Forecasting Aggregate Retail Sales: The Case of South Africa*

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Abstract

Forecasting aggregate retail sales may improve portfolio investors' ability to predict movements in the stock prices of the retailing chains. Therefore, this paper uses 26 (23 single and 3 combination) forecasting models to forecast South Africa's aggregate seasonal retail sales. We use data from 1970:01 - 2012:05, with 1987:01-2012:05 as the out-of-sample period. Unlike, the previous literature on retail sales forecasting, we not only look at a wider array of linear and nonlinear models, but also generate multi-steps-ahead forecasts using a real-time recursive estimation scheme over the out-of-sample period, to mimic better the practical scenario faced by agents making retailing decisions. In addition, we deviate from the uniform symmetric quadratic loss function typically used in forecast evaluation exercises, by considering loss functions that overweight forecast error in booms and recessions. Focusing on the single models alone, results show that their performances differ greatly across forecast horizons and for different weighting schemes, with no unique model performing the best across various scenarios. However, the combination forecasts models, especially the discounted mean-square forecast error method which weighs current information more than past, produced not only better forecasts, but were also largely unaffected by business cycles and time horizons. This result, along with the fact that individual nonlinear models performed better than linear models, led us to conclude that theoretical research on retail sales should look at developing dynamic stochastic general equilibrium models which not only incorporates learning behaviour, but also allows the behavioural parameters of the model to be state-dependent, to account for regime-switching behaviour across alternative states of the economy.

Key Words: seasonality, weighted loss, retail sales forecasting, combination forecasts, South Africa

JEL Classification: C32, C53, E32

1. Introduction

Forecasting, in general, is a difficult task, but is, perhaps, more challenging for emerging economies. This is primarily because of the fact that emerging economies are subject to various structural changes more often than developed economies (Aye et al., forthcoming). Against this backdrop, this paper is the first of its kind to examine the ability of different linear and nonlinear models in forecasting retail sales of an emerging economy, namely South Africa. The management of retail sales is of paramount importance to retail organisations and retail policy makers. Due to competition and globalization, sales forecasting plays a prominent role as part of the commercial enterprise (Xiao and Qi, 2008). Most retailers are constantly struggling to reduce their cost and increase profits. An accurate sales forecasting system is an efficient way to achieve

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these goals as reliable prediction of sales can improve the quality of business strategy. Forecasting of the future demand is central to the planning and operation of retail business at both macro and micro levels. At the organizational level, forecasts of sales are essential inputs to many decision activities in various functional areas such as marketing, sales, and production/purchasing, as well as finance and accounting (Mentzer and Bienstock, 1998; Zhang, 2009). Sales forecasts also provide basis for regional and national distribution and replenishment plans. For profitable retail operations, accurate demand forecasting is crucial in organizing and planning purchasing, production, transportation, and labour force, as well as after sales services (Zhang, 2009). Therefore, the ability of retailing managers to estimate the probable sales quantity in the next period, can lead to improved customers' satisfaction, reduced destruction of products, increased sales revenue and more effective and efficient production plan (Chen and Ou, 2011a, 2011b). Forecasting in the retail industry has basically been done either using the individual or aggregate retail sales. Industry forecasts are especially useful to big retailers who may have a greater market share (Alon et al., 2001). For the retailing industry, Peterson (1993) showed that large retails are more likely to use time-series methods and prepare industry forecasts, while small retails emphasize judgmental methods and company forecasts. Better forecasts of aggregate retail sales can improve the forecasts of individual retailers because changes in their sales levels are often systematic (Peterson, 1993). More accurate forecasts of aggregate retail sales may improve portfolio investors' ability to predict movements in the stock prices of retailing chains (Barksdale and Hilliard, 1975; Thall; 1992; Alon et al., 2001). However, poor forecasting would result in redundant or insufficient stock that will directly affect the revenue and competitive position (Agrawal and Schorling, 1996).

Given the critical role of retail sales and the importance of its forecasting, this study is set out to forecast South Africa's aggregate retail sales. The decision to use South Africa as a representative for emerging economies, emanates due to the readily available data, and that too for a prolonged period (1970-2012) and higher (monthly) frequency, on retail sales. The period under study is long enough to accommodate major events in not only South Africa, but the dominant economies around the world, which in turn, had an impact on the emerging markets. Specifically speaking, the retail industry in South Africa is classified under the tertiary sector and falls within the wholesale and retail sub-sector (also known as the trade sub-sector). In 2011, the tertiary sector contributed 69.1 percent to the country's economy. The wholesale and retail trade subsector contributed approximately 13.7 percent to the economy. The retail trade and repairs of goods made the largest contribution (45 percent) within the wholesale and retail trade sub-sector (IHS, Global Insight, 2012; Gauteng Province: Provincial Treasury Quarterly Bulletin, 2012). This indicates that the retail industry drives the performance of the trade sub-sector. The retail industry contributes about 5.7 percent of total GDP. The retail industry is among the top industries in the country in terms of the share of employed labour force. The industry's share of employment to the national total has been fluctuating around 7 percent. The highest contribution made by the retail industry to employment was in 2006 when it reached 7.9 percent. In 2010, 7.2 percent of employed people were in the retail industry. This placed the retail industry as the fifth largest employer in the country. At first place was, households at 10.5 percent, followed by other business activities at 10.1 percent. Third place was held by education, which accounted for 7.5 percent of total employment and in fourth place was public administration and defence which accounted for 7.2 percent (IHS, Global Insight, 2012; Gauteng Province: Provincial Treasury Quarterly Bulletin, 2012). The South Africa retail industry is one of largest retail industry in the Sub Saharan region that presents profitable investment opportunities for new players (RNCOS, 2011). The Global Retail Development Index (GRDI) annual publication ranks the top developing countries for retail expansion internationally where countries are ranked on a 100 point scale. A higher ranking translates to a greater urgency for retailers to enter the specific country. The GRDI scores are based on country and business risk,

market attractiveness, market saturation and time pressure variables. In 2011, South Africa was ranked 26th out of 30 developing countries with a score of 42.2, a deterioration from the 24th rank of 2010 (41.7). At the top of the rankings was Brazil with a GRDI score of 71.5. Focusing on the individual components of GRDI, South Africa scored 46.9 percent on market attractiveness, 89.3 on country and business risk, 15.2 on market saturation and 17.2 on time pressure. However, South Africa dropped out from the 2012 rankings because of market saturation of international retailers compared to other countries in the GRDI (Kearney, 2011, 2012). These statistics indicate the important role of the retail industry in South Africa, and hence, justifies the need to forecast such a variable. Furthermore, from a macroeconomic perspective, forecast of retail sales, with it being a good proxy for consumption (Garrett, et al., 2004; Case et al., 2005, 2012; Zhou, 2012), is likely to provide an early indication (being at a monthly frequency), as to where the general economy (Gross Domestic Product (GDP)) might be heading, given that consumption is the dominant component of GDP. Not surprisingly, like most, if not all, emerging and developing economies, consumption data in South Africa is only available at the quarterly frequency.

As is well known, retail sales data present strong seasonal variations and its forecasts remains an important problem for forecasters. How to best deal with seasonal time series and which seasonal model is the most appropriate for a given time series are still largely unsolved (Zhang and Cline, 2007). Some of the international studies on retail sales forecasting, as would be evident from the detailed literature review section below, attempt to select the optimal forecasting model by comparing forecasts from single artificial neural networks (ANNs) models with one or two traditional methods such as exponential smoothing, moving average (MA), autoregressive and integrated moving average (ARIMA), seasonal ARIMA (SARIMA) and generalized autoregressive conditional heteroskedastic (GARCH) models. While the forecasting ability of the traditional models is limited by their assumption of a linear behaviour, and thus, not always satisfactory (Zhang, 2003), the ANNs, which offer an alternative, by taking into account both endogenous and exogenous variables and allowing arbitrary non-linear approximation functions to be derived (learned) directly from the data are not without limitations and criticisms (Moreno et al., 2011). ANNs lack a theoretical foundation and a systematic procedure for the construction of the model, comparable to the classical approximations such as the Box-Jenkins methodology (Box and Jenkins, 1976). As a result, the construction phase of the model involves the experimental selection of a wide number of parameters by trial and error. According to Moreno et al. (2011), the most criticised aspect in the use of ANN focuses on the study of the effect and significance of the input variables of the model, due to the fact that the value of the parameters obtained by the network does not possess a practical interpretation, unlike classical statistical models. As a consequence, ANN models have been presented to the user as a 'black box' as it is not possible to analyze the role played by each of the input variables in the forecast carried out. However, attempts are being made to overcome these criticisms (Hansen, et al., 1999; Montaño and Palmer, 2003; Palmer et al., 2008).

From the foregoing, it is obvious that all commonly used single forecasting models (traditional or ANNs) have their own characteristics, strengths and weaknesses. Further, when employing a single model, only a certain point of the effective information can be used, showing that the range of information sources is insufficient (Wan et al., 2012). Single model will also be affected by the model's set conditions and other factors. These factors may deteriorate the accuracy of individual forecasting methods and increase the size of errors. Combining the different forecasts averages such errors (Makridakis, 1989). The origin of forecast combination dates back to the seminal work of Bates and Granger (1969). Empirical findings in general show that combining improves forecasting accuracy and reduces the variance of post-sample forecasting errors (Makridakis and Winkler, 1983) and this holds true in statistical forecasting, judgmental estimates

and when averaging statistical and subjective predictions (Clemen, 1989). In this regard, most studies on retail sales forecasting has attempted to improve forecasts from single models by combining forecasts from two or atmost three ANN models, primarily (see Chang and Wang 2006; Doganis et al., 2006; Aburto and Weber, 2007; Chen et al. 2009; Chen and Ou, 2011a, 2011b; Ni and Fan, 2011).

