

MAXIMUM HEAT TRANSFER RATE DENSITY FROM DUCTS WITH WRINKLED ENTRANCES

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ABSTRACT

This paper reports an increase in the heat transfer rate density by using wrinkled entrance regions in ducts with laminar flow. The heat transfer rate density is increased by taking advantage of the presence of relatively isothermal fluid in the entrance regions. In order to stimulate a more complete thermal interaction between walls and fluid, the square entrances are wrinkled on the perimeter, at the one-third and the two-third positions. The new structure has two degrees of freedom. The fluid flow through the ducts is forced by the imposed pressure difference across the duct. Numerical simulations document the effects of the dimensionless pressure drop on the optimized configurations and show a fifteen percent enhancement in heat transfer rate density.

INTRODUCTION

The use of progressively smaller length scales for heat transfer in ducts and channels is attracting considerable interest [1-17]. The need to install more and more heat transfer rate in a fixed volume has led to various new methods and designs. One method that is drawing interest is constructal design [1]. The method focuses on the generation of optimal flow architecture as a mechanism by which systems achieve maximum transport density under constraints. Constructal design offers strategies for the pursuit of configuration, such that architectural features that have been found to be beneficial in the past can be refined and incorporated in more complex systems [2 - 4].

The constructal design of packages of channels began with Bejan and Sciubba [5], which determined the optimal spacing for arrays of parallel plates using the asymptotes method. The application was the cooling of electronics packages. Optimal spacings for heat and fluid flow structures have been developed for several classes of flow configurations: stacks of parallel plates, staggered plates, cylinders in cross-flow, and pin fins arrays with impinging flow. Optimal spacings have been determined for natural

convection and for forced convection with specified overall pressure difference. Most recently Yilmaz et al. [17] and Muzychka [15], determined analytically the optimal duct dimensions and maximum heat transfer per unit volume for parallel-plates channels, rectangular channels, elliptic ducts, circular ducts, polygonal ducts, and triangular ducts.

In this paper we propose a new concept to improve the constructal design for the cooling of a duct. We start with a square packing of channels, Fig 1. The size of the square ducts is to be selected numerically subject to a fixed pressure drop across the packing. The entranced regions of the duct do not participate fully in the heat transfer enterprise, because of the presence of relatively isothermal (cold) fluid that has not interacted thermally with the walls. Next, the duct entrance is wrinkled to stimulate a more complete thermal interaction between walls and fluid. The new structure has two degrees of freedom: the length of the wrinkled entrance, and the length or the spacing of the square ducts. These geometric features are optimized for maximum volumetric heat transfer density.

