

VALIDATION OF THE THERMAL EQUILIBRIUM ASSUMPTION IN PERIODIC FREE CONVECTION IN POROUS CHANNEL USING DARCY-BRINKMAN-FORCHHEIMER MODEL

M. M. Awad¹, M. S. Tahat², and A. F. Khadrawi³

¹College of Engineering and Islamic Architecture, Umm Al-Qura University, Makkah, Saudi Arabia

²Al-Qunfudah Engineering College, Umm Al-Qura University, Makkah, Saudi Arabia

³Ajloun University College, Al-Balqa' Applied University, B. O. Box: 33, Ajloun, Jordan

¹Author for correspondence, E-mail: mmawad@uqu.edu.sa

ABSTRACT

Examination of the thermal equilibrium assumption in the periodic natural convection channel flow is investigated numerically. The periodic disturbance in the free convection flow is due to a periodic thermal disturbance imposed on the channel walls. The Darcy-Brinkman-Forchheimer model is used to describe the flow inside the porous channel. A scripted finite element model builder and numerical solver, FlexPDE, is used to solve the equations. Five dimensionless parameters are found to have significant effect on the local thermal equilibrium assumption. These parameters are thermal conductivity, volumetric Nusselt number, thermal diffusivity ratio, amplitude of thermal disturbance and frequency of the thermal disturbance. The effects of these parameters on the local thermal equilibrium assumption are investigated. The volumetric Nusselt number is found to have the most significant effect on this assumption.

INTRODUCTION

Flow through porous media is very important because the increasing demands for oil, water, and food produced in an environmentally sound manner have placed emphasis on their production manner. Flow through porous media has been the topic of a large number of investigations during the past few decades. This continuing interest is due to the presence of porous media in many disciplines such as biophysics (e.g. life processes like flow in the lung and the kidney), chemical engineering (e.g. adsorption, chromatography, dispersion of chemical contaminants through water saturated soil, filtration, flow in packed columns, ion exchange, migration of moisture in grain storage system, reactor-engineering, separation processes in chemical industries, solidification of casting), hydrology (e.g. movement of trace pollutants in water systems, recovery of water for drinking and irrigation, salt water encroachment into fresh water reservoirs), petroleum engineering (e.g. crude oil production, displacement of oil with gas, fluid flow in geothermal reservoirs, water and miscible solvents including surface-active agent solutions and description of reservoirs), and soil physics (e.g. movement of water, nutrients, and

pollutants into plants). These applications are discussed in details in the recent books by Nield and Bejan [1], Ingham and Pop [2], Ingham et al. [3], and Vafai [4].

This paper contains three major sections. The first section presents a review on this topic, which is presently available in the open literature. Next, the validation of the thermal equilibrium assumption in periodic free convection in porous channel is examined using Darcy-Brinkman-Forchheimer model. To achieve this goal, the effect of many parameters on the absolute temperature differences was investigated. These parameters include: thermal conductivity, volumetric Nusselt number, thermal diffusivity ratio, amplitude of thermal disturbance and frequency of the thermal disturbance. Finally, a discussion of the obtained results is presented.

NOMENCLATURE

A	dimensionless Forchheimer coefficient $(F\rho_f^2L^4g\beta\Delta T)/(\mu_{eff}^2K^{0.5})$
AR	aspect ratio
Bi	volumetric Biot number
c	specific thermal capacity
c_a	acceleration coefficient tensor
C_a	modified acceleration coefficient tensor, $c_a\alpha_s\rho_f/\mu_{eff}$
CR	solid-to-fluid total thermal capacity ratio
Da	Darcy number, $(K^*\mu_{eff})/(L^2\mu_f)$
F	Forchheimer coefficient
Fo	Forchheimer number
Gr	Grashof number
h	volumetric heat transfer coefficient
H	coefficient
k	thermal conductivity
K	thermal conductivity ratio, $(1-\varepsilon)k_s/(\varepsilon k_f)$
K^*	permeability
Kn	Knudsen number
$2L$	channel width
Nu	volumetric Nusselt number, $hL^2/((1-\varepsilon)k_s)$
Pe	Peclet number
Pr	Prandtl number
Ra	Rayleigh number
t	time
t_o	reference time, L^2/α_s
T	temperature
u	axial velocity
U	dimensionless axial velocity, u/u_o

u_o	reference velocity, $(\rho_f L^2 g \bar{\beta} \Delta T) / \mu_{eff}$
x	transverse coordinate
X	dimensionless transverse coordinate, x/L
y	axial coordinate
Y	dimensionless axial coordinate, y/L

Special characters

α	thermal diffusivity ratio, α_f / α_s
α_f	thermal diffusivity of the fluid
α_s	thermal diffusivity of the solid
β	amplitude of the thermal disturbance, $T_w / (T_w - T_\infty)$
$\bar{\beta}$	coefficient of thermal expansion
ΔT	temperature difference, $T_w - T_\infty$
ε	porosity
λ	wave ratio
μ	dynamic viscosity
Π	nonequilibrium parameter
θ	dimensionless temperature, $(T - T_\infty) / (T_w - T_\infty)$
τ	dimensionless time, t / t_o
ω	dimensionless frequency of the thermal disturbance, $\bar{\omega} L / \alpha_s$
$\bar{\omega}$	frequency of the thermal disturbance
ζ_{max}	dimensionless channel length

Subscripts

eff	effective
f	fluid
m	modified
s	solid
w	wall condition
∞	ambient condition

LITERATURE REVIEW

Due to wide use of porous media, many studies had been conducted to describe flow through it and many models are used to simulate the thermal transport, losses, interference with the media. The first effort was started by Darcy in 1856 with his well known empirical equation. Darcy [5] revealed a proportionally relationship between the instantaneous discharge rate through a porous medium, the dynamic viscosity of the fluid and the pressure drop over a given distance. The total discharge is equal to the product of the permeability of the medium, the cross-sectional area to flow, and the pressure gradient in the flow direction, all divided by the dynamic viscosity of the fluid. The negative sign is needed in the Darcy's law because fluids flows from high pressure to low pressure.

Deviations due to form drag were addressed and rectified by Forchheimer [6]. For very high velocities in porous media, inertial effects can also become significant. In this case, the Forchheimer's equation should be used. The Forchheimer's equation has two terms. The first term is the usual Darcy term and the second term, known as Forchheimer term, is the inertial term. The Forchheimer term is used to account for the non-linear behavior of the pressure difference versus velocity data.

Brinkman [7,8] extended Darcy model for high porosity media. Brinkman [7] gave a calculation of the viscous force, exerted by a flowing fluid on a dense swarm of particles. His model underlying these calculations was that of a spherical particle embedded in a porous mass. The researcher described

the flow through this porous mass using a modification of Darcy's equation. Such a modification was necessary to obtain consistent boundary conditions. He obtained a relation between permeability and particle size and density. He compared his results with an experimental relation due to Carman [9].

