

NON ITERATIVE MODEL FOR STEAM CONDENSATION IN PRESENCE OF NON-CONDENSABLE GASES INSIDE PASSIVE CONTAINMENT COOLING VERTICAL TUBES

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ABSTRACT

The modelling of condensation in presence of non-condensable gases is of relevance for the design of passive containment cooling condenser of the third generation of Passive Nuclear Power Plants. Fast and accurate methods of predictions for condensation in presence of non-condensable gases are necessary in order to be implemented in the thermal-hydraulic codes without slowing down the computational speed of these codes. In this paper we present a mechanistic model for condensation in presence of non-condensable gases inside vertical tubes. In this model we take into account the influence of the non-condensable gases over the liquid side heat transfer without any iteration to calculate the liquid-steam interfacial temperature. The trick is to perform a set of Taylor expansions for the main physical magnitudes as viscosity, steam mass fraction and so on. We also consider the interfacial shear stress exerted by the steam-non-condensable mixture flow over the condensate layer thickness. The calculation of the condensate layer thickness can be performed with the help of the mass, energy and momentum conservation equations and can be achieved without any iteration following the method of Munoz-Cobo et al [1,2]. The new proposed mechanistic model solves explicitly the real interfacial temperature by means of a cubic or a quartic equation depending on the degree of approximation that has been chosen. Moreover, as the main non-condensable effects can be accounted for in the heat and mass transfer processes, the new model will be more realistic. The model has been validated with the Vierow experimental data, obtaining a total average relative error, for the fourth order equation method model, of 21% with 268 experimental points at different conditions.

INTRODUCTION

Modelling of thermal-hydraulic phenomena driven by natural circulation is a matter of concern in the nuclear industry. A great number of new generation reactors use the so-called passive safety systems, which can work without the

action of an operator within a considerable time interval (usually, at least 72 hours after the initial event).

One of these reactors is the ESBWR (Economic and Simplified Boiling Water Reactor), whose Passive Containment Cooling System (PCCS) is based on the condensation of the steam released into the containment after a primary system blow-down. The steam will move upward by natural circulation, dragging some air from the containment. This gas mixture will come into a set of vertical tubes which are immersed in a water pool situated at a higher level, the heat released by condensation is transferred to this pool.

NOMENCLATURE

C_{pl}	[J/Kg]	Liquid specific heat at constant pressure
D	[m^2 / s]	Diffusion coefficient
d	[m]	Internal tube diameter
f		Friction factor
h	[W/m ² K]	Heat transfer coefficient
h_c	[W/m ² K]	Condensation heat transfer coefficient
h_{fg}	[J/Kg]	Specific phase change enthalpy
h_{pfg}	[J/Kg]	Specific phase change enthalpy plus subcooling to the average temperature of the condensate layer
h_g	[W/m ² K]	Gas boundary layer heat transfer coefficient
h_s	[W/m ² K]	Sensible heat transfer coefficient
h_l	[W/m ² K]	Liquid film heat transfer coefficient
h_m	[Kg/m ² s]	Mass Transfer coefficient
k	[W/mK]	Thermal conductivity
M_m	[Kg/Kgmol]	Gas mixture molecular weight
M_V	[Kg/Kgmol]	Steam molecular weight
\dot{m}_c	[Kg/m]	Condensation mass flux moving to the interface
Nu	[--]	Nusselt Number
q''	[W/m ²]	Heat flux
Sc	[--]	Schmidt Number
St	[--]	Stanton number
T	[K]	Temperature
u_i	[m/s]	Liquid velocity at the interface
u_m	[m/s]	Mixture velocity
W	[--]	Mass fraction

Special characters

Γ [Kg/ms] Condensate mass flow rate per unit of circular length

δ	[m]	Condensate film thickness
μ	[N s/m ²]	Dynamic Viscosity
ρ	[Kg/m ³]	Density
τ	[Pa]	Shear stress

Subscripts

a	Non condensable
b	Calculated at the bulk temperature or conditions
g	Steam-non condensable mixture
i	Interface
l	Condensate
m	Relative to mass transfer
r	Value for the real case
s	sensible
sat	saturation
v	Vapour or steam
w	Wall conditions

There have traditionally existed two main ways of modelling the heat and mass transfer inside tubes. The first one is the empirical approach followed by Vierow and Kunh [3,4] based on deriving a correlation from experimental data, which considers the non-condensables effect over the heat transfer as a degradation factor with respect to the pure steam case, theoretically calculated from Nusselt theory. Usually, this correlation is a function of the non-condensable mass fraction and the gas mixture Reynolds number.