Against this background, the academic contribution of this paper to the literature on forecasting retail sales involves considering a wider range of forecasting models (twenty three) unlike previous studies. This set of models includes not only the commonly used ANNs and traditional linear models whose weaknesses have been enumerated above, but also other nonlinear methods with good statistical and theoretical foundations. In addition, we also use recent emerging Bayesian techniques, which permit prior distributions on the coefficients of the models by allowing the researcher to quantify the uncertainty about parameter values, given the observed data. Using a large number of models gives us a greater chance of selecting the 'best' among the set with very little or no bias. Further, the study also contributes by providing a forecast combination of these different model forecasts. The theory of forecast combination suggests that methods that weight better-performing forecasts more heavily will perform better than simple combination forecasts, and that further gains might be obtained by introducing time variation in the weights or by discounting observations in the distant past (Stock and Watson, 2004). This study uses not only the simple methods, but also more complex and efficient, forecast combination techniques not previously used in the retail sales forecasting literature. Also, the studies relating to forecasting retail sales, have used a fixed parameter estimation method, whereby, the parameters estimated over the in-sample is kept fixed for the out-ofsample as well, when generating forecasts. Given that, the retail sales series, in general, is highly volatile, the models should be estimated recursively over the out-of-sample horizon to produce forecasts, by identifying an in-sample first, where the series is likely to be more stable. We not only perform recursive estimations, but also, unlike in the literature, look beyond one-step-ahead forecasting horizons by producing multi-steps results for short, medium and longer forecasting horizons. Further, the performance of a particular model may be determined by the type of evaluation criteria employed. The use of improper criteria to evaluate forecasts may result in poor forecasting performance (van Dijk and Franses, 2003). While the use of the standard root mean square error (RMSE) and Harvey et al. (1997) modified Diebold Mariano (MDM) tests are widely accepted in the forecasting literature (including the literature on retail sales forecasting), the recent recession has demonstrated that a good forecast of a rather extreme event might be of special interest beyond that of minimizing an average squared error. Hence, we extend previous studies on retail sales forecasting by considering loss functions that overweight the forecast errors in either booms or recessions or both.

Given the above forecasting set up, the results obtained (discussed in details later) have important implications for the retail sales industry. Clearly, given the complexity involved in forecasting and that too a highly seasonal (and hence volatile) variable, the industry is likely to benefit from a wide array of linear and nonlinear models than just few models in their possession. This is primarily because, different models capture different aspects of the series (be it nonlinearity or seasonality) at different points of the forecasting horizon. In addition, forecasting during periods of boom and recession, also affect the success of the forecasting models, since predictability during periods of calm is vastly different from periods of turmoil. Finally, since forecast combination methods reduce forecast variability across horizons, as well as periods of booms or recessions, it is not only important for the retail sector to have a wide array of models, but also the knowledge of forecast combination methods. This is of pivotal importance to the retail industry, since combination approaches are likely to provide more accurate forecasts not only over short- to long-horizons, but also less variable forecasts across

extreme periods, thus allowing the sector to plan better in terms of, organizing and planning input decisions in the production process, transportation, inventory management, as well as, after sales services, which in turn, altogether is likely to lead to higher profitability.

The rest of the paper is organized as follows. The next section discusses the literature on retail sales forecasting. The data and econometric methodology is discussed in Section 3. The empirical results are reported in Section 4, while, Section 5 concludes.

2. Literature

In this section, we provide a review of empirical studies on retail (aggregate and individual) sales forecasting with a view to confirming the contribution of the current study as already highlighted in the introductory part. Alon (1997) found that the Winters' exponential smoothing model forecasts aggregate retail sales more accurately than the simple exponential and Holt's models and that it accurately forecasts individual product sales, company sales, income statement items, and aggregate retail sales. Alon et al. (2001) compared the performance of artificial neural networks (ANN) to the traditional time series models namely Winters' exponential smoothing, ARIMA models and multivariate regression, using monthly aggregate retail sales data for the U.S. Their results based on mean absolute percentage error (MAPE) suggested that the ANN methods produce the best results as they were able to "capture the dynamic nonlinear trend and seasonal patterns, as well as the interactions between them." Chu and Zhang (2003) compare the out-of-sample forecasting performance of linear (ARIMA with time series, regression with dummy variables, and regression with trigonometric variables) and nonlinear (neural networks) seasonal forecasting models for the U.S. monthly aggregate retail sales from January 1985 to December 1999. They found that the neural network estimated using deseasonalized data outperformed the rest of the models based on three performance measures (the root mean squared error (RMSE), the mean absolute error (MAE) and the (MAPE). They also found that although seasonal dummy variables can be useful for predicting retail sales, their performance may not be robust and that trigonometric models are not useful in aggregate retail sales forecasting. Frank et al. (2003) using U.S. annual data from 1997-2000 on women's apparel sales evaluated the forecasting performance of three different forecasting models namely single seasonal exponential smoothing, Winters' three parameter model, and ANNs. Their result indicates that ANN model outperform the other two models based on R² evaluation. Doganis et al. (2006) presented an evolutionary sales forecasting model which is a combination of two artificial intelligence technologies, namely the radial basis function and genetic algorithm (GA-RBF). The methodology is applied to sales data of fresh milk provided by a major manufacturing company of daily product in Greece and the findings from different formulations of the model was compared to linear (AR, ARMA, RLS, Holt-Winters) models. Their findings show that the adaptive formulation of the combined neural network model had the least MAPE, showing that models that allow correction of itself as new information becomes available are able to forecast sales more accurately. Chang and Wang (2006) integrated fuzzy logic and artificial neural network into the fuzzy back-propagation network (FBPN) for sales forecasting in Printed Circuit Board (PCB) industry in Taiwan. The results from FBPN are compared to those of Grey Forecasting (GF), Multiple Regression Analysis (MRA) and Back-propagation network (BPN). The experimental results indicate that the Fuzzy back-propagation approach outperforms the other three different forecasting models in Mean Absolute Percentage Error (MAPE) measures.

Aburto and Weber (2007) presented a hybrid intelligent system combining ARIMA model and MLP neural networks for demand forecasting and found that the MLP outperformed the ARIMA model while the hybrid model outperformed the individual models based on MAPE and normalized MSE. They also show that a replenishment system for a Chilean supermarket

based on improved forecast accuracy, leads simultaneously to fewer sales failure and lower inventory levels. Joseph et al. (2007) examine out-of-sample forecasts of aggregate sales using 3month treasury bills interest rate in NeuroSolutions environment referenced against forecasts of linear regression models. Two types of dynamic neural network models trained with the Levenberg-Marquardt back propagation algorithm under supervised learning were used. The neural network models outperform the linear regression models. Au et al. (2008) illustrated evolutionary neuron network for sales forecasting and showed that when guided with the BIC and the pre-search approach, the non-fully connected neuron network can converge faster and more accurate in forecasting time series than the fully connected neuron network and traditional SARIMA model based on MSE criterion. Sun et al. (2008) also developed different sales forecasting models in fashion retailing in Hong Kong. They applied ELM neural network model to investigate the relationship between sales amount and some significant factors which affect demand. The results demonstrate that the proposed methods outperform the back-propagation neural network model. Ali et al. (2009) explored forecasting accuracy versus data and model complexity trade-off in the grocery retailing sales forecasting problem, by considering a wide spectrum in data and technique complexity. The experiment results indicate that simple time series techniques perform very well for periods without promotions. However, for periods with promotions, regression trees with explicit features improve accuracy substantially.

Chen et al. (2009) developed the GMFLN forecasting model by integrating GRA and MFLN neural networks. The experimental results indicated that the proposed forecasting model outperforms the MA, ARIMA and GARCH forecasting models of the retail goods. Gil-Alana et al. (2010) examine whether retail sales forecasts can be better explained in terms of a model that incorporates both long run persistence and seasonal components in a fractional differencing framework than models that use integer degrees of differentiation. They find that retail sales forecasts are better explained in terms of a long memory model that incorporates both persistence and seasonal components. Chen and Ou (2011a, 2011b) developed a Grey relation analysis with extreme learning machine (GELM) model for forecasting future daily sales of fresh food retail industry in Taiwan. Using the MSE and MAD statistics, they show that the GELM model outperforms the standard statistical time series model, GARCH, as well as two other artificial neural network (GBPN, and GMFLN) models. Ni and Fan (2011) proposed a two-stage dynamic forecasting model, which is a combination of the ART model and error forecasting model based on neural network to improve the accuracy of fashion retail forecasting. However, their results are not compared to other forecasting models.

As can be seen from the above, most of the studies reviewed, emphasized the importance of different forms of neural network models (hence, nonlinearity in general) and compared the forecast with a few linear forecasting models. These studies evaluate the forecasts from different models using the standard loss function, which is essentially minimizing an average squared error, to show that ANN models, in general, tend to outperform standard linear models. However, in this study, we consider twenty (26) seasonal forecasting (23 single and 3 combined) models for aggregate retail sales, and we employ forecast evaluation techniques with different weighting schemes to see how each model performs in times of booms and recessions. Generally speaking, we extend the literature on retail sales studies by considering a wide array of linear and nonlinear models beyond neural network methods. This is important, given the criticisms on neural network models, with primary concern being the fact that it is not theoretically founded. Moreover, we do not only perform recursive estimations to capture the volatilities inherent in retail sales, but also, unlike in the literature, look beyond one-step-ahead forecasting horizons by producing multi-steps results for short, medium and longer forecasting horizons.

At this stage, it is important to emphasize further the role of a recursive forecasting scheme, conducting multi-steps-ahead forecasts, and forecast combination. The recursive approach mimics the real-time forecasting scenario of a retailer when making any decision of the production chain, in the sense that a retailer can only use available (in-sample) information on retail sales. In addition, forecasting retail sales is complicated by the problem that a retailer, in real time, must reach decisions regarding various aspects under uncertainty concerning the "optimal" forecasting model. The real-time forecasting approach resolves this problem by assuming that a retailer uses a search-and-updating technique to predict retail sales. The search part requires that a retailer, in every period of time when a decision must be reached, estimates a large number of forecasting models and then identifies an "optimal" model by means of some model selection criteria based on available data, after identifying in and out-of-sample periods. The updating part, in turn, requires that a retailer re-estimates the forecasting models whenever new information on retail sales becomes available. The real-time forecasting approach, thereby, does not only account for model uncertainty but also for the possibility that the optimal forecasting model may change over time. As discussed earlier, given a series as volatile as retail sales, changes in the optimal forecasting model are very likely due to structural breaks and regime shifts, and something that we see below across forecasting horizons and also periods of extremes in the discussion of our results. Previous studies, which relied on a once-off estimation of a model or a small set of models over an identified in-sample, is thus likely to be misspecified, since these studies not only reduce the possibility of choosing a better model, but also because they fail to account for parameter instability, and hence, model uncertainty. In light of this, it is highly possible, that the observation made in the literature that a specific model always performs better than a benchmark over the entire out-of-sample horizon is spurious, since economic conditions change, and it is impossible for a single forecasting model to capture all the dynamics over the out-of-sample horizon. As far as multi-steps-ahead forecasts are concerned, clearly retailing decisions are not only made over the short-run, but also medium to long-run. Hence, multi-steps-ahead forecasts, based on a recursive approach (unlike the constant parameter method), which accounts for parameter and model uncertainty, is more realistic in nature and is also likely to be more precise. Finally, forecast combination approaches allows us to reduce the problem of model uncertainty, since this provides us a statistical approach to combine the best features of the various models used, given model uncertainty over the out-of-sample horizon. However, it is also important to emphasize that forecast combination is not likely to perform well, if the array of models used is limited in number. In other words, unlike the literature, our approach of using large number of individual models, recursive estimation, multi-steps-ahead forecasts and forecast combination is a more realistic and an ideal way of conducting a forecasting exercise for a volatile series like retail sales.

