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## EFFECT OF THERMOPHYSICAL PROPERTIES MODELS ON THE PREDICTING OF CONVECTIVE HEAT TRANSFER OF NANOFLUIDS WITH CONSIDERING NANOPARTICLES MIGRATION

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### ABSTRACT

In order to study the heat transfer behavior of the nanofluids, precise values of thermal and physical properties such as specific heat, viscosity and thermal conductivity of the nanofluids are required. There are a few well-known correlations for predicting the thermal and physical properties of nanofluids which are often cited by researchers to calculate the convective heat transfer behaviors of the nanofluids. Each researcher has used different models of the thermophysical properties in their works. The aim of the present paper is to study the convective heat transfer of nanofluids containing low volume concentration of  $Al_2O_3$  nanoparticles with a regard to the migration of nanoparticles due to Brownian diffusion and thermophoresis. To do this, a two-component model has been used and a numerical study on laminar flow of alumina-water nanofluid through a constant wall temperature tube has been performed. Two different models have been adopted for predicting the thermophysical properties of nanofluids. All of the properties are assumed to be temperature as well as particle concentration dependent. The effects of these models on the predicted value of the convective heat transfer of nanofluid and the migration of nanoparticles have been discussed in detail.

### INTRODUCTION

With progresses of thermoscience and thermal engineering, many efforts have been devoted to heat transfer enhancement. An effective way of improving the thermal conductivity of fluids is to suspend small solid particles in the fluids. Traditionally, solid particles of micrometer or millimeter sizes were mixed in the base liquid. Although the solid additives may improve heat transfer coefficient, practical uses are limited because the micrometer and/or millimeter-sized particles settle rapidly, clog flow channels, erode pipelines and cause severe pressure drops.

In recent years, modern technologies have permitted the manufacturing of metallic particles down to the nanometer

scale, which in turn created a new class of fluids, called nanofluids.

The concept of nanofluids refers to a new kind of heat transport fluids by suspending nanoscaled metallic or nonmetallic particles in base fluids. This new class of nanotechnology-based heat transfer fluids exhibits thermal properties superior to those of their base fluids or conventional particle fluid suspensions. Thus, nanofluids have interesting potential and can be used to replace common fluids for advanced thermal applications; however they are still in the early stages of development.

### NOMENCLATURE

$C$	[J/kg K]	nanofluid specific heat
$d$	[m]	pipe diameter
$D_B$	[m <sup>2</sup> /s]	Brownian diffusion coefficient
$D_T$	[m <sup>2</sup> /s]	thermal diffusion coefficient
$h$	[W/m <sup>2</sup> K]	heat transfer coefficient
$k$	[W/m.K]	nanofluid thermal conductivity
$k_B$	[J/K]	Boltzmann constant
$Re$	[-]	Reynolds number
$x, y$	[m]	Cartesian axis directions

#### Special characters

$\varphi$	[-]	nanoparticle volumetric fraction
$\mu$	[Pa.s]	viscosity
$\rho$	[kg/m <sup>3</sup> ]	nanofluid density
$\tau_w$	[Pa]	shear stress at the wall

#### Subscripts

$bf$	base fluid
$eff$	effective
$in$	inlet
$m$	mean
$out$	outlet
$p$	nanoparticle
$w$	wall

#### Superscripts

-	averaged property
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To apply the nanofluid to practical heat transfer processes, more studies on its flow and heat transfer feature are needed. A number of experimental studies were performed to measure the thermal conductivity and the viscosity of nanofluids (e.g. see [1-4]), while some studies reported the correlations for predicting the thermal conductivity, density, viscosity and specific heat of the nanofluids.

