Appendix A

Derivation of the elliptic equation directly from the σ coordinate quasi-elastic equations

In this appendix, the elliptic equation (3.81) is derived directly from the σ coordinate equations (3.64) to (3.68). Multiplying (3.66) by s gives:

$$s\frac{R}{g}\frac{D}{Dt}(\frac{\omega T}{p}) + gs + s^2\frac{\partial \phi}{\partial \sigma} = 0. \tag{A.1}$$

Taking $(\partial/\partial x)$ of (3.64), $(\partial/\partial y)$ of (3.65) and $(\partial/\partial \sigma)$ of (A.1), and adding the three resulting equations gives:

$$\begin{aligned}
&\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial}{\partial \sigma}\left(s^2\frac{\partial \phi}{\partial \sigma}\right) - \sigma\frac{\partial \phi}{\partial \sigma}\left(\frac{\partial^2 \ln p_s}{\partial x^2} + \frac{\partial^2 \ln p_s}{\partial y^2}\right) \\
&-\sigma\left[\frac{\partial \ln p_s}{\partial x}\left(\frac{\partial^2 \phi}{\partial x \partial \sigma}\right) + \frac{\partial \ln p_s}{\partial y}\left(\frac{\partial^2 \phi}{\partial y \partial \sigma}\right)\right] = -\frac{\partial}{\partial x}\left(\frac{Du}{Dt}\right) - \frac{\partial}{\partial y}\left(\frac{Dv}{Dt}\right) \\
&-\frac{\partial}{\partial \sigma}\left[\frac{p}{p_s T}\frac{D}{Dt}\left(\frac{\omega T}{p}\right)\right] + f\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - u\frac{df}{dy} - \frac{\partial}{\partial \sigma}(sg)\,.
\end{aligned} \tag{A.2}$$

In order to simplify (A.2), it is necessary to expand the terms $-\partial\left(Du/Dt\right)/\partial x$, $-\partial\left(Dv/Dt\right)/\partial y$ and $-\partial\left[(p/p_s T)\,D\left(\omega T/p\right)/Dt\right]\partial\sigma$. First note that from expanding the total derivatives of the two components of the horizontal wind, it follows that:

$$-\frac{\partial}{\partial x}\left(\frac{Du}{Dt}\right) = -\frac{D}{Dt}\left(\frac{\partial u}{\partial x}\right) - \left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial \dot\sigma}{\partial x}\frac{\partial u}{\partial \sigma}\right) \tag{A.3}$$

and

$$-\frac{\partial}{\partial y}\left(\frac{Dv}{Dt}\right)=-\frac{D}{Dt}\left(\frac{\partial v}{\partial y}\right)-\left(\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}+\frac{\partial\dot{\sigma}}{\partial y}\frac{\partial v}{\partial\sigma}\right). \tag{A.4}$$

Finding an appropriate expansion for $-\partial\left[(p/p_sT)\,D\,(\omega T/p)\,/Dt\right]\partial\sigma$ requires a more extensive procedure. From the use of relationship (3.69), it follows that

$$\frac{p}{p_sT}\frac{D}{Dt}(\frac{\omega T}{p})=\frac{p}{p_sT}\frac{D}{Dt}\left[\frac{Tp_s}{p}\left(\sigma\frac{D\ln p_s}{Dt}+\dot{\sigma}\right)\right]$$

$$=\frac{p}{p_sT}\left[\frac{D}{Dt}\left(\frac{Tp_s\sigma}{p}\frac{D\ln p_s}{Dt}\right)+\frac{D}{Dt}\left(\frac{Tp_s}{p}\dot{\sigma}\right)\right]$$

$$=\underbrace{\sigma\frac{D^2\ln p_s}{Dt^2}}_{A}+\underbrace{(\frac{p}{p_sT})\frac{D}{Dt}(\frac{Tp_s\sigma}{p})\frac{D\ln p_s}{Dt}}_{B}+\underbrace{(\frac{p}{p_sT})\frac{D}{Dt}(\frac{Tp_s}{p}\dot{\sigma})}_{C}. \tag{A.5}$$

Noting that

$$B=\left[\dot{\sigma}+\frac{\sigma p}{p_sT}\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\frac{D\ln p_s}{Dt} \tag{A.6}$$

and

$$C=\frac{D\dot{\sigma}}{Dt}+\dot{\sigma}\left(\frac{p}{p_sT}\right)\frac{D}{Dt}\left(\frac{Tp_s}{p}\right) \tag{A.7}$$

it follows that

$$\frac{\partial A}{\partial\sigma}=\frac{D^2\ln p_s}{Dt^2}+\sigma\frac{\partial}{\partial\sigma}\left(\frac{D^2\ln p_s}{Dt^2}\right), \tag{A.8}$$

$$\frac{\partial B}{\partial\sigma}=\frac{\partial}{\partial\sigma}\left\{\left[\dot{\sigma}+\frac{\sigma p}{p_sT}\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\frac{D\ln p_s}{Dt}\right\} \tag{A.9}$$

and

$$\frac{\partial C}{\partial\sigma}=\frac{D}{Dt}\left(\frac{\partial\dot{\sigma}}{\partial\sigma}\right)+\left[\frac{\partial u}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial x}+\frac{\partial v}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial y}+\frac{\partial\dot{\sigma}}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial\sigma}\right]$$
$$+\frac{\partial}{\partial\sigma}\left[\dot{\sigma}\left(\frac{p}{p_sT}\right)\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]. \tag{A.10}$$

Equation (A.10) is derived from expanding the total derivative of $\dot{\sigma}$ in (A.7). From combining (A.8) to (A.10) it is obtained that:

$$-\frac{\partial}{\partial\sigma}\left[\frac{p}{p_s T}\left(\frac{D}{Dt}(\frac{\omega T}{p})\right)\right] = -\frac{D}{Dt}\left(\frac{\partial\dot{\sigma}}{\partial\sigma}\right) - \left[\frac{\partial u}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial x} + \frac{\partial v}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial y} + \frac{\partial\dot{\sigma}}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial\sigma}\right]$$

$$-\frac{\partial}{\partial\sigma}\left[\dot{\sigma}\left(\frac{p}{p_s T}\right)\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right] - \frac{D}{Dt}\left(\frac{D\ln p_s}{Dt}\right) - \sigma\frac{\partial}{\partial\sigma}\left(\frac{D^2\ln p_s}{Dt^2}\right)$$

$$-\frac{\partial}{\partial\sigma}\left\{\left[\dot{\sigma} + \frac{\sigma p}{p_s T}\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\frac{D\ln p_s}{Dt}\right\}. \tag{A.11}$$

