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THERMOMAGNETIC CONVECTION IN TWO- AND THREE-DIMENSIONAL CHANNELS USING THE LATTICE BOLTZMANN METHOD

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ABSTRACT

The influence of a magnetic field on heat transfer is studied by using the lattice Boltzmann method for a magnetic fluid (ferrofluid) flowing through a two-dimensional micro channel; to analyse the effect of the sidewalls upon the flow and heat transfer, the three-dimensional version of the micro channel is also studied. This problem is of considerable interest when dealing with cooling of micro-electronic devices. The magnitude of the magnetic force is controlled by changing the electrical current through a dipole. The results indicate that the flow is relatively uninfluenced by the magnetic field until its strength is large enough for the Kelvin body force to overcome the viscous force. It was observed that the magnetic force was able to change the flow field and increase the heat transfer in the channel.

INTRODUCTION

Thermomagnetic convection has been identified as a viable approach for augmenting and controlling the convective heat transfer. To this purpose, colloidal suspensions containing magnetic nano-particles, known as ferrofluids, are required, and their motion is controlled through external magnetic fields [1,2].

In the absence of an external magnetic field, these particles are oriented randomly in the carrying fluid; however, once an external magnetic field is applied to the suspension, the nano-particles align with it. These suspensions, in general, exhibit normal liquid behaviour coupled with super paramagnetic properties. This leads to the possibility of controlling the properties and the flow of these liquids with relatively moderate magnetic field strengths. This magnetic control has enabled numerous developments dealing with electrical, mechanical, biomedical and thermal engineering applications, and these suspensions are already being considered as the next-generation heat transfer fluids as they offer the possibility of achieving heat transfer rates much higher than those of conventional fluids and fluids containing micro-sized non-magnetic metallic particles [1-5]. The force generated between the magnetic field and the homogeneously distributed magnetic particles enables the manipulation of the ferrofluid. The direction and intensity of the interaction force are dependent on the orientation and strength of the magnetic field.

The present work aims to study the enhancement of laminar ferrohydrodynamic convection due to an imposed magnetic field to two- and three-dimensional channel flows using the lattice Boltzmann method (LBM) on D2Q9 and D3Q19 lattice [6], respectively.

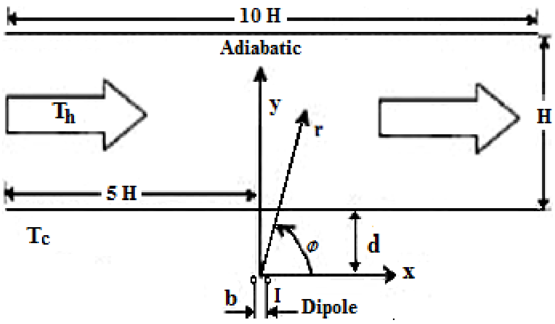
Finlayson [7] in 1970 explained how an external magnetic field imposed on a ferrofluid with varying magnetic susceptibility, e.g., due to a temperature gradient, results in a non-uniform magnetic body force, which leads to thermomagnetic convection. This leads to the possibility of controlling the magnetic properties and the flow of these liquids with magnetic fields of moderate intensity.

The numerical studies of heat transfer with magnetic fluids reported in the literature are relatively scarce and invariably use as governing equations the Navier-Stokes and energy equations. The majority of the studies available in the literature have considered uniform magnetic fields- an assumption which does not correspond to reality, when heat transfer applications are concerned [8,9]. In this work a non-uniform magnetic field will be considered and the variation of the magnetic susceptibility with temperature will be taken into account in the simulation. Ganguly *et al.* [10] did a comprehensive study on thermomagnetic convection in two-dimensional channels using Navier-Stokes equations.

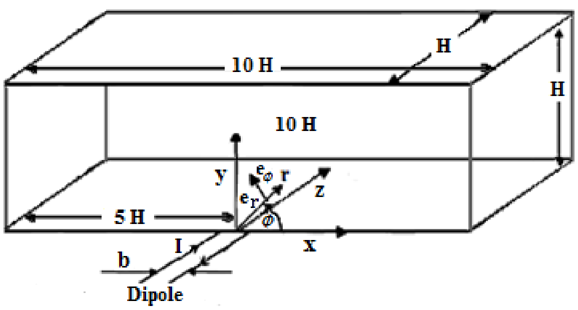
In the present study, the study starts with the simulation of 2-D thermomagnetic convection in a small scale channel using LBM; then, the methodology is extended to a 3-D small scale channel with the purpose of further validating the approach used and also to investigate the effect of the sidewalls upon the fluid flow and heat transfer.