The second modelling way is based on considering the different phenomena that are involved in the heat and mass transfer processes; this mechanistic method has been used by Ghiaasiaan, Collier and Sparrow [5,6,7]. Two paths can be distinguished related with this method: the first one consists of a numerical approach based on solving the mass, momentum, energy, and conservation species over the gas boundary layer and the film boundary layer; the second one is based on a heat balance equation -usually at the interface- between the latent plus the sensible heat contributions, and the heat transferred through the condensate film to the wall, where different corrections are applied in order to consider the other phenomena. This second way relies generally on the HMTA (heat and mass transfer analogy) to calculate the mass transfer coefficient.

With respect to the first kind of mechanistic modelling, Sparrow and Mynkowycz [7,8] developed one of the earliest investigation on condensation in presence of non-condensable gases.

Concerning the models that are based on an energy balance at the interface, this kind of models use to differ among them mainly depending on how the mass transfer coefficient - and therefore the condensation heat transfer- is computed, and secondary, which simplifications and correlations are used. Regarding on the mass transfer (mass flux) expressions, a recent sensitivity study has been performed by Malet [9], where different mass flux formulations were compared with experimental data, basically, the Chilton and Bird formulation [10,11] and the Collier and Stephan formulation [6,12]. These two expressions differ in the mass transfer coefficient, and while the Chilton coefficient considers the heat and mass transfer correction due to suction effects, Collier considers film's theory hypothesis, which states that the convective non-

condensable gases term towards the interface through the gas-diffusion boundary layer is equal in magnitude to the diffusion term towards the bulk. These differences make that the Collier's expression results in values lower than 40% with respect to Chilton's ones. But as the mass flux expression is embodied in the condensation model, usually this term is not separately considered.

Other widely used condensation heat transfer coefficient uses the Peterson condensation conductivity [13] This author integrates the one-dimensional gas-vapor diffusion mass equation -considering also the convective term- over the gas diffusion boundary layer, to obtain an average gas-vapour velocity, and with the use of the HMTA, the author obtains the condensation heat transfer coefficient. The difference with Collier's model lies in the particular form of Peterson's condensation heat transfer coefficient, based on both the use of the ideal gas law and the Clausius-Clapeyron equation to transform the difference in the partial pressures of the steam along the boundary layer in a difference of temperatures.

PHENOMENOLOGY OF THE PROCESS

Once the gaseous mixture enters into the tube, whose wall has a temperature below the saturation temperature at the steam partial pressure in the bulk, the vapour will start condensing, and a thin liquid film will quickly be established (filmwise regime). Then, a condensation mass flux (together with the noncondensables) will be stated due to the suction caused by the phase change. Due to the very small solubility of the non-condensable gases in the water at the typical PCCS performance conditions, they will start accumulating closely to the interface, so giving place to a gas boundary layer thermal resistance. In this new situation, the vapour flux will move towards the interface due to the combination of two mechanisms: on the one hand, because of the phase change suction (obviously modified with respect to the pure steam case) due to a total pressure gradient, and on the other hand, because of the vapour diffusion by the Fick's law through the gas boundary layer, due to the existing molar fraction gradient between the bulk and the interface. The result is a degradation of the heat and mass transfer process. This degradation is physically characterized by a reduction of the condensation mass flux, or, equivalently, of the interfacial vapor mass fraction.

Therefore, the influence of the noncondensables over the transfer process is double: first, by means of a new thermal resistance, and second (also in order of importance), by means of a liquid side thermal resistance variation, through a modification of the interfacial temperature.

Because the condensation in presence of non-condensable gases is a complex phenomenon in which a set of phenomena are involved, is highly suitable, in order to model this phenomenon, to make a distinction between those of primary and secondary importance. With respect to the primary order group, the most important phenomenon is the vapor condensation and, therefore, the condensation flux moving toward the interface. The heat transfer flux from the interface to the wall, which passes through the liquid film, has also great importance. Some of the second order group phenomena would

be the interfacial shear stress, different for each phase, because the condensed vapour momentum is transferred to the liquid side, causing a shrinkage of the film; the rippled structure of the liquid film; the mist formation, that consists of an anticipated condensation of vapour before reaching the interface, forming liquid droplets within the gas, etc.

All these phenomena can be considered in a mechanistic model that starts from a heat balance equation at the interface, and putting in the second order group phenomena as an addition over the main model. This model assumes that the total heat transfer from the gas bulk to the wall is characterized by a set of thermal resistances. First, there exist two thermal resistances in parallel in the gas diffusion boundary layer, one for the sensible and the other one for the latent heat. The resultant thermal resistance is coupled in series with the liquid film thermal resistance, usually considered as a purely conductive transfer (so neglecting the convective term). Therefore, obtaining these two heat transfer coefficients (liquid and gas layers) will represent the basis for the model efficiency.

MODEL CHARACTERIZATION

Nowadays, several integral models for the condensation in presence of noncondensables inside vertical tubes can predict the experimental data with a good degree of accuracy. Even so, the modelling encounters different problems that have to be faced:

1. On the one hand, the mechanistic model approach is usually more precise than the empirical approach one [2], so the latter is not highly recommended, besides of accounting only for a restricted range of physical conditions.

