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OPTIMISATION OF CONJUGATE TRIANGULAR COOLING CHANNELS WITH INTERNAL HEAT GENERATION

Olakoyejo O.T., Bello-Ochende T.* and Meyer J.P.
*Author for correspondence
Department of Mechanical and Aeronautical Engineering,
University of Pretoria, Pretoria,
Private Bag X20, Hatfield 0028,
South Africa.
E-mail: Tunde.Bello-Ochende@up.ac.za

ABSTRACT

This work presents a three dimensional geometric optimisation of a conjugate cooling channel in forced convection with an internal heat generation within the solid for isosceles right triangular channel and equilateral triangular channel configurations. The isosceles right triangle and equilateral triangle are special case of triangle which can easily and uniformly be packed and arranged to form a larger constructs.

The configurations were optimised in such a way that the peak temperature was minimised subject to the constraint of fixed global volume of solid material. The cooling fluid is driven through the channels by the pressure difference across the channel. The structure has channel height, width and channel to channel spacing as degrees of freedom as design variables. The shape of the channel is allowed to morph to determine the best configuration that gives the lowest thermal resistance. A gradient-based optimisation algorithm is applied in order to search for the best optimal geometric configurations that improve thermal performance by minimising thermal resistance for a wide range of dimensionless pressure difference. This optimiser adequately handles the numerical objective function obtained from CFD simulations. The effect of porosities, applied pressure difference and heat generation rate on the optimal aspect ratio and channel to channel spacing are reported. There are unique optimal design variables for a given pressure difference

The numerical results obtained are in agreement with the theoretical formulation using scale analysis and method of intersection of asymptotes

Results obtained show that the effects of dimensionless pressure drop on minimum thermal resistance are consistent with those obtained in the open literature.

INTRODUCTION

Constructal theory and design [1, 2] have been adopted as an optimisation technique for the development of a procedure that is sufficiently allocating and optimising a fixed global space constraint using a physical law (constructal law). The method seeks to optimise the flow architecture that predicts the flow and thermal fluid behaviour in a structure that is subject to a global volume constraint. Bejan [1, 2] stated this law as: *For a finite-size system to persist in time (to live), it must evolve in such a way that it provides easier access to the imposed (global) currents that flow through it.*

The application of this theory started with Bejan and Sciubba [3], who obtained a dimensionless pressure difference number for optimal spacing of board to board of an array of parallel plate to channel length ratio and a maximum heat transfer density that can be fitted in a fixed volume in an electronic cooling application using the method of intersection asymptotes. This body of knowledge has been applied in all facets of lives; from humanity and nature to science and engineering [4-8].

In this paper our focus is on the original engineering application of Constructal theory, which is the geometric and shape optimisation especially in heat transfer analysis [9-11]. The advantage of constructal law in the engineering field is that flow architecture is not assumed in advance of the optimisation process, but is its consequence by allowing the structure to morph [12]. The applications of this theory have been reviewed most recently by the work of Bejan and Lorente [13], in which under certain global constraints, the best architecture of a flow system can be archived as the one that gives less global flow resistances, or allows high global flow access. In other words, the shapes of the channels and unit structure that is subject to global constraint are allowed to morph. The optimisation of heat exchangers and multiscale devices by constructal theory was also, recently reviewed and summarised by Luo Fan [14].

Yilmaz *et al.* [15] studied the optimum shape and dimensions for convective heat transfer of laminar flow at constant wall temperatures for ducts with parallel plate, circular, square and equilateral triangle geometries. Approximate equations were derived in the form of maximum dimensionless heat flux and optimum dimensionless hydraulic diameter in terms of the duct shape factors and the Prandlt number (*Pr*).

Da Silva et al. [16], optimised the space allocation on a wall occupied by discrete heat sources with a given heat generation rate by forced convection using the method of constructal theory in order to minimise the temperature of the hot spot on the wall.