3. Data and Methodology

We use monthly aggregate sales data for South Africa covering the period 1970:01 to 2012:05 making a total of 509 observations. The period covers a number of economic events thereby capturing both the boom and the recession periods in South Africa. The data is sourced from Statistics South Africa. The full data set is split into two. We use data from 1970M1-1986M6 (204 observations) for in-sample. Data from 1987:01-2012:05 (305 observations) is used for the out-of-sample period. The plot of the seasonally adjusted aggregate retail sales series is shown in Figure 1 while its growth rate is plotted in Figure 2. There is a noticeable seasonal variation in the data. Figure 1 shows that retail trade sales follow a particular pattern annually. Every December, retail sales figures spiked-upward and in January, a contraction occurred. This trend is explained by the tendencies of households to shop more during the December month since

most people are on holiday or have received bonuses. In the month of January, consumer spending reduces as people prepare to go back to work or school and also pay off short-term debts incurred in December. The overall trend is an increase in retail trade sales. Figure 2 also depicts strong volatility with the highest peak in January 1987 (8.6%); thus justifying our choice of 1987:01-2012:05 as the out-of-sample period.

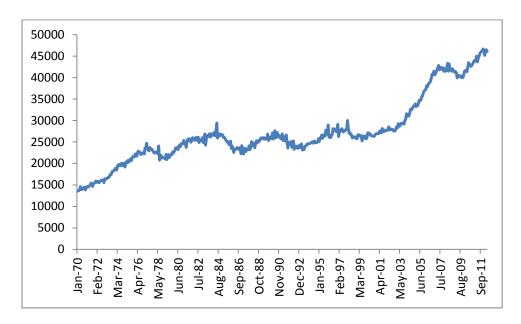


Figure 1: Aggregate seasonal retail sales series in million rand

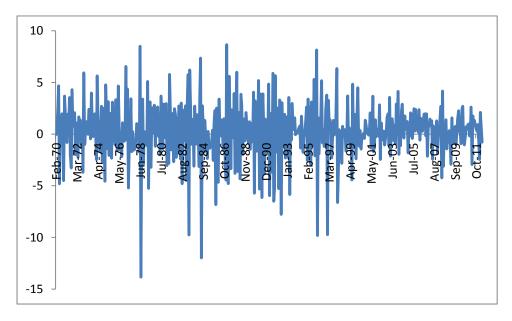


Figure 2: Growth rate of aggregate seasonal retail sales

3.1 Forecasting Models

A model is identified using the in-sample data and then the same model is recursively reestimated and 1 to 24 step-ahead forecasts are obtained recursively over the out-of-sample period. Only the parameters are re-estimated in the recursive forecasting, but identified model structure is kept constant. We have two classes of models. The acronyms and brief description of the models we used are presented in Table 1.¹ The first class consists of 17 models with seasonal dummy variables. This is equivalent to deterministic seasonal adjustment. For instance for the ARIMA model we estimate,

$$\phi(L)\Delta^d y_t = \mu + \sum_{s=1}^{11} \gamma_s d_{s,t} + \theta(L)\varepsilon_t \tag{1}$$

where y is the log of the aggregate retail sales and $d_{s,t}$ is a dummy variable taking value of one for month s. At each recursive estimation, step dummy are included in the regression and forecasts of seasonal component is easily obtained from $\mu + \sum_{s=1}^{11} \gamma_s d_{s,t}$. The models are presented

in panel A of Table 1. When joint estimation of the seasonal component and non-seasonal component is not feasible, for Genetic Algorithm (GA) method for instance, seasonal component is pre-estimated using linear regression and non-seasonal component is forecasted in

a second step; and final forecasts are recovered by adding $\mu + \sum_{s=1}^{11} \gamma_s d_{s,t}$. The second class consists

of 9 full seasonal models. The models are presented in panel B of Table 1. In all models, data is log of first differences, since there is a unit root. Level forecast are recovered from the forecasts of the growth rates. All model order is selected using BIC. In each case, forecasts were made at four horizons: 1, 4, 12 and 24 months.

Table 1: Model description and specification

| S/N | Code | Description and specification |
|-----|--------|--|
| | | A. Models with seasonal dummy variables. |
| 1. | RW | Random walk, equivalent to ARIMA(0,1,0) |
| 2. | ARIMA | Autoregressive integrated moving average, estimated model is ARIMA(2,1,0) |
| 3. | ARFIMA | Autoregressive fractionally integrated moving average, estimated model is ARFIMA(2,1+d,0) |
| 4. | BARIMA | Bayesian ARIMA model parameters are estimated to minimize the 24-step MSE once over the |
| | | out-of-sample period. We start with a long model with ARIMA $(p,1,q)$ and where $p,q \le 12$. |
| | | Estimates arising from minimizing 24-step MSE are used as informative priors in the |
| | | recursive estimation. |
| 5. | BCAR | Bias corrected AR model, the estimated model is AR(2) with first differencing. The method |
| | | we used is described in Stine and Shaman (1989). |
| 6. | MSAR | Markov Switching autoregressive model, estimated model is MS-AR(2) with 2 regimes and |
| | | first differencing. |
| 7. | SETAR | Self-exciting threshold autoregressive model, estimated model is SETAR (k,p,d) , with $k=2$ (# |
| | | of regimes), $p=2$ (autoregression order) and $d=1$ (delay order). |
| 8. | LSTAR | Logistic smooth transition autoregressive model, estimated model is LSTAR (k,p,d) , with $k=2$ |
| | | (# of regimes), $p=2$ (autoregression order) and $d=1$ (delay order). |
| 9. | ARANN | Autoregressive artificial neural network. Autoregressive order is 2. We use 3 hidden layers. |
| | | The ANN is multi-layer perceptron (MLP) feed-forward network with hyperbolic-tangent |
| | | (tansig) activation function for the hidden layers and a linear activation function for the |
| | | output layer. |

¹ However, given the pivotal role of forecast combination in this paper, detailed descriptions of the forecast combination models are given in the next sub-section.

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| 10. | NPAR | Fully non-parametric (auto) regression, it is an autoregressive model with lag order equal to 2. |
|-----|---------|--|
| 11. | SPAR | Semi-parametric (auto)regression, it is an autoregressive model with lag order equal to 2. |
| 12. | GARCH | Generalized Autorregressive Conditional Heteroskedasticity model. We use ARIMA(2,1,0)-EGARCH(1,1) model. |
| 13. | GA | This is the Genetic Algorithm based forecasting. Two lags are used as inputs (see Szpiro, 1997 for the approach we used). Function appriximation is terminated at a maximum step of 3000. |
| 14. | FUZZY | Evolutionary Fuzzy Modeling. The approach is taken from Peña -Reyes (2004). Fuzzy fitting uses 200 population and 60000 generations. |
| 15. | DISC | Discounted forecast combination. The discount factor we used is 0.50. |
| 16. | PC | Principal components forecast combination. We used maximum of 4 principal components based on Bai and Ng (2002) method. |
| 17. | MEAN | Simple mean forecast combination. |
| | | B. Full seasonal models |
| 1. | SRW | Seasonal random walk, ARIMA(0,1,0)(0,1,0), so both seasonal and regular random walk components exist. |
| 2. | HW | Holt-Winters methods, tree smoothing parameters are estimated. |
| 3. | TBATS | State space exponential smoothing model with trigonometric seasonal component (See Hyndman et al. 2002). |
| 4. | SARANN | Seasonal autoregressive ANN. The ANN is multi-layer perceptron (MLP) feed-forward network with hyperbolic-tangent (tansig) activation function for the hidden layers and a linear activation function for the output layer. We use the approach in Taskaya-Temizel and Casey (2005) to set the number of delays (AR order). A total of 9 hidden layers are used. |
| 5. | SUTSEA | Seemingly unrelated structural time series model with local trend and additive seasonal component (see Harvey, 2006). |
| 6. | SUTSET | Seemingly unrealted structural time series model with local trend and trigonometric seasonal component (see Harvey, 2006). |
| 7. | SARIMA | Seasonal ARIMA, the estimated model is SARIMA(2,1,0)(2,0,0). |
| 8. | BSARIMA | Bayesian SARIMA model parameters are estimated to minimize the 24-step MSE once over the out-of-sample period. We start with a long model with ARIMA $(p,1,q)(P,0,Q)$ and where $p,q,P,Q \le 4$. Estimates arising from minimizing 24-step MSE are used as informative priors in the recursive estimation. |
| 9. | SARFIMA | Autoregressive fractionally integrated moving average, estimated model is ARFIMA $(2,1+d,0)(2,0,0)$. |

Each model we use in our study has one or more features. Full seasonal models attempt to capture seasonal variation using parameterization suitable for stochastic and complicated seasonality. Models with seasonal dummies assume that seasonal variation is deterministic. RW and SRW models serve as benchmark models. We can split these models into two basic groups. The first group is a class of linear time series model and these include ARIMA, ARFIMA, BARIMA, BCAR, HW, TBATS, SUTSEA, SUTSET, SARIMA, BSARIMA, and SARFIMA. These models are the most commonly used class of models for modeling linear short- and longmemory time series. ARFIMA and SARFIMA models assume a fractionally integrated time series and captures long-memory. BARIMA and BSARIMA models are based on Bayesian estimation of the parameters and also the order of the models are chosen based on informative priors to improve the out-of-sample forecasting errors. BCAR corrects bias in autoregressive parameter estimations, which may improve forecasting performance in cases where parameter estimates may be biased due to small sample and deviations from assumptions. The second group of models can be classified as nonlinear models broadly. The nonlinear models include MSAR, SETAR, LSTAR, ARANN, NPAR, SPAR, GARCH, GA, FUZZY, and SARANN. These models capture various types of nonlinearities and may have better forecasting performance, if the underlying time series has nonlinear dynamics. Regime switching models MSAR, SETAR, and LSTAR are well known in the literature and best fits to cases where a time series follows asymmetric dynamics like recessions and booms. MSAR models the regime switching based on latent regime variable that follows a first order Markov process, and therefore has unexplained switching. SETAR and LSTAR models have explained switching and therefore regime switching follows a known structure. SETAR models assumes a swift switching that is completed in one period while the LSTAR model assumes a smooth switching in and out of a regime that spreads to more than one period. The GARCH model assumes autoregressive conditionally heteroscedastic error term and suitable time varying variance. ARANN, NPAR, SPAR, GA, FUZZY, and SARNN models do not assume any known parametric functional form and successfully approximates quite general nonlinear functions. Moreover, we also form a combination of the different models using three combination methods not previously used in any of the retail sales forecasting papers to ensure that the combination model that best fits the data is selected

