	0.0364	0.0013283	81.8%	12.2%	3.3%	2.809	6
0.475	42-22	-0.3320	0.0080	0.0706	0.4947	0.0178	0.0359	0.0012910	82.1%	11.7%	3.4%	2.798	7
0.450	42-23	-0.4020	0.0010	0.0221	0.4248	0.0147	0.0347	0.0012034	89.2%	14.0%	3.2%	2.430	18
0.425	42-24	-0.6620	0.0000	0.0000	0.4563	0.0139	0.0305	0.0009278	81.9%	10.9%	2.8%	2.351	21
0.400	42-25	-0.6680	0.0040	0.0441	0.3616	0.0110	0.0304	0.0009221	72.5%	10.1%	2.1%	2.049	35
0.375	42-26	-0.3520	0.0270	0.1985	0.4520	0.0161	0.0356	0.0012650	69.2%	10.3%	2.4%	2.585	13
0.350	42-27	-0.3730	0.0220	0.1764	0.4811	0.0169	0.0352	0.0012387	81.5%	13.3%	3.2%	2.692	9
0.325	42-28	-0.3150	0.0300	0.2103	0.4287	0.0155	0.0362	0.0013121	68.5%	7.2%	2.6%	2.516	16
0.300	42-29	-0.4230	0.0030	0.0353	0.4735	0.0163	0.0343	0.0011782	88.9%	12.1%	3.3%	2.611	12
0.275	42-30	-0.4210	0.0070	0.0686	0.4566	0.0157	0.0344	0.0011805	70.2%	6.4%	2.4%	2.544	14
0.250	42-31	-0.5800	0.0010	0.0221	0.4246	0.0135	0.0317	0.0010069	91.8%	12.8%	2.9%	2.301	23
0.225	42-32	-0.5590	0.0010	0.0221	0.3788	0.0122	0.0321	0.0010287	90.4%	11.8%	2.8%	2.160	30
0.200	42-33	-0.5690	0.0020	0.0294	0.3709	0.0118	0.0319	0.0010178	83.4%	9.6%	2.9%	2.129	31
0.175	42-34	-0.1580	0.3500	1.0000	0.4633	0.0182	0.0392	0.0015357	76.5%	9.2%	2.6%	2.833	4
0.150	42-35	-0.7290	0.0010	0.0221	0.3552	0.0105	0.0295	0.0008681	91.2%	12.8%	2.7%	2.002	38
0.125	42-36	-0.5950	0.0000	0.0000	0.3725	0.0117	0.0315	0.0009917	87.1%	12.5%	2.6%	2.120	32
0.100	42-37	-0.4980	0.0000	0.0000	0.4387	0.0145	0.0331	0.0010934	80.6%	12.1%	3.0%	2.410	19
0.075	42-38	-0.5470	0.0020	0.0294	0.4076	0.0132	0.0323	0.0010409	71.0%	8.1%	2.7%	2.264	26
0.050	42-39	-0.3590	0.0310	0.2103	0.3765	0.0133	0.0354	0.0012563	81.2%	9.0%	2.6%	2.274	25
0.025	42-40	-0.6000	0.0010	0.0221	0.4036	0.0127	0.0314	0.0009871	89.8%	12.6%	2.9%	2.217	27
0.000	42-41	-0.5540	0.0030	0.0353	0.3929	0.0126	0.0322	0.0010340	87.6%	12.0%	2.8%	2.210	28
L/S 0.000	42	-	-	-	0.5659	0.0240	0.0424	0.0017987	126.1%	24.08%	5%	3.702	1

The results from the test on variances can therefore be used to disaggregate set P into two tuples. The first tuple P1 comprises those portfolios in set P found to be significantly different from the L/S zero beta portfolio (beta 0.600, beta 0.575, beta 0.550) and the second tuple P2 contains those portfolios in set P for which the null hypothesis was not rejected (beta 0.525, beta 0.500, beta 0.475)

Tuples P1 and P2 can be further characterised using insights drawn from Tancaret et al (2012) with regard to the type of replication required being either 1) the precise capture of target beta in hedging, benchmarking or as the passive element in a core satellite approach applications with strict tracking error mandates or 2) to generate reasonable hedge fund mimicking risk return corresponding risk premia but focused more on performance and diversification benefits profile.

Tuple P1 portfolios exhibit variances of return which are lower than that for the long/short zero beta portfolio and can be distinguished by their relatively lower industry concentration indices, better top 5 ranking representation by cumulative return index value. Tuple P2, which has the lowest portfolio betas on the other hand exhibits variance of return profiles which are statistically indistinguishable from that of the zero beta portfolio and can be noted for its higher industry concentration and active share values as well as the highest ranked long portfolio in respect of cumulative index returns value and Sharpe ratio.

For the replication objective of generating reasonable hedge fund mimicking risk return premia focusing on performance and diversification benefits, tuple P1 may suffice as a potential solution set together with tuple P2 depending on the appetite for risk. However in respect of the second Tancaret et al (2012) replication objective requiring precise capture of beta target with strict tracking error mandates, only tuple P2 can be considered and within it the portfolio beta 0.525 represents the lowest beta portfolio with the highest reward to risk characteristics and portfolio beta 0.475 represents the lowest beta portfolio which is viable with moderate reward to risk requirements. Although portfolio beta 0.450 represents the lowest feasible low beta portfolio, it is excluded from the solution set owing to it unfavourable risk / return characteristics and relatively low cumulative index returns value.

6.6 Research Question Six – Portfolio weights

Portfolio weights: Do the optimal portfolio weights of securities approximate the simplified analytic equation by Clarke et al. (2011)?

