# DEA APPLIED TO A GAUTENG SAMPLE OF PUBLIC HOSPITALS[1]

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*Abstract*
The research presented in this paper provides an analysis of the delivery of a few health care services by the public sector in Gauteng, South Africa. The data for the study was especially difficult to collect, suggesting the need for hospital level data information systems, as well as staff who are trained to analyze the information collected. The empirical results from the analysis suggest that services provided by small-scale medical facilities waste fewer resources, while medical centres offering more technical services, such as surgeries, also appear to deliver medical services more efficiently.

*Keywords: Data Envelopment Analysis and Non-parametric*

## 1. INTRODUCTION

The provision of healthcare services represents a large component of many African government's health budgets. South Africa is no exception and, with the recent passage of legislation promising to deliver anti-retroviral medicines free of charge to anyone needing the medication, the healthcare services budget will represent an even larger component of South Africa's governmental expenditure.[2] If health-care expenditures represented the only area in which the government purse was under pressure, then it might be possible, even though inappropriate, to be unconcerned with the efficient delivery of public healthcare services. However, healthcare services represent just one of many public sector service delivery concerns in the country. Other public sector projects competing with healthcare services include, but are not limited to: providing clean water and sanitation to a large part of the population, improving the transportation and communication infrastructure, providing improved stadium infrastructure for the 2010 FIFA World Cup,

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[2] The health budget for 2005/06 is estimated at R9.8bn, and is set to rise to R10.7bn in 2006/07 and Rl 1.2bn in 2007/08. Hospital services encompass more than 80% of that budget: estimated to be R7.4bn in 2005/06, R7.9bn in 2006/07, and R8.2bn in 2007/08, National Treasury (2005).

raising the standard and delivery of education at all levels, and reducing the level of crime across the country.

Given the large number of investment and current expenditure projects making up the public budget and remaining on the public's wish list, it is imperative that the public sector carefully examines whether or not the public budget is providing all it can. Recent municipality audit evidence suggests that much more can be done to improve the delivery of public sector services. Although auditing can provide very useful information regarding the precise allocation of inputs in the delivery of certain services, many audits cannot or do not assess the *effectiveness* of those input allocations. Furthermore, when audits are used to ascertain the effectiveness of inputs in the delivery of services, those audits are often (and inevitably) one dimensional; so an audit may be an incomplete approach to the measurement of service delivery effectiveness. Hollingsworth and Parkin (1995) suggest that traditional efficiency indices and performance indicators are subject to manipulation and other sorts of problems. Empirical approaches to measuring efficiency, on the other hand, are more difficult to manipulate, and can provide a more accurate measurements of efficiency.

In this paper, we present research into the efficiency of the delivery of public healthcare. The measurement approach that we use is Data Envelopment Analysis (DEA), which can be used to compare multiple multi-dimensional service delivery outlets. DEA has often been used to examine the delivery of healthcare services. The primary reason for its popularity is the fact that it is one of the few empirical techniques capable of handling multiple inputs and multiple outputs in the same specification. Furthermore, one of DEA's greatest advantages is that it does not specify the function of interest to the analysis, which in this case is a production function, nor does it make an explicit assumption about the distribution of error terms, although there is an implicit assumption — as discussed below.

The rest of this paper proceeds as follows. In section 2 we discuss the most recent relevant research in the field. The theoretical model, as well as its associated advantages and pitfalls, will be considered in section 3- The data used in the analysis is presented in section 4, while section 5 contains the results from the analysis. Finally, we concude with a few recommendations for future research and potential policy implications in section 6.

## 2. RELEVANT LITERATURE

Efficiency, which occurs in various forms in economics, has a rich history in economics and underscores all of economic thinking. Despite the importance of efficiency in economics, the ability to measure it has only recently been developed. In production economics, efficiency takes on two forms, technical efficiency, where firms produce the most output possible with their current set of inputs, and allocative efficiency, where input prices determine the least costly mix of inputs capable of producing along the technically efficient frontier. Theoretically, profit-maximizing and cost-minimizing firms are assumed to achieve technical efficiency in both the short run and the long run, as long as markets are unfettered. However, in the theory of production related to hospitals, cost-minimizing or profit-maximizing behaviour is not necessarily the *modus operandi?*

[3] Research by Newhouse (1970) and Evans (1971) represent early forays into alternative optimizing behaviour.

Despite the fact that pure efficiency or absolute efficiency may not be the expected result when considering public hospital production, due for example to the fact that costs are covered by the national purse, eliciting more and better health care from available resources remains an important public goal. For that reason, the relative efficiency of public hospital production has implications for public policy Unfortunately, hospitals produce a multiple of intermediate goods, all of which go towards the improvement of the patient's health, a good that cannot be easily quantified. Due to the difficulty surrounding the measurement of true hospital output, requiring the measurement of a multiplicity of intermediate outputs, the analysis of hospital production often focuses on the production of intermediate goods; see for example Grosskopf and Valdmanis (1987) and Sexton, Lieken, Nolan, Liss, Hogan and Silkman (1989).

DEA has been applied in a number of hospital efficiency studies. The various analyses in the literature include comparisons of efficiency across ownership types: Grosskopf and Valdmanis (1987) and Valdmanis (1992) compare public and not-for-profit hospital efficiency. Similarly, a number of studies have been conducted to determine the effect of financing on efficiency; Gruca and Nath (2001) and Steinmann and Zweifel (2003) represent two such examples. O'Niell (1998) and Grosskopf, Margaritis and Valdmanis (2001) compare teaching and non-teaching hospital performance, while Hofmarcher, Paterson and Riedel (2002) compare within and across hospital performance across medical fields. Hospital congestion has been examined by Valdmanis, Kumanarayake and Lertiendumrong (2004). Dacosta-Claro and Lapierre (2003), amongst others, have examined returns to scale, while McCallion, Glass, Jackson, Kerr and McKillop (2000), amongst others, have examined differences in performance based on hospital size. All of the preceding studies have been performed within one country or one area of a country; however, differences in efficiency across countries have been studied by Mobley and Magnussen (1998). Given the amount of data available for this study, the analysis in this paper focuses on the simpler comparisons surrounding returns to scale as well as efficiency differences between different types of hospitals.

In earlier research, validity of DEA relied upon simple dynamic and static comparisons. For example, Parkin and Hollingsworth (1997) examine whether or not efficiency scores change profoundly from one year to the next. In another analysis, O'Niell (1998) extends DEA to multi-factor productivity indexes, which can then be compared to more aggregated DEA indexes. Furthermore, Steinmann and Zweifel (2003) examine whether or not the estimated scores are sensitive to the use of inpatient days as an input or as an output.[4] However, many validity issues in DEA are addressed through the introduction of probabilistic notions. For example, Cooper, Li, Seiford, Tone, Thrall and Zhu (2001) discuss the input and output variations required to move firms onto and off the efficiency frontier. Olesen and Pietersen (2002), in a similar vein, describe the measurement of probabilistic assurance regions in DEA. Validity analysis undertaken in this research is based on a number of simple comparisons, especially in the comparison of efficiency scores across a wide range of input and output combinations.