From substituting (A.3), (A.4) and (A.11) into (A.2), and applying the continuity equation (3.67), it follows that:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial}{\partial\sigma}(s^2\frac{\partial\phi}{\partial\sigma}) - \sigma\frac{\partial\phi}{\partial\sigma}\left(\frac{\partial^2\ln p_s}{\partial x^2} + \frac{\partial^2\ln p_s}{\partial y^2}\right)$$

$$-\sigma\left[\frac{\partial\ln p_s}{\partial x}\left(\frac{\partial^2\phi}{\partial x\partial\sigma}\right) + \frac{\partial\ln p_s}{\partial y}\left(\frac{\partial^2\phi}{\partial y\partial\sigma}\right)\right] =$$

$$+\left(\underbrace{-\frac{\partial u}{\partial x}\frac{\partial u}{\partial x}}_{B_{2a}} - \frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\underbrace{-\frac{\partial\dot{\sigma}}{\partial x}\frac{\partial u}{\partial\sigma}}_{B_{4a}}\right) + \left(-\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\underbrace{-\frac{\partial v}{\partial y}\frac{\partial v}{\partial y}}_{B_{2b}}\underbrace{-\frac{\partial\dot{\sigma}}{\partial y}\frac{\partial v}{\partial\sigma}}_{B_{4b}}\right)$$

$$+\left[\underbrace{-\frac{\partial u}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial x}}_{B_{4c}}\underbrace{-\frac{\partial v}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial y}}_{B_{4d}}\underbrace{-\frac{\partial\dot{\sigma}}{\partial\sigma}\frac{\partial\dot{\sigma}}{\partial\sigma}}_{B_3}\right] - \frac{\partial}{\partial\sigma}\left[\dot{\sigma}\left(\frac{p}{p_s T}\right)\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\underbrace{-\sigma\frac{\partial}{\partial\sigma}\left(\frac{D^2\ln p_s}{Dt^2}\right)}_{B_1}$$

$$-\frac{\partial}{\partial\sigma}\left\{\left[\dot{\sigma} + \frac{\sigma p}{p_s T}\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\frac{D\ln p_s}{Dt}\right\} + f\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) - u\frac{df}{dy} - \frac{\partial}{\partial\sigma}(sg). \tag{A.12}$$

In order to simplify (A.12), the labeled terms need to be written in alternative form. Firstly, note that

$$B_1 = -\sigma\frac{\partial}{\partial\sigma}\left(\frac{D^2\ln p_s}{Dt^2}\right) = -\sigma\left[\frac{\partial\ln p_s}{\partial x}\frac{\partial}{\partial\sigma}\left(\frac{Du}{Dt}\right) + \frac{\partial\ln p_s}{\partial y}\frac{\partial}{\partial\sigma}\left(\frac{Dv}{Dt}\right)\right]$$

$$-2\sigma\left[\frac{\partial u}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial x}\right) + \frac{\partial v}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial y}\right)\right]. \tag{A.13}$$

By expanding the total derivatives of u and v in (A.13), it is obtained that:

$$B_1 = -\sigma\frac{\partial}{\partial\sigma}(\frac{D^2\ln p_s}{Dt^2}) = \sigma f\left(\frac{\partial u}{\partial\sigma}\frac{\partial\ln p_s}{\partial y} - \frac{\partial v}{\partial\sigma}\frac{\partial\ln p_s}{\partial x}\right)$$

$$+\sigma\left(\frac{\partial^2\phi}{\partial x\partial\phi}\frac{\partial\ln p_s}{\partial x} + \frac{\partial^2\phi}{\partial y\partial\phi}\frac{\partial\ln p_s}{\partial y}\right) - \sigma\frac{\partial}{\partial\sigma}\left(\sigma\frac{\partial\phi}{\partial\sigma}\right)\left[\left(\frac{\partial\ln p_s}{\partial x}\right)^2 + \left(\frac{\partial\ln p_s}{\partial y}\right)^2\right]$$

$$-2\sigma\left[\frac{\partial u}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial x}\right) + \frac{\partial v}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial y}\right)\right]. \tag{A.14}$$

From making use of the continuity equation (3.67) it follows that:

$$B_{2a} + B_{2b} = 2\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\left(\frac{D\ln p_s}{Dt} + \frac{\partial\dot\sigma}{\partial\sigma}\right) \tag{A.15}$$

From differentiating relationship (3.69) with respect to σ it follows that

$$\frac{\partial\dot\sigma}{\partial\sigma} = \frac{\partial}{\partial\sigma}\left(\frac{p}{p_s}\Omega\right) - \frac{D\ln p_s}{Dt} - \sigma\left(\frac{\partial u}{\partial\sigma}\frac{\partial\ln p_s}{\partial x} + \frac{\partial v}{\partial\sigma}\frac{\partial\ln p_s}{\partial y}\right). \tag{A.16}$$

It follows, from applying the continuity equation, that

$$B_3 = -\left(\frac{\partial\dot\sigma}{\partial\sigma}\right)^2 = \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)\frac{\partial\dot\sigma}{\partial\sigma} + \frac{D\ln p_s}{Dt}\frac{\partial\dot\sigma}{\partial\sigma}. \tag{A.17}$$

Making use of (3.69) to write $\dot\sigma$ in terms of ω and $D\ln p_s/Dt$ implies that

$$B_{4a}+B_{4b}+B_{4c}+B_{4d} = -2\left(\frac{\partial u}{\partial\sigma}\frac{\partial\dot\sigma}{\partial x} + \frac{\partial v}{\partial\sigma}\frac{\partial\dot\sigma}{\partial y}\right) = -2\left[\frac{\partial u}{\partial\sigma}\frac{\partial}{\partial x}\left(\frac{p}{p_s}\Omega\right) + \frac{\partial v}{\partial\sigma}\frac{\partial}{\partial y}\left(\frac{p}{p_s}\Omega\right)\right]$$

$$+2\sigma\left[\frac{\partial u}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial x}\right) + \frac{\partial v}{\partial\sigma}\frac{D}{Dt}\left(\frac{\partial\ln p_s}{\partial y}\right)\right]$$

$$+2\sigma\frac{\partial u}{\partial\sigma}\left(\frac{\partial u}{\partial x}\frac{\partial\ln p_s}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial\ln p_s}{\partial y}\right) + 2\sigma\frac{\partial v}{\partial\sigma}\left(\frac{\partial u}{\partial y}\frac{\partial\ln p_s}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\ln p_s}{\partial y}\right). \tag{A.18}$$

By substituting (A.14), (A.15), (A.17) and (A.18) into (A.12), it follows, after some cancellations between terms, that:

$$\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial}{\partial\sigma}(s^2\frac{\partial\phi}{\partial\sigma})-\sigma\frac{\partial\phi}{\partial\sigma}\left(\frac{\partial^2\ln p_s}{\partial x^2}+\frac{\partial^2\ln p_s}{\partial y^2}\right)$$

$$+\sigma\frac{\partial}{\partial\sigma}\left[\left(\frac{\partial\ln p_s}{\partial x}\right)^2+\left(\frac{\partial\ln p_s}{\partial y}\right)^2\right]-2\sigma\left[\frac{\partial\ln p_s}{\partial x}\left(\frac{\partial^2\phi}{\partial x\partial\sigma}\right)+\frac{\partial\ln p_s}{\partial y}\left(\frac{\partial^2\phi}{\partial y\partial\sigma}\right)\right]=$$

$$2\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial u}-\frac{\partial v}{\partial x}\frac{\partial u}{\partial y}\right)\underbrace{+2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\left(\frac{\partial\dot\sigma}{\partial\sigma}\right)+\frac{D\ln p_s}{Dt}\frac{\partial\dot\sigma}{\partial\sigma}+\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\left(\frac{D\ln p_s}{Dt}\right)}_{C_2}$$

$$\underbrace{-2\left[\frac{\partial u}{\partial\sigma}\frac{\partial}{\partial x}\left(\frac{p\omega}{p_s}\right)+\frac{\partial v}{\partial\sigma}\frac{\partial}{\partial y}\left(\frac{p\omega}{p_s}\right)\right]}_{C_1}$$

