HEFAT2007
5th International Conference of Heat Transfer, Fluid Mechanics, and Thermodynamics
1-4 July 2007, Sun City, South Africa
GM1

Stationary convection and internal gravity waves in compressible liquids: influence of piston effect

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Abstract

Fundamental equations of a compressible viscous heat-conducting fluid are derived. The rigorous linear stability analysis has been used for the onset of thermal convection and internal gravity waves. Particular emphasis is placed upon the influence of a thermo-acoustic waves (piston effect) on these phenomena. .This recently found contra-intuitive speeding up of thermal equilibration at constant volume replaced the traditional slowing down in a strongly compressible liquid at constant pressure. Our analysis shows that for the onset of convection the results are coincide with those at fixed pressure proving thereby that the piston effect does not influence the. thermodynamic phenomenon of free convection. However, the dynamic phenomenon of propagation of the internal gravity waves is essentially dependent on the piston effect.

1. Introduction

Convective instability and internal gravity waves in liquids are phenomena related to the presence of a significant height density inhomogeneity. Such non-homogeneity may appear as result of gravity or of an external temperature gradient ∇T, or by both of these. Oscillating wave motion in liquid is associated with the disturbing the mechanical equilibrium by the displacement of any liquid element. Gravity and temperature gradient re-establish the equilibrium acting in the same direction when the heating is from above, and in opposite direction if the heating is from the bottom upwards.

Gravity induced stratification usually occurs in large scale geophysical objects. However, similar behavior takes place also near the liquid-gas critical points where the overall change of the density may reach 10% in a centimeters height layer. Our approach is equally applied to geophysical objects and near-critical fluid, where the compressibility plays an essential role. However, we will not compare the strength of singularity near the critical point of different terms in appropriate equations in order to simplify these equations, as it usually done, in order to maintain the generality of our analysis. Consider the layer of a liquid restricted by horizontal planes $z=-L/2$ and $z=L/2$ with infinite extent in the horizontal $x-y$ plane, subjected to a vertical gradient of temperature in the gravitational field $g\hat{z}$, where $\hat{z}$ is the unit vector in the upward direction. The two important equilibrium properties of a liquid approaching the liquid-gas critical point are the huge (theoretically infinite) increase of the compressibility and of the thermal expansion. The latter leads to a strong vertical stratification of density, if the heat is added from below, say by applying an external temperature gradient. If the temperature gradient does not exceed some critical value, the liquid remains in mechanical equilibrium. However, if this condition is not satisfied, a vertical motion of the liquid (free convection) develops tending to equalize the temperature throughout the fluid. Convection is hindered by the compressibility of the liquid and dissipative processes. The large stratification leads to an appearance of the horizontal oscillations of the liquid (internal gravity waves). Along with the stationary convection, an oscillatory branch of convective instability may appear in non-compressible liquid [1], but this is beyond the scope of our analysis.

````Both convection and internal gravity waves are described by the same system of hydrodynamic equations, and after the use of boundary conditions, one obtains the equation for the eigenvalues of the problem of the form

$$f\left(\nabla T,\omega,k\right)=0 \tag{1}$$

where $\omega$ and $k$ are the frequency and the wave vector of a perturbation. For convection this equation describes the critical temperature gradient $\left(\nabla T\right)_{cr}$ at which the convection (monotonic motion with $\omega=0$) comes into play. On the other hand, for the gravity waves, equation (1) presents the dispersion relation, $\omega=\omega\left(k\right)$, for given temperature gradient. It is evident that the time-depeent problem of internal gravity waves is more complicated that the stationary problem of the onset of convection.

The problem of convection and internal gravity waves in near-critical fluids came to our attention a long time ago [2], [3], [4], [5]. The necessity of considering this effect again stems from the recently proposed outstanding theoretical idea of speeding up of the critical dynamics [6], which was subsequently found experimentally [7].