One of the most interesting insights to be drawn from the Clarke et al. (2011) analytic equation for the derivation of weights shown below is that the stocks whose individual market betas β_i are greater than the threshold beta β_L are excluded from the minimum variance portfolios.

$$w_i = \frac{\sigma_{LMV}^2}{\sigma_{\varepsilon i}^2}\left(1 - \frac{\beta_i}{\beta_L}\right) \; for\; \beta_i < \beta_L \quad else\; w_i = 0$$

As one of the aims of this research was to determine the extent to which mean variance optimised portfolios would exhibit number of stocks and portfolio weight distribution patterns similar to those of minimum variance portfolios under Clarke et al. (2011) generalisations, imputed beta threshold values β_L were computed to enable the analysis. With β_L = 0.7442 under the first optimisation and 0.7185 on rebalancing, the respective Clarke et al. (2011) equivalent portfolio with corresponding $w_i$weights constructed showed considerable departures from the mean variance optimised portfolios. The table below highlights the differences in weights

Table 19 : Highlighted weight difference comparisons between rebalanced beta constrained optimised portfolios and the Clarke et al (2011) minimum variance equivalents

24 Month Rebalance 31 Dec 2012

	number of counters	15	14	16	13	15	20	29	14	21	20	18
	target beta	1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	4.6%	1.5%	0.0%	0.0%	0.0%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
AMS	Anglo American Plat Ltd	5.5%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	0.0%	0.0%	0.0%	0.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	10.0%	0.9%	10.0%	3.3%	10.0%
APN	Aspen Pharmacare Hldgs Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	1.4%	10.0%	10.0%	0.9%	10.0%
BIL	BHP Billiton plc	10.0%	10.0%	10.0%	10.0%	4.8%	0.0%	0.3%	0.0%	0.0%	0.0%	0.0%
BVT	Bidvest Ltd	0.0%	0.0%	3.8%	0.0%	0.1%	0.1%	0.0%	10.0%	5.1%	4.5%	0.0%
DSY	Discovery Ltd	0.0%	0.0%	0.4%	0.9%	7.6%	5.3%	3.2%	9.2%	10.0%	7.3%	10.0%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.0%	0.0%	0.0%	4.6%	0.5%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	3.8%	10.0%	10.0%	10.0%	10.0%	9.6%	10.0%
IMP	Impala Platinum Hlgs Ltd	0.0%	9.5%	4.3%	0.7%	0.0%	0.0%	0.9%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	1.9%	0.0%	0.0%	0.0%	0.1%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.8%	0.0%	0.0%	0.2%	0.0%
IPL	Imperial Holdings Ltd	2.8%	10.0%	5.2%	4.8%	1.2%	0.0%	5.5%	0.0%	0.0%	0.0%	0.0%
MDC	Mediclinic Internat Ltd	0.0%	0.1%	1.1%	9.9%	10.0%	10.0%	10.0%	9.5%	9.6%	10.0%	10.0%
MPC	Mr Price Group Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	0.8%	10.0%	3.8%	10.0%	7.6%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.8%	0.0%	1.0%	1.9%	0.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.4%	2.9%	9.1%	3.7%	10.0%
NPN	Naspers Ltd -N-	10.0%	10.0%	10.0%	10.0%	10.0%	5.2%	1.9%	0.0%	0.3%	0.0%	0.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	5.7%	5.2%	10.0%	0.3%	10.0%	8.4%
OML	Old Mutual plc	10.0%	5.4%	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%
REM	Remgro Ltd	10.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.0%	0.0%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.9%	8.4%	0.5%	1.3%	1.0%
SAB	SABMiller plc	5.8%	3.4%	10.0%	10.0%	8.7%	3.2%	0.0%	0.0%	0.5%	0.0%	0.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.4%	0.6%	3.6%	3.6%	0.5%
SHF	Steinhoff Int Hldgs Ltd	10.0%	10.0%	9.3%	3.7%	0.0%	0.0%	0.2%	0.0%	0.2%	0.1%	0.0%
SHP	Shoprite Holdings Ltd	0.0%	10.0%	10.0%	10.0%	10.0%	10.0%	3.3%	10.0%	10.0%	4.4%	10.0%
SLM	Sanlam Limited	0.0%	0.1%	1.1%	10.0%	10.0%	10.0%	4.2%	0.0%	1.8%	10.0%	1.1%
SOL	Sasol Limited	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	0.0%	0.0%	0.0%	0.0%
TBS	Tiger Brands Ltd	0.0%	0.0%	1.8%	0.0%	3.8%	10.0%	6.1%	8.4%	8.9%	10.0%	10.0%
WHL	Woolworths Holdings Ltd	10.0%	10.0%	10.0%	10.0%	10.0%	10.0%	1.1%	0.0%	5.1%	4.5%	1.0%
		100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

