

## Supersonic Nozzle Flow in the Two-Phase Ejector as Water Refrigeration System by Using Waste Heat

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### ABSTRACT

In the near future the usage of waste heat in houses will be increased. The development of refrigeration system using waste heat is particularly needed for summer season consumptions. The ejector cycle can directly convert the thermal energy into the compression mechanical energy. Moreover, water which is one of the cleanest refrigerants can be used in the ejector cycle. As the steam ejector has been researched [1-3] as air conditioning system, the huge energy needed for large latent heat for water. We propose that the high-speed steam and water mixture, so-called two-phase flow, can be utilized instead of steam. It is theoretically shown that the heat need for this system will be reduced by the latent heat of hot water in two-phase flow, if the two-phase in the ejector flow has the same potential of acceleration and compression as steam. And the nozzle efficiencies for the two-phase flow of water are also obtained by the experiment.

### INTRODUCTION

Nowadays, saving the energy became one of the most important issues in the world since the fossil fuel will dry up in the not so long future and especially in Japan, the nuclear energy may be unavailable after the Great East Japan Earthquake. Then the clean energy such as the solar panel and the fuel cells are expected to be introduced into a large amount of houses for the electrical source. As the energy conversion efficiency of those power systems is not so high, the low temperature waste heat will be generated in houses. The low level energy must be recovered to increase the net efficiency of the power systems. In winter season, the low level waste heat in houses is available for the house heating or for the hot water supply. On the other hand, there is no need for the low temperature heat in summer time. But, if this heat had been used by air conditioning system, the problem would be solved.

One solution is the ejector compression refrigeration cycle [4] which enables us to convert the low level heat into the mechanical compression energy. If the hot water is used for the

refrigerant, the environment load will be reduced more because water is friendly, safe and coexistent material for human being, as the house cooling system will be widely dispersed over our society. While in usual air conditioning system, electric power is used to compress the refrigerant vapour, high pressure steam heated by the waste heat in the water ejector system are accelerate to the supersonic state and the kinetic energy of the steam is used. Although the energy conversion efficiency is low, this system has a possibility to prevail because the originally abandoned waste heat changes to the useful energy.

In order to improve this system, we propose to utilize the high-speed two-phase flow instead of steam. A large amount of heat must be needed to obtain steam for latent heat of water. The p-h diagrams for the ejector refrigeration system are drawn in Figure 1. This shows that the thermal energy of high temperature and high pressure water is converted into the kinetic energy to compress the low pressure and low temperature vapour. If the acceleration and compression by two-phase flow is possible in the ejector, the heat need for this system will be reduced by the latent heat of hot water in two-phase flow.

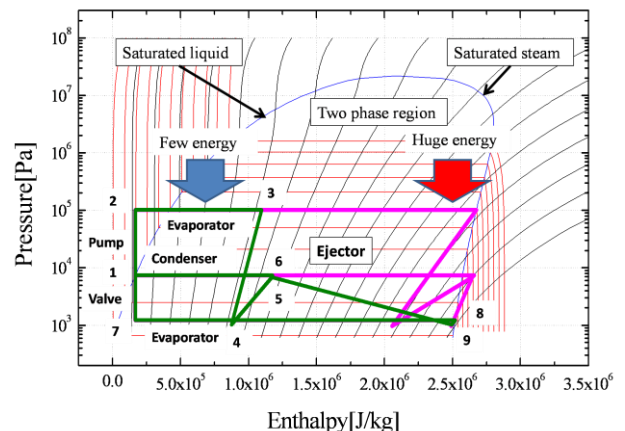


Figure 1 P-h diagram of water

## OBJECTIVE

The steam ejector refrigeration cycle using water as refrigerant had been studied. The huge energy, however, is necessary for saturated water to change into vapour state to use this cycle. Two-phase flow ejector cycle can save more energy compared with steam ejector cycle.

There is almost no research about the nozzle of two-phase flow ejector for water. The object of present study is to obtain the performance of the water two-phase flow nozzle. From the results of this paper, optimized inlet conditions will be obtained and will apply to the design of two-phase ejector refrigeration system.