3.1.1 Forecast Combination Methods

Three forecast combination methods are considered: the simple forecasts (MEAN), the discounted MSFE (DISC) and the principal component (PC) methods. Our selection of these three is based on the good performance as reported in previous studies. The forecast combination methods differ in the way they use historical information to compute the combination forecast and in the extent to which the weight given an individual forecast is allowed to change over time. Some of the combining methods require a holdout period to calculate the weights used to combine the individual model forecasts, and we use the first P_0 observations from the out-of-sample period as the initial holdout period following Rapach and Strauss (2010). The combination forecasts of y_{t+h}^h made at time t, $\hat{y}^h_{CB,t+h|t}$, typically are a linear combination of the individual model forecasts

$$\hat{y}_{CB,t+h|t}^{h} = \sum_{i=1}^{n} w_{i,t} \hat{y}_{i,t+h|t}^{h}$$
(2)

where $\sum_{i=1}^n w_{i,t} = 1$. When the weights, $\{w_{i,t}\}_{i=1}^n$, are estimated, we use the individual out-of-sample forecasts and y_{t+h}^h observations available from the start of the holdout out-of-sample period to time t. For each of the combining methods, we compute combination forecasts over the post-holdout out-of-sample period. This leaves us with a total of $P_h = P - (h-1) - P_0$ combination forecasts, $\{\hat{y}_{CB,t+h|t}^h\}_{t=R+P_0}^{t-h}$, available for evaluation².

Simple Combination Forecasts

The simple combination forecasts compute the combination forecast without regard to the historical performance of the individual forecasts. Stock and Watson (1999, 2003, 2004) find that simple combining methods work well in forecasting inflation and output growth using a large number of potential predictors. Stock and Watson (2004) noted that there seems to be little difference between the mean and the trimmed mean forecast performance while the median typically has somewhat higher relative MSFE than either the mean or trimmed mean. Therefore, we consider the mean combination forecast (MEAN). The mean combination forecast simply involves setting $w_{i,t} = 1/n$ (i = 1,...,n) in (2). Thus, the simple combining methods do not require a holdout out-of-sample period.

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² We use 1987:01-1996:12 as the initial hold-out out-of sample period.

Discounted MSFE combination forecasts

Following Stock and Watson (2004) and Rapach and Strauss (2010), we consider a combining method where the weights in (2) are a function of the recent historical forecasting performance of the individual models. The discounted MSFE combination (DISC) *h*-step-ahead forecast method has the form (2) where the weights are:

$$w_{i,t} = m_{i,t}^{-1} / \sum_{j=1}^{n} m_{j,t}^{-1}$$
(3)

where
$$m_{i,t} = \sum_{s=R}^{t-h} \gamma^{t-h-s} (y_{s+h}^h - \hat{y}_{i,s+h|s}^h)^2$$
 (4)

and γ is a discount factor. When $\gamma = 1$, there is no discounting, and (3) produces the optimal combination forecast derived by Bates and Granger (1969) for the case where the individual forecasts are uncorrelated. When $\gamma < 1$, greater importance is attached to the recent forecasting accuracy of the individual models. We consider γ value of 0.5. The results are the same with a γ value of 0.70. Although, this seems to be a low discount factor, however, it may due to the seasonal time series we are forecasting and recent past is the most important, weights given to past forecast required to decline very fast in our case.

Principal component forecast combination

Principal component forecast combination (PC) requires (i) recursively computing the first few principal components of estimated common factors of the panel of forecasts, (ii) estimating a regression of $y_{s+h|s}^h$ onto these principal components, and (iii) forming the forecast based on this regression (Stock and Watson, 2004). Reduction of many forecasts to a few principal components provides a convenient method for allowing some estimation of factor weights, yet reduces the number of weights that must be estimated. This method has been used by Chan et al. (1999), Stock and Watson (2004) and Rapach and Strauss (2010) among others. One motivation for use of PC is that, recent work on large forecasting models suggests that large macroeconomic data sets are well described by a few common dynamic factors that are useful for forecasting, and that the common factors can be estimated by principal components (Forni, et al., 2000, 2003; Stock and Watson, 1999, 2002, 2004).

The principal component forecasts are constructed as follows. Let $\hat{F}^h_{1,s+h|s},...,\hat{F}^h_{m,s+h|s}$ for s=R,...,t denote the first m principal components of the uncentered second-moment matrix of the individual model forecasts, $\hat{y}^h_{i,s+h|s}$ (i=1,...,n;s=R,...,t). To form a combination forecast of y^h_{t+h} at time t based on the fitted principal components, we estimate the following regression model $y^h_{s+h} = \theta_1 \hat{F}^h_{1,s+h|s} + ... + \theta_m \hat{F}^h_{m,s+h|s} + v^h_{s+h}$ (5)

where s = R,...,t-h. The combination forecast is given by $\theta_1 \hat{F}_{1,s+h|s}^h + ... + \theta_m \hat{F}_{m,s+h|s}^h$, where $\hat{\theta}_1,...,\hat{\theta}_m$ are OLS estimates of $\theta_1,...,\theta_m$, respectively, in (5). We use the IC_{p3} information criterion developed by Bai and Ng (2002) to select m (considering a maximum value of 4) when calculating combination forecasts using the PC method. Bai and Ng (2002) show that familiar information criteria such as the Akaike information criterion (AIC) and Schwarz Bayesian

information criterion (SIC) do not always consistently estimate the true number of factors, and they develop alternative criteria that consistently estimate the true number of factors under more general conditions. In extensive Monte Carlo simulations and using a large sample size as in our study, Bai and Ng (2002) find that the IC_{p3} criterion performs well in selecting the correct number of factors.

3.2 Forecast Evaluation using Weighted Loss Functions

The standard period-t loss function used in most of the forecast evaluation literature is the squared forecast error

$$L_{i,t} = e_{i,t}^2 \tag{6}$$

where $e_{i,t} = y_t - y_{i,t}^f$ is the forecast error of model i, y_t is the realization of the target variable, y_t aggregate retail sales in our case, $y_{i,t}^f$ is the value predicted by model i. Comparing the average loss difference of two competing models 1 and 2 implies computing their mean squared forecast errors

$$MSFE_i = \frac{1}{P} \sum_{t=T+1}^{T+P} e_{i,t}^2, \qquad i = 1, 2,$$
 (7)

over the forecast period T+1 to T+P and choosing the model with the smaller MSFE.

However, according to Carstensen et al. (2010), there are many occasions in which different loss functions can make more sense for the applied forecaster but also for the user of a forecast such as a politician or the CEO of a company. For instance, the case of the recent recession which demonstrated that a good forecast of a rather extreme event might be of special interest beyond that of minimizing an average squared error. Consequently, banks could have taken earlier measures to shelter against the turmoil, governments could have started stimulus packages in time, and firms might have circumvented their strong increase in inventories. This is in line with van Dijk and Franses (2003) argument that a weighted squared forecast error can be used to place more weight on unusual events when evaluating forecast models. Following van Dijk and Franses (2003) and Carstensen et al. (2010), we use a weighted squared forecast error. Hence, the loss function in (6) can be re-specified as:

$$L_{i,t}^{w} = w_t e_{i,t}^2 (8)$$

where the weight w_t is specified as

- 1. $w_{left,t} = 1 \hat{F}(y_t)$, where $F(\cdot)$ is the cumulative distribution function of y_t , to overweight the left tail of the distribution. This gives rise to a "recession" loss function.
- 2. $w_{right,t} = \hat{F}(y_t)$, to overweight the right tail of the distribution. This gives rise to a "boom" loss function.
- 3. $w_{tail,t} = 1 \hat{F}(y_t) / \max(\hat{F}(y_t))$, where $F(\cdot)$ is the density function of y_t , which allows to focus on both tails of the distribution given rise to both recession and boom loss function.

When equal weights, $w_t = 1$ are imposed, the weighted loss function (8) collapses to the standard loss function (6) giving rise to the conventional "uniform" loss function.

To evaluate a forecast model i over a forecast period T+1 to T+P using the weighted loss function simply requires calculating the weighted mean squared forecast error

$$MSFE_{i} = \frac{1}{P} \sum_{t=T+1}^{T+P} w_{t} e_{i,t}^{2},$$
 (9)

In order to compare, say, model i to a benchmark model 0, one calculates the weighted loss difference

$$d_{i,t} = L_{0,t}^{w} - L_{i,t}^{w} = w_{t}e_{0,t}^{2} - w_{t}e_{i,t}^{2}$$

$$\tag{10}$$

and averages over the forecast period

$$\overline{d}_{i} = \frac{1}{P} \sum_{t=T+1}^{T+P} d_{i,t} = \frac{1}{P} \sum_{t=T+1}^{T+P} w_{t} e_{0,t}^{2} - \frac{1}{P} \sum_{t=T+1}^{T+P} w_{t} e_{i,t}^{2}$$
(11)

We use this weighted loss and analyse the forecast accuracy of different models with respect to the different weighting schemes introduced above. There is large number of tests proposed in the literature to analyse whether empirical loss differences between two or more competing models are statistically significant. The most influential and most widely used is the pairwise test introduced by Diebold and Mariano (1995). In this study, we employ the modified Diebold-Mariano (MDM) test proposed by Harvey et al. (1997), which corrects for small sample bias. MDM test is a pairwise test designed to compare two models at a time, say, model *i* with benchmark model 0. The null hypothesis of the MDM test is that of equal forecast performance,

$$E[d_{i,t}] = E[L_{0,t}^{w} - L_{i,t}^{w}] = 0 (12)$$

Following Harvey et al. (1997), we use the modified Diebold-Mariano test statistic

$$MDM = \sqrt{\frac{P+1-2h+h(h-1)/P}{P}} \frac{\overline{d}_i}{\sqrt{\hat{V}(\overline{d}_i)}}$$
(13)

where h is the forecast horizon and $\hat{V}(\bar{d}_i)$ is the estimated variance of series $d_{i,t}$. The MDM test statistic is compared with a critical value from the t-distribution with P-1 degrees of freedom.