$$+2\sigma\frac{\partial u}{\partial\sigma}\left(\frac{\partial u}{\partial x}\frac{\partial\ln p_s}{\partial x}+\frac{\partial v}{\partial x}\frac{\partial\ln p_s}{\partial y}\right)+2\sigma\frac{\partial v}{\partial\sigma}\left(\frac{\partial u}{\partial y}\frac{\partial\ln p_s}{\partial x}+\frac{\partial v}{\partial y}\frac{\partial\ln p_s}{\partial y}\right)$$

$$\sigma f\left[\frac{\partial u}{\partial\sigma}\frac{\partial\ln p_s}{\partial y}-\frac{\partial v}{\partial\sigma}\frac{\partial\ln p_s}{\partial x}\right]+f\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)-u\frac{df}{dy}-\partial\left(sg\right)$$

$$\underbrace{-\frac{\partial}{\partial\sigma}\left[\dot\sigma\left(\frac{p}{p_sT}\right)\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]}_{C_3}\underbrace{-\frac{\partial}{\partial\sigma}\left\{\left[\dot\sigma+\frac{\sigma p}{p_sT}\frac{D}{Dt}\left(\frac{Tp_s}{p}\right)\right]\frac{D\ln p_s}{Dt}\right\}}_{C_4}. \quad \text{(A.19)}$$

It may finally be noted that

$$C_1=-\frac{2}{p_s}\left[\frac{\partial u}{\partial\sigma}\frac{\partial}{\partial x}\left(p\Omega\right)+\frac{\partial v}{\partial\sigma}\frac{\partial}{\partial y}\left(p\Omega\right)\right]+\frac{2p\omega}{p_s}\left(\frac{\partial\ln p_s}{\partial x}\frac{\partial u}{\partial\sigma}+\frac{\partial\ln p_s}{\partial y}\frac{\partial v}{\partial\sigma}\right), \quad \text{(A.20)}$$

$$C_2=2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\left(\frac{\partial\dot\sigma}{\partial\sigma}\right)+\frac{D\ln p_s}{Dt}\frac{\partial\dot\sigma}{\partial\sigma}+\left(-\frac{\partial\dot\sigma}{\partial\sigma}-\frac{D\ln p_s}{Dt}\right)\left(\frac{D\ln p_s}{Dt}\right)$$

$$=2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\left(\frac{\partial\dot\sigma}{\partial\sigma}\right)-\left(\frac{D\ln p_s}{Dt}\right)^2=2\frac{D\ln p_s}{Dt}\frac{\partial\dot\sigma}{\partial\sigma}+\left(\frac{D\ln p_s}{Dt}\right)^2 \quad \text{(A.21)}$$

and

$$C_3 + C_4 = -\frac{\partial}{\partial \sigma}\left(\dot{\sigma}\frac{D \ln p_s}{Dt}\right) - \frac{\partial}{\partial \sigma}\left[\left(\frac{p}{p_s T}\right)\frac{D}{Dt}\left(\frac{T p_s}{p}\right)\frac{p\omega}{p_s}\right]$$

$$= -\frac{\partial}{\partial \sigma}\left(\dot{\sigma}\frac{D \ln p_s}{Dt}\right) + \frac{1}{\gamma}\left[\frac{p\omega}{p_s}\frac{\partial \omega}{\partial \sigma} + \frac{\partial (p\omega)}{\partial \sigma}\frac{\omega}{p_s}\right] - \frac{p\omega}{p_s}\frac{\partial}{\partial \sigma}\left(\frac{D \ln p_s}{Dt}\right)$$

$$-\frac{\partial}{\partial \sigma}(p\Omega)\frac{1}{p_s}\frac{D \ln p_s}{Dt}$$

$$= -2\frac{\partial \dot{\sigma}}{\partial \sigma}\frac{D \ln p_s}{Dt} - 2\frac{p\omega}{p_s}\left(\frac{\partial u}{\partial \sigma}\frac{\partial \ln p_s}{\partial x} + \frac{\partial v}{\partial \sigma}\frac{\partial \ln p_s}{\partial y}\right) - \left(\frac{D \ln p_s}{Dt}\right)^2$$

$$+\frac{1}{\gamma}\left[\frac{p\omega}{p_s}\frac{\partial \omega}{\partial \sigma} + \frac{\partial (p\omega)}{\partial \sigma}\frac{\omega}{p_s}\right]. \qquad \text{(A.22)}$$

Equations (A.19) to (A.22) may now be substituted in (A.18). After some cancellations and reorganization of the terms, the elliptic equation (3.81) is obtained.

Appendix B

Alternative derivation of the linearized elliptic equation

Linearizing equation (3.81) gives

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial}{\partial \sigma}\left(s_0^2 \frac{\partial \phi}{\partial \sigma}\right) + \frac{\partial}{\partial \sigma}\left(s_0 \frac{T^{'}}{T_0} g\right) = 0. \tag{B.1}$$

Substituting wave-like solutions of the form (3.106) into (B.1) leads to

$$\frac{d}{d\sigma}\left(\frac{\sigma p_0 + p_T}{p_0}\right)^2 \frac{d\hat{\phi}'}{d\sigma} - H_0^2 k^2 \hat{\phi}' + H_0^2 \frac{d}{d\sigma}\left(s_0 \frac{\hat{T}'}{T_0} g\right) = 0. \tag{B.2}$$

Here

$$H_0^2 \frac{d}{d\sigma}\left(s_0 \frac{\hat{T}'}{T_0} g\right) = \frac{H_0^2 g}{T_0}\left(s_0 \frac{d\hat{T}'}{d\sigma} + \frac{T^{'}}{H_0}\right). \tag{B.3}$$

From (3.110) it follows that

$$\frac{dT^{'}}{d\sigma} = -\kappa\left[\left(\frac{p_0}{\sigma p_0 + p_T}\right)\left(\frac{1}{ikc}\right)\left(\frac{d\hat{\dot\sigma}}{d\sigma} - ikc\hat{\pi}\right)\right] T_0 +$$

$$\kappa\left[\left(\frac{p_0}{\sigma p_0 + p_T}\right)^2 \left(\frac{1}{ikc}\right)\left(\hat{\dot\sigma} - ikc\sigma\hat{\pi}\right)\right] T_0 = 0. \tag{B.4}$$

Making use of (3.110) and (B.4) to substitute for $\hat{T}^{'}$ and $d\hat{T}^{'}/d\sigma$ in (B.3) gives:

$$H_0^2 \frac{d}{d\sigma}\left(s_0 \frac{\hat{T}'}{T_0} g\right) = -H_0 g\kappa\left(\frac{1}{ikc}\right)\left(\frac{d\hat{\dot\sigma}}{d\sigma} - ikc\hat{\pi}\right). \tag{B.5}$$

Substituting from (3.107) and (3.109) gives

$$H_0^2 \frac{d}{d\sigma}\left(s_0 \frac{\hat{T}'}{T_0} g\right) = \frac{H_0 g \kappa}{c^2} \hat{\phi}' = H_0^2 \frac{N^2}{c^2} \hat{\phi}'. \tag{B.6}$$

From substituting (B.6) in (B.2) equation (3.113) is obtained.