2. At the same time, the mechanistic models result to be much more complex when they have to be implemented in a thermal-hydraulic code, producing also a slowdown on the computation process caused by their iterative nature (due to the inclusion of the interfacial parameters in the calculations).

In order to solve these issues, No and Park [14] developed a fully non-iterative nature model in which the condensation heat transfer coefficient for the gas layer is formulated not as a function (either explicit or implicit) of the interfacial parameter. Linking this formulation with some of the models for the liquid side heat transfer, an integral model is finally obtained.

The problem in the No and Park model is that, in order to keep the non-iterative nature of the model, they have to compute the liquid heat transfer coefficient considering the interfacial temperature as the saturated vapour temperature at its vapour partial pressure at the bulk conditions, i.e., $T_i = T_{sat}(P_{v,b})$. In fact, the interfacial temperature drops to a value that already does not depend only on the vapor partial pressure in the bulk, but also depends on the mass diffusion process through the gas boundary layer, which causes a vapor concentration gradient in such a way that the interfacial vapor mass fraction (and so, the vapor partial pressure) is lower due to the non-condensable gas accumulation near the interface.

This means that the influence of the non-condensable gases over the liquid film thermal resistance is not considered. This simplification gives place to a variation of the liquid heat transfer, because both the thickness layer and the liquid thermal

conductivity strongly depend on the liquid properties (through the temperature gradient through the film, the viscosity, the enthalpy, the density, etc.).

In order to solve this problem, the variation of the liquid heat transfer produced by the interfacial temperature variation (due to the presence of non-condensable gases) has been considered, obtaining an equation whose solution is the real interfacial temperature. Thus, the model will be able to account for the non-condensable effects with a better degree of accuracy and without any kind of iteration.

In this paper, an explanation of the theoretical model, considering different methods to compute the liquid variables, is presented. Then, some comparisons with the Vierow experimental data [3] have been performed, not only with the new models, but with the No and Park model too, which has been computed with different correlations for the mass transfer, in order to assess the sensitivity of the used mass correlations, as well as the enhancement of the proposed contributions. Finally, some conclusions will be stated.

3.1. The physical model

For solving the heat and mass transfer problem, a heat balance equation at the interface -with a steady state condition- is formulated as follows:

$$q_t'' = h_l(T_i - T_w) = h_g(T_b - T_i) \quad (1)$$

where h_l and h_g are the liquid and gas heat transfer coefficients, respectively, and T_b , T_i , and T_w are the temperatures at the bulk, interface, and wall respectively. Therefore, the heat transfer from the bulk gas to the wall tube will consist of two thermal resistances coupled in series:

$$1/h_t = 1/h_l + 1/h_g \quad (2)$$

where h_t is the total heat transfer coefficient.

With respect to the liquid side thermal resistance, owing to its very thin thickness (from tens to hundreds of microns), the heat transfer coefficient is expressed as follows:

$$h_l = \frac{k_l}{\delta} \quad (3)$$

Where k_l is the liquid thermal conductivity, and δ is the liquid film thickness.

For the liquid film thickness calculation, we have used the Muñoz-Cobo et al. expression [1]. This model modifies the thickness computed by Nusselt accounting for the next issues:

1. The condensation does not take place over a flat vertical plate, but in the internal side of a vertical tube.
2. The interfacial shear stress, considering the vapor condensation influence (the momentum transport of the condensation flux).

An advantage of this model is that it does not need to iterate in order to obtain the film thickness.

With respect to the gas-mixture boundary layer, the condensing heat transfer and the sensible heat transfer due to the temperature jump between the bulk and the interface must be accounted for. Therefore, the heat transfer coefficient of the gas phase from the bulk to the interface is written as follows:

$$h_g = h_c + h_s \quad (4)$$

where we have followed the Colburn-Hougen model [15], expressing the gas-vapor boundary layer coefficient as a parallel combination of a sensible and a condensation coefficient. Here, h_c stands for the condensation heat transfer coefficient, and h_s for the sensible heat contribution. For the sensible heat transfer, we have used a Stanton correlation which will be shown below.

The condensation heat transfer can be expressed as:

$$h_c = \frac{\dot{m}_{cond}'' h_{pfg}}{(T_b - T_i)} \quad (5)$$

where the condensation mass flux, \dot{m}_{cond}'' , has been obtained as the result of a convective plus a diffusive contribution using the Fick's law:

$$\dot{m}_{cond}'' = \left[-\rho_g D \nabla W_v \right]_i + (W_v \dot{m}'')_i = -h_m \frac{W_{v,i} - W_{v,b}}{1 - W_{v,i}} \quad (6)$$

where we have considered that the condensate surface is impermeable to the non-condensable gases, and we have linearized the mass fraction evolution through the boundary layer.