Jang and Choi [5, 6] proposed and modeled the Brownian-motion induced nanoconvection as a key nanoscale mechanism governing the thermal behaviour of nanofluids. In their model, the effects of various parameters such as the ratio of the thermal conductivity of nanoparticles to that of a base fluid, volume fraction, nanoparticle size, and temperature on the effective thermal conductivity of nanofluids were included. Prasher et al. [7] introduced a Brownian-motion-based convective-conductive model which predicts the right trend with respect to different parameters such as nanoparticle volume fraction, nanoparticle diameter, and temperature. Xue and Xu [8] derived an expression for the effective thermal conductivity of nanofluids with interfacial shells. Their expression was not only dependent on the thermal conductivity of the solid and liquid and their relative volume fraction, but also depended on the particle size and the interfacial properties. They compared the theoretical results with the experimental data for the effective thermal conductivity of CuO-water and CuO-ethylene glycol nanofluids and found good agreement between them. Masoumi et al. [9] introduced a new equation for calculating the nanofluid viscosity by considering the Brownian motion of nanoparticles. In their work the relative velocity between the base fluid and nanoparticles has been taken into account. Their equation presented the nanofluid viscosity as a function of the temperature, the mean nanoparticle diameter, the nanoparticle volume fraction, the nanoparticle density and the base fluid physical properties.

Heris et al. [10, 11] investigated the convective heat transfer coefficient of Al<sub>2</sub>O<sub>3</sub>-water and CuO-water nanofluids for laminar flow in annular tube under a constant wall temperature boundary condition. The thermal and physical properties of the nanofluids were calculated using the following formulas: the Xuan and Roetzel [12] equation for specific heat, the Einstein equation for viscosity, and Yu and Choi [13] correlation for thermal conductivity. The results showed that the heat transfer coefficient increased with an increasing Peclet number and increasing volume fraction while Al<sub>2</sub>O<sub>3</sub>-water nanofluid showed larger enhancement than CuO-water nanofluids.

Rea et al. [14] measured the pressure drop and convective heat transfer coefficient of alumina-water and zirconia-water nanofluids in a flow loop with a vertical heated tube. They measured the viscosity and thermal conductivity of nanofluids experimentally and reported correlations derived by curve fitting for their experimental data. Their experimental results showed that the heat transfer coefficients in the entrance region and in the fully developed region increase by 17% and 27%, respectively, for alumina-water nanofluid at 6 vol % with respect to pure water. Also, the zirconia-water nanofluid heat transfer coefficient increases by approximately 2% in the entrance region and 3% in the fully developed region at 1.32 vol %.

It is important to note that the actual amount of experimental data regarding the nanofluid properties, in particular thermal conductivity and dynamic viscosity, remains quite limited. Therefore, in order to estimate such properties, researchers often turned to available formulas either derived from the classical theory of two-phase mixtures or based on semi-empirical models. Such approximations induce important discrepancies in the determination of nanofluids thermal properties and can cause considerable errors when assessing the heat transfer and pressure drop of nanofluids in various thermal applications. Thus, understanding the thermophysical properties of nanofluid is essential before using nanofluids in practical applications.

In the present paper, we investigate the effect of two different models used to predict nanofluid thermal conductivity and dynamic viscosity on their thermal and hydrodynamic performance for laminar forced convection in a tube with uniform wall temperature. The nanofluid under study is Al<sub>2</sub>O<sub>3</sub>-water mixture. A nonhomogenous model proposed by Buongiorno [15] is used for numerical simulations.

## MATHEMATICAL MODELING

### Thermophysical properties of nanofluids

The formulas selected for the thermophysical properties of the nanofluid in the present simulation are as follows;

The density of nanofluids is predicted by mixing theory as [15],

$$\rho = \varphi\rho_p + (1 - \varphi)\rho_{bf} \quad (1)$$

The specific heat of nanofluid is calculated as [15],

$$C_p = \frac{\varphi\rho_p C_{p,p} + (1 - \varphi)\rho_{bf} C_{p,bf}}{\rho} \quad (2)$$

### Thermal conductivity

Among the various formulas of the thermal conductivity for nanofluid presented in the literature, here two correlations are used which have the following form;

$$k = k_{bf}(1 - \varphi) + \beta k_p \varphi + C_1 \frac{d_{bf}}{d_p} k_{bf} \text{Re}_{d_p}^2 \text{Pr} \varphi \quad (3)$$

$$k = k_{bf}(T)(1 + 4.5503\varphi) \quad (4)$$

where  $C_1$ ,  $\beta$ , and  $\varphi$  are a proportional constant, a constant related to Kapitza resistance and volume fraction of nanoparticle, respectively.

Equation (3), is proposed by Jang and Choi [5, 6], can predict the enhancement of the effective thermal conductivity for nanofluids in terms of nanoparticles concentration, size and temperature. This model can predict well the effective thermal conductivity of water-Al<sub>2</sub>O<sub>3</sub> nanofluids over a very wide concentration range from 0.01 to 5 vol. % [16].

