Viscosity of nanofluids based on an artificial intelligence model

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Abstract

By using an FCM-based Adaptive neuro-fuzzy inference system (FCM-ANFIS) and a set of experimental data, models were developed to predict the effective viscosity of nanofluids. The effective viscosity was selected as the target parameter, and the volume concentration, temperature and size of the nanoparticles were considered as the input (design) parameters. To model the viscosity, experimental data from literature were divided into two sets; a train and a test data set. The model was instructed by the train set and the results were compared with the experimental data set. The predicted viscosities were compared with experimental data for four nanofluids, which were Al_2O_3, CuO, TiO_2 and SiO_2, and with water as base fluid. The viscosities were also compared with several of the most cited correlations in literature. The results, which were obtained by the proposed FCM-ANFIS model, in general compared very well with the experimental measurement.

Nomenclature

d_p	nanoparticle average diameter (m and nm)
T	temperature (°C and K)
T_0	reference temperature (°C)
m	system factor
C	correction factor
R	thickness of capping layer (m)
r	radius of nanoparticle (m)
n	number of data points
X_p	predicted value
X_a	actual (experimental) data

Greek letters

ϕ	volume concentration
ϕ_e	effective volume concentration

μ_{nf}	viscosity of nanofluid (mPa.s)
μ_{bf}	viscosity of base fluid (mPa.s)
α, β, γ	empirical constant
δ	distance between the center of nanoparticles (m)
ρ_p	particle density (Kg/m^3)

Abbreviation

FCM-ANFIS	FCM-based Neuro-Fuzzy Inference System
MAE	Mean Absolute Error
MRE	Mean Relative Error
RMSE	Root Mean Squared Error

Keyword

Nanofluid; Effective viscosity; FCM-based adaptive neuro-fuzzy inference system (FCM-ANFIS); Particle size; Volume concentration; Temperature

1. Introduction

Viscosity is one of the most important thermophysical properties of nanofluids, especially in thermal applications where heat transfer and fluid flow occur. Changes in viscosity properties in industrial applications influence the pumping power required as well as the convective heat transfer coefficients. Therefore, accurate information on the viscosity properties of nanofluids is essential [1]. Although the heat capacity and density of nanofluids can be predicted accurately it is challenging to determine with great accuracy, the viscosity of nanofluids due to hydrodynamic interactions and particle-particle interactions of nanoparticles in dispersions [2].

The study of nanofluids as next-generation heat transfer fluids has received significant attention since the work of Masuda et al. [3] and Choi [4] has been followed by a large number of published papers over the past two decades. However, most of this work concentrated on potential applications, nanofluid synthesis and thermal conductivity prediction models [5]. Only a few studies [6 – 8] have been conducted on the viscosity of nanofluids as summarised below.

Pak and Cho [6] measured the viscosity of nanofluids containing different sizes of γ-Al_2O_3- and TiO_2 nanoparticles in a water base for different volume concentrations. They observed that the viscosity of the nanofluids decreased asymptotically as the temperature increased and the rate of the decrease became larger with an increase in volume concentration.

Nguyen et al. [7] reported some experimental data for the viscosity of Al_2O_3- (with an average diameter of 36 and 47 nm) and CuO (with an average diameter of 29 nm) nanoparticles mixed in water from ambient temperature to 75°C at different volume concentrations from 1 to 9%. Their results showed that the nanoparticle size effect is more significant for high volume concentrations in comparison with volume concentrations below 4%.

Chen et al. [8] experimentally investigated the rheological properties of TiO_2 and titanate nanotubes in water and ethylene glycol base fluids at volume concentrations below 2%. They observed that base fluids have an important effect on the rheological properties of nanofluids. Nanofluids containing an ethylene glycol base fluid showed a Newtonian behaviour, while the nanofluids containing a water-based fluid showed a non-Newtonian behaviour. Recently, artificial intelligent techniques have become increasingly prevalent for solving complex engineering problems in different application areas with a considerable reduction in computational time [9]. Artificial intelligent techniques, which are known as fuzzy logic, neural networks and genetic algorithms, are among the systems which transfer the knowledge and rules that exist beyond the input data into the network structure by data processing [10]. Kurt and Kayfeci [11] developed an artificial neural network model to predict the thermal conductivities of ethylene glycol/water-based nanofluids by taking into account temperatures, volume concentrations and densities. Papari et al. [12] modelled the thermal conductivity of single-wall carbon nanotubes and multi-wall carbon nanotubes dispersed into several base fluids by using a diffusion neural network.