Here, W_v is the steam mass fraction computed at the interface (i subscript) or at the bulk (b subscript), \dot{m}'' is the gas-vapor flux moving to the interface, and D is the diffusion coefficient of the vapour in the air. Then, h_m , which is the mass transfer coefficient, will be:

$$h_m = \frac{\rho_g D}{\delta_g} \quad (7)$$

where δ_g is the gas-vapor diffusion boundary layer thickness. By means of the HMTA, we have used the same Stanton correlation mentioned above for obtaining h_m .

The iterative nature of the model lies in the fact that the interfacial temperature is implicit, on the one hand, in the liquid film thermal-hydraulic properties calculation, as well as in the friction factor and in the condensation mass flux, and on the other hand, in the Nusselt liquid film thickness.

To avoid the iteration procedure on the gas layer side, No and Park [14] have computed the condensation heat transfer coefficient in such a way that the temperature and the steam mass fraction at the interface are set as a function of bulk parameters. Therefore, the gas-vapour boundary layer thermal resistance will be stated in terms of known variables.

The problem will lie in the influence of the non-condensable gases that takes place over the liquid film through the variation of the interface temperature value, as it will be shown below.

In order to avoid the interface temperature implicit and explicit contribution, we have developed a simple method which leads us to an equation where the interfacial temperature influence is only explicit, so we can directly solve the equation to obtain the real value of this temperature. Finally, taking up again equation (1), the total heat transfer will be computed.

With respect to the condensation heat transfer computation, rearranging the heat balance equation (1) with the help of equation (4), it is possible to obtain the following equation:

$$(T_b - T_i) = \frac{h_l}{h_l + h_c + h_s} (T_b - T_w) \quad (8)$$

Expanding the mass fraction of steam at the interface $W_{v,i}$ in Taylor series of $(T_b - T_i)$, and retaining the first order term and using equation (8), yields:

$$W_{v,i} \approx W_{v,b} + \left. \frac{\partial W_v}{\partial T} \right|_b (T_i - T_b) \quad (9)$$

Therefore we can write:

$$1 - W_{v,i} = 1 - W_{v,b} + \frac{h_l}{h_l + h_c + h_s} (T_b - T_w) \left. \frac{\partial W_v}{\partial T} \right|_b \quad (10)$$

Using now equations (5), (6), (8), and (10), and after some simple calculations, the following expression for the condensation heat transfer is obtained:

$$h_c = h_m h_{pfg} \frac{\left. \frac{\partial W_v}{\partial T} \right|_b}{\left(1 - W_{v,b} + \frac{h_l}{h_l + h_c + h_s} (T_b - T_w) \left. \frac{\partial W_v}{\partial T} \right|_b \right)} \quad (11)$$

where the subcooling effect has been considered inside No and Park formulation. Making use of the Clausius-Clapeyron equation, it is possible to expand the partial derivative:

$$\frac{\partial W_v}{\partial T} = \frac{\partial W_v}{\partial P_v} \frac{\partial P_v}{\partial T} = \frac{1}{P} N_A \frac{\partial P_v}{\partial T} \approx \frac{h_{pfg} \rho_v}{PT} N_A \quad (12)$$

where N_A is a constant depending on the steam and NC-gas molar weights and mass fractions:

$$N_A = \frac{M_a M_v}{[M_a (1 - X_v) + M_v X_v]^2} \quad (13)$$

Finally, a second degree equation in h_c with constant coefficients is obtained, with solutions given by

$$h_c = \frac{-B \pm |B| \sqrt{1 - 4AC/B^2}}{2A} \quad (14)$$

Where:

$$\begin{aligned} A &= 1 - W_{v,b} & B_{2T} &= \frac{h_{pfg} \rho_v}{PT} (T_b - T_w) N_A \\ B &= H_1 A + H_2 B_{2T} \\ C &= H_1 B_{3T} & B_{3T} &= \frac{g h_{pfg}^2 \rho_v}{PT} N_A \\ H_1 &= h_l + h_s \text{ and } H_2 = h_l \end{aligned} \quad (15)$$

Because A is always positive, and C is always negative, then we have only one real solution.

As it has been stated above, in order to keep the non-iterative nature of the model, No and Park considered the interface temperature as the saturated vapour temperature at its vapour partial pressure i.e $T_i = T_{sat}(P_{v,b})$, which is true only for the pure steam case. But if non-condensable gases are present, then the interfacial temperature will drop to a value that will not depend only on the vapour partial pressure in the bulk, but also

on the accumulation of non-condensable gases at the interface dragged by the convective current and partially diffused away from the interface (see Fig. 1). This will cause a vapour concentration gradient in such a way that the vapour mass fraction at the interface (therefore, the vapour partial pressure) will be lower than in the case without the mass diffusion process (the pure steam case). We have considered that the total pressure remains constant through the gas boundary layer.