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The slowing-down of the dynamic processes near the liquid-gas critical points is the hallmark of the critical phenomena. This is just the feature which makes so difficult to get reliable equilibrium experimental data near the critical points. The slowing-down arises from the vanishing of the kinetic coefficients at the critical point, such as the heat conductivity which is inversely proportional to the specific heat at constant pressure, a quantity which is strongly singular at the critical point. However, the very interesting phenomena of a speeding-up of dynamic processes in critical fluids at constant volume, in contrast to the slowing-down, has been predicted theoretically [6]. The usual experimental set-up is that of a closed cell filled with fluid, where, after changing the temperature at the bottom or around the cell, one waits for the establishing of thermal equilibrium inside the cell. The slow diffusive heat propagation is responsible for the critical slowing-down. It turns out, however, [6] that a fast thermal equilibrium is established through the thermo-acoustic effect, where the temperature change at the surface induces acoustic waves which result in a fast change of the pressure, and, hence, of the temperature everywhere in the fluid. This effect is also called the ”piston effect”, since the thermal boundary layer generated near the heated bottom of the cell produces, like a piston, a pressure on the fluid confined in the constant volume.

The aim of this article is to calculate the influence of the piston effect on the onset of convection and on the internal gravity waves. We consider these two phenomena together since both of them are described by the hydrodynamic equations which we will analyze in the next section for a layer of a compressible liquid subjected to vertical gravity and temperature fields

Nomenclature

T Temperature\
ρ Density
p Pressure
S Entropy
Q Heat
c_p Specific heat at constant pressure
c_v Specific heat at constant volume
α_T Compressibility
β Thermal expansion coefficient
ν Shear viscosity
ς Bulk viscosity
λ Thermal conductivity
κ_p Thermal diffusivity at constant pressure
κ_v Thermal diffusivity at constant volume
g Gravitational acceleration
A Temperature gradient
ω Frequency
k Wave vector

2. Main equations

The hydrodynamic equations are nothing but the conservation laws for mass, momentum and energy, namely the continuity equation

$$\frac{d\dot{\rho}}{dt} + \dot{\rho} div \dot{v} = 0, \tag{2}$$

the Navier-Stokes equation

$$\frac{d\dot{v}}{dt} = -\frac{1}{\overline{\rho}}\nabla \dot{p} - \frac{g\dot{\rho}}{\overline{\rho}}\hat{z} + \nu\nabla^2\dot{v} + \left(\frac{\nu}{3}+\zeta\right)\nabla div\dot{v}\,, \tag{3}$$

and the heat conductivity equation

$$\frac{dQ}{dt} \equiv \rho\dot{T}\frac{dS}{dt} = \lambda\nabla^2\dot{T} \tag{4}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \left(v\nabla\right),$ and all the parameters have their obvious meaning. We restrict our discussion to the linear analysis which justifies the neglect of $\left(v\nabla\right)v$ term in Eq. (3) as well as the viscous dissipation and the viscous stress tensor in hydrodynamic equations.

The temperature $\dot{T}$, velocity $\dot{v}$, pressure $\dot{p}$, and density $\dot{\rho}$ are perturbed around their equilibrium values

$$\dot{T} = \overline{T} + T_0(z) + T; \qquad \dot{p} = \overline{p} + p_0(z) + p; \qquad \dot{\rho} = \overline{\rho} + \rho_0(z) + \rho; \qquad \dot{v} = 0 + v \tag{5}$$

where, in the presence of gravity and the imposed temperature gradient,