Also, Bello-Ochende et al. [17] conducted a three-dimensional optimisation of heat sinks and cooling channels with heat flux using scale analysis and the intersection of asymptotes method based on constructal theory to investigate and predict the design and optimisation of the geometric configurations of the cooling channels. Rocha et al. [18] and Biserni et al. [19] applied the theory to optimise the geometry of C- and H-shaped cavities respectively that intrude into a solid conducting wall in order to minimise the thermal resistance between the solid and the cavities. Muzychka [20] studied and analysed the optimisation of microtube heat sinks and heat exchangers for maximum thermal heat transfer by using a multiscale design approach. In his analysis, he was able to show that through the use of interstitial microtubes, the maximum heat transfer rate density for an array of circular tubes increased. Reis *et al.* [21] optimised the internal configurations of parallel plate and cylindrical channels using contructal theory to understand the morphology of particle agglomeration and the design of air-cleaning devices.

The recent comment by Meyer [22] on the latest review of constructal theory by Bejan and Lorente [23] shows that the constructal law's application in all fields of educational design is a wide road to future advances.

This paper focuses on the study of three-dimensional, laminar forced convection cooling of triangular solid structures. It examines the optimisation of a fixed and finite global volume of solid materials with an array of rectangular cooling channels, which experience a uniform internal heat generation which will result in the minimal global thermal resistance. The objective is the building of a smaller construct to form a larger construct body that will lead to the minimisation of the global thermal resistance or, inversely, the maximisation of the heat transfer rate density (the total heat transfer rate per unit volume). This is achieved by forcing a coolant to the heated spot in a fast and efficient way so as to drastically reduce the peak temperature at any point inside the volume that needs cooling. The optimisation process is carried out numerically under total fixed volume and manufacturing constraints.

This study is an extension of our previous work [24, 25] on the constructal theory for the cylindrical, square and rectangular configurations with internal heat generation, where we showed that the minimised peak temperature is a function of the geometry and shape. Triangular shapes are considered separately because of the unique nature of the internal configurations.

NOMENCLATURE

Be	[-]	Dimensionless pressure drop number
P	[Pa]	Pressure
Re	[-]	Reynolds number
Pr	[-]	Prandtl number
q''	$[W/m^2]$	Heat flux
C_P	$[J/kgK]$	Specific heat at constant pressure
T	$[^0C]$	Temperature
T_{max}	$[^0C]$	Peak temperature
T_{in}	$[^0C]$	Inlet temperature
H	[m]	Structure height
R	[-]	Thermal resistance
V	[m^3]	Structure volume
W	[m]	Structure width
L	[mm]	Axial length
LFOPC	[-]	Leapfrog Optimisation Program for Constrained Problems
V_{el}	[m^3]	Elemental volume
V_c	[m^3]	Channel volume
w	[mm]	Elemental width
h	[mm]	Elemental height
I-R	[-]	Isosceles right
Equi	[-]	Equilateral
d_h	[mm]	Hydraulic diameter
s	[mm]	Channel-to-channel spacing
N	[-]	Number of channels
x, y, z	[m]	Cartesian coordinates
n	[-]	Normal

Greek symbols

k	$[W/mK]$	Thermal conductivity
α	$[m^2/s]$	Thermal diffusivity
μ	$[kg.s/m]$	Viscosity
ν	$[m^2/s]$	Kinematics viscosity
ρ	$[kg/m^3]$	Density
∞		Far extreme end
ϕ	[-]	Porosity
Δ	[-]	Difference
i	[-]	Mesh iteration index
γ	[-]	Convergence criterion

Subscripts

0	Initial extreme end
f	Fluid
in	Inlet
max	Maximum
Min	Minimum
opt	Optimum
out	Outlet
s	Solid

COMPUTATIONAL MODEL

The physical configuration is shown schematically in Fig. 1. The system consists of parallel cooling channels of length, L of fixed global volume, V for the two configurations. The internal heat generation in the solid material is q_s'''. The body is cooled by forcing a single-phase cooling fluid (water) from the left side through the parallel cooling channels. The flow is driven along the length L, of the circular channel with a fixed pressure difference ΔP.An elemental volume, v_{el} , consisting of a cooling channel and the surrounding solid was used for analysis because of the assumption of the symmetrical heat distribution inside the structure. However, the elemental volume v_{el} is not fixed and is allowed to morph by varying cooling channel shape v_c for fixed porosity. The heat transfer in the elemental volume is a conjugate problem, which combines heat conduction in the solid and the convection in the working fluid. These two modes of heat transfer are coupled together through the continuity of heat flux at the solid-fluid interface.