Later, Brinkman [8] extended his previous calculation of the permeability of a swarm of particles to closely packed particles. The researcher introduced an approximate model for the fluctuations of the mean density of the particles surrounding a central one and adjusted so as to represent the experimental results. His refined model served as a basis for an extension of the calculation to particles that themselves were permeable such as occur in catalyst pellets.

It is should be noted that the Brinkman's equation has two viscous terms. The first term is the usual Darcy term and the second term is analogous to the Laplacian term, which appears in the Navier-Stokes equation. The second term is known as the Brinkman term, which is used to account for transitional flow between boundaries. In the second term, Brinkman set the effective viscosity and the fluid viscosity equal to each other, but in general that is not true. Actually, Brinkman did not just add another term to Darcy's law but he obtained a relationship between the permeability and the porosity for an assembly of spheres a "self-consistant" procedure. Lundgren [10] mentioned that the Brinkman's equation was valid only when the porosity was greater than 0.6. Therefore, this requirement is highly restrictive because most naturally occurring porous media have porosities less than 0.6.

Greenkorn [11] presented a review article on steady flow through porous media. The researcher reviewed the fundamentals of steady flow through porous media. His paper discussed the pseudotransport coefficients permeability, capillary pressure, and dispersion and related these coefficients to the geometry of porous media. Also, it discussed single-fluid flow, multifluid immiscible flow, and multifluid miscible flow including the influences of heterogeneity, nonuniformity, and anisotropy of media.

Hamdan [12] reviewed the leading models of single-phase fluid flow through porous media and discussed the boundary conditions associated with these models. The researcher derived entry conditions to a porous channel that were compatible with the various flow models when the flow was fully developed. He made comparison of these entry profiles for various flow parameters with the corresponding entry condition when the flow was governed by the Navier-Stokes equations.

Hamdan and Ford [13] considered fluid flow through porous channels and in free-space into a two-dimensional sink in an attempt to determine the critical (finite) channel length that was compatible with the entry conditions to the channel derived in Part I of the work [12]. The researchers analyzed the effect of the presence of a porous matrix on the critical channel length for various types of porous media and the various models describing the flow. They discussed the influence of the entry conditions to the channel on flow structure for the different models and compared with the results of flow in a regular (nonporous) channel as governed by the Navier-Stokes equations.

Kladias and Prasad [14] reported experimental results for natural convection in a horizontal porous cavity of aspect ratio (AR) = 5, and heated from below. The researchers covered a wide range of governing parameters using careful selection of beads size, solid material, and fluid. Their results fully supported the influences of fluid flow parameters (Rayleigh and Prandtl numbers (Ra and Pr)), porous matrix structure parameters (Darcy and Forchheimer numbers (Da and Fo)), and the conductivity ratio as predicted by the formulation based on the Darcy-Brinkman-Forchheimer (DBF) equations of motion. The DBF flow model, with variable porosity and variable thermal conductivity in the wall regions, predicted reasonably well in comparison with the experimental data. However, the difference between the predictions and the measurements increased as the ratio of solid-to-fluid thermal conductivity became very large.

Vafai and Kim [15] revisited in their work the description of the fluid mechanics at the interface between a fluid layer and a porous medium that was first investigated by Beavers and Joseph [16]. The researchers presented an exact solution describing the interfacial fluid mechanics using a Brinkman-Forchheimer-extended Darcy equation (generalized momentum equation). They discussed briefly the influences of the Darcy number and inertia parameter.

Nield [17] presented a critique of the ability of the Brinkman-Forchheimer equation to adequately model flow in a porous medium and at a porous-medium/clear-fluid interface. The researcher demonstrated that certain terms in the equation as commonly used require modification, and that there was a difficulty when using this equation to deal with a stress boundary condition.

Vafai and Kim [18] analyzed many philosophical points with respect to the momentum equation in a porous medium. The researchers showed that many erroneous/irrelevant issues were put forward in the previous work of Nield [17]. A porous medium/clear fluid interface was best dealt with by the Brinkman-Forchheimer-extended Darcy formulation and the continuity of velocities and stresses at the interface. The influence of porosity variation was not required for a high-porosity medium but should be considered for a dense porous medium.

It should be noted that the three articles listed above have appeared in the International Journal of Heat and Fluid Flow and discussed the limitations of the Brinkman-Forchheimer-extended Darcy model. Later, both Nield [19] and Vafai and Kim [20] agreed to present closure statements on the Brinkman-Forchheimer-extended Darcy model in order to assist interested journal readers in the comparison of their two viewpoints presented in these publications.

Kaloni and Guo [21] presented a theoretical study of the problem of steady nonlinear double-diffusive convection through a porous medium. The researchers used the Brinkman-Forchheimer model to represent the porous medium. They gave a variational formulation to deal with the weak solution and the existence, regularity, and discussed uniqueness results.

Chen and Vafai [22] presented the phenomenological analysis of free surface transport through porous media using the Brinkman-Forchheimer-extended Darcy model. The

researchers employed a finite difference scheme using the marker-and-cell (MAC) method to investigate the momentum and energy transport in a porous channel involving free surface transport phenomena. They took into account the interfacial tension effect at the free surface in their analysis. Their investigation constituted one of the first numerical investigations of the free surface momentum and energy transport through porous media using the MAC method as well as one of the first studies on Non-Darcian effects on free surface transport in porous media. They compared fully developed velocity and temperature fields for saturated as well as unsaturated porous channels, for cases with different Darcy numbers and verified against existing analytical solutions. They presented temporal free surface distributions for cases with various Darcy numbers and Reynolds numbers. Also, they explored the influence of the free surface transport in porous media on the energy transfer. They found that the boundary and inertial influences had a significant effect on the free surface transport through porous media and that the surface tension effects became insignificant for the Reynolds number based on the square root of permeability was greater than 1.

In the literature, extensive work has been done by Al-Nimr with his coworkers in porous media. For example, Alkam and Al-Nimr [23] presented a numerical simulation for the transient forced convection in the developing region of a cylindrical channel partially filled with a porous substrate. The researchers attached the porous substrate to the inner side of the cylinder wall that was exposed to a sudden change in temperature. They used the Brinkman-Forchheimer-extended Darcy model to describe the flow within the porous domain. They studied the effects of many parameters on the hydrodynamic and thermal characteristics of their problem. These parameters included the porous substrate thickness, Darcy number, and Forchheimer coefficient. The results of their model showed that the existence of the porous substrate might improve the Nusselt number at the fully developed region by a factor of 8. However, there was an optimum thickness of the porous substrate beyond which no significant improvement in the Nusselt number was achieved. Also, they included the macroscopic inertial term in the porous domain momentum equation due to its significant effect in their work. They found that the steady state time increased as the substrate thickness increased up to a certain limit and then the steady state time decreased upon further increase in the substrate thickness. In addition, increasing the Forchheimer coefficient and decreasing the Darcy number increased the steady state time.