The geometric configurations for two- and three dimensional cases are shown in Figures 1(a,b). The channel-flows are influenced by a magnetic field that can be described by the Maxwell's equations. For the two-dimensional case,

Figure 1a, a heated ferrofluid flows through a small channel, in which, for simulation purposes, the third dimension is infinite. The lower channel wall is considered to be an isothermal heat sink and the upper wall is adiabatic. For the three dimensional case, Figure 1b, a simulation of a hot ferrofluid flowing through a square channel is conducted. The bottom channel wall is considered to be an isothermal heat sink while the other walls are adiabatic. Line dipoles are placed adjacent to the isothermal walls of the two- and three-dimensional channels, H/2 from the lower wall, providing the external magnetic fields.



a) Two-dimensional configuration



b) Three-dimensional configuration

Figures 1. Schematic of two- and three-dimensional channels, a) and b) respectively, and position of the line dipole magnet.

NUMERICAL METHOD

The magnetic field is governed by the Maxwell's relations in static form [10], namely:

$$\nabla.B = 0 \quad , \quad \nabla \times H = 0 \tag{1}$$

where B is the magnetic field inside the ferrofluid due to the line dipole, which can be expressed as follows [10]:

$$B = \mu_0(1+\chi)m\left[\frac{\sin\theta}{r^2}e_r - \frac{\cos\theta}{r^2}e_\theta\right] \tag{2}$$

where μ_0 is the magnetic permeability of the free space, m denotes the magnetic dipole moment of the electromagnet coil per unit length, and χ is the susceptibility of the magnetic fluid. H and B are related by the following relation:

$$H = \frac{1}{\mu_0(1+\chi)}B \tag{3}$$

The magnetic fluid susceptibility, χ, varies with the temperature according to the function:

$$\chi = \frac{\chi_0}{1+\beta(T-T_0)} \tag{4}$$

where T_0 is the reference temperature and χ_0 is the magnetic field susceptibility for T_0 .

The ferrofluid becomes polarized in the presence of the external magnetic field by a magnetization process. Forces are generated on each particle in the ferrofluid due to the interaction of the fluid polarization and the external magnetic field. These forces can be modeled as a body force acting on the homogeneous ferrofluid: $F = (M.\nabla)B$. The Kelvin body force is simplified to the format proposed in [11] by defining an effective pressure, $P^* = P - \frac{\mu_0\chi_0}{2}H^2$, yielding:

$$F = \frac{1}{2}\mu_0\chi_0[1-\beta(T-T_0)]\nabla(H.H) + \mu_0\chi_0^2\beta(H.\nabla T)H \tag{5}$$

This body force is added to the single relaxation time lattice Boltzmann equation (LBE) D2Q9, for 2-D case, and D3Q19, for 3-D case, which can be written, when the LBGK collision operator [12] is used, as follows:

$$f_i(x+c_i\delta t, t+\delta t) - f_i(x,t) = -\frac{\delta t}{\tau}\left[f_i(x,t) - f_i^{eq}(x,t)\right] + F \tag{6}$$

$$g_i(\vec{x}+c_i\delta t, t+\delta t) - g_i(\vec{x},t) = -\frac{\delta t}{\tau_g}\left[g_i(\vec{x},t) - g_i^{eq}(\vec{x},t)\right] \tag{7}$$

where τ and τ_g are the momentum and energy relaxation times, respectively; δt is the lattice time step, related to the lattice length scale, δx, as $c=\delta x/\delta t$; f_i and g_i are the density and energy density distribution functions, respectively, which have direction-wise components on each lattice site and represents the probability of finding a fluid particle at position x at time t with direction e_i [6]. The discrete velocity is $c_i=c\ e_i$, where i=9 for D2Q9 and i=19 for D3Q19.