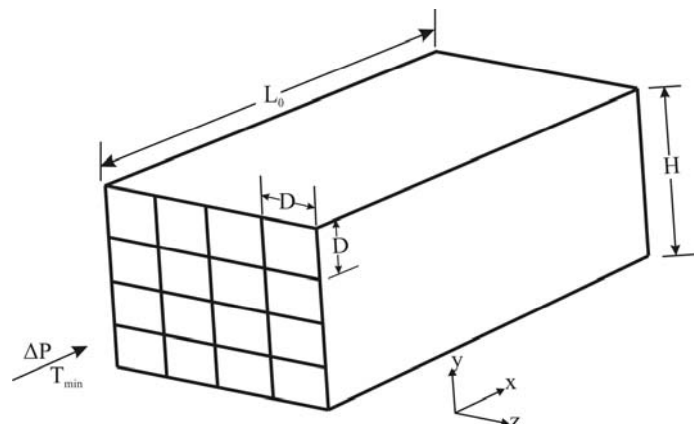


Figure 1 Packing of channels with square cross-sections

NOMENCLATURE

Be	[-]	dimensionless pressure drop, Eq. (6)
D	[m]	side of square cross-section
H	[m]	height of the duct
k	[W/mK]	thermal conductivity
L_0	[m]	length in the flow direction
L_1	[m]	flow length of the the wrinkled section
P	[Pa]	pressure
Pr	[-]	Prandtl number
q	[W]	heat transfer rate
q'''	[-]	heat transfer rate density
\tilde{q}	[-]	dimensionless heat transfer density
Re	[-]	Reynolds number
T	[K]	temperature
T_{wall}	[K]	wall temperature
T_{min}	[K]	free-stream temperature
U	[m/s]	mean velocity
u, v, w	[m/s]	velocity component
z, y, z	[m]	cartesian coordinates
Greek letters		
α	[m ² /s]	thermal diffusivity,
μ	[kg/s m]	viscosity,
γ	[-]	convergence criterion
Subscripts		
L_0		length
max		maximum
min		minimum
opt		optimum
wall		wall
∞		free-stream
Superscript		
~		dimensionless

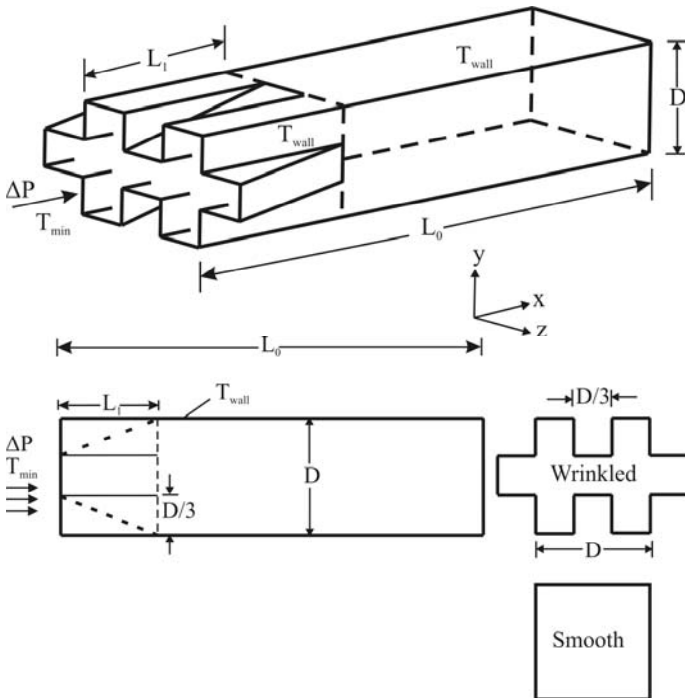


Figure 2 Entrance of a square channel wrinkled at the 1/3 and 2/3 positions.

MODEL

Consider the three dimensional configuration of an elemental channel with a wrinkled perimeter around a square cross-section. Figure 2 shows the entrance of one square channel after each side was wrinkled at the one-third and two-thirds positions. The walls of the channels (including the wrinkled portions) are maintained at the temperature T_{max} . The channel is cooled by a single-phase fluid at T_{min} , which is forced into the channel by a specified pressure difference (ΔP). The coolant is modelled as a Newtonian fluid with constant properties. The dimensionless equations for the steady-state flow of mass, momentum and energy are

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} + \frac{\partial \tilde{w}}{\partial \tilde{z}} = 0 \quad (1)$$

$$\frac{Be}{Pr} \left(\tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} + \tilde{w} \frac{\partial \tilde{u}}{\partial \tilde{z}} \right) = -\frac{\partial \tilde{P}}{\partial \tilde{x}} + \nabla^2 \tilde{u} \quad (2)$$

$$\frac{Be}{Pr} \left(\tilde{u} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} + \tilde{w} \frac{\partial \tilde{v}}{\partial \tilde{z}} \right) = -\frac{\partial \tilde{P}}{\partial \tilde{y}} + \nabla^2 \tilde{v} \quad (3)$$

$$\frac{Be}{Pr} \left(\tilde{u} \frac{\partial \tilde{w}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{w}}{\partial \tilde{y}} + \tilde{w} \frac{\partial \tilde{w}}{\partial \tilde{z}} \right) = -\frac{d\tilde{P}}{d\tilde{z}} + \nabla^2 \tilde{w} \quad (4)$$