Equation (4) is an empirical correlation which derived for the alumina-water nanofluid used in the references [14, 17].

### Dynamic viscosity

For the effective dynamic viscosity of nanofluid, the model proposed in [9], and an empirical correlation derived by Rea et al. [14] as well as Williams et al. [17] are considered here and can be, respectively, expressed as

$$\mu = \mu_{br}(T) + \frac{\rho_p V_B d_p^2}{72C\delta} \quad (5)$$

$$\mu = \mu_{br}(T) \exp[4.91\varphi/(0.2092 - \varphi)] \quad (6)$$

In equation (5),  $V_B$  and  $\delta$  are Brownian velocity and the distance between the centres of particles, respectively. These are given as below

$$V_B = \frac{1}{d_p} \sqrt{\frac{18k_B T}{\pi \rho_p d_p}} \quad (7)$$

$$\delta = \sqrt[3]{\frac{\pi}{6\varphi}} d_p \quad (8)$$

Moreover,  $C$  is a correction factor can be calculated by the experimental data [9].

It should be mentioned here that, as can be seen, all of the nanofluid properties are as a function of temperature and nanoparticle concentration. The reason of such adoption resides in the fact that, as will be explained in the next section, a non-homogenous two-component model [15] has been chosen for numerical simulation of the convective heat transfer of nanofluid.

## GOVERNING EQUATIONS

The nanofluid is treated as a two-component mixture (base fluid + nanoparticles) with the following assumptions [15]:

- 1- incompressible flow,
- 2- no chemical reactions,
- 3- negligible external forces,
- 4- dilute mixture ( $\varphi \ll 1$ ),
- 5- negligible viscous dissipation,
- 6- negligible radiative heat transfer,
- 7- nanoparticles and the base fluid are in thermal equilibrium locally.

Therefore, under the above assumptions, the continuity equation for the nanofluid, nanoparticle continuity equation, nanofluid momentum equation, and nanofluid energy equation, for a steady, two-dimensional laminar flow can be expressed, respectively, as [15]:

$$\nabla \cdot V = 0 \quad (9)$$

$$V \cdot \nabla \varphi = \nabla \cdot \left[ D_B \nabla \varphi + D_T \frac{\nabla \cdot T}{T} \right] \quad (10)$$

$$\rho(V \cdot \nabla V) = -\nabla P + \nabla \cdot \mu \nabla V \quad (11)$$

$$(\rho C_p) V \cdot \nabla T = \nabla \cdot k \nabla T \quad (12)$$

In equation (10),  $D_B$  represents the Brownian diffusion coefficient, given by the Einstein-Stokes's equation, and  $D_T$  represents the thermophoretic diffusion coefficient of the nanoparticles:

$$D_B = \frac{k_B T}{3\pi \mu d_p} \quad (13)$$

$$D_T = \left( \frac{0.26k}{2k + k_p} \right) \left( \frac{\mu}{\rho} \right) \varphi \quad (14)$$

The phenomenon of thermophoresis in gases is well understood and can be calculated accurately. However, for liquids the converse is true. Unfortunately there is very little

experimental data and considerable confidence is placed on the experimental data of McNab and Meisen [18] which were obtained for particle sizes greater than 1  $\mu\text{m}$  in diameters. However, the thermophoretic velocity appears to be independent of particle size; therefore, extrapolation to smaller sizes may be appropriate. Thus, the thermophoretic diffusion coefficient of the nanoparticles,  $D_T$ , can be represented as Eq. (14). Also,  $K_B$  is the Boltzmann's constant ( $K_B = 1.3807 \times 10^{-23}$  J/K).

## NUMERICAL METHOD AND BOUNDARY CONDITIONS

The system of governing equations (9)-(12) constitute a complete set of equations from which  $V$  (velocity vector field),  $P$  (pressure field),  $\varphi$  (particle volume fraction distribution), and  $T$  (temperature field) can be calculated. These equations are solved by control volume approach.

