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LAMINAR SEMI-POROUS CHANNEL ELECTRICALLY CONDUCTING FLOW UNDER MAGNETIC FIELD

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ABSTRACT

Laminar electrically conducting fluid flow in various conduits under different magnetic fields has received great attention in recent years due to its various applications for biomedical (i.e.: blood filtration in artificial kidney), thermal (i.e.: cooling of turbine blades), chemical (i.e.: food processing), environmental (i.e.: dust separation) and nuclear (i.e.: ionization control) purposes.

Present paper studies flow characteristics of electrically conducting fluid under uniform magnetic field in the small gap between uniformly moving lower plate and a fixed parallel semi porous plate that governed by dimensionless Hartman number (*Ha*), and Reynolds number (*Re*). The weighted residual Least Squares Method (*L.S.M.*) is used to solve the two dimensional governing simulation equations.

In the range $Re < 1.0$ and $Ha < 1.0$, neither *Ha* nor *Re* has noticeable effect on vertical flow velocity *V*. The rate of *V* is linear within the gap and vanishes in the vicinity of both plates. Fluid flow rate *q* leaving out through the semi porous upper plate shows significant dependency on both *Ha* and *Re*, where it decreases with increasing either *Ha* or *Re* due to the dependency of the horizontal velocity *U* on both *Ha*, and *Re*.

In the ranges $1.0 < Re, Ha < 10$ both *Ha* and *Re* also still have minor effects on *V*. At higher *Re* the results show higher shear stress and lower *U* values in vicinity of lower plate, signifying a reluctant fluid flow that does not follow the speeding up of the moving lower plate. At $Ha = 10$, the effect of *Re* on *U* diminishes to its lowest limit, and the flow suffers an almost oscillating nature in the upper *75%* of the gap between the plates, and a very high shear stress is in the lower *25%* of the gap.

Present results agree well with other published results that had used Galerkin method, numerical methods and Homotopy analysis method.

Keywords: Generalized laminar viscous flow; Semi-porous channel; Uniform magnetic field; Weighted Least Squares method.

INTRODUCTION

The flow problem in porous channels/tubes has received great attention in recent years because of its various applications in different engineering branches. Examples include blood filtration in artificial kidney, blood flow in capillaries and oxygenations, transpiration, cooling of turbine blades, lubrication of ceramic machine parts, food processing, electronics cooling, gaseous diffusion, magnet-hydro dynamic applications, the extraction of geothermal energy, pollution control by dust collection and nuclear reactor ionization control and cooling systems. Much of the credit of progress in the field of flow in porous channel/tubes goes to the pioneer research work undertaken by Berman [1] who described an exact solution of the Navier-Stokes equation for steady two-dimensional laminar viscous incompressible flow in a channel with porous walls driven by uniform, steady suction or injection at the walls. Over the years, several authors [2-9] have used Berman’s solution [1] as a benchmark to their numerical or theoretical investigations for solving the flow problem in channel of a semi-permeable membrane.

The influence of magnetic field over a laminar viscous flow in a semi-porous channel gas has been studied analytically and numerically [10-11]. In recent years, much attention has been paid to develop new methods for analytic solutions of such governing equations. These methods include the decomposition methods [10], perturbation methods [11], and homotopy analysis methods (HAM) [11]. Hayata et al. [12] investigated the channel flow of a third order fluid that is electrically conducting in the presence of a magnetic field applied transversely to the porous walls of a channel. They developed an expression for velocity by the homotopy analysis method (HAM). More recently, Makinde and Chinyoka [13] have numerically solved the governing equations using a semi-implicit finite difference scheme in order to determine the transient heat transfer in channel flow of an electrically conducting variable viscosity fluid in the presence of a magnetic field and thermal radiation. Numerical analysis of this flow problem was developed by Desseaux [14], who examined a two-dimensional laminar boundary-layer flow of a conducting Newtonian fluid in a semi-porous channel with a possible moving boundary in the presence of a transverse magnetic field. The governing ordinary differential equations were analyzed using the cross flow Reynolds number as a perturbation parameter and then numerical integration was applied to obtain the solution. Ziabakhsh and Domairry [15] solved the laminar viscous flow in a semi-porous channel in the presence of a uniform magnetic field using the homotopy analysis method. They presented detailed equations to relate the velocity to the distance normal to the plates. The Adomian decomposition method (ADM) was used by Ganji and Ganji [16] to compute an approximation for the solution of the system of nonlinear differential equations governing the problem. Their results of the (ADM) were compared with solutions of the numerical method (NM), homotopy perturbation method (HPM), and variation iteration method (VIM). The results have revealed that their method was very effective and simple.