$$Be \left(\tilde{u} \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} + \tilde{w} \frac{\partial \tilde{T}}{\partial \tilde{z}} \right) = \nabla^2 \tilde{T} \quad (5)$$

where $\nabla^2 = \partial^2 / \partial \tilde{x}^2 + \partial^2 / \partial \tilde{y}^2 + \partial^2 / \partial \tilde{z}^2$, and the dimensionless pressure difference is defined as [11, 12]:

$$Be = \frac{\Delta P L_0^2}{\mu \alpha} \quad (6)$$

The non-dimensionalization of the governing equations is based on defining the variables

$$\tilde{x}, \tilde{y}, \tilde{z} = \frac{(x, y, z)}{L_0}, \quad \tilde{P} = \frac{P}{\Delta P} \quad (7)$$

$$(\tilde{u}, \tilde{v}, \tilde{w}, \tilde{U}) = \frac{(u, v, w, U)}{\Delta P L_0 / \mu}, \quad \tilde{T} = \frac{T - T_{min}}{T_{wall} - T_{min}} \quad (8)$$

The boundary conditions are

$$\tilde{P} = 1, \quad \tilde{v} = \tilde{w} = 0, \quad \tilde{T} = 0 \text{ at } \tilde{x} = 0 \quad (9)$$

$$\tilde{u} = \tilde{v} = \tilde{w} = 0, \quad \tilde{T} = 1 \text{ on the walls} \quad (10)$$

$$\tilde{P} = \frac{\partial \tilde{u}}{\partial \tilde{x}} = \frac{\partial \tilde{v}}{\partial \tilde{x}} = \frac{\partial \tilde{w}}{\partial \tilde{x}} = \frac{\partial \tilde{T}}{\partial \tilde{x}} = 0 \text{ at } \tilde{x} = \tilde{L}_0 \quad (11)$$

$$\tilde{D} = \frac{D}{L_0} \quad (12)$$

We are interested in the geometric arrangement of the wrinkled packing for which the overall heat transfer rate per unit of flow volume is maximum:

$$q''' = \frac{\text{total heat transfer rate from walls to stream}}{\text{volume of one duct}} = \frac{q}{L_0 D^2} \quad (13)$$

In dimensionless form, the above definition can be rewritten as

$$\tilde{q} = \frac{qL_0}{k(T_{\text{wall}} - T_{\infty})D^2} \quad (14)$$

We solved Eqs. (2) - (5) by using a commercial finite volume code [18]. The domain was discretized using tetrahedral elements, and the governing equations were integrated on every control volume. Second order schemes were used for the diffusive terms. The pressure-velocity coupling was performed with the SIMPLE procedure. Convergence was obtained when the normalized residuals for the mass and momentum equations were smaller than 10^{-4} , and the residual of the energy equation became smaller than 10^{-8} .

To obtain accurate numerical results, several mesh refinement tests were conducted. The monitored quantity was the overall heat transfer rate density, which was computed with Eq. (14). Convergence was established based on the criterion

$$\gamma = \frac{|\tilde{q}_j - \tilde{q}_{j-1}|}{|\tilde{q}_j|} \leq 0.02 \quad (15)$$

where j is the mesh iteration index, such that j increases when the mesh is more refined. When the criterion is satisfied, the $j-1$ mesh is selected as the converged mesh. A mesh size of 0.005 per unit length in the y and z directions, and a mesh size of 0.01 per unit length in the axial direction were found to satisfy the criterion chosen in Eq. (15).