Neural networks, as well as fuzzy logic approaches, have advantages and deficiencies. However, a neuro-fuzzy system that is created by the combination of an artificial neural network and a fuzzy logic approach can recover the weaknesses of these two methods and create an efficient method to model engineering systems. The neuro-fuzzy method uses learning approaches derived from an artificial neural network in order to find the appropriate fuzzy membership functions and fuzzy rules. An adaptive neuro-fuzzy inference system (ANFIS) is one of the neuro-fuzzy systems in which a learning algorithm is aligned with an integrated learning approach [13]. Mehrabi et al.

[14] developed two different models based on an FCM-based neuro-fuzzy inference system (FCM-ANFIS) and a genetic algorithm-polynomial neural network (GA-PNN) approach to model the thermal conductivity ratio of Al_2O_3-water nanofluids as function of particle size, volume concentration and temperature.

The modelling technique employed in the present paper is FCM-ANFIS. This method uses a neural network and fuzzy logic advantages for modelling the viscosity of nanofluids. It is the purpose of this paper to introduce the FCM-ANFIS method for predicting the viscosity of nanofluids as a function of particle size, volume concentration and temperature.

2. FCM-based neuro-fuzzy inference system modelling technique

Various structures have been suggested to establish a fuzzy system with neural networks but among them ANFIS, which was developed by Jang [15], is one of the most important ones. In the ANFIS system, the neural network and fuzzy logic approaches are combined, as it can produce accurate results that will include both fuzzy intellect as well as simulation capabilities of a neural network.

The ANFIS structure is organised into two parts; an introductory and concluding part, which are linked together by a set of rules. There are five distinct layers in the structure of an ANFIS network, which form a multilayer network. The first layer performs fuzzy formation and the second layer performs fuzzy rules. The third layer performs normalisation of membership functions, the fourth layer is the conclusive part of fuzzy rules and the last layer calculates network outputs. Detailed information about ANFIS network structure and each layer function is given in Mehrabi et al. [13].

There are three different structure identification methods for an ANFIS model; grid partitioning, subtractive clustering and fuzzy C-means clustering. Each structure identification method consists of six different steps, which are selecting the input variables, input space partitioning, choosing the number and the kind of membership functions for input variables, creating fuzzy rules, premise and conclusion parts of fuzzy rules, and selecting the initial parameters for membership functions.

In this paper, the fuzzy C-means clustering (FCM) method is selected as it can identify the promised membership functions of the ANFIS model. The detailed information about the FCM identification method was previously used by the present authors and is fully described in Ref. [14].

3. Effective parameters on viscosity of nanofluids

There are several parameters that influence the viscosity of nanofluids; namely temperature, volume concentration and thickness of the nanolayer, as well as the nanoparticle geometrical properties such as nanoparticle size, shape, aspect ratio and interparticle spacing. Empirical investigations have been conducted on the effect of electromagnetic fields, electro-viscous, dispersion energy and settling time on the viscosity of nanofluids as well as the influence of base fluid properties such as density and polarity [16]. Among these parameters, the three important ones which were chosen for this study are particle size, volume concentration and temperature.