In a recent study [17], Abdel-Rahim et al. solved the laminar viscous flow in a semi-porous channel under uniform magnetic field using the weighted residual Galerkin method to solve the governing equations for low values of flow and magnetic effects. They presented the flow characteristics in terms of graphical representations and fitted equations relating flow velocities and flow rates in terms of Reynolds number and Hartman number. Talmage et al. [18] treated the flow of a conducting fluid in a toroidal duct under influence of transverse magnetic field , and concluded that inertial effects in magneto-hydrodynamic (MHD) duct flows can lead to unexpected flow patterns. They mentioned that this mechanism exists in MHD pumps, flow meters, sea water two-phase flow propulsion systems and power conversion systems such as liquid-metal sliding electrical contacts for homo-polar devices. Figueroa et al. [19] reported experimental observations and numerical comparisons of laminar flow in thin horizontal layer of electrolyte under unidirectional electric current and of small permanent magnetic field. Their results indicated that, except in the zone above the lateral edges of the magnet, no recirculating flows had appeared and vertical velocity components were negligible. Narasimhan [20] constructed continuum theory of an electrically conducting nonlocal viscous fluid flow between two non-conducting parallel plates under a transverse magnetic field. He analytically and numerically investigated the velocity and shear stress fields under varying magnetic field. Ferdows [21] investigated the steady laminar boundary flow over an impulsively stretching surface enclosed by strong magnetic field due to its industrially increasing importance. Kalita [22] discussed unsteady channel flow of electrically conducting viscous liquid flow containing non-conducting small dust particles between two parallel plates in the presence of a transverse magnetic field. His solutions under different values of Hartman number, pressure gradient, dust concentration and time have shown that the velocity of dust particles was always greater than that of the liquid especially near the axis of the channel. Experimental ultrasonic investigation conducted by Nakamura et al. [23] on two-dimensional channel flow of a magnetic fluid subject to magnetic fluid have concluded that the ferromagnetic particles have showed an aggregation along the direction of the magnetic field and have formed clusters. Eguía et al. [24] studied the effects of temperature on viscosity, electric conductivity, Reynolds number and particle concentration on the unsteady MHD flow of a dusty, electrically conducting fluid between parallel plates under uniform perpendicular magnetic field using the network simulation method. They studied velocity and temperature for different values of viscosity, magnetic field parameters, different particle concentrations and different upper wall velocities. Turkyilmazoglu [25] conducted analytical solution of the boundary layer flow of a steady, laminar, incompressible, viscous and electrically conducting fluid due to a rotating disk using homotopy analysis method. They computed the mean velocity profiles corresponding to a wide range of magnetic strength.

The objective of the current work is to study the flow characteristics of the laminar semi-porous channel electrically conducting flow subjected to a high magnetic field and under high values of Reynolds number. The results are to be presented, discussed and compared with other published data.

NOMENCLATURE

a,b, ..., h coefficients for *V* and *U* equation.
B magnetic field.
Ha Hartman number.
L plate length.
P_x pressure component in x-direction.
P_y pressure component in y-direction.
q flow rate though upper porous wall.
Re Reynolds Number.
U dimensionless velocity component in x-direction u.
u velocity component in x-direction.

u_o	uniform velocity of lower plate in x-direction.
u_x	x-derivative of fluid horizontal velocity.
V	dimensionless velocity in y-direction.
v	velocity component in *y*-direction.
v_y	y-derivative of fluid vertical velocity.
v_∞	normal velocity at the porous wall.
V'	derivative of dimensionless velocity component in y-direction.
x, y	horizontal and vertical coordinates.
δ	channel gap.
ν	fluid kinematic viscosity.
ρ	fluid density.
σ	fluid electrical conductivity.

ANALYSIS

A Mathematical Model: Consider the laminar two-dimensional stationary flow of an electrically conducting incompressible viscous fluid in a semi-porous channel as shown in Fig. 1. The channel is made of one stationary infinite porous upper plate and a horizontal plate uniformly moving in the x-direction at a distance δ from the upper plate where the plate length is much greater than the distance (L ≫ δ). A uniform magnetic field *B* is assumed to be applied in the *y*-direction.