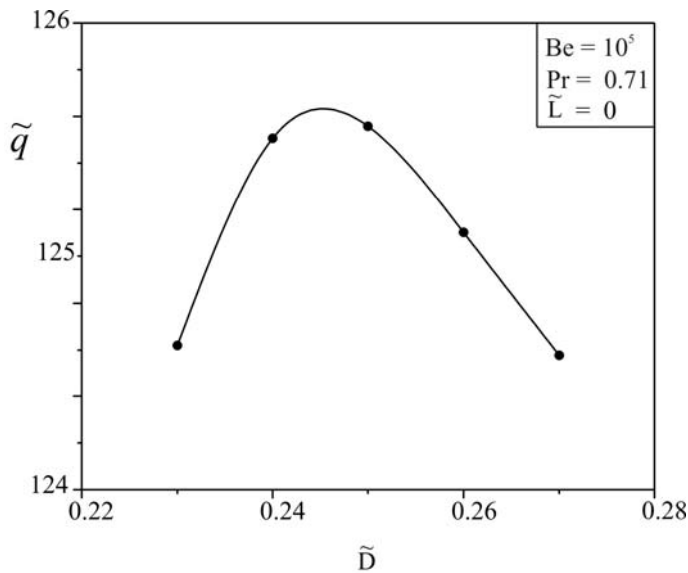


Figure 3 Effect of the channel size \tilde{D} (smooth) on the heat transfer density

RESULTS: CONSTRUCTAL DUCT WITH WRINKLED ENTRANCE

We started the optimization procedure by considering the case in which the mouth of the square channel is smooth, without wrinkles. In this case the only one degree of freedom is the duct size, \tilde{D} . Figure 3 shows the selection of the channel size and the heat transfer rate density, and indicates that an optimal size exists for a channel with a square packing.

The optimal square channel was found for a fixed dimensionless pressure drop number Be in the range $10^5 - 10^8$, and a Prandtl number of 0.71. These initial results were compared with the available literature to validate the numerical code. The procedure was repeated several times to cover the

range $10^5 \leq Be \leq 10^8$. Figure 3 shows that the variation of channel sizes reveals a maximum of the heat transfer rate density. The figure shows that the optimum size for $Be = 10^5$ is located between $\tilde{D} = 0.24$ and 0.25 on the abscissa.

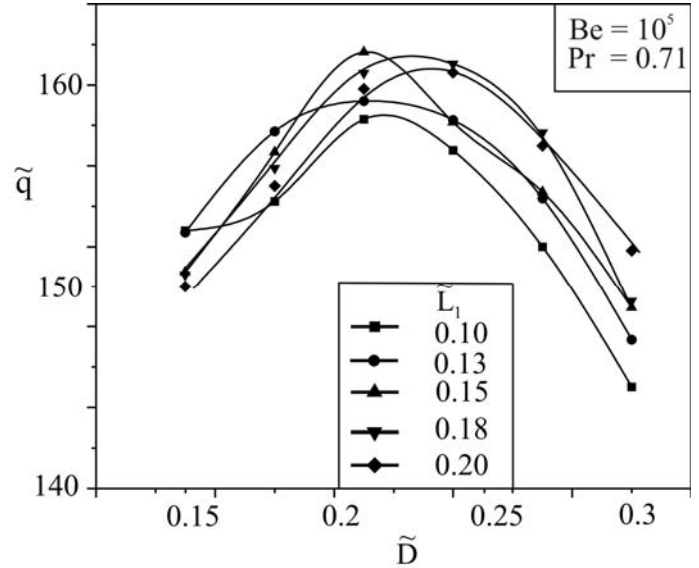


Figure 4 The effect of \tilde{L}_1 and \tilde{D} (wrinkled) on the heat transfer density.