3.1 Effect of particle size

Namburu et al. [17] measured the viscosity of nanofluids containing three different sizes of silicon dioxide nanoparticles with diameters of 20, 50 and 100 nm over a temperature range from -35 to 50°C at volume concentrations of 2, 4, 6 and 10%. Their results showed that the viscosity decreased as the particle size increased. Lu and Fan [18] conducted an experimental and numerical investigation into the viscosity of Al_2O_3-nanoparticles with average diameters of 35, 45 and 90 nm in water and ethylene glycol-based suspensions. They observed the same results as Namburu et al. [17], namely that the viscosity decreased as the particle sizes increased. Pastoriza-Gallego et al. [19] reported viscosity measurements of water containing CuO nanoparticles with average diameters of 33±13 and 11±3 nm, temperatures from 10 - 50ºC, and volume concentrations from 0.16 - 1.17%. They also observed that for a constant volume concentration, the nanofluid samples with smaller average particle sizes had a larger viscosity.

3.2 Effect of volume concentration

Most of the viscosity data of nanofluids in the literature exhibited the trend that as the volume concentration of the particles increased, the effective viscosity also increased [1,7, 17, 18, 20-22]. Chevalier et al. [23] measured the relative viscosity of nanofluids containing three different sizes of silicon dioxide nanoparticles with diameters of 35±3, 94±5 and 190±8 nm at different volume concentrations up to 7%. They also observed that the relative viscosity increased as the volume concentration increased. Duangthongsuk and Wongwises [24] reported viscosity measurements of water containing TiO_2 nanoparticles with average particle diameters of 21 nm at three different temperatures, which were 15, 25 and 35°C. They conducted their experimental work with a parallel-plate rotational rheometer at five different volume concentrations ranging from 0.2 to 2%. They observed the same result as Chevalier et al. [23], namely that the relative viscosity increased as the volume concentration increased.

3.3 Effect of temperature

Chen et al. [25] measured the viscosity of distilled water, ethylene glycol, glycerol and silicone oil suspensions with different multi-wall carbon nanotube volume fractions as a function of temperature by using a plate-and-cone viscometer. They studied the temperature effect on the viscosity at temperatures from 5 to 65ºC and they observed that the viscosity decreased as the temperature increased. Lee et al. [26] reported viscosity measurements of distilled water containing silicon carbide nanoparticles at temperatures between 28 and 72ºC. They observed that the viscosity decreased as the temperature increased. The experimental results published by Prasher et al. [27] and Chen et al. [28, 29] showed that the relative viscosity of Al_2O_3-propylene glycol and TiO_2-water nanofluids is independent of temperature at temperatures between 30 to 60 ºC and 20 to 60ºC, respectively. However, the experimental data of Lee et al. [26] do not correspond to this observation that the relative viscosity is independent of temperature. In literature there is no discussion of the effect of temperature on the effective viscosity of nanofluids which decreased as temperature increased.

4. Experimental data used for the training and the testing procedure

Nguyen et al. [7] investigated the viscosity of Al_2O_3-water and CuO-water nanofluids with a piston-type viscometer. They studied the temperature and volume concentration effects of the viscosity of Al_2O_3-water nanofluids with average diameters of 36 and 47 nm as well as CuO-water nanofluids with an average diameter of 29 nm.

Their experiments covered a wide range of temperatures from 21 - 70°C, for four nanoparticle volume concentrations.

Tavman et al. [30] experimentally investigated the viscosity of SiO_2-water and Al_2O_3-water nanofluids prepared with 12 and 30 nm average diameters of silicon dioxide and alumina nanoparticle, respectively. They conducted their experimental work at seven temperatures from 20 - 50°C, for different volume concentrations. Lee et al. [31] measured the viscosity of Al_2O_3-water nanofluid by using an oscillation-type viscometer. They dispersed Al_2O_3-powder with an average diameter of 30 nm into water and measured the viscosity of Al_2O_3-water nanofluid sizes over a range of temperatures from 21 - 39°C, at low volume concentrations.

Duangthongsuk and Wongwises [24] reported some experimental data for viscosity of TiO_2, with an average diameter of 21 nm, in a water-based nanofluid for three different temperatures of 15, 25 and 35°C, at volume concentrations of 0.2, 0.6, 1, 1.5 and 2%. Turgut et al. [32] reported their measurements for the viscosity of TiO_2-water suspensions with a nominal diameter of 21 nm for four different volume concentrations up to 3% over a temperature range from 13 to 55°C.

