

**Theory and applications of univariate distribution-free Shewhart, CUSUM
and EWMA control charts**

by

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Declaration

I declare that the dissertation/thesis, which I hereby submit for the degree MSc (Mathematical Statistics) at the University of Pretoria, is my own work and has not previously been submitted by me for a degree at this or any other tertiary institution.

Signature: _____

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Summary

Statistical quality control charts originated in the late 1920's by Shewhart (1926, 1931 and 1939). Their applications in various disciplines have been ever-increasing. Although most control charts are distribution-based, recent literature witnessed the development of a considerable number of distribution-free or nonparametric control charts.

The purpose of this thesis is to present the concepts and introduce the researcher to the essentials of univariate nonparametric control charts. Various properties of nonparametric control charts are comprehensively discussed and concepts are clearly explained. Proofs and detailed calculations have been given to help the reader to study and understand the subject more thoroughly. This text contains a wide variety of illustrative examples to give an overall picture of how nonparametric control charts are used. Both simulated and real data examples have been integrated throughout the text. Since most practical problems are too large to be solved using hand calculations, some type of statistical software package is required to solve these problems. There are several excellent statistical packages available and in this thesis we make use of Microsoft Excel, SAS, Minitab, Mathcad and Mathematica to construct (almost all) the tables in this thesis. We point out that a number of Mathematica programs are provided by Chakraborti and Van de Wiel (2003) by means of the website www.win.tue.nl/~markvdw.

The aim throughout is to convey the concepts of univariate nonparametric control charts in a way that readers will find attractive and interesting. Since the majority of nonparametric procedures, to be distribution-free, require a continuous population, only variables control charts are covered. We only consider control charts for monitoring the location of a process, since very few nonparametric charts are available for monitoring the spread. In this thesis we consider the three main classes of control charts: the Shewhart, CUSUM and EWMA control charts and their refinements. The text is divided into several chapters. An introduction to nonparametric control charts is presented in Chapter 1. A discussion of some of the advantages of nonparametric control charts is included while pointing out some of the disadvantages. In Chapter 2 we describe the Shewhart-, CUSUM- and EWMA-type sign control charts with (and without) warning limits. In Chapter 3 we describe the Shewhart-, CUSUM- and EWMA-type signed-rank control charts with (and

without) runs-type signalling rules. The Shewhart-type sign-like control chart with (and without) signalling rules is considered in Chapter 4. In Chapter 5 we consider the Shewhart-type signed-rank-like control chart. Finally, in Chapter 6 we consider the Shewhart- and CUSUM-type Mann-Whitney-Wilcoxon control charts. We considered decision problems under both Phase I and Phase II (see Section 1.5 for a distinction between the two phases). In all the sections of this thesis we considered Phase II process monitoring, except in Section 6.2 where a CUSUM-type control chart for the preliminary Phase I analysis of individual observations based on the Mann-Whitney two-sample test is proposed. In the last chapter we have some concluding remarks along with some ideas for future research.

Table of contents

1. Introduction	11
1.1. Notation	11
1.2. Distribution of chance causes	12
1.3. Nonparametric or distribution-free	12
1.4. Nonparametric control charts	13
1.5. Terminology and formulation	14
1.6. Shewhart-type charts	15
1.7. CUSUM-type charts	16
1.8. EWMA-type charts	16
 Section A: Monitoring the location of a process when the target location is specified (Case K)	
2. Sign control charts	17
2.1. The Shewhart-type control chart	17
2.1.1. Introduction	17
2.1.2. Definition of the sign test statistic	17
2.1.3. Plotting statistic	19
2.1.4. Determination of control limits	21
2.1.5. Run length distribution	27