Finally, it is important to note that the analysis undertaken and reported in this paper is not the first to consider Africa or South Africa, although it is the first to examine hospitals in Gauteng. Zere, Mclntyre and Addison (2001) used data from the former

[4] They argue that inpatient days, which are often used as an output in DEA, might better represent an input, since patients are using their days in the hospital to recuperate.

Cape Province and the current Western Cape Province covering the years 1992 to 1998. The data they used was different from the data used in this research, which could explain why they measure average efficiency to be lower than the average estimates provided here. Importantly since their data only covers the Western Cape, it is unclear whether or not their results are representative of healthcare delivery at a national level. Although the exact emphasis of the analysis presented in this paper is different from that presented by Zere *et al.* (2001), it is hoped that the empirical results will help fill the gap in research that exists, regarding the effective delivery of healthcare services in South Africa.

## 3. DATA ENVELOPMENT ANALYSIS

Data Envelopment Analysis, based on the radial measure of efficiency originally developed by Farrell (1957) and extended by many others, including Charnes, Cooper and Rhodes (1978), Banker, Charnes and Cooper (1984), and Fare, Grosskopf and Lovell (1985), is the empirical model applied in this paper. Although the model is empirical, in the sense that observations determine the estimates, the model is non-parametric, since neither a functional form nor an empirical error distribution is assumed.[5] Although there are few specification assumptions, Newhouse (1994) argues that frontier estimation models should be treated cautiously because inputs and outputs are difficult to measure; certain strong and non-testable hypotheses regarding noise and inefficiency distributions must be made, while limited degrees of freedom require too much aggregation of the data. Despite Newhouse's (1994) concerns, it is possible that overarching tendencies can be uncovered in the empirical analysis and, therefore, the analysis can provide some guidance for managerial improvement within hospitals.

An illustration of the intuition behind DEA is provided in Fig. 1, in which five combinations of weighted inputs and weighted outputs (see below) are illustrated as A

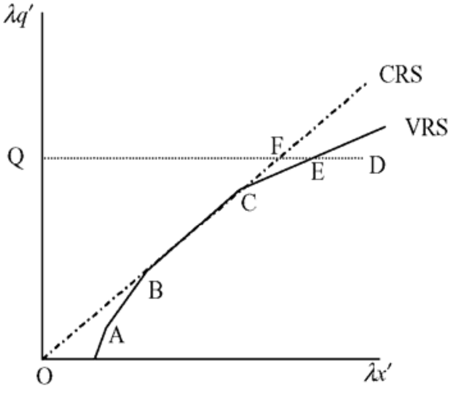


*Figure 1. Constant Returns to Scale (CRS) Compared to Variable Returns to Scale (VRS) Calculated Via Data Envelopment Analysis*

' According to Banker (1993), under certain assumptions, DEA is a maximum likelihood estimator, and, when error distributions are either half-normal or exponential, standard statistical tests can be conducted using the DEA estimates.

through E. In the long run, under constant returns to scale (CRS) technology, combinations B and C are technically and scale efficient, while A, D and E are inefficient. On the other hand, under variable returns to scale (VRS) technology, in the short run, combinations A through D are technically efficient, while E remains inefficient. Therefore, combination E is inefficient in the short run as well as the long run. Total inefficiency for E, denoted by the horizontal distance EF, can be explained by scale inefficiency (the horizontal distance FG) and technical inefficiency (the horizontal distance EG). Furthermore, the technical inefficiency of E is determined by a convex combination of C and D; therefore C and D represent technological peers of E.

In order to formalize the illustration, consider public hospitals, denoted by *i = {\, 2, . . . , I},* which produce outputs $q_i^j$, for 7 = {1,2,...,/} using inputs $x_i^k$, for *k* = {1, 2 . . . , *K*. The preceding technology can be used for the creation of an index, determined by the ratio of a weighted sum of the inputs to a weighted sum of the outputs. DEA assumes this efficiency index or ratio, denoted *byE,* must lie in the unit simplex for all firms, so that:[6]

$$E_i = \frac{\sum_{j=i}^{J} \phi_i^j q_i^j}{\sum_{k=1}^{K} \omega_i^k x_i^k} \in [0,1] \forall_i \in \{1, 2, \ldots, I\}. \tag{1}$$

As noted in equation (1), the output weights and the input weights, which both must be non-negative, are hospital specific. Assuming that all other hospitals must also meet the restriction in equation (1), the efficiency score for each hospital is chosen so that the relative weights allow for the most favourable view of the hospital. Although equation (1) is non-linear and the constraints, also given in equation (1), are non-linear, the efficiency score for each hospital can be determined by a linear programme.

Confining the consideration to relative weights, such that either the relative input weights or the relative output weights determine the efficiency score, will lead to the following linear programme, given in equation *(2)7*

$$\begin{aligned} &Min\colon E_0 = \theta_0 \\ &\sum_{i=1}^{I} x_i^k \lambda_i = x_i^0 \theta_0 - s_i^k \ \ \forall k \in \{1, 2, \ldots, K\} \\ &\sum_{i=1}^{I} \lambda_i q_i^j = q_0^j + r_i^j \ \ \forall j \in \{1, 2, \ldots, J\} \\ &\text{subject to: } \lambda_i, s_i, r_i \geq 0 \ \forall i \in \{1, 2, \ldots, I\} \end{aligned} \tag{2}$$

Intuitively, $\theta_0$ represents the smallest proportional reduction in inputs used by firm 0 to keep it on the frontier determined by a convex combination of the inputs used by all firms in the data set. Furthermore, the output produced by firm 0 cannot exceed the same convex combination of outputs produced by all the firms in the data set, where r and s measure slackness in the constraints. Programme (2), which allows for the convex

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[6] An excellent intuitive description of the technique can be found in Parkin and Holfingsworth (1997), while a more technical, but readable description, can be found in Holfingsworth, Dawson and Maniadakis (1999). Equations (1), (2), and (3) are adapted from these two papers, amongst others.