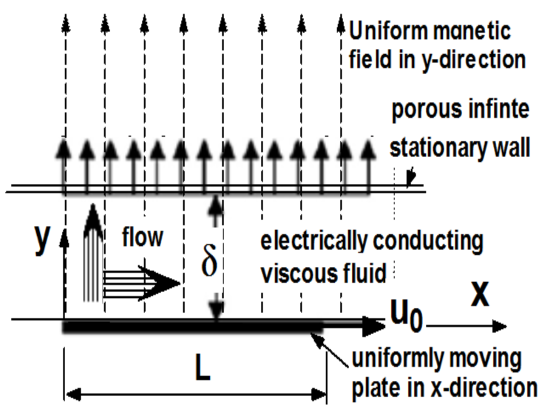

Fig.1 Schematic representation of the two-dimensional flow domain.

The fluid properties (ν, ρ, and σ) are assumed to be constants. The uniform velocity of the plate is u_o, while the normal velocity at the porous wall is V_∞. By neglecting the electrical field and gravity forces, the governing equations over the flow domain and the appropriate boundary conditions, [15], are:

$$u_x + v_y = 0 \tag{1}$$

$$u.u_x + v.v_y = -\frac{P_x}{\rho} + \nu\left(u_{xx} + u_{yy}\right) - u.\frac{\sigma.B^2}{\rho} \tag{2}$$

$$u.v_x + v.v_y = -\frac{P_y}{\rho} + \nu\left(v_{xx} + v_{yy}\right) \tag{3}$$

$$y = \{0, \delta\} : u = \{u_o, 0\}, v = \{0, v_\infty\} \tag{4}$$

In the above equations, subscripts signify partial differentiation. Using the assumptions, non-dimensional similarity transformations introduced by [13] and the method of separation of variables, then the above equations in dimensionless forms reduce to:

$$V^{IV} + \mathrm{Re}.V.V''' - \left(Ha^2 + \mathrm{Re}.V'\right).V'' = 0 \tag{5}$$

$$U'' + \mathrm{Re}.V.U' - \left(Ha^2 + \mathrm{Re}.V'\right).U = 0 \tag{6}$$

$$y = \{0,1\} : V = \{0,1\}, V' = \{0,0\}, U = \{1,0\} \tag{7}$$

In the above equations, the two dimensionless numbers: Reynolds number $\mathrm{Re}\ (=\frac{\delta.U}{\nu})$ and the Hartman number $Ha\ (= B.h\sqrt{\frac{\sigma}{\rho.\nu}})$ that represent the effects of dynamic forces and the magnetic forces respectively are assumed to be constants [2].

B. Solution by Weighted Residual Least Squares Method (L.S.M.): The weighted residual least square method has been used to solve the above system of equations. This method can basically be summarized, (e.g.: [26, 27] as follows:

(i) Assign some presumed values for *Re* number and *Ha* number and substitute them in the above equations.

(ii) Assume solution sets with unknown coefficients α_{ij} and β_{ij} for each of *V* and *U* (or alternatively *V'* and *U*) in the forms of functions in *y* so that: (1) they satisfy the boundary conditions given by Equ. (7) and (2) they should be chosen as a linear combination of basic functions selected from of a linearly independent set. Present study uses the following forms:

$$V_{m_1 m_2} = \alpha_{ij}.y^i.(1-y)^j \quad , \quad \{i,j\} = \{(0,m_1), (0,m_2)\} \tag{8}$$

$$U_{n_1 n_2} = \beta_{kl}.y^k.(1\text{-}y)^l \quad , \quad \{k,l\} = \{(0,n_1), (0,n_2)\} \tag{9}$$

Repeated indices signify summation process and the integer numbers are positive, i.e.: m_1, m_2, n_1, $n_2 > 0$.

(iii) Substitute the above solution set into Equs. (5 - 6) to formulate the following two residual equations:

$$EV = V_{m_1 m_2}^{IV} + \mathrm{Re}.V_{m_1 m_2}.V_{m_1 m_2}''' - (Ha^2 + \mathrm{Re}.V_{m_1 m_2}').V_{m_1 m_2}'' \tag{10}$$

$$EU = U''_{n_1n_2} + \mathrm{Re}.V_{m_1m_2}.U'_{n_1n_2} - (Ha^2 + \mathrm{Re}.V'_{m_1m_2}).U_{n_1n_2} \tag{11}$$

These equalities will be zero if the assumed solutions are found to be the exact ones, i.e.: $V = V_{m_1m_2}$ and $U = U_{n_1n_2}$.