## NOMENCLATURE

$w$	[W/kg]	Specific work needed for compression
$h$	[J/kg]	Specific enthalpy
$s$	[J/(kgK)]	Entropy
$q$	[W/kg]	Specific heat
$p$	[Pa]	Pressure
$u$	[m/s]	Velocity
$A$	[m <sup>2</sup> ]	Area
$G$	[kg/s]	Mass flow rate
$z$	[m]	Coordinate along flow from the throat
$x$	[-]	Quality
$g$	[-]	Mass flow ratio
$COP$	[-]	Coefficient Of Performance

Special characters		
$\rho$	[kg/m <sup>3</sup> ]	Density

Subscripts		
$s$		Saturated state (for Pressure)
$d$		Driving flow
$s$		Suction flow
$c$		Critical condition
$ex$		Experimental result
$th$		Isentropic Homogeneous Equilibrium model

## COEFFICIENT OF PERFORMANCE

The schematic diagram of the ejector refrigeration system is shown in Figure 2. This cycle consists of two loops. One is the loop from condenser to pump, high temperature evaporator, ejector and then back to condenser. The other is the loop from condenser to expansion valve, low temperature evaporator, ejector, back to condenser. The high temperature evaporator works by the waste heat, and the low temperature evaporator acts as the heat sink of refrigeration cycle. The former loop is similar to the conventional steam engine cycle, and the latter to vapour compression refrigeration cycle. Then the ejector operates as both the turbine in thermal engine and the compressor in air-conditioner.

We calculate the thermal conversion efficiency of system as the following equation of Coefficient of Performance, COP.

$$COP = \frac{\text{Heat absorbed at the L - evaporator}}{\text{Heat absorbed at the H - evaporator}}$$

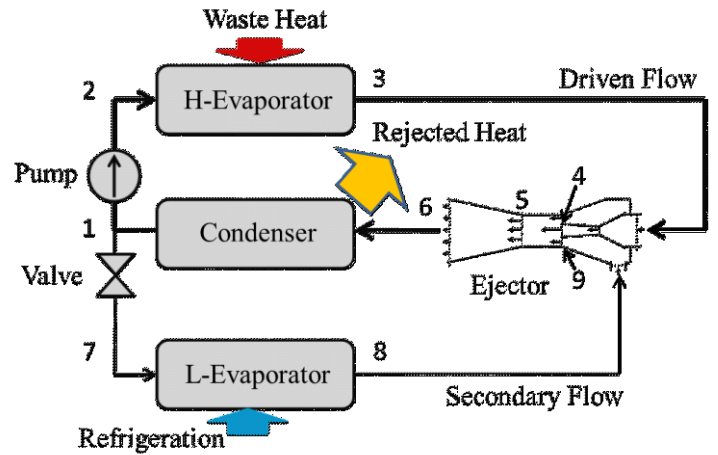


Figure 2 Ejector refrigeration system of this study

This turns the product of the thermal efficiency of engine and the COP of the air-conditioner refrigeration cycle if there is no friction loss at the connection rod between the engine and compressor.

It is assumed that the pump, evaporators and condenser are ideal. The inlet condition of the pump denoted by under suffix 1, and outlet 2. The pumping work  $w_{21}$  for unit mass flow is written as the enthalpy difference.

$$w_{21} = h_2 - h_1, \quad s_2 = s_1 \quad (1)$$

Where  $h$  is enthalpy and  $s$  entropy.

The outlet of high temperature evaporator is denoted by 3. The thermal energy absorbed  $q_{32}$  is also expressed by enthalpy gain.

$$q_{32} = h_3 - h_2, \quad p_3 = p_2 \quad (2)$$

The pressure  $p_2$  must lower than the saturation pressure of waste heat  $p_s(100\text{ }^\circ\text{C})$ , but we use equal temperature for ideal case, where the saturation pressure  $p_s(T)$  is the function of temperature,  $T$ . The inlet and outlet of low temperature of evaporator are denoted by 7 and 8, respectively. The same equations are obtained as equation (2).

$$q_{87} = h_8 - h_7, \quad p_8 = p_7 \quad (3)$$

$p_7$  is equals to  $p_s(10\text{ }^\circ\text{C})$  which is saturation pressure of air-conditioner. The point between outlet of the diffuser and the inlet of condenser is denoted by 6. At the condenser,  $q_{61}$  is released to the atmosphere.

$$q_{61} = h_6 - h_1, \quad p_6 = p_1 \quad (4)$$

Then  $p_6$  is set to  $p_s(40\text{ }^\circ\text{C})$  from the temperature of atmosphere.