Minimum Variance Clarke et al (2011) equivalent

		4	6	9	7	10	12	16	13	15	16	15
		1.000	0.900	0.800	0.700	0.600	0.500	0.400	0.300	0.200	0.100	0.000
AGL	Anglo American plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
AMS	Anglo American Plat Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
ANG	Anglogold Ashanti Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	7.7%	8.7%	7.1%	6.8%	6.9%	6.6%
APN	Aspen Pharmacare Hldgs Ltd	16.8%	14.1%	11.4%	9.3%	7.5%	6.3%	7.1%	5.8%	5.6%	5.7%	5.4%
BIL	BHP Billiton plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
BVT	Bidvest Ltd	0.0%	0.0%	9.8%	0.0%	6.5%	5.4%	6.1%	5.0%	4.8%	4.9%	0.0%
DSY	Discovery Ltd	0.0%	0.0%	24.0%	19.6%	15.9%	13.2%	15.0%	12.3%	11.8%	11.9%	11.4%
FSR	Firstrand Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	0.0%	0.0%	0.9%	0.9%
GRT	Growthpoint Prop Ltd	0.0%	0.0%	0.0%	0.0%	23.4%	19.4%	22.1%	18.1%	17.3%	17.6%	16.7%
IMP	Impala Platinum Hlgs Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INL	Investec Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
INP	Investec plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
IPL	Imperial Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
MDC	Mediclinic Internat Ltd	0.0%	58.6%	47.2%	38.6%	31.2%	26.0%	29.6%	24.1%	23.1%	23.5%	22.3%
MPC	Mr Price Group Ltd	35.4%	29.7%	23.9%	19.6%	15.8%	13.2%	15.0%	12.2%	11.7%	11.9%	11.3%
MTN	MTN Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NED	Nedbank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.3%	2.7%	2.6%	2.6%	2.5%
NPN	Naspers Ltd -N-	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
NTC	Netcare Limited	0.0%	0.0%	0.0%	0.0%	0.0%	6.8%	7.7%	6.3%	6.0%	6.1%	5.8%
OML	Old Mutual plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
REM	Remgro Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
RMH	RMB Holdings Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.7%	2.2%	2.1%	2.1%	2.0%
SAB	SABMiller plc	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SBK	Standard Bank Group Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	2.6%	2.1%	2.0%	2.0%	1.9%
SHF	Steinhoff Int Hldgs Ltd	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
SHP	Shoprite Holdings Ltd	40.3%	33.9%	27.3%	22.3%	18.1%	15.0%	17.1%	14.0%	13.4%	13.6%	12.9%
SLM	Sanlam Limited	0.0%	12.4%	10.0%	8.2%	6.6%	5.5%	6.3%	0.0%	4.9%	5.0%	4.7%
SOL	Sasol Limited	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
TBS	Tiger Brands Ltd	0.0%	0.0%	32.7%	0.0%	21.6%	18.0%	20.5%	16.7%	16.0%	16.3%	15.5%
WHL	Woolworths Holdings Ltd	3.7%	3.1%	2.5%	2.0%	1.6%	1.4%	1.6%	0.0%	1.2%	1.2%	1.2%
		96.2%	151.9%	188.9%	119.6%	148.2%	137.6%	166.5%	128.6%	129.4%	132.4%	121.1%

From the table above one of the observations is that the weights arising from the analytic equation are generally higher than those in the optimised portfolio but are concentrated on just a few stocks as a significant majority of the securities have zero weights. As the weights depend on ex-ante portfolio variance and the threshold beta, relatively higher values of ex-ante variance σ_{LMV}^2 coupled with somewhat high threshold beta values in the higher target beta portfolios results in the exclusion of more stocks , there is a concentration of weights in the smaller qualifying stocks. This is consistent to some degree with the equations. However the extent to which the single market index model upon which the Clarke et al (2011) is based, underestimates σ_{LMV}^2 appears to also account for the larger than expected differences in weights as differences in correlation factors outside of market risk factor play a role in the departure from specification.

The presence of weight constraints in the mean variance portfolios from -10% to 10% also plays a part in the differences. Furthermore considering the threshold beta limit , there is still a relatively sizable proportion of securities that remain in the optimised portfolio with $\beta_i > \beta_L$ represented by the proportion of the difference between the number of stocks in the optimised portfolio and the

Clarke et al (2011) equivalent. Although the general behaviour of the optimised portfolios does mimick minimum variance portfolios particularly towards the beta 0.450 mark, the degree to which Clarke et al (2011) observations that the majority of optimisation candidate securities are zero weighted in the minimum variance portflio is somewhat tempered in the extent that a generalization to the mean variance optimised portfolios may be fully applied given some of the constraints. The key principle that low beta stocks dominate the long-only portfolio does hold however in line with Clarke et al(2011). At the level of individual weights, given the above as well as the comparative tabulated analysis of the differences in weight and the common size comparison results in chapter 5, shows relative underweighting in the mean variance portfolio as well as the relatively higher number of stocks in comparison to the imputed minimum variance equivalent. Accordingly the individual optimised weights are not well approximated by the Clarke et al (2011) analytic equation for weights.

6.7 Research Question Seven – Minimum variance behaviour

Minimum Variance Behaviour: Does the beta constraint force the long only portfolio to behave similar to a minimum variance portfolio rather than a mean variance efficient portfolio?

One of the constraints imposed by the analytic equation $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ is that to reduce variance properly requires the exclusion of stocks with betas above the threshold beta. If the proposed mean variance optimised portfolios that are the subject of this research were fully compliant with the Clarke et al.(2011) requirements and the approximated $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2}$ ratio of 80 percent , then the lowest portfolio beta that could be achievable would essentially be determined from the equation as follows:

$$\beta_p = \beta_L \left(\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} \right)$$