Haddad [24] presented analytical solutions for fully developed natural convection in open-ended vertical channels partially filled with porous substrates. The researcher investigated four fundamental boundary conditions and computed the corresponding fundamental solutions. He obtained these four fundamental boundary conditions by combining every of the two conditions of one boundary maintained at uniform heat flux and one at uniform wall temperature; in every condition the opposite boundary was kept adiabatic or isothermal at the inlet fluid temperature. He gave expressions for the fully developed velocity, temperature,

mixing cup temperature, volumetric flow rate, friction coefficient, and Nusselt number for every fundamental solution. Such fully developed values were approached, in a given channel, when the height-to-gap width ratio was sufficiently large. These values represented the limiting conditions and provided analytical checks on numerical solutions for steady and transient developing flows.

Rees and Vafai [25] investigated the free convection boundary layer flow of a Darcy-Brinkman fluid that was induced by a constant-temperature horizontal semi-infinite surface embedded in a fluid-saturated porous medium. The researchers showed that both the Darcy and Rayleigh numbers (Da and Ra) might be scaled out of the boundary layer equations, leaving a parabolic system of equations with no parameters to vary. They studied the equations using both numerical and asymptotic methods. Near the leading edge, the boundary layer had a double-layer structure: a near-wall layer, where the temperature adjusted from the wall temperature to the ambient and where Brinkman influences dominated, and an outer layer of uniform thickness that was a momentum-adjustment layer. Further downstream, these layers merged, but the boundary layer eventually regained a two-layer structure; in this case, a growing outer layer existed that was identical to the Darcy-flow case for the leading order term, and an inner layer of constant thickness resided near the surface, where the Brinkman term was important.

Chen [26] developed theoretical analysis and systematic computational simulations of the hydrodynamics and heat transfer associated with Newtonian and non-Newtonian fluids in porous media, and conducted a practical application to innovative porous media heat exchanger design for heat transfer enhancement. The researcher used the Brinkman-Forchheimer-extended Darcy model and its modified form for power-law fluids for modeling different transport phenomena within the porous media. He included the non-Darcy influences of inertia and boundary, and other important physical phenomena including flow channeling and thermal dispersion in the model. He considered both fibrous media and packed beds that were the two most widely used engineered porous materials. Based on the theoretical and computational studies, all the physical phenomena described in these mathematical models and numerical simulations were validated with existing experimental, numerical and theoretical results.

Chen [26] conducted a new theoretical analysis of fully developed non-Darcy forced convection in a two-dimensional channel containing a fibrous medium saturated with a power-law fluid. The researcher obtained closed-form boundary layer solutions using the integral method for velocity profiles, temperature fields and fully developed Nusselt number. His theoretical solutions could be used to predict primary characteristics of physical phenomena associated with forced convection of non-Newtonian fluids in porous media, and were convenient to serve as a benchmark for more complicated numerical solutions. He conducted a detailed computational study of non-Darcy forced convection in a two-dimensional porous channel saturated with a power-law fluid. He performed systematic parametric studies to analyze the influences of power law index, Darcy number (or particle diameter) and

Reynolds number on flow and heat transfer in the channel. His results showed that the combined use of a highly permeable porous matrix with a shear thinning fluid appeared to be promising as a heat transfer augmentation technique. Also, he conducted a preliminary numerical simulation for three-dimensional non-Darcy forced convection in a square porous duct saturated with a Newtonian fluid. He found that channeling phenomena and thermal dispersion influences were reduced considerably in a three-dimensional duct compared with results for a two-dimensional channel. For the practical application to innovative porous media heat exchanger designs for heat transfer enhancement, he performed a detailed numerical investigation of two-dimensional laminar forced convection in a sintered porous channel with inlet and outlet slots. He found very good agreement between his numerical predictions and the corresponding experimental measurements. His results showed that a smaller particle diameter generated higher heat transfer enhancement while a larger particle diameter led to more efficient performance based on heat transfer enhancement per unit pumping power. He generated empirical correlations of the length-averaged Nusselt number and the friction factor for efficient design.

Rees and Pop [27] studied the influence of adopting a two-temperature model of microscopic heat transfer on the classical Cheng and Minkowycz [28] vertical free convection boundary-layer flow in a porous medium. The researchers found that this model that allowed the solid and fluid phases not to be in local thermal equilibrium to modify substantially the behavior of the flow relatively close to the leading edge, where the boundary layer was comprised of two distinct asymptotic regions. The numerical simulation of the developing boundary-layer relied heavily on near-leading-edge analysis to provide suitable boundary conditions. At increasing distances from the leading edge the difference between the temperatures of the fluid and solid phases decreased to zero that corresponded to thermal equilibrium between the phases; this was confirmed by an asymptotic analysis.

Chan et al. [29] presented a laminar flow model for natural convection with porous. The researcher used the Brinkman-Forchheimer extended Darcy's equation to model the porous media. Meanwhile, they used the Boussinesq-Oberbeck approximation to simulate the influences of natural convection. First, they validated the results obtained from their model, and found to be in good agreement with existing cases. Then, they used to analyze the heat flow of a vertically mounted porous heat sink. They found that, at high Rayleigh numbers (Ra), the heat transfer characteristics were strongly dependent on the Darcy number (Da), while the porosity had very little effect on the outcome. Though the present code was tailored for microelectronic component cooling, its applications might span various disciplines, from petroleum engineering to metal casting.

Chan et al. [30] presented a numerical solution for laminar and turbulent convective heat transfer in a backward-facing step channel through a porous insert. In addition to the Navier-Stokes equation for the fluid region, the researchers introduced the Brinkman-Forchheimer-extended Darcy equation into the numerical solver to model the porous

medium. In the turbulent flow scenarios, they used a two-equation k - ε model with wall function for both the fluid region and the porous medium. The results obtained from their study concurred with existing benchmarks. For the turbulent flow results, they observed that the amount of flow resistance offered by the porous insert was more heavily dependent on its width than the permeability.

Mohamad [31] discussed the results of two-dimensional, steady-state, natural convection for a range of controlling parameters the modified Rayleigh number (Ra_m , Rayleigh parameter based on the permeability of the medium), thermal conductivity ratio, and porosity. The researcher found that the equilibrium model could not be justified for a certain range of parameters, particularly when the thermal conductivity of the solid phase was equal to or higher than the thermal conductivity of the fluid phase and for nonequilibrium parameter (II) was lower than one. The deviation from the equilibrium condition was significant in regions of high regions. The difference between the solid and fluid temperatures could exceed 30% at certain locations. In addition, the porosity of the medium affected the temperature fluid of the system.