[7] This program is actually the dual, although the primal problem is easily formulated.

combinations to be chosen freely, is equivalent to an assumption of CRS; see Charnes *et al.* (1978). However, if the convex combinations are further constrained, as in equation (3), VRS technology is assumed; see Banker *et al.* (1984).[8]

$$\sum_{i=1}^{I} \lambda_i = 1 \tag{3}$$

Regardless of whether CRS or VRS is assumed, and both will be considered in this research, a public hospital is defined as efficient if and only if (i) $\overline{\theta}_i = 1$ and (ii) $\overline{r_i}^j = \overline{s_i}^j = 0$, where an over-bar represents a solution value. One of the most useful features of the analysis is the fact that the efficiency measure is invariant to the choice of units in the measurement, although it is not invariant to either the number of inputs or the number of outputs used in the analysis. Unfortunately, neither the input slacks, *s,* nor the output slacks, r, are invariant to the units of analysis; see Steinmann and Zweifel (2003). As can be seen in Fig. 1 and the discussion above, the input contraction required to make combination E technically efficient is a ratio of the horizontal distances G and E, and that ratio is unit free; however, the input slack for the same problem would be determined by the horizontal distance EG, and that total distance would depend upon the unit of measure along the input axis.[9]

The models represented in equations (2) and (3) are applied using different subsets of inputs and outputs. The goal of the analysis presented here is to learn if public hospital efficiency is general, suggesting that certain public hospitals are more poorly managed than others, or if public hospital efficiency might be input-output specific, suggesting that certain hospitals undertake certain services or use certain inputs more efficiently than others. Additional analysis of the DEA outcomes will be undertaken to determine if different types of hospitals are generally more or less efficient.

## 4. THE DATA

The data used in the analysis is primary data collected during 2004. All of the public hospitals in the province of Gauteng were contacted.[10] There are 29 public hospitals in the province, one of which is a women's hospital, another is a long-term rehabilitation centre, while another is an academic hospital, so that they were not included in the analysis. Of the remaining 26 hospitals, only 14 provided data on some of the inputs and outputs desired for the investigation. Unfortunately, not all of the hospitals could be used in all of the analyses, due to the fact that some hospitals did not provide complete information, e.g., some hospitals do not offer surgery or, if offered, data was not provided.

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[8] Although CRS technology results from the fact that output can only be doubled if inputs are doubled, which is a one-to-one relationship, suggesting that the restriction in equation (3) ought to relate to CRS, rather than VRS, that comparison is not correct. Instead, the restriction limits output expansion beyond the best firm and output contraction below the worst firm, given current input combinations. For a more thorough discussion see Valdmanis (1992).

[9] The slacks can be made invariant to the choice of units *via* a reciprocal measure of efficiency, as well as the inclusion of upper bounds on the input and output weights, Steinmann and Zweifel (2003).

[10] Gauteng is the wealthiest province in South Africa. It includes the business capital of the country, in Johannesburg and Sandton, as well as the executive branch of the national government, in Pretoria.

The participating hospitals provided monthly data on inputs and outputs, as far back as 1999, in a few cases; generally, though, the hospitals provided monthly data for the preceding year, 2003, and up to six months or more of the investigating year, 2004. The data provided by those 14 hospitals varied in detail, and, therefore, the analysis was forced to focus on data commonalities. Three input variables were available from all of the hospitals: physicians (doctors and specialists), nurses, and active beds. In addition, up to four output variables were available from the hospitals: total admissions, inpatient visits, outpatient days and total surgeries. The most complete information was available for admissions and inpatient days, although it was not available for all hospitals at all times.

As can be garnered from the 62% hospital response rate,[11] the willingness or ability to participate in the study was limited. In many cases, hospital CEOs or other administrators provided initial consent to the study, but were later forced to recant, because they did not have staff that could provide us with the data, or because their hospital board had, in the meantime, rejected the research participation application. In many other cases, approval was granted, but data collection could not proceed, due to staff turnover. The average waiting time, between initial data request and final receipt of the data, was 4.35 months (Kibambe and Koch, 2005). As was clear from the data collection efforts, many of the hospitals lacked the necessary information systems or the staff to manage the information systems, and data often had to be transcribed from numerous sources, if it was available. Despite these difficulties, some of the public hospitals in the province were able to provide data back to 1999, suggesting that some hospitals had adequate information systems, and the staff were adequately trained to work with the systems.[12] A further concern, raised by the poor response rate, is the potential that the sample is selective, i.e. only the hospitals efficient enough to collect their own data were willing to participate. If the observations were from a selected sample, then the results reported below would only be representative of the sample, rather than being representative of the entire province.

A summary of the data is provided in Table l.[13] The monthly output data has been averaged over each year for all of the hospitals, for which at least part of the year's data was available. In addition, for hospitals from which more than one year of data was available, each year's data is counted as a separate observation in the sample. Due to the restructuring of the monthly data, the 14 hospitals could be reorganized into 42 different observations. The data in the table is presented by size of hospital, as measured by the number of active beds, where the median value of 220 was chosen as the cut-off between large and small hospitals. The table includes the input and output variable averages and standard deviations as well as the number of non-zero responses for that particular input or output.

The presentation of the data in Table 1 highlights a number of important issues. As already mentioned, all hospitals were able to provide data on all of the inputs, but not for

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[1] The calculated response rate was based on the fact that 18 hospitals, out of 29, responded positively to data requests. As already mentioned in the text, three of those hospitals were removed from the data, due to the specialist nature of their services, while one of the hospitals offered a single annual observation that turned out to be incorrect.

[12] In related research, Kibambe and Koch (2005) find a strong positive relationship between the hospital's ability to provide data and certain measures of efficiency.

[13] Data is not provided by hospital, even under moniker, in order to prevent any hospital from being singled out in the analysis.

*Table 1. Summary Statistics of Analysis Data*

| Input Variable | Large Hospitals (n = 20) Active Beds > 220 Average | Non-zero Observations | Small Hospitals (n = 22) Active Beds < 221 Average | Non-zero Observations |
|---|---|---|---|---|
| Active beds | 762.94 (304.4) | 20 | 151.90 (35.7) | 22 |
| Medical doctors & Specialists | 192.28 (114.4) | 20 | 12.51 (3.9) | 22 |
| Nurses | 920.67 (407.0) | 20 | 124.95 (20.4) | 22 |
| **Output Variable** | | | | |
| Outpatient Visits | 7763.48 (15926.9) | 12 | 1678.02 (2126.7) | 10 |
| Total admissions | 1572.62 (1843.4) | 18 | 944.43 (471.8) | 22 |
| Inpatient days | 17163.94 (9975.9) | 18 | 3470.05 (1295.0) | 22 |
| Theater case/Surgeries | 127.53 (327.6) | 6 | 117.66 (95.6) | 15 |

*Note:* Standard deviations are in parenthesis.
*Source:* Authors' calculations from primary data collected for a subset of public hospitals in Gauteng province from 1999 to 2004.

the outputs. Also, there is a notable difference between large and small hospitals in terms of input usage as well as output usage. However, if you consider simple ratios of inputs to outputs, larger hospitals appear to use relatively more inputs than smaller hospitals in producing each of the inputs. For example, large hospitals use 5-0 (763/152) times more nurses than smaller hospitals, although total admissions is only 1.1 times larger (128/118); total outpatient days produced is only *4.6* (7763/1678) times larger; and for inpatient days the ratio is 4.9 (17164/3470). These results suggest that there may be important hospital scale effects, further supporting the need to compare CRS and VRS technologies.