The forecasting performance of a candidate forecast is also evaluated by comparing its out-of-sample RMSE to the benchmark forecast following Chan et al (1999), Stock and Watson (2004) and Rapach and Strauss (2010). The benchmark forecast used here is from the random walk (RW) model. If the candidate forecast has a relative RMSE less than one, then it outperformed the RW benchmark over the forecast period.

RMSE is simply the square root of the mean square error (MSE), which is the most frequently used forecast error measure by the academicians and practitioners (Carbone and Armstrong, 1982). RMSE is equivalent to root mean square percentage error when the forecasted series is in

logs, which is the case in this study. RMSE is scale dependent and would not be recommended to use to compare methods for a group of series. Chatfield (1992) point out that it is perfectly reasonable to evaluate forecasts from different models by the RMSE for single series. We compare forecasting performance of a group of models for a single series and using RMSE does not have any disadvantage over the other forecast error measures. Further, Zellner (1986) points out that the use of the mean of the predictive probability density function for a series, is optimal relative to a squared error loss function and the MSE or RMSE, and hence, the RMSE is an appropriate measure to evaluate performance of forecasts, when the mean of the predictive probability density function is used, as it happens to be in our case when estimating the nonlinear and Bayesian models.

4. Empirical Results

In this section, we report the results from all the 26 aggregate retails forecasting models. We first present the uniform, boom, recession, and boom and recession weighted RMSE and their corresponding ranks. These results are presented in Tables 2, 3, 4 and 5 for horizons of 1, 4, 12 and 24, respectively. The rankings in most -but by far not in all- cases differ greatly between boom and recession periods and even at different forecast horizons. In general, models with seasonal dummy variables seem to have smaller RMSE than full seasonal models. Also as a general result, the average forecast errors based on the uniform weighting scheme are strongly driven by the forecast errors made during booms which are substantially higher than during recessions. This holds true for all models and forecast horizons. It implies that improvements in terms of model building should aim at better predictions of boom periods.

Table 2: Root Mean Squared Forecast Errors (*h*=1)

| | Unif | orm | Boo | m | Reces | sion | Ta | il |
|--------|--------|------|--------|------|--------|------|--------|------|
| Model | RMSE | Rank | RMSE | Rank | RMSE | Rank | RMSE | Rank |
| RW | 0.0252 | 15 | 0.0193 | 15 | 0.0074 | 13 | 0.0112 | 14 |
| DISC | 0.0139 | 1 | 0.0109 | 1 | 0.0039 | 2 | 0.0057 | 2 |
| PC | 0.0180 | 3 | 0.0136 | 3 | 0.0058 | 3 | 0.0086 | 3 |
| MEAN | 0.0209 | 4 | 0.0154 | 8 | 0.0067 | 4 | 0.0101 | 5 |
| ARIMA | 0.0217 | 12 | 0.0166 | 14 | 0.0068 | 5 | 0.0100 | 4 |
| ARFIMA | 0.0213 | 9 | 0.0159 | 12 | 0.0070 | 7 | 0.0103 | 6 |
| BARIMA | 0.0264 | 16 | 0.0205 | 16 | 0.0074 | 14 | 0.0114 | 16 |
| BCAR | 0.0213 | 8 | 0.0157 | 9 | 0.0072 | 10 | 0.0106 | 9 |
| MSAR | 0.0150 | 2 | 0.0121 | 2 | 0.0038 | 1 | 0.0057 | 1 |
| SETAR | 0.0212 | 7 | 0.0153 | 7 | 0.0070 | 8 | 0.0105 | 8 |
| LSTAR | 0.0209 | 5 | 0.0153 | 6 | 0.0069 | 6 | 0.0103 | 7 |
| ARANN | 0.0215 | 11 | 0.0158 | 11 | 0.0072 | 11 | 0.0107 | 10 |
| NPAR | 0.0210 | 6 | 0.0148 | 4 | 0.0073 | 12 | 0.0109 | 12 |
| SPAR | 0.0218 | 13 | 0.0159 | 13 | 0.0071 | 9 | 0.0108 | 11 |
| GARCH | 0.0374 | 24 | 0.0282 | 24 | 0.0117 | 22 | 0.0177 | 22 |
| GA | 0.0215 | 10 | 0.0152 | 5 | 0.0078 | 16 | 0.0113 | 15 |

| FUZZY | 0.0219 | 14 | 0.0157 | 10 | 0.0076 | 15 | 0.0111 | 13 |
|---------|--------|----|--------|----|--------|----|--------|----|
| SRW | 0.0325 | 21 | 0.0235 | 22 | 0.0109 | 21 | 0.0161 | 21 |
| HW | 0.0285 | 18 | 0.0221 | 19 | 0.0095 | 17 | 0.0131 | 17 |
| TBATS | 0.0367 | 23 | 0.0273 | 23 | 0.0124 | 24 | 0.0180 | 24 |
| SARANN | 0.0329 | 22 | 0.0222 | 20 | 0.0118 | 23 | 0.0180 | 23 |
| SUTSEA | 0.0292 | 19 | 0.0228 | 21 | 0.0096 | 18 | 0.0133 | 18 |
| SUTSET | 0.1031 | 26 | 0.0738 | 26 | 0.0378 | 26 | 0.0537 | 26 |
| SARIMA | 0.0278 | 17 | 0.0208 | 17 | 0.0098 | 19 | 0.0136 | 19 |
| BSARIMA | 0.0471 | 25 | 0.0338 | 25 | 0.0179 | 25 | 0.0248 | 25 |
| SARFIMA | 0.0297 | 20 | 0.0214 | 18 | 0.0105 | 20 | 0.0150 | 20 |

Notes: This Table reports the root MSFEs and the corresponding ranking for each forecasting horizon and weighting scheme.

Interestingly, the combination forecasts especially the DISC and PC models outperform the single or individual forecast models. The outstanding performance of the DISC appears to be robust to both the weighting scheme and forecast horizons taking the first rank in 12 cases out of 16 and 2^{nd} for the remaining 4 cases. This implies that the DISC model has the smallest RMSE in general. Following closely to the DISC is the PC model. However, we observe that at medium and longer term horizons (h=12 and h=24), the PC model's performance for either the recession forecasts or tail forecasts is not quite impressive as it takes the rank of between 6th and 18th for these cases. Another interesting finding in this study with respect to RMSE evaluation criterion is that the more sophisticated forecast combination methods outperformed the simple mean combination method unlike other studies cited previously.

Table 3: Root Mean Squared Forecast Errors (*h*=4)

| | Unif | orm | Boo | m | Reces | sion | Ta | Tail | | |
|--------|--------|------|--------|------|--------|------|--------|------|--|--|
| Model | RMSE | Rank | RMSE | Rank | RMSE | Rank | RMSE | Rank | | |
| RW | 0.0334 | 15 | 0.0239 | 15 | 0.0127 | 18 | 0.0176 | 16 | | |
| DISC | 0.0227 | 1 | 0.0174 | 1 | 0.0071 | 1 | 0.0104 | 1 | | |
| PC | 0.0276 | 2 | 0.0201 | 2 | 0.0102 | 4 | 0.0143 | 3 | | |
| MEAN | 0.0298 | 11 | 0.0220 | 11 | 0.0105 | 6 | 0.0150 | 6 | | |
| ARIMA | 0.0292 | 5 | 0.0228 | 14 | 0.0094 | 2 | 0.0133 | 2 | | |
| ARFIMA | 0.0291 | 4 | 0.0215 | 8 | 0.0105 | 7 | 0.0148 | 5 | | |
| BARIMA | 0.0348 | 16 | 0.0264 | 16 | 0.0111 | 13 | 0.0161 | 14 | | |
| BCAR | 0.0295 | 8 | 0.0213 | 6 | 0.0112 | 15 | 0.0157 | 13 | | |
| MSAR | 0.0300 | 12 | 0.0225 | 13 | 0.0099 | 3 | 0.0144 | 4 | | |
| SETAR | 0.0295 | 7 | 0.0214 | 7 | 0.0104 | 5 | 0.0151 | 7 | | |
| LSTAR | 0.0292 | 6 | 0.0212 | 4 | 0.0106 | 11 | 0.0151 | 9 | | |
| ARANN | 0.0297 | 9 | 0.0216 | 9 | 0.0110 | 12 | 0.0155 | 12 | | |
| NPAR | 0.0289 | 3 | 0.0206 | 3 | 0.0105 | 9 | 0.0152 | 11 | | |
| SPAR | 0.0297 | 10 | 0.0217 | 10 | 0.0106 | 10 | 0.0152 | 10 | | |
| GARCH | 0.0415 | 22 | 0.0324 | 23 | 0.0111 | 14 | 0.0177 | 17 | | |

| GA | 0.0310 | 14 | 0.0212 | 5 | 0.0131 | 19 | 0.0178 | 18 |
|---------|--------|----|--------|----|--------|----|--------|----|
| FUZZY | 0.0301 | 13 | 0.0223 | 12 | 0.0105 | 8 | 0.0151 | 8 |
| SRW | 0.0414 | 21 | 0.0311 | 22 | 0.0136 | 21 | 0.0194 | 21 |
| HW | 0.0381 | 20 | 0.0294 | 20 | 0.0135 | 20 | 0.0181 | 20 |
| TBATS | 0.0471 | 23 | 0.0341 | 24 | 0.0162 | 23 | 0.0240 | 23 |
| SARANN | 0.0477 | 24 | 0.0289 | 19 | 0.0178 | 24 | 0.0290 | 24 |
| SUTSEA | 0.0378 | 19 | 0.0300 | 21 | 0.0120 | 16 | 0.0166 | 15 |
| SUTSET | 0.1087 | 26 | 0.0798 | 26 | 0.0389 | 26 | 0.0549 | 26 |
| SARIMA | 0.0371 | 17 | 0.0277 | 18 | 0.0126 | 17 | 0.0180 | 19 |
| BSARIMA | 0.0698 | 25 | 0.0487 | 25 | 0.0278 | 25 | 0.0388 | 25 |
| SARFIMA | 0.0376 | 18 | 0.0272 | 17 | 0.0136 | 22 | 0.0195 | 22 |

Notes: see notes to Table 2.