Anoop et al. [33] measured the viscosity of alumina-water nanofluids by using a cone-plate viscometer. They used alumina nanoparticles with an average diameter of 95 nm for their experiments and reported the results for volume concentrations of 1, 2, 4 and 6% for the temperature range of 20 - 50°C.

Pastoriza-Gallego et al. [34, 19] published their experimental investigation of the effect of temperature variation and volume concentration on viscosity of Al_2O_3-water suspensions. Al_2O_3-nanoparticles with 8 and 43 nm average diameters were mixed with water at seven different volume concentrations ranging from 0.13 - 2.9% and the resulting suspensions were evaluated at temperatures ranging from 10 - 60°C. Furthermore, they measured the viscosity of CuO-water nanofluids with 33 and 11 nm nanoparticle sizes for eight different temperatures (10, 15, 20, 25, 30, 40 and 50°C) at low volume concentrations ranging from 0.16 to 1.17%.

Kwek et al. [35] conducted an experimental investigation into the variation in temperature and volume concentration on the viscosity of Al_2O_3-water suspensions. Al_2O_3-nanoparticles with an average diameter of 25 nm were used at 2 and 3% volume concentrations and the results were given at five temperatures ranging from 15 - 55°C.

Table 1 : The most cited correlations of nanofluids viscosity

Model	Correlation	Remark
Einstein [37]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .(1 + 2.5\,\phi)$	Valid for very low volume concentrations ($\phi \leq 0.02$) and spherical particles
Brinkman [38]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .\left(\frac{1}{(1-\phi)^{2.5}}\right)$	
Batchelor [39]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .(1 + 2.5\,\phi + 6.5\,\phi^2)$	
Abu-Nada *et al.* [40]	$\mu_{\mathrm{Al_2O_3}} = \exp\,(3.003 - 0.04203\,T - 0.5445\,\phi + 0.0002553\,T^2 + 0.0524\,\phi^2 - \frac{1.622}{\phi})$ $\mu_{\mathrm{CuO}} = -0.6967 + \left(\frac{15.937}{T}\right) + 1.238\,\phi + \left(\frac{1356.14}{T^2}\right) - 0.259\,\phi^2 - 30.88\left(\frac{\phi}{T}\right) - \left(\frac{19652.74}{T^3}\right) + 0.01593\,\phi^3 + 4.38206\left(\frac{\phi^2}{T}\right) + 147.573\left(\frac{\phi}{T^2}\right)$ $\mu_{\mathrm{H_2O}} = -81.1 + 98.75\ln(T) - 45.23\ln^2(T) + 9.71\ln^3(T) - 0.946\ln^4(T) + 0.03\ln^5(T)$	The viscosity in these equations is expressed in centi poise (cP), the temperature in $^{\mathrm{o}}$C.
Abedian and Kachanov [42]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .\left(\frac{1}{1-2.5\,\phi}\right)$	Newtonian fluid with a single rigid spherical particle
Masoud Hosseini *et al.* [43]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .\exp\left[m + \alpha\left(\frac{T}{T_0}\right) + \beta(\phi) + \gamma\left(\frac{d_{\mathrm{P}}}{1+R}\right)\right]$ $\alpha = -0.485\,, \beta = 14.94, \gamma = 0.0105$ $m = 0.72\,, T_0 = 20\,^{\mathrm{o}}\mathrm{C}, R = 1\ \mathrm{nm}$	For Al_2O_3-Water nanofluids (Based on Nguyen *et al.* [7] experimental data)
Ward model [44]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .[1 + (2.5\,\phi) + (2.5\,\phi)^2 + (2.5\,\phi)^3 + (2.5\,\phi)^4 + \cdots]$	
renewed Ward (RW) model [44]	$\mu_{\mathrm{nf}} = \mu_{\mathrm{bf}} .[1 + (2.5\,\phi_e) + (2.5\,\phi_e)^2 + (2.5\,\phi_e)^3 + (2.5\,\phi_e)^4 + \cdots]$ $\phi_e = \phi.\left(1 + \frac{h}{r}\right)^3$	

Fedele et al. [36] experimentally measured the viscosity of titanium oxide nanoparticles with an average diameter of 76 nm in a water-based suspension. A cone-plate-type viscometer was used to measure the viscosity at different temperatures ranging from 10 to 70°C.