Individuals expect a number of different services from hospitals; primarily, they expect to be healthier upon exit than arrival. It is likely, though, that the individual's expectation for health improvement is a strong determinant of hospital usage.[14] For that reason, the best measure of hospital production is the amount of improvement obtained by the patient. However, data on health improvement does not exist. Despite the lack of data on one measure of output quality, it would still be possible to control for quality in other ways, if data on the medical centre's case-mix could be garnered. Given the difficulty in obtaining basic data on hospital outputs, however, it should not be surprising that case-mix data was not obtained. Therefore, the results presented below focus on the data that has been made available.

## 5. THE RESULTS

The main results from DEA applied to the Gauteng public hospital dataset are presented in Tables 2 through *7*■ Tables 2 through 6 present a comparison between efficiency measured against CRS to the efficiency calculated against VRS, for each of the possible

[14] Research by Leonard, Mliga and Mariam (2002) shows that these quality of care perceptions are very important for determining health centre bypass behaviour, where individuals bypass a closer health facility in order to seek health care from farther away, in rural Africa.

*Table 2. Summary Results for Single Output DEA Models*

| Assumed Model | Inputs | | | Outputs | | | | | | Retruns to Scale | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatient Visits | Inpatient Days | Admissions | Surgeries | Average Efficiency | Numbef of Technically Efficient Public Hospitals | Number of IRS | Number of CRS | Number of DRS |
| CRS (n = 39) | x | x | | | | x | | 54.1 | 3 | | | |
| VRS (n = 39) | x | x | | | | x | | 71.4 | 10 | 5 | 9 | 25 |
| CRS (n = 39) | x | | x | | | x | | 45.8 | 3 | | | |
| VRS (n = 39) | x | | x | | | x | | 63.6 | 6 | 8 | 6 | 25 |
| CRS (n = 39) | | x | x | | | x | | 57.6 | 3 | | | |
| VRS (n = 39) | | x | x | | | x | | 69.5 | 10 | 11 | 10 | 18 |
| CRS (n = 39) | x | x | x | | | x | | 58.1 | 5 | | | |
| VRS (n = 39) | x | x | x | | | x | | 72.6 | 13 | 6 | 13 | 20 |
| CRS (n = 39) | x | x | | | x | | | 77.6 | 3 | | | |
| VRS (n = 39) | x | x | | | x | | | 88.8 | 11 | 8 | 10 | 21 |
| CRS (n = 39) | x | | x | | x | | | 69.6 | 3 | | | |
| VRS (n = 39) | x | | x | | x | | | 81.4 | 8 | 10 | 8 | 21 |
| CRS (n = 39) | | x | x | | x | | | 62.0 | 1 | | | |
| VRS (n = 39) | | x | x | | x | | | 83.9 | 8 | 12 | 8 | 19 |
| CRS (n = 39) | x | x | x | | x | | | 77.8 | 3 | | | |
| VRS (n = 39) | x | x | x | | x | | | 90.0 | 13 | 7 | 13 | |
| CRS (n = 21) | x | x | | x | | | | 52.0 | 2 | | | |
| VRS (n = 21) | x | x | | x | | | | 70.0 | 8 | 6 | 3 | 12 |
| CRS (n = 21) | x | | x | x | | | | 37.9 | 1 | | | |
| VRS (n = 21) | x | | x | x | | | | 65.8 | 7 | 0 | 2 | 19 |
| CRS (n = 21) | | x | x | x | | | | 51.8 | 2 | | | |
| VRS (n = 21) | | x | x | x | | | | 67.3 | 3 | 5 | 3 | 13 |
| CRS (n = 21) | x | x | x | x | | | | 53.4 | 2 | | | |
| VRS (n = 21) | x | x | x | x | | | | 71.7 | 8 | 5 | 3 | 13 |
| CRS (n = 21) | x | x | | | | | x | 68.6 | 2 | | | |
| VRS (n = 21) | x | x | | | | | x | 89.1 | 5 | 2 | 5 | 14 |
| CRS (n = 21) | x | | x | | | | x | 71.6 | 2 | | | |
| VRS (n = 21) | x | | x | | | | x | 86.8 | 6 | 6 | 6 | 9 |
| CRS (n = 21) | | x | x | | | | x | 68.6 | 2 | | | |
| VRS (n = 21) | | x | x | | | | x | 84.4 | 5 | 2 | 5 | 14 |
| CRS (n = 21) | x | x | x | | | | x | 73.7 | 3 | | | |
| VRS (n = 21) | x | x | x | | | | x | 90.3 | 7 | 2 | 7 | 12 |

*Source:* Authors' calculations from DEA analysis on subset of Gauteng public hospitals.

input-output combinations, if there is enough data. Table 7, on the other hand, contains the results for non-parametric statistical tests of potential population differences. Due to the limited availability of data, as discussed in the previous section, some of the results in each of the tables are based on rather small samples and, therefore, those results should be treated cautiously.

*5.1. Single Outputs*

Initially, the efficiency scores for public hospitals were separately computed for each output at the hospital level. The calculations were conducted assuming both CRS and VRS; the results are summarized in Tables 2 and 3- The first column of each table lists the technology assumption used in the analysis as well as the number of public hospitals included in the analysis. In Table 2, The next group of columns, headed by "Inputs" and "Outputs", show, by means of an x' in the column, which inputs and outputs were included in the analysis. Finally, the last few columns provide the average relative efficiency attained by the public hospitals in the analysis, the number of hospitals in the sample to have attained an efficiency score of 1, and the number of hospitals to be

