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ON HEAT CONDUCTION IN A PERIODICALLY STRATIFIED MEDIUM WITH SLANT LAMINATION

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ABSTRACT

The paper deals with the two-dimensional stationary temperature distribution problem for a composite medium. The nonhomogenous medium is assumed to be a composite with micro-periodically stratified structure. The elementary unit of composite is a two-layered laminae. The ideal thermal condition on interfaces is assumed. The layering is inclined with an arbitrary angle to the boundary planes. In this paper the two cases of considered medium are shown:

- Layer with periodically structure with given constant temperature on upper and lower boundary surface;
- Half-space with periodically structure with given constant temperature in boundary surface.

The considered problem are solved within the framework of the homogenized model with microlocal parameters given by Woźniak (1987), Matysiak and Woźniak (1986). The plane problems of periodically stratified medium with slant layering heated by given boundary temperature are solved analytically. The influence of thermal and geometrical properties on temperature distribution in analysed medium was investigated.

INTRODUCTION

The heat conduction problem for micro-periodical layered composites is very important from the engineering point view. The knowledge the distribution of temperature and heat flux in this kind of composites is necessary for thermoelastic problems. Many monographs and papers have been devoted to the modelling of heat conduction problems in periodically stratified bodies, for example [1-7, 12-13, 17, 21, 23]. The large number of elementary unit repeated periodically causes the complication of calculation by use analytical and numerical methods. It seems to be intentional to use some approximated model. One of them is the homogenized model with microlocal parameters given by Woźniak [22], Matysiak and Woźniak [17-18]. This model, derived by using the nonstandard analysis combined with some a priori postulated assumptions, is described by unknowns: macro-temperature and thermal microlocal parameters. The main merit of the model is that the thermal continuity conditions on interfaces are satisfied.

The application of homogenized model with microlocal parameters for layered composites with parallel or vertical boundary surface have been analysed in the papers [8, 14, 19, 24, 20]. The accuracy and applicability of the homogenized model can be found in the papers [9-11, 16].

This paper deals with the heat conduction problem in layered composites with periodically repeated laminae. The inclined layering with the under arbitrary angle to the boundary planes is considered. The boundary surface are assumed to be kept at given temperatures.

The homogenized model with microlocal parameters given by Woźniak [22], Matysiak and Woźniak [17-18] is applied to describe distribution of temperature and heat flux. For solving the considered stationary and plane boundary value problem two coordinate systems are introduced: the first one is connected with the layering and the second one is connected with the boundaries. Some special cases of temperature distributions in the periodically stratified layer and half-space will be discussed and it will be presented in the form of figures. A wider analysis of the results for formulated problem can be found in the papers [26], [27].

NOMENCLATURE

a	[m]	the half of length of heated range;
(x,y)	[m]	Cartesian coordinates connected with the layering;
$(\tilde{x},\ \tilde{y})$	[m]	Cartesian coordinates connected with the boundary
K_1, K_2	[$Wm^{-1}K^{-1}$]	coefficients of thermal conductivity of the subsequent component of the body

$\tilde{K},\ K^*$	$[\mathrm{Wm^{-1}K^{-1}}]$	effective thermal modulus on the homogenized model with microlocal parameters
α	[rad]	angle of inclination of layering to axis $\tilde{x}$
δ_1, δ_2	[m]	thickness of the layers being the constituents of composite
$\delta = \delta_1 + \delta_2$	[m]	thickness of fundamental unit
$\eta = \delta_1 / \delta$	[-]	saturation coefficient of fundamental unit by the first kind of material
θ	[K]	macro-temperature in homogenized model with microlocal parameters
$i = 1,\ 2$	[-]	kind of sublayer; $i = 1$ the first kind or $i = 2$ the second kind of the subsequent layers

FORMULATION OF THE PROBLEM

Consider medium is a composite with periodic structure repeated periodically. Nonhomogeneous layer is composed by two-layered conductors with assumption that the ideal thermal connection between the component of elementary unit. The layering is inclined to boundary surface with arbitrary angle α, (see Fig. 1). Considered body will be described in two Cartesians coordinate systems: the first one (x, y, z) is such that y – axis is parallel to the layering (see Fig. 1), and the second one $(\tilde{x},\ \tilde{y},\ \tilde{z})$ such that the axis $0\tilde{x}$ is directed according to the boundary.