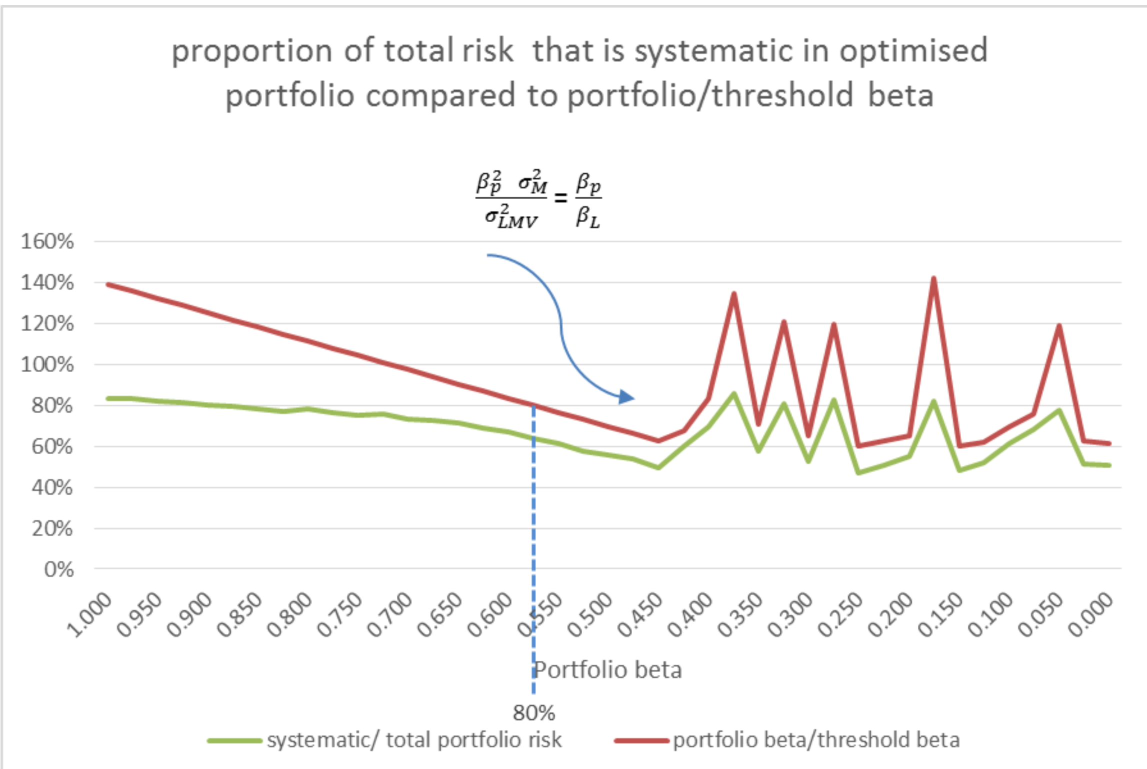
$$= 0.7185 \ x \ 0.8$$

$$= 0.5758$$

For the initial optimisation $\beta_p = 0.5748$.

As the graph below shows it is around the 0.575 target beta level at which the convergence of β_p and $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2}$ should occur. The trajectory that is instead apparent is that the optimisation tends towards convergence until the optimisation threshold of 0.450 is reached.

Figure 39: Implied convergence point for equality of relationship between the equality of proportion of total risk that is systematic (80 %) to the portfolio beta/threshold beta ratio across rebalanced optimisation portfolios with calculated threshold beta of 0.7185 to give a portfolio beta of 0.5758



The other implication of this finding is that the theoretical limits for minimum variance efficiency as it were, with respect to the optimisations that this research was concerned with can be regarded as breached at target beta levels approximately below 0.575. This implied limit appears to be stable through the infrequent rebalancing as evidenced by the close similarity of the calculated values.

6.7.1 Research Question Seven (a)

Systematic risk: Do any of the portfolios conform to the 80 to 90% range of portfolio risk being systematic that Clarke et al. (2011) find for minimum variance portfolios?

Clarke et al (2011) found most of the analytically tractable long only minimum variance risk (80 to 90%) to be associated with general market exposure. As shown in the preceding calculation tables and graphs, target beta 0.975 to 0.65 in the initial optimization and target beta 0.1 to 0.725 portfolios after rebalancing approximately fall within the 80 to 90 % range of portfolio risk being systematic. At the higher beta target levels, there is greater exposure to market risk therefore a higher proportion of risk is expected to be systematic, indeed for the optimised portfolios the proportion is well above 80 percent initially. This however is not the analytically suitable range for the exploration of low beta optimisations that are the subject of this research.

There is a clear mathematical tension between the quadratic character of $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2}$ which tends to curve below the suggested 80 percent threshold at the high target betas, whereas the portfolio beta to threshold beta ratio $\frac{\beta_p}{\beta_L}$ appears to hold well into viable ranges of target beta. This has the effect of turning $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ into an inequality of the form $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} < \frac{\beta_p}{\beta_L}$ with an asymptotic profile. The higher portfolio variances σ_{LMV}^2 possibly obtaining from the underestimation of total risk arising from the use of a single index model to construct the co-variance matrix may partially explain the departure from equality as non-market security correlations are not fully factored. The other reason possibly relates to the mean variance optimisation function's return maximization component resulting in a relatively higher proportion of riskier securities featuring in the solution set than would be the case in minimum variance efficiency settings. As Stoyanov (2011) points out, minimum variance portfolio optimisations tend to lower risk to suboptimal levels at the expense of the returns and also that the optimisation process typically results in portfolios that do not properly exploit correlation and are highly over-weighted with low volatility stocks. This would explain the relatively larger solution set relative to the Clarke et al (2011) equivalent.

If the underlying reasons for the inequality relate to sub optimisation arising from minimum variance constraints then possibly there could be scope for modification of the Clarke et al (2011) equation to the form : $\frac{\beta_p^2 \ \sigma_M^2}{k \ \sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ were k is a function or constant that adjusts σ_{LMV}^2 for mean variance settings to more fully enable the powerful analytics that the original equation provides for minimum variance portfolios.