*Table 3. Summary of Single Output DEA Slack Estimates*

| Assumed Model | Number of Public Hospitals with Input Slacks | | | Number of Public Hosptials with Output Slacks | | | | |
|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatient Visits | Inpatient Days | Admissions | Surgeries | Average Efficiency |
| CRS (n = 39) | 6 | 11 | | | | | | 54.1 |
| VRS (n = 39) | 4 | 13 | | | | 19 | | 71.4 |
| CRS (n = 39) | 23 | | 9 | | | | | 45.8 |
| VRS (n = 39) | 4 | | 12 | | | 22 | | 63.6 |
| CRS (n = 39) | | 6 | 15 | | | | | 57.6 |
| VRS (n = 39) | | 10 | 10 | | | 14 | | 69.5 |
| CRS (n = 39) | 17 | 9 | 16 | | | | | 58.1 |
| VRS (n = 39) | 7 | 13 | 18 | | | 17 | | 72.6 |
| CRS (n = 39) | 2 | 5 | | | | | | 77.6 |
| VRS (n = 39) | 4 | 7 | | | 9 | | | 88.8 |
| CRS (n = 39) | 3 | | 5 | | | | | 69.6 |
| VRS (n = 39) | 9 | | 6 | | 4 | | | 81.4 |
| CRS (n = 39) | | 28 | 8 | | | | | 62.0 |
| VRS (n = 39) | | 12 | 15 | | 3 | | | 83.9 |
| CRS (n = 39) | 3 | 10 | 31 | | | | | 77.8 |
| VRS (n = 39) | 6 | 7 | 17 | | 6 | | | 90.0 |
| CRS (n = 21) | 3 | 1 | | | | | | 52.0 |
| VRS (n = 21) | 3 | 12 | | 12 | | | | 70.0 |
| CRS (n = 21) | 17 | | 3 | | | | | 37.9 |
| VRS (n = 21) | 10 | | 9 | 12 | | | | 65.8 |
| CRS (n = 21) | | 1 | 6 | | | | | 51.8 |
| VRS (n = 21) | | 10 | 5 | 12 | | | | 67.3 |
| CRS (n = 21) | 11 | 1 | 11 | | | | | 53.4 |
| VRS (n = 21) | 8 | 13 | 11 | 12 | | | | 71.7 |
| CRS (n = 21) | 1 | 8 | | | | | | 68.6 |
| VRS (n = 21) | 0 | 4 | | | | | 3 | 89.1 |
| CRS (n = 21) | 0 | | 1 | | | | | 71.6 |
| VRS (n = 21) | 0 | | 6 | | | | 3 | 86.8 |
| CRS (n = 21) | | 5 | 8 | | | | | 68.6 |
| VRS (n = 21) | | 4 | 7 | | | | 3 | 84.4 |
| CRS (n = 21) | 1 | 8 | 5 | | | | | 73.7 |
| VRS (n = 21) | 2 | 5 | 9 | | | | 4 | 90.3 |

*Source:* Authors' summary of slack results from DEA applied to Gauteng public hospitals.

classified as operating under increasing returns to scale, constant returns to scale, and decreasing returns to scale, respectively.[15] In Table 3, however, the headings are slightly different. Table 3 provides information on the slacks,[16] which are calculated in the DEA.[17] Therefore, the second group of columns provides information on the number of hospitals in the sample, which required further input reductions, while the third group of columns provides the number of hospitals in the sample requiring output expansions. The last column in Table 3 reiterates the calculated average efficiency score in the sample.

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[15] Returns to scale calculations are only available under the VRS model assumption.

[16] Due to the non-invariance of input slacks in this model, the information provided is the total number of hospitals in the sample with observed positive slack values for each input and output used in the calculation.

[17] The input slacks represent additional reductions, beyond the proportional reduction calculated by the efficiency score, required to keep a firm's input on the convex combination of all firms' inputs. Output slacks are similarly calculated.

*Table 4. Summary Results for Multiple Output DEA*

| Assumed Model | Inputs | | | Outputs | | | | | | Returns to Scale | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatient Visits | Inpatient Days | Admissions | Surgeries | Average Efficiency | Numbef of Technically Efficient Public Hospitals | Number of IRS | Number of CRS | Number of DRS |
| CRS (n = 37) | x | x | | | x | x | | 85.4 | 9 | | | |
| VRS (n = 37) | x | x | | | x | x | | 91.8 | 14 | 7 | 13 | 17 |
| CRS (n = 37) | x | | x | | x | x | | 75.0 | 5 | | | |
| VRS (n = 37) | x | | x | | x | x | | 84.8 | 9 | 8 | 9 | 20 |
| CRS (n = 37) | | x | x | | x | x | | 79.5 | 6 | | | |
| VRS (n = 37) | | x | x | | x | x | | 89.5 | 14 | 14 | 14 | 9 |
| CRS (n = 37) | x | x | x | | x | x | | 87.0 | 12 | | | |
| VRS (n = 37) | x | x | x | | x | x | | 92.7 | 16 | 7 | 16 | 14 |
| CRS (n = 21) | x | x | | x | x | | | 85.0 | 6 | | | |
| VRS (n = 21) | x | x | | x | x | | | 93.8 | 13 | 6 | 9 | 6 |
| CRS (n = 21) | x | | x | x | x | | | 83.7 | 3 | | | |
| VRS (n = 21) | x | | x | x | x | | | 94.8 | 10 | 8 | 6 | 7 |
| CRS (n = 21) | | x | x | x | x | | | 80.9 | 4 | | | |
| VRS (n = 37) | | x | x | x | x | | | 90.0 | 8 | 7 | 8 | 6 |
| CRS (n = 21) | x | x | x | x | x | | | 87.5 | 6 | | | |
| VRS (n = 21) | x | x | x | x | x | | | 95.7 | 14 | 5 | 10 | 6 |
| CRS (n = 20) | x | x | | | x | | x | 91.4 | 7 | | | |
| VRS (n = 20) | x | x | | | x | | x | 98.0 | 13 | 2 | 13 | 5 |
| CRS (n = 20) | x | | x | | x | | x | 92.3 | 8 | | | |
| VRS (n = 20) | x | | x | | x | | x | 97.8 | 12 | 3 | 12 | 5 |
| CRS (n = 20) | | x | x | | x | | x | 80.2 | 4 | | | |
| VRS (n = 20) | | x | x | | x | | x | 93.3 | 9 | 2 | 9 | 9 |
| CRS (n = 20) | x | x | x | | x | | x | 93.0 | 8 | | | |
| VRS (n = 20) | x | x | x | | x | | x | 98.9 | 14 | 2 | 14 | 4 |
| CRS (n = 19) | x | x | | x | | x | | 70.3 | 3 | | | |
| VRS (n = 19) | x | x | | x | | x | | 83.9 | 13 | 1 | 9 | 9 |
| CRS (n = 19) | x | | x | x | | x | | 69.3 | 4 | | | |
| VRS (n = 19) | x | | x | x | | x | | 83.7 | 9 | 5 | 5 | 9 |
| CRS (n = 19) | | x | x | x | | x | | 73.5 | 4 | | | |
| VRS (n = 19) | | x | x | x | | x | | 83.3 | 10 | 1 | 9 | 9 |
| CRS (n = 19) | x | x | x | x | | x | | 74.1 | 4 | | | |
| VRS (n = 19) | x | x | x | x | | x | | 85.6 | 13 | 1 | 9 | 9 |
| CRS (n = 19) | x | x | | | | x | x | 87.8 | 5 | | | |
| VRS (n = 19) | x | x | | | | x | x | 97.8 | 10 | 1 | 10 | 8 |
| CRS (n = 19) | x | | x | | | x | x | 82.9 | 4 | | | |
| VRS (n = 19) | x | | x | | | x | x | 96.7 | 10 | 4 | 10 | 5 |
| CRS (n = 19) | | x | x | | | x | x | 88.2 | 5 | | | |
| VRS (n = 19) | | x | x | | | x | x | 97.0 | 10 | 3 | 10 | |
| CRS (n = 19) | x | x | x | | | x | x | 90.3 | 7 | | | |
| VRS (n = 19) | x | x | x | | | x | x | 98.9 | 12 | 1 | 12 | 6 |

*Source:* Authors' calculations from DEA analysis on subset of Gauteng public hospitals.