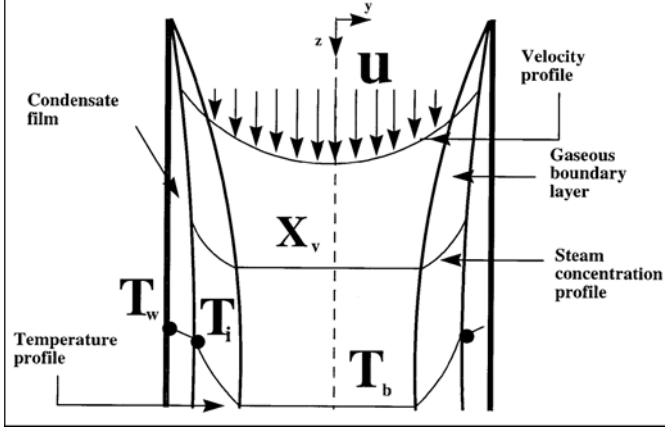


Fig. 1. Temperature, steam concentration, and velocity profiles in condensation with non-condensable gases inside vertical tubes.

Following this approach, the influence of the non-condensable gases over the liquid film thermal resistance will be neglected.

In order to avoid this simplification, let us consider again equation number (1). If this is called the real heat transfer flux, denoted as q_r'' , and the heat transfer flux for the pure steam case is called q_i'' , then:

$$\begin{aligned} q_r'' &= q_i'' \frac{q_r''}{q_i''} = h_{l,t} (T_{sat,b} - T_w) \frac{h_l}{h_{l,t}} \frac{(T_i - T_w)}{(T_{sat,b} - T_w)} \\ &= \underbrace{\frac{h_l}{h_{l,t}} \frac{(T_i - T_w)}{(T_{sat,b} - T_w)}}_{\text{degradation factor}} h_{l,t} (T_{sat,b} - T_w) \end{aligned} \quad (16)$$

where $h_{l,t}$ is the liquid film heat transfer coefficient for the pure steam case, and $T_{sat,b}$ is the vapor saturation temperature at its partial pressure in the bulk. The difference between $h_{l,t}$ and h_l will lie in how the interfacial temperature and, therefore, the liquid temperature, are computed. Relative to the nomenclature, we denote $T_{l,b}$ to the liquid temperature assuming that the interfacial temperature is the saturated vapour temperature at its vapour partial pressure, and $T_{l,i}$ to the liquid temperature considering T_i as an unknown variable. The liquid temperature is computed as follows:

$$\begin{aligned} T_{l,b} &= T_w + f \cdot (T_{sat,b} - T_w) \\ T_{l,i} &= T_w + f \cdot (T_i - T_w) \end{aligned} \quad (17)$$

where f has been taken as 0.31, [6].

Following the empirical approach, the problem consists of solving a theoretical -instead of an empirical- degradation factor, not for the heat transfer coefficient, but for the heat transfer itself.

The ratio between both liquid heat transfer coefficients can be expressed as follows:

$$\frac{h_l}{h_{l,t}} = \frac{k_l(T_{l,i})}{k_l(T_{l,b})} \frac{\delta(T_{l,b})}{\delta(T_{l,i})} = \text{rat}k_l \text{ rat}\delta \quad (18)$$

where $\text{rat}k_l$ and $\text{rat}\delta$ stand for the thermal conductivity and film thickness ratio. Expanding now $\text{rat}\delta$ using Muñoz-Cobo et al [1] expression for the condensate layer thickness inside a pipe with shear stress forces, it is obtained after some approximations the following result:

$$\text{rat}\delta = (\text{rat}\delta_N^*) \frac{\left[2 + \frac{4l_i(T_{l,i})\text{rat}\delta_N^* + m_i(T_{l,i})}{3\delta_p(T_{l,b})} \right]^{1/3}}{\text{den}} \quad (19)$$

Where δ_p is the condensate film thickness that is obtained neglecting the shear stress, and is calculated as in reference [1], and the ratio of Nusselt thicknesses can be approximated by:

$$\text{rat}\delta_N^* = \frac{\delta_N^*(T_{l,b})}{\delta_N^*(T_{l,i})} \approx \left[\text{rat}\mu \frac{(T_{sat,b} - T_w)}{(T_i - T_w)} \right]^{1/4} \quad (20)$$

where the denominator polynomials in the expression for δ of reference [1], have been simplified, taking only their constant coefficient -since x_p (ratio of condensate thickness to radius) is a very small value-, and the coefficients a_l , b_l , and c_l , have already been substituted by their respective values. The coefficients l_i and m_i , depend on the interfacial shear stress and are given in reference [1]. Finally, den is given by:

$$\text{den} = \delta_p(T_{l,b}) + \frac{4l_i(T_{l,b})}{3\delta_p(T_{l,b})} + \frac{m_i(T_{l,i})}{2} \quad (21)$$

From the classical expression of Nusselt film thickness, we arrive to the following approximate expression:

$$\text{rat}\delta_N^* \approx \left[\text{rat}\mu \frac{(T_{sat,b} - T_w)}{(T_i - T_w)} \right]^{1/4} \quad (22)$$

where $\text{rat}\mu$ is the viscosity ratio.