The process in the expansion valve is isenthalpic as usual assumption.

$$h_7 = h_1 \quad (5)$$

The ejector is supposed to be divided into four parts. They are the driving nozzle, the suction nozzle, the mixing section and the diffuser. The process in nozzles, mixing and diffuser is assumed to be ideal and then isentropic. We denote outlet of the driving and the suction nozzle and mixing section as 4, 9 and 5, respectively.

$$\frac{u_4^2}{2} = (h_3 - h_4), s_3 = s_4 \quad (6)$$

$$\frac{u_9^2}{2} = (h_9 - h_8), s_9 = s_8 \quad (7)$$

$$h_6 = h_5 + \frac{u_5^2}{2}, s_6 = s_5 \quad (8)$$

The kinetic energies at the inlets and outlet of ejector are neglected, because of the large inlet and outlet flow areas.

In the mixing section, mass, momentum and energy conservations hold, and following equations are obtained.

$$\rho_4 u_4 A_4 + \rho_9 u_9 A_9 = \rho_5 u_5 A_5 \quad (9)$$

$$\rho_4 u_4^2 A_4 + \rho_9 u_9^2 A_9 + A_4 p_4 + A_9 p_9 = \rho_5 u_5^2 A_5 + A_5 p_5 \quad (10)$$

$$\frac{u_4^2}{2} + h_4 + \frac{u_9^2}{2} + h_9 = \frac{u_5^2}{2} + h_5 \quad (11)$$

As the outlets of two nozzles are connected to mixing section, the relations below must be hold.

$$p_9 = p_4, \quad A_4 + A_9 = A_5 \quad (12)$$

And the mass flow rate of driving and suction flow is defined by  $G_d$  and  $G_s$ .

$$G_d = \rho_4 u_4 A_4, G_s = \rho_9 u_9 A_9, \quad \text{where } g = G_s/G_d \quad (13)$$

All thermal parameters in this system are determined by the 2 variable  $g$  and  $p_4$ . For example, the area  $A_4$  is determined by  $\rho_4$  from the continuity equations and then finally by  $p_4$ .

In this system, the outlet pressure of the ejector must be saturation pressure of atmosphere. The ratio of mass flow rate is determined by this restriction.

And the pressure  $p_4$  is decided as the COP of this system obtains the maximum value.

The calculated relationship between the qualities at inlet of the ejector and the COP of two phase flow ejector as water refrigerant is shown in Figure 3. As mentioned above, the temperature of waste heat, atmosphere and air-conditioner is set to 100, 40 and 10 °C Figure 3 indicates that the COP gets higher with the decreasing quality and maximum value at considerably low quality. The best inlet quality for this condition is about 0.3. This means that the two-phase flow ejector system of the air conditioner using the waste heat is better solution compared with steam ejector.

## EXPERIMENTAL EQUIPMENT

The experimental apparatus shown in Figure 4 consists of high pressure tank, heater, mixing section, two-phase flow nozzle, condenser and flow meter. The tank is covered by thick thermal insulator. The connecting tubes between apparatuses are covered by neoprene foam rubbers to prevent the thermal loss. The temperature inside the tank is regulated to be constant by the control unit. When the temperature inside the tank became steady at 150 °C, saturated steam flows out from the upper side of the tank and saturated liquid from the lower side. Controlling the each flow by the valve, and coordinating the mass flux of the each phase, we obtain the arbitrary quality of the two-phase flow. The mass flow of saturated steam is measured by capillary flow meter and the total mass flow at outlet after the condenser is monitored by Coriolis flow meter in liquid state. Then the flow rate of saturated liquid can be calculated by subtracting the steam flow from the total flow. As a result, the quality at the nozzle inlet is obtained.

As saturated steam has specific volume about 1000 times bigger than saturated liquid, the diameter of the liquid inlet at the mixing section is designed to be much smaller than that of the steam inlet in order to make the velocity of both flows same order. Moreover, from the standpoint of the inter phase heat transfer at the surface of the droplet, it is needed to make the diameter of droplet smaller to obtain larger heat transfer area per unit mass of liquid phase.