The results in Table 2 show that the efficiency scores, as expected, depend upon the input combinations used to produce the output, as well as the choice of output. The results in the table also show that the efficiency score rises when the model specification is relaxed. In these models, the relaxation occurs in two dimensions. In the seventh and eighth rows of each eight-row block in the table, three inputs are used to produce each output, as opposed to the two inputs assumed in the first six rows. In each block in the table, the average efficiency score is higher in the last two rows than in any of the first six rows. Also, the model is relaxed with the CRS assumption, the results of which appear in the even rows of each block in the table. As expected, adjusting the model from CRS to VRS increases the number of efficient hospitals in the sample, which is part of the

*Table 5. Summary of Multiple Output DEA Slack Estimates*

| Assumed Model | Number of Public Hospitals with Input Slacks | | | Number of Public Hosptials with Output Slacks | | | | |
|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatients Visits | Inpatients Days | Admissions | Surgeries | Average Efficiency |
| CRS (n = 37) | 2 | 6 | | | 1 | 6 | | 85.4 |
| VRS (n = 37) | 3 | 6 | | | 7 | 16 | | 91.8 |
| CRS (n = 37) | 12 | | 6 | | 5 | 12 | | 75.0 |
| VRS (n = 37) | 6 | | 6 | | 5 | 18 | | 84.8 |
| CRS (n = 37) | | 11 | 7 | | 4 | 10 | | 79.5 |
| VRS (n = 37) | | 6 | 10 | | 6 | 6 | | 89.5 |
| CRS (n = 37) | 4 | 5 | 15 | | 0 | 6 | | 87.0 |
| VRS (n = 37) | 4 | 6 | 15 | | 5 | 15 | | 92.7 |
| CRS (n = 21) | 0 | 1 | | 11 | 4 | | | 85.0 |
| VRS (n = 21) | 0 | 6 | | 10 | 5 | | | 93.8 |
| CRS (n = 21) | 6 | | 3 | 16 | 0 | | | 83.7 |
| VRS (n = 21) | 3 | | 8 | 14 | 6 | | | 94.8 |
| CRS (n = 21) | | 12 | 4 | 12 | 4 | | | 80.9 |
| VRS (n = 21) | | 6 | 6 | 12 | 6 | | | 90.0 |
| CRS (n = 21) | 5 | 8 | 7 | 11 | 4 | | | 87.5 |
| VRS (n = 21) | 3 | 8 | 7 | 10 | 6 | | | 95.7 |
| CRS (n = 20) | 0 | 5 | | | 0 | | 3 | 91.4 |
| VRS (n = 20) | 1 | 2 | | | 6 | | 3 | 98.0 |
| CRS (n = 20) | 0 | | 5 | | 0 | | 4 | 92.3 |
| VRS (n = 20) | 0 | | 4 | | 4 | | 5 | 97.8 |
| CRS (n = 20) | | 8 | 7 | | 4 | | 8 | 80.2 |
| VRS (n = 20) | | 4 | 6 | | 4 | | 7 | 93.3 |
| CRS (n = 20) | 0 | 7 | 8 | | 0 | | 4 | 93.0 |
| VRS (n = 20) | 1 | 2 | 5 | | 4 | | 3 | 98.9 |
| CRS (n = 19) | 6 | 8 | | 9 | | 0 | | 70.3 |
| VRS (n = 19) | 1 | 9 | | 9 | | 5 | | 83.9 |
| CRS (n = 19) | 8 | | 5 | 8 | | 0 | | 69.3 |
| VRS (n = 19) | 3 | | 11 | 9 | | 9 | | 83.7 |
| CRS (n = 19) | | 2 | 5 | 8 | | 0 | | 73.5 |
| VRS (n = 19) | | 6 | 2 | 9 | | 5 | | 83.3 |
| CRS (n = 19) | 9 | 6 | 9 | 8 | | 0 | | 74.1 |
| VRS (n = 19) | 3 | 9 | 6 | 9 | | 6 | | 85.6 |
| CRS (n = 19) | 4 | 4 | | | | 3 | 7 | 87.8 |
| VRS (n = 19) | 3 | 2 | | | | 6 | 5 | 97.8 |
| CRS (n = 19) | 0 | | 6 | | | 5 | 6 | 82.9 |
| VRS (n = 19) | 1 | | 4 | | | 6 | 5 | 96.7 |
| CRS (n = 19) | | 4 | 7 | | | 3 | 8 | 88.2 |
| VRS (n = 19) | | 2 | 5 | | | 2 | 5 | 97.0 |
| CRS (n = 19) | 4 | 4 | 6 | | | 3 | 6 | 90.3 |
| VRS (n = 19) | 3 | 2 | 6 | | | 6 | 4 | 98.9 |

*Source:* Authors' summary of slack results from DEA applied to Gauteng public hospitals.

explanation for the increased average relative efficiency observed in the sample.[18] Average relative efficiency across CRS calculations varies from a low of 37-9% up to a maximum of 77-8%, while average relative efficiency in VRS models varies from a low of 63-6% up to a maximum of 90.3%.

From an economic perspective, the results presented in the table are less obvious. Essentially there are two implications contained in Table 2, subsequently supported in

[18] The rest of the increased average is due to the fact that all the remaining hospitals in the sample cannot have a lower efficiency score under VRS than CRS. Only some of the hospitals will actually rise to full efficiency, though; see Fig. 1 for an illustration.