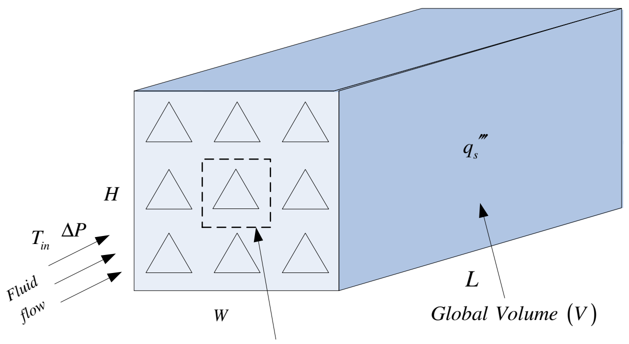

Figure 1. Three-dimensional parallel square channels across a slab with heat flux from one side and forced flow from the other side

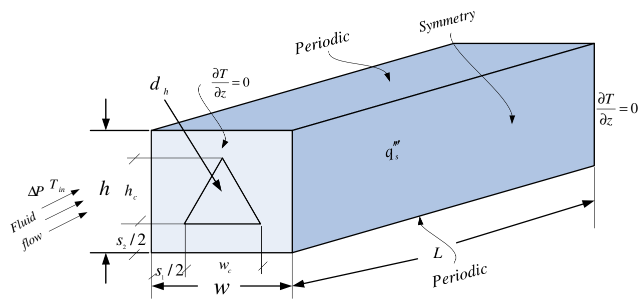

Figure 2. The boundary conditions of the three-dimensional computational domain of the elemental volume

Design variables for isosceles triangle

In Fig. 2, an elemental volume, v_{el}, constraint is considered to be composed of an elemental cooling channel of hydraulic diameter, d_h, and the surrounding solid of thickness s (spacing between channels) and these variables are defined as:

$$w = h, \quad v_{el} = w^2 L, \quad h_c = \frac{w_c}{2}, \quad d_h = \frac{w_c^{\,2}}{w_c + \sqrt{2}w_c}, \quad v_c = \frac{w_c^{\,2}}{4}L \tag{1}$$

However, the void fraction or porosity of the unit structure can be defined as:

$$\phi = \frac{v_c}{v_{el}} = \frac{w_c}{4w} \tag{2}$$

Design variables for equilateral triangle

In Fig. 2, an elemental volume, v_{el}, constraint is considered to be composed of an elemental cooling channel of hydraulic diameter, d_h, and the surrounding solid of thickness s (spacing between channels) and these variables are defined as:

$$w = h, \quad v_{el} = w^2 L, \quad h_c = \frac{\sqrt{3}}{2}w_c, \quad d_h = \frac{1}{\sqrt{3}}w_c, \quad v_c = \frac{\sqrt{3}}{4}w_c^{\,2}L \tag{3}$$

However, the void fraction or porosity of the unit structure can be defined as:

$$\phi = \frac{v_c}{v_{el}} = \frac{\sqrt{3}}{4}\frac{w_c}{w} \tag{4}$$

For a fixed length of the channel, the cross-sectional area of the structure is

$$A_s = HW \tag{5}$$

Therefore, the number of channels in the structure arrangement Therefore, the total number of channels in the structure arrangement for the two configurations can be defined as:

$$N = \frac{HW}{hw} = \frac{HW}{(h_c + s_2)(w_c + s_1)} \tag{6}$$

Some other assumptions imposed on the two triangular configurations model are, the solid structure top and bottom boundaries of the domain correspond to periodic boundary conditions, the left and right side of the solid surfaces were taken as symmetry boundary conditions. All the outside walls were taken as plane of symmetry of the solid structure and were modelled as adiabatic as shown in Figure 2.