The macroscopic density (ρ), velocity ($\mathbf{u}$), and temperature, (T) on each lattice site are calculated as follow:

$$\rho = \sum_i f_i \qquad \rho\mathbf{u_i} = \sum_i c_i f_i \qquad T = \sum_i g_i$$

The kinematic viscosity and the thermal diffusivity are given by:

$$\upsilon = \left(\tau - \frac{1}{2}\right)c_s^2 \tag{8}$$

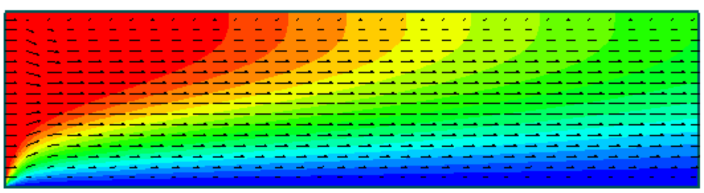
$$\alpha = \left(\tau_g - \frac{1}{2}\right)c_s^2 \tag{9}$$

RESULTS AND DISCUSSIONS

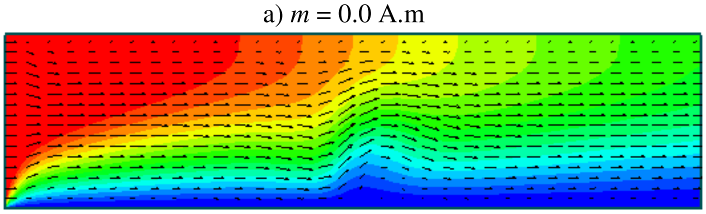
The density and dynamic viscosity of the ferrofluid at the reference temperature of 300 K are set equal to 1180 kg/m^3 and 1.0×10^{-3} kg/ms, respectively. The Prandtl number, *Pr*, and fluid compressibility, β, are taken as 5.5 and 5.6×10^{-4} K^{-1}, respectively. The inlet fluid temperature is 350 K whereas the cold lower wall is maintained at 300 K and the other walls are adiabatic. The flow is assumed to be thermally and hydrodynamically fully developed at the end of the channel. The size of the two-dimensional channel is 0.2 mm × 2 mm and the three-dimensional channel is a square channel with the size of 0.2 mm × 0.2 mm × 2 mm. The numerical solution should be independent of the lattice size, and to this purpose, a 51×501 and a 51×51×501 lattice density were found to be adequate to establish lattice-independent solutions for the range of the parameters used in the present study for two-dimensional and three-dimensional channels, respectively.

The presence of the magnetic dipole will disturb the flow motion in the channel and, as a consequence, the temperature distribution in the channel along with the heat transfer rate at its bottom wall. For *m*= 0.001 A.m a very small change in the fluid flow can be observed, and when this value is exceeded, the Kelvin body force was able to overcome the viscous force.

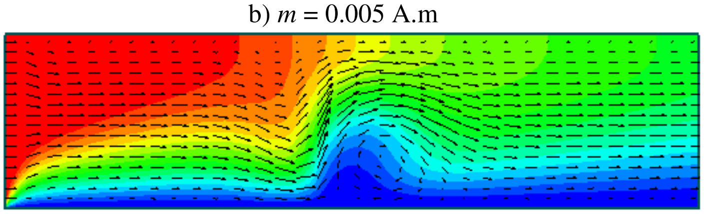
The simulated flow field and temperature profile in the two-dimensional channel are shown in Figures 2(a-c) for three different dipole strengths of *m* = 0.0, 0.005, and 0.010 A.m, respectively. The dimensionless velocity and temperature are calculated as: $u^* = \dfrac{u}{u_{in}}$ and $T^* = \dfrac{T - T_c}{T_h - T_c}$, respectively. The inlet dimensionless velocity is equal to one, $u^*_{in} = 1.0$.