Appendix C

The linearized elliptic equation under transformations Z and F

The transformation relationship (3.114) implies that for any function $G = G(\sigma)$, it holds that

$$\frac{dG}{d\sigma} = \frac{dG}{dZ}\frac{dZ}{d\sigma} = \left(-H_0 \frac{p_0}{\sigma p_0 + p_T}\right)\frac{dG}{dZ}. \tag{C.1}$$

From (3.114) it may also be noted that

$$\frac{d\sigma}{dZ} = -\frac{1}{H_0}\left(\frac{\sigma p_0 + p_T}{p_0}\right). \tag{C.2}$$

Equation (3.113) may written alternatively as:

$$2\left(\frac{\sigma p_0 + p_T}{p_0}\right)\frac{d\hat{\phi}'}{d\sigma} + \left(\frac{\sigma p_0 + p_T}{p_0}\right)^2 \frac{d^2\hat{\phi}'}{d\sigma^2} + H_0^2\left(\frac{N^2}{c^2} - k^2\right)\hat{\phi}' = 0. \tag{C.3}$$

From applying (C.1), (C.3) transforms to:

$$-2H_0\frac{d\hat{\phi}'}{dZ} - H_0\left(\frac{\sigma p_0 + p_T}{p_0}\right)\frac{d}{dZ}\left[-H_0\left(\frac{p_0}{\sigma p_0 + p_T}\right)\frac{d\hat{\phi}'}{dZ}\right] + H_0^2\left(\frac{N^2}{c^2} - k^2\right)\hat{\phi}' = 0. \tag{C.4}$$

Applying transformation relationship (3.115) to (C.4) leads to:

$$-2H_0\frac{dF}{dZ}\exp^{z/2H_0} - F\exp^{z/2H_0} + H_0^2\left(\frac{1}{4H_0^2}F + \frac{1}{H_0}\frac{dF}{dZ} + \frac{d^2F}{dZ^2}\right)\exp^{z/2H_0}$$

$$+H_0\frac{dF}{dZ}\exp^{z/2H_0}+\frac{1}{2}F\exp^{z/2H_0}+H_0^2\left(\frac{N^2}{c^2}-k^2\right)F\exp^{z/2H_0}=0, \tag{C.5}$$

which reduces to the required relationship (3.116).

Appendix D

Alternative derivation of the Lamb wave frequency equation

Relationship (3.118) may also be consistently obtained from considering the boundary conditions that apply to $\dot{\sigma}$, $\hat{\phi}'$ and $d\hat{\phi}'/d\sigma$. The details are as follows.

At the lower boundary, where $\sigma = 1$ and $\dot{\sigma} = 0$ by definition, it follows from substituting (3.112) and (3.126) in (3.127) that:

$$c^2 H_0^2 \left(k^2 - \frac{N^2}{c^2}\right)\hat{\pi} = AH_0\left(\mu - \frac{1}{2H_0}\right)\left(\frac{p_0+p_T}{p_0}\right)^{H_0(\mu+\frac{1}{2H_0})} + BH_0\left(-\mu - \frac{1}{2H_0}\right)\left(\frac{p_0+p_T}{p_0}\right)^{H_0(-\mu+\frac{1}{2H_0})}. \tag{D.1}$$

Substituting (3.126) into (D.1), and applying (3.129) gives:

$$\left(\mu^2 - \frac{1}{4H_0^2}\right)\left[\left(\frac{p_0+p_T}{p_0}\right)^{H_0(\mu+\frac{1}{2H_0})} - \left(\frac{p_T}{p_0}\right)^{2H_0\mu}\left(\frac{p_0+p_T}{p_0}\right)^{H_0(-\mu+\frac{1}{2H_0})}\right] =$$

$$\left(k^2 - \frac{N^2}{c^2}\right)\left\{\left[\left(\frac{p_0+p_T}{p_0}\right)^{H_0(\mu+\frac{1}{2H_0})} - (\left(\frac{p_T}{p_0}\right)^{2H_0\mu}\left(\frac{p_0+p_T}{p_0}\right)^{H_0(-\mu+\frac{1}{2H_0})}\right] + \left[\left(\frac{p_T}{p_0}\right)^{2H_0\mu}\left(\frac{p_T}{p_0}\right)^{H_0(-\mu+\frac{1}{2H_0})} - \left(\frac{p_T}{p_0}\right)^{H_0(\mu+\frac{1}{2H_0})}\right]\right\}. \tag{D.2}$$

Noting that the last two terms in (D.2) cancel, it follows that:

$$\left[\left(\mu^2 - \frac{1}{4H_0^2}\right) - \left(k^2 - \frac{N^2}{c^2}\right)\right]\left[(\frac{p_0 + p_T}{p_0})^{H_0(\mu + \frac{1}{2H_0})} - (\frac{p_T}{p_0})^{2H_0\mu}(\frac{p_0 + p_T}{p_0})^{H_0(-\mu + \frac{1}{2H_0})}\right] = 0 \tag{D.3}$$

which implies (3.118).

Appendix E

Applying the continuity equation for the case of solutions with sinusoidal variation in height

Substituting (3.143) in (3.117) gives:

$$\hat{\pi} = \frac{1}{c^2}\left(A\int_0^1 \exp^{imH_0 \ln X} X^{-1/2} d\sigma + B\int_0^1 \exp^{-imH_0 \ln X} X^{-1/2} d\sigma\right). \quad \text{(E.1)}$$

Both integrals appearing in (E.1) have the form $\int_0^1 \exp^{ia \ln X} X^{-1/2} d\sigma$, with a a real number and $X = (\sigma p_0 + p_T)/p_0$, and

$$\int_0^1 \exp^{ia \ln X} X^{-1/2} d\sigma = \int_0^1 \cos(a \ln X) X^{-1/2} d\sigma + i\int_0^1 \sin(a \ln X) X^{-1/2} d\sigma. \quad \text{(E.2)}$$

The two integrals implied by (E.2) may both be evaluated by using integration by parts, which gives:

$$\left(1+4a^2\right)\int_0^1 \cos(a \ln X) X^{-1/2} d\sigma = \left[2X^{1/2}\cos(a \ln X)\right]_{\sigma=0}^{\sigma=1} + \left[4aX^{1/2}\sin(a \ln X)\right]_{\sigma=0}^{\sigma=1}$$

$$= 2\left\{\left(\frac{p_0+p_T}{p_0}\right)^{1/2}\cos\left[a \ln \frac{p_0+p_T}{p_0}\right] - \left(\frac{p_T}{p_0}\right)^{1/2}\cos\left[a \ln \frac{p_T}{p_0}\right]\right\} +$$

$$4a\left\{\left(\frac{p_0+p_T}{p_0}\right)^{1/2}\sin\left[a \ln \frac{p_0+p_T}{p_0}\right] - \left(\frac{p_T}{p_0}\right)^{1/2}\sin\left[a \ln \frac{p_T}{p_0}\right]\right\} \quad \text{(E.3)}$$

and

$$\left(1+4a^2\right)\int_0^1 \sin\left(a\ln X\right)X^{-1/2}d\sigma = \left[2X^{1/2}\sin\left(a\ln X\right)\right]_{\sigma=0}^{\sigma=1} - \left[4aX^{1/2}\cos\left(a\ln X\right)\right]_{\sigma=0}^{\sigma=1}$$

$$= 2\left\{\left(\frac{p_0+p_T}{p_0}\right)^{1/2}\sin\left[a\ln\frac{p_0+p_T}{p_0}\right] - \left(\frac{p_T}{p_0}\right)^{1/2}\sin\left[a\ln\frac{p_T}{p_0}\right]\right\} -$$

$$4a\left\{\left(\frac{p_0+p_T}{p_0}\right)^{1/2}\cos\left[a\ln\frac{p_0+p_T}{p_0}\right] - \left(\frac{p_T}{p_0}\right)^{1/2}\cos\left[a\ln\frac{p_T}{p_0}\right]\right\}. \qquad \text{(E.4)}$$

Substituting results (E.3) and (E.4) in (E.2) leads to:

$$\int_0^1 \exp^{ia\ln X} X^{-1/2}d\sigma = \frac{1}{c^2\left(1+4a^2\right)}\left[(2-4ai)\left(\frac{p_0+p_T}{p_0}\right)^{1/2}\exp^{ia\ln[(p_0+p_T)/p_0]}\right.$$

$$\left. + (-2+4ai)\left(\frac{p_T}{p_0}\right)^{1/2}\exp^{ia\ln(p_T/p_0)}\right] \qquad \text{(E.5)}$$

Applying result (E.5) in (E.1) gives the required equation (3.145).