We can generally infer that the relative performance of the DISC model is unaffected by the specific economic conditions. Another model that seems to perform fairly well is the MSAR. This is particularly so for the shortest (ranking 1^{st} for recession and tail forecasts and 2^{nd} for boom and uniform forecasts) and longest term forecasts (with a rank of 3 for both uniform and tail forecasts and 5 for both boom and recession forecasts). However, for the rest of the models, the rankings in most cases differ greatly between boom and recession periods and even at different forecast horizons. Take the GARCH model for instance: while it seems to be the most useful model for recession forecasts with a rank of 1 at b=24; it ranks 21^{st} for the boom forecasts. The same model ranks 22^{nd} and 24^{th} for the recession and boom forecasts respectively at b=1, 23^{rd} and 14^{th} at b=4 and 25^{th} and 2^{nd} at b=12.

Table 4: Root Mean Squared Forecast Errors (*h*=12)

| | Unif | orm | Boo | m | Reces | sion | Ta | il |
|--------|--------|------|--------|------|--------|------|--------|------|
| Model | RMSE | Rank | RMSE | Rank | RMSE | Rank | RMSE | Rank |
| RW | 0.0577 | 18 | 0.0335 | 7 | 0.0297 | 23 | 0.0393 | 23 |
| DISC | 0.0400 | 1 | 0.0278 | 1 | 0.0159 | 1 | 0.0221 | 1 |
| PC | 0.0469 | 2 | 0.0299 | 2 | 0.0209 | 10 | 0.0289 | 6 |
| MEAN | 0.0483 | 4 | 0.0315 | 3 | 0.0205 | 7 | 0.0289 | 5 |
| ARIMA | 0.0478 | 3 | 0.0325 | 5 | 0.0189 | 3 | 0.0271 | 2 |
| ARFIMA | 0.0506 | 6 | 0.0322 | 4 | 0.0228 | 18 | 0.0313 | 18 |
| BARIMA | 0.0512 | 9 | 0.0337 | 9 | 0.0218 | 15 | 0.0304 | 10 |
| BCAR | 0.0521 | 10 | 0.0325 | 6 | 0.0240 | 21 | 0.0328 | 20 |
| MSAR | 0.0509 | 7 | 0.0344 | 13 | 0.0209 | 11 | 0.0293 | 7 |
| SETAR | 0.0524 | 12 | 0.0355 | 17 | 0.0209 | 9 | 0.0298 | 9 |
| LSTAR | 0.0522 | 11 | 0.0341 | 11 | 0.0222 | 17 | 0.0310 | 17 |
| ARANN | 0.0529 | 15 | 0.0337 | 10 | 0.0236 | 19 | 0.0326 | 19 |
| NPAR | 0.0526 | 14 | 0.0348 | 14 | 0.0215 | 13 | 0.0306 | 12 |
| SPAR | 0.0525 | 13 | 0.0349 | 16 | 0.0219 | 16 | 0.0307 | 13 |

| GARCH | 0.0598 | 24 | 0.0443 | 25 | 0.0179 | 2 | 0.0283 | 4 |
|---------|--------|----|--------|----|--------|----|--------|----|
| GA | 0.0595 | 21 | 0.0348 | 15 | 0.0305 | 25 | 0.0402 | 24 |
| FUZZY | 0.0541 | 16 | 0.0366 | 20 | 0.0218 | 14 | 0.0309 | 14 |
| SRW | 0.0590 | 20 | 0.0417 | 22 | 0.0203 | 5 | 0.0305 | 11 |
| HW | 0.0587 | 19 | 0.0388 | 21 | 0.0238 | 20 | 0.0339 | 22 |
| TBATS | 0.0612 | 25 | 0.0363 | 19 | 0.0299 | 24 | 0.0402 | 25 |
| SARANN | 0.0509 | 8 | 0.0342 | 12 | 0.0212 | 12 | 0.0298 | 8 |
| SUTSEA | 0.0597 | 23 | 0.0421 | 24 | 0.0204 | 6 | 0.0310 | 16 |
| SUTSET | 0.1122 | 26 | 0.0775 | 26 | 0.0430 | 26 | 0.0616 | 26 |
| SARIMA | 0.0595 | 22 | 0.0417 | 23 | 0.0206 | 8 | 0.0309 | 15 |
| BSARIMA | 0.0554 | 17 | 0.0356 | 18 | 0.0243 | 22 | 0.0338 | 21 |
| SARFIMA | 0.0495 | 5 | 0.0335 | 8 | 0.0196 | 4 | 0.0281 | 3 |
| | | | | | | | | |

Notes: see notes to Table 2.

If we focus on different horizons, we can easily pick out the best three models for recession or boom forecasts. For example, at the shortest term horizon (b=1), the top three models for booms are DISC, MSAR and PC models in that order while the top three models for recessions are MSAR, DISC and PC models. At the 4-month horizon, the top three models for booms are DISC, PC and NPAR models while the top three models for recessions are DISC, ARIMA and MSAR models. At the 12-month horizon, the top three models for booms are DISC, PC and MEAN models while the top three models for recessions are DISC, GARCH and ARIMA models. At the longest term horizon (h=24), the top three models for booms are PC, DISC and ARFIMA models while the top three models for recessions are GARCH, DISC and SETAR models. In practice, the choice of an appropriate model may depend on both the forecast horizon and on the specific loss function, since as shown by our results some single models are more suited for short term horizons, while other models perform better in the longer-run. Moreover, given that the performance of the single models differ depending on the economic condition in which the country is in, therefore forecasters who particularly dislike forecast errors during recessions should use a slightly different set of models than forecasters who are more interested in correct prediction during booming markets. This is consistent with the findings in Carstensen et al.(2010).

Table 5: Root Mean Squared Forecast Errors (h=24)

| | Unif | orm | Boo | m | Reces | sion | Ta | Tail | | |
|--------|--------|------|--------|------|--------|------|--------|------|--|--|
| Model | RMSE | Rank | RMSE | Rank | RMSE | Rank | RMSE | Rank | | |
| RW | 0.0997 | 19 | 0.0544 | 9 | 0.0550 | 24 | 0.0714 | 23 | | |
| DISC | 0.0692 | 1 | 0.0454 | 2 | 0.0326 | 2 | 0.0416 | 1 | | |
| PC | 0.0823 | 2 | 0.0451 | 1 | 0.0451 | 18 | 0.0581 | 17 | | |
| MEAN | 0.0857 | 6 | 0.0536 | 8 | 0.0407 | 11 | 0.0544 | 11 | | |
| ARIMA | 0.0851 | 5 | 0.0533 | 6 | 0.0406 | 10 | 0.0539 | 10 | | |
| ARFIMA | 0.0873 | 7 | 0.0520 | 3 | 0.0426 | 15 | 0.0571 | 15 | | |
| BARIMA | 0.0848 | 4 | 0.0535 | 7 | 0.0391 | 9 | 0.0530 | 7 | | |
| BCAR | 0.0877 | 8 | 0.0520 | 4 | 0.0424 | 14 | 0.0572 | 16 | | |
| MSAR | 0.0831 | 3 | 0.0528 | 5 | 0.0374 | 5 | 0.0510 | 3 | | |

| SETAR | 0.0899 | 15 | 0.0597 | 19 | 0.0359 | 3 | 0.0514 | 4 |
|---------|--------|----|--------|----|--------|----|--------|----|
| LSTAR | 0.0881 | 11 | 0.0559 | 12 | 0.0387 | 8 | 0.0536 | 8 |
| ARANN | 0.0895 | 14 | 0.0552 | 11 | 0.0408 | 12 | 0.0561 | 13 |
| NPAR | 0.0880 | 10 | 0.0573 | 15 | 0.0369 | 4 | 0.0520 | 5 |
| SPAR | 0.0879 | 9 | 0.0567 | 13 | 0.038 | 6 | 0.0529 | 6 |
| GARCH | 0.0947 | 18 | 0.0675 | 21 | 0.032 | 1 | 0.0481 | 2 |
| GA | 0.1029 | 20 | 0.0583 | 16 | 0.0549 | 23 | 0.0716 | 24 |
| FUZZY | 0.0926 | 16 | 0.0617 | 20 | 0.0382 | 7 | 0.0538 | 9 |
| SRW | 0.1202 | 24 | 0.0854 | 24 | 0.0471 | 20 | 0.0659 | 20 |
| HW | 0.1159 | 22 | 0.0800 | 22 | 0.0496 | 22 | 0.0669 | 21 |
| TBATS | 0.1064 | 21 | 0.0567 | 14 | 0.0566 | 25 | 0.0754 | 25 |
| SARANN | 0.0885 | 12 | 0.0548 | 10 | 0.0431 | 16 | 0.0569 | 14 |
| SUTSEA | 0.1211 | 25 | 0.0855 | 25 | 0.0475 | 21 | 0.0670 | 22 |
| SUTSET | 0.1504 | 26 | 0.1024 | 26 | 0.0610 | 26 | 0.0862 | 26 |
| SARIMA | 0.1166 | 23 | 0.083 | 23 | 0.0462 | 19 | 0.0644 | 19 |
| BSARIMA | 0.0939 | 17 | 0.0586 | 18 | 0.0435 | 17 | 0.0590 | 18 |
| SARFIMA | 0.0894 | 13 | 0.0586 | 17 | 0.0409 | 13 | 0.0545 | 12 |

Notes: see notes to Table 2.

Next we also evaluate the forecasting models based on their RMSE relative to the benchmark RW forecast.3 If the relative RMSE of any model is less than 1, then it outperformed the RW model. Almost all the models with seasonal dummy variables outperformed the benchmark RW model whereas the RW model outperformed all the full seasonal models at the 1-month and 4month horizons. This is robust to different weighting schemes. However, at 12-month and 24month horizons both full seasonal models and models with seasonal dummy variables outperformed the RW model especially for the recession and tail forecasts. It is also observed that the DISC combined forecast has substantial gains over both the benchmark RW and the rest individual models. For instance, the RMSE for DISC model is lower than the RMSE for the RW model by about 43% and 48%, respectively for the boom and recession forecasts at horizon one. However, this gain reduces as one progress to longer horizons. Looking at horizon 24, the gain relative to RW model reduces to 26% and 41%, respectively, for the boom and recession forecasts. MSAR is the best individual model at horizon one with an improvement of 37% and 48%, respectively, for the boom and recession forecasts. At the 24-month horizon, the best individual performing model for the recession forecasts is GARCH with an improvement of 42% over the RW model whereas for the boom period the RMSE of the former is 24% higher than the later. The best performing individual model (ARFIMA) for the boom forecasts improves upon the RW model by only 4% at b=24. Overall, the performance of the models relative to the RW model differs by both forecast horizon and different weighting schemes.