Al-Nimr and Kiwan [32] investigated analytically the validity of the local thermal equilibrium assumption in the periodic forced convection porous channel flow. The researchers presented closed form expressions for the temperatures of the fluid and solid domains and for the criterion that ensured the validity of the local thermal equilibrium assumption. They found that four dimensionless parameters controlled the local thermal equilibrium assumption. These parameters are the porous domain void fraction (ε), the volumetric Biot number (Bi), the dimensionless frequency (ω), and the solid-to-fluid total thermal capacity ratio (CR). The criterion that secured the validity of the local thermal equilibrium assumption within 5% error, was found to be $\omega CR(1-\varepsilon)/Bi < 0.05$.

Al-Nimr and Abu-Hijleh [33] investigated analytically the validity of the local thermal equilibrium assumption in the transient forced convection channel flow. The researchers presented closed form expressions for the temperatures of the fluid and solid domains and for the criterion that insured the validity of the local thermal equilibrium assumption. They found that four dimensionless parameters controlled the local thermal equilibrium assumption. These parameters were the porosity (ε), the volumetric Biot number (Bi), the dimensionless channel length (ξ_{max}) and the solid to fluid total thermal capacity ratio (CR). They investigated the qualitative and quantitative aspects of the influences of these four parameters on the channel thermal equilibrium relaxation time.

Al-Nimr and Khdrawi [34] investigated analytically the problem of transient free convection in domains partly filled with porous substrates using Laplace transformation technique. The researchers considered four configurations that were subject to an isothermal heating boundary condition. They adopted the Brinkman-extended Darcy model to describe the hydrodynamics behavior of the porous domain.

Khdrawi and Al-Nimr [35] investigated numerically the local thermal equilibrium assumption in the transient natural convection channel flow. The researchers used the

Darcy-Brinkman-Forchheimer model to model the flow inside the porous domain. They examined the influence of various parameters on the validity of the local thermal equilibrium assumption. They found that the volumetric Nusselt number had the most significant effect on the local thermal equilibrium assumption.

Rees [36] studied the influence of adopting a two-temperature model of microscopic heat transfer on the classical Cheng-Minkowycz [28] vertical free convection boundary-layer flow in a porous medium. Such a model that allowed the solid and fluid phases not to be in local thermal equilibrium was found to modify substantially the behavior of the flow relatively close to the leading edge. A companion paper dealt with the (parabolic) boundary-layer theory, but his work investigated in detail how elliptical effects were manifested. This was undertaken by solving the full equations of motion, rather than the boundary-layer approximation. In general, the researcher found that at any point in the flow, the temperature of the solid phase was higher than that of the fluid phase, and therefore that the thermal field of the solid phase was of greater extent than that of the fluid phase. He characterized the microscopic inter-phase heat transfer using the coefficient (H) and showed that these thermal non-equilibrium effects were strongest when H was small.

Abu-Hijleh et al. [37] investigated numerically the validity of the local thermal equilibrium assumption in the transient forced convection channel flow. The researchers included axial conduction in both fluid and solid domains. They found that five dimensionless parameters controlled the local thermal equilibrium assumption. These parameters were the thermal diffusivity ratio (α_s/α_f), the volumetric Nusselt number (Nu), the dimensionless channel length (ξ_{max}), Peclet number (Pe), and the solid to fluid total thermal capacity ratio (CR). They investigated the qualitative and quantitative aspects of the influences of these five parameters on the channel thermalization time.

Haddad et al. [38] investigated analytically the validity of the local thermal equilibrium assumption in natural convection over a vertical flat plate embedded in porous medium. Their study was based on the two-phase (Schumann) model, using the Brinkman term (no-slip condition) to cover the flow. The researchers found that there were four dimensionless parameters controlling the local thermal equilibrium assumption: the volumetric Biot number (Bi), the modified Rayleigh number (Ra_m), the modified Darcy number (Da), and the ratio of effective to dynamic viscosity (μ_{eff}/μ). They investigated the influences of these parameters and developed a correlation equation to determine the region where the local thermal equilibrium assumption was valid.

Chan and Lien [39] derived rigorously a k - ε model, based on the work of Lee and Howell [40], based on time average of spatially averaged Navier-Stokes equations. Then, the researchers employed the model to solve for a flow in a backward-facing step channel with a porous insert. They modified the numerical solver from the STREAM code [41], and validated it against the experimental data of Seegmiller and Driver [42]. Then, they used the code to perform simulation for cases with a porous insert. The resistance of the porous insert

could be altered by changing its permeability, Forchheimer's constant, or thickness. Their goal was to examine the effect of every parameter on the resulting flow and turbulent kinetic energy distributions. They discovered that, by increasing the resistance of the insert, flow eventually entered a transitional regime towards relaminarization. This was due to the contribution of Darcy's and Forchheimer's terms in the governing equations, and modifying these two terms changed the levels of turbulence generation term. Generally speaking, lowering permeability or raising Forchheimer's constant, causing the flow to relaminarize. Meanwhile, if the pore size was reasonably large to sustain turbulence within the porous media, increasing thickness reduced but did not eliminate the turbulent activity in the porous insert.

Haddad et al. [43] investigated analytically the validity of the local thermal equilibrium assumption in natural convection over a vertical flat plate embedded in a porous medium. Their study was based on the two-phase (Schumann) model, using the Darcy model (slip condition) to govern the flow. The researchers found that there were three dimensionless parameters controlling the local thermal equilibrium assumption: the Biot number (Bi), the modified Rayleigh number (Ra_m), and the Darcy number (Da). They investigated the influences of these parameters and developed a correlation equation to determine the region where the local thermal equilibrium assumption was valid.

Khashan et al. [44] investigated numerically the forced convection heat transfer flows in a tube filled with a fluid-saturated porous medium. The researchers considered steady state incompressible flows with isothermal tube walls along with a uniform inlet approach velocity and temperature conditions. In addition, they considered the generalized form of the momentum equation by accounting for the solid boundary and the Forchheimer quadratic inertial effects without invoking the boundary layer approximations. Moreover, they simulated the energy transport using the two-equation model that accounted separately for the local fluid and solid temperatures. They obtained their numerical solution through the application of the finite volume method. They tested the validity of the local thermal equilibrium (LTE) over a wide domain of the employed dimensionless parameters, namely; the Peclet number, Biot-like number, effective fluid-to-solid thermal conductivity ratio, Reynolds number, Forchheimer dimensionless coefficient and Darcy number. They examined the validity of the LTE condition for the full tube length and upon excluding the first tube diameter length.

Khadrawi et al. [45] investigated analytically the validity of the local thermal equilibrium assumption in the periodic free convection channel flow. The researchers considered two cases in their study. They included transverse conduction in the solid domain in the first case while they included transverse conduction in the fluid domain in the second case. The periodic disturbance in the free convection flow was due to a periodic thermal disturbance imposed on the channel walls. They used the Darcy-Brinkman model to model the flow inside the porous domain. They found that four dimensionless parameters controlled the local thermal equilibrium assumption in the first case and five parameters

controlled the local equilibrium assumption in the second case. They derived the criteria that secured the validity of the local thermal equilibrium assumption.