Bibliography

[1] Admirat P, Goyer GG, Wojtiw L, Carte EA, Roos D and Lozowki EP (1985) A comparitive study of hailstorms in Switzerland, Canada and South Africa. *J. of Climatol.* **5** 35-51.

[2] Arakawa A and Lamb VR (1977) Computational design of the basic dynamical processes of the UCLA general circulation model. *Methods of Computational Physics,* Vol. 17, Academic Press, 174-265, 337 pp.

[3] Asselin R (1972) Frequency filter for time integration. *Mon. Wea. Rev.* **100** 487-490.

[4] Bates JR (1984) An efficient semi-Lagrangian and alternating direction implicit method for integrating the shallow water equations. *Mon. Wea. Rev.* **112** 2033-2047.

[5] Bates JR and McDonald A (1982) Multiply upstream, semi-Lagrangian advective schemes: analysis and application to a multi-level primitive equation model. *Mon. Wea. Rev.* **110** 1831-1842.

[6] Bates JR, Li Y, Brandt SF, McCormick SF and Ruge J (1995) A global shallow-water numerical model based on the semi-Lagrangian advection of potential vorticity. *Quart. J. Roy. Meteor. Soc.* **121** 1981-2005.

[7] Benoit R, Desgagne M, Pellerin P, Desjardins S and Chartier Y (1997) The Canadian MC2: A semi-Lagrangian, semi-implicit wide-band atmospheric model suited for fine-scale process studies and simulation. *Mon. Wea. Rev.* **125** 2382-2415.

[8] Burden RL and Faires JD (1993) *Numerical analysis.* PWS Publishing company, Boston. 768 pp.

[9] Brugge R and Moncrieff MW (1985) The effect of physical processes on numerical simulations of two-dimensional cellular convection. *Beitr. Phys. Atmos.* **58** 417-440.

[10] Burger AP (1991) The potential vorticity equation: from planetary to small scale. Tellus **43A**, 191-197.

[11] Burger AP and Riphagen HA (1978) *Analysis of energy-consistency of simple weather prediction models.* CSIR Special Report, SWISK 1, 55 p.

[12] Burger AP and Riphagen HA (1979) The lower boundary condition and energy consistency in primitive and filtered models. *J. Atmos. Sci.* **36** 1436-1449.

[13] Burger AP and Riphagen HA (1981) Available potential energy and simplifications to the adiabatic equation. *Tellus* **33** 505-508.

[14] Burger AP and Riphagen HA (1990) The basic equations in meteorological dynamics - a reexamination of unsimplified forms for a general vertical coordinate. *Beitr. Phys. Atmosph.* **63** 151-164.

[15] Burger AP and Riphagen HA (1999) Energy conservation for a general vertical coordinate - a reexamination of unsimplified forms. *Beitr. Phys. Atmosph.* **72** 25-50.

[16] Carpenter KM (1979) An experimental forecast using a nonhydrostatic mesoscale model. *Quart. J. Roy. Meteor. Soc.* **105** 629-655.

[17] Carte AE and Held G (1978) Variability of hailstorms on the South African Plateau. *J. Appl. Meteorol.* **17** 365-373.

[18] Caya D and Laprise R (1999) A semi-implicit semi-Lagrangian climate model: the Canadian RCM.

[19] Clark TL and Peltier W (1997) On the evolution and stability of finite-amplitude mountain waves. *J. Atmos. Sci.* **34** 1715-1730.

[20] Crimp SJ, Lutjeharms JRE and Mason SJ (1998) Sensitivity of a tropical-temperate trough to sea-surface temperature anomalies in the Agulhas retroflection region. *Water SA* **24** 93-100.

[21] Clark TL and Peltier W (1977) On the evolution and stability of finite-amplitude mountain waves. *J. Atmos. Sci.* **34** 1715-1730.

[22] Cullen MJP (1990) A test of a semi-implicit integration technique for a fully-compressible non-hydrostatic model. *Quart. J. Roy. Meteor. Soc.* **116** 1253-1258.

[23] Daley R (1988) The normal modes of the spherical non-hydrostatic equations with the application to the fitering of acoustic modes. *Tellus* **40A**, 96-106.

[24] Davies T, Staniforth A, Wood N and Thuburn J (2003) Validity of the anelastic and other equation sets as inferred from normal-mode analysis. *Quart. J. Roy. Meteor. Soc.* **129** 2761-2775.

[25] Davies T, Cullen MJP, Malcolm AJ, Mawson MH, Staniforth A, White AA and Wood N (2005) A new dynamical core for the Met Office's global and regional modelling of the atmosphere. *Quart. J. Roy. Meteor. Soc.* **131** 1759-1782.

[26] Davies-Jones RP (1983) The onset of rotation in thunderstorms. Preprints, 13th Conference of Severe Local Storms, Tulsa, Okla., American Meteorological Society, Boston, 215-218.

[27] De Coning E and Adam BF (2000) The tornadic thunderstorm events during the 1998-1999 South African Summer. *Water SA* **26** 361-376.

[28] De Coning E, Adam BF and Banitz L (2000) A severe weather event on 29 December 1997: Synoptic and mesoscale perspectives. *Water SA* **26** 137-146.

[29] De Villiers MP (1998) *Clear air turbulence over the republic of South Africa.* MSc-thesis, University of Pretoria, 136 p.

[30] De Villiers MP (2001) Clear air turbulence over South Africa. *Meteorol. Appl.* **8** 119-126.

[31] Doswell III CA (1991) A review for forecasters on the application of hodographs to forecasting severe thunderstorms. *NWA Digest* **16** 2-16.

[32] Droegemeier KK (1985) *The numerical simulation of thunderstorm outflow dynamics.* Ph.D. dissertation, University of Illinois at Urbana-Champaign, 695 pp.

[33] Droegemeier KK and Wilhemson (1987) Numerical simulations of thunderstorm outflow dynamics. Part I: Outflow sensitivity experiments and turbulent dynamics. *J. Atmos. Sci.* **44** 1180-1210.

[34] Dudhia J (1993) A nonhydrostatic version of the Penn-State-NCAR mesoscale model: Validation tests and simulation of an Atlantic cyclone and cold front. *Mon. Weather. Rev.* **121** 1493-1513.

[35] Dudhia J and Bresch JF (2002) A global version of the PSU-NCAR mesoscale model. *Mon. Wea. Rev.* **130** 2989-3007.

[36] Durran DR (1989) Improving the anelastic approximation. *J. Atmos. Sci.* **46** 1453-1461.