To evaluate whether the above findings are statistically significant, we employ the weighted version of modified Diebold-Mariano pair-wise test. The null hypothesis of the MDM test is that of equal forecast performance. The result is reported in Tables 6-7.⁴ The columns with heading "+" indicate the number of times a specific model significantly outperforms its competitors. The columns with heading "-" indicate the number of times a specific model is outperformed by its

³ The relative values are essentially the ratio of each model to the RW model. We do not present the results here but they are available upon request.

⁴ We report only the rankings and show the best model in bold. The MW-DM statistics with the p-values are available from authors upon request.

competitors. Recalling we have 26 forecasting models, a rank of 25 is therefore the maximum a specific model can either outperform other models or be outperformed by other models. At the 1-month horizon, the DISC and the MSAR models significantly outperform the rest competing models 24 times and were not significantly dominated by any other model. This simply implies that these two models yield significantly smaller losses than their competitors. The next good performing model is the PC model. These results are robust to the different weighting schemes. At horizon 4, the DISC model significantly outperforms the rest 25 models and is not outperformed by any other irrespective of the weighting scheme used. Following the DISC model is the PC model. A similar result holds at the 12-month horizon with the exception that the PC model did not perform equally well for the recession and tail forecasts. At horizon 24, the DISC model is again the best performing model. This is followed by PC model for the boom forecasts and BARIMA model for the uniform and recession forecasts. The worst performing model at all horizons and weighting schemes is the SUTSET as it never outperform any model significantly but is rather significantly dominated by other models.

Table 6: Summary of Modified Diebold-Mariano Forecast Accuracy Tests(h=1 and h=4)

| | | | | j | b=1 | | | | <i>b</i> =4 | | | | | | | |
|---------|-----|------|----|----|-----|--------|----|-----|-------------|------|----|----|-----|--------|----|------|
| | Uni | form | Во | om | Rec | ession | Т | ail | Uni | form | Во | om | Rec | ession | T | 'ail |
| Model | + | _ | + | - | + | _ | + | _ | + | _ | + | _ | + | _ | + | - |
| RW | 10 | 14 | 8 | 14 | 11 | 12 | 10 | 9 | 10 | 14 | 11 | 12 | 7 | 13 | 4 | 13 |
| DISC | 24 | 0 | 24 | 0 | 24 | 0 | 24 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 |
| PC | 23 | 2 | 23 | 2 | 23 | 2 | 23 | 2 | 21 | 1 | 18 | 1 | 21 | 1 | 20 | 1 |
| MEAN | 12 | 3 | 12 | 3 | 13 | 3 | 13 | 3 | 12 | 2 | 12 | 2 | 13 | 3 | 12 | 3 |
| ARIMA | 12 | 3 | 12 | 6 | 12 | 3 | 15 | 3 | 12 | 1 | 12 | 2 | 18 | 1 | 22 | 1 |
| ARFIMA | 12 | 3 | 13 | 3 | 14 | 3 | 15 | 3 | 13 | 2 | 12 | 1 | 15 | 2 | 15 | 2 |
| BARIMA | 6 | 15 | 5 | 15 | 9 | 15 | 7 | 12 | 6 | 14 | 5 | 15 | 8 | 13 | 6 | 13 |
| BCAR | 12 | 3 | 13 | 3 | 13 | 4 | 13 | 3 | 13 | 2 | 12 | 1 | 13 | 5 | 12 | 4 |
| MSAR | 24 | 0 | 24 | 0 | 24 | 0 | 24 | 0 | 12 | 2 | 11 | 2 | 13 | 1 | 12 | 1 |
| SETAR | 12 | 3 | 12 | 3 | 12 | 3 | 12 | 3 | 12 | 2 | 12 | 1 | 13 | 1 | 12 | 2 |
| LSTAR | 12 | 3 | 13 | 3 | 16 | 3 | 14 | 3 | 14 | 1 | 14 | 1 | 15 | 2 | 12 | 3 |
| ARANN | 12 | 3 | 12 | 3 | 12 | 5 | 12 | 6 | 12 | 2 | 12 | 1 | 13 | 5 | 12 | 4 |
| NPAR | 12 | 3 | 13 | 3 | 12 | 3 | 11 | 3 | 13 | 1 | 12 | 1 | 13 | 2 | 12 | 3 |
| SPAR | 12 | 3 | 12 | 4 | 12 | 3 | 11 | 3 | 12 | 2 | 12 | 3 | 13 | 3 | 12 | 3 |
| GARCH | 2 | 21 | 2 | 22 | 2 | 21 | 2 | 20 | 3 | 16 | 2 | 18 | 3 | 14 | 3 | 13 |
| GA | 12 | 3 | 12 | 3 | 11 | 7 | 10 | 9 | 12 | 6 | 12 | 2 | 7 | 13 | 5 | 13 |
| FUZZY | 12 | 3 | 12 | 3 | 11 | 4 | 10 | 5 | 12 | 3 | 11 | 3 | 13 | 3 | 12 | 3 |
| SRW | 3 | 19 | 3 | 18 | 3 | 20 | 2 | 20 | 3 | 19 | 2 | 18 | 3 | 18 | 3 | 16 |
| HW | 5 | 15 | 4 | 14 | 6 | 16 | 7 | 15 | 3 | 15 | 3 | 15 | 3 | 16 | 4 | 13 |
| TBATS | 2 | 20 | 2 | 21 | 2 | 20 | 2 | 20 | 2 | 22 | 2 | 20 | 2 | 22 | 2 | 22 |
| SARANN | 2 | 17 | 4 | 16 | 2 | 19 | 2 | 19 | 2 | 16 | 2 | 15 | 2 | 18 | 2 | 19 |
| SUTSEA | 4 | 15 | 4 | 16 | 6 | 16 | 7 | 15 | 4 | 15 | 3 | 15 | 5 | 13 | 5 | 4 |
| SUTSET | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 |
| SARIMA | 6 | 14 | 7 | 14 | 7 | 15 | 7 | 15 | 4 | 15 | 5 | 15 | 5 | 13 | 5 | 13 |
| BSARIMA | 1 | 24 | 1 | 24 | 1 | 24 | 1 | 24 | 1 | 24 | 1 | 24 | 1 | 24 | 1 | 24 |

| SARFIMA | 5 | 15 | 5 | 14 | 5 | 17 | 5 | 19 | 4 | 15 | 5 | 15 | 3 | 16 | 3 | 15 |
|---------|---|----|---|----|---|----|---|----|---|----|---|----|---|----|---|----|

Notes: The columns "+" indicate the number of times a specific model significantly outperforms its competitors. The columns "-" indicate the number of times a specific model is outperformed by its competitors.

Overall, there appears to be no single model that performs relatively better than other single models at all forecast horizons and for all weighting schemes. It is MSAR at horizon 1 for all weighting schemes, ARFIMA for recession and tails forecasts and LSTAR for uniform and boom and also recession forecasts at horizon 4. At horizon 12, it is ARIMA for recession and ARFIMA for boom forecasts. At horizon 24, it is ARFIMA for the boom and BARIMA model for the uniform and recession forecasts. However, the combination forecasts, especially the DISC model forecast is the best at all horizons no matter which weighting scheme is employed. These findings confirm the superiority of combined forecasts over individual forecasts for forecasting South Africa's aggregate retail sales. This is graphically confirmed in Figure 3, where we plot the actuals and the forecasts at different horizons from the discounted MSFE combination method under the uniform loss weighting scheme.⁵ As can be seen, the forecasts closely track the actuals for all the horizons considered. Overall then, academics and practitioners should not depend on a single forecasting model as no single model uniformly dominates others, but at the same time should have a wide array of models rather than just a few for the sake of efficiency gains, since for a series as volatile as retail sales, different models tend to capture different aspects of the unknown data generating process that defines the structure of retail sales. This fact is highlighted by the better performance of the principal component forecast combination approach relative to that of the simple mean-based forecast combination method. The principal component approach uses a statistical procedure to provide more weight to the forecasts from those models for which the forecasts tend to move closely, and this is likely to be the case for models which produce better forecasts, since otherwise this approach would not have performed better than the mean-based combination approach. However, the outstanding performance of the discounted MSFE combination method, highlights the importance of using forecast information sets that are more recent. The fact that we needed to discount the past quite strongly (using a discount factor of 0.50), from an empirical point of view is probably due to the fact that the series is characterized by volatility and strong seasonal patterns to the extent that forecasts information from distant past is of little value. However, from a theoretical perspective, realizing that the retail sales series that we are working with comprises of realized values (hence are market clearing or equilibrium retail sales), is possibly an indication of learning on behalf of the retailers. In other words, retailers tend to update their information sets valuing current information more than recent past to better predict the future, realizing that economic conditions and demand are continuously changing over time, and the supply decisions need to change accordingly to accommodate such dynamism. At this stage, it is important to point out that unlike previous studies that have combined forecasts, while forecasting various variables of interest, and have found the simple mean-based forecast combination method to often perform better than more sophisticated combination methods, like the principal components and discounted MSFE approaches (see Rapach and Strauss, 2010 for a detailed discussion), we find the relatively more statistically rigorous discounted MSFE to be the standout forecast combination approach. This we believe could emanate from the theoretical reasons involving learning discussed above.

The nonlinearity, especially of parametric nature, seems to be important and should always be taken into account. While MSAR model tend to perform better relative to the other forms of

⁵ See appendix 1 for similar plots for the best single models.

nonlinear models at short horizons, the LSTAR model tends to stand out at longer horizons. As discussed earlier, while for the MSAR model the regime-switching is based on latent regime variable that follows a first order Markov process, and therefore has unexplained switching, the LSTAR model has explained switching and therefore regime-switching follows a known structure, with smooth transition in and out of a regime that spreads to more than one period. Theoretically this result is important for us, as it more than likely suggests the development of dynamic stochastic general equilibrium models to capture the nonlinear dynamics of equilibrium retail sales, with parameters of the model defining consumer and producer choices being state-dependent, to account for regime-switching behaviour across alternative states of the economy. Lastly, from an empirical perspective, attempting to model the seasonal variation with models that have complicated seasonal features does not seem to worth the effort, as lots of the nonlinearity due to seasonal behaviour seems to be captured by the nonlinear, but non-seasonal, models already.