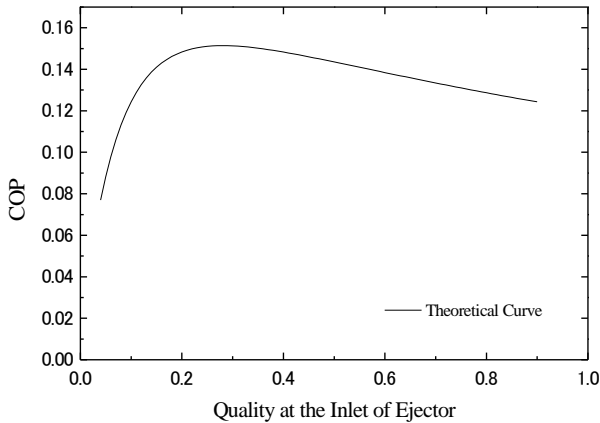


Figure 3 COP of the ejector refrigeration system

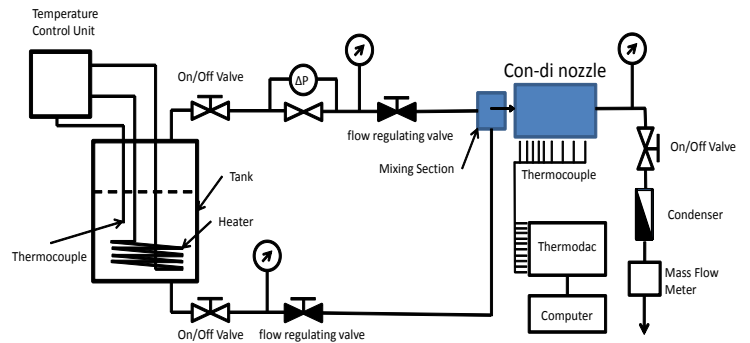
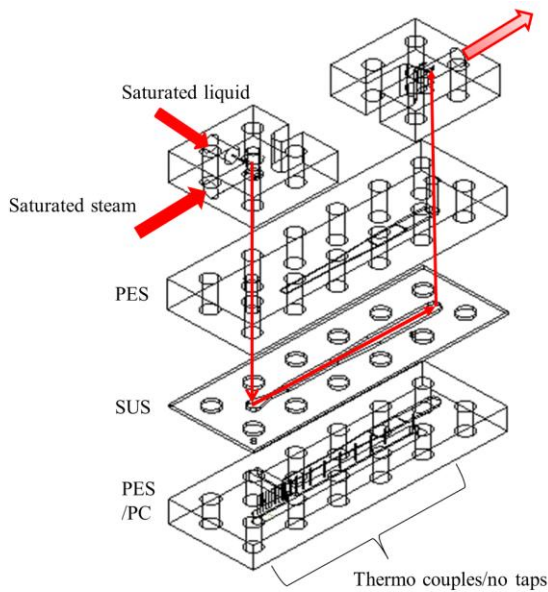


Figure 4 Experimental Apparatus

The testing convergent-divergent nozzle is designed for the flow rate of 3.0 g/s and the quality of 0.2 at the inlet of nozzle by considering the capacity of high pressure tank. For this case, the pressure inlet condition is 0.27 MPa, and temperature 130 °C. Figure 5 shows the geometries of nozzle. It is made of Poly Ether Sulfone, which has low thermal conductivity. The geometries at outlet of the nozzle were designed as to apply to the two-phase flow ejector in the future.

There are 5 pressure taps at convergent section and 4 at divergent section on the side wall of the nozzle. K type thermocouples are inserted into those pressure taps and pressure distributions are obtained to evaluate the performance of the nozzle.

Furthermore, to carry out the visualization experiment of the flow in the nozzle, the side wall of another set is also made of Polycarbonate with no taps.