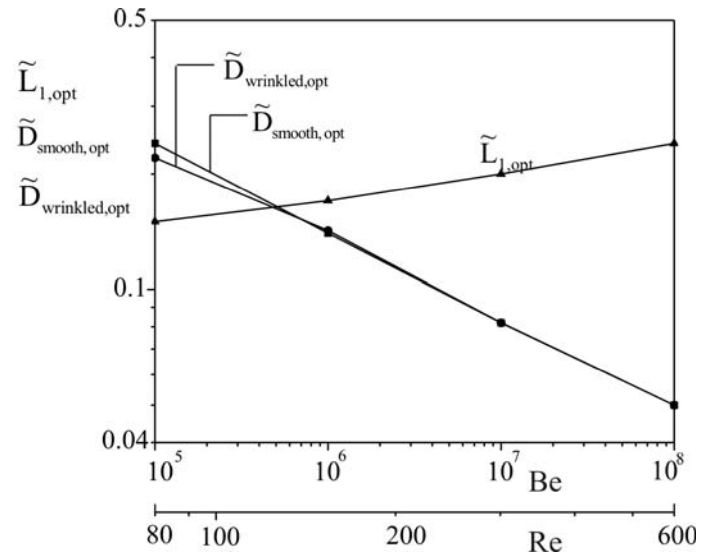


Figure 5 Effect of dimensionless pressure difference on the optimized length scales

To enhance the heat transfer rate density, we wrinkled the mouths of the square channel in order to bring the walls closer to the unused core fluid. The total volume of the wrinkled duct is the same as that of the smooth duct. The new structure has two degrees of freedom. The dimensionless length of the fold and the side of the square cross-section are defined as

$$\tilde{L}_1 = \frac{L_1}{L_0} \quad (16)$$

The optimization was executed in the following sequence: We first assumed values of \tilde{D} and \tilde{L}_1 , using as initial

guess the optimal values of \tilde{D} were obtained when there are no wrinkles. The optimization was performed in two nested loops, because of the two degrees of freedom, \tilde{D} and \tilde{L}_1 . In the inner loop, the values of Be , Pr , and \tilde{D} (wrinkled) were specified, and \tilde{L}_1 was varied until a maximum heat transfer rate density was reached. The outer loop required that the inner loop was repeated for several values of \tilde{D} , such that all the possible combinations of \tilde{D} and \tilde{L}_1 were analyzed, as shown in Fig. 4. The resulting heat transfer rate density was maximized with respect to both \tilde{D} and \tilde{L}_1 . The above procedure was repeated for Be in the range $10^5 \leq Be \leq 10^8$.

Figure 5 shows the behaviour of the optimized length scales as the Be number changes. The optimized length scale ($\tilde{L}_{1,opt}$) increases with the dimensionless pressure drop number. The slenderness of the channel also increases in this direction. The optimal duct size decreases as the dimensionless pressure drop increases. The optimal duct sizes for the wrinkled and smooth ducts are almost the same. The results presented in Fig. 5 are correlated by

$$\begin{aligned}\tilde{D}_{smooth,opt} &\cong 3.27Be^{-0.24} \\ \tilde{D}_{wrinkled,opt} &\cong 2.73Be^{-0.22} \\ \tilde{L}_{opt} &= 0.067Be^{0.068}\end{aligned}\quad (17)$$

This behaviour is in agreement with previous results for optimal spacings for square channels [15]. The assumption of laminar flow is validated by calculating the Reynolds number based on the mean velocity and optimal duct size based on the smooth ducts.

$$Re = \frac{\rho U D_{smooth,opt}}{\mu} = \tilde{U} \tilde{D}_{smooth,opt} \frac{Be}{Pr}\quad (18)$$

Equation (18), shows the relationship between the dimensionless pressure drop number Be , the mean velocity, the optimal duct size and the Prandtl number. By determining \tilde{U} and $\tilde{D}_{smooth,opt}$ numerically, we find that the Reynolds numbers of the flows lie in the laminar regime (Fig. 5). Expressed in terms of Re , the results are correlated as

$$\begin{aligned}\tilde{D}_{smooth,opt} &\cong 8.16Re^{-0.80} \\ \tilde{D}_{wrinkled,opt} &\cong 6.54Re^{-0.77} \\ \tilde{L}_{opt} &= 0.051Re^{0.24}\end{aligned}\quad (19)$$