If we now raise to the third power the heat balance equation (1), and we approximate $\text{rat}k_l \approx 1$, we obtain:

$$h_g^3 (T_b - T_i)^3 = \frac{(q_i'')^3 (T_i - T_w)^3}{\text{den}^3 (T_b - T_w)^3} \text{rat}\delta_N^{*3} (b + c \text{rat}\delta_N^*) \quad (23)$$

where we have defined:

$$\begin{aligned} b &= 2 + m_i / 2 \\ c &= \frac{4}{3} \frac{l_i}{\delta_p(T_{l,b})} \end{aligned} \quad (24)$$

In expressions (24) m_i and l_i have been evaluated at the bulk temperature of the condensate layer i.e. $T_{l,b}$.

Operating in equation (23), on account of equation (22) and the following approximation:

$$\left(1 / \text{rat}\mu \frac{(T_i - T_w)}{(T_{sat,b} - T_w)} \right)^{1/4} \approx 1 \quad (25)$$

One gets:

$$h_g^3 (T_b - T_i)^3 = \frac{(q_i'')^2 (T_i - T_w)^2}{den^3 (T_b - T_w)^3} (T_{sat,b} - T_w) rat\mu (b + c) \quad (26)$$

Then, after a little algebra the following cubic equation for the interfacial temperature T_i is obtained:

$$c_1 T_i^3 + c_2 T_i^2 + c_3 T_i + \{T_b^3 D_c / rat\mu - T_w^2\} = 0 \quad (27)$$

where the constants c_1 , c_2 and c_3 are given by

$$c_1 = -\frac{D_c}{rat\mu}, c_2 = \left[\frac{3T_b}{rat\mu} - 1 \right], c_3 = -\frac{3T_b^2 D_c}{rat\mu} + 2T_w \quad (28)$$

$$D_c = A_c / (b + c), \text{ and } A_c = \left[\frac{h_g den (T_b - T_w)}{q_i''} \right]^3 \frac{1}{(T_{sat,b} - T_w)}, \quad (29)$$

As the discriminant of equation (27) is always positive, there is only one real solution.

Once the interfacial temperature is known, for knowing the heat transfer flux it will be enough to take up the heat balance equation (1), and to substitute h_g by its respective value.

With respect to the viscosity ratio, $rat\mu$, two alternatives have been considered. The direct way of resolution is through a numerical approach, knowing that the temperature range is limited by $T_{l,b}$ and $T_{l,i}$. In this way, we could implement an approximated ratio. This has been the adopted solution for the cubic equation method.

Other way of solving the viscosity ratio would be to expand the liquid viscosity at $T_{l,i}$ by means of a Taylor series:

$$\mu(T_{l,i}) \approx \mu(T_{l,b}) + \left. \frac{\partial \mu}{\partial T} \right|_{l,b} (T_{l,i} - T_{l,b}) \quad (30)$$

where the simplification made in the Taylor expansion is valid since the temperature range is small. Applying equation (17):

$$(T_{l,i} - T_{l,b}) = f(T_i - T_{sat,b}) \quad (31)$$

then, the viscosity can be set as a function of the interfacial temperature:

$$\mu(T_{l,i}) \approx \mu(T_{l,b}) - \left. \frac{\partial \mu}{\partial T} \right|_{l,b} f(T_{sat,b} - T_i) \quad (32)$$

where the partial derivative can be obtained from any correlation which sets the viscosity as a function of the temperature.

If equation (32) is substituted in equation (26), then a quartic equation for T_i is obtained.

The most important correlations implemented in the model are the followings:

i) The Wallis' expression for the interfacial friction factor[16]:

$$f_i = f_g \left(1 + 360 \frac{\delta}{d} \right) \quad (33)$$

where f_g is the friction factor of the steam and non-condensable mixture for smooth tube wall:

$$f_g = 0.079 Re_g^{-0.25} \text{ for } Re_g > 2300$$

$$f_g = \frac{16}{Re_g} \text{ for } Re_g < 2300 \quad (34)$$

ii) The Reynolds-Colburn analogy and its equivalent form by means of the HMTA:

$$St = \frac{f_i}{2} \cdot Pr^{-2/3}$$

$$St_m = \frac{f_i}{2} \cdot Sc^{-2/3} \quad (27) \quad (35)$$

A complete discussion of the heat and mass transfer analogy and its application to condensation problems has been given by ambrosini et Al [17].