The fundamental problem under consideration is the numerical optimisation of d_h and S, which corresponds to the minimum resistance of a fixed volume for a given pressure drop. The optimisation is evaluated from the analysis of the extreme limits of $(0 \le d_h \le \infty)$ and the extreme limits of $(0 \le s \le \infty)$. The optimal values of the design variables within the prescribed interval of the extreme limits exhibit the minimum thermal resistance. The temperature distribution in the model was determined by solving the equation for the conservation of mass, momentum and energy numerically. The discretised three-dimensional computational domain of the configuration is shown in figure. 3. The cooling fluid was water, which was forced through the cooling channels by a specified pressure difference ΔP across the axial length of the structure. The fluid is assumed to be in single phase, steady and Newtonian with constant properties. Water is more promising than air, because air-cooling techniques are not likely to meet the challenge of high heat dissipation in electronic packages [26, 27].

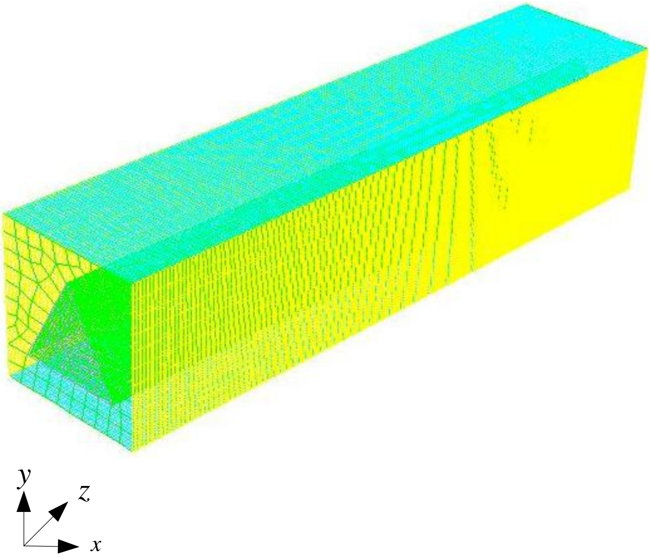

Figure 3. The discretised 3-D computational domain for triangular cooling channel

The governing differential equations used for the fluid flow and heat transfer analysis in the unit volume of the structure are:

$$\nabla \cdot \vec{u} = 0 \quad (7)$$

$$\rho(\vec{u} \cdot \nabla \vec{u}) = -\nabla P + \mu \nabla^2 \vec{u} \quad (8)$$

$$\rho_f C_{Pf} (\vec{u} \cdot \nabla T) = k_f \nabla^2 T \quad (9)$$

Energy equation for a solid given as:

$$k_s \nabla^2 T + q_s''' = 0 \quad (10)$$

The continuity of the heat flux at the interface between the solid and the liquid is given as:

$$k_s \left.\frac{\partial T}{\partial n}\right| = k_f \left.\frac{\partial T}{\partial n}\right| \quad (11)$$

A no slip boundary condition is specified at the wall of the channel, $\vec{u} = 0$, at the inlet (x = 0), $u_x = u_y = 0$, $T = T_{in}$ and

$$P = \frac{Be \alpha u}{L^2} + P_{out} \quad (12)$$

where, Be is the dimensionless pressure difference called Bejan number [28,29].

At the outlet ($x = L$), zero normal stress, $P_{out} = 1 \ atm$

At the solid boundaries,

$$\nabla T = 0 \quad (13)$$

The measure of performance is the minimum global thermal resistance, which could be expressed in a dimensionless form as:

$$R_{\min} = \frac{k_f \left(T_{\max} - T_{in}\right)_{\min}}{q_s''' L^2} \quad (14)$$

And it is a function of the optimised design variables and the peak temperature.

$$R_{\min} = f\left(AR, d_h, s_1, s_2, \phi, \left(T_{\max}\right)_{\min}\right) \quad (15)$$

R_{min} is the minimised thermal resistance for the optimised design variables. The inverse of R_{min} is the optimised overall global thermal conductance.