$$\nabla T_0 = -A; \qquad \nabla p_0 = -\rho_0 g; \qquad \nabla \rho_0 = -\overline{\rho}\beta\nabla T_0 + \alpha_T \nabla p_0 = \overline{\rho}\beta A - \overline{\rho}\alpha_T g \tag{6}$$

with

$$\alpha_T = \left(\frac{\partial \rho}{\partial p}\right)_T; \qquad \beta = -\frac{1}{\overline{\rho}}\left(\frac{\partial \rho}{\partial T}\right)_p \tag{7}$$

Then, equation (2) takes the following form

$$\frac{\partial \rho}{\partial t} = -\overline{\rho}\, div\, v - v_z \nabla \rho_0 \tag{8}$$

For the time-independent case equation (8) reduces to

$$div\, v = -v_z \frac{\nabla \rho_0}{\overline{\rho}} \tag{9}$$

All our analysis is based on the assumption of the local equilibrium,

$$\rho = \alpha_T\, p - \overline{\rho}\, \beta T, \tag{10}$$

Let us now exclude the v_x and v_y components from the vector equation (3). Differentiate the $x-$ and $y-$projections of equation (3) with respect to x and y, respectively, and combine the resulting equations. Then, by using (10), one obtains

$$\frac{1}{\overline{\rho}\alpha_T}\nabla_1^2 \rho + \frac{\beta}{\alpha_T}\nabla_1^2 T = \left(\nu\nabla^2 - \frac{\partial}{\partial t}\right)\left(div\, v - \frac{\partial v_z}{\partial z}\right) + \left(\frac{\nu}{3} + \varsigma\right)\nabla_1^2 div\, v \tag{11}$$

where $\nabla_1^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, and for the time-independent case

$$\nu\nabla^2 div\, v - \nu\nabla^2 \frac{\partial v_z}{\partial z} + \left(\frac{\nu}{3} + \varsigma\right)\nabla_1^2 div\, v = \frac{1}{\overline{\rho}\alpha_T}\nabla_1^2 \rho + \frac{\beta}{\alpha_T}\nabla_1^2 T \tag{12}$$

If one applies to (3) the vector operator $curlcurl = graddiv - \nabla^2$, all $\nabla -$terms will vanish, and the $z-$projection of $curlcurl(\rho g \hat{z})$ leads to $-\nabla_1^2 \rho$. Finally, one gets

$$\left(\nu\nabla^2 - \frac{\partial}{\partial t}\right)\left(\nabla^2 v_z - \frac{\partial}{\partial z} div\, v\right) = \frac{g}{\overline{\rho}}\nabla_1^2 \rho \tag{13}$$

which, for time-independent case reduces to

$$\nu\nabla^4\, v_z - \nu\nabla^2 \frac{\partial}{\partial z} div\, v = \frac{g}{\overline{\rho}}\nabla_1^2 \rho \tag{14}$$

By this means we obtained four equations ((4), (8), (11), and (13)) for four variables $(div\, v,\, v_z,\, \rho \text{ and } T)$. Substituting $div\, v$ from Eq. (8) into (11) and (13), one can bring these equations to the following forms

$$\begin{gathered} \left(\frac{1}{\overline{\rho}\alpha_T}\nabla_1^2 - \frac{S}{\overline{\rho}}\frac{\partial}{\partial t}\right)\rho + \frac{\beta}{\alpha_T}\nabla_1^2 T - R\frac{\partial v_z}{\partial z} - \frac{\nabla \rho_0}{\overline{\rho}} S v_z = 0 \\ \left(\frac{g}{\overline{\rho}}\nabla_1^2 - \frac{R}{\overline{\rho}}\frac{\partial^2}{\partial z \partial t}\right)\rho + R\left(\nabla^2 + \frac{\nabla \rho_0}{\overline{\rho}}\frac{\partial}{\partial z}\right)v_z = 0 \end{gathered} \tag{15}$$

Two operators R and S which appear in the latter equation is defined as

$$R = \frac{\partial}{\partial t} - \nu\nabla^2; \qquad S = R - \left(\frac{\nu}{3} + \zeta\right)\nabla_1^2 \tag{16}$$

Eliminating ρ from equations (15), one obtains

$$
\frac{R^2}{\bar{\rho}}\frac{\partial^3 v_z}{\partial z^2 \partial t}+\left[g+\frac{\nabla\rho_0}{\alpha_T\bar{\rho}}\right]R\nabla_1^2\frac{\partial v_z}{\partial z}-
$$

$$
-\left(RS\frac{\partial}{\partial t}\nabla^2-\frac{\nabla\rho_0}{\bar{\rho}}gS\nabla_1^2-\frac{1}{\alpha_T}R\nabla_1^2\nabla^2\right)v_z-\frac{\beta}{\alpha_T}\left(g\nabla_1^2+R\frac{\partial^2}{\partial z\partial t}\right)\nabla_1^2 T=0 \tag{17}
$$

which for time-independent case takes the following form

$$