**Figure 5** Structure of two-phase flow nozzle

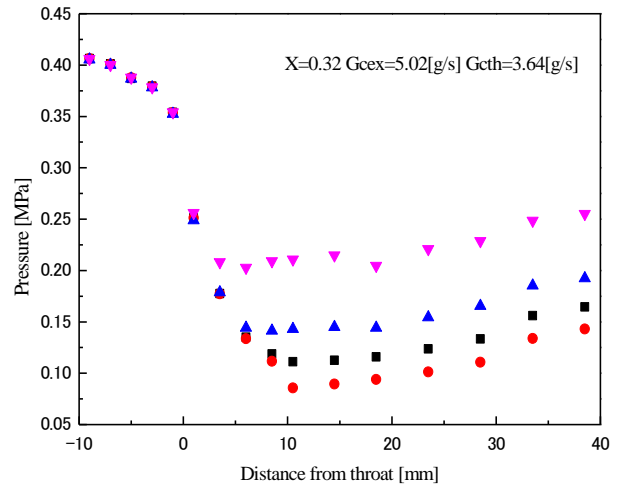
**EXPERIMENTAL RESULT**

The experiment is carried out by changing the quality of the nozzle inlet. The flow rate of steam and hot liquid from the high pressure tank and the static pressure distributions along the nozzle are measured for each experiment. The range of the inlet quality is 0.3-1.0. At quality of 0.3, the highest COP for the two-phase ejector is obtained by the theory in previous section. The pressure at the outlet of the nozzle is also changed to examine how high the pressure recovers behind the nozzle. The outlet pressure ranges from 0.09 to 0.22 MPa.

**Dependence of the nozzle outlet pressure**

One of the most typical pressure distributions along the convergent-divergent nozzle are shown in Figure 6.  $z=0$  mm

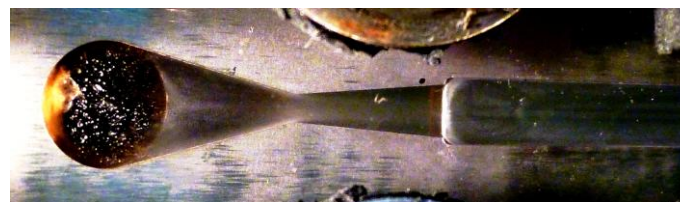
correspond to the nozzle throat and  $z = 10$  mm to nozzle outlet. When the outlet pressure became higher, the flat pressure distributions appear at the divergent section of the nozzle. But pressures at the convergent section are not changed by the outlet pressure. The flat pressure distribution is thought to be two-phase flow shock waves. Gas dynamics shock waves always have the steep increase in pressure, while two-phase flow shock waves has flat distribution because of large inertia of liquid phase [5].



**Figure 6** Pressure Distribution

**Appearance of the shock wave**

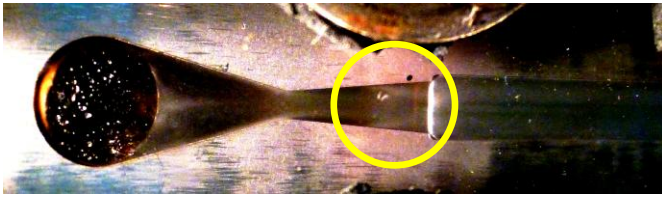
To make sure the appearance of two-phase flow shock waves in the flow channel, the visualization experiment has done. The picture is taken by the digital camera while the outlet pressures of the nozzle are changed by the outlet valve. The typical pictures of this experiment are shown in Figure 7 to 9. The inlet condition for the quality is 0.83, and outlet pressures are 0.10, 0.15 and 0.25 MPa for Figure 7, 8, 9 respectively.



**Figure 7** Visualization Picture with 0.10 MPa at outlet



**Figure 8** Visualization Picture with 0.15 MPa at outlet



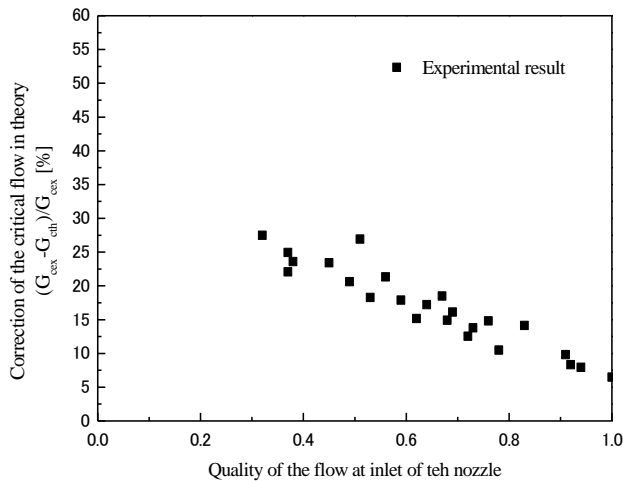
**Figure 9** Visualization Picture with 0.25 MPa at outlet

There is no apparent change of the contrast by two-phase flow along the channel in Figure 7. But, the slight contrast change normal to flow direction by two-phase flow shock wave appears at the downstream of the nozzle in Figure 8. When the outlet pressure becomes higher in Figure 9, the contrast change by the two-phase flow wave intrudes into the divergent section of the nozzle. The positions of the contrast change in those figures coincide with the start point of flat pressure distributions in Figure 6. It is evident from this experiment that the two-phase flow shock waves have flat pressure distributions.