iii) A correction for the entrance effect due to Sidiq [18]:

$$Nu = \left(1 + \frac{0.8 (1 + 70000 Re_g^{-1.5})}{z/d} \right) Nu_0 \quad (36)$$

iv) The suction mass effect is taken into account calculating the interfacial shear stresses acting on the mixture side and the condensing fluid side by means of the expressions [1,16]:

$$\tau_{im} = \tau_{i0} \frac{a'}{e^{a'} - 1} \quad (37)$$

$$\tau_{if} = \tau_{i0} \frac{a' e^{a'}}{e^{a'} - 1} \quad (38)$$

Where a' is the ratio of the mass flow condensing per unit area in the tube to the mass flux hitting the interface and rebounding after giving up its momentum to the interface. τ_{i0} is the interfacial shear stress in the absence of phase change.

RESULTS AND VALIDATION

A set of comparisons were carried out with the Vierow steam plus non-condensable gas condensation experiments inside vertical tubes. We have compared the following models:

- 1-The No and Park model.
- 2-The No and Park model using the Reynolds-Colburn analogy instead of the Dipprey's correlation for the convection and condensation (by means of HMTA) heat transfer coefficients.
- 3-The cubic equation model; where an approximation has been used to account for the viscosity variation.
- 4-The quartic equation model.

In order to perform the comparisons, the models have been implemented in a computer program written in FORTRAN 77 language.

We have compared the average relative error of the different models with a total of 268 points in 22 gas-vapour experiments from Vierow database. The total gas-vapor tests run were actually 25, but two of them are lack of interest in order to validate a model because their results were anomalous, as Vierow verified.

Some of the main consequences related with the model comparisons are the followings:

1-Fig. 2 displays a general comparison of the experimental heat transfer coefficient versus the calculated one computed with the quartic equation method. As it can be seen, most of the points are within an error of $\pm 20\%$.

2-We have computed the average relative errors (268 points) for the different models with the following merit function:

$$\varepsilon_r (\%) = \frac{1}{n} \sum_{j=1}^n \left| \frac{h_{cal,j} - h_{exp,j}}{h_{exp,j}} \right| 100 \quad (39)$$

The model which gives a better prediction, from this general comparison, is the quartic equation method, which gives an average error of 21.09%.

3.-The average relative errors for all the measured points of run numbers 30, 31, and 32 are most of the times inside a 10% range, excluding only the No and Park model using Dipprey's correlation.

4-Some comparisons are shown in Fig. 3. From the test comparisons depicted, it can be stated that the quartic equation method gives better results than the other methods.

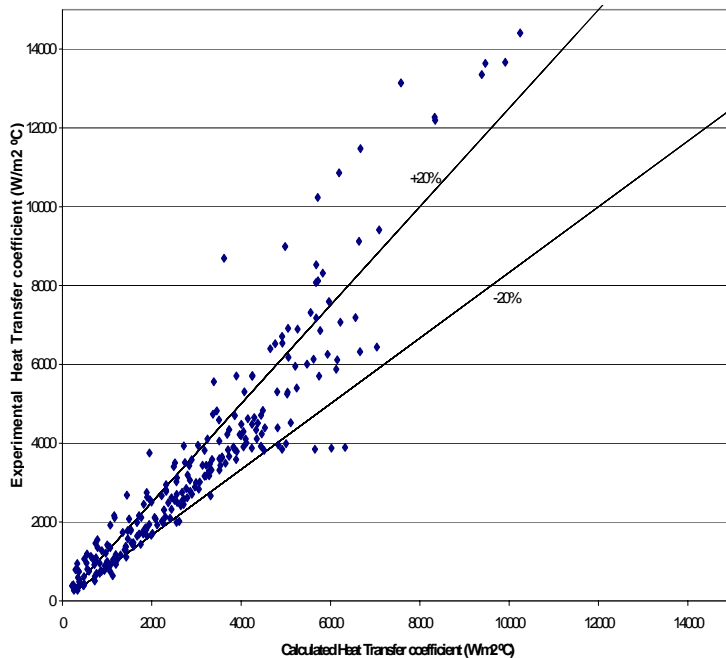


Fig. 2. Experimental and calculated heat transfer coefficient with the quartic equation method.