(iv) Use the partial derivative of the above residual equations with respect to each unknown coefficient as weight for its respected residual equation to form the weighted residual Least Squares equations.

(v) Equate to zero the integrals of the weighted equations over the y-domain {0, 1} to get algebraic equations relating the unknown coefficients. These integrals have the form:

$$LS(V_{\alpha_{ij}}) = \int_0^1 EV.\frac{\partial EV}{\partial \alpha_{ij}}.dy = 0 \ \ , \{i, j\} = \{(0, m_1), \ (0, m_2)\} \tag{12}$$

$$LS(U_{\gamma_{kl}}) = \int_0^1 EU.\frac{\partial EU}{\partial \beta_{kl}}.dy = 0 \ \ , \{k, l\} = \{(0, n_1), (0, n_2)\} \tag{13}$$

(vi) Solve the resulting set of the simultaneous algebraic equations to calculate the unknown coefficients that correspond to the assumed values of *Re* number and of *Ha* number.

(vii) Substitute the results into the residual equations to calculate the values of the errors, and compare them to presumed acceptable error limits. Reject the assumed equations if the test fails, and repeat the above steps for other assumed solution equations.

(viii) If the error test passes, accept the solutions equations and repeat steps (iv – vi) for other values of *Re* number and *Ha* number.

C. Flow rate (q): The flow rate of the fluid through the porous upper plate (q) can be calculated by equating its value to the main stream flow rate in the horizontal direction. For unit width, this is given as:

$$q = \int_0^1 U.dy \tag{14}$$

RESULTS AND DISCUSSION

Many trail solutions are assumed for V (or alternatively V') and U that fulfill the boundary conditions given by Equ. (7). The coefficients of these assumed trials are determined using the L.S.M. solution steps given above. The trial solutions that gave acceptable results are:

$$V' = a.y.(1-y) + b.y^2.(1-y)^2 + c.y^3.(1-y)^3 + d.c.y^4.(1-y)^4 \tag{15a}$$

which by integration, results in value of V as:

$$V = \frac{1}{2}a.y^2 - \frac{1}{3}(a-b).y^3 - \frac{1}{3}(2.b-c).y^4 + \frac{1}{5}(b-3.c+d).y^5 + \frac{1}{6}(3.c-4.d).y^6 - \frac{1}{7}(c-6.d).y^7 - \frac{1}{8}d.y^7 + \frac{1}{8}d.y^9 \tag{15b}$$

And the horizontal velocity is expressed as:

$$U = e.(1-y) + f.(1-y)^2 + g.(1-y)^2 + g.(1-y)^3 + h.(1-y)^4 \tag{16}$$

Using the above equations as solutions of the differential equations has resulted in errors, as given by Eqs. (12 -13), less than -7.3E-12.

I. Flow characteristics in the range $0.0 \le \mathrm{Re} \le 1.0$ *and* $0.0 \le Ha \le 1.0$

I.1. U and V characteristics: The variations of the horizontal and vertical flow velocities; U and V; over the gap between the plates are displayed in Fig.2 and Fig.3 for values of Reynolds number *Re* and Hartman number *Ha* in the range (0.0, 1.0). At a value of Re = 1.0, Fig.2 shows that the vertical flow velocity V is almost independent from the effect of *Ha*. This velocity has an almost zero rates of variation of with y in the vicinity of both the lower and upper plates, and this rate is almost linear at its highest value inside the gap between the plates. The value of this vertical velocity starts from zero at the lower plate to its maximum value at the upper plate, i.e. where the fluid leaves the perforation. The Hartman number *Ha* has a minor

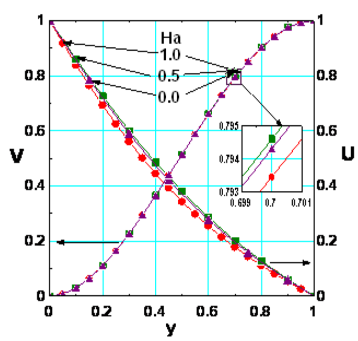

Fig.2 Effect of *Ha* on *V* and *U* at *Re*=1.0 over the whole domain of y.

