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ANALYTICAL STUDY OF NATURAL CONVECTION IN A HEATED VERTICAL CHANNEL

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ABSTRACT

In the present work, natural convection of air in heated vertical channels is studied. On the basis of the conception of *induced-forced analogy*, a simple and efficient analytical method of heat transfer and flow calculations is developed. General heat-transfer correlations in the form $Nu = Nu(Re,Pr)$ are obtained for the air flow. The velocity (Reynolds number) is obtained by solving a momentum equation for induced convection taking into account energy balance for this case. It is shown that the developed method is simple and clear when applied to the problem under consideration. In this calculation, the initially known heat input (heat transfer rate or wall temperature) and hydraulic parameters are used. The developed method is applied to the problem natural convection of air in a symmetrically or asymmetrically heated vertical channel, which has allowed us to make a comparison with the literary data. The analogy and difference between the natural convection in a heated channel and a heated vertical plate in an infinite volume is demonstrated. A criterion of transition between the two is presented, and it is shown that this transition is smooth.

INTRODUCTION

In the present work, natural induced convection of air is studied. We use the term "induced" for the natural convection in channels with a physically obvious direction of a vented flow, as distinct from the free natural convection or, shortly, free convection in an infinite volume.

A conception of *induced-forced analogy* is offered in the present paper. In general, we obtain induced heat-transfer correlations in the form $Nu = Nu(Re,Pr)$ (*Re*-based analysis). The induced flow parameters must be defined, because they are initially unknown, in contrast to the case of forced convection. For this analysis, we have to consider the fact that an induced flow is initiated by the heating surface itself, and not by an external source. This fact determines the presence of an additional connection between thermal characteristics and air flow. Velocity (Reynolds number) calculation is realized by solving a momentum equation for induced convection taking into account the energy balance for this case. The constructive properties of the whole path of the flow are accounted for in hydraulic calculation similarly to those of forced convection. Therefore, the values of hydraulic coefficients play an important role in calculations, especially since the friction coefficient for the induced convection can differ from the case of forced convection. It should be pointed out that the calculations of hydraulic characteristics of the system are not the subject of our study. In our calculations we use data of Zvirin [1] for the friction coefficients for the induced convection flow, and recommendations of Idelchik [2] regarding local resistance factors.

Next, universal heat-transfer correlations $Nu(Re,Pr)$ are utilized. This procedure is basically the same as that for the forced convection. Note that heat-transfer correlations $Nu(Re)$ for induced convection do not coincide with heat-transfer correlations for forced convection, too.

Basic ideas of the present approach were first presented by Dubovsky et al. [3]. The approach has been thoroughly developed and verified by Dubovsky [4].

Several investigations were devoted to induced convection in a two-dimensional completely heated vertical channel. It is the simplest, although rather important case, which deserved greater attention. The results of a great number of experiments comprehensively reflecting the problem under study were presented by Aung et al. [5], Bar-Cohen and Rohsenow [6], Manca and Nardini [7], Auletta and Manca [8]. The generalization of experimental data, as mentioned by Auletta and Manca [8], was realized in Nu_b- Ra_b^* coordinates, where b is the channel spacing and $Ra_b^* = \dfrac{g\beta\rho_0^2 q'' b^5}{k\mu^2 L}Pr$ is a

NOMENCLATURE

a	[-]	Linear scale factor
A_c	[m^2]	Cross-section area
A_s	[m^2]	Heating surface area
b	[m]	Channel spacing
C	[-]	Constant
c_p	[J/kg K]	Specific heat of air
d_H	[m]	Hydraulic diameter of the channel
f	[-]	Friction coefficient
Fr	[-]	Froude number
g	[m/s^2]	Gravitational acceleration
Gr	[-]	Grashof number
Gr^*	[-]	Modified Grashof number
h	[W/m^{2}K]	Average heat transfer coefficient based on temperature difference ΔT
H	[m]	Height
k	[W/mK]	Thermal conductivity
L	[m]	Length of the channel
n	[-]	Number of heated sides in a two-dimensional channel
Nu	[-]	Nusselt number based on heat transfer coefficient h
P	[Pa]	Pressure
ΔP	[Pa]	Pressure difference
Pr	[-]	Prandtl number
q	[W]	Heat input
q''	[W/m^2]	Heat flux
Ra	[-]	Rayleigh number
Ra^*	[-]	Modified Rayleigh number (Elenbaas number)
Re	[-]	Reynolds number
T	[K]	Temperature
t	[s]	Time
ΔT	[K]	Temperature difference T_{av}-T_0
υ	[m/s]	Average air velocity
z	[m]	Vertical coordinate