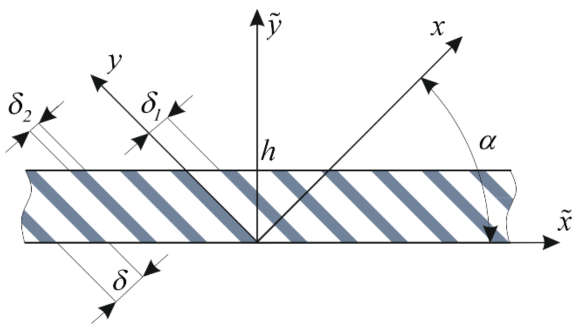

Figure 1 Scheme of cross section of considered composite layer

Let δ_1, δ_2 be the thicknesses of the layers being the constituents of composite, and $\delta = \delta_1 + \delta_2$ be the thickness of repeated lamina, see Fig. 1. Let K_1, K_2 be the thermal conductivity of the subsequent component of the composite, and h the thickness of the layer. Let the upper boundary plane $\tilde{y} = h$ be kept at given temperature $\theta_0(\tilde{x}),\ \tilde{x} \in R,$ and the lower boundary $\tilde{y} = 0$ be kept at zero temperature. Thus, the considered problem is two-dimensional and stationary. Formulated problem can be the basis to obtain the solution for case of half-space problem with heating on the boundary surface $\tilde{y} = 0$ and assumed that the thickness of layer tends to infinity.

HOMOGENIZED MODEL WITH MICROLOCAL PARAMETERS

The heat conduction for composite material with microperiodic layered structure can be described in framework of classical heat conduction equation with strong oscillating coefficients. In this case the solution of formulated problem need to satisfy the boundary condition and ideal thermal connections on the interfaces (it is assumed that the conditions on the interface are perfect). This approaches is rater very complicated. The natural way is apply the some approximation methods. One of them is homogenized model with microlocal parameters given by Woźniak [23] and applied to multilayered composites by Matysiak and Woźniak [19].

The governing relations of the model are described in the coordinates (x, y) connected with the layering, see Fig. 1 and see: [8-11, 14-20, 24]. Let $T(x, y)$ denote the temperature.

The temperature and its gradient within the homogenized model is predicted in the form:

$$\begin{aligned} &T(x,y) = \theta(x,y) + h(x)q(x,y) \approx \theta(x,y), \\ &\frac{\partial T(x,y)}{\partial x} \approx \frac{\partial \theta(x,y)}{\partial x} + h'(x)q(x,y), \\ &\frac{\partial T(x,y)}{\partial y} \approx \frac{\partial \theta(x,y)}{\partial y}, \end{aligned} \tag{1}$$

where $\theta(x, y)$ is the macro-temperature, $q(x, y)$ is unknown microlocal parameter, and $h(x)$ is the shape function for two-layered composites which allow to fulfill the ideal thermal condition on the interfaces is given in the form

$$h(x) = \begin{cases} x - 0.5\delta_1 & \text{for} \quad 0 \le x \le \delta_1 \\ \dfrac{-\eta x}{1-\eta} - 0.5\delta_1 + \dfrac{\delta_1}{1-\eta} & \text{for} \quad \delta_1 \le x \le \delta, \end{cases}$$
$$h\,(x + \delta) = h(x), \tag{2}$$

where

$$\eta = \frac{\delta_1}{\delta}. \tag{3}$$

General equations of the homogenized model for stationary two-dimensional problem take the form, see [9-10]:

$$\tilde{K}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) + [K]\frac{\partial q}{\partial x} = 0\ , \quad \hat{K}\, q = -[K]\frac{\partial \theta}{\partial x}. \tag{4}$$

where

$$\begin{aligned} &\tilde{K} = \eta K_1 + (1-\eta)K_2\,, \\ &[K] = \eta(K_1 - K_2), \\ &\hat{K} = \eta K_1 + \frac{\eta^2}{1-\eta}K_2\,, \end{aligned} \tag{5}$$