Table 7: Summary of Modified Diebold-Mariano Forecast Accuracy Tests (h=12 and h=24)

| | <i>b</i> =12 | | | | | | | | <i>h</i> =24 | | | | | | | |
|---------|--------------|----|------|----|-----------|----|------|----|--------------|----|------|----|-----------|----|------|----|
| Model | Uniform | | Boom | | Recession | | Tail | | Uniform | | Boom | | Recession | | Tail | |
| | + | _ | + | _ | + | _ | + | _ | + | _ | + | _ | + | _ | + | _ |
| RW | 1 | 9 | 6 | 2 | 1 | 17 | 1 | 16 | 1 | 3 | 5 | 2 | 1 | 18 | 1 | 18 |
| DISC | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 24 | 0 | 25 | 0 | 25 | 0 |
| PC | 22 | 1 | 22 | 1 | 10 | 1 | 9 | 1 | 12 | 1 | 24 | 0 | 8 | 1 | 5 | 1 |
| MEAN | 21 | 1 | 19 | 1 | 15 | 1 | 13 | 1 | 11 | 1 | 12 | 2 | 10 | 1 | 10 | 1 |
| ARIMA | 15 | 1 | 10 | 1 | 19 | 1 | 17 | 1 | 10 | 1 | 11 | 2 | 10 | 1 | 10 | 1 |
| ARFIMA | 13 | 3 | 15 | 2 | 8 | 5 | 7 | 3 | 8 | 1 | 16 | 2 | 7 | 3 | 5 | 4 |
| BARIMA | 10 | 3 | 8 | 3 | 8 | 3 | 7 | 1 | 13 | 1 | 15 | 2 | 13 | 1 | 10 | 1 |
| BCAR | 7 | 4 | 14 | 2 | 4 | 8 | 4 | 9 | 8 | 2 | 15 | 2 | 5 | 3 | 5 | 4 |
| MSAR | 7 | 3 | 6 | 3 | 8 | 2 | 8 | 1 | 11 | 1 | 10 | 2 | 12 | 1 | 9 | 1 |
| SETAR | 5 | 3 | 6 | 6 | 8 | 1 | 9 | 1 | 8 | 1 | 6 | 12 | 10 | 1 | 6 | 1 |
| LSTAR | 6 | 3 | 9 | 5 | 6 | 3 | 6 | 3 | 9 | 1 | 8 | 4 | 10 | 1 | 7 | 2 |
| ARANN | 4 | 5 | 8 | 5 | 4 | 13 | 4 | 12 | 7 | 3 | 8 | 5 | 5 | 6 | 5 | 6 |
| NPAR | 5 | 4 | 5 | 5 | 7 | 3 | 5 | 4 | 9 | 1 | 8 | 5 | 11 | 1 | 8 | 1 |
| SPAR | 5 | 4 | 7 | 5 | 7 | 3 | 6 | 3 | 10 | 1 | 8 | 5 | 10 | 1 | 10 | 1 |
| GARCH | 1 | 16 | 1 | 18 | 3 | 1 | 3 | 1 | 5 | 6 | 5 | 16 | 8 | 1 | 8 | 1 |
| GA | 1 | 15 | 1 | 5 | 1 | 18 | 1 | 18 | 1 | 14 | 5 | 6 | 1 | 18 | 0 | 18 |
| FUZZY | 3 | 8 | 2 | 11 | 4 | 5 | 4 | 4 | 6 | 10 | 6 | 13 | 8 | 3 | 5 | 2 |
| SRW | 1 | 9 | 1 | 17 | 2 | 3 | 1 | 2 | 1 | 18 | 1 | 21 | 1 | 15 | 1 | 5 |
| HW | 1 | 13 | 1 | 11 | 2 | 12 | 1 | 8 | 1 | 18 | 1 | 21 | 1 | 18 | 1 | 17 |
| TBATS | 1 | 17 | 1 | 14 | 1 | 21 | 1 | 18 | 1 | 17 | 5 | 9 | 0 | 18 | 0 | 18 |
| SARANN | 10 | 4 | 7 | 3 | 7 | 4 | 8 | 3 | 8 | 3 | 10 | 2 | 8 | 4 | 5 | 4 |
| SUTSEA | 1 | 8 | 1 | 17 | 1 | 2 | 1 | 2 | 1 | 18 | 1 | 21 | 1 | 14 | 1 | 5 |
| SUTSET | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 25 | 0 | 24 | 0 | 23 |
| SARIMA | 1 | 8 | 1 | 17 | 2 | 4 | 1 | 4 | 1 | 18 | 1 | 21 | 1 | 15 | 1 | 5 |
| BSARIMA | 2 | 10 | 5 | 7 | 4 | 13 | 3 | 13 | 6 | 11 | 5 | 9 | 5 | 9 | 4 | 9 |
| SARFIMA | 11 | 2 | 8 | 3 | 13 | 1 | 10 | 1 | 7 | 4 | 6 | 7 | 8 | 1 | 8 | 1 |

Notes: The columns "+" indicate the number of times a specific model significantly outperforms its competitors. The columns "-" indicate the number of times a specific model is outperformed by its competitors.

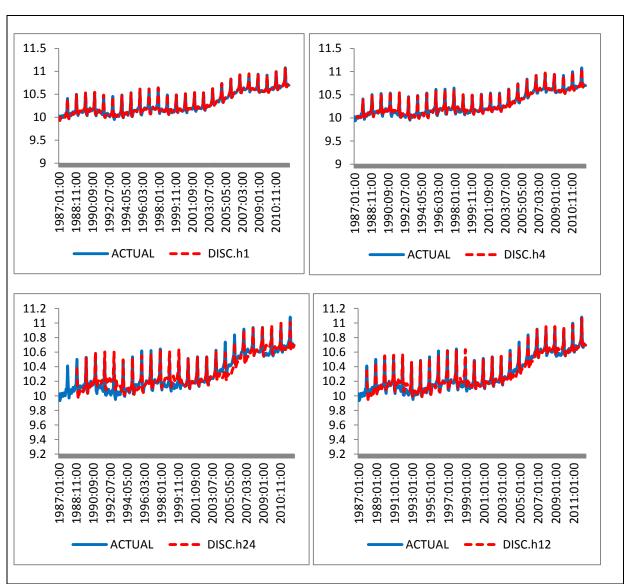


Figure 3. Actual and DISC forecast of retail sales at different horizons for the uniform weighting scheme

Note: DISC.h1, DISC.h4, DISC.h12 and DISC.h24 represents forecasts from the discounted forecast combination model at horizons 1, 4, 12 and 24, respectively.

5. Conclusion

In this paper, we assess the forecasting performance of 26 models of South Africa's aggregate seasonal retail sales over 1987:01 – 2012:05 out-of-sample period. The recent recession has demonstrated that a good forecast of a rather extreme event might be of special interest beyond that of minimizing an average squared error. Hence, we allowed for departures from the uniform symmetric quadratic loss function typically used in forecast evaluation exercises. We overweighed forecast errors during periods of high or low growth rates to check how the indicators perform during booms and recessions, i.e., in times of particularly high demand for good forecasts. Specifically, we use van Dijk and Franses (2003) weighted MSFE and weighted modified MDM tests to evaluate forecasts from the 26 different forecasting models. We estimated two broad classes of modes: 17 models with seasonal dummy variables and 9 full seasonal models. In general, the models with seasonal dummy variables produce better forecasts than full seasonal

models. Most of the models performed better than the random work benchmark. The most widely used nonlinear model in the retail sales forecasting literature, namely the ANN, model is consistently outperformed by other types of nonlinear models which are more strongly theoretically founded. From the analysis, it is difficult to identify a specific individual model as the best for forecasting South Africa's aggregate retail sales. Some single models are well suited for booms while others are well suited for recessions and this differ across forecast horizons. However, the combination forecasts offer ways of incorporating and culling information from a larger number of forecasting models. This group of models turned out to outperform the individual models in general. Specifically, the discounted combination forecast model (DISC) outperform all the single models and the other two combination forecasts (simple mean and principal component) models and the performance is largely unaffected by specific economic or business cycle situation or forecast horizon.

The findings in this study demonstrate the need to include a wide array of linear and nonlinear models than just few models when forecasting retails sales, since each model captures different aspects of the series at different forecasting horizons. Moreover, it is also important to consider forecasting during both periods of booms and recessions, since the predictability of the single models during periods of calm is vastly different from periods of turmoil. Moreover, the identification of the best model for forecasting retail sales, which happens to be the combined model, based on discounted MSFE in this case, helps to provide accurate forecasts not only over short- to long-horizons, but also less variable forecasts across extreme periods. This should, in turn, enable businesses and investors to make efficient inventory management, proper business plans and strategies and also optimal portfolio resource allocation decisions, all of which are likely to affect profitability. Further, government officials would require such accurate aggregate retail sales forecast in designing and implementing optimal public policy for the retailing industry which will subsequently benefit both consumers and businesses. The fact that better retail sales forecasting gives an indication for the path of consumption and also a vital preinflationary indicator has implications for policy makers and investors. For instance, if the growth of retail sales is stalled or slowing, it could signal a recession, because of the significant role personal consumption plays in the growth of the economy. More so, a sudden rise in retail sales in the midst of a business cycle, would cause the Reserve Bank to increase interest rate at least in the short term to curtail any possible inflation. This might negatively affect financial assets such as stocks and bonds as well as investors' future cash flows. Therefore, accurate forecast of and timely policy intervention in the retail industry based on the best econometric model is crucial for economic growth and stability.

This current paper adds to the literature on forecasting retail sales empirically, primarily, but also hint towards possible theoretical models that needs to be developed to capture appropriately the dynamics of equilibrium retail sales, based on the results obtained. From an applied perspective, unlike the previous literature on retail sales forecasting, we not only look at a wider array of linear and non-linear models, but also conduct multi-steps-ahead forecasts based on a real-time (recursive) scenario, as is likely to be faced by the agents. In addition, we also look at the performance of the empirical models across different phases of the equilibrium retail sales growth. Our results indicating the outstanding performance of the discounted MSFE combination methods and the importance of nonlinearity, highlights the need for developing dynamic stochastic general equilibrium models for retail sales which not only incorporates learning behaviour, but also provides frameworks that allows the behavioural parameters of the model to be state-dependent, to account for regime-switching behaviour across alternative states of the economy. Nevertheless, the current paper adopts an univariate approach, with retail sales being predicted by its past values only, either in a linear or nonlinear fashion. However, as indicated by Dias et al. (2010), retail sales are likely to be affected by large number of economic

variables. Hence, future research would aim at using linear and nonlinear models of forecasting retail sales involving macroeconomic and financial variables as possible predictors, and, in turn comparing the results with the various univariate models discussed in this paper. This, in turn, would allow us to strengthen our theoretical conclusions, which at this stage are, to some extent, quite conjectural in nature.

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Appendix 1: Actual and forecast of retail sales at different horizons for the uniform weighting scheme for the best single models