### Evaluation of the Critical Mass flow rate

As shown in Figure 6, pressure profiles at the convergent section of the nozzle are not changed by the back pressure. This means that the mass flow rate is not varied with nozzle outlet pressure, because the velocity and the quality at the throat maintain at constant values. In fact, the measured flow rate by Coriolis flow meter is not changed. Therefore, the critical condition is established and we call this flow rate as critical mass flow rate.

The critical mass flow rate does not depend on back pressure but depend on inlet conditions. This is also true from our experiment. The measured critical mass flow rate increased



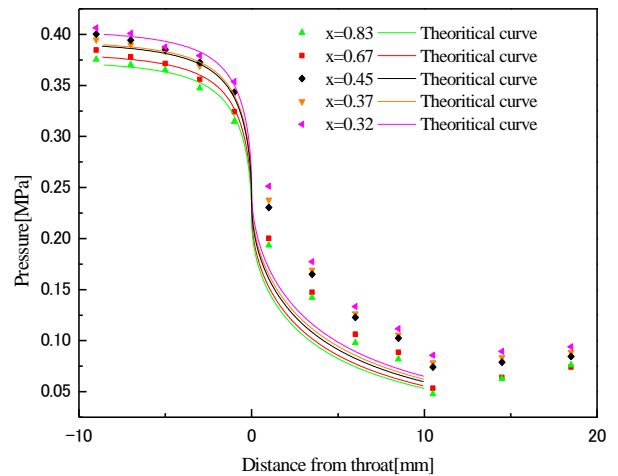
**Figure 10** Difference of critical mass flow between experiment and the theory

with the decreasing inlet quality. The dependence of quality is shown in Figure 10.  $G_{th}$  is the mass flow rate predicted by Isentropic Homogenous Equilibrium theory. This model assumes the temperature and the velocity of each phase are equal to each other and the isentropic velocity of two-phase flow at the nozzle throat corresponds with the sound speed of two-phase flow.  $G_{ex}$  is the measured mass flow rate by our experiment. The differences between  $G_{ex}$  and  $G_{th}$  are also increased by the inlet quality. Figure 10 indicates that the non-equilibrium phenomena between phases are increased at small inlet quality. The results of our experiment shown in Figure 6 are well agreed with the study of Isbin et al., Starkman et al. and Akagawa [6]. It means the flow of this experiment could be considered as the homogeneous flow.

### The pressure distribution for different inlet quality

Figure 11 shows the static pressure distribution in the nozzle when the qualities at inlet were arbitrary changed with a fixed the pressure at outlet. The solid lines in Figure 11 are the predicted pressure profile by Isentropic Homogenous Equilibrium model. The measured pressures are higher than that predicted ones. The departures of experimental data from theory are increased by the decreasing quality. This tendency is the same as that of the critical flow rate shown in Figure 10. It is thought that the non-equilibrium phenomena between phases are occurred for low quality.

The pressure recovery is observed at the downstream of nozzle. This comes from the compression work of two-phase flow ejector. The kinetic energy of two-phase flow is converted into the pressure energy behind the nozzle where correspond to the mixing section and the diffuser. If the vapours from the evaporator were sucked at the nozzle outlet, they would compressed by this pressure increase by the two-phase flow ejector.



**Figure 11** Pressure Distribution

### The energy conversion efficiency of the nozzle

The speeds of two-phase flow flushed out from the nozzle are estimated from the pressure profiles shown in Figure 11.

The reactions on the nozzle are calculated from the pressure distribution and this force is put to equal to momentum increase of two-phase flow. The energy conversion efficiencies of the nozzle are plotted against the inlet quality in Figure 12. The energy conversion efficiency is gradually decreased with the inlet quality. As mentioned above, the non-equilibrium phenomena are increased for small quality.