[37] Dutton JA and Fichtl GH (1969) Approximate equations of motion for gases and liquids. *J. Atmos. Sci.* **26** 241-254.

[38] Eckart C (1960) The hydrodynamics of oceans and atmospheres. Pergamon Press, Oxford.

[39] Eliassen A (1949) The quasi-static equation of motion with pressure as independent variable. *Geophs. Publ.* **17** 1-44.

[40] Engelbrecht FA (2000) Nested climate modelling over southern Africa with a semi-Lagrangian limited area model. MSc-thesis, University of Pretoria, 134 pp.

[41] Engelbrecht FA (2005) Simulations of climate and climate change over southern and tropical Africa with the conformal-cubic atmospheric model. In *Potential Impacts and Vulnerabilities of Climate Change on Hydrological Responses in southern Africa,* ed. R.E. Schulze, chap. 4. WRC Report **1430/1/05**. Water Research Commission, Pretoria.

[42] Engelbrecht FA and Rautenbach CJ deW (2000) Perspective for nested climate modelling over southern Africa. *SA Tydskrif vir Natuurwetenskap en Tegnologie* **19** 47-51.

[43] Engelbrecht FA, Rautenbach CJ deW, McGregor JL and Katzfey JJ (2002) January and July climate simulations over the SADC region using the limited-area model DARLAM. *Water SA,* **28**, 361-374.

[44] Fischer G. (1965). On a finite difference scheme for solving the non-linear primitive equations for a barotropic fluid with application to the boundary current problem. *Tellus* **4** 405-412.

[45] Fujita TT (1973a) Experimental classification of tornadoes in FPP scale. *SMRP Research paper* No **98** University of Chicago, Illinois.

[46] Fujita TT (1973b) Tornadoes around the world. *Weatherwise* **26** 56-83.

[47] Fujita T and Grandoso H (1968) Split of a thunderstorm into anticyclonic and cyclonic storms and their motion as determined from numerical model experiments. *J. Atmos. Sci.* **35** 1070-1096.

[48] Gal-Chen T and Somerville RCJ (1975) On the use of a coordinate transformation for the solution of the Navier-Stokes equations. *ibid.* **17** 209-228.

[49] Gallus WA Jr and Rancic M (1996) A nonhydrostatic version of NMC's regional Eta model. *Quart. J. Roy. Meteor. Soc.* **122** 495-513.

[50] Garstang M, Kelbe BE, Emmitt GD and London WB (1987) Generation of convective storms over the escarpment of northeastern South Africa. *Mon. Wea. Rev.* **115** 429-443.

[51] Goliger AM, Milford RV, Adam BF and Edwards M (1997) *Inkanyamba - Tornadoes in South Africa.* United Litho, ISBN 0-7988-5417-0.

[52] Haltiner GJ (1971) *Numerical Weather Prediction.* New York, Wiley.

[53] Haltiner GJ and Williams RT (1980) *Numerical prediction and Dynamic Meteorology (2nd edition).* John Wiley and sons. 477 p.

[54] Harrison MSJ (1984) A generalized classification of South African summer rain-bearing synoptic systems. *J. Climatol.* **4** 547-560.

[55] Harrison MSJ (1986) *A synoptic climatology of South African rainfall variations.* Ph.D. thesis. University of the Witwatersrand.

[56] Held G (1978) The probability of hail in relation to radar echo heights on the South African Highveld. *J. Appl. Meteorol.* **17** 755-762.

[57] Held G (1982) Comparison of radar observations of a devastating hailstorm and a cloudburst at Jan Smuts airport. In: Agee EM and Asai T (eds.) *Cloud Dynamics.* D Reidel Publishing Company, Hamburg, Germany. 273-284.

[58] Hewitson B, Tadross M and Jack C (2005) Scenarios from the University of Cape Town. In *Potential Impacts and Vulnerabilities of Climate Change on Hydrological Responses in southern Africa,* ed. R.E. Schulze, chap. 3. WRC Report **1430/1/05**. Water Research Commission, Pretoria.

[59] Holton JR (1992) *An Introduction to Dynamic Meteorology (3rd edition).* Academic press limited. 511 pp.

[60] Hoskins BJ and Pearce RP (1983) *Large-scale dynamical processes in the atmosphere.* London Academic Press.

[61] Hsu W-R and Sun W-Y (2001) A time-split forward-backward numerical model for solving a nonhydrostatic and compressible system of equations. *Tellus* **53A** 279-299.

[62] Janjic ZI, Gerrity JP Jr and Nickovic S (2001) An alternative approach to nonhydrostatic modeling. *Mon. Wea. Rev.* **129** 1164-1178.

[63] Janjic Z.I. and Wiin-Nielsen A (1977) On geostrophic adjustment and numerical procedures in a rotating fluid. *J. Atmos. Sci.* **34** 297-310.

[64] Johnson M (1978) *The structure of vorticity in cumulonimbus convection: a numerical study.* Ph.D. thesis, University of London.

[65] Joubert AM, Katzfey JJ, McGregor JL and Nguyen KC (1999) Simulating mid-summer climate over southern Africa using a nested regional climate model. *J. Geophys. Res.* **104** 19015-19025.

[66] Juang H.-M. H. (1992) A spectral fully compressible nonhydrostatic model in hydrostatic sigma coordinates: Formulation and preliminary results. *Meteorol. Atmos. Phys.* **50** 75-88.

[67] Jury MR, Pathack B, Rautenbach CJ deW and Van Heerden J (1996) Drought over South Africa and Indian Ocean SST: Statistical and GCM results. *The Global Atmosphere and Ocean System* **4** 47-63.

[68] Jury MR, Mulenga H and Rautenbach CJ deW (2000) Tropical Atlantic variability and Indo-Pacific ENSO: Statistical analysis and numerical simulation. *The Global Atmosphere and Ocean System* **7** 107-124.

[69] Kalnay-Rivas E and Hoitsma D (1979) Documentation of the fourth-order GLAS model. *NASA Tech. Memo* **80608**, 126 pp.

[70] Kaplan ML, Zack JW, Wang VC and Tuccillo JJ (1982) Initial results from a mesoscale atmospheric simulation system and comparisons with the AVE-SESAME I dataset. *Mon. Wea. Rev.* **110** 1564-1590.

[71] Kasahara A (1974) Various vertical coordinate systems used for numerical weather prediction. *Mon. Wea. Rev.* **102** 504-522.

[72] Klemp JB (1987) Dynamics of tornadic thunderstorms. *Annu. Rev. Fluid Mech.* **19** 369-402.

[73] Laprise R (1992) The Euler equations of motion with hydrostatic pressure as an independent variable. *Mon. Weather Rev.* **122** 3-26.

[74] Laprise R, Caya D, Bergeron G and Giguere M (1997) The formulation of Andre Robert MC2 (Mesoscale compressible Community) model. *The Andre J. Robert Memorial Volume (C Lin, R Laprise and H Ritchie, Eds.), companion volume to Atmos.-Ocean*, **35** 195-220.

[75] Laprise R (1998) Semi-implicit semi-Lagrangian fully elastic nonhydrostatic model formulation. *ECMWF Seminar 1998.*

[76] Leslie LM and Purser RJ (1991) High-order numerics in an unstaggered three-dimensional time-split semi-Lagrangian forecast model. *Mon. Weather Rev.* **119** 1612-1623.

[77] Lipps FB and Hemler RS (1982) A scale analysis of deep moist convection and some related numerical calculations. *J. Atmos. Sci.* **51** 3549-3565.