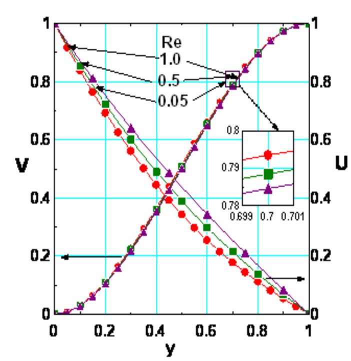

Fig.3 Effect of *Re* on *V* and *U* at *Ha* =1.0 over the whole domain of y.

effect on the horizontal velocity *U* near both the plates, and this effect increases towards the center of the gap. This horizontal velocity *U* is at its maximum value which is identical to that of the lower plate, and decreases to zero value towards the upper fixed plate. The shear stress is very high in the vicinity of the lower plate (exemplified by the high rate of variation of *U* with respect to *y*), and is at its lowest value near the upper plate. Figure 3 shows the effects of Reynolds number *Re* on the horizontal and vertical flow velocities at a value of *Ha =1.0*. As discussed in Fig.2, Re has almost no effect on the vertical velocity *V* and has an appreciable effect on the horizontal velocity within the gap. Again, characteristics of these two velocities in Fig.3 are the same as discussed in regards to Fig.2.

I.2. Flow rate *q* through porous wall: The flow rate *q* through the upper porous plate, as calculated from Eq. (14) shows a significant dependency on both *Ha*, and *Re* as presented in Fig. 4. For a constant value of *Ha*, *q* decreases with increasing the value of *Re*. Similar trends of *q* is shown with increasing *Ha* at constant value of *Re*. The reason is attributed to the fact that the velocity *U* is mainly dependent on both *Ha*, and *Re* as presented in Figs. 2 and 3. The general conclusion from this figure is that the lower the values of both *Re* and *Ha* numbers the higher the flow rate is.

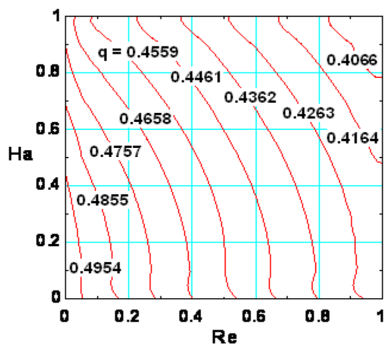

Fig.4 Contour values of the flow rate *q* as dependent on *Re* and *Ha* values.

II. *U*, *V* and *V'* Flow characteristics in the range $1.0 \le Re \le 10$ *and* $0.0 \le Ha \le 10$

Figure 5(a), (b) and (c) show the variations of the horizontal flow velocity; *U*, vertical flow velocity; *V* and its rate; *V'* over the gap; *y* for the designated ranges of *Re* and *Ha* numbers. Figure 5(a) shows that both *Ha* and *Re* have appreciable effect on the horizontal velocity *U*. For values of *Ha = 0.0, 3.0* and 5.0, the higher the value of *Re* the lower this velocity is, and the higher the shear stress is in the vicinity of the lower plate (as exemplified be the higher slope near *y=0.0*). This behavior of *U* means that the fluid is reluctant to follow the speeding up of the moving lower plate. At a value of *Ha = 10*, the effect of *Re* on *U* diminishes to its lowest limit. At this value of *Ha* the flow has an almost oscillating nature in about the upper *75%* of the gap. This is exhibited by the alternating positive and negative values of this velocity. At this value of *Ha*, the shear stress is very high in the lower *25%* of the gap. In Fig. 5(b), neither *Ha* number nor *Re* number has appreciable effects on the vertical velocity *V* along the whole range of *y*, where its value starts from zero at the lower plate to its maximum value at the upper plate. However, as shown in Fig. 5(c), these two dimensionless numbers have noticeable effects on the velocity rate especially in the center of the gap, where its value is about *V' =1.5* at *Ha = 0.0* and decreases with the increase of *Ha* for all values of *Re*. At a value of *Ha = 10,* the velocity rate flattens in the center of the gap with a value of about *V' = 1.2*. This decrease in the rate of the vertical velocity supports the concluded nature of the oscillating flow for high values of *Re* and *Ha* numbers, as previously explained in Fig. 5(a).