Khashan and Al-Nimr [46] assessed the validity of the local thermal equilibrium assumption in the non-Newtonian forced convection flow through channels filled with porous media. The researchers solved the problem numerically using local thermal non-equilibrium and non-Darcian models. They utilized numerical solutions obtained over broad ranges of representative dimensionless parameters to map conditions at which the local thermal equilibrium assumption could or could not be employed. They identified the circumstances of a higher modified Peclet number, a lower modified Biot number, a lower fluid-to-solid thermal conductivity ratio, a lower power-law fluid index, and a lower microscopic and macroscopic frictional flow resistance coefficients, as unfavorable circumstances for the local thermal equilibrium (LTE) condition to hold. They presented quantitative LTE validity maps that reflected the proportional influence of every parameter as related to others.

Haddad et al. [47] studied numerically steady laminar forced convection gaseous slip-flow through parallel-plates microchannel filled with porous medium under Local Thermal Non-Equilibrium (LTNE) condition. The researchers considered incompressible Newtonian gas flow that was hydrodynamically fully developed while thermally was developing. They used the Darcy-Brinkman-Forchheimer model embedded in the Navier-Stokes equations to model the flow within the porous domain. Their study reported the influence of many operating parameters on velocity slip and temperature jump at the wall. Mainly, their study demonstrated the influences of: Knudsen number (Kn), Darcy number (Da), Forchheimer number (Fo), Peclet number (Pe), Biot number (Bi), and effective thermal conductivity ratio (K_{eff}) on velocity slip and temperature jump at the wall. They gave results in terms of skin friction and Nusselt number. It was found that the skin friction: (1) increased as Darcy number increased; (2) decreased as Forchheimer number or Knudsen number increased. Heat transfer was found to (1) decreased as the Knudsen number, Forchheimer number, or effective thermal conductivity ratio increased; (2) increased as the Peclet number, Darcy number, or Biot number increased.

Chen et al. [48] carried out a numerical investigation for steady, free convection inside a cavity filled with a porous medium. The cavity had vertical wavy walls that were isothermal. The top and bottom horizontal straight walls were kept adiabatic. Their numerical method was based on the finite-volume method with body-fitted and nonorthogonal grids. The researchers used a generalized model that included a Brinkman term, a Forchheimer term, and a nonlinear convective term. They carried out studies for a range of wave ratio (λ) = 0-1.8, aspect ratio (AR) = 1-5, Darcy number (Da) = 10^{-1} - 10^{-6} , and Darcy-Rayleigh number (Ra) = 10 - 10^5 . They presented their results in the form of streamlines, isotherms, and local and average Nusselt numbers. Their generalized model that considered viscous, inertia, and convective effects enabled results to be obtained for a wider range of Darcy and Rayleigh numbers.

Khadrawi et al. [49] investigated semianalytically the transient thermal behavior of a homogeneous composite domain described by the hyperbolic heat-conduction model. The composite domain consisted of a matrix (fluid domain) and inserts (solid domain), every of which was made of various materials. The researchers considered two cases. They included the conduction in the solid domain in the first case, and the conduction in the fluid domain in the second case. They determined the range of parameters, within which the use of the thermal equilibrium assumption in transient flow in porous channel as described by a hyperbolic heat-conduction model. Also, they investigated the influence of various parameters that affected the local thermal equilibrium assumption under the influence of the hyperbolic heat conduction model.

Umavathi [50] analyzed a fully developed free convection flow of immiscible fluids in a vertical channel filled with a porous medium in the presence of source/sink. The researcher modeled the flow using the Darcy-Brinkman-Forchheimer equation model. He included the viscous and Darcy dissipation terms in the energy equation. The channel walls were maintained at two different constant temperatures. He assumed that the transport properties of both fluids to be constant. He employed continuous conditions for velocity, temperature, shear stress, and heat flux of both fluids at the interface. He solved the resulting coupled nonlinear equations analytically using regular perturbation method and numerically using finite difference method. He obtained the velocity and temperature profiles in terms of porous parameter, Grashof number (Gr), viscosity ratio, width ratio, conductivity ratio, and heat generation or heat absorption coefficient. It was found that the presence of porous matrix and heat absorption reduced the flow field.

Singh et al. [51] addressed the transient as well as non-Darcian effects on laminar natural convection flow in a vertical channel partially filled with porous medium. The researchers did their analysis using Forchheimer-Brinkman extended Darcy model to simulate momentum transfer within the porous medium. For the momentum equation, two regions were coupled by equating the velocity and shear stress. For the thermal energy equation, they took matching of the temperature and heat flux. They obtained approximate solutions using perturbation technique. Also, they obtained and depicted graphically variations in velocity field with Darcy number (Da), Grashof number (Gr), kinematic viscosity ratio, distance of interface and variations in temperature distribution with thermal conductivity ratio, distance of interface. In addition, they derived the skin-friction and rate of heat transfer at the channel walls and tabulated the numerical values for different physical parameters.

Tasnim et al. [52] conducted analytical studies on the flow and thermal fields of unsteady compressible viscous oscillating flow through channels filled with porous media representing stacks in thermoacoustic systems. The researchers used the Brinkman-Forchheimer-extended Darcy model to describe the flow in the porous material. They obtained analytical expressions for oscillating velocity, temperature, and energy flux density after linearizing and solving the governing differential equations with long wave, short stack, and small

amplitude oscillation approximations. Also, they conducted experimental work to verify the temperature difference obtained across the porous stack ends. They designed and constructed a thermoacoustic heat pump to produce the experimental results, where reticulated vitreous carbon (RVC) was used as the stack material. They obtained a very good agreement between the modeling and their experimental results. Also, they compared the expression of temperature difference across the stack ends obtained in their study with the existing thermoacoustic literature. Their proposed expression surpassed the existing expression available in the literature. The system of equations developed in their study was a helpful tool for thermal engineers and physicist to design porous stacks for thermoacoustic devices.

From the literature review, it is clear that there are many models of the conservation of momentum equation like the Darcian model, and the non-Darcian extension models are widely used to describe the fluid flow in porous medium. Also, many researchers applied the Brinkman-Forchheimer extended Darcy model for modeling various transport phenomenon through the porous media. In addition, other investigators extended their work to comply with the important cases such as channeling, two phase flow, turbulent flow, thermal dispersion, non Darcy effect, and the effect of different parameters on convection process. In porous research, the Forchheimer extended Darcy model is preferred because Darcy law does not adequately reflect the fluid flow through the porous media. This true in turn affect of proper determination of the actual heat transfer that take place. Moreover, Darcy law does not take in account solid boundaries and it neglects the inertial effects.

The local thermal equilibrium assumption in transient forced convection porous channel flow has been investigated analytically. This is accomplished by focusing on the operating conditions required for both the solid and fluid domains to approximately attain the same temperature, and as a result, the local thermal equilibrium assumption is secured.

The velocity and temperature profiles are obtained in terms of porous parameter, Grashof number (Gr), viscosity ratio, width ratio, conductivity ratio, and heat generation or heat absorption coefficient. It is found that the presence of porous matrix and heat absorption reduces the flow field.

MATHEMATICAL FORMULATION

Consider the problem of the periodic free convection fluid flow in open-ended vertical porous channel. The fluctuations in the hydrodynamics and thermal behaviors of the channel are due to the harmonic fluctuations in the wall temperature of the channel. Referring to Figure 1, and using the dimensionless parameters given in the Nomenclature, the momentum and energy equations are given as:

$$-C_a \frac{\partial U}{\partial \tau} + \frac{\partial^2 U}{\partial Y^2} - \frac{U}{Da} - AU^2 - \theta_f = 0 \quad (1)$$

$$\frac{\partial \theta_s}{\partial \tau} = \frac{\partial^2 \theta_s}{\partial Y^2} + Nu(\theta_f - \theta_s) \quad (2)$$

$$\alpha \frac{\partial \theta_f}{\partial \tau} = \frac{\partial^2 \theta_f}{\partial Y^2} + Nu K (\theta_s - \theta_f) \quad (3)$$

where $U = U(\tau, Y)$, $\theta_f = \theta_f(\tau, Y)$, $\theta_s = \theta_s(\tau, Y)$.

Equations (1-3) assume the following initial and boundary conditions:

$$U(0, Y) = \theta_f(0, Y) = \theta_s(0, Y) = 0.0$$

$$\frac{\partial U}{\partial Y}(\tau, 0) = \frac{\partial \theta_f}{\partial Y}(\tau, 0) = \frac{\partial \theta_s}{\partial Y}(\tau, 0) = 0.0 \quad (4)$$

$$U(\tau, 1) = 0.0$$

$$\theta_f(\tau, 1) = \theta_s(\tau, 1) = \beta \sin(\omega\tau)$$

In Equations (1-4), subscripts f and s refer to the fluid and solid domains, respectively, β stands for the relative amplitude of oscillations. The other parameters appearing in Eqs. (1-4) are defined as:

$$U = \frac{u}{u_o}, \quad Y = \frac{y}{L}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \tau = \frac{t}{t_o}$$

Axial thermal diffusion is not included in Eqs. (2-3) because it is of insignificant effect as compared to transverse diffusion and may be neglected in channel having low width to length aspect ratio. Also, axial thermal diffusion may be neglected for gases.

The harmonic fluctuations in the imposed wall temperatures are the only source for the disturbances in the thermal and hydrodynamics behaviors of the channel flow.

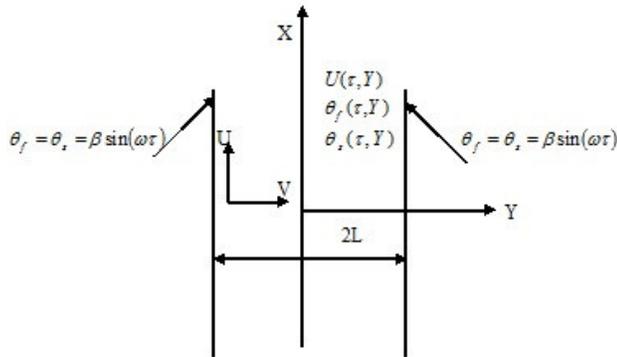


Figure 1 Schematic diagram of the problem under consideration

SOLUTION METHODOLOGY

The governing equations have been solved numerically by means of FlexPDE program [53]. FlexPDE is a “scripted finite element model builder and numerical solver”. It performs

the operations necessary to turn a description of partial differential equations system into a finite element model, and solve the system. FlexPDE is also, a “problem solving environment”, because it performs the entire range of functions necessary to solve partial differential equation system: a mesh generator for building finite element meshes and a finite element solver to find solutions. It is a fully integrated PDE solver, combining several modules to provide a complete problem solving system.

A symbolic equation analyzer expands defined parameters and relations, performs spatial differentiation, and symbolically applies integration by parts to reduce second order terms to create symbolic Galerkin equations. It then differentiates these equations to form the Jacobian coupling matrix. A mesh generation module constructs a triangular finite element mesh over an arbitrary two-dimensional problem domain. In three-dimensional problems, the 2D mesh is extruded into a tetrahedral mesh covering an arbitrary on non-planar layers in the extrusion dimension. A finite element numerical analysis module selects an appropriate solution scheme for steady-state, time-dependent or eigenvalue problems, with separate procedures for linear and nonlinear systems. Finite element basis may be either quadratic or cubic. An error estimation procedure measures the adequacy of the mesh and refines the mesh wherever the error is large. The system iterates the mesh refinement and solution until a user-defined error tolerance is achieved. In addition, FlexPDE uses an adaptive time step control that tries to adjust to changes in the time behavior of the system. An abrupt change in the boundary condition could be slightly over-stepped if it does not substantially change the apparent smoothness of the solution, but it should clearly be caught within the next time step. Time step will be repeatedly halved and re-run if a sudden change causes the solution to change suddenly.

In our case the number of nodes and the degree of freedom are 320 and 960 respectively. Also, the maximum error is 3.591E-3 and the root mean square error (RMS) is 1.373E-3.

RESULTS AND DISCUSSION

The effects of different parameters on the validity of the local thermal equilibrium assumption are investigated in Figures 2-9.

Figure 2 shows the effect of the volumetric Nusselt number (Nu) on the transient behavior of the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at the following values: $\alpha = 0.1$, $\beta = 10$, $\omega = \pi/6$, $Da = 0.01$, $K = 0.1$, $A = 10$, $Y = 0.5$, and different values of the volumetric Nusselt number (Nu) of 0.1, 1, 10, and 100 respectively. As predicted, the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ decreases as the volumetric Nusselt number (Nu) increases. Increasing Nu enhances the heat transfer between the fluid and the solid matrix and this in turn shortens the time required to attain local thermal equilibrium. Also, Figure 2 shows that maximum deviation of the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ occurs at different time as Nu changes. This is predicted because the phase lag between θ_s and θ_f is very sensitive to Nu . The maximum

deviation between the fluid and solid matrix temperature may reach 15% of the amplitude of the thermal distribution (β) = 10 at $Nu = 0.1$.

Figure 3 shows the effect of the volumetric Nusselt number (Nu) on the spatial distributions of the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at the following values: $A = 10$, $Da = 0.01$, $K = 0.1$, $\beta = 10$, $\alpha = 0.1$, $\tau = 1$, $\omega = \pi/6$, and different values of the volumetric Nusselt number (Nu) of 0.1, 1, 10, and 100 respectively. It can be seen that the effect of the volumetric Nusselt number (Nu) on the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ is insignificant at large values of Nu . Also, Figure 3 shows that the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ reaches its minimum value at $Y = 1$ for different values of the volumetric Nusselt number (Nu). Thus, the local thermal equilibrium may be secured in locations far away from the heated (or cooled) boundaries.