This method is based on the spatial integration of the conservation equations over finite control volumes, converting the governing equations to a set of algebraic equations. The algebraic "discretized equations", resulting from this spatial integration process, are sequentially solved throughout the physical domain considered by combining the line-by-line procedure and the well-known *TDMA* technique. Pressure and velocity were coupled using Semi Implicit Method for Pressure Linked Equations [SIMPLE] [19].

Because of symmetry, only half of the flow field has been considered as shown in fig. 1. The grid spacing of the domain was tested and it was found that further increase in total number of nodal points caused no substantial changes in the calculation results.

The following boundary conditions were used for solving the coupled non-linear partial differential equations given above. At the channel inlet section, a fully developed velocity profile, a uniform temperature profile,  $T_{in}$ , and uniform particle distribution,  $\varphi_{in}$ , are specified. The length of the computational domain is chosen large enough so that the flow exits the channel with fully developed velocity and temperature profiles. Therefore, at the outlet section, the zero normal gradient condition is applied.

On the wall surface of the tube, the no-slip boundary condition and constant wall temperature were imposed. Due to Impermeability, the total nanoparticle mass flux at the wall is zero, i.e.,

$$\left[ D_B \nabla \varphi + D_T \frac{\nabla \cdot T}{T} \right]_{\text{wall}} = 0 \quad (15)$$

On the tube centre of the pipe, the symmetry boundary condition, the zero normal gradient for all variables, was applied.

## RESULTS AND DISCUSSION

In the present study, first the code has been tested for pure water flow mode and the results compared with the available data in the literature for the classical problem of the laminar flow of pure water in tubes, i.e.,  $\varphi = 0$ . To do this, it is assumed that flow enters the pipe with uniform velocity and temperature and the length of the computational domain is chosen large enough that the flow exits the tube with fully developed velocity and temperature profiles. A constant temperature is applied as the wall thermal boundary condition.

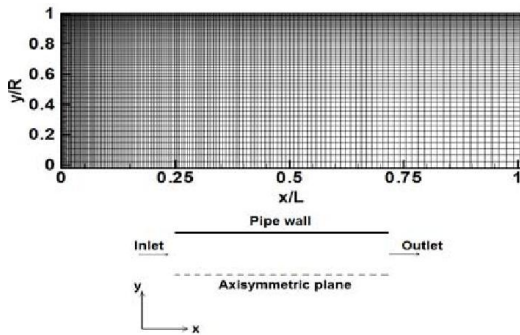


Figure 1 Grid layout and geometry of problem

As can be seen in Figure 2, the velocity profile obtained in the fully developed region of the tube is in complete agreement with the well-known quadratic profile, i.e.,  $u/U_{in} = 2[1-(y/R)^2]$ , obtained from the analytical solution of the Navier-Stokes equations. Moreover, the local Nusselt number obtained from our computer program is compared with the following Shah equation [20, 21] of the local Nusselt number in a circular tube for a constant wall temperature in Figure 3,

$$Nu_x = \begin{cases} 1.077x_*^{-1/3} - 0.70 & x_* \leq 0.01 \\ 3.657 + 6.874(10^3 x_*)^{-0.488} e^{-57.2x_*} & x_* > 0.01 \end{cases} \quad (16)$$

where  $Nu_x = h(x)d_{tube} / k_{water}$ ,  $x_* = [(x / d_{tube}) / (Re_{d_{tube}} Pr)]$ .

Again, a good agreement is apparently displayed between the results obtained by our computer code and that of Shah Eq. [20, 21]. Thus, these results confirm the validity of the computational scheme used in the present investigation.

For the purposes of the present study we have considered the following two combinations of relations for the calculation of the nanofluid properties.

Eq. (3) for the conductivity and Eq. (5) for the viscosity. This combination is identified as model I.

Eq. (4) for the conductivity and Eq. (6) for the viscosity. This combination is identified as model II.