Special characters

β	[K^{-1}]	Volumetric thermal expansion coefficient
Θ	[K]	Relative air temperature
μ	[kg/m s]	Dynamic viscosity
ρ	[kg/m^3]	Air density
ξ	[-]	Overall local resistance factor
ξ_0	[-]	Overall resistance factor

Subscripts

a	Air
av	Average
b	Channel spacing
D	Hydraulic diameter
eff	Effective
f	Friction
L	Heated length
out	Outlet
q	Heated
0	Ambient or overall

modified Rayleigh number (Elenbaas number). The general form of the correlations was

$$Nu_b = C\, Ra_b^{*\ 0.2} \tag{1}$$

where the constant value C is equal to 0.6 [5], 0.73 [6], 0.68 [7], 0.775 [8].

Developing equation (1), one can easily obtain $\dfrac{hb}{k} = C\left(\dfrac{g\beta\rho_0^2 q''}{k\mu^2 L} Pr\right)^{0.2} b$; where the channel spacing b is obviously cancelled. It is easy to check that these correlations are reduced to a well-known correlation of free convection for a vertical plate in an infinite volume (Raithby and Hollands [9]):

$$Nu_L = 0.624\, Ra_L^{*\ 0.2} \tag{2}$$

with a slight increase in the constant coefficient. The magnitude Ra_L^* is determined as $Ra_L^* = \dfrac{g\beta\rho_0^2 q'' L^4}{k\mu^2} Pr$. Thus, these correlations of equation (1) do not take into account differences between the channel and infinite volume. Note that this problem is absent in the benchmark paper involving the induced natural convection in a vertical channel by Elenbaas [10].

We demonstrate the simplicity and clearness of the developed method application to the problem under study and a good agreement of the calculated results with experimental data presented in [5]÷[8] and [10]. At the same time, we show the role of channel spacing.

Joint research of the applicability area of our method and the transition from the induced convection in a channel to the free convection in an infinite volume has shown that the limit of applicability of our method and the limit between the induced and free convection are the same limit. This limit corresponds to the value $\frac{b}{L} Ra_L^{0.25} = 12$, and the magnitude Ra_L is determined as $Ra_L = \dfrac{g\beta\rho_0^2 \Delta T\, L^3}{\mu^2} Pr$.

GENERAL APPROACH

Momentum equation and energy balance

We have to find a method of calculating the air flow velocity (Reynolds number) as a function of known values with the account for energy balance.

Attention is drawn to the fact that in natural circulation loops we deal with the induced convection analogously to open-ended vented channels. Zvirin [1] calculated fluid velocity by solving momentum equation for a natural loop and used these flow parameters in the heat-transfer analysis. In our general approach, we analyze a one-dimensional momentum equation for induced convection, too, but with boundary conditions of the ambient air.

A local one-dimensional momentum equation was written for an induced flow by Zvirin [1]. This equation for a vertical flow is

$$\rho\frac{\partial\upsilon}{\partial t} = \frac{\partial P}{\partial z} - \rho g - 4\frac{0.5 f\rho_0\upsilon^2}{d_H} \tag{3}$$

where P is pressure, υ is flow velocity, g is gravity acceleration, z is the vertical coordinate, and d_H is the hydraulic diameter of the flow channel. Shear stress at the wall is expressed by $1/2f\rho_0\upsilon^2$, where f is the friction coefficient. It is assumed that the Boussinesq approximation is valid, i.e., the density ρ is treated as constant except for the buoyancy force term. The density has a reference value ρ_0. Hence, the average velocity υ is uniform over the channel height.