*Table 6. Summary Results for Additional Multiple Output DEA*

| Assumed Model | Inputs | | | Outputs | | | | | | Returns to Scale | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatient Visits | Inpatient Days | Admissions | Surgeries | Average Efficiency | Numbef of Technically Efficient Public Hospitals | Number of IRS | Number of CRS | Number of DRS |
| CRS (n = 19) | x | x | | x | x | x | | 92.2 | 7 | | | |
| VRS (n = 19) | x | x | | x | x | x | | 97.3 | 14 | 5 | 10 | 4 |
| CRS (n = 19) | x | | x | x | x | x | | 93.2 | 5 | | | |
| VRS (n = 19) | x | | x | x | x | x | | 98.7 | 12 | 6 | 8 | 5 |
| CRS (n = 19) | | x | x | x | x | x | | 89.5 | 6 | | | |
| VRS (n = 19) | | x | x | x | x | x | | 94.9 | 10 | 5 | 9 | 5 |
| CRS (n = 19) | x | x | x | x | x | x | | 95.0 | 8 | | | |
| VRS (n = 19) | x | x | x | x | x | x | | 98.7 | 14 | 4 | 10 | 5 |
| CRS (n = 19) | x | x | | | x | x | x | 95.8 | 12 | | | |
| VRS (n = 19) | x | x | | | x | x | x | 98.7 | 14 | 1 | 14 | 4 |
| CRS (n = 19) | x | | x | | x | x | x | 95.1 | 11 | | | |
| VRS (n = 19) | x | | x | | x | x | x | 99.2 | 16 | 1 | 16 | 2 |
| CRS (n = 19) | | x | x | | x | x | x | 93.5 | 9 | | | |
| VRS (n = 19) | | x | x | | x | x | x | 97.8 | 13 | 3 | 13 | 3 |
| CRS (n = 19) | x | x | x | | x | x | x | 97.1 | 14 | | | |
| VRS (n = 19) | x | x | x | | x | x | x | 99.7 | 16 | 0 | 16 | 3 |

*Source:* Authors' calculations from DEA analysis on subset of Gauteng public hospitals.

Table 3- The first implication is that public hospitals in Gauteng, according to the analysis, are more likely to be operating under decreasing returns to scale than either increasing returns to scale or constant returns to scale. Such a result suggests that public hospitals have too many inputs; however, that would be a naive interpretation of the results. Due to the fact that trained doctors and nurses have become some of the most common emigrants from South Africa, it might be expected that public hospitals had too many active beds, given the population of doctors and nurses. Intuitively, returns to scale are determined by the fixed input which is, in most of the calculations, active hospital beds.[19] Table 3 provides anecdotal evidence that, in fact, there may be too many beds relative to medical professionals. The second implication taken from the results in Table 2 is that hospitals providing inpatient services and surgeries, and those hospitals that were able to provide actual inpatient day and surgery numbers, are more similar to each other than hospitals only able to provide data on admissions and outpatient days; however, neither group is necessarily more or less likely to be more efficient than the other. The output slacks in Table 3 further support the implied similarity between certain types of hospital production in the sample.[20] The number of hospitals with output slacks for inpatient days and surgeries are smaller than the number of hospitals with output slacks for admissions and outpatient days.

[19] Input and output slacks, residuals from the analysis, show that, with few exceptions, additional reductions in bed inputs are not required, once the efficiency score has been calculated, in order to keep bed inputs on the convex combination; see equation (2). On the other hand, doctors, and especially nurses, are slack more often. In other words, active beds are driving the efficiency score, so that returns to scale are strongly influenced by the ability of beds to translate into output.

[20] With only one output, the CRS model will not yield output slacks; rather, there must be at least two outputs.

*Table 7. Summary of Non-parametric Tests of Distribution Equivalence Across a Selected Subsample of Gauteng Public Hospitals*

| Aussumed Model | Inputs | | | Outputs | | | | Kruskal-Wallis Chi-Sq Values | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Beds | Doctors | Nurses | Outpatient Visits | Inpatient Days | Admissions | Surgeries | Large and Small Hospitals the Same? | Offer or not Offer Surgery, the same? | Offer Outpatient Services or not, the same? |
| CRS (n = 39) | x | x | | | | x | | 13.8 | 8.9 | 0.4 |
| VRS (n = 39) | x | x | | | | x | | 6.1 | 6.7 | 0.5 |
| CRS (n = 39) | x | | x | | | x | | 14.5 | 0.9 | 0.1 |
| VRS (n = 39) | x | | x | | | x | | 17.0 | 1.1 | 0.0 |
| CRS (n = 39) | | x | x | | | x | | 16.2 | 7.1 | 0.1 |
| VRS (n = 39) | | x | x | | | x | | 6.3 | 5.2 | 0.0 |
| CRS (n = 39) | x | x | x | | | x | | 16.7 | 6.8 | 0.0 |
| VRS (n = 39) | x | x | x | | | x | | 8.1 | 5.1 | 0.1 |
| CRS (n = 39) | x | x | | | x | | | 2.7 | 0.2 | 0.4 |
| VRS (n = 39) | x | x | | | x | | | 4.0 | 5.3 | 0.0 |
| CRS (n = 39) | x | | x | | x | | | 2.3 | 0.4 | 1.6 |
| VRS (n = 39) | x | | x | | x | | | 9.6 | 1.2 | 1.8 |
| CRS (n = 39) | | x | x | | x | | | 0.7 | 7.9 | 0.8 |
| VRS (n = 39) | | x | x | | x | | | 0.3 | 9.0 | 0.5 |
| CRS (n = 39) | x | x | x | | x | | | 2.9 | 0.2 | 0.4 |
| VRS (n = 39) | x | x | x | | x | | | 3.9 | 4.1 | 0.0 |
| CRS (n = 37) | x | x | | | x | x | | 9.0 | 6.2 | 0.7 |
| VRS (n = 37) | x | x | | | x | x | | 1.1 | 3.7 | 0.5 |
| CRS (n = 37) | x | | x | | x | x | | 4.3 | 0.3 | 3.0 |
| VRS (n = 37) | x | | x | | x | x | | 2.9 | 1.9 | 0.2 |
| CRS (n = 37) | | x | x | | x | x | | 13.4 | 10.2 | 0.7 |
| VRS (n = 37) | | x | x | | x | x | | 1.7 | 6.1 | 0.0 |
| CRS (n = 37) | x | x | x | | x | x | | 10.3 | 5.9 | 0.8 |
| VRS (n = 37) | x | x | x | | x | x | | 2.4 | 4.9 | 0.0 |

*Source:* Chi-squared values computed via STATA 8.2 SE kwallis command. Data taken from DEA results summarized in Tables 2 and 4.
*Note:* Critical values for Kruskal-Wallis tests at 5% confidence is 3.84.

*5-2. Dual Outputs*

Tables 4 through 6 contain summary information for DE Analysis undertaken for multiple output combinations, using the same input combinations discussed in the preceding subsection. The information contained in Tables 4 and 6 is the same as the information contained in Table 2 and, therefore, their column headings follow the same pattern; Table 5 contains the same information as Table 3 and, therefore, the two tables have equivalent column headings.[21]

The empirical results in each of the last three tables show, as expected and shown before, that increasing the model's flexibility cannot reduce the average efficiency score in the sample, because no single efficiency score can be lowered. For example, more public

[21] Although there are actually six potential two-output combinations, there were only 12 observations in the sample when outpatient days and surgeries were combined, for that reason, there are only five two-output combinations listed in Tables 4 and 5. Similarly, although there are three three-output combinations and one four-output combination available in the data, including both outpatient days and surgery in the output combinations resulted in two few observations; therefore, there are only two three-output combinations presented in Table 6.

hospitals, regardless of the combination investigated, are determined to be efficient under VRS than CRS: CRS DEA averages range from 70.3% to 90.3%, while the VRS DEA averages range from 83-3% to 98.9%. Finally, efficiency averages are higher in Table 4 than in Table 2, due to the inclusion of an additional output in the mix. Once again, these efficiencies are relative in the sense that the higher average does not absolutely imply a more efficient set of public hospitals. Rather, it could also imply a more uniform set of observations, which are actually less efficient, overall. As with the single output analysis, decreasing returns to scale are relatively more common than increasing returns to scale, although constant returns to scale are more common than either increasing or decreasing returns to scale.