In this paper, all the above-mentioned experimental results were used to model the viscosity of nanofluids using the FCM-ANFIS approach. The design variables (input parameters) chosen for the nanoparticles were the average diameter, volume concentration and temperature. The results of the FCM-ANFIS models were compared against experimental data [19, 30, 33-36] and the most cited correlations from literature that are shown in Table 1.

5. Prediction models

A total of 536 input-output experimental data points obtained from literature [7, 19, 30-36] were used to establish four different prediction models (Model I to Model IV) for viscosity.

For the first model (Model I), Al_2O_3-water nanofluid experimental data were used to create a model for predicting the viscosity of Al_2O_3-water nanofluids. The experimental data were divided into two subsets as 80% for training and 20% for testing purposes.
The same procedure was used to establish the second and third models (Model II and Model III) for predicting the viscosity of CuO-water and TiO_2-water nanofluids,

Table 2: Statistical criteria used for the analysis of the results

Statistical criterion	Equation
Mean absolute error	$\mathrm{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left\|X_p - X_a\right\|$
Mean relative error	$\mathrm{MRE}(\%) = \frac{100}{n}\sum_{i=1}^{n}\left(\frac{\left\|X_p - X_a\right\|}{X_a}\right)$
Root mean square error	$\mathrm{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(X_p - X_a\right)^2}$

respectively. For the fourth model (Model IV), the viscosity of SiO_2-water nanofluid was determined but without any experimental data in the training section. The model was established with the input-output experimental data points for the Al_2O_3-water, CuO-water and TiO_2-water models.

Three different statistical criteria given in Table 2 were used to determine how well the FCM-ANFIS proposed models could predict the viscosity of nanofluids corresponding to various values of inlet variables.

6. Results and discussion

Fig. 1 shows the experimental results of Kwek et al. [35] compared with the FCM-ANFIS model (Model I) and correlations for an Al_2O_3-water nanofluid with a particle size of 25 nm, volume concentration of 2% at temperature ranging from 15 - 55 ^{o}C.

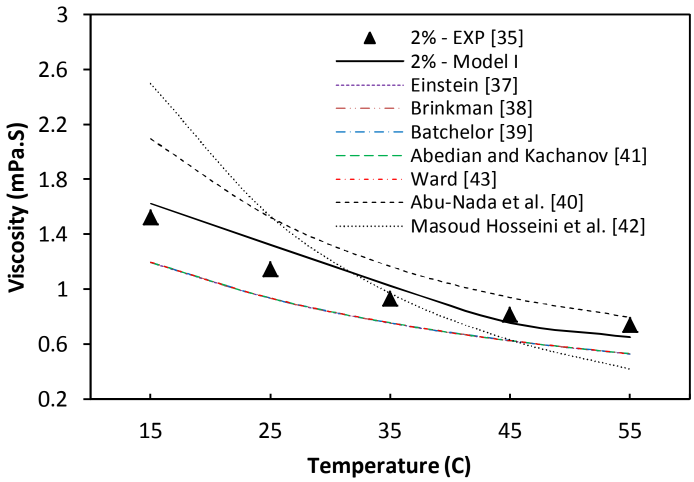

Fig.1 Comparison between the experimental data of Kwek et al. [35] with Model I and correlations from literature for an Al_2O_3-water nanofluid, with an average particle size of 25 nm at a volume concentration of 2%.

Model I is in good agreement with the experimental data (MAE = 0.10, MRE = 10% and RMSE = 0.11). The proposed FCM-ANFIS model is well matched with the experimental data in comparison with the correlations, especially in the low temperature range from 15 - 35^{o}C.