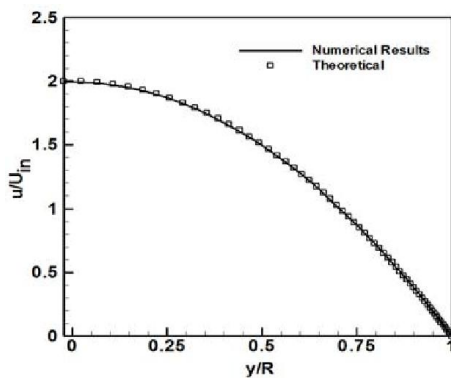


Figure 2 Comparison of predicted velocity profile for fully developed laminar flow through a tube with analytical solution at  $Re=1000$

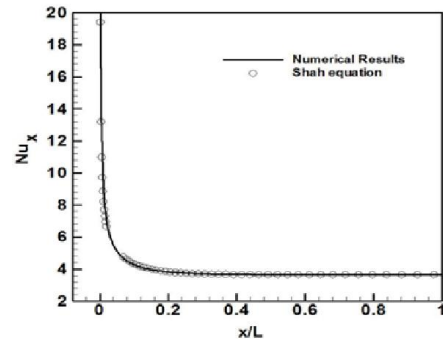


Figure 3 Comparison of Shah Equation and numerical results for local Nusselt number of pure water

In both models the density and the specific heat of the nanofluid is calculated from equations (1) and (2), respectively.

Before starting to determine the effects of the different thermophysical property correlations of the nanofluids on the predicted convective heat transfer coefficient of the nanofluid, comparisons between the two models for calculating the viscosity and thermal conductivity of nanofluids are made. The results of this comparison are illustrated in Figs. 4-7.

Figures 4 and 5 show the variations between the effective viscosity and thermal conductivity of nanofluid and temperature at a fixed volume fraction  $\phi = 0.03$ . As follows from this Figures, the trend of variation for both models are identical, while model I predicts the higher values.

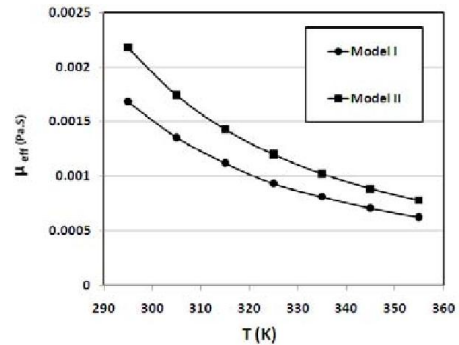


Figure 4 Comparison of the viscosity equations of nanofluid ( $\phi = 0.03$ )

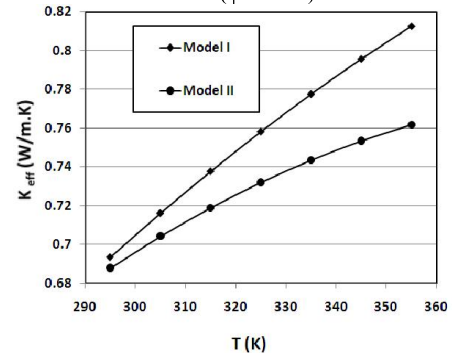


Figure 5 Comparison of the thermal conductivity equations of nanofluid ( $\phi = 0.03$ )

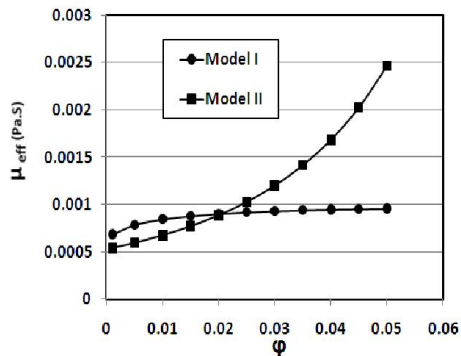


Figure 6 Comparison of the viscosity equations of nanofluid (T = 325K)

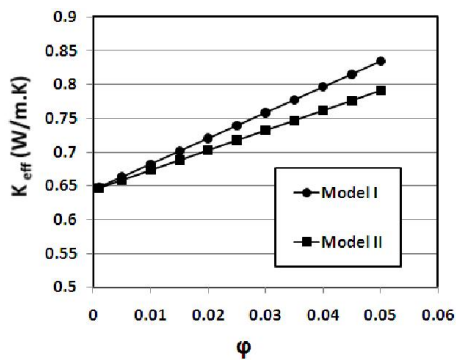


Figure 7 Comparison of the thermal conductivity equations of nanofluid (T = 325K)

Figures 6 and 7 show the variations between the effective viscosity and thermal conductivity of nanofluid and volume fraction at a fixed temperature  $T = 325\text{K}$ . The results show that the two models give different values and trend is not the same. Thus, it can be clearly seen that the different models give differing results when used to predict the thermophysical properties of nanofluid.