A steady-state solution of Eq. (3) for an open-ended vertical channel, with due regard for local hydraulic resistance is

$$\Delta P = g\int_0^L \rho\, dz + \frac{1}{2}\left(\xi + f\,\frac{L}{d_H}\right)\rho_0 \upsilon^2 \quad (4)$$

where ΔP is the pressure difference between the outlet and inlet, and ξ is the overall local resistance factor.

The pressure difference between the outlet and inlet of the channel is the weight of the environmental air column of the height L per unit area ($g\rho_0 L$). Denote the average air density in the channel as

$$\rho_{av} = \frac{1}{L}\int_0^L \rho\, dz \quad (5)$$

with the corresponding average air temperature T_{av}.

Solving Eq. (4) in the framework of the Boussinesq approximation $\rho_0 - \rho_{av} = \beta\rho_0\left(T_{av} - T_0\right)$ yields:

$$g\beta H_{eff}\Theta_a = \frac{1}{2}\left(\xi + f\,\frac{L}{d_H}\right)\upsilon^2 \quad (6)$$

where a relative outlet air temperature $\Theta_a = T_{out} - T_0$, and we define the *effective height of the channel* H_{eff} as

$$H_{eff} = \frac{T_{av} - T_0}{\Theta_a} L \quad (7)$$

A detailed analysis carried out by Dubovsky [8] indicates that the effective height of the channel in a general case is close to the difference between the heights of the outlet of the channel and the heated plate centre.

$$\upsilon A_c \rho_0 c_p \Theta_a = q'' A_s \quad (8)$$

where A_c is the cross-section area of the heated channel and A_s is the area of the heating surface with an average flux $q'' = \frac{q}{A_s}$ for the overall heat input q. Thus,

$$\Theta_a = \frac{q''}{\upsilon\rho_0 c_p}\frac{A_s}{A_c} \quad (9)$$

Equation (6) yields:

$$\upsilon^3 = \frac{2}{\xi + f\,\dfrac{L}{d_H}}\,\frac{g\beta q''}{\rho_0 c_p}\frac{A_s}{A_c} H_{eff} \quad (10)$$

We use the Reynolds number and modified Grashof number based on the length of the heated plate H_q (for the case without adiabatic extension, H_q is equal to L): $Re_L = \frac{\upsilon\rho_0 H_q}{\mu}$, $Gr_L^* = \frac{g\beta\rho_0^2 q'' H_q^4}{k\mu^2}$, and Eq. (10) yields:

$$Re_L^3 = \frac{2}{\xi_0} Gr_L^* \frac{k}{\mu c_p}\frac{H_{eff}}{H_q}\frac{A_s}{A_c} \quad (11)$$

where we define the overall resistance factor ξ_0 as a sum of local and frictional resistance factors $\xi_0 = \xi + f\,\frac{L}{d_H}$.

Finally,

$$Re_L = \left(\frac{2}{\xi_0}\frac{H_{eff}}{H_q}\frac{A_s}{A_c}\frac{Gr_L^*}{Pr}\right)^{1/3} \quad (12)$$

The right-hand side of equation (12) generally contains a dimensionless hydraulic resistance term ($2/\xi_0$), dimensionless buoyancy term (H_{eff}/H_q), and dimensionless heat input term (A_s/A_c)(Gr^*_L/Pr). For the case without adiabatic extension, as mentioned above, the buoyancy term is equal to 0.5.

Heat-transfer correlations for the induced convection of air

In this section we present heat-transfer correlations as a power dependence of the *Nu* number on the *Re* number. This analogy with forced convection analysis is the central tenet of our concept. The existence of *Nu-Re* correlation with geometric parameters defined solely for a heated plate (heated channel) independently of other geometrical parameters (such as, for instance, parameters of the channel inlet-outlet, height and spacing of adiabatic extension and so on) demonstrates the advantage of our concept. It should be recalled that previous well-known correlations based on natural convection, such as *Nu-Gr* correlations, depend on some additional geometrical parameters and are applicable to a particular channel design only. In our approach, a general hydraulic characteristic of the whole channel is used for the induced flow calculation only, and specific heat-transfer experiments are not needed for every individual case.