2.1.6. One-sided control charts	29
2.1.6.1.Lower one-sided control charts	29
2.1.6.2.Upper one-sided control charts	31
2.1.7. Two-sided control charts	37
2.1.8. Summary	38
2.2. The Shewhart-type control chart with warning limits	38
2.2.1. Introduction	38
2.2.2. Markov chain representation	40
2.2.3. One-sided control charts	41
2.2.3.1.Upper one-sided control charts	41
2.2.3.2.Lower one-sided control charts	49
2.2.4. Two-sided control charts	51
2.2.5. Summary	63
2.3. The tabular CUSUM control chart	64
2.3.1. Introduction	64
2.3.2. One-sided control charts	68
2.3.2.1.Upper one-sided control charts	68
2.3.2.2.Lower one-sided control charts	86
2.3.3. Two-sided control charts	91
2.3.4. Summary	106
2.4. The EWMA control chart	106
2.4.1. Introduction	106
2.4.2. The proposed EWMA sign chart	109
2.4.3. Markov-chain approach	111
2.4.4. Summary	120

3. Signed-rank control charts	121
3.1. The Shewhart-type control chart	121
3.1.1. Introduction	121
3.1.2. Definition of the signed-rank test statistic	121
3.1.3. Plotting statistic	122
3.1.4. Determination of chart constants	125
3.1.5. Summary	129
3.2. The Shewhart-type control chart with runs-type signalling rules	129
3.2.1. Introduction	129
3.2.2. Example	130
3.2.3. Summary	132
3.3. The tabular CUSUM control chart	133
3.3.1. Introduction	133
3.3.2. One-sided control charts	133
3.3.2.1. Upper one-sided control charts	133
3.3.2.2. Lower one-sided control charts	161
3.3.3. Two-sided control charts	168
3.3.4. Summary	187
3.4. The EWMA control chart	188
3.4.1. Introduction	188
3.4.2. The proposed EWMA signed-rank chart	188
3.4.3. Markov-chain approach	189
3.4.4. Summary	196

Section B: Monitoring the location of a process when the target location is unspecified or unknown (Case U)

4. Sign-like control charts	197
4.1. The Shewhart-type control chart	197
4.1.1. Introduction	197
4.1.2. Preliminary	198
4.1.3. Probability of no signal	201
4.1.4. Determination of chart constants	202
4.1.5. The median chart	204
4.1.6. Control charts for other percentiles	205
4.1.7. Properties of order statistics	206
4.1.8. One-sided control charts	207
4.1.8.1. Lower one-sided control charts	207
4.1.8.2. Upper one-sided control charts	217
4.1.9. Two-sided control charts	227
4.1.10. Run-length distribution and <i>ARL</i> under some alternatives	238
4.1.10.1. Location alternatives	239
4.1.10.2. Scale alternatives	239
4.1.10.3. Location-scale alternatives	239
4.1.10.4. Lehmann alternatives	240
4.1.10.5. Proportional hazards alternatives	240
4.1.10.6. Summary	240
4.2. The Shewhart-type control chart with runs-type signalling rules	240
4.2.1. Introduction	240
4.2.2. Example	241
4.2.3. Summary	244



5. Signed-rank-like control charts	245
5.1. The Shewhart-type control chart	245
5.1.1. Introduction	245
5.1.2. Definition of the signed-rank-like test statistic	245
5.1.3. Distribution-free properties	249
5.1.4. Simulation study	253
5.1.5. Comparisons	255
5.1.6. The tabular CUSUM control chart	263
5.1.7. The EWMA control chart	264
5.1.8. Summary	265
6. Mann-Whitney-Wilcoxon control charts	266
6.1. The Shewhart-type control chart	266
6.1.1. Introduction	266
6.1.2. Plotting statistic	267
6.1.3. Properties of the run-length distribution	268
6.1.4. The computation of the signal probability	272
6.1.5. Saddlepoint approximations	273
6.1.6. Monte Carlo simulation	276
6.1.7. Determination of chart constants	282
6.1.8. Control chart performance	287
6.1.9. Summary	293
6.2. The tabular Phase I CUSUM control chart	294
6.2.1. Introduction	294
6.2.2. Determination of chart constants	297
6.2.3. Performance comparison	298
6.2.4. Summary	308



7. Concluding remarks	309
Appendix A	311
Appendix of theorems and proofs	
Appendix B	325
Appendix of computer programs	
References	361