Figure 4 shows the effect of the thermal conductivity ratio (K) on the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at the following values: $\tau = 1$, $\beta = 10$, $\omega = \pi/6$, $Da = 0.01$, $A = 10$, $Y = 0.5$, and different values of the thermal diffusivity ratio (α) and the volumetric Nusselt number (Nu) of 0.1, and 10 respectively. From Eq. (3), it is clear that K has the same effect as Nu on the validity of the local thermal equilibrium assumption. Increasing the coefficient of $(\theta_s - \theta_f)$ in Eq. (3) secures the validity of the local thermal equilibrium assumption. This implies that increasing K will secure the local thermal equilibrium assumption. From Figure 4, it can be seen that the effect of K on the local thermal equilibrium assumption is more significant at small values of Nu .

Figure 5 shows the effect of ω on the spatial distributions of the difference between the fluid and solid temperatures. The temperature difference $|\theta_s - \theta_f|$ increases as ω increases. The ability of one domain to sense the thermal fluctuations carried by the other domain becomes weak as the frequency of these fluctuations increases. Also, the effect of the disturbance frequency on the temperature deviation is more significant at locations far away from the heating locations.

Figure 6 shows the effect of thermal diffusivity ratios (α) on the transient behavior of the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at the following values: $A = 10$, $K = 0.1$, $Nu = 0.1$, $Da = 0.01$, $Y = 0.5$, $\omega = \pi/6$, $\beta = 10$, and different values of thermal diffusivity ratios (α) of 1000, 100, 50, 10, 1, 0.1, and 0.01 respectively. It is clear that the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ increases as α increases especially at large values of α . The validity of the local thermal equilibrium assumption may be secured for small values of α . The effect of α on the absolute temperature difference is insignificant within the range $\alpha < 1$. Small values of α imply that the fluid has very high thermal diffusivity or very low thermal capacity. This means that the fluid does not store large amounts of energy and it diffuses it to the solid domain. As a result, both domains have very close temperatures that in turn enhance the local thermal equilibrium assumption.

Figure 7 shows the effect of thermal diffusivity ratio (α) on the absolute temperature difference between the solid and

fluid $|\theta_s - \theta_f|$ at the following values: $Y = 0.5$, $Da = 0.01$, $A = 10$, $\omega = \pi/6$, $\beta = 10$, $\tau = 1$, and different values of Nu and K of 0.1 and 10 respectively. It can be seen that the local thermal equilibrium may be secured at small values of α and large values of K and Nu . The influence of α on this assumption is insignificant at large values of Nu and K . Moreover, the effect of α on the local thermal equilibrium assumption is insignificant at both very small and very large ratios of α .

Figure 8 shows the effect of the amplitude of the thermal disturbance (β) on the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at the following values: $Da = 0.01$, $K = Nu = 0.1$, $A = 10$, $Y = 0.5$, $\alpha = 10$, $\tau = 1$, and $\omega = \pi/6$. As predicted, the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ is linearly proportional amplitude of the thermal disturbance (β). Local thermal equilibrium assumption is secured in application involve weak thermal disturbances.

Figure 9 shows the effect of dimensionless frequency of the thermal disturbance (ω) on the absolute temperature difference between the solid and fluid $|\theta_s - \theta_f|$ at different Nu numbers at the following values: $\tau = 1$, $Y = 0.5$, $Da = \alpha = K = 0.1$, $\beta = A = 10$, $\omega = \pi/6$, $\pi/4$, and 1 respectively. The influence of Nu and ω on the absolute temperature difference $|\theta_s - \theta_f|$ is shown from another point of view in Figure 9. It is clear that the effect of Nu number on the absolute temperature difference is insignificant at large values of Nu . This is justified because the time required for both solid and fluid domain to attain the same temperature is proportional to q^{-1} , where q is the convective heat transfer between the fluid and solid domain. This convective heat transfer is proportional to h that is the volumetric convective heat transfer coefficient. As a result, the absolute temperature difference $|\theta_s - \theta_f|$ is proportional to h^{-1} or to Nu^{-1} . Also, it is clear that the absolute temperature difference increases as ω increases. However, the influence of ω on the validity of the thermal equilibrium assumption is insignificant at large value of Nu .

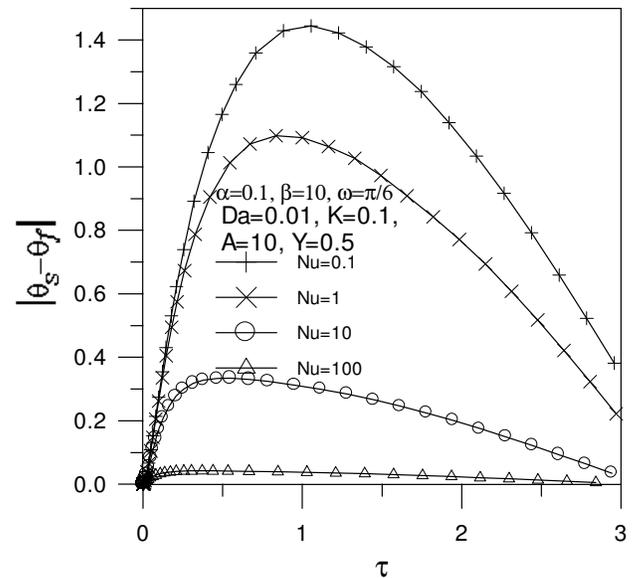


Figure 2 Transient behavior of the difference between the fluid and solid temperature at different Nu

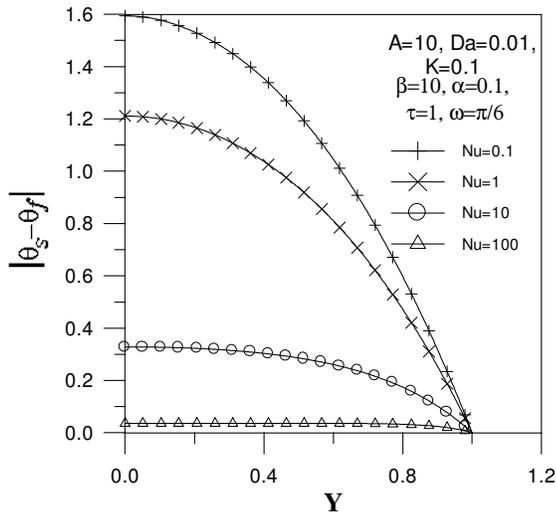


Figure 3 Spatial distributions of the difference between the fluid and solid temperature at different Nu

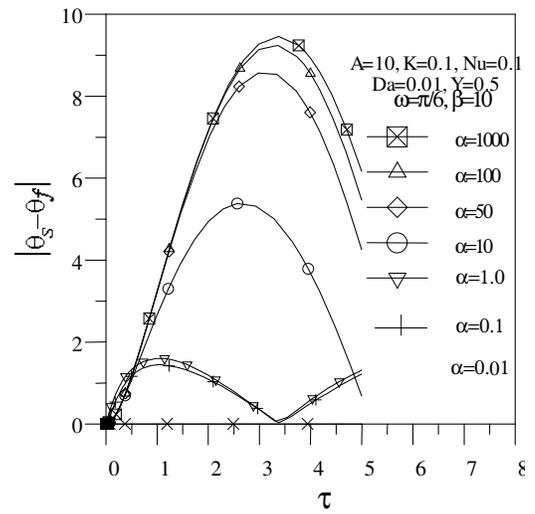


Figure 6 Transient behavior of the difference between the fluid and solid temperatures at different α

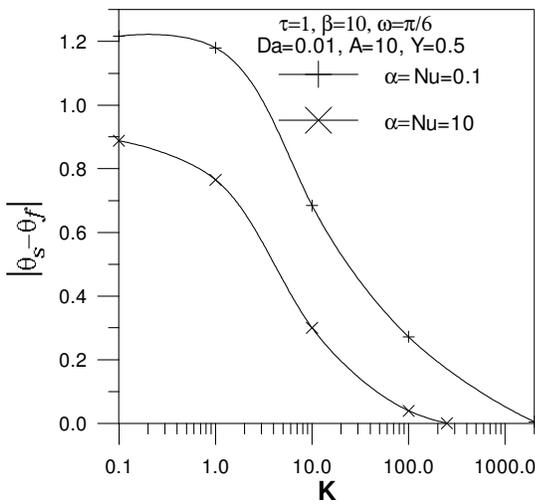


Figure 4 Effect of K on the difference between the fluid and solid temperature at different Nu numbers

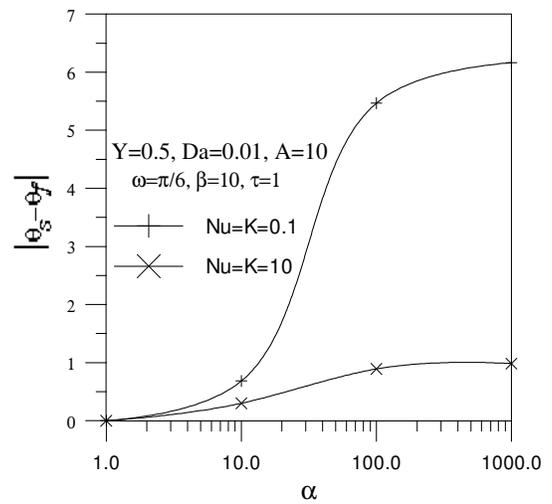


Figure 7 Effect of α on the difference between the fluid and solid temperature at different Nu and K

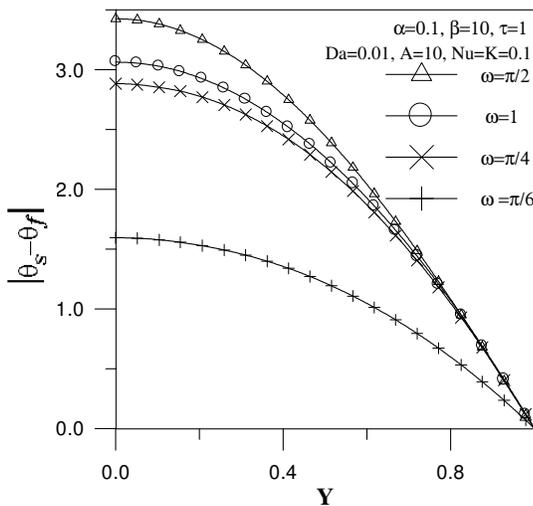


Figure 5 Spatial distributions of the difference between the fluid and solid temperature at different ω

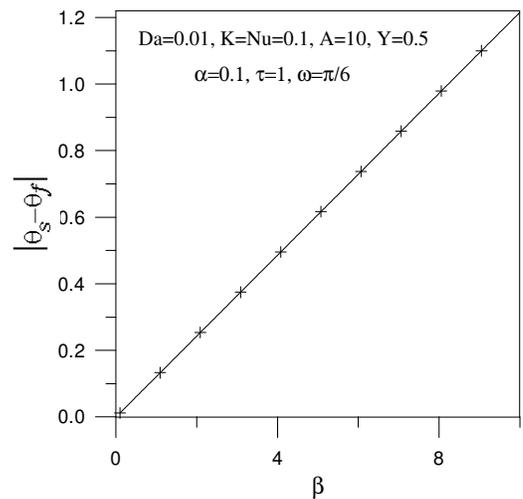


Figure 8 Effect of β on the difference between the fluid and solid temperatures

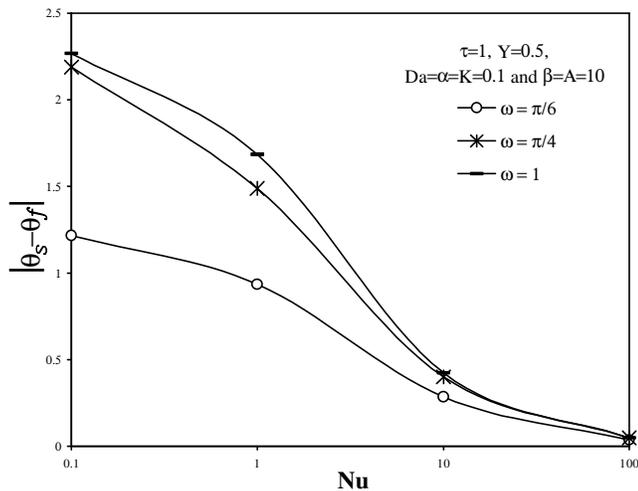


Figure 9 Effect of ω on the difference between the fluid and solid temperature at difference Nu numbers

SUMMARY AND CONCLUSIONS

The Darcy-Brinkman-Forchheimer model is used to investigate the local thermal equilibrium assumption in the periodic free convection porous channel. It is found that the volumetric Nusselt number, thermal conductivity and diffusivity ratios, fluctuation frequencies and amplitudes have significant effects on the local thermal equilibrium assumption. The local thermal equilibrium assumption may be secured in applications involve large values of Nu and K and small values of α , ω and β . The local thermal equilibrium assumption is secured in locations far away from the heating (or cooling) sources. The effects of α and K on the thermal equilibrium assumption is insignificant at very small values of α and very large values of K . At very large values of Nu , the effect of other different parameters on the local thermal equilibrium assumption is insignificant.

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