[78] Mahrt L (1986) On the shallow motion approximations. *J. Atmos. Sci.* **43** 1036-1044.

[79] McDonald A (1984) Accuracy of multiply-upstream, semi-Lagrangian advection schemes. *Mon. Wea. Rev.* **112** 1267-1275.

[80] McDonald A (1986) A semi-Lagrangian and semi-implicit two time-level integration scheme. *Mon. Wea. Rev.* **114** 824-830.

[81] McGregor (1987) Accuracy and initialization of a two-time level split semi-Lagrangian model. *Short- and Medium-Range Numerical Weather Prediction,* T. Matsuno, Ed., Special Volume of the J. Meteor. Soc. Japan, 233-246.

[82] McGregor JL (1993) Economical determination of departure points for semi-Lagrangian Models. *Mon. Wea. Rev.* **121** 221-230.

[83] McGregor JL (1996) Semi-Lagrangian advection on conformal-cubic grids. *Mon. Wea. Rev.* **124** 1311-1322.

[84] McGregor JL and Dix MR (2001) The CSIRO conformal-cubic atmospheric GCM. *IUTAM Symposium on Advances in Mathematical Modelling of Atmosphere and Ocean Dynamics,* P.F. Hodnett, Ed., Kluwer, 197-202.

[85] McGregor JL, Nguyen KC and Katzfey JJ (2002) Regional climate simulations using a stretched-grid global model. *Research activities in Atmospheric and Oceanic Modelling* Rep. **32**, WMO Tech Doc. 1105, 2 pp.

[86] McGregor JL (2005) Geostrophic adjustment for reversibly staggered grids. *Mon. Wea. Rev.* **133** 1119-1128.

[87] Mendez-Nunez LR and Carroll JJ (1994) Application of the MacCormack scheme to atmospheric nonhydrostatic models. *Mon. Wea. Rev.* **122** 984-1000.

[88] Mesinger F and Arakawa A (1976) *Numerical Methods used in Atmospheric Models.* Volume 1. GARP Publications Series No. 17 64 pp.

[89] Miller MJ (1974) On the use of pressure as vertical coordinate in modelling convection. *Quart. J. Roy. Meteor. Soc.* **100** 155-162.

[90] Miller MJ and Bets AK (1977) Travelling convective storms over Venezuela. *Mon. Weather Review.* **105** 833-848.

[91] Miller MJ and Pearce RP (1974) A three-dimensional primitive equation model of cumulonimbus convection. *Quart. J. Roy. Meteor. Soc.* **100** 133-154.

[92] Miller MJ and Thorpe AJ (1981) Radiation conditions for the lateral boundaries of limited-area numerical models. *Quart. J. Roy. Meteor. Soc.* **107** 615-628.

[93] Miller MJ and White AA (1984) On the nonhydrostatic equations in pressure and sigma coordinates. *Quart. J. Roy. Meteor. Soc.* **110** 515-533.

[94] Miranda PMA and James IN (1992) Nonlinear three-dimensional effects on gravity-wave drag: Splitting flow and breaking waves. *Quart. J. Roy. Meteor. Soc.* **118** 1057-1081.

[95] Miranda PMA and Valente MA (1997) Critical level resonance in three-dimensional flow past isolated mountains. *J. Atmos. Sci.* **54** 1574-1588.

[96] Moncrieff MW and Miller MJ (1976) The dynamics and simulation of tropical cumulonimbus and squall lines. *Ibid.* **102** 373-394.

[97] Ogura Y and Charney JC (1962) A numerical model of thermal convection in the atmosphere. *Proc. Int. Symp. Numerical Weather Prediction,* Tokyo, Japan, Meteor. Society of Japan, 431-451.

[98] Ogura Y and Phillips NA (1962) Scale analysis of deep and shallow convection in the atmosphere. *J. Atmos. Sci.* **19** 173-179.

[99] Orlanski I (1976) A simple boundary condition for unbounded hyperbolic flows. *J. Comput. Phys.* **21** 251-269.

[100] Phillips NA (1957) A coordinate system having some special advantages for numerical forecasting. *J. Meteor.* **14** 184-185.

[101] Pielke RA (1984) *Mesoscale Meteorological Modeling.* Academic Press, San Diego, USA.

[102] Pudykiewicz J and Staniforth A (1984) Some properties and comparitive performance of the semi-Lagrangian method of Robert in the solution of the advection-diffusion equation. *Atmos.-Ocean,* **22**, 283-308.

[103] Purnell DK (1976) Solution of the advective equation by upstream interpolation with a cubic spline. *Mon. Wea. Rev.* **104** 42-48.

[104] Purser RJ (1987) The filtering of meteorological fields. *J. Climate Appl. Meteor.* **26** 1764-1769.

[105] Purser RJ and Leslie LM (1988) A semi-implicit semi-Lagrangian finite-difference scheme using high-order spatial differencing on a nonstaggered grid. *Mon. Wea. Rev.* **116** 2069-2080.

[106] Rautenbach CJ deW (1999) *Introduction of a hybrid vertical co-ordinate to an atmospheric general circulation model.* PhD-thesis, University of Pretoria, 149 pp.

[107] Rautenbach CJ deW (2003) Seasonal climate predictions with a coupled atmosphere-ocean general circulation model. *Water Research Commission Report,* No **904/1/03** 82 pp.

[108] Rautenbach CJ deW, Engelbrecht FA, Engelbrecht CJ, Ndarandana T and McGregor JL (2005) *Regional model development for simulating atmospheric behavior and rainfall over southern Africa.* Water Research Commission, Pretoria, RSA, Report 1261/1/05.

[109] Richtmyer RD and Morton KW (1967) *Difference methods for initial value problems.* Wiley-Interscience.

[110] Riishogaard LP, Cohn SE, Li Y and Menard R (1998) The use of spline interpolation in semi-Lagrangian transport models. *Mon. Wea. Rev.* **126** 2008-2016.

[111] Riphagen HA (1984) *The implementation of a split explicit weather prediction model for the Southern Hemisphere.* MSc dissertation, University of Pretoria, South Africa, November 1984, 164 p.

[112] Riphagen HA (1989) High latitude filtering for the global prediction model. *In: Proc. Sixth Annual Conference of the S.A. Society for Atmospheric Sciences,* Pretoria, 26-27 October, p 14.

[113] Riphagen HA (1993) Meteorological modelling in South Africa. *In: Proc. Tenth Annual conference of the S.A. Society for Atmospheric Sciences,* Midrand, 13-14 October, p 18.

[114] Riphagen HA (1999) The Eta model in South Africa. *International ETA Model Newsletter,* no. 2, February 1999.

[115] Riphagen HA and Burger AP (1978) *A non-integrated, adiabatic three-level filtered weather prediction model.* CSIR Special Report, SWISK 7, 26 p.

[116] Riphagen HA and Burger AP (1986) Comments on the computational stability of Gadd's adjustment and advection schemes for a split explicit model. *Quart. J. Roy. Met. Soc.* **112** 276-282.

[117] Riphagen HA, Bruyere CL, Jordaan W, Poolman ER and Gertenbach JD (2002) Experiments with the NCEP regional ETA model at the South African Weather Bureau, with emphasis on terrain presentation and its effect on precipitation predictions. *Mon. Wea. Rev.* **130** 1246-1263.

[118] Ritchie H (1987) Semi-Lagrangian advection on a Gaussian grid. *Mon. Wea. Rev.* **115** 608-619.