6.7.2 Research Question Seven (b)

> **Cross sectional variation of portfolio weights:** Does the cross sectional variation in weights of stocks in the portfolio depend on the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$ and market beta β_i parameters as in the Clarke et al. (2011) case?

No significant relationship in respect of the cross sectional variation in weights of stocks in the mean variance optimised portfolios against the individual stock ex-ante unsystematic variances of returns could be established through regression in this study as indicated in chart X. The presence of high ex-ante unsystematic variance is expected to lower the weight of optimal stocks in the portfolio and in the case of this research it appears that this relationship has been less pronounced owing to the reality that weights for mean variance efficient optimisation also depend on the expected returns Markowitz (1952). There is also the existence of other constraints in the respect of the minimum and maximum weight thresholds imposed and the target portfolio beta requirement that affect the interaction of optimal weights with ex-ate unsystematic variance.

However a strong of relationship between optimal weights regressed and beta was detected. These findings especially in respect of beta are in line with the analysed case for minimum variance portfolio wherein according to Clarke et al (2011) optimal weights show relatively stronger alignment to beta related risk (systematic) rather than security specific risk (unsystematic) due to natural leanings towards optimal diversification. The extent to which the CAPM criticism of low beta high return stocks that Ward and Muller (2012) observe on the JSE possibly also affects the relationships analytical relationships is not clear from the results.

There is therefore some room within which the analytically derived generalizations from Clarke et al (2011) are applicable to the mean variance optimisation case. Key principles, including that the optimal set is comprised of a small proportion of available stocks and that the majority of those stocks are comprised of low beta stocks appear to hold in the analysed mean variance portfolios to a large degree. The extent to which a threshold beta strictly governs the inclusion of higher beta stocks in the optimal set is seen to be moderate, as it is tempered by factors including constraints and the return maximization component of the mean variance optimisation function which results in the inclusion of otherwise beta threshold disqualified stocks ($\beta_i > \beta_L$) in the solution set.

6.8 Research Question Eight- Long/Short Hedge Fund Replication

> Is there an optimal level set of low beta constrained long only equity portfolios that can generate a risk return profile to satisfy investors interested in a controlled range of exposure to exploitable broad systematic risk from unit beta down to beta neutrality without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral, but focused more on performance and diversification benefits?

The aim of the research has been primarily to isolate the lowest beta portfolios that effectively mimick the characteristics of the long short zero beta portfolio in order to facilitate cost effective risk management without the disadvantages and constraints of leveraging , shorting and fees associated with traditional long/short equity market neutral portfolios. The scope within which investors interested in controlled exposure to exploitable broad systematic risk on a calibrated scale from unit beta down to beta neutrality could be provided with satisfactory alternatives was of key interest particularly for those applications without the strict requirement of necessarily achieving a precise hedge fund beta target of market neutral, but focused more on achievable performance and diversification benefits.

The path to realising those aims depended on exploring and obtaining an in depth theoretical and empirical understanding of the interactions of key risk and return measures and other pertinent characteristic features of beta constrained long equity portfolios as they are mean variance optimised and constrained towards market beta neutrality.

The analysis of the results of this study suggests that indeed tangible insights have been drawn with which to answer affirmatively that there exists a set of finely calibrated low beta mean variance optimised portfolios from which investors can obtain controlled exposure to low levels of broad systematic risk approaching beta neutrality with significant risk adjusted performance and diversification benefits, excluding the disadvantages of the typical long/short hedge equity neutral funds.

Accordingly the insights are summarized in respect of 1) achievable limits to long only mean variance optimisation for replicating long/short equity market beta neutral portfolios, 2) the general behaviour of possible long only low beta alternatives under mean variance optimisation and beta constraining towards neutrality in respect of key risk return measures and characteristics, and 3) the solution set of low beta portfolio opportunities.

6.8.1 Feasible optimisation limit

On the basis of finely calibrated beta scale of 1.000 down to 0.000 with decrements of 0.025 a target long only equity beta portfolio set of 41 was created , plus a 42^{nd} long/short equity zero beta portfolio. Through optimisation an attempt was made to optimise the entire set of 41 target portfolios from beta 1.000 down to 0.000 for comparison with the optimised long / short beta 0.000 portfolio. Optimisation to precise target beta was achieveable up to beta 0.450. As beta is a proxy for systematic risk (Sharpe, 1964), the possibility of minimum variance characteristics generalized by Clarke et al (2011) emerging was investigated. The theoretical minimum achievable target beta implied by those generalisation for the optimisation in this study was estimated to be 0.575. The difference in the two limits is explained mainly through influences of the return maximization component that is retained in the mean variance optimisation. Thus the limit of 0.450 is considered to be empirically sound for the JSE/FTSE Top 40 sample that was used.

6.8.2 General behaviour of target beta portfolios

As target beta is decremented towards the market neutrality but within the scope of the lowest feasible threshold of 0.450, characterizing constructs such and active share and the industry concentration index exhibit strongly quadratic relationship to the target beta constraint whereas tracking error displays a very strong first order linear relationship. The number of shares in those portfolios fluctuates and the relationship to the target beta is considered to be complex and possibly multi order polynomial.