NUMERICAL PROCEDURE AND GRID ANALYSIS

The simulation work began by fixing the length of the channel, applied pressure difference, porosity, The internal heat generation and material properties and we kept varying the values of the elemental volume and hydraulic diameter of the channel in order to identify the best (optimal) internal configuration that minimised the peak temperature. The numerical solution of the continuity, momentum and energy Eqs. (7) - (10) along with the boundary conditions (11) - (13) was obtained by using a three-dimensional commercial package FLUENT™ [30], which employs a finite volume method. The details of the method were explained by Patankar [31]. FLUENT™ was coupled with geometry and mesh generation package GAMBIT [32] using MATLAB [33] to allow the automation and running of the simulation process. After the simulation had converged, an output file was obtained containing all the necessary simulation data and results for the post-processing and analysis. The computational domain was discretised using hexahedral/wedge elements. A second-order upwind scheme was used to discretise the combined convection and diffusion terms in the momentum and energy equations. The SIMPLE algorithm was then employed to solve the coupled pressure-velocity fields of the transport equations. The solution is assumed to have converged when the normalised residuals of the mass and momentum equations fall below 10^{-6} and while the residual convergence of energy equation was set to less than 10^{-10}. The number of grid cells used for the simulations varied for different elemental volume and porosities. However, grid independence tests for several mesh refinements were carried out to ensure the accuracy of the numerical results. The convergence criterion for the overall thermal resistance as the quantity monitored is:

$$\gamma = \frac{\left|\left(T_{\max}\right)_i - \left(T_{\max}\right)_{i-1}\right|}{\left|\left(T_{\max}\right)_i\right|} \le 0.01 \quad (16)$$

where i is the mesh iteration index. The mesh is more refined as i increases. The $i-1$ mesh is selected as a converged mesh when the criterion (16) is satisfied.

NUMERICAL RESULTS

In this section, we present results for the case when the elemental volume was in the range of $0.025 \text{ mm}^3 \le v_{el} \le 5 \text{ mm}^3$ and the porosities ranged between $\phi = 0.2$ and a fixed length of L = 10 mm and fixed applied dimensionless pressure differences of $\Delta P = 50$ kPa . The internal heat generation within the solid was taken to be fixed at 100 kW/m^3. The thermophysical properties of water [34] used in this study were based on water at $300K$ and the inlet water temperature was fixed at this temperature.

Figures 5 and 6 show the existence of an optima hydraulic diameter and optimal elemental volume of the structure that minimised the peak temperature at any point in the channel for the for the two types of triangular configurations studied. Figure 5 shows the peak temperature as a function of the channel hydraulic diameter at this fixed pressure difference. It shows that there exists an optimal channel hydraulic diameter, which lies in the range $0.005 \le d_h/L \le 0.02$ minimising the peak temperature.