a) *m* = 0.0 A.m



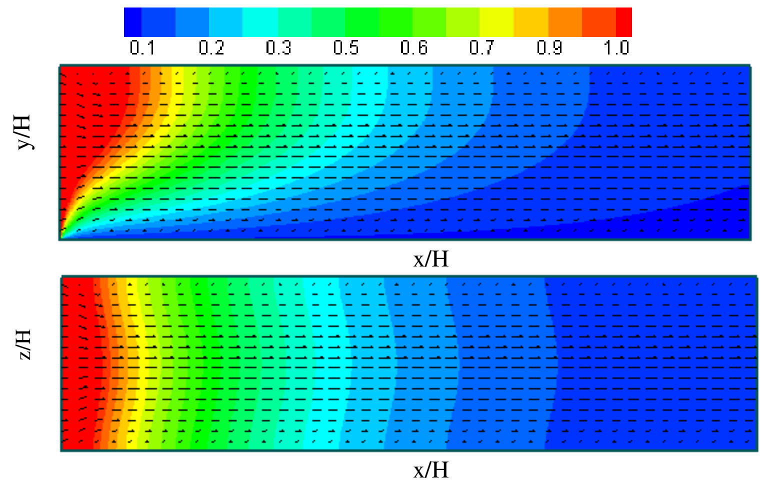
b) *m* = 0.005 A.m



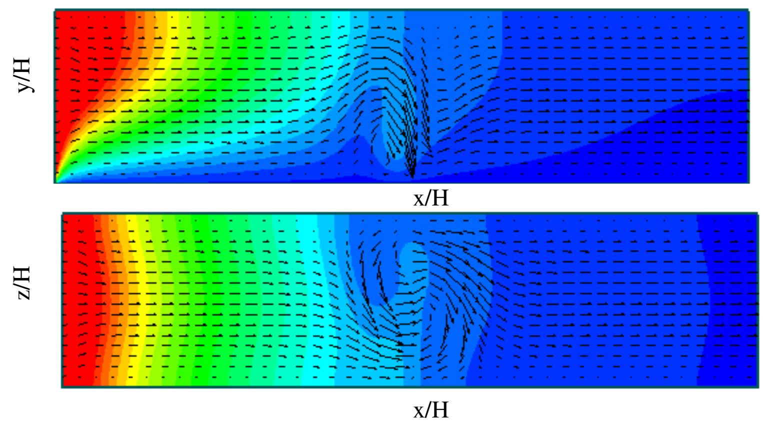
c) *m* = 0.010 A.m

Figures 2. Dimensionless velocity and temperature profiles for different electromagnet dipole strengths for two-dimensional channel, a) m = 0.0, b) m = 0.005, c) m = 0.010 A.m.

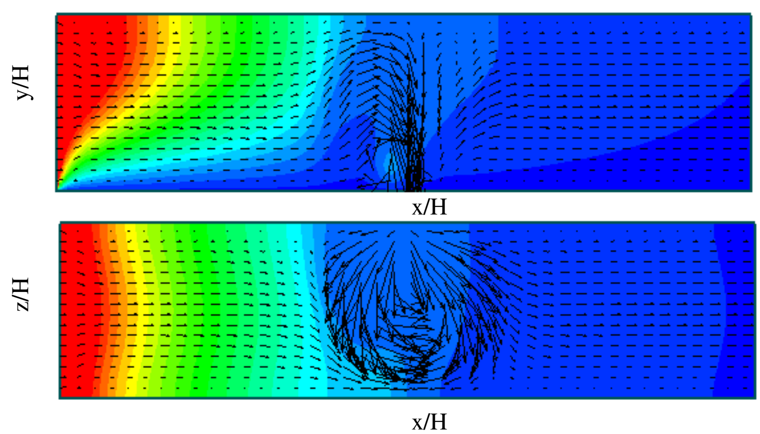
For the three-dimensional channel, the changes in the flow and the temperature for the planes in the streamwise direction and halfway the sidewalls (*y/H)* and the top and bottom walls (*z/*H) of the channel are shown in Figures 3(a-c) for *m*=0.0, 0.005, 0.01A.m.



a) *m* = 0.0 A.m



b) *m* = 0.005 A.m



c) *m* = 0.010 A.m

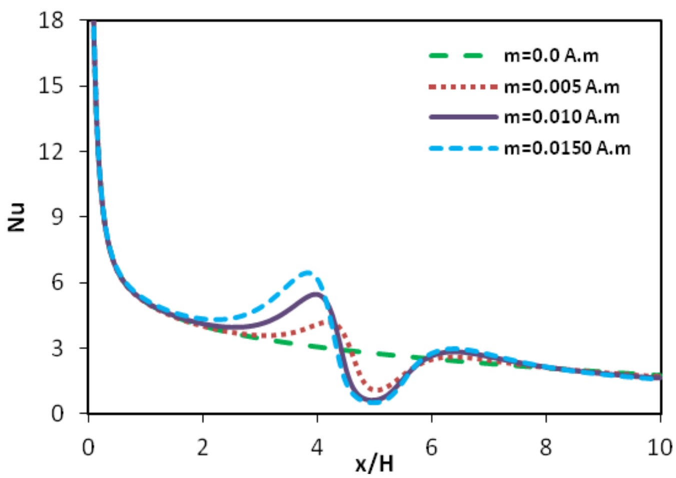
Figures 3. Dimensionless velocity and temperature profiles for different electromagnet dipole strengths for three-dimensional channel, a) m = 0.0, b) m = 0.005, c) m = 0.010 A.m.

Since the susceptibility of the colder fluid is larger than that in the warmer regions and the thermal boundary layer is not symmetric about the dipole location, a circulation zone is formed. This alters the advection of the energy, which causes the change of the temperature distribution in the fluid and enhances the heat transfer. Effects on the flow and temperature profile increase with increasing values of the dipole strength.

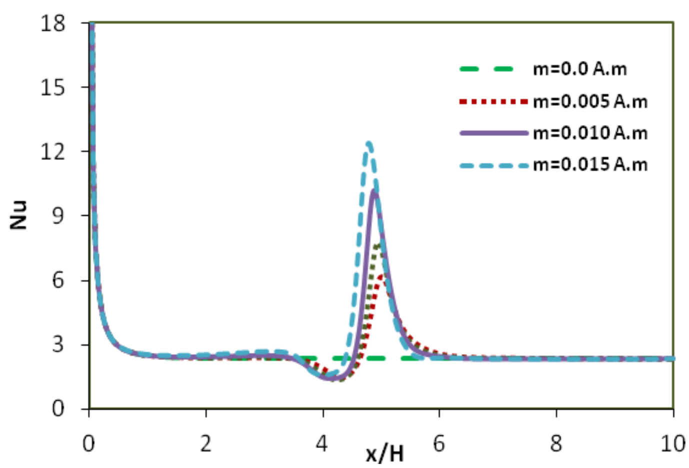
The predicted values of the local Nusselt number on the cold walls of the 2-D and 3-D cases for a range $0 \le m \le 0.015$ A.m are shown in Figures 4(a,b). The local Nusselt number was calculated based on the following equation:

$$Nu = D_h \frac{1}{T_h - T_c} \frac{dT}{dy}\Big|_{y=0} \tag{10}$$

where D_h is the hydraulic diameter. For the three dimensional channel, the local Nusselt number is the local average Nusselt number in the z direction of the cold wall.



a) Two-dimensional channel



b) Three-dimensional channel

Figures 4. Variation of the local Nusselt numbers for different magnetic fields.

The magnetic effects on the flow are localized, because the field intensity and its gradients diminish away from the magnetic dipole. Observation of Figures 4(a,b) indicates the local value of Nusselt number rises sharply in the region before the dipole. Downstream of the dipole, the formation of a vortex of colder fluid causes the decrease of heat transfer and, consequently, the reduction in Nusselt number.

The maximum and minimum peaks in the local Nusselt number distribution, Figure 4b, are closer together compared to those values for the two dimensional case (Figure 4a). This effect is explained by presence of the sidewalls. Specifically, the averaging of the local Nusselt number in the direction normal to the sidewalls reflects the effect – in the vicinity of the sidewalls, the reduction in velocity leads to decreasing heat transfer rates, which are not compensated by the acceleration of the core flow in the three dimensional case.

As a concluding remark, the numerical results clearly indicate significant changes of the temperature and velocity fields when an external magnetic field is applied to the ferrofluid flow. Of particular relevance is the finding that even relatively weak magnetic fields can yield a substantial increase in heat transfer for small scale devices.

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