Figure 8 shows the computed average wall heat transfer coefficient ratio  $\bar{h}_r = \bar{h} / \bar{h}_{of}$  (ratio of the averaged wall heat transfer coefficient for nanofluid to the averaged wall heat transfer coefficient for pure water).

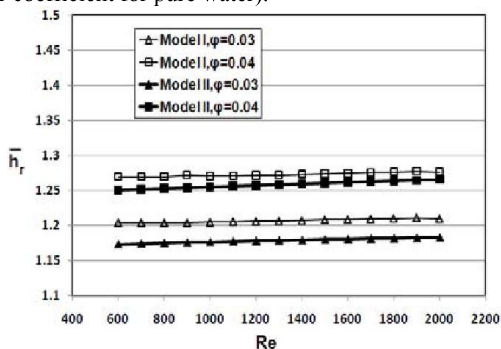
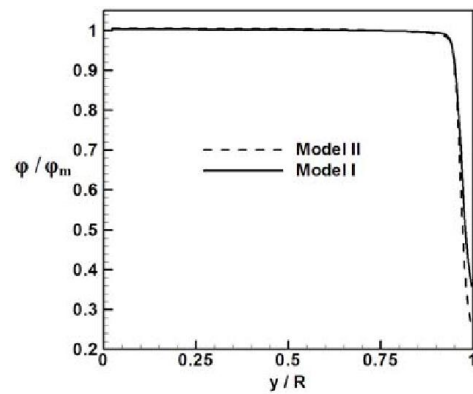


Figure 8 Effect of nanoparticle volume fraction on the averaged heat transfer coefficient ratio

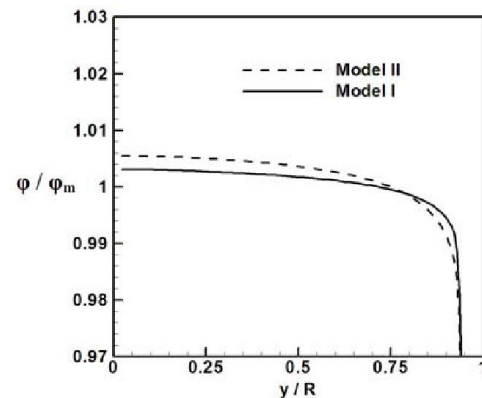
As is seen in Fig. 8, addition of alumina nanoparticles to pure water effectively enhances the convective heat transfer in laminar regime. The level of heat transfer increase depends on the volume concentration. Moreover, Model I predicts the higher values for heat transfer augmentation.

This study is based on the model developed by Buongiorno [15]. The difference of this model with single-phase traditional ones is existence of the nanoparticle continuity equation, i.e., Eq. (10). This equation states that nanoparticles can move homogeneously with the fluid, but they also possess a slip velocity relatively to the fluid, which is due to Brownian diffusion and thermophoresis. This slip phenomenon causes a non-uniform distribution of thermal conductivity and viscosity field and reduces the thermal boundary layer thickness.

Figure 9 shows the effect of using of two different models on nanoparticle concentration distribution. This figure shows that two adopted different models lead to substantial different results near the wall of the tube, while there is no significant effect at the centre of the tube (see Fig. 9 (b)).



(a)



(b)

Figure 9 Radial particle concentration distribution at  $Re = 1800$  and  $\phi_m = 0.04$

## CONCLUSION

The effects of the thermophysical properties model on the predicted values of the convective heat transfer coefficient and particle distribution profiles in  $\text{Al}_2\text{O}_3$ -water nanofluid flowing in a horizontal tube were numerically investigated.

As no generally accepted correlations currently exist for determining the nanofluid dynamic viscosity and thermal conductivity as a function of temperature and volume fraction, simultaneously, two different models were opted to use. In the first model the combination of two theoretical models for predicting the viscosity and thermal conductivity of nanofluid were used, while in the second one a combination of two empirical correlations were used.

From the obtained results, it can be concluded that the different models of the thermophysical properties of nanofluid give different results, especially for nanoparticle distribution along the radius of the tube. Therefore, the type of models used in simulation of nanofluid may be very important.

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