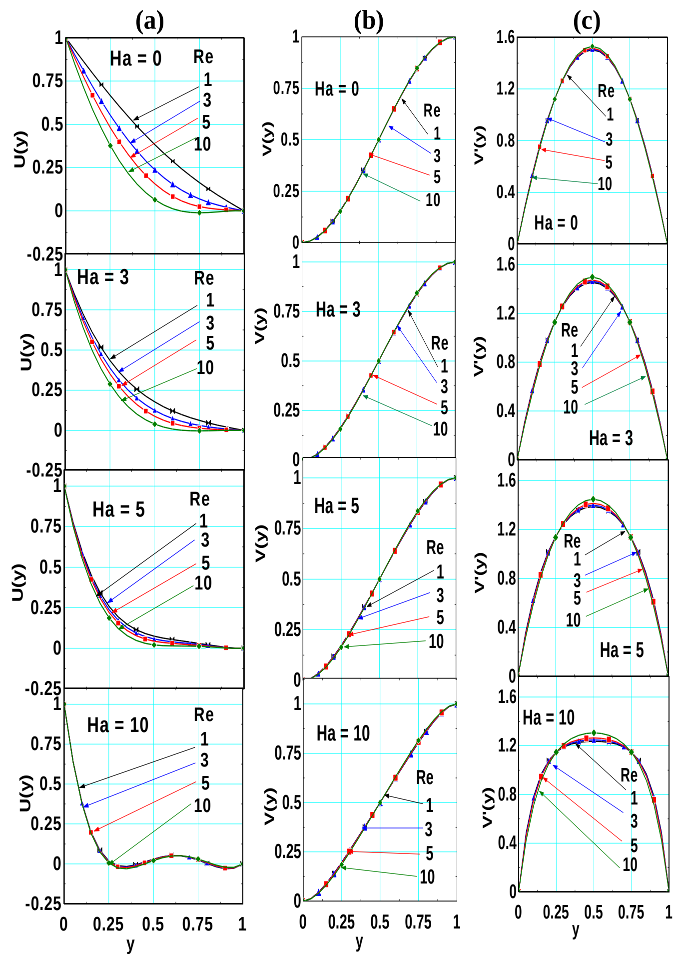

Fig.5 Variations of *U*, *V* and *V'* with *y* at different values of *Re* and *Ha*.

Figures 6(a), (b) and (c) show the effect of *Ha* and *Re* on the flow velocities *U*, *V* and its rate *V'*. Figure 6(a) shows that the value of *Ha* results in oscillating flow in the upper *75%* of the gap irrespective of the value of *Re*. In this figure, the slight increase in shear stress with the increase of *Re* was on the expense of the value of the velocity, which suffers a slight decrease at all values of *Ha*. Again, Fig. 5b above shows a slight effect on vertical velocity for all values of *Re* and *Ha*. As compared to slight effect of *Re* on *V'* at constant *Ha* displayed in Fig. 5(c) before, at constant values of *Re* Fig.6 (c) shows great dependency of *V'* on *Ha* number.

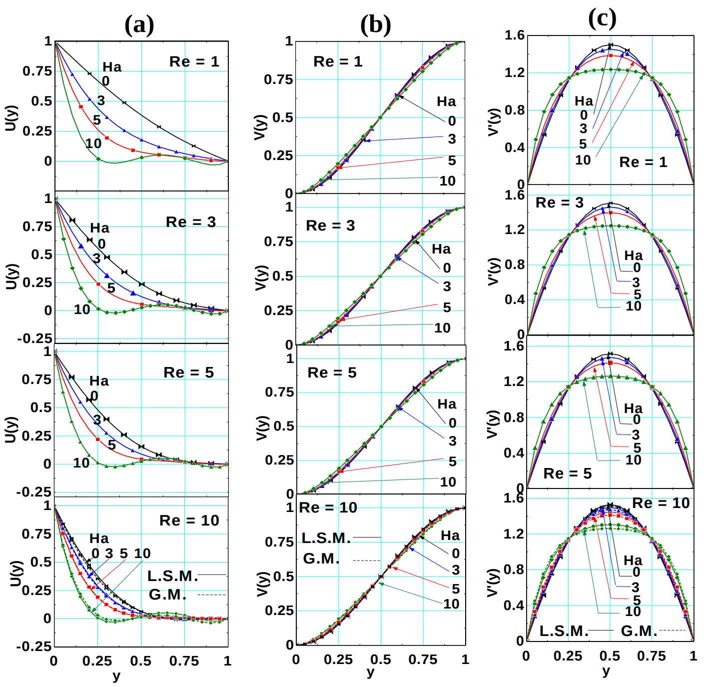

Fig.6 Variations of *U*, *V* and *V'* with *y* at different values of *Re* and *Ha* and comparisons of results of LSM with Galerkin method.