\nu\nabla^2\left[g+\frac{\nabla\rho_0}{\alpha_T\bar{\rho}}\right]\frac{\partial v_z}{z}+\left\{\left(g\frac{\nabla\rho_0}{\bar{\rho}}\left[\nu\nabla^2+\left(\frac{\nu}{3}+\zeta\right)\nabla_1{}^2\right]+\frac{1}{\alpha_T}\nu\nabla^4\right)\right\}v_z=-\frac{g\beta}{\alpha_T}\nabla_1{}^2 T \tag{18}
$$

Let us look for a solution of hydrodynamic equation (17) for the vertical component of the velocity v_z and the temperature T in the form

$$
v_z=V(z)\exp\left[i\left(k_x x+k_y y\right)+i\omega t\right]
$$

$$
T=\tau(z)\exp\left[i\left(k_x x+k_y y\right)+i\omega t\right] \tag{19}
$$

On substituting (19) into (16), (17)) and (18), one gets

$$
R_\omega=i\omega-\nu\nabla^2;\qquad S_\omega=i\omega-\nu\nabla^2+\left(\frac{\eta}{3}+\zeta\right)k^2 \tag{20}
$$

$$
i\omega R_\omega{}^2\frac{d^2V_z}{dz^2}-R_\omega k^2\left[g+\frac{\nabla\rho_0}{\bar{\rho}\ \alpha_T}\right]\frac{dV_z}{dz}-
$$

$$
-\left\{\left(\frac{k^2}{\alpha_T}+i\omega S_\omega\right)R_\omega\nabla^2+\frac{\nabla\rho_0}{\bar{\rho}}gS_\omega k^2\right\}V_z-\frac{\beta}{\alpha_T}\left(gk^2-i\omega R_\omega\frac{d}{dz}\right)k^2\tau=0 \tag{21}
$$

where $k^2=k_x^2+k_y^2$, or, for time-independent case,

$$
\nu\nabla^2\left[g+\frac{\nabla\rho_0}{\alpha_T\bar{\rho}}\right]\frac{dV_z}{dz}+\left\{\frac{\nu}{\alpha_T}\nabla^4+g\frac{\nabla\rho_0}{\bar{\rho}}\left[\nu\nabla^2-\left(\frac{\nu}{3}+\zeta\right)k^2\right]\right\}V_z-
$$

$$
-\frac{g\beta}{\alpha_T}k^2\tau=0 \tag{22}
$$

The transformation of equation (4) will be slightly different for $S(p,T)$ and $S(\rho,T)$ depending which pair, p,T or ρ,T , is chosen as the fundamental thermodynamic variables, and we will leave the analysis of this equation to the next section.

3. Onset of convection

3.1. Fixed pressure

This case has been already investigated in [2]. For T and p as independent variables, one gets

$$
\nabla S_0=-\left(\frac{\partial S}{\partial T}\right)_p A+\left(\frac{\partial S}{\partial p}\right)_T\rho_0 g
$$

$$
S=\left(\frac{\partial S}{\partial T}\right)_p p+\left(\frac{\partial S}{\partial p}\right)_T T \tag{23}
$$

On substituting (23) into linearized equation (4), one obtains

$$
\bar{\rho}T\left[\left(\frac{\partial S}{\partial T}\right)_p\frac{\partial T}{\partial t}+\left(\frac{\partial S}{\partial p}\right)_T\frac{\partial p}{\partial t}\right]+v_z\left[-\left(\frac{\partial S}{\partial T}\right)_p A+\left(\frac{\partial S}{\partial p}\right)_T\bar{\rho}g\right]=\lambda\nabla^2 T \tag{24}
$$

Using the well-known thermodynamic equalities [8]

$$\left(\frac{\partial s}{\partial T}\right)_p = \frac{c_p}{T} \quad ; \quad \left(\frac{\partial s}{\partial p}\right)_T = -\frac{c_p}{T}\left(\frac{\partial T}{\partial p}\right)_s \quad ; \quad \left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial T}{\partial p}\right)_\rho \left(1-\frac{c_v}{c_p}\right) \tag{25}$$

one can, after the substitution of (23) into (4), rewrite the latter equation in the following form

$$\frac{\partial T}{\partial t} - \left(\frac{\partial T}{\partial p}\right)_\rho \left(1-\frac{c_v}{c_p}\right)\frac{\partial p}{\partial t} = \kappa_p \nabla^2 T + AV_z\left(1-\delta\right) \tag{26}$$

and, for the time-independent equation,

$$\kappa_p \nabla^2 T + A\left(1-\delta\right)V_z = 0 \tag{27}$$

where

$$\delta = \frac{g\alpha_T}{\beta A}\left(1-\frac{c_v}{c_p}\right) = \frac{\bar{\rho} g}{A}\left(\frac{\partial T}{\partial p}\right)_\rho \left(1-\frac{c_v}{c_p}\right); \qquad \kappa_p = \frac{\lambda}{\bar{\rho} c_p} \tag{28}$$

The instability of a stationary state occurs when ω becomes positive, i.e., the boundary of stability is defined by condition $\omega = 0$. To find the critical temperature gradient which defines the onset of convection one has to consider two ordinary differential equations, (22) and (27) for V_z and T. The boundary conditions for (22) and (27) depend on the types of boundaries. For both rigid boundaries