Eliminating the microlocal parameter q from $(4)_1$ we obtain

$$\tilde{K}^{-1}K^{*}\frac{\partial^{2}\theta}{\partial x^{2}}+\frac{\partial^{2}\theta}{\partial y^{2}}=0, \tag{6}$$

where

$$K^{*}=\frac{K_{1}K_{2}}{(1-\eta)K_{1}+\eta K_{2}}. \tag{7}$$

The heat flux vector in a layer of the i - th, i = 1,2, kind expressed in the coordinates (x,y) has the form

$$\mathbf{q}^{(i)}\equiv\left(q_{x}^{(i)},q_{y}^{(i)},0\right)=\left(-K^{*}\frac{\partial\theta}{\partial x},\ -K_{i}\frac{\partial\theta}{\partial y},\ 0\right). \tag{8}$$

The considered boundary value problem it will be solved in the coordinates $(\tilde{x},\ \tilde{y})$, see Fig. 1. The following relations between coordinates $(\tilde{x},\ \tilde{y})$ and (x,y) hold

$$\begin{aligned}x&=\tilde{x}\cos\alpha+\tilde{y}\sin\alpha,\\ y&=-\tilde{x}\sin\alpha+\tilde{y}\cos\alpha.\end{aligned} \tag{9}$$

The equation (6) expressed in coordinates $(\tilde{x},\ \tilde{y})$ takes the form

$$\begin{aligned}&\frac{\partial^{2}\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{x}^{2}}\left(\tilde{K}^{-1}K^{*}\cos^{2}\alpha+\sin^{2}\alpha\right)+\\ &+\frac{\partial^{2}\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{x}\partial\tilde{y}}\left(\tilde{K}^{-1}K^{*}-1\right)\sin2\alpha+\\ &\frac{\partial^{2}\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{y}^{2}}\left(\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha\right)=0.\end{aligned} \tag{10}$$

SOLUTION OF THE PROBLEM

Boundary conditions for considered problem of layer can by write in the form:

$$\begin{aligned}&\theta(\tilde{x},0)=0,\\ &\theta(\tilde{x},h)=\vartheta(\tilde{x}),\quad x\in R,\end{aligned} \tag{11}$$

where $\vartheta(\tilde{x})$ satisfies

$$\lim_{\tilde{x}\to\pm\infty}\vartheta(\tilde{x})=0. \tag{12}$$

and it is taken into account that

$$\vartheta(-\tilde{x})=\vartheta(\tilde{x}),\quad x\in R. \tag{13}$$

Solution of formulated problem will by obtained by integral Fourier transform methods

$$\bar{\theta}(\gamma,\tilde{y})=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\theta(\tilde{x},\tilde{y})e^{-i\tilde{x}\gamma}\mathrm{d}\tilde{x},\quad i=\sqrt{-1}, \tag{14}$$

The temperature in Fourier integrals form with satisfy the boundary conditions (11) of the composite layer has the following form

$$\begin{aligned}&\theta(\tilde{x},\ \tilde{y})=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}\bar{\vartheta}(\gamma)\frac{\sinh(\omega\gamma\tilde{y})}{\sinh(\omega\gamma h)}\\ &\cos\left\{\gamma\left[(\tilde{y}-h)\frac{\sin2\alpha\left(\tilde{K}^{-1}K^{*}-1\right)}{2\left(\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha\right)}-\tilde{x}\right]\right\}\mathrm{d}\gamma.\end{aligned} \tag{15}$$

where

$$\omega=\frac{\sqrt{\tilde{K}^{-1}K^{*}}}{\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha}\ . \tag{16}$$

The heat fluxes in the directions x and y given by (8) can be rewritten by using variables $\tilde{x},\ \tilde{y}$ in the form

$$\begin{aligned}q_{x}^{(j)}&=-K^{*}\left(\cos\alpha\ \frac{\partial\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{x}}+\sin\alpha\frac{\partial\tilde{\theta}(\tilde{x},\ \tilde{y})}{\partial\tilde{y}}\right),\\ q_{y}^{(j)}&=-K_{i}\left(-\sin\alpha\frac{\partial\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{x}}+\cos\alpha\frac{\partial\tilde{\theta}(\tilde{x},\ \tilde{y})}{\partial\tilde{y}}\right).\end{aligned} \tag{17}$$

