

# Adaptive parameter estimation for an energy model of belt conveyor with DC motor<sup>†</sup>

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## ABSTRACT

In practice, design parameters of a belt conveyor are likely drifting away from their design values by maintenance, readjustment, retrofit, abrasion and circumstance change. For the purpose of energy optimization, these parameters should be estimated through experiments. In this paper, a new energy model of a DC motor driven belt conveyor is presented. Then, based on an adaptive observer, a parameter estimation algorithm is derived. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values can also be concluded. Compared with the existing methods, our methods can be implemented by measuring only the feed rate of the belt conveyor and the angular velocity of the rotor of the DC motor.

**Key Words:** parameters estimation, energy model, belt conveyor, observer, DC motor

## I. Introduction

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Belt conveyors have high transfer capacity and long transfer distance. They are widely used to transfer bulk material in mining, metallurgical and coal industry. According to the report in [1], about 10% of the total maximum power demand in South Africa is to handle materials, where up to 40% energy cost is borne by the operational cost of the belt conveyor systems [2]. Therefore, it has great significance to improve energy efficiency of belt conveyors by reducing the energy consumption of material handling.

There are four levels to improve energy efficiency of a belt conveyor: performance, operation, equipment, and technology [3]. It is easy to achieve higher energy efficiency by introducing highly efficient equipment [4, 5, 6, 7, 8]. However, extra investment is needed to retrofit or replace the equipment. At the operation level, many methods are proposed to improve energy efficiency for the belt conveyors [9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. For example, the authors in [17] proposed an optimal switching control and a variable speed drive based optimal control to reduce the energy consumption of belt conveyors. In [18], an analytical energy model is proposed. It has four coefficients which can be estimated through the algorithms such as least square (LSQ) [19] and recursive least square (RLSQ) [20]. After obtaining the energy model, an optimization is also done at operational level with two performance indicators, energy cost and energy consumption. However, in order to estimate these four coefficients, power of motor  $P_M$ , feed rate  $T$  and belt speed  $V$  should be measured. Recently, in order to estimate unknown states and unknown constant parameters, adaptive observers have made great progress [21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

In this paper, a new energy model of belt conveyor with DC motor is introduced. Then, an adaptive observer is designed for the model. In order to identify the four coefficients of the energy model, the feed rate  $T$  of the belt conveyor and the angular velocity  $w_m$  of the rotor of the DC motor should be measured on-line. Then, based on the adaptive observer, a parameter estimation algorithm is derived. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values can also be concluded. Simulation results show the validity of our methods.

This paper is organized as follows. The analytical energy model of belt conveyors in [18], the model of a DC motor, and the adaptive observers design are reviewed in Section II, respectively. In Section III, we present a new energy model of belt conveyors with DC motor, an adaptive observer for this model, and a parameter estimation algorithm. In Section IV, an example is given to show the validity of our new methods. Section V presents the conclusion.

## II. Preliminaries

### 2.1. An analytical energy model of belt conveyors

A typical belt conveyor is shown in Fig. 1. As in [18], an analytical energy model of the belt conveyor is given as follows

$$P_T - \frac{V^2 T}{3.6} = \bar{\theta}_1 T^2 V + \bar{\theta}_2 V + \bar{\theta}_3 \frac{T^2}{V} + \bar{\theta}_4 T, \quad (1)$$

where  $P_T$  is the mechanical power,  $V$  denotes the belt speed (m/s),  $T$  is the feed rate (t/h),  $\bar{\theta}_1$ ,  $\bar{\theta}_2$ ,  $\bar{\theta}_3$  and  $\bar{\theta}_4$  are four parameters. In practice, these four parameters often drift away by maintenance, readjustment, retrofit, abrasion and circumstance change. For the purpose of energy optimization, these four parameters are estimated by both an off-line and an on-line parameter estimation schemes based on  $P_T$ ,  $V$  and  $T$  measured on-line and off-line, respectively in [18].

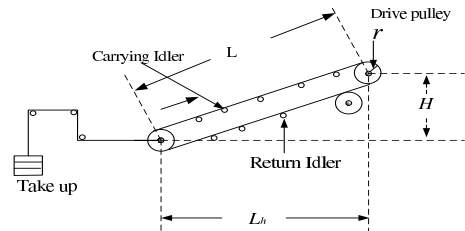


Fig. 1. Typical profile of belt conveyors.

## 2.2. A DC motor

The dynamics of a DC motor are given by [31]

$$\begin{cases} J_m \frac{d\omega_m}{dt} + b_m \omega_m &= k_p i_f i_a - T_L, \\ L_f \frac{di_f}{dt} + R_f i_f &= e_f, \\ L_a \frac{di_a}{dt} + R_a i_a &= e_a - k_c i_f \omega_m, \end{cases} \quad (2)$$

where  $J_m$  denotes the mass moment of inertia of the motor,  $\omega_m$  is its angular velocity,  $k_p$  is the torque constant,  $b_m$  is the damping coefficient,  $T_L$  is the presence of some external load,  $L_a$  and  $L_f$  are its inductances,  $R_a$  and  $R_f$  are its resistances,  $i_a$  and  $i_f$  are its currents,  $k_c$  is proportional constant to the flux and the angular velocity of the motor,  $e_a$  and  $e_f$  are two separate potentials are used to power the armature and filed, respectively. The corresponding circuit is shown in Fig. 2.

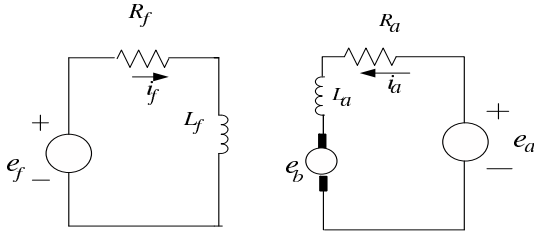


Fig. 2. Circuit diagram of separately excited DC motor.

## 2.3. Adaptive observers

Adaptive observers can be used to estimate unknown parameters. Now, let us review the adaptive observers design. Consider the following system in adaptive observer form [26]

$$\begin{cases} \dot{z} &= \bar{A}_0 z + \gamma(y, u) + \bar{b} \beta^T(y, u, t) \theta, \\ y &= \bar{C}_0 z, \end{cases} \quad (3)$$

where  $z \in \mathcal{R}^n$ ,  $y \in \mathcal{R}$ ,  $u \in \mathcal{R}^m$ ,  $\theta \in \mathcal{R}^p$ ,  $\gamma(y, u)$  a smooth function mapping  $\mathcal{R} \times \mathcal{R}^m \rightarrow \mathcal{R}^n$ ,  $\bar{A}_0 =$

$$\begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \quad \bar{b} = [\bar{b}_1, \dots, \bar{b}_n]^T \in \mathcal{R}^n \text{ is given}$$

such that the polynomial  $\bar{b}_1 s^{n-1} + \dots + \bar{b}_n$  is Hurwitz and  $\bar{b}_1 > 0$ ,  $\bar{C}_0 = [1 \ 0 \ \cdots \ 0]$ ,  $\beta^T(y, u, t) \theta = \sum_{i=1}^p \theta_i \beta_i(y, u, t)$ ,  $\beta_i(y, u, t)$  ( $i = 1, \dots, p$ ) are continuous functions and uniformly bounded for every  $(y, u)$  bounded. A global adaptive observer with parameter convergence is designed in [26] as follows:

$$\begin{cases} \dot{\hat{z}} &= \bar{A}_0 \hat{z} + \bar{K} \bar{C}_0 e + \gamma(y, u) + \bar{b} \beta^T(y, u, t) \hat{\theta}, \\ \dot{\hat{\theta}} &= \Gamma \beta(y, u, t) \bar{C}_0 e, \end{cases}$$

where  $\Gamma$  is any symmetric positive definite matrix,  $e = z - \hat{z}$ ,  $\bar{K} = \frac{1}{\bar{b}_1} (\bar{A}_0 \bar{b} + \lambda \bar{b})$  with  $\lambda$  an arbitrary positive real. Since  $(\bar{A}_0, \bar{b}, \bar{C}_0)$  satisfies the strictly positive real condition, then, for any symmetric definite matrix  $\bar{Q}$ , there exist a symmetric positive definite matrix  $\bar{P}$ , a positive real  $\bar{d}$  such that [26]

$$(\bar{A}_0 + \bar{K} \bar{C}_0)^T \bar{P} + \bar{P} (\bar{A}_0 + \bar{K} \bar{C}_0) < -\bar{d} \bar{Q}, \quad \bar{P} \bar{b} = \bar{C}_0^T.$$

## III. A new energy model

In this section, we make the following assumption: the belt is non-slip;  $T$  and  $\omega_m$  are measured on-line. We shall also assume a constant potential  $e_f$  and assume that the circuit is operating at steady state so that  $e_f = i_f R_f$ , yielding a constant field current  $i_f$ . Therefore, we have

$$\begin{cases} J_m \frac{d\omega_m}{dt} + b_m \omega_m &= k_T i_a - T_L, \\ L_a \frac{di_a}{dt} + R_a i_a &= e_a - k_b \omega_m, \end{cases} \quad (4)$$

where  $k_T = k_p i_f$ ,  $k_b = k_c i_f$ .

When the DC motor is employed to drive the conveyor belt, then,

$$T_L = F_U r, \quad V = 2\pi r \omega_m, \quad (5)$$

where  $r$  is the radius of the rotor and  $F_u$  is the peripheral driving force of the belt conveyor and can be calculated by the following equation [18]

$$F_U = \frac{VT}{3.6} + \frac{T^2}{6.48\rho b_1^2} + \{gfQ[L \cos \delta + L(1 - \cos \delta)(1 - \frac{2Q_B}{Q})] + k_3 + C_{Ft}\} + k_1 \frac{T^2}{V^2} + \left( \frac{gL \sin \delta + gfL \cos \delta}{3.6} + k_2 \right) \frac{T}{V}, \quad (6)$$

where  $f$  is the artificial friction factor,  $L$  is the center-to-center distance (m),  $Q = Q_{RO} + Q_{RU} + 2Q_B$ ,  $Q_{RO}$  is the unit mass of the rotating parts of carrying idler rollers (kg/m),  $Q_{RU}$  is the unit mass of the belt of rotating parts of the return idler rollers (kg/m),  $Q_B$  is the unit mass of the belt (kg/m),  $\delta$  is the inclination angle ( $^\circ$ ),  $\rho$  is the bulk density of materia (kg/m<sup>3</sup>),  $b_1$  is the width between the skirt boards (m),  $k_1$ ,  $k_2$ ,  $k_3$  are constants coefficients which relate to the structural parameters of the belt conveyor,  $C_{Ft}$  is a constant. From (5) and (6), we have

$$T_L - \frac{2\pi r^2 T w_m}{3.6} = \frac{r T^2}{6.48 \rho b_1^2} + r \{gfQ[L \cos \delta + L(1 - \cos \delta)(1 - \frac{2Q_B}{Q})] + k_3 + C_{Ft}\} + k_1 r \frac{T^2}{4\pi^2 r^2 w_m^2} + \left( \frac{gL \sin \delta + gfL \cos \delta}{3.6} + k_2 \right) \frac{T}{2\pi r w_m}. \quad (7)$$

Let  $\theta_1 = \frac{r}{6.48\rho b_1^2}$ ,  $\theta_2 = r\{gfQ[L \cos \delta + L(1 - \cos \delta)(1 - \frac{2Q_B}{Q})] + k_3 + C_{Ft}\}$ ,  $\theta_3 = \frac{k_1 r}{4\pi^2 r^2}$ ,  $\theta_4 = \left( \frac{gL \sin \delta}{3.6} + \frac{gfL \cos \delta}{3.6} + k_2 \right) \frac{1}{2\pi r}$ ,  $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T$ , and

$$\psi(T, w_m) = \begin{bmatrix} T^2 & 1 & \frac{T^2}{w_m^2} & \frac{T}{w_m} \end{bmatrix}^T.$$

Then, a new energy model of belt conveyor with DC motor is given as follows

$$\begin{cases} J_m \frac{d\omega_m}{dt} + b_m \omega_m = k_T i_a - \frac{2\pi r^2 T w_m}{3.6} - \psi^T(T, w_m) \theta, \\ L_a \frac{di_a}{dt} + R_a i_a = e_a - k_b \omega_m, \end{cases} \quad (8)$$

or

$$\dot{z} = \tilde{A}z + \tilde{B}u + \tilde{b} \frac{\psi^T(T, w_m)}{J_m} \theta, \quad (9)$$

$$\text{where } \tilde{A} = \begin{bmatrix} -\frac{b_m}{J_m} & \frac{k_T}{J_m} \\ -\frac{k_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix}, \tilde{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, z = \begin{bmatrix} w_m \\ i_a \end{bmatrix}, u = \begin{bmatrix} -\frac{2\pi r^2 T w_m}{3.6 J_m} \\ \frac{1}{L_a} e_a \end{bmatrix}.$$

For the conveyors with permanent instruments for  $T$  and  $w_m$ , the real-time data can be accessed through the supervisory control and data acquisition system (SCADA). Therefore, let  $w_m$  is the output of the system (8), *i.e.*,

$$y = w_m = Cz, \quad (10)$$

where  $C = [1 \ 0]$ . We obtain that

$$\text{rank} \begin{bmatrix} C \\ C\tilde{A} \end{bmatrix} = \text{rank} \begin{bmatrix} 1 & 0 \\ -\frac{b_m}{J_m} & \frac{k_T}{J_m} \end{bmatrix} = 2,$$

which means that  $(\tilde{A}, C)$  is observable. Then, the coordinate transformation

$$\tilde{z} = Q_1 z = \begin{bmatrix} 1 & 0 \\ -\frac{b_m}{J_m} & \frac{k_T}{J_m} \end{bmatrix} z$$

can transform (9), (10) into the following canonical form [32]

$$\begin{cases} \dot{\tilde{z}}_1 = \tilde{z}_2 + a_1 y - \frac{2\pi r^2 T w_m}{3.6 J_m} + \frac{1}{J_m} \psi^T(T, y) \theta, \\ \dot{\tilde{z}}_2 = a_2 y - \frac{2R_a \pi r^2 T w_m}{3.6 L_a J_m} + \frac{k_T}{J_m L_a} e_a + \frac{R_a}{L_a J_m} \psi^T(T, y) \theta, \\ y = \tilde{z}_1, \end{cases} \quad (11)$$

where  $a_1 = -\frac{b_m}{J_m} - \frac{R_a}{L_a}$ ,  $a_2 = -\frac{R_a b_m}{L_a J_m} - \frac{k_T k_b}{J_m L_a}$ . Let  $b = [1, b_2]^T$  is a vector such that

$$s + b_2$$

is a Hurwitz polynomial. Consider the following filter transformation [26]

$$\begin{cases} x_1 = \tilde{z}_1, \\ x_2 = \tilde{z}_2 - \sum_{i=1}^4 \eta_i \theta_i, \end{cases} \quad (12)$$

where

$$\eta_i = -b_2 \eta_i - \frac{b_2}{J_m} \psi_i(T, \omega_m) + \frac{R_a}{L_a J_m} \psi_i(T, \omega_m), \quad (13)$$

where  $\eta_i(t_0) = 0$ ,  $\psi_i(T, \omega_m)$  is the  $i$ th component of  $\psi(T, \omega_m)$  ( $i = 1, \dots, 4$ ). It transforms (11) into the following system

$$\begin{cases} \dot{x}_1 &= x_2 + a_1\omega_m - \frac{2\pi r^2}{3.6J_m}Tw_m \\ &+ (\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\theta, \\ \dot{x}_2 &= a_2\omega_m - \frac{2R_a\pi r^2}{3.6L_aJ_m}Tw_m + \frac{k_T}{J_mL_a}e_a \\ &+ b_2(\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\theta, \end{cases} \quad (14)$$

where  $\eta = [\eta_1, \eta_2, \eta_3, \eta_4]^T$ . For the system (14) which is in adaptive observer form, an adaptive observer can be designed as follows [26]

$$\begin{cases} \dot{\hat{x}}_1 &= \hat{x}_2 + k_1e_1 + a_1\omega_m - \frac{2\pi r^2}{3.6J_m}Tw_m \\ &+ (\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\hat{\theta}, \\ \dot{\hat{x}}_2 &= k_2e_1 + a_2\omega_m - \frac{2R_a\pi r^2}{3.6L_aJ_m}Tw_m \\ &+ \frac{k_T}{J_mL_a}e_a + b_2(\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\hat{\theta}, \end{cases} \quad (15)$$

and

$$\dot{\hat{\theta}} = \Gamma(\frac{1}{J_m}\psi(T, \omega_m) + \eta)e_1, \quad (16)$$

where  $e_1 = x_1 - \hat{x}_1$ , and  $k_1 = b_2 + \lambda$ ,  $k_2 = b_2\lambda$ ,  $\lambda > 0$ ,  $\Gamma$  is any symmetric positive matrix. From (14) and (15), (16), it follows that

$$\begin{cases} \dot{e}_1 &= e_2 - k_1e_1 + (\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\tilde{\theta}, \\ \dot{e}_2 &= -k_2e_1 + b_2(\frac{1}{J_m}\psi(T, \omega_m) + \eta)^T\tilde{\theta}, \end{cases} \quad (17)$$

and

$$\dot{\tilde{\theta}} = -\Gamma(\frac{1}{J_m}\psi(T, \omega_m) + \eta)e_1, \quad (18)$$

where  $e = x - \hat{x}$ ,  $\tilde{\theta} = \theta - \hat{\theta}$ .

Let us now state and prove the main results of this paper.

**Theorem 1** For the energy model of belt conveyor with DC motor (8), there exists filter transformation (12), (13) to transform (8) into (14). Moreover, for the system (15), (16), if  $k_1$  and  $k_2$  are selected such that  $(A, b, C)$  satisfies the strictly positive real condition,

where  $A = A_0 + KC$ ,  $K = [k_1, k_2]^T$ , then,  $\|\hat{\theta}(t) - \theta\|$  is uniformly bounded.

**Proof:** Using the same method as in [26], we can obtain the result.

In order to ensure that  $\hat{\theta}(t)$  converges to the desired value, the following result is needed.

**Lemma 1** Consider the following system

$$\begin{cases} \dot{\eta} &= A_1\eta + B_1\frac{\psi(T, \omega_m)}{J_m}, \\ \bar{y} &= C_1\eta + D_1\frac{\psi(T, \omega_m)}{J_m}, \end{cases} \quad (19)$$

where  $A_1 = \text{diag}\{-b_2, -b_2, -b_2, -b_2\}$ ,  $B_1 = \text{diag}\{-b_2 + \frac{R_a}{L_a}, -b_2 + \frac{R_a}{L_a}, -b_2 + \frac{R_a}{L_a}, -b_2 + \frac{R_a}{L_a}\}$ ,  $C_1 = I$ ,  $D_1 = I$ . If there exist  $T_0 > 0$ ,  $k_p > 0$  such that  $\psi(T, \omega_m)$  satisfies the following persistence excitation condition

$$\int_t^{t+T_0} \psi(T(\tau), \omega_m(\tau))\psi^T(T(\tau), \omega_m(\tau))d\tau > k_p I, \quad (20)$$

then, there exists  $k'_p > 0$  such that

$$\int_t^{t+T_0} \bar{y}(\tau)\bar{y}^T(\tau)d\tau > k'_p I. \quad (21)$$

**Proof:** It follows from (19) that

$$C_1(SI - A_1)B_1 + D_1 = (\frac{s + \frac{R_a}{L_a}}{s + b_2})I,$$

which implies that the system (19) is stable and minimal phase. By Lemma 2.6.7 in [33], we obtain the result.

**Theorem 2** For the energy model of belt conveyor with DC motor (8), there exists filter transformation (12), (13) to transform (8) into (14). Moreover, for the system (15), (16), if  $k_1$  and  $k_2$  are selected such that  $(A, b, C)$  satisfies the strictly positive real condition, and there exist  $T_0 > 0$  and  $k_p > 0$  such that the

condition (20) holds, then, we have  $\lim_{t \rightarrow \infty} \|\theta - \hat{\theta}(t)\| = 0$ .

**Proof:** From (13), we have

$$\eta_i(t) = e^{-b_2 t} \int_{t_0}^t e^{b_2 \tau} \left( -\frac{b_2}{J_m} + \frac{R_a}{L_a J_m} \right) \psi_i(T(\tau), \omega_m(\tau)) d\tau.$$

It is obvious that  $\psi(T, \omega_m)$  is bounded for every  $(T, \omega_m)$  bounded. Therefore,  $\eta_i(t)$  ( $i = 1, 2, 3, 4$ ) are bounded for every  $(T, \omega_m)$ , which implies that  $(\frac{1}{J_m} \psi(T, \omega_m) + \eta)$  is bounded for every  $(T, \omega_m)$  bounded. Along the trajectory of the system (17), (18), calculate the derivative of the following Lyapunov function

$$V(e) = e^T P e + \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \quad (22)$$

we have

$$\begin{aligned} \left. \frac{dV(e)}{dt} \right|_{(17),(18)} &= e^T (A^T P + P A) e \\ &\quad + 2e^T P b \left( \frac{1}{J_m} \psi(T, y) + \eta \right)^T \tilde{\theta} \\ &\quad - \tilde{\theta}^T \left( \frac{1}{J_m} \psi(T, y) + \eta \right) e_1 < -de^T Q e. \end{aligned}$$

Thus,  $\|e(t)\|$  and  $\|\tilde{\theta}(t)\|$  are uniformly bounded for any  $t > t_0$ . Moreover,  $(\frac{1}{J_m} \psi(T, y) + \eta)$  is uniformly bounded, then,  $\|\dot{e}(t)\|$  is also uniformly bounded. Since  $V(e)$  is a uniformly bounded non-increasing function

$$\lim_{t \rightarrow \infty} \int_{t_0}^t e^T(\tau) Q e(\tau) d\tau < V(t_0) - V(\infty) < V(\infty),$$

By Barbalat Lemma [26], we have

$$\lim_{t \rightarrow \infty} \|e(t)\| = 0. \quad (23)$$

From (18) and (23), we have

$$\lim_{t \rightarrow \infty} \dot{\tilde{\theta}}(t) = 0.$$

Then, there exists a constant  $\tilde{\theta}^*$  such that

$$\lim_{t \rightarrow \infty} \tilde{\theta}(t) = \tilde{\theta}^*.$$

Therefore, for any  $\varepsilon > 0$ , there exists  $t_1 > 0$  such that

$$\|\tilde{\theta}(t) - \tilde{\theta}^*\| < \varepsilon, \forall t > t_1. \quad (24)$$

Now, we will prove that  $\tilde{\theta}^* = 0$  by contradiction.

Assume that  $\tilde{\theta}^* \neq 0$ .

Consider the following function

$$\varphi(\tilde{\theta}(t), t) = \frac{1}{2} [\tilde{\theta}^T(t + T_0) \Gamma^{-1} \tilde{\theta}(t + T_0) - \tilde{\theta}^T(t) \Gamma^{-1} \tilde{\theta}(t)],$$

which is bounded. The time derivative of  $\varphi(\tilde{\theta}(t), t)$  is given as

$$\begin{aligned} \frac{d\varphi(\tilde{\theta}(t), t)}{dt} &= \tilde{\theta}^T(t + T_0) \Gamma^{-1} \dot{\tilde{\theta}}(t + T_0) - \tilde{\theta}^T(t) \Gamma^{-1} \dot{\tilde{\theta}}(t) \\ &= \int_t^{t+T_0} \frac{d}{d\tau} (\tilde{\theta}^T(\tau) \Gamma^{-1} \dot{\tilde{\theta}}(\tau)) d\tau \\ &= - \int_t^{t+T_0} \frac{d}{d\tau} (\tilde{\theta}^T(\tau) \bar{y}(\tau) e_1(\tau)) d\tau \\ &= \int_t^{t+T_0} e_1(\tau) \bar{y}^T(\tau) \Gamma \bar{y}(\tau) e_1(\tau) d\tau \\ &\quad - \int_t^{t+T_0} \tilde{\theta}^T(\tau) \dot{\bar{y}}(\tau) e_1(\tau) d\tau \\ &\quad - \int_t^{t+T_0} \tilde{\theta}^T(\tau) \bar{y}(\tau) (e_2(\tau) - k_1 e_1(\tau)) d\tau \\ &\quad - \int_t^{t+T_0} \tilde{\theta}^T(\tau) \bar{y}(\tau) \bar{y}^T(\tau) \tilde{\theta}(\tau) d\tau. \end{aligned}$$

Note that  $\dot{\bar{y}}(\tau) = (\frac{1}{J_m} \dot{\psi}(T(\tau), y(\tau)) + \dot{\eta}(\tau))$  is uniformly bounded for every  $(T, \omega_m)$  bounded, and  $\lim_{t \rightarrow \infty} e(t) = 0$ , and (21), (24) hold, then, there exists  $M > 0$  such that when  $t > t_1$ , we have

$$\begin{aligned} \frac{d\varphi(\tilde{\theta}(t), t)}{dt} &< M \int_t^{t+T_0} (e_1^2(\tau) + e_2(\tau)^2) d\tau \\ &\quad - \int_t^{t+T_0} \tilde{\theta}^{*T} \bar{y}(\tau) \bar{y}^T(\tau) \tilde{\theta}^* d\tau \\ &\quad - 2 \int_t^{t+T_0} \tilde{\theta}^{*T} \bar{y}(\tau) \bar{y}^T(\tau) (\tilde{\theta}(\tau) - \tilde{\theta}^*) d\tau \\ &\quad - \int_t^{t+T_0} (\tilde{\theta}(\tau) - \tilde{\theta}^*)^T \bar{y}(\tau) \bar{y}^T(\tau) (\tilde{\theta}(\tau) - \tilde{\theta}^*) d\tau \\ &< M \int_t^{t+T} (e_1^2(\tau) + e_2(\tau)^2) d\tau \\ &\quad - \frac{k'_p}{2} \tilde{\theta}^{*T} \tilde{\theta}^* < -\frac{k'_p}{4} \tilde{\theta}^{*T} \tilde{\theta}^*, \forall t > t_1, \end{aligned}$$

which contradicts the boundedness of  $\varphi(\tilde{\theta}(t), t)$ .

Therefore,  $\lim_{t \rightarrow \infty} \tilde{\theta}(t) = 0$ . The proof is completed.

#### IV. Simulation Results

We test the proposed adaptive parameter estimation (13), (15), (16) by simulation with parameters  $b_2 = 4.0$ ,  $\lambda = 3$ ,  $\Gamma = \text{diag}\{180, 180, 180, 180\}$  and with four coefficients  $\theta_1 = 1$ ,  $\theta_2 = 0.3$ ,  $\theta_3 = 3.5$ ,  $\theta_4 = 2.1$ , for a DC motor, whose parameters are:  $k_T = 0.1$ ,  $k_b = 0.1$ ,  $b_m = 0.4$ ,  $J_m = 0.05 \text{ Kgm}^2$ ,  $R_a = 15 \text{ Ohm}$ ,  $L_a = 1.0 \text{ H}$ ,  $r = 0.01 \text{ m}$ ,  $e_a = 380 \text{ V}$ . The initial condition of (14), (15), and (16) are given by  $(0.01, 0.7)$ ,  $(0.8, 0.2)$  and  $(0.1, 0.1, 0.1, 0.1)$ , respectively. It should be noted that it is difficult to check the inequality (20) holds. In practice, if the feed rate  $T$  does not change much, a complete determination of all the parameters is impossible. In order to estimate all the parameters, one should sufficiently disturbing the feed rate  $T$  during the period of estimation. In this example, we choose the feed rate  $T = 0.09(8 + 5 \sin(10t + 1) + 2 \cos(-5t + 2) + \sin(20t)) + 0.3(8.6 + 2 \cos(-5t + 2) + \sin(15t + 0.4) + \sin(20t) + 4 \sin(t)) |(\sin(4t + 0.5))| \text{ kg/s}$  ( $0 \leq t \leq 60s$ ). The simulation results are shown in Fig. 3-Fig. 6.

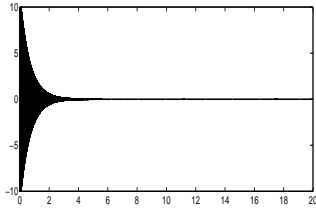


Fig. 3. Trajectory of  $\tilde{\theta}_1$ .

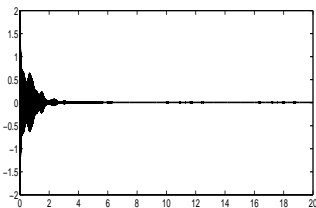


Fig. 4. Trajectory of  $\tilde{\theta}_2$ .

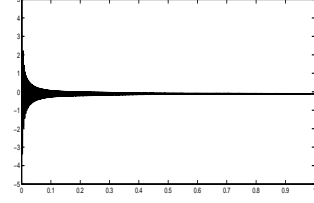


Fig. 5. Trajectory of  $\tilde{\theta}_3$ .

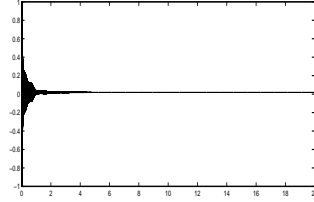


Fig. 6. Trajectory of  $\tilde{\theta}_4$ .

To test the algorithm against measurement noise, a band limited white noise is added to  $y_1$ . The results are demonstrated in Fig. 7-Fig. 10.

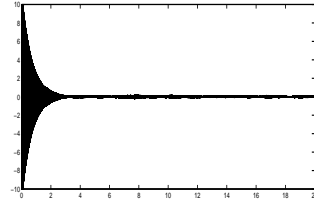


Fig. 7. Trajectory of  $\tilde{\theta}_1$  with band limited white noise.

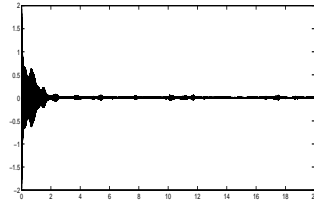


Fig. 8. Trajectory of  $\tilde{\theta}_2$  with band limited white noise.

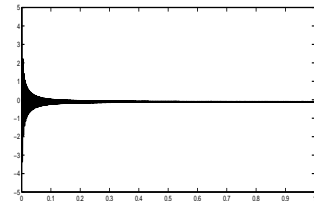


Fig. 9. Trajectory of  $\tilde{\theta}_3$  with band limited white noise.

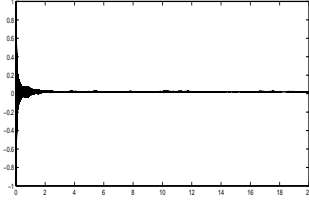


Fig. 10. Trajectory of  $\tilde{\theta}_4$  with band limited white noise.

Practically, the parameters may drift away during the belt conveyor operates. For example,  $\theta_1 = 1.2$ ,  $\theta_2 = 0.3$ ,  $\theta_3 = 3.5$ ,  $\theta_4 = 2.3$ , using the same initial conditions and the feed rate  $T$ , we implement the adaptive identifier for 30s. Fig. 11- Fig. 14 show the simulation results.

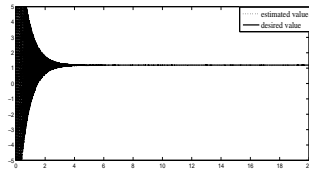


Fig. 11. Trajectories of  $\theta_1$  and  $\hat{\theta}_1$ .

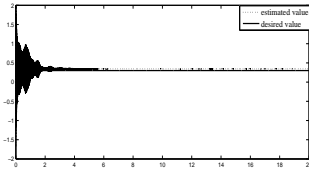


Fig. 12. Trajectories of  $\theta_2$  and  $\hat{\theta}_2$ .

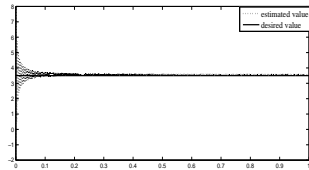


Fig. 13. Trajectories of  $\theta_3$  and  $\hat{\theta}_3$ .

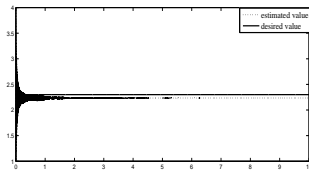


Fig. 14. Trajectories of  $\theta_4$  and  $\hat{\theta}_4$ .

## V. Conclusion

In this paper, a new energy model of a belt conveyor driven by a DC motor was presented, which lumped all the parameters into four coefficients. Then, an adaptive observer was designed to estimate the unknown parameters. In addition, under a persistent excitation condition, the convergence of the parameters to the desired values could also be concluded. Compared with the existing methods, our methods could be implemented by measuring only the feed rate of the belt conveyor and the angular velocity of the rotor of the DC motor.

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