$$V_z = \frac{dV_z}{dx} = T = 0 \qquad at\ z = \pm L/2 \tag{29}$$

Two limit cases (Schwarzchild and Rayleigh) can be easily obtained from equations (22) and (27). If the compressibility is taken into account only in (27), while viscosity and thermal conductivity are neglected, one obtains the Schwarzchild criteria for a compressible liquid $\alpha = 1$. or

$$A_{Schw} = \bar{\rho} g\left(\frac{\partial T}{\partial p}\right)_\rho \left(1-\frac{c_v}{c_p}\right) \tag{30}$$

The second limit case, the Rayleigh criterion corresponds to so-called Boussinesq approximation wherein the compressibility is taken into account only in the $\rho_0 \beta T$ term in the Navier-Stokes equation (3). Then, equations (22) and (27) take the form

$$\left(\frac{d^2}{dz^2} - k^2\right)^2 V_z = \frac{A\beta g k^2}{\kappa_p \nu} T \; ; \qquad \left(\frac{d^2}{dz^2} - k^2\right)T = -V_z \tag{31}$$

Zero value of the determinant of Eqs. (31) leads to the Rayleigh criterion [2].

$$A_{Ra} = \gamma_0 \frac{\kappa_p \nu}{\beta g L^4} \tag{32}$$

where the coefficient γ_0 slightly depends on the boundary conditions. For rigid boundaries (29), $\gamma_0 = 1707.8,$ and for free boundaries $\gamma_0 = 657.5.$

Hence, in two limit cases one obtains the Rayleigh and Schwarzchild criteria. By using the Galerkin variational method, the critical temperature gradient of the onset of convection has been found in [2]. The appropriate equation (2.5) from [2] for the critical temperature gradient A_{cr} determining the onset of convection, can be rewritten as

$$A_{cr} = \left[\frac{\delta L_1}{\beta} + \frac{\kappa_p \nu}{\beta g L^4}\left(\gamma_0 + \gamma_1 L^2 g^2 \alpha_T^2\right)\right]\left(1+\eta_1 \frac{\kappa_p \nu \alpha_T}{L^2}\right)^{-1} \tag{33}$$

or

$$A_{cr} = \left[A_{Ra} + A_{Schw} + \frac{\gamma_1 \kappa_p \nu g \alpha_T^2}{\beta L^2}\right]\left(1+\frac{\gamma_1 \nu \kappa_p \alpha_T}{L^2}\right)^{-1} \tag{34}$$

where, for solid boundaries, $\gamma_1 = 70.5.$ Corrections to limit cases (30) and (32) depend on the relations between the layer height L and the characteristic lengths $\left(\frac{\gamma_1 \kappa_p \nu L_2^2}{g\beta}\right)^{1/2}$ and $\left(\frac{\gamma_1 \kappa \nu_p L_2}{g}\right)^{1/2}$.

3.2. Fixed volume

In real experiments one keeps the volume constant rather than the pressure. For the analysis of this case, one uses the well-known thermodynamic relations [8]

$$\frac{dS}{dt}=\left(\frac{\partial S}{\partial T}\right)_v\frac{dT}{dt}+\left(\frac{\partial S}{\partial v}\right)_T\frac{dv}{dt}=\frac{c_v}{T}\frac{dT}{dt}+\frac{c_p-c_v}{T}\left(\frac{\partial T}{\partial\rho}\right)_p\frac{d\rho}{dt} \tag{35}$$

which, after substitution in linearized equation (4), gives

$$\frac{\partial T}{\partial t}-Av_z-\frac{1}{\beta}\left(1-\frac{c_p}{c_v}\right)div\, v=\kappa_v\nabla^2T\ ;\qquad \kappa_v=\frac{\lambda}{\rho c_v} \tag{36}$$

The time-independent form of the last equation, after using Eq. (9), is

$$\kappa_v\nabla^2T+Av_z+\left(1-\frac{c_p}{c_v}\right)\frac{\nabla\rho_0}{\beta\overline{\rho}}v_z=0 \tag{37}$$

The last term in (37) comes from the compressibility. Indeed, for a non-compressible liquid far away from the critical point, $c_p\approx c_v$, and the last term in (37) vanishes. Moreover, the specific heat at constant volume appears in (37), as distinguished from that at constant pressure in (27), as one would expect. One can rewrite equation (37) in a form similar to (27),