Risk and risk adjusted returns measures including the variances, the Sharpe ratio, the information ratio and Treynor ratio show that the value can be controlled effectively as whilst reducing portfolio beta. Although the variances have been shown to be somewhat high, the higher level of returns and finely reduced systematic risk exposure provide reward to risk effectiveness. The Sharpe ratios increase towards the low beta target. Although the equality of Sharpe ratios (Ledoit & Wolf, 2008) test was not conclusive, triangulation using the cumulative index returns value as well as the Ledoit and Wolf (2011) robust test for differences in variances enabled the full and effective characterisation of the beta target spectrum. The testing of differences between variances using another robust test from Ledoit and Wolf (2011) yielded strong positive results at the 5 percent significance level adjusted for Benjamini and Yekutieli (2001) familywise errors, which enabled a cogent differentiation of the relevant target beta portfolios for inclusion into a solution set. An analysis of the different risk return metrics, active share, industry concentration index, tracking error, cumulative index return value indicated the potential solution set to be from beta 0.475 to beta 0.600.

Individual optimised weights were found not to be well approximated by the Clarke et al (2011) analytic equation for weights. The weights arising from the analytic equation were found to be generally higher than those in the optimised portfolio and more relatively more concentrated on just a few stocks as a significant majority of the securities have zero weights. However at the portfolio level and globally across target betas, Clarke et al (2011) principles and generalisations on portfolio behaviour are seen to be pertinent. The imputed threshold beta equations provide a useful approximation of ideal levels of systematic risk to total risk that can be managed in a optimised portfolio. To a certain extent the individual level of beta composition of low beta portfolios can be understood base on the threshold beta. Furthermore through the knowledge that mean variance optimised portfolios exhibit number of stocks and portfolio weight distribution patterns similar to those of minimum variance portfolios under Clarke et al. (2011) generalisations, together with the use of the analytic equations can be used to effectively characterize the level of departure from minimum variance efficiency of mean variance optimised portfolios.

The convergence of the equation $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ has been demonstrated and how it can be useful in characterizing the theoretical limits to minimum variance efficiency as well as the estimation of the limits to the feasible mean variance optimised low beta portfolio. Furthermore the study has been able to identify and explain the mathematical tension between the quadratic character of the level of systematic to total risk $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2}$ which tends to curve below the suggested 80 percent threshold at the high target betas, in comparison to the portfolio beta to threshold beta ratio $\frac{\beta_p}{\beta_L}$ which exhibits a linear relationship. Thus an inequality of the form $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} < \frac{\beta_p}{\beta_L}$ with an asymptotic profile is identified whose resolution can be accounted for by the single index model misspecification of total risk σ_{LMV}^2 and mean variance optimisation function's return maximization component resulting in a relatively higher proportion of riskier securities featuring in the solution set.

Finally the cross sectional variation in weights of stocks in the optimal portfolios has been shown to be heavily dependent market beta β_i parameters as in the Clarke et al. (2011) case but not so much on the ex-ante unsystematic variance of returns $\sigma_{\varepsilon i}^2$. This lack of a clear relationship between weights and unsystematic variance is attributed to the role of expected returns in optimisation as well as other constraints in the respect of the minimum and maximum weight thresholds imposed and the target portfolio beta requirement affecting the interaction of optimal weights with ex-ate unsystematic variance

6.8.3 Solution set of low beta portfolios.

On the basis of the analysis of the different risk return metrics, active share, industry concentration index, tracking error, cumulative index return value together with the use of robust statistical tests including the Ledoit and Wolf (2008) test for equality of Sharpe ratios and the Ledoit and Wolf (2011) robust test for variances a potential solution set (P) of low beta portfolio alternatives extending from beta 0.475 to beta 0.600 was identified, characterized and disaggregated in to definitive solution tuples P1 (beta 0.600, beta 0.575, beta 0.550) and P2(beta 0.525, beta 0.500, beta 0.475).

Tuple P1 portfolios exhibit variances of return which are statistically different at the 5 percent significance level and lower than that of the long/short zero beta portfolio and can be distinguished by their relatively lower industry concentration indices, better top 5 ranking representation by cumulative return index value. Tuple P2, has the lowest portfolio betas and exhibits variance of return profiles which are statistically indistinguishable at the 5 percent significance level from that of the zero beta portfolio and can be noted for its higher industry concentration and active share values as well as the highest ranked long portfolio in respect of cumulative index returns value and Sharpe ratio.

In terms of Tancaret et al (2012) replication objectives 1) the precise capture of target beta in hedging, benchmarking or as the passive element in a core satellite approach applications with strict tracking error mandates or 2) to generate reasonable hedge fund mimicking risk return corresponding risk premia but focused more on performance and diversification benefits profile.

For the replication objective of generating reasonable hedge fund mimicking risk return premia focusing on performance and diversification benefits, tuple P1 may suffice as a potential solution set together with tuple P2 depending on the appetite for risk. However in respect of the second replication objective requiring precise capture of beta target with strict tracking error mandates, only tuple P2 can be considered and within it the portfolio beta 0.525 represents the lowest beta portfolio with the highest reward to risk characteristics and portfolio beta 0.475 represents the lowest beta portfolio which is viable with moderate reward to risk requirements. Although portfolio beta 0.450 represents the lowest feasible low beta portfolio, it is excluded from the solution set owing to it unfavourable risk / return characteristics and relatively low cumulative index returns value.

7 CONCLUSION

7.1 Principal findings

The research undertaken has been able to rigorously characterize the performance of a beta constrained initially well diversified portfolio of equity stocks under mean variance optimised conditions through forcing beta from a market identical value of one down towards market neutral zero, and drawn the key insights regarding the interactions and behavior on key risk and return metrics and variables that include active share, tracking error, the industry concentration index, the Sharpe ratio, the information ratio and the Treynor ratio in general to the extent that the following cogent and theoretically defensible conclusions can be drawn.