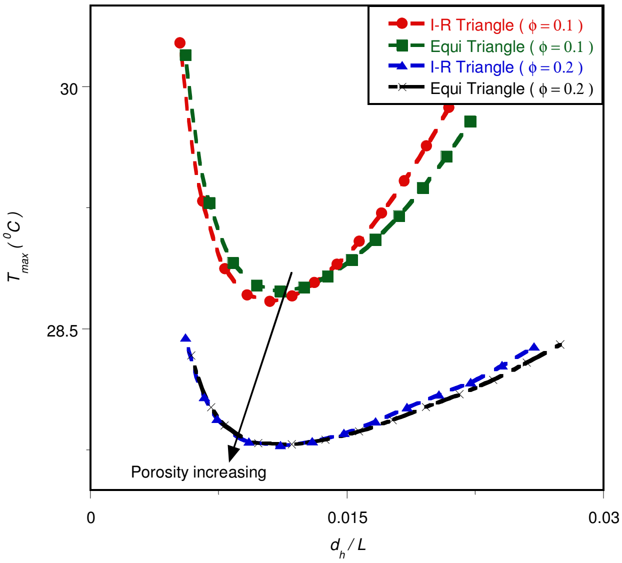

Figure 5 Effect of optimised hydraulic diameter d_h, on the peak temperature

Also, the elemental volume of the structure has a strong effect on the peak temperature as shown in Figure 6. The minimum peak temperature is achieved when the optimal elemental volume of the structure that minimised the peak temperature and this lies in the range of 0.2 mm^3 $\le v_{el} \le$ 2.5 mm^3 . Any further increase or decrease in the design variable beyond the optimal values indicates that the working fluid is not properly engaged in the cooling process, which is detrimental to the global performance of the system. The results also, show that the optimal arrangement of the elemental volume for the entire structure at this fixed pressure difference should be very small in order to achieve better cooling.

MATHEMATICAL OPTIMISATION

In this section, we introduce an optimisation algorithm that will search and identify the optimal design variables at which the system will perform best. A numerical algorithm, Dynamic-Q [35], is employed and incorporated into the finite volume solver and grid (geometry and mesh) generation package by using MATLAB for more efficient and better accuracy in determining the optimal performance.

The Dynamic-Q is a multidimensional and robust gradient-based optimisation algorithm, which does not require an explicit line search. The technique involves the application

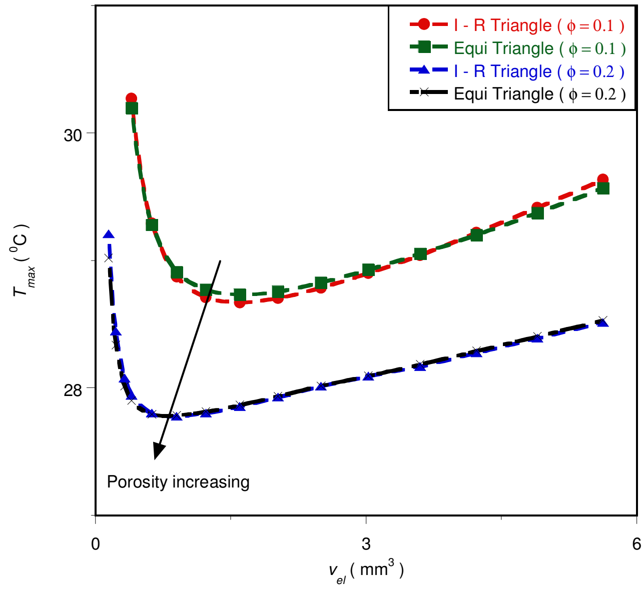

Figure 6 Effect of optimised elemental volume, v_{el}, on the peak temperature

of a dynamic trajectory LFOPC optimisation algorithm to successive quadratic approximations of the actual problem [36]. The algorithm is also specifically designed to handle constrained problem where the objective and constraint functions are expensive to evaluate. The details of the Dynamic-Q and applications can be found in open literature [35-40].

OPTIMISATION PROBLEM

Design variable constraints

The constraint ranges for the optimisation are:

$$0.025\text{mm}^3 \le v_{el} \le 5\text{mm}^3, \quad 0.1 \le \phi \le 0.2,$$
$$0 \le h_c \le h, \quad 0 \le w_c \le w, \quad 0 \le d_h \le w, \quad (17)$$
$$0 \le s_1 \le w, \quad 0 \le s_2 \le w$$

The design and optimisation technique involves the search for and identification of the best channel layout that minimises the peak temperature, T_{max} such that the minimum thermal resistance between the fixed volume and the cooling fluid is obtained with the desired objectives function. The hydraulic diameter and the channel spacing and elemental volume of the square configuration were considered as design variables. A number of numerical optimisations and calculations were carried out within the design constraint ranges given in (17) and the results are presented in the succeeding section in order to show the optimal behaviour of the entire system. The elemental volume of the structure was in the range of 0.4 mm^3 to 5 mm^3. The optimisation process was repeated for applied dimensionless pressure differences (Be) that correspond to ΔP = 5 kPa to ΔP = 50 kPa.