From (3.8) it follows that

$$\begin{aligned}&\frac{\partial\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{x}}=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}\gamma\bar{\vartheta}(\gamma)\frac{\sinh(\omega\gamma\tilde{y})}{\sinh(\omega\gamma h)}\times\\ &\times\sin\left\{\gamma\left[(\tilde{y}-h)\beta-\tilde{x}\right]\right\}\gamma\,\mathrm{d}\gamma,\\ &\frac{\partial\theta(\tilde{x},\ \tilde{y})}{\partial\tilde{y}}=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}\bar{\vartheta}(\gamma)\left\{\frac{\cosh(\omega\gamma\tilde{y})}{\sinh(\omega\gamma h)}\omega\gamma\times\right.\\ &\times\cos\left\{\gamma\left[(\tilde{y}-h)\beta-\tilde{x}\right]\right\}-\beta\sin\left\{\gamma\left[(\tilde{y}-h)\beta-\tilde{x}\right]\right\}\times\\ &\left.\times\frac{\sinh(\omega\gamma\tilde{y})}{\sinh(\omega\gamma h)}\gamma\beta\right\}\mathrm{d}\gamma.\end{aligned} \tag{18}$$

where

$$\beta=\frac{\sin2\alpha\left(\tilde{K}^{-1}K^{*}-1\right)}{2\left(\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha\right)}. \tag{19}$$

REMARKS

Special case for half-space with slant lamination can be taken from Eq. (15) the macro-temperature $\theta(\tilde{x},\ \tilde{y})$ can be written in the form

$$\begin{aligned}&\theta(\tilde{x},\ \tilde{y})=\sqrt{\frac{2}{\pi}}\int_{0}^{\infty}\bar{\vartheta}(\gamma)\exp\left[-\frac{\sqrt{\tilde{K}^{-1}K^{*}}\gamma\tilde{y}}{\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha}\right]\\ &\cos\left[\gamma\left(\tilde{y}\frac{\sin2\alpha\left(\tilde{K}^{-1}K^{*}-1\right)}{2(\tilde{K}^{-1}K^{*}\sin^{2}\alpha+\cos^{2}\alpha)}-\tilde{x}\right)\right]\mathrm{d}\gamma,\end{aligned} \tag{22}$$

where

$$\bar{\vartheta}(\gamma)=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\vartheta(\tilde{x})\cos(\gamma\,\tilde{x})\,\mathrm{d}\tilde{x}. \tag{23}$$

NUMERICAL RESULTS AND DISCUSSION

Consider the following boundary temperature distribution

$$\vartheta(\tilde{x})=\theta_0 H\left(|\tilde{x}|-a\right), \tag{24}$$

where θ_0 and a are constants, $H(\cdot)$ is the Heaviside`s step function. For the numerical analysis of temperature and heat flux distributions the following dimensionless variables are introduced:

$$\tilde{x}^*=\tilde{x}/a\,,\tilde{y}^*=\tilde{y}^*/a\,, \tag{25}$$

and let $\tilde{\delta}=\delta/a$ be dimensionless thickness of the fundamental lamina. The inverse Fourier transform is calculated numerically and the obtained results are presented in the forms of figures. Figure 2 shows the isothermal lines for first problem connected with the layer. The distribution of dimensionless temperature on first figures shows the results for homogeneous layer. The next three figures shows the distributions for three cases of arbitrary angle $\alpha=0;\pi/4;\pi/2$ and $K_1/K_2=8$.