7.1.1 The use of systematic constrained mean variance optimisation procedure delivered viable low beta hedge fund replicas

A systematic algorithmic process of market beta constrained mean variance optimisation with minimal rebalancing at 24 months can be used to produce a finely calibrated and well characterised set of viable low beta mean variance optimised long equity portfolios that are able to offer cost effective alternatives to long short and market neutral portfolios for investors seeking controlled market exposure to systematic risk without the added risks and costs that come with hedging products.

7.1.2 Low beta hedge fund alternatives from JSE Top 40 sample effectively isolated and fully characterized to risk /return replication objectives in line with Tancaret et al (2012)

In the case of the JSE Top 40 over a 48 month testing period, this systematic optimisation process was used to isolate a solution set (P) of low beta portfolio alternatives extending from a target beta value of 0.475 to a beta value of 0.600 which was identified, characterised and disaggregated into definitive solution tuples P1 (beta 0.600, beta 0.575, beta 0.550) and P2 (beta 0.525, beta 0.500, beta 0.475).

a. Tuple P1 portfolios exhibit variances of return which are statistically different at the 5 percent significance level and lower than that of the long/short zero beta portfolio and can be distinguished by their relatively lower industry concentration indices, better top 5 ranking representation by cumulative return index value.

b. Tuple P2, has the lowest portfolio betas and exhibits variance of return profiles which are statistically indistinguishable at the 5 percent significance level from that of the zero beta portfolio and can be noted for its higher industry concentration and active share values as well as the highest ranked long portfolio in respect of cumulative index returns value and Sharpe ratio.

What it particularly important to note is that the hedge fund alternative solution set is able to go beyond just satisfying the needs of the investor with a Tancaret et al (2012) replication objective two requirement of gaining performance and diversification benefits from reasonable hedge fund mimicking risk return corresponding risk premia without a strict requirement for hedging target beta accuracy. For objective two, Tuple P1 may suffice as a solution set together with tuple P2 depending on an investor's appetite for risk.

Tuple P2 on the other hand can also be used to satisfy Tancare et al (2012) replication objective one requirements involving the precise capture of target beta, in hedging, benchmarking and even passive core satellite approach applications with strict tracking error mandates. Within P2 the portfolio beta 0.525 represents the lowest beta portfolio with the highest reward to risk characteristics.

7.1.3 Out of sample testing of developed hedge fund alternative tuples shows consistent and significant benchmark outperformance figures

The real world value creation potential of these tuples is significant as evidenced by the cumulative index value results quantitatively and qualitatively through visual inspection (Ward and Muller, 2012). For instance a ranking of the portfolios indicated that the low beta threshold portfolios ranging from beta target 0. 525 down to 0.475 would have significantly outperformed the JSE All share index and JSE/FTSE Top 40. In the solution set the beta target 0.525

portfolio would have produced a cumulative index value of 2.89 against a value of 3.8 for the long /short portfolio and 1.51 for the JSE All share index.

7.1.4 Robust statistical techniques and analytics of key risk return variables and measures underpin the findings

The isolation of P1 and P2 is underpinned by a triangulation of robust statistical tests including Ledoit and Wolf (2008) tests for equality of Sharpe ratios, Ledoit and Wolf (2011) test for equality of variances, cumulative returns index value (Ward and Muller, 2012) analysis and combined with an analysis of key portfolio variables including industry concentration and active share.

7.1.5 The behaviour of beta constrained mean variance long only equity portfolios has been fully characterized with a body of knowledge created in terms of key risk and reward measures and variables and also mathematically in respect of Clarke et al (2011) analytic equations

a. Beta constrained mean variance optimised portfolios exhibit some minimum variance behaviour that can be characterized in terms of Clarke et al's (2011) analytic equations relating to minimum threshold beta and the ratio of systematic to total risk. In particular as the minimum feasible optimizable beta target is approached:

   The mathematical relationship $\frac{\beta_p^2 \ \sigma_M^2}{\sigma_{LMV}^2} > \frac{\beta_p}{\beta_L}$ has been shown to converge asymptotically.

b. The minimum feasible optimizable target of beta can be analytically estimated in terms of mean and minimum variance effectiveness using Clark et al's (2011) set of analytic equations.

c. As target beta is decremented towards the market neutrality but within the scope of the lowest feasible threshold of 0.450, characterizing constructs such and active share and the industry concentration index exhibit strongly quadratic relationship to the target beta constraint whereas tracking error displays a very strong first order linear relationship. The number of shares in those portfolios fluctuates and the relationship to the target beta is considered to be complex and possibly multi order polynomial.

7.2 Implications for management

The implications of this research for management and stakeholders include but are certainly not limited to the following:

7.2.1 Global application and product development opportunities

The findings in respect of the development of equity market neutral hedge fund replicas are at the cutting edge of investment finance, generalisable and able to be applied across different capital markets. The principle and mathematical relationships explored are universal and not at all limited to the JSE. Therefore alternative hedge fund product development opportunities to take advantage of the well characterized behaviour of beta constrained portfolios and insights from this research can be applied in other bourses such as the New York stock exchange or the London Stock Exchange.