The inclusion of an additional input, as compared to the results in Table 2 and 3, however, makes the interpretation of input and output slacks more difficult. The input and output slacks for the dual output DEA models are presented in Table 5- Unlike in the single output case, there are no obvious patterns. In the single output models, admissions and outpatient days were associated with a larger number of observed output slacks. When either admissions or outpatient days are combined with inpatient days, the same result holds; however, when either outpatient days or admissions are combined with another output, including each other, there are fewer observed output slacks. A similar story emerges regarding input slacks, also shown in Table 5- With few exceptions, as in Table 3, there are fewer positive active bed slacks than with other inputs, which could increase the count of decreasing returns to scale observations, especially when compared to increasing returns to scale. However, the second set of outcomes presented in each table, in particular, suggests very similar numbers of increasing returns and decreasing returns observations, despite the small number of observed active bed input slacks.

The final DEA table, Table 6, presents the three-output combination DEA results. A detailed discussion of the results will not be undertaken here, given the similarity with results already presented, as well as the fact that very few public hospitals in the sample were deemed to be inefficient. However, the table does, once again, continue to reveal the increase in calculated efficiency likely to result from the increase in model flexibility. Although there is some support for the continued presence of decreasing returns to scale over increasing returns to scale, the numerical differences are less pronounced than in Tables 2 and 4.

*53. Efficiency Differences*

The analysis concludes with a comparison of the differences in measured efficiency across hospital populations. Due to the fact that many of the calculations involved small numbers of observations, these final comparisons are based only on the sets of results for which there were a minimum of 30 observations. In other words, the comparison is for the first two single output DEA models (presented in Tables 2 and 3) as well as the first of the dual output DEA models (presented in Tables 4 and 5). Using the available sample data to distinguish between *(i)* large and small hospitals, *(ii)* hospitals offering outpatient services, and *(Hi)* hospitals offering surgical services, a non-parametric test is used to statistically differentiate the populations, if they can be differentiated. Table 7 contains the $\chi^2_1$-statistic associated with a Kruskal-Wallis non-parametric test which, as its null hypothesis, assumes efficiency scores are drawn from the same population.[22]

[22] The critical value for the test, using 5% confidence, is 3.84.

When the sample is split by hospital size, where more than 220 beds are defined as a large hospital, the Kruskal-Wallis test rejects the null hypothesis in all but 9 of 24 cases.[23] However, in all but one of the 24 cases, when the sample is split based upon whether or not the medical centre offers outpatient services, the Kruskal-Wallis test accepts the null hypothesis.[24] If, on the other hand, hospitals are split according to whether or not they provided data on surgical services, the null hypothesis of equal populations was accepted in 8 of 24 and, therefore, not accepted in 16 of the 24.

In conclusion, the organization and provision of services at large hospitals are often statistically different from the organization and provision of services at small hospitals. In fact, the average efficiency score is higher in small hospitals, suggesting that smaller hospitals organize their production activities more efficiently[25] Furthermore, hospitals offering data on surgical procedures are often statistically different from those not offering such services, where again the tendency is towards improved efficiency[26] However, there does not appear to be any difference between medical centres providing data on outpatient services compared to those centres providing that data.

## 6. CONCLUSION AND RECOMMENDATIONS

An incomplete sample of Gauteng public hospitals was used for the purpose of generating efficiency scores using a linear programming technique referred to as Data Envelopment Analysis. The data was difficult to obtain due to participation reluctance as well as information system inadequacy. Although every attempt was made to include all public hospitals in the analysis, approximately 50% of the population could not be included in the analysis. Due to the limited participation, which could have been selective in nature, the results from the preceding analysis should be treated cautiously. The broadest empirical conclusions to be extracted from the analysis are from a small set of the DE Analyses that were employed in the research. For this subset of analyses, there is a statistical difference between small and large medical centres as well as between centres offering and not offering surgical procedures. The statistical difference between large and small hospitals is consistent with another broad observation that public hospitals in Gauteng more commonly operate under decreasing returns to scale than under increasing returns to scale. Decreasing returns to scale could be due to the emigration of qualified medical professionals, or it could be related to the need to hold excess capacity in case of a large-scale negative health event.

[23] The 24 cases are based on the number of input-output combinations for which there were at least 37 observations. See those combinations in Tables 2 through 6.

[2] In actual fact, it is not clear whether outpatient services are or are not provided; rather it is only clear that the medical facility did not make data on outpatient visits available.

[25] Although a table of these averages is not provided, the average efficiencies for the sample of small hospitals using beds and doctors to produce admissions were 75.6 (CRS) and 91.9 (VRS), compared to the large hospital averages of 29.1 (CRS) and 47.6 (VRS). The difference in averages across many of the other model specifications is similarly large.

[26] Medical centres providing data on surgeries, averaged 64.2 (CRS) and 71.1 (VRS) per cent efficiency, compared to 51.4 (CRS) and 67.0 (VRS), for those centres that did not, in the case of producing admissions using medical doctors and nurses. Similar results obtain in other model specifications.

Regarding efficiency, according to the single output estimates, where there are a reasonably large number of observations, surgeries and inpatient days are more efficiently produced than outpatient visits and admissions. However, the relatively improved efficiency could obtain because the medical centres providing surgeries and inpatient services are more uniform than the medical centres providing outpatient services and admissions; in particular, it is true that all medical centres admit patients, which suggests that if there are differences between centres, that heterogeneity will be most acute across total admissions.

Finally, from a public policy perspective, the availability of data for analysis is a concern. The empirical approach used in this paper, as discussed in earlier sections, is not without flaws. Despite those flaws, DEA, when used carefully, can be used to guide resource allocation in multiple output production units, as long as the data used in the analysis is representative of the production process and can be compared to appropriate peer production units. The data used in this analysis may not be completely representative of either the production process or the public hospital population; therefore, it is absolutely necessary that medical centres across the country be encouraged, if not required, to develop and implement data warehousing systems, so that additional research on this topic can be undertaken. Furthermore, those warehousing systems should be equivalent across the entire public health delivery system

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