Figure 6 shows the variation of the maximal heat transfer rate density for the wrinkled and the smooth ducts. Both increase with the dimensionless pressure difference. When Be is in the range $10^5 - 10^6$, the heat transfer density increases by 17 % in going from the optimized smooth duct to the optimized wrinkled duct. The increase is 13% for Be in the range $10^7 - 10^8$. The results of Fig. 6 are correlated by

$$\begin{aligned}q_{max,smooth} &\cong 0.38Be^{1/2} \\ q_{max,wrinkled} &\cong 0.45Be^{1/2}\end{aligned}\quad (20)$$

The trend is similar to the results of Muzychka [15] and the earlier work on the optimal spacing for convection [1]. The 13 - 17 % increase in heat transfer rate density is consistent with the

increase achieved in Ref. [3] by inserting shorter (optimal) plates in the entrance region of stacks of parallel-plates channels.

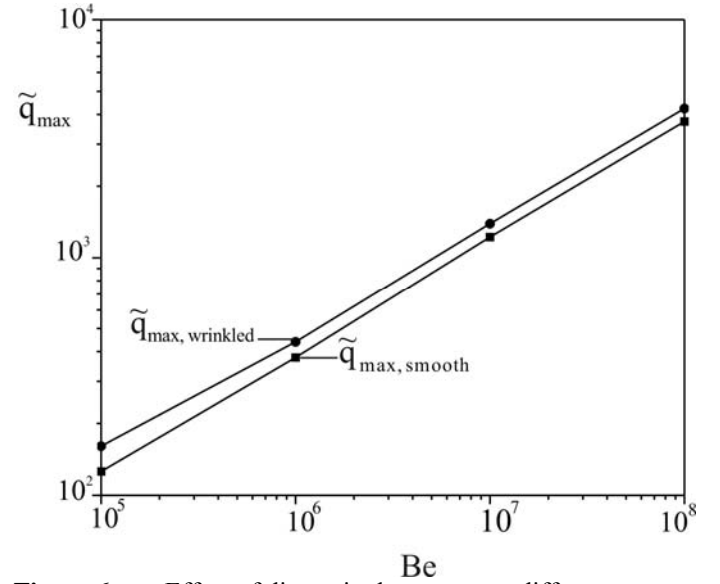


Figure 6 Effect of dimensionless pressure difference on the heat transfer rate density.

Figure 7 show the temperature distributions of optimized wrinkled ducts at several axial locations for $Be = 10^5$ and $Pr = 0.71$. The figure shows the evolution of the temperature distributions as the axial distance increases. The temperature ranges between two colours, red ($\tilde{T}=1$) and blue ($\tilde{T} = 0$). As the axial distances increases, the colour at the central plane of the wrinkled ducts changes from blue at the wrinkled entrance to red at the exit (square).

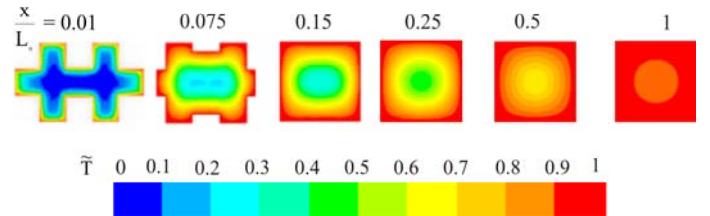


Figure 7 The temperature distribution at the optimize entrance, midsection and outlet of the wrinkled ducts for $Be = 10^5$.

CONCLUSION

In this paper, we described a new design for increasing the heat transfer rate density by using wrinkled entrance regions in ducts with laminar forced convection. The method consists of wrinkling the perimeter of the square entrances, and by extending these deformations gradually downstream to a length (\tilde{L}_1) that is optimized. This new geometry allows a more complete heat transfer interaction between the walls and the fluid. The results show that the heat transfer rate density increases with the imposed pressure difference (Be). In the wrinkled geometry the optimal length of the fold increases with

Be, in accord with results reported in the literature for multi-scale packages of parallel plates. The results show an increase of 15 % in heat transfer rate density in going from smooth ducts to wrinkled ducts.

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