7.2.2 Innovation and creative disruption

The financial products involving cost effective hedge fund replication that potentially emanate from the findings of this research have the potential to take away a significant section of the value chain from the hedge fund industry as it currently stands. Intensity of competition within the investment management industry is increasing as the state of the playing field is changed continuously by new alternatives such as the one proposed. Investment managers should be prepared for the ongoing trend on innovative and competitive disruption to be spurred by innovations such as the one presented in this study. The innovations from this research can thus help investors and investment managers to navigate the powerful and inevitable commoditization trends that are gripping the global investment management industry

7.2.3 Risk management

The findings of this research represent a contribution to the body of knowledge of practical and theoretically grounded investment and financial risk management that may contribute in some way to the development and refinement of sophisticated risk management techniques that make more effective use of investment funds.

7.2.4 Portfolio management practices

The low portfolio rebalancing represents an opportunity for significant savings to be realised from changes in practice in the domain of hedge fund management. Whereas industry practice has typically involved significant frequent rebalancing in order to recalibrate risk, the stability of the low beta portfolios under infrequent rebalancing allows for a change in thinking and progressive practices to deal with unnecessary turnover and the concomitant added transaction costs.

7.2.5 Human machine interaction

The process that is used in this research is highly systemised and algorithmic in nature to the extent that it can effectively be implemented end to end by machines. This reduces the need to pay excessive fees to hedge fund managers who are unable to add value beyond closet indexing as described by Cremers and Petajiisto (2008). More importantly the introduction of such innovations has implications for managers in terms of understanding and preparing for the changing topology of job structures and opportunities that would be available in the financial services and investment management industry as more and more traditional jobs begin to be taken over by machines on the back of systematisation of complex financial services and risk management applications. There is thus an opportunity to define new roles for humans in work process value addition within the investment management domain amongst many others.

7.2.6 Obsolescence of the hedge fund manager

The development of this study's findings adds to the further disintermediation of the hedge fund industry through effective lower cost alternatives to what are typically opaque and at times inordinately risky and expensive investment structures. The transparency and flexibility of the proposed solution presents investors with an opportunity to gain exposure to a sophisticated hedge fund strategy at the fee levels that are competitively below those charged by mutual funds even.

7.2.7 Increased participation of small investors

The opportunities for individual retail investors who previously were unable to access hedge fund strategies owing to prohibitive costs and opacity of information are broadened by the introduction of cost effective and innovative investment products such as the low beta long only hedge fund replicant portfolios developed from this research.

7.3 Limitations of the research

Some of the limitations of the research include the following:

7.3.1 The research focuses only on low rebalancing hedge fund replication

The focus on low rebalancing type of hedge funds does not fully address the potential illiquidity issues for short term investors that could arise as a result of the naturally longer holding periods. The issue of flexibility compared to short holding period fund structure that typical mutual funds may offer has not been fully explored and addressed in this research.

7.3.2 Focus on only equity market neutral replication

There are many other hedge fund strategies for which the need exists for cost effective replication exists, that are strikingly different from the equity market neutral hedge strategy profile such as for instance the convertible arbitrage or event driven (Fung & Hsieh, 2002). The extent to which the findings of this research can be generalized to the different hedge fund strategy species is limited.

7.3.3 Capital Asset Pricing Model framework critique

This research uses CAPM concepts extensively. The extent to which the CAPM low beta high returns phenomenon applies and possibly affects the application of the findings of this research in different capital markets is not clear.

7.3.4 A highly systematic process is used which precludes the opportunity for human judgement and skill to be applied

The systematic process, whilst it has advantages in respect of predictability and reliability of outputs and broad application, may be vulnerable to the inherent weakness of not being able to fully exploit human insights particularly within the context of rapidly changing information and practices. Undoubtedly there are still many swathes of the investment management value chain where there is still no substitute for human intelligence and professional judgement.

7.3.5 Use of historical inputs

Although highly conservative shrinkage of the covariance matrix was implemented to stabilize errors in the optimisation procedures the use of adjusted historical time series inputs does not completely suppress errors or change of regime artefacts that could affect the quality of optimisation. It is possible that with the use of more human judgement and conditioned information in the formulation of risk and return estimates, the optimisation results could be significantly improved.

7.4 Recommendations for future research

7.4.1 High frequency trading

It would be interesting for related research to be conducted which addresses how low beta alternative optimisation can incorporate very high frequency rebalancing for possible applications in high frequency trading markets.

7.4.2 Refinement of Clarke et al (2011) analytic equations for mean variance optimisation

The mathematical relationships in the Clarke et al (2011) analytical equations for weights in minimum variance portfolios present opportunities for further inquiry and possible derivation of a substantive simplified mean variance optimisation under constraints equivalent for single index model optimisations. In particular the modification of the Clarke et al (2011) equation to the form : $\frac{\beta_p^2 \ \sigma_M^2}{k \ \sigma_{LMV}^2} = \frac{\beta_p}{\beta_L}$ were k is a function or constant that interacts with the standard deviation of the long only portfolio σ_{LMV}^2 for mean variance settings to more fully enable the application of the original equation in constrained mean variance settings.

7.4.3 Performance and active management diagnostics

Research into a more practical active management and return diagnostics for algorithmic applications would help provide richer insights into how constraints contribute to a reduction in portfolio return losses under mean variance optimisation.

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9 APPENDIX 1: ETHICAL CLEARANCE LETTER

Gordon Institute of Business Science
University of Pretoria

Dear Innocent Siziba

Protocol Number: **Temp2015-01162**

Title: **The implications of forcing beta from one down towards beta neutrality on key risk and return and other measures in long only mean variance efficient equity portfolios**

Please be advised that your application for Ethical Clearance has been APPROVED.

You are therefore allowed to continue collecting your data.

We wish you everything of the best for the rest of the project.

Kind Regards,

Adele Bekker