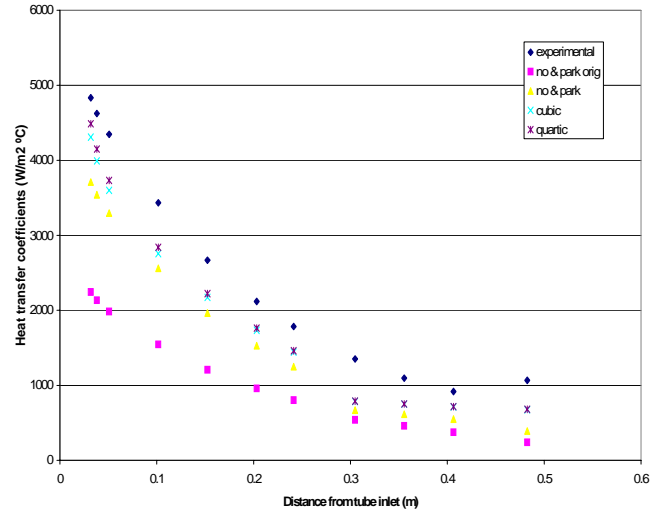


Figure 3. Variation of the heat transfer coefficients along the length of the tube computed using the different models for Vierow run number 9.

5-The total improvement of the model when considering the two new contributions is close to 7%. The fact of accounting for the variation of the film thickness - and therefore the liquid side heat transfer coefficient-, considering the viscosity variation by means of a numerical approach, yields an improvement over the total error of about a 4%. The inclusion of a more realistic viscosity variation due to a different interfacial temperature is less than 3%.

6-However, there is a major shift when any of the models is compared with the No and Park original model. This model presents the main difference of using Dipprey's correlation instead of Reynolds-Colburn analogy in order to compute the mass transfer coefficient. This means that the primary variable of the model, upon the heat transfer process is based on, is the condensation mass flux calculation.

7-In all models, the condensation mass flux is computed as the product of a concentration gradient and a transfer coefficient, which is solved using the HMTA, by means of an empirical correlation for the heat process. Depending on the used correlation for the mass transfer coefficient, the error of the model can change even beyond 50%. Thereby, the use of the HMTA must be analyzed in its theoretical basis. Some studies have already been performed by Herranz et al [19], but without stressing the sensibility over the different empirical correlations used in the HMTA (in this sense, some comparisons performed in this paper can be seen as a "sensitivity study"). The adequacy of the main hypothesis of the analogy should also be investigated. This hypothesis states that the concentration and the temperature boundary layer thicknesses are the same.

8-In general, all models underestimate the heat transfer coefficient. This is basically because all of them have not included some other phenomena which improve the transfer processes. The main phenomena not accounted for the rippled structure of the film surface; and the mist formation.

9-The models can correctly predict the heat transfer coefficient evolution, even at the lower part of the tube where the heat transfer is not dominated by the condensation term, which is important from a theoretical point of view, although the heat transfer coefficient values are not so high. In Fig. 3 it is possible to see how the curve starts achieving a constant value, instead of keep decreasing, so reproducing the abrupt change that takes place in the heat transfer process.

10-It is important to say that the Vierow experimental conditions set a noncondensables mass fraction much lower than usual. It ranges from 0.86% to 13%, while the most of databases Siddique [20], Park [21], Maheshwari [22] have a range that starts on values of 10%, and can have a maximum of even 60%. This makes that the non-condensable gases effect over the liquid film thickness will be even higher than can be expected observing the average errors of the No and Park model comparing with the cubic and quartic method models.

11- A comparison of CPU elapsed time between iterative and non-iterative models has been performed. The calculations show a saving of 71.4% in CPU time when using the non-iterative model developed in this paper.

CONCLUSIONS

Some contributions for modelling the condensation in presence of non-condensable gases in vertical tubes have been developed. The new models have a non-iterative nature, and at the same time, they can account for the main non-condensable effects over the heat and mass transfer processes. In these models, the real interface temperature can be explicitly obtained.

We have obtained two models, depending on how the liquid viscosity is computed. The first model is based on solving a cubic degree equation, where the viscosity has been achieved by a numerical approach. The second model is based on solving a quartic equation, where the viscosity has been accounted for by means of a Taylor series expansion.

The results of both models have been compared with the Vierow experimental data and with the No and Park model, both the original one and that which has the same correlations implemented in our model, so the new contributions could be measured.

The fact of considering the liquid heat transfer coefficient variation due to the non-condensable gases effect yields an improvement close to 7%.

The high sensitivity of the models with respect to the mass transfer correlation has been stated, since the error of the models changes more than 50%, depending on the used correlation. This means that the heat and mass transfer analogy has to be carefully used.

The validation results show a better agreement for the quartic method model, more precisely, a 21.09% total average relative error for 268 points -where only the anomalous cases run by Vierow have been neglected-, while the other methods that have been compared present a higher error that ranges from 23.72% for the cubic equation model and 27.33% for the No and Park with Reynolds-Colburn analogy correlation using HMTA for the mass transfer coefficient, to 47.63% for the No and Park original model.

The importance of this type of models is based on their use in large numerical simulation codes for the third generation of passive nuclear power plants. These kinds of plants use passive containment cooling condensers. The simulation of a transient for a small loss of coolant accident need to simulate 24 hours of real time, for this case the condensation model subroutines are called hundred of thousand times so an small saving of computational time is very important.

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