[119] Robert A (1966) The integration of a low order spectral form of the primitive meteorological equations. *J. Meteorol. Soc. Jpn.* **44** 237-245.

[120] Robert A (1969) The integration of a spectral model of the atmosphere by the implicit method. Pp. VII-19-VII-24 in Proceedings WMO/IUGG Symposium on Numerical Weather Prediction. Japan Meteorol. Agency.

[121] Robert A (1981) A stable numerical integration scheme for the primitive meteorological equations. *Atmos.-Ocean* **19** 35-46.

[122] Robert A (1982) A semi-Lagrangian and semi-implicit numerical integration scheme for the primitive meteorological equations. *J. Meteor. Soc. Japan* **60** 319-325.

[123] Robert A (1993) Bubble convection experiments with a semi-implicit formulation of the Euler equations. *J. Atmos. Sci.* **50** 1865-1873.

[124] Room R (1989) The general form of dynamical equations of the atmosphere in the isobaric coordinate space. Pp. 368-371 *in Proceedings of the Estonian Academy Sci. Phys. Math* **38**

[125] Room R (1990) General form of the equations of atmospheric dynamics in isobaric coordinates. *Izvestiya, Atmospheric and Ocean Physics* **26** 9-14.

[126] Room R (1998) Acoustic filtering in nonhydrostatic pressure coordinate dynamics: A variational approach. *J. Atmos. Sci.* **46** 1154-1164.

[127] Room R, Miranda PMA and Thorpe AJ (2001) Filtered non-hydrostatic models in pressure-related coordinates. *Quart. J. Roy. Meteor. Soc.* **127** 1277-1292.

[128] Rotunno R (1981) On the evolution of thunderstorm rotation. *Mon. Wea. Rev.* **109** 171-180.

[129] Salmon R and Smith LM (1994) Hamiltonian derivation of the nonhydrostatic pressure-coordinate model. *Q. J. R. Meteor. Soc.* **120** 1409-1413.

[130] Schoenstadt AL (1978) A tranfer function analysis of numerical schemes used to simulate geostrophic adjustment. *NPS Rept.* NP.-53-79-001, Naval Postgraduate School, Monterey. CA, 93943, 44 pp.

[131] Shapiro R (1975) Linear filtering. *Math. Comp.* **29** 1094-1097.

[132] Shutts GJ and Gray MEB (1994) A numerical modelling study of the geostrophic adjustment process following deep convection. *Q. J. R. Meteorol. Soc.* **120** 1145 1178.

[133] Skamarock WC, Klemp JB, Dudhia J, Gill DO, Barker DM, Wang W and Powers JG (2005) A description of the Advanced Research WRF Version 2. *NCAR Technical Note,* TN-468+STR, 88 pp.

[134] Smolarkiewicz PK, Margolin LG and Wyszgrodzki AA (2001) A class of nonhydrostatic global models. *J. Atmos. Sci.* **58** 349-364.

[135] Straka JM, Wilhelmson RB, Wicker LJ, Anderson JR and Droegemeier KK (1993) Numerical solutions of a nonlinear density current: A benchmark solution and comparisons. *Int. J. Num. Methods Fluids* **17** 1-22.

[136] Tanguay MA, Robert A and Laprise R (1990) A semi-implicit semi-Lagrangian fully compressible regional forecast model. *Mon. Wea. Rev.* **118** 1970-1980.

[137] Tapp MC and White PW (1976) A nonhydrostatic mesoscale model. *Quart. J. Roy. Meteor. Soc.* **102** 277-296.

[138] Tennant WJ, Riphagen HA, Gertenbach JD, De Villiers MP and Rae KJ (1997). The effect of radiosonde and buoy reduction on numerical prediction products at the South African Weather Bureau. *Proceedings of CGC/WMO Workshop on the Impact of Various Observing Systems on Numerical Weather Prediction,* Geneva, 7-9 April, World Weather Watch Technical Report No. 18, WMO/TD No. 686, 165-183.

[139] Thorpe AJ and Miller MJ (1978) Numerical simulations showing the role of the downdraught in cumulonimbus motion and splitting. *Quart. J. Roy. Meteor. Soc.* **104** 873-893.

[140] Thorpe AJ, Miller MJ and Moncrieff MW (1982) Two-dimensional convection in non-constant shear: a model of mid-latitude squall lines. *Ibid.* **108** 739-762.

[141] Thuburn J, Wood N and Staniforth A (2002a) Normal modes of deep atmospheres. I: Spherical geometry. *Quart. J. Roy. Meteor. Soc.* **128** 1771-1792.

[142] Thuburn J, Wood N and Staniforth A (2002b) Normal modes of deep atmospheres. II: f-F plane geometry. *Quart. J. Roy. Meteor. Soc.* **128** 1793-1806.

[143] Tritton DJ (1988) *Physical fluid dynamics,* 2nd edn. Oxford University Press, Oxford, UK.

[144] Van Heerden J, Rautenbach CJ deW and Truter MM (1995) Techniques for seasonal and longer term rainfall prediction in South Africa. *Water Research Commission Report* No **373/1/92**, 1-51.

[145] Visser PJM (2000) The storm-structure-severity method for the identification of convective storm characteristics with conventional radar. *Meteorol. Appl.*

[146] Weismann ML and Klemp JB (1982) The dependence of numerically simulated convective storms on vertical wind shear and buoyancy. *Mon. Wea. Rev.* **110** 504-520.

[147] White AA (1989) An extended version of a nonhydrostatic, pressure coordinate model. *Quart. J. Roy. Meteor. Soc.* **115** 1243-1251.

[148] White AA and Bromley RA (1995) Dynamically consistent, quasi-hydrostatic equations for global models with a complete representation of the Coriolis force. *Quart. J. Roy. Meteor. Soc.* **121** 399-418.

[149] White AA, Hoskins BJ, Roulstone I and Staniforth A (2005) Consistent approximate models of the global atmosphere: shallow, deep, hydrostatic, quasi-hydrostatic and nonhydrostatic. *Quart. J. Roy. Meteor. Soc.* **131** 2081 2107.

[150] Wicker LJ and Skamarock WC (2002) Time-splitting methods for elastic models using forward time schemes. *Mon. Wea. Rev.* **130** 2088-2097.

[151] Wilhelmson RB and Ericksen JH (1977) Direct solutions for Poisson's equation in three dimensions. *J. Comput. Phys.* **25** 319-331.

[152] Wilhelmson R and Ogura Y (1972) The pressure perturbation and the numerical modelling of a cloud. *J. Atmos. Sci.* **29** 1295-1307.

[153] Winninghoff FJ (1968) On the adjustment toward a geostrophic balance in a simple primitive equation model with apllication to the problems of initialization and objective analysis. Ph.D. thesis, University of California, Los Angeles, 161 pp.

[154] Wood N and Staniforth A (2003) The deep-atmosphere Euler equations with a mass-based vertical coordinate. *Q. J. R. Meteorol. Soc.* **129** 1289-1300.

[155] Xue M (1989) *A Nonhydrostatic Numerical Model in Sigma-coordinates and Simulations of Mesoscale Phenomena.* PhD Thesis, Reading University, 258 pp.

[156] Xue M and Thorpe AJ (1991) A mesoscale numerical model using the nonhydrostatic pressure-based sigma-coordinate equations: Model experiments with dry mountain flows. *Mon. Wea. Rev.* **119** 1168-1185.