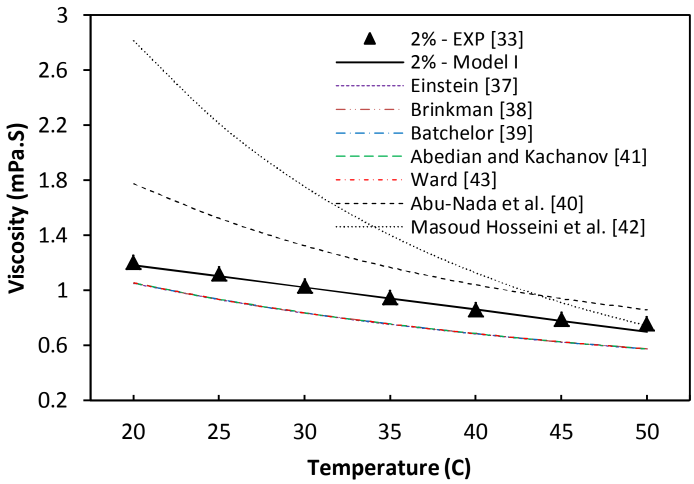

Fig.2 Comparison between the experimental data of Anoop et al. [33] with Model I and correlations from literature for an Al_2O_3-water nanofluid, with an average particle size of 95 nm at a volume concentration of 2%.

Fig. 2 shows the experimental results of Anoop et al. [33] compared with the FCM-ANFIS model (Model I) and the correlations from literature for a particle size of 95 nm and a volume concentration of 2% for an Al_2O_3-water nanofluid. The FCM-ANFIS model is in very good agreement with the experimental data (MAE = 0.020, MRE = 2.2% and RMSE = 0.026) and predicts the viscosities better than any of the correlations.

Fig. 3 shows a comparison between the experimental results of Pastoriza-Gallego et al. [34], the FCM-ANFIS model (Model I) and correlations for an Al_2O_3-water nanofluid with a particle size of 43 nm and a volume concentration of 1.4%. The FCM-ANFIS model (MAE = 0.023, MRE = 2.6% and RMSE = 0.025) corresponds very well with the experimental data although the correlations of Brinkman [38], Batchelor [39], Abedian and Kachanov [41], and Ward [43] also correspond well with the experimental measurements.

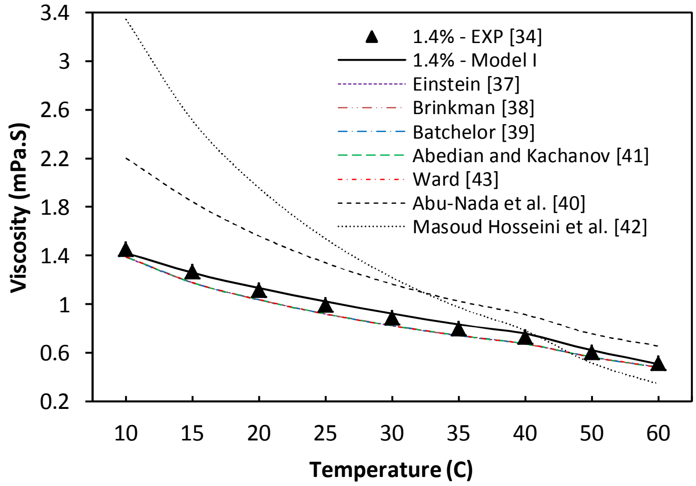

Fig.3 Comparison between the experimental data of Pastoriza-Gallego et al. [34] with Model I and correlations for an Al_2O_3-water nanofluid, with an average particle size of 43 nm at a volume concentration of 1.4%.

In Fig. 4, the experimental results of Tavman et al. [30] are compared with those of the FCM-ANFIS model (Model I) and the correlations for an Al_2O_3-water nanofluid with a particle size of 30 nm, and a volume concentration of 0.5%. In general, the FCM-ANFIS model matches the data (MAE = 0.095, MRE = 11% and RMSE = 0.097) better than any of the other correlations.