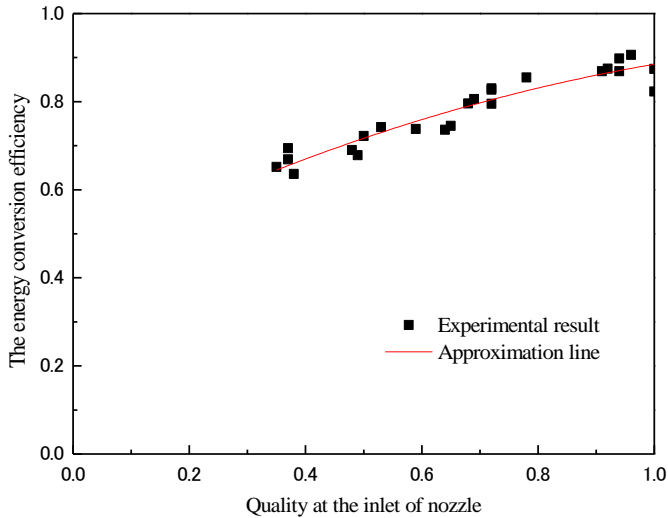


Figure 12 Energy conversion efficiency

### Evaluation of COP of the ejector cycle with the used nozzle

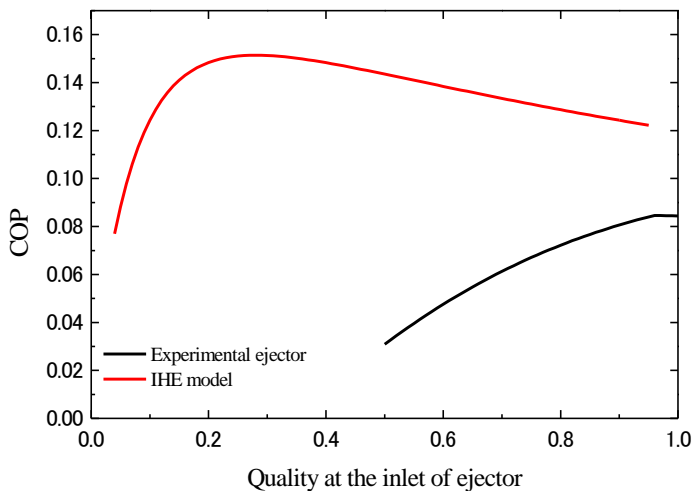


Figure 13 COP of the ejector cycle with the used nozzle

At this stage, it is worthwhile to evaluate the performance of the ejector air conditioning system with the nozzle used in the experiment. Theoretical COP resented in previous section is calculated by using the energy conversion efficiency of the nozzle shown in Figure 11. The resultant of COP is plotted by solid line in Figure 13. The dotted line that is the same as in

Figure 3 is also plotted to compare. There is large difference between two lines. This means that the energy conversion efficiency of the nozzle has large impact on the COP of the ejector system. The considerably small COP at low quality is caused by the low efficiency of the nozzle.

This result shows that the two-phase flow nozzle with much higher energy conversion efficiency is needed to develop the high COP ejector cycle using water refrigerant.

### CONCLUSION

The nozzle performance of two-phase flow ejector air conditioning system is experimentally examined in this study. And the COP of the ejector refrigeration system is theoretically estimated. Following results are obtained.

- (1) It is theoretically elucidated that the efficiency of ejector refrigeration system using waste heat is improved by using two-phase flow nozzle.
- (2) The present visualization experiment shows that the two-phase flow shock wave appears with the flat pressure distributions in the divergent part of the nozzle.
- (3) The non-equilibrium phenomena between phases are occurred for low quality. And this lowers the energy conversion efficiency.
- (4) Since the COP of the ejector refrigeration cycle using waste heat is largely affected by the nozzle efficiency, the nozzle used in this study has not ability to apply to the two-phase ejector refrigeration system.

The two-phase flow ejector is theoretically expected to attain the high COP compared with the steam ejector. However, it is elucidated from our experiments that the nozzle efficiency for two-phase flow rapidly decrease with the decreasing quality. The high efficiency two-phase flow nozzle must be developed for the refrigeration system using water as a refrigerant.

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