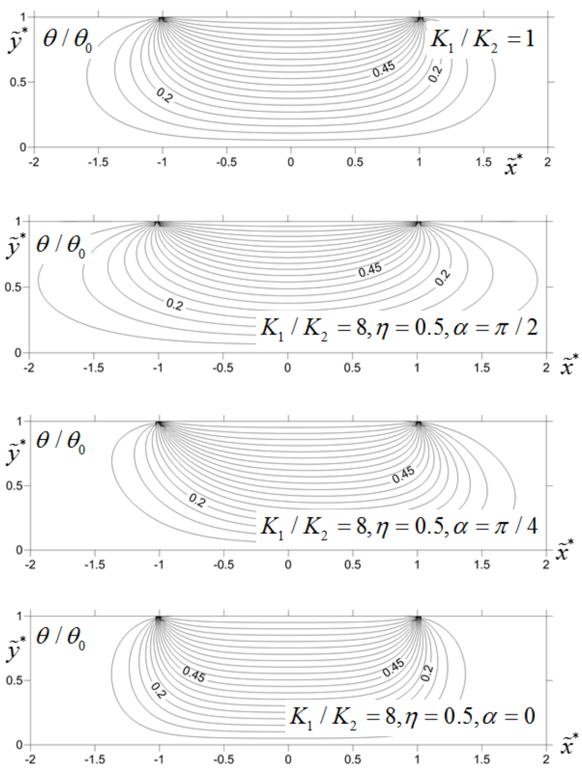

Figure 2 The dimensionless isothermal lines in the periodically layer

It is seen that the highest values of temperature at this point are achieved for $\alpha=0$ (the layering is perpendicular to the boundary). Next figures 3 shows the dimensionless isothermal lines for solution connected with half-space. The first figure shows the result for homogeneous half-space. The next figures present the dimensionless distributions of temperature for three cases of angle $\alpha=0;\pi/4;\pi/2$ and $K_1/K_2=8$. We can observe the same situation for the highest values of temperature for the case when the layering is perpendicular to the boundary surface.

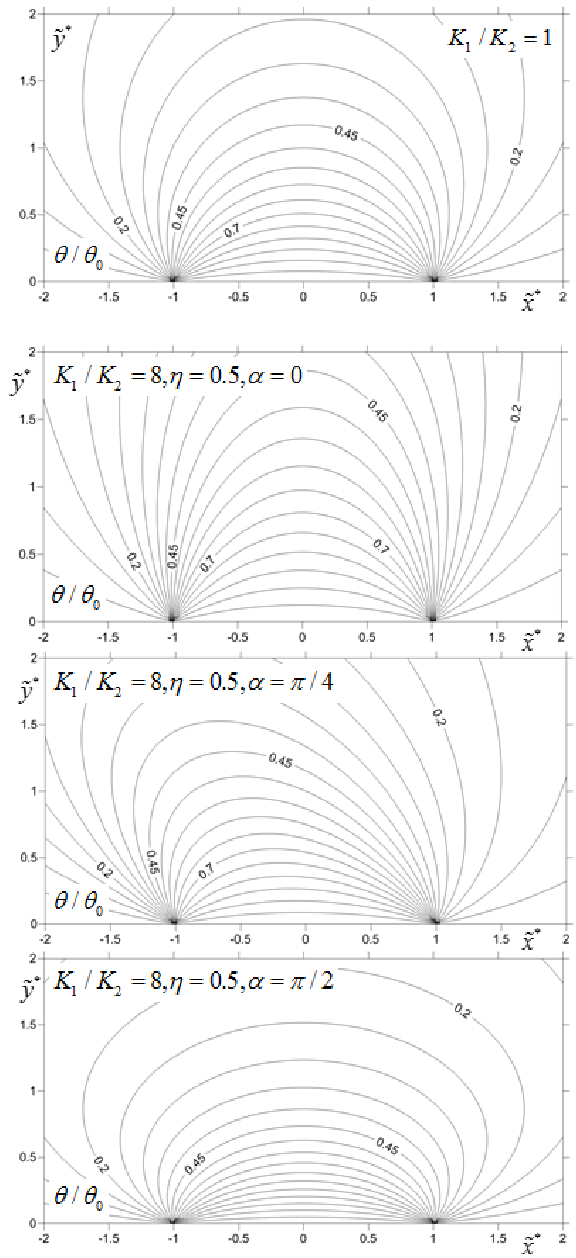

Figure 3 The dimensionless isothermal lines in the periodically half-space

CONCLUSIONS

The paper presents the applications of the homogenized model with micro-local parameters to the boundary value problems for periodically layered composites with slant layering to boundaries. General equation of this model combined with the coordinate transformation permit to solve problems of heat conduction in the composites with slant lamination. The obtained results for temperature possess the same characteristics as adequate solutions within the framework of the classical equation of heat conduction.

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