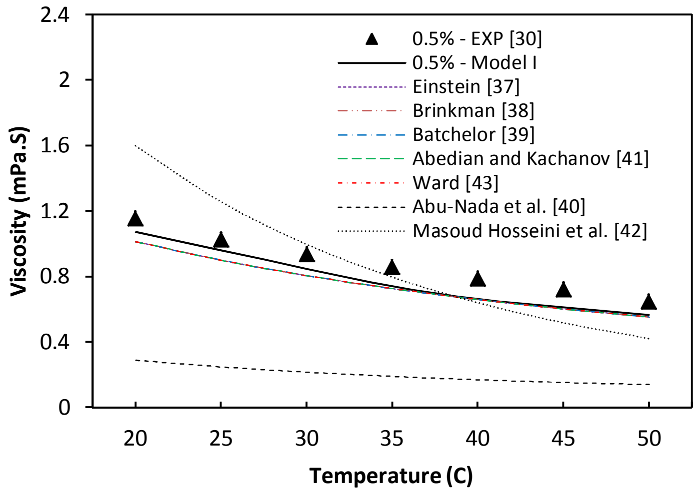

Fig.4 Comparison between the experimental data of Tavman et al. [30] with Model I and correlations for an Al_2O_3-water nanofluid, with an average particle size of 30 nm at a volume concentration of 0.5%.

Based on the results in Figures 2 to 4, it can be concluded that for Al_2O_3-water nanofluids, in general, the FCM-ANFIS, Model I, predicts the viscosities better than those of the correlations in literature.

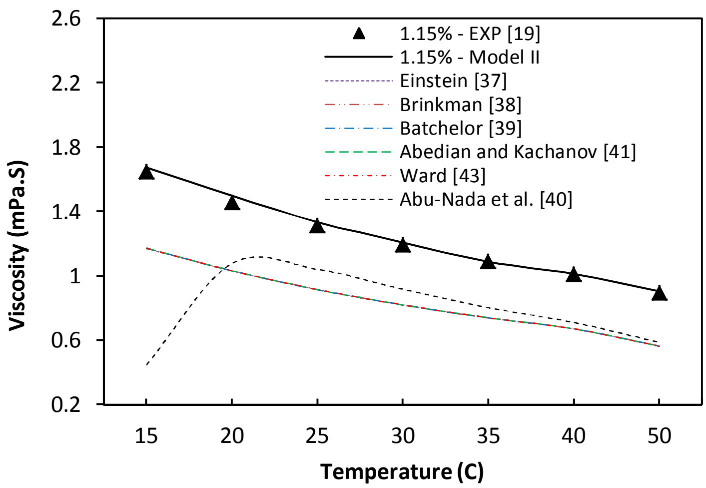

Fig.5 Comparison between the experimental data of Pastoriza-Gallego et al. [19] with Model II and correlations for a CuO-water nanofluid, with an average particle size of 11±3 nm at a volume concentration of 0.5%.

In Fig. 5, the experimental data of Pastoriza-Gallego et al. [19] is compared with the predictions of the FCM-ANFIS model (Model II) and with correlations for a CuO-water nanofluid with particle sizes of 11±3 nm and a volume concentration of 1.15%. The model predicts the viscosities the best when compared with the measurements (MAE = 0.018, MRE = 1.3% and RMSE = 0.022). All the models significantly under predict the experimental data.

In Fig. 6, the experimental data of Fedele et al. [36] are compared with the modelled values of the FCM-ANFIS model (Model III) and correlations for a TiO_2-water nanofluid with particle size of 76 nm and a volume concentration of 5.54%. The renewed Ward [44] model and Model III (MAE = 0.22, MRE = 20% and RMSE = 0.24) show a better agreement with the experimental data in comparison with other correlations. The renewed Ward correlation predicts better results than Model III.

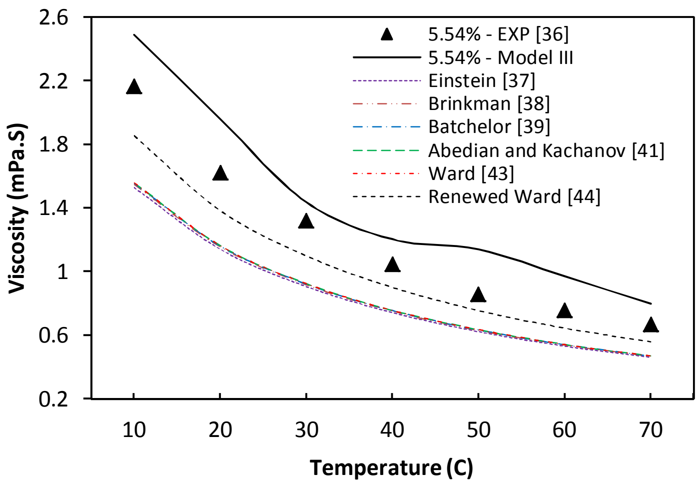

Fig.6 Comparison between the experimental data of Fedele et al. [36] with Model III and correlations for a TiO_2-water nanofluid, with an average particle size of 76 nm at a volume concentration of 5.54%.

Fig. 7 compares the experimental measurements of Tavman et al. [30] with those of the FCM-ANFIS model (Model IV) and the correlations for an SiO_2-water nanofluid with particle size of 12 nm and a volume concentration of 1.85%. As was mentioned before, there were no experimental data for the viscosity of SiO_2-water nanofluid in the FCM-ANFIS model-training procedure. The FCM-ANFIS model trend matches the experimental data the best, while all the correlations significantly under predicted the experimental data.

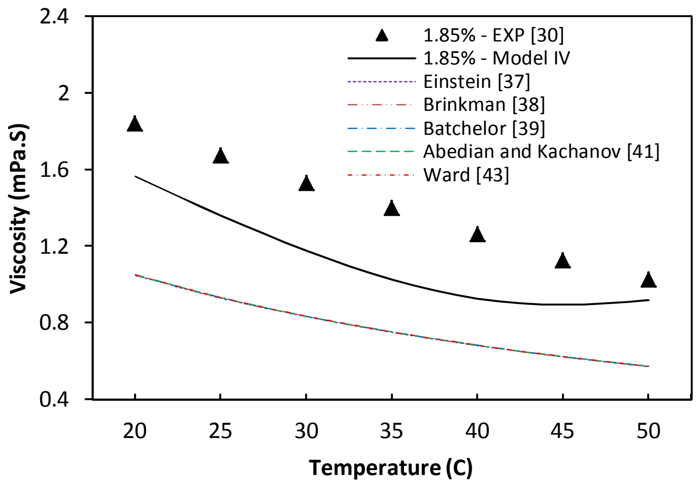

Fig.7 Comparison between the experimental data of Tavman et al. [30] with Model IV and correlations for an SiO_2-water nanofluid, with an average particle size of 12 nm at a volume concentration of 1.85%.

7. Conclusions

The FCM-ANFIS approach was used for modelling the viscosity of nanofluids as function of particle size, volume concentration and temperature. In the FCM-ANFIS method, which consists of a neural network combined with a fuzzy logic approach, the fuzzy C-means clustering was used as an identification method. The adaptive neuro-fuzzy inference system (ANFIS) used neural network and fuzzy logic approaches at the same time to combine the advantages of each method to achieve a better performance.

A literature review of experimental data of the viscosity of nanofluids showed that particle size, volume concentration and temperature were the three most important variables that determined viscosity. Therefore, 536 experimental data points for Al_2O_3, CuO, TiO_2 and SiO_2 nanoparticles with water as base fluid were obtained from literature to model the viscosity of nanofluids by using the data point as input-output for the FCM-ANFIS method.

The results of the FCM-ANFIS method were compared with the experimental data points and with several well-cited correlations from literature and in almost all cases, the proposed FCM-ANFIS models were in very good agreement with the experimental data. This study showed the ability of artificial intelligent methods for modelling engineering problems containing nanofluids based on input-output experimental data, published in the literature.

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