$$\kappa_v\nabla^2T+Av_z\left(1-\delta_1\right)=0 \tag{38}$$

where, by using (6), one gets

$$\delta_1=\frac{\nabla\rho_0}{A\beta\overline{\rho}}\left(\frac{c_p}{c_v}-1\right)=\left(\frac{c_p}{c_v}-1\right)\left[\frac{g\rho}{A}\left(\frac{\partial T}{\partial p}\right)_\rho-1\right] \tag{39}$$

Just as for constant pressure, the onset of convection at constant volume neglecting the dissipation is defined by condition $\delta_1=1$ which gives the Schwarzchild criteria just as (30) in the case of the constant pressure

Far away from the critical point, $c_p/c_v\approx1$, i.e., $\delta_1\approx0$, and one obtains the Rayleigh criterion. In the intermediate case one has to use the Galerkin method. However, there is no need for additional calculations. Indeed, comparison of the heat conductivity equations (27) and (38) shows that the results for the case of constant volume coincide with those for constant pressure if one replace δ by δ_1. Therefore, as it follows from Eq. (33), the critical temperature gradient A_{cr} determining the onset of convection for the constant volume, is given by

$$A_{cr}=\left[\delta_1A+\frac{\kappa_v\nu}{\beta gL^4}\left(\gamma_0+\gamma_1L^2g^2\alpha_T^2\right)\right]\left(1+\eta_1\frac{\kappa_v\nu\alpha_T}{L^2}\right)^{-1} \tag{40}$$

Substituting (39) into (40), one obtains

$$A_{cr}=\left(A_{Schw}+A_{Ra}+\frac{\gamma_1\kappa_p\nu g\alpha^2}{\beta L^2}\right)\left(1+\frac{\gamma_1\kappa_p\nu\alpha}{L^2}\right)^{-1} \tag{41}$$

The final result (41) for the liquid at constant volume is identical to that at constant pressure, Eq. (34).

4. Internal gravity waves

Solution of the time-dependent hydrodynamic equations describing the gravity waves is much more complicate than the preceding analysis of the time-independent equations sufficient for an analysis of free convection. We will bring the full analysis of the gravity waves elsewhere, restricting our consideration here to demonstrating the fact that, contrary to the free convection, the piston effect exerts some influence on the propagation of the gravity waves in compressible liquids.

The full system of hydrodynamic equations contains two equations connecting V_z (z) and τ (z). Another equation, in addition to (21), is obtained by substituting (19) into (36) and the second equation in (15) which gives

$$\begin{gathered}i\omega\tau-AV_z+\frac{1}{\beta\overline{\rho}}\left(1-\frac{c_p}{c_v}\right)\left(i\omega\rho+\nabla\rho_0v_z\right)=\kappa_v\nabla^2\tau\\ \left(k^2\frac{g}{\overline{\rho}}-i\omega\frac{R_\omega}{\overline{\rho}}\frac{d}{dz}\right)\rho-R_\omega\left(\nabla^2+\frac{\nabla\rho_0}{\overline{\rho}}\frac{d}{dz}\right)V_z=0\end{gathered} \tag{42}$$

which, after excluding ρ from these two equations, leads to

$$\left[\left(i\omega-\kappa_v\nabla^2\right)\left(i\omega\frac{R_\omega}{\overline{\overline{\rho}}}\frac{d}{dz}-k^2\frac{g}{\overline{\overline{\rho}}}\right)\right]\tau-$$
$$-\left\{\frac{i\omega R_\omega}{\beta\overline{\overline{\rho}}}\left(1-\frac{c_p}{c_v}\right)\left(\nabla^2+\frac{\nabla\rho_0}{\overline{\overline{\rho}}}\frac{d}{dz}\right)+\left[A-\frac{1}{\beta\overline{\overline{\rho}}}\left(1-\frac{c_p}{c_v}\right)\nabla\rho_0\right]\left(+i\omega\frac{R_\omega}{\overline{\overline{\rho}}}\frac{d}{dz}-k^2\frac{g}{\overline{\overline{\rho}}}\right)\right\}V_z=0 \quad (43)$$

We postpone the full analysis of the cumbersome equations (21) and (43) till the feature publication. However, as one can see from these equations, the piston effect described in these equations by the terms incorporated factor $\left(1-c_p/c_v\right)$, essentially influences the propagation of the gravity waves.

