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Inventory management under uncertainty: A military application

by

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24041328

A dissertation submitted in partial fulfillment of the requirements for the degree of

Masters of Engineering (Industrial Engineering)

in the

Faculty of Engineering, Built Environment and Information Technology

University of Pretoria
South Africa

January 2011

Abstract

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Inventory management under uncertainty is a widely researched field and many different types of inventory models have been used to address inventory problems in practice [1, 10, 11, 26, 50, 35]. However, there is a lack of published studies focusing on inventory planning in environments, such as the military, that are characterised by uncertainty as a result of extreme events.

A critical area in military decision support is inventory management. Planning for stock levels in particular can be a daunting task, due to the uncertainty associated with the future. The military is typically an environment where improbable events can have massive impacts on operations; and the availability of the correct amount of stock can enhance the responsiveness, efficiency, and preparedness of the military, and ultimately save human lives. On the other hand, excessive stock — especially ammunition — can result in huge monetary losses through damages, stock degradation, and stock obsolescence. Excessive ammunition also poses a risