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LAMINAR FREE CONVECTION UP A VERTICAL POROUS PLATE WITH SUCTION

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ABSTRACT

A heated vertical plate generates a buoyant boundary layer that rises up its surface, growing in width as it rises. If suction is applied to the surface of the plate then at some height the buoyant layer ceases to grow and its width no longer varies in the vertical direction. Analytic expressions exist for this fully developed region based on the assumption of uniform suction. If however the suction is through holes that have a diameter or spacing that is significant with respect to the thickness of the thermal boundary layer, then the assumption of uniform suction is questionable. This paper uses CFD to model the flow for this case of non-negligible hole diameter and spacing, and presents results for how the heat transfer differs from the case of uniform suction.

NOMENCLATURE

c_p	[J/kg.K]	Specific heat
D	[m]	Hole diameter
q'	[W/m]	Heat flow per unit length
q''	[W/m^2]	Heat flux
P	[m]	Hole pitch
u_0	[m/s]	Suction velocity (flow rate/plate surface area)
V	[-]	Non-dimensional vertical velocity
y	[m]	Distance from bottom of plate

Special characters

α	[m^2/s]	Thermal diffusivity of the fluid
δ_T	[m]	Nominal width of thermal boundary layer
δ_0	[m]	Reference length based on equation (4)
θ	[-]	Non-dimensional temperature
υ	[m^2/s]	Kinematic viscosity of fluid
ρ	[kg/m^3]	Density

Subscripts

bl	Boundary layer
norm	Normalised with respect to solution for uniform suction
w	Wall
0	Reference
∞	Ambient

INTRODUCTION

Free convection of a fluid heated by a vertical plate is a classical problem of buoyancy induced convective heat transfer. The plate heats the adjacent fluid resulting in a buoyant heated layer that rises up the plate. For a laminar plate held at a constant temperature the width of this boundary layer grows as $y^{1/4}$, where y is the distance from the bottom of the plate [1], with fluid being entrained into the boundary layer from the surroundings.

If a uniform suction is applied to the plate then the growth of the boundary layer is decreased, until a fully developed region is reached where the rate of entrainment of fluid into the boundary layer equals the rate of suction at the wall. Above this height the boundary layer is vertically homogeneous, with the velocity and temperature profiles no longer varying with height [2]. The temperature profile in this region is only dependent on the ratio of wall suction to the thermal diffusivity, whilst the profile of vertical velocity further depends on the Prandtl number of the fluid and the Rayleigh number of the flow and thermal conditions.

The theory proposed by Parikh et al. in [2] is for uniform suction and has been applied to the modelling of transpired solar collectors, which are perforated panels heated by the sun where air is heated by the panel and then drawn in through the perforations [3]. When the holes and their spacing are less than the thickness of the boundary layer then uniform suction may be a good approximation of the flow. However, for a typical solar collector the hole size and spacing may be an order of magnitude larger than the boundary layer thickness, and the validity of the approximation is doubtful.

In this paper we first review the theory for convection with uniform suction. We then use a CFD model to examine how the properties of the flow differ when the suction can no longer be assumed uniform, but instead is through holes that have a size and spacing that is similar to the thickness of the buoyant boundary layer.

FREE CONVECTION WITH UNIFORM SUCTION

As discussed in the introduction, a heated plate generates a layer of buoyant fluid that rises up the surface of the plate. This boundary layer entrains fluid as it rises, growing in width. However, if suction is applied to the plate there reaches a height where the rate of entrainment matches the suction through the plate, resulting in a fully developed region where the width of the plume no longer varies in the vertical axis.

In this fully developed region of the boundary layer the scales for the flow and heat transfer are the temperature difference between the plate and the surroundings, $\Delta T = T_w - T_\infty$, the mean suction velocity u_0, the Prandtl number Pr, and the Rayleigh or Grashof number (Ra or Gr). The temperature may be non-dimensionalised as:

$$\theta = \frac{T - T_\infty}{\Delta T} \tag{1}$$

For the case of uniform suction u_0, the temperature profile in the fluid is given by [2] as:

$$\theta = \exp\left(-\frac{u_0}{\alpha}x\right) \tag{2}$$

Since the flow is independent of height, a reference length needs to be defined to non-dimensionalise the distance from the wall, and to allow the definition of a Rayleigh number. A suitable characteristic width of thermal boundary layer is:

$$\delta_T = \frac{1}{\Delta T}\int_0^\infty T\,dx \tag{3}$$

Using this expression to define a reference length δ_0, the temperature profile given in equation (2) gives:

$$\delta_0 = \frac{\alpha}{u_0} \tag{4}$$

The expression for the temperature profile then becomes:

$$\theta = \exp\left(-\frac{x}{\delta_0}\right) \tag{5}$$

The horizontal velocity is uniformly $-u_0$. It can be shown that for a fluid with a Prandtl number of 1 the vertical velocity has a profile of:

$$V = \frac{v}{u_0 Ra_\delta} = \frac{x}{\delta_0}\exp\left(-\frac{x}{\delta_0}\right) \tag{6}$$

Here the Rayleigh number Ra_δ is based on δ_0. These temperature and velocity profiles are shown in Figure 1. It is interesting to note that the temperature profile depends only upon δ_0, the ratio of the thermal diffusivity and the suction velocity. The profile of the dimensional vertical component of velocity depends additionally on the Prandtl number, and for a Prandtl number of 1 has a magnitude proportional to the Rayleigh number.

For a vertical wall that is being used as a solar collector, two heat flows are of interest. Firstly we are interested in the rate of heat transfer from the plate to the fluid. Since the flow is fully developed and does not vary in the vertical direction, this must equal the enthalpy of the suction fluid drawn through the plate, if we take the reference enthalpy of the ambient fluid to be zero.

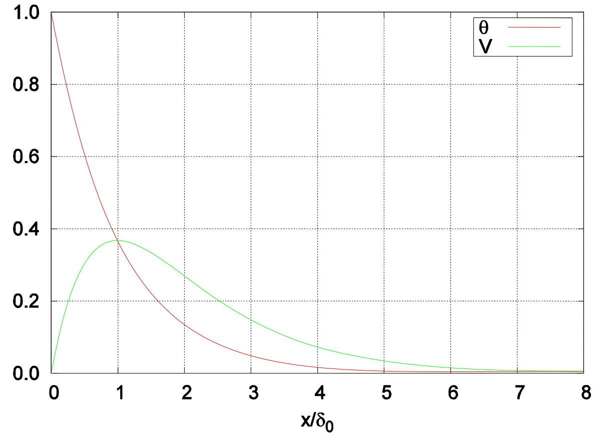

Figure 1 Profiles of temperature and vertical velocity for a buoyant boundary layer on a plate with uniform suction.

Using the temperature profile given in (2), the heat flux from the plate to the fluid is:

$$q''_{conv} = k\left.\frac{dT}{dx}\right|_{x=0} = \rho c_p \alpha \frac{\Delta T}{\delta_0}\exp\left(-\frac{x}{\delta_0}\right) = \rho c_p u_0 \Delta T \tag{7}$$

Since the fluid at the wall is at the same temperature as the wall, then the enthalpy flux of the suction fluid is:

$$q''_{suction} = -\rho c_p u_0 \Delta T \tag{8}$$

These are seen to be equal and opposite; the negative sign for (8) is due to the fluid leaving the domain.

The second heat flow of interest is the vertical heat flow rate of the fluid rising in the buoyant boundary layer. This energy is lost to the environment when the boundary layer leaves the collector as a buoyant plume at the top of the plate. This vertical flow of thermal energy is solely due to convection and is calculated by integrating the product of the mass flow and enthalpy profiles. The profiles in equations (5) and (6) therefore give a vertical heat flow per unit width of:

$$q'_{bl} = \rho c_p \int_0^\infty vT\,dx = \tfrac{1}{4}\rho c_p u_0 Ra_\delta \delta_T \Delta T \tag{9}$$

For a plate held at a constant temperature, increasing the suction velocity would increase the heat flow from the plate. However, for a solar collector the incident radiation that heats the plate is a constant, so increasing the velocity would simply decrease the plate temperature. However, there may be a benefit to decreasing the plate temperature in this manner, since decreasing the plate temperature would reduce the loss due to radiation.

For the plate as a whole the vertical heat flow rate of the buoyant boundary layer is the overall convective heat loss. To reduce this loss either the temperature difference ΔT or the thermal boundary layer width δ_T should be minimised, the latter requiring maximising the suction velocity.

FREE CONVECTION WITH DISCRETE SUCTION

The analysis in the previous section was based on a panel that had uniform suction across its surface. However, in general solar collectors do not have uniform suction but instead are constructed from sheet metal with holes drilled or punched through the surface. If the diameter of the holes, D, and their spacing or pitch, P (Figure 2), are small compared to the width of the thermal boundary layer then uniform suction may be a good approximation. However, if these dimensions are equal or greater than the boundary layer thickness then we would expect the discrete nature of the suction to have some effect on the flow.

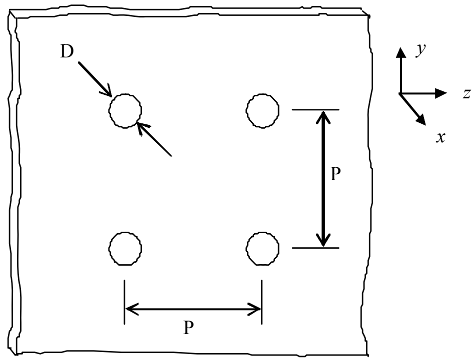

Figure 2 Hole geometry for plate

A test rig at the University of Auckland to model solar collectors has holes of 1.5 mm diameter spaced at a pitch of 20 mm [4]. For typical experiments the temperature difference between the plate and the surrounds was approximately 10 K for suction velocities that varied between 10 and 20 mm/s. This gives a thermal boundary layer thickness of approximately 2 to 1 mm, which is of the order of the hole diameter and an order of magnitude less than the hole pitch. Clearly for such a flow condition the discrete nature of the suction can no longer be ignored. A question remains regarding how having non-uniform suction changes the heat transfer to the fluid drawn through the holes, the thermal boundary layer thickness, and the thermal energy contained in the wall plume.

To model the effect of this non-uniform or discrete suction, the flow was calculated for the geometry shown in Figure 3. The same non-dimensionalisation as was used as for the case of uniform suction, and the flow and temperature fields are again both dependant on the nominal boundary layer thickness δ_0, the Prandtl number, and the Rayleigh or Grashof number. In addition the flow fields also depend on the diameter D and pitch P of the holes in the plate.

Again we are interested in the heat transfer from the plate to the fluid, and the vertical heat flow rate of the buoyant boundary layer. These quantities can be normalised with respect to their values for the case of uniform suction to see how far they depart from this idealised case.

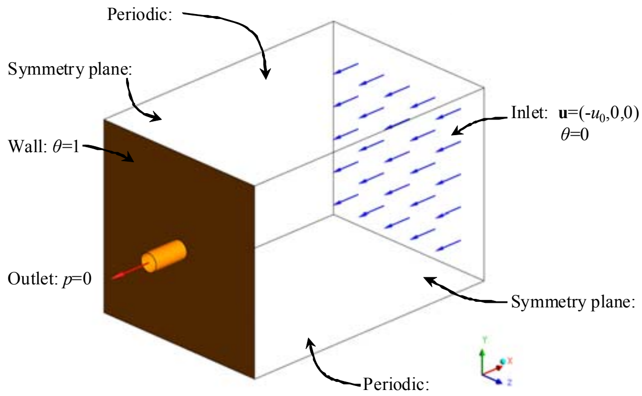

Figure 3 Boundary conditions for the CFD model.

THE NUMERICAL MODEL

The free convection flow was modelled using the CFD package CFX 12.1, the flow being modelled as a laminar constant property flow with buoyancy being modelled using the Boussinesq approximation.

The geometry of a representative domain is shown in Figure 3. An inlet condition at the high x boundary has fluid entering the domain with a constant horizontal velocity of $-u_0$, zero vertical velocity, and a temperature T_∞. Symmetry conditions were imposed at the high and low z boundaries, and the flow was periodic across the upper and lower y boundaries. The plate was modelled as an isothermal no-slip wall at temperature T_w. Fluid could only exit the domain through the end of the pipe where a constant static pressure outlet condition was imposed. Since we were only interested in the heat transfer from the front of the plate, the perimeter of the hole was modelled as an adiabatic no-slip wall.

For comparison with the analytic model for uniform suction flow, the calculations were performed for a fluid with a Prandtl number of 1. Calculations were performed for Rayleigh numbers in the range 1 to 1000, with hole diameters of D/δ_0 of 0.1 to 10, and hole pitches of 2 D to 10 D. These could be compared with analytic solutions for uniform suction which was considered to be the limiting case of $D/\delta_0 = 0$.

RESULTS

Physical reasoning suggests that as $D/\delta_T \rightarrow 0$ and $P/\delta_T \rightarrow 0$ the flow should tend to the solution for uniform suction. However, as the hole diameter or pitch increases the solution should deviate from this model.

We will first consider the heat flux from the plate, which is equal to the enthalpy flow rate of the suction fluid divided by the area of the plate. For hole diameters and pitches that are less than the boundary layer thickness we would expect the suction heat flux to be given by equation (8). However, if the hole diameter is of the same magnitude or larger than the thermal boundary layer one might assume that not all of the fluid drawn into the hole is drawn from within the thermal boundary layer, with some being drawn from the cooler exterior fluid. The suction of this colder fluid would be expected to decrease the enthalpy of the suction flow.

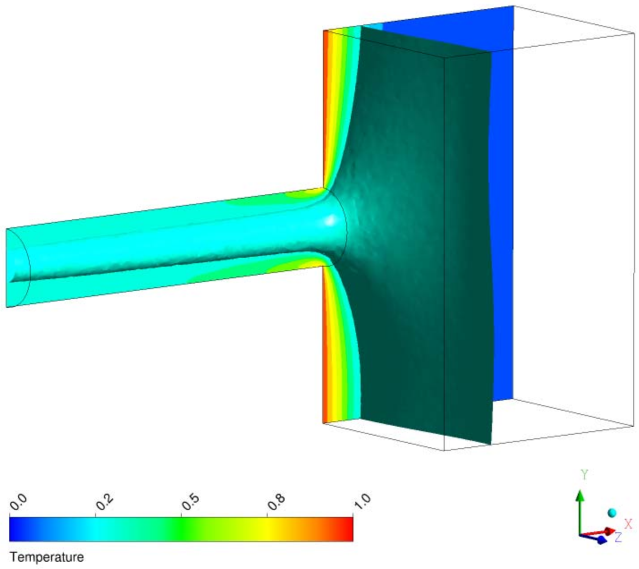

Figure 4 Temperature profiles along vertical centreline and isotherm of $\theta = 0.3$ for $D/\delta_0 = 5$, $P/D = 5$ and $Ra_\delta = 1$.

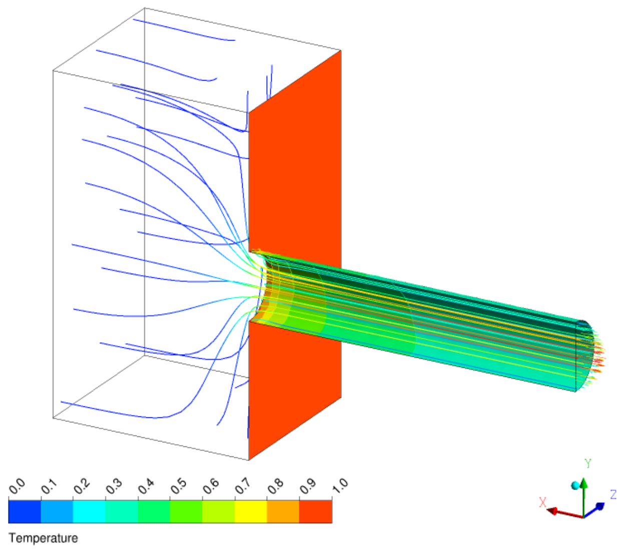

Figure 5 Streamlines and wall temperatures for $D/\delta_0 = 5$, $P/D = 5$ and $Ra_\delta = 1$.

Increasing the hole pitch would have the effect of increasing the velocity at the mouths of the holes and would have a similar effect, with fluid again being drawn in from outside the thermal boundary layer.

These assumptions are borne out in Figure 6 which shows that increasing the hole diameter decreases the suction heat flux, as does increasing the pitch. The flow structure for the case of $D/\delta_0 = 5$, $P/D = 5$ is shown in Figure 5, showing fluid being drawn in from outside the boundary layer, whilst the temperature contours and isotherm in Figure 4 show cold fluid entering the mouth of the hole.

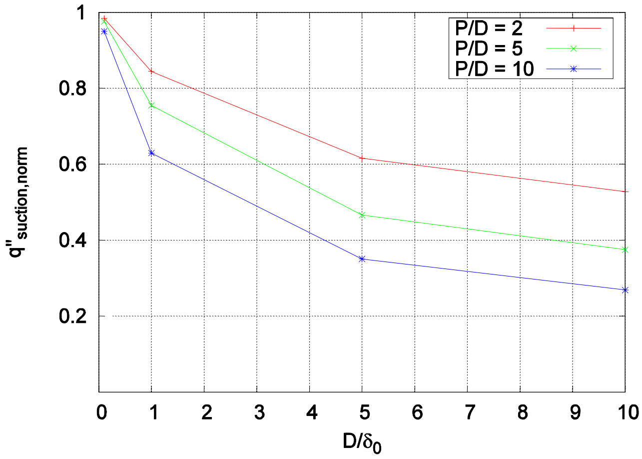

Figure 6 Normalised suction heat flux for $Ra_\delta = 100$.

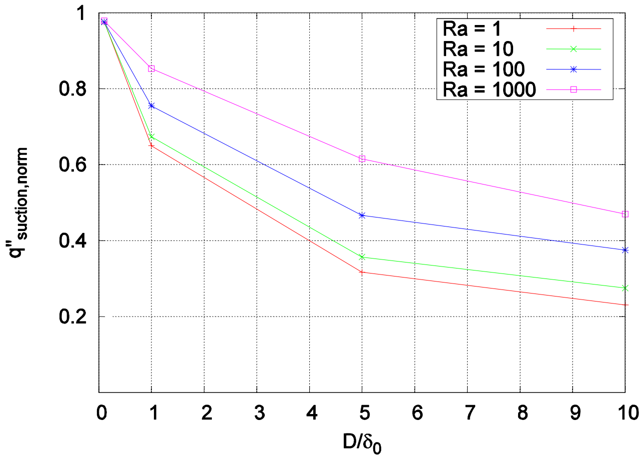

Figure 7 Normalised suction heat flux for hole pitch $P/D = 5$.

Figure 7 shows the effect of varying the Rayleigh number of the thermal boundary layer. Increasing the Rayleigh number (which for a given plate and fluid would correspond to increasing the temperature difference or decreasing the suction velocity) increases the enthalpy of the flow drawn into the hole. This is thought to be due to the higher vertical velocity of the high Rayleigh number flows which creates a jet of warm fluid across the mouth of the hole increasing the temperature of the inlet flow. Figure 8 shows the jet of warm fluid across the mouth of the hole, whilst Figure 9 reveals how the streamlines for the suction flow are drawn from the wall flow, and not from the flow external to the thermal boundary layer. One interesting effect shown in Figure 9 is the formation of a roll in the mouth of the hole, with the flow resembling driven cavity flow.

Figure 10 shows that increasing the hole diameter and pitch has the effect of increasing the thickness of the thermal boundary layer. This increase in boundary layer thickness is understandable when one considers that increasing the hole diameter or pitch reduces the heat flux from the plate surface. All things being equal, this decrease in heat flux must be matched by a corresponding decrease in temperature gradient, which results in a thicker thermal boundary layer.

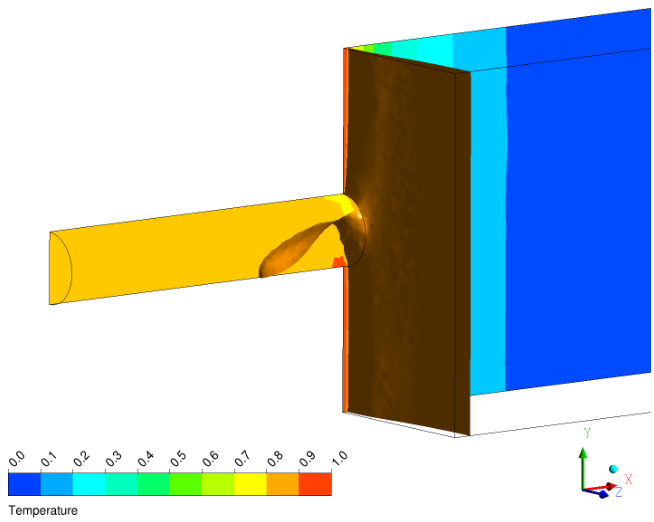

Figure 8 Temperature profiles along vertical centreline and isotherm of $\theta = 0.87$ for $D/\delta_0 = 1$, $P/D = 5$ and $Ra_\delta = 1000$.

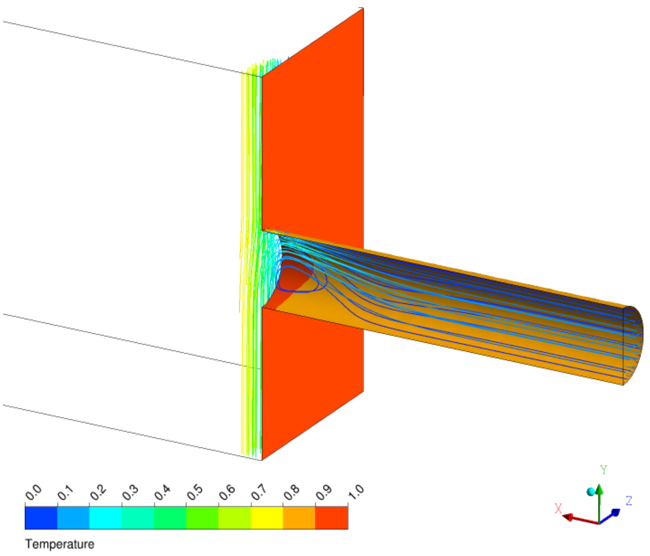

Figure 9 Streamlines and wall temperatures for $D/\delta_0 = 1$, $P/D = 5$ and $Ra_\delta = 1000$.

Finally the effect of hole diameter and pitch upon the vertical heat flow rate in the boundary layer is plotted in Figure 11. The increased heat flow rate over that for uniform suction may be explained by the increased depth of the thermal boundary layer. This results in both a higher mean temperature, and so a higher mean enthalpy, and a higher flow rate due to larger buoyancy forces.

This last diagram is perhaps the most important. For a solar collector the rate of energy lost from the collector is the sum of radiation back to the environment, and the heat flow rate of the buoyant plume at the top of the collector. For the cases considered here, the collector with discrete suction has up to 30 times the convective loss of the case of uniform suction.

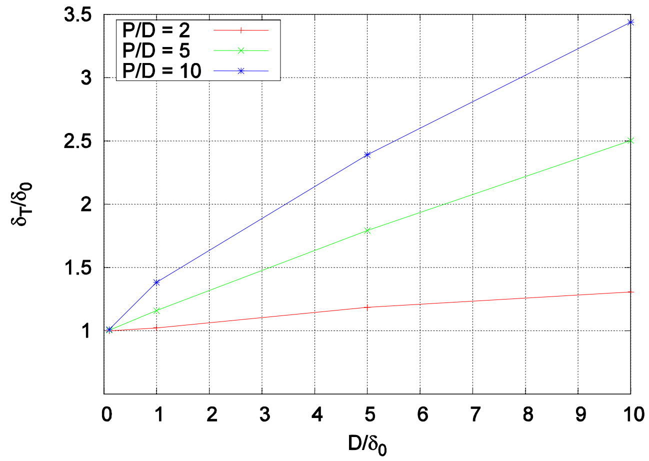

Figure 10 Thickness of thermal boundary layer with varying hole diameter and pitch.

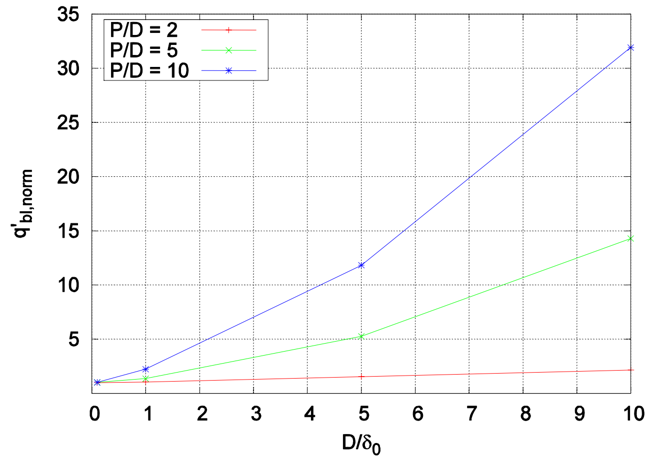

Figure 11 Boundary layer vertical heat flow per unit width of the plate for varying hole diameter and pitch.

CONCLUSIONS

The thermal boundary layer and buoyant wall flow up a heated plate have been modelled for the case of uniform suction, and suction through discrete holes. It is shown that compared to the case of uniform suction, when for hole diameters and spacing are of the order of the thermal boundary layer thickness or larger, the heat flux from the plate is decreased, the thickness of the thermal boundary layer increases, and the vertical heat flow rate of the boundary layer increases. This would result in a decrease in the rate that energy is captured by a solar collector.

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