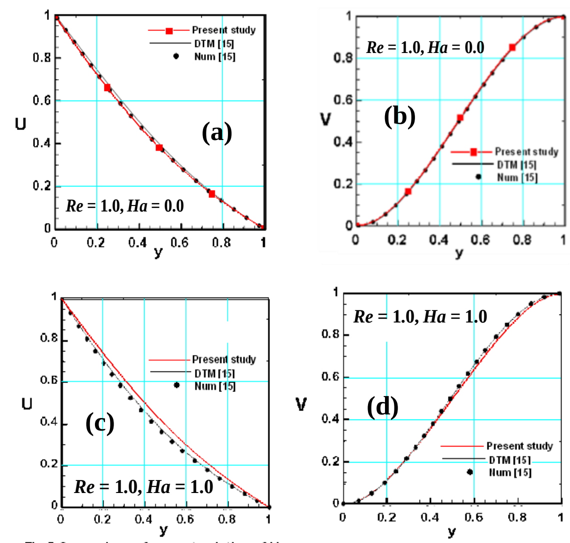

Fig.7 Comparison of present results of *U* and *V* with published results.

Shown comparisons between those predicted results of [15] and present results indicate that there is a slight difference between the value of *U*, while *V* values show better agreement.

III. Comparison with Galerkin Method Results [17]:

Present results of solving the governing differential equations by the weighted residual least square method (L.S.M.); shown by solid lines, with their solution [17] by Galerkin Method (G.M.); shown by dotted lines, are shown in lower figures of both Fig. 6(a), (b) and (c). The displayed figures show a very good agreement between the results of these two methods, signifying the creditability of the solution by both methods. It should be noted that although both methods belong to the weighted residual solutions methods, they do not necessarily give comparable results unless the system of differential equations is stable. This signifies that the controlling equations of the present simulation is completely stable, and any solution results by any of these two methods will logically represent the behavior of the flow system.

IV. Comparison with Published Results of Homotopy Analytical Method and Numerical Method [15]:

Comparisons of present results of values of *U* and *V* with other published results for some specific values of *Re* and *Ha* are shown in Figs. 7(a), (b), (c) and (d). The variations of dimensionless velocity components *U* and *V* versus vertical distance *y* at *Re = 1.0* and *Ha = 0* are presented in Figs. 7(a) and (b). A good agreement between present results and those predicted by [15] is shown in the figures. The published work had used two solution methods, namely: (i) Homotopy analytical method (DTM) and (ii) Numerical method (Num). The results by both these two methods agree well with present results. The variations *V*, and *U* versus vertical distance y at *Re = 1.0* and *Ha = 1* are presented in Figs. 7(c) and (d).

CONCLUSIONS

Weighted residual Least Squares Method is used to solve the two dimensional flow of electrically conducting fluid in a channel under uniform magnetic field between uniformly moving lower plate and a fixed parallel semi-porous plate. The flow is governed by the two dimensionless numbers Hartman number (*Ha*), and Reynolds number (*Re*). In the range $Re <$ 1.0 and $Ha <$ 1.0, neither *Ha* nor *Re* has noticeable effect on vertical flow velocity *V*. The rate of this velocity is linear within the gap and vanishes in the vicinity of both the lower and upper plates.

The fluid flow rate *q* through the upper porous plate shows significant dependency on both *Ha* and *Re*, where it decreases with increasing the value of either *Ha* or *Re* due to the dependency of the horizontal velocity *U* on both *Ha*, and *Re*.

In the ranges 1.0 < *Re*, *Ha* < 10 both *Ha* and *Re* have appreciable effect on the horizontal velocity *U*, where the higher *Re* the lower this velocity is, and the higher the shear stress is in the vicinity of the lower plate. This behavior of *U* means that the fluid is reluctant to follow the speeding up of the moving lower plate upon the increase of *Re*. At a value of *Ha* = 10, the effect of *Re* on *U* diminishes to its lowest limit, and the flow suffers an almost oscillating nature in the upper *75%* of the gap between the plates. At this value of *Ha*, the shear stress is very high in the lower *25%* of the gap.

Present results agree well with elsewhere 3 different published results that have used weighted residual Galerkin method (G.M.), numerical method and Homotopy analytical method.

Future work is needed to consider variations in both the magnetic field and the electrical conductivity and viscosity of the fluid. Also the effect of pressure gradient needs further investigations.

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