A numerical analysis of more than 200 cases of induced convection was performed for various structural versions of the channel with different adiabatic extensions and heated plates, including its arrangement within complicated enclosures. The effects of chimney height, heat losses and radiation heat transfer, together with geometric and thermal scaling, were analyzed and substantiated by these results. The method and verification of the numerical calculations are described in the previous work (Dubovsky [4]). Finally, the following general *Nu-Re* correlations for an induced convection of air (Figure 1) were obtained:

$$\begin{aligned} Nu_L &= 2.05 Re_L^{0.4} Pr^{0.4} \qquad \textit{laminar flow} \\ Nu_L &= 0.12 Re_L^{0.75} Pr^{0.5} \qquad \textit{turbulent flow} \end{aligned} \quad (13)$$

These general correlations contain the Nusselt and Reynolds numbers based on the length of the heating surface. Note that for free convection in an infinite volume, the size used in dimensionless groups is the length of the heating surface, too.

In our previous work [4] it is shown that the laminar-turbulent limit criterion is Re_D (analogously to a forced pipe and tube flow), rather than Re_L. It is not surprising, because Re_D distinguishes laminar and turbulent flows in a channel in case of hydraulic calculation. Thus, it is not surprising that in the general graph in Figure 1 for 2000 < Re_L < 3000, the flow can be both laminar (for Re_D <2300), and turbulent (for Re_D > 2300).

Accordingly, the importance of both Re_D and Re_L and their different meanings are demonstrated. This dualism (partial correspondence to either forced or free convection) is the most specific quality of naturally induced flows in channels.

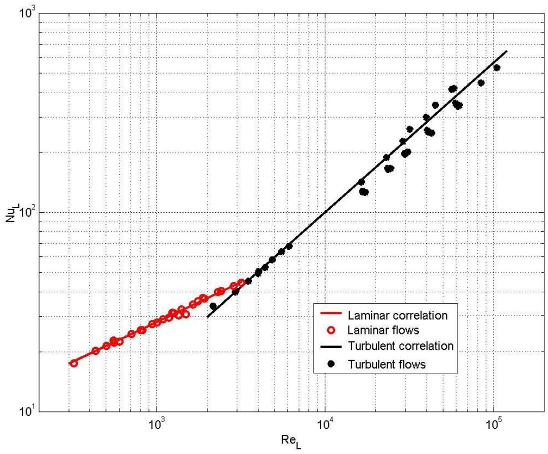

Figure 1. General graph of heat-transfer correlation

Simplification of the equations and reference case usage

Equations (12) - (13) generally allow obtaining the solution of the problem. By consideration of each separate case, the equations become simpler.

For the *laminar airflow* we assume that the local resistance is unimportant, and the friction coefficient f is inversely related to the Reynolds number ($f = \dfrac{C_f}{Re_D}$).

$$Nu_L = 2.05\left(\frac{2}{C_f}\frac{d_H^2}{H_q^2}\frac{H_{eff}}{H_q}\frac{A_s}{A_c}Ra_L^*\right)^{1/5} \tag{14}$$

where the modified Rayleigh number is based on the heated plate length, $Ra_L^* = Gr_L^* Pr$.

For the two-dimensional case without adiabatic extension, with regard for hydraulic data of Zvirin [1], equation (14) yields:

$$Nu_L = 0.892\left(n\frac{b}{L}Ra_L^*\right)^{1/5} \tag{15}$$

where b is the channel spacing and n is the number of heated sides; n = 2 for symmetrical (complete) heating and n = 1 for asymmetric heating.

Modifications of the relation (15) for the case of heat input considered as the known wall temperature instead of the known heat transfer rate, as well as for the case of dimensionless groups based on channel spacing b instead of plate length L, may be very useful:

$$Nu_L = 0.867\left(n\frac{b}{L}Ra_L\right)^{1/4} \tag{16}$$

$$Nu_b = 0.892\left(n\frac{b}{L}Ra_b^*\right)^{1/5} \tag{17}$$

$$Nu_b = 0.867\left(n\frac{b^2}{L^2}Ra_b\right)^{1/4} \tag{18}$$

where $Ra_b = \dfrac{g\beta\rho_0^2\Delta T b^3}{\mu^2}Pr$. Elenbaas number Ra_b^* and the Rayleigh number Ra_L were defined above.

Note that Eq. (17) demonstrates that a correct processing of experimental results in Nu_b- Ra_b^* coordinates should be carried out with respect to $\dfrac{b}{L}Ra_b^*$ value.

Now we consider a reference case that may be obtained from the study of experimental or numerical calculations. For this set of conditions, it is possible to perform a direct recalculation of another case with a different heat input or/and geometrically scaled system. The general forms of Eqs. (15), (17) and Eqs. (16), (18) in terms of physical parameters are, respectively:

$$h \sim \left(q''/a\right)^{1/5}; \quad h \sim \left(\Delta T/a\right)^{1/4} \tag{19}$$

where a is a linear scale factor.

Turbulent flow

$$Nu_L = 0.12\left(\frac{2}{\xi_0}\frac{H_{eff}}{H_q}\frac{A_s}{A_c}Ra_L^*\right)^{1/4} \tag{20}$$

For the same two-dimensional case without adiabatic extension, equation (20) yields:

$$Nu_L = 0.12\left(\frac{n}{\xi_0}\frac{L}{b}Ra_L^*\right)^{1/4} \tag{21}$$

If we assume that the local resistance is unimportant (similarly to the laminar case), equation (21) yields

$$Nu_L = 0.143\left(\frac{n}{f}Ra_L^*\right)^{1/4} \tag{22}$$

This result is independent of the channel spacing b if the friction coefficient f is constant.

If the overall resistance factor ξ_0 is independent of the linear scale factor, the general form of the relations for turbulent flows

$$h \sim (q'')^{1/4}; \quad h \sim (\Delta T)^{1/3}, \tag{23}$$

is also independent of the linear scale factor.

Validation of the results

The equations (12) - (13) generally allow receiving a correct solution of the problem.

Now we compare our analytical results with experimental data of the two-dimensional case without adiabatic extension [5]-[8].

According to Auletta and Manca [8], the generalization of experimental data of [5], [6], [7] and [8] was realized in Nu_b-Ra_b^* coordinates. The comparison of these results with calculations using equations (12)-(13) in the same coordinates is demonstrated in Figure 2.

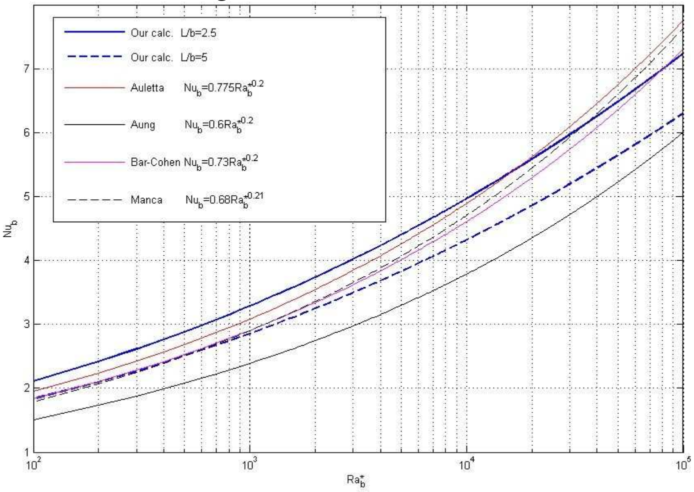

Figure 2. Comparison of our analytical predictions to experimental data of a fully heated two-dimensional channel

In the benchmark paper [10], Elenbaas obtains the correlation for a symmetrical heating case in the following form:

$$Nu_b = \frac{1}{24}\frac{b}{L}Ra_b\left(1-\exp\left[-\frac{35}{\frac{b}{L}Ra_b}\right]\right)^{0.75} \qquad (24)$$

A comparison with our results in Nu_b – Ra_b coordinates is shown in Figure 3.

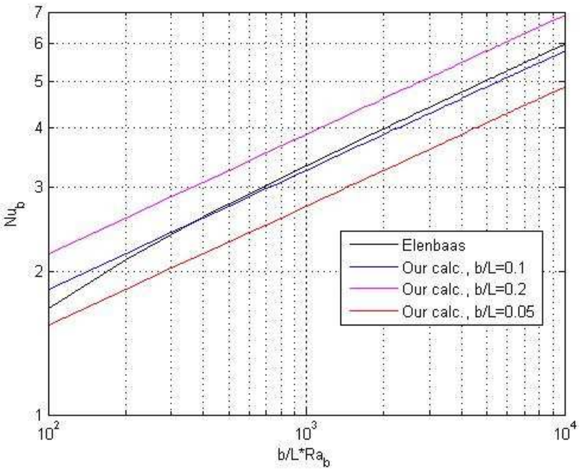

Figure 3. Comparison of Nusselt-Rayleigh relationships of Elenbaas with our calculations

Note that in the strict sense we compare two different cases – the general case examined by Elenbaas with the case of only frictional resistance without local hydraulic one. Nevertheless, the comparison demonstrates a good qualitative and quantitative correspondence.

LIMIT OF THE APPROACH AND TRANSITION FROM INDUCED CONVECTION TO FREE CONVECTION

As shown in our previous work (Dubovsky [4]), a criterion for the limits of the method in the general case is the value of the specific Froude number

$$Fr = \frac{2\upsilon^2}{g\beta d_H \Theta_a} > 4 \qquad (25)$$

Now we analyze a two-dimensional vertical channel involving a heated plate and an opposite plate. The opposite plate located at the distance b may be adiabatic (n = 1) or with the same heating (n = 2). Obviously, with increasing distance b, the induced convection in the channel finally passes into a free convection in an infinite volume. The following problems are studied:

- to define the limit between the induced convection in a channel and free convection in an infinite volume;
- to concretize the applicability limit of the analytical approach (equation (25)) for this case;
- to find out whether these limits are the same or not.

Numerical calculations were carried out in the following range: L = 0.1÷0.6m, b = 0.005÷0.2m, ΔT = 2÷100°C, q'' = 4÷200W/m^2. The results are processed with respect to the value $b/n/L \bullet Ra_L^{0.25}$ and presented in Figures 4 and 5.

In Figure 4 the value $Nu_L/(Re_L Pr)^{0.4}$ is shown in Y-direction. It is shown that this value is actually constant close to 2.05, as it should be according to the equation (13) for the induced convection under the conditions defined by the relation (25).

In Figure 5 the value Nu_L/Nu_{free} is shown in Y-direction, where Nu_{free} is determined by the well-known relation for a heated plate in an infinite volume [11]:

$$Nu_{free} = 0.59 Ra_L^{0.25} \qquad (26)$$

One can easily see that for $b/n/L \bullet Ra_L^{0.25}$>12, the heat transfer corresponds to a free convection in an infinite volume.

The range

$$3 < \frac{b}{nL} Ra_L^{0.25} \leq 12 \qquad (27)$$

corresponds to the induced convection in a channel both in Figure 4 and in Figure 5. In this range, the numerical results and the suggested analytical calculations for the induced convection are in a very good agreement.

For $b/n/L \bullet Ra_L^{0.25}$ < 3, the calculation of the heat transfer coefficient using temperature difference ΔT as for free convection, is physically incorrect.

Summing up, it is possible to assert that the applicability range of the suggested analytical method extends up to the transition of the induced convection into a free convection. Such transition occurs at $b/n/L \bullet Ra_L^{0.25}$ = 12.