Effect of applied pressure difference on optimised geometry and minimised thermal resistance

Figures 7 show the minimised dimensionless global thermal resistance as a function of dimensionless pressure difference at different porosity for the two triangular configurations. The results show that the minimised dimensionless global thermal resistance monotonically decreases as the dimensionless pressure difference increases.

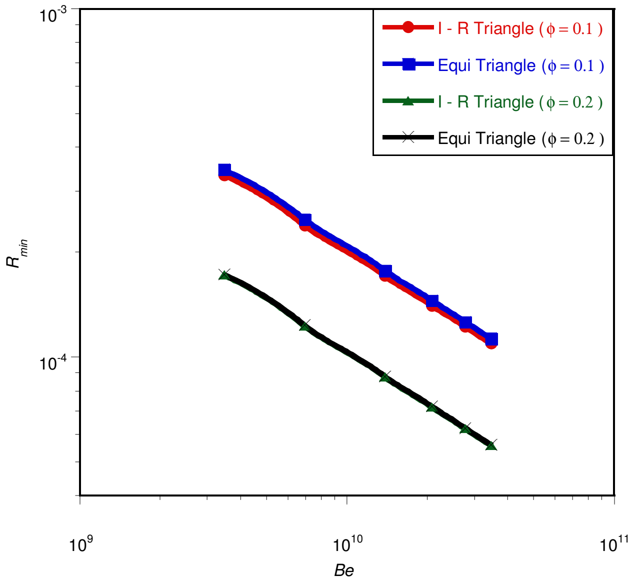

Figure 7. The effect of dimensionless pressure difference on the minimised thermal resistance

Also Figures 8 and 9 show the optimal behaviours of the geometry with respect to applied dimensionless pressure difference (or Bejan number) at different porosity for the two triangular configurations. The Figure 8 show that the optimal hydraulic diameter decreases as the dimensionless pressure differences increase and there exists a unique optimal geometry for each of the applied dimensionless pressure differences for the configurations. The trend is in agreement with previous work [24, 40].

The optimal channel spacing ratio (s_1/s_2) remains unchanged and insensitive to the performance of the system whereas regardless of the dimensionless pressure difference number for the two triangular configurations as shown in Figure 9. This constant value could be described as allowable spacing due to manufacturing constraints. This implies that the closer the channels are to one another, the better the effective cooling ability of the global system.

Figure 10a and 10b show the temperature contours of the elemental structure and of the inner wall of the cooling channel with cooling fluid triangular configurations. The blue region indicates the region of low temperature and the red region indicates that of high temperature

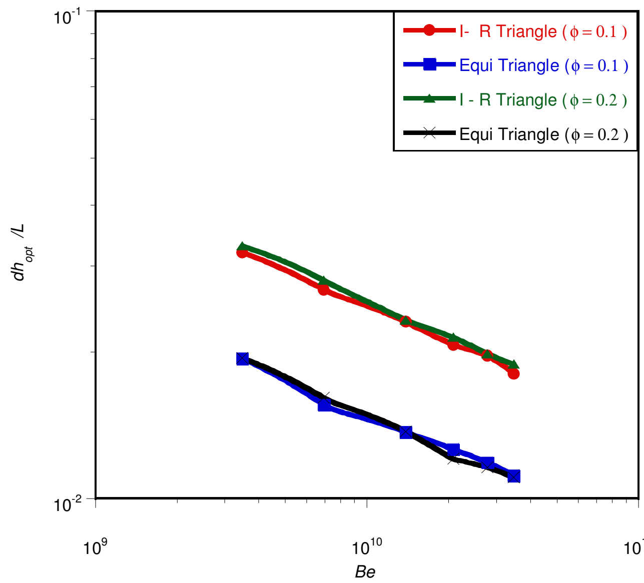

Figure 3. The effect of dimensionless pressure difference on the optimised hydraulic diameter

