

Determining the best apparent resistivity versus frequency definition for a magnetotelluric sounding : a comparison between two statistical techniques

By

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Various different statistical reduction techniques were used to determine the best curves that would fit through apparent resistivity and impedance phase versus frequency data. The major problem in processing magnetotelluric data is the presence of manmade electromagnetic noise. This noise causes outliers to appear on the data and as a result does not always have a Gaussian distribution. Most of the conventional reduction techniques like those based on the L_1 - and L_2 -norm assume that the noise in the data is normally distributed. To address this problem two additional techniques were applied to the data, namely the robust M-estimation technique, and the L_p norm technique. The robust M-estimation technique minimises a loss function with a known distribution. Different weights are applied to the impedance data depending on the position in the error distribution. The adaptive L_p norm technique uses the real distribution of the data to determine the value of p used in the reduction.

These methods were first tested on synthetic data and then applied to real data collected in the Northern Cape Province of South Africa. The synthetic tests showed the L_1 – norm and L_p – norm to provide good results. It also became clear that the adaptive L_p -norm method is more susceptible to the starting impedance values than the robust M-estimation technique. When applied to real magnetotelluric data, very similar results were obtained from all the techniques when the data quality was relatively good. For bad quality data, the robust M-estimation method gave the best results. It is clear that the effectiveness of the statistical techniques is dependent on the quality of the data.

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H	Magnetic intensity	A/m
D	Electric displacement	C/m ²
E	Electric field	V/m
J	Current density	A/m ²
q	Charge	C
t	Time	s
f	Frequency	hz
ϵ_0	Electric permittivity of free space	F/m
P	Polarisation	C/m ²
Φ	Magnetic flux	Wb
ξ	Electromotive force	V
σ	Conductivity	mho/m
μ_0	Magnetic permeability of free space	H/m
ω	Angular frequency	rad/s
δ	Skindepth	m
Z_{ij}	Impedance	ohm
ρ_{ij}	Apparent resistivity	ohm.m
ϕ_{ij}	Phase	degree