5. Discussion

We have performed the standard stability analysis of a layer of near-critical fluid with fixed volume. It turns out that the onset of convection in this case is coinciding with that in the case of constant pressure. There are already indications in the literature [13] that the piston effect does not influence the onset of convection. Our rigorous stability analysis supports this statement which is attributable to the influence of the piston effect on the dynamic rather than on the thermodynamic behavior. Indeed, the propagation of the gravity waves is strongly influenced by the piston effect. A few restriction of our analysis should be emphasized. Firstly, we consider all thermodynamic derivatives entering the hydrodynamic equations, as constant in a fluid which implies that there is comparatively small stratification along a fluid, i.e., not too close to the critical point. Secondly, we considered the stability of a stationary state with a constant temperature gradient along the height of a fluid, ignoring the transient processes. As a result of the appearance of boundary layers at the liquid boundaries, a fluid might become unstable at the boundary layers before the stationary temperature gradient is established [14],[15]. Some other common approximations (infinite dimensions in horizontal directions, infinite heat conductivity of planes surrounding a fluid, etc) have been made. However, we used an analytical approach, which avoids the numerical calculations widely used in analysis of convection ([16] and references therein)

References

[1] V. A. Steinberg, Oscillating convective instability of compressible liquid, Izv. AN SSSR - Gas Dyn. nu. 3 (1971).

[2] M. Gitterman and V. Steinberg, Criteria of occurrence of free convection in compressible viscous heat-conducting fluid, J. Appl. Math..Mech. **34**, 305 (1970).

[3] M. Gitterman and V. Steinberg, Criteria for commencement of convection in a liquid close to a critical point, High. Temp. **8,** 754 (1970).

[4] M. Gitterman and V. Steinberg, Internal gravity waves and convectional instability in liquids, Izv. AN SSSR- Gas Dyn . nu.2, 238 (1972).

[5] M. Gitterman, Rev. Mod. Phys. Hydrodynamics of fluids near a critical point, **50,** 85 (1978).

[6] A. Onuki and R. A. Ferrel, Adiabatic heating effect near the gas-liquid critical point, Physica A **164**, 245 (1989) ; B. Zappoli et. al., Anomalous heat transport by the piston effect in superfluids under zero gravity, Phys. Rev. A **41**, 2264 (1990); H. Boukari et. al., Critical speeding up in pure fluids, Phys. Rev. **A** 41, 2260 (1990).

[7] R. P. Behringer, A. Onuki, and H. Meyer, Thermal equilibrium of fluid near the liquid-vapor critical point, J. Low Temp.Phys **81**, 71 (1990); H. Meyer and F. Zhong, Equilibration and other dynamic properties of fluid near the liquid-vapor critical point, Compt. Rend., Mecanique **332**, 327 (2004).

[8] L. Landau and E. Lifshitz, Statistical physics, [Pergamon, London 1958].

[9] L. Landau and E. Lifshitz, Hydrodynamics, [Pergamon, London 1959]

[10] H. Lamb, Hydrodynamics, [Dover, New York 1945].

[11] M. Gitterman, Study of the critical state of a liquid using gravity waves, Sov. Phys.-Doklady, **12**, 1141 (1968).

[12] K. Ishii, T. Maekawa, H. Azuma, S Yoshihara, and M. Onishi, Effect of gravitational acceleration on temperature wave propagation in a critical fluid, Appl. Phys. Lett.,**72**, 16 (1999).

[13] P.Carles and B.Ugurtas, The onset of free convection near the liquid-vapor critical point. Part 1 :Stationary initial state, Physica D **126**, 69 (1999) ; P. Carles, The onset of free convection near the liquid-vapor critical point. Part 2 Unsteady heating, Physica D **147**, 36 (2000)

[14] L. El Khouri and P. Carles,. Scenarios for the onset of convection close to the critical point, Phys. Rev E **66**, 066309 (2002).

[15] T. Maekawa, et. al., Onset of buoyancy convection in a horizontal layer of a supercritical fluid heated from below, J. Phys. A **37**, 7955 (2004).

[16] A. Furukawa and A. Onuki, Convective heat transport in compressible fluids , Phys. Rev. E **66**, 016302 (2002).