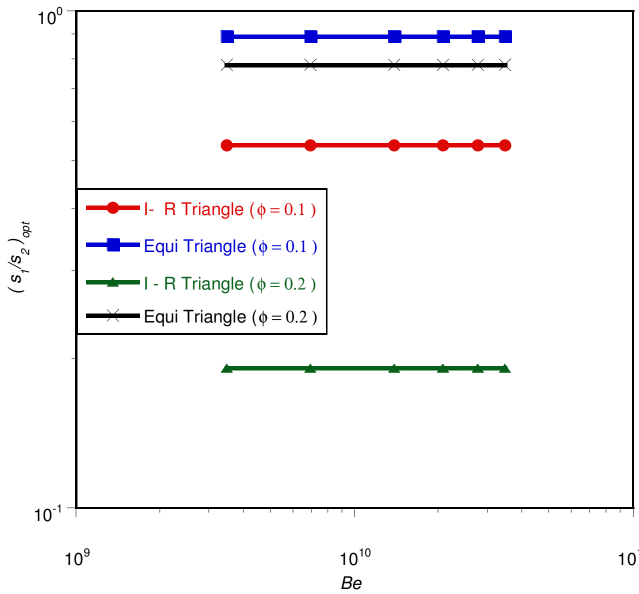

Figure 9. The effect of dimensionless pressure difference on the optimised spacing

CONCLUSION

This paper studied the numerical optimisation of geometric structures of cooling volumes with internal heat generation for isosceles right triangular channel and equilateral triangular channel cross-sections based on constructal theory. The effects of different geometrical parameters such as the hydraulic diameters, and channel spacing and elemental volume were comprehensively studied. The results showed that there is an optimal geometry for the two channel configurations considered which minimises the peak temperature and hence thermal resistance.

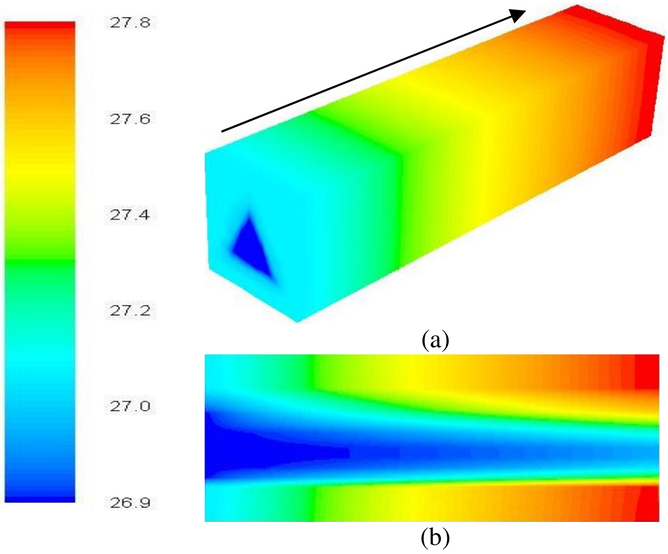

Figure 10. Temperature distributions (a) on the unit structure and (b) on the cooling fluid and inner wall, and unit structure

Also, the numerical result shows that the dimensionless global thermal resistance for the two triangular configurations are almost the same, though that of isosceles right triangular is slightly lower than that of equilateral triangular. The numerical analysis also showed that the optimised geometry and minimised thermal resistance are function of the dimensionless pressure difference for different porosites. This shows the existence of unique optimal design variables (hydraulic diamters) for a given applied dimensionless pressure number for each configuration. The results also show that the minimised peak temperature decreases as the porosity increases.

The optimal channel spacing ratio (s_1/s_2) remains unchanged and insensitive to the performance of the system whereas regardless of the dimensionless pressure difference number for the two triangular configurations. This constant value could be described as allowable spacing due to manufacturing constraints. This implies that the closer the channels are to one another, the better the effective cooling ability of the global system.

The use of the optimisation algorithm coupled to the CFD package made the numerical results to be more robust with respect to the selection of optima structures' geometries, internal configurations of the flow channels and dimensionless pressure difference.

Future work will investigate the analytical optimisation of this study to confirm this numerical solution

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