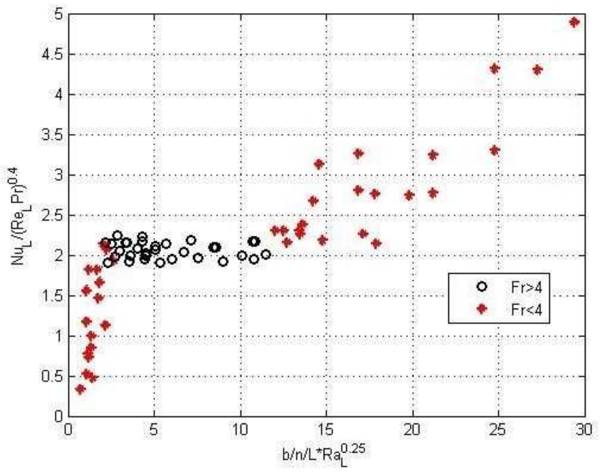

Figure 4. Ratio $Nu_L/(Re_L Pr)^{0.4}$ as a function of $b/n/L{\bullet}Ra_L^{0.25}$

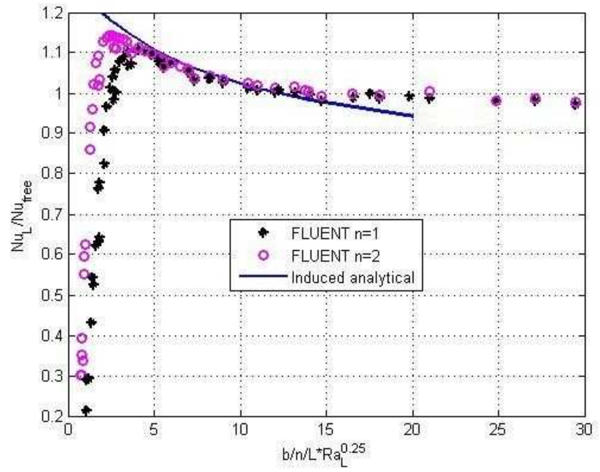

Figure 5. Ratio $Nu_L/(0.59Ra_L^{0.25})$ for different cases

SUMMARY

A conception of *induced-forced analogy* is suggested in the present paper. General heat-transfer correlations in the form $Nu = Nu(Re,Pr)$ (*Re*-based analysis) were obtained for the induced air flow. A preliminary calculation of the velocity (Reynolds number) is realized by solving a momentum equation for the induced convection taking into account the energy balance for this case. In this calculation, initially known parameters (the Grashof number) are used, as well as known hydraulic resistance factors.

Simplified relations for the laminar and turbulent cases are obtained. The existence of a reference case is considered. For a typical flow, simple formulas for a direct recalculation of another case with a changing heat input or/and geometrically scaled system are demonstrated.

The developed method is applied to the problem of induced convection in a symmetrically or asymmetrically heated vertical channel that allowed us to make a comparison with the literary data. Results of the calculation are in good agreement with the experimental data presented in [5]-[8] and [10]. The point is that recently suggested heat transfer correlations [5]-[8] do not differ, basically, from the known correlations for a free convection in an infinite volume. The one and only difference consists in the use of dimensionless groups based on channel spacing, whereas for a free convection the dimensionless groups are based on the plate height. The difference disappears after the transformation of correlations to the same Nusselt and Rayleigh numbers. This effect has a simple physical explanation. Actually, it is shown above that the limit between the induced convection and free convection in an infinite volume occurs at $b/n/L{\bullet}Ra_L^{0.25}=12$. For $b/n/L{\bullet}Ra_L^{0.25} > 12$, the heat transfer coincides with the free convection in an infinite volume. Accordingly, at $b/n/L{\bullet}Ra_L^{0.25} < 12$, the transition to our relations for the induced convection occurs very smoothly.

Thus, it is possible to recommend data generalization of the induced convection in channels with respect to the Nusselt – Rayleigh numbers based on the height of a heated plate and in comparison with the well-known free convection in an infinite volume.

It is noteworthy that the results of the presented theoretical analysis can find a wider application and are not limited by the problem under study. In particular, they can be applied to the cases of three-dimensional heated channels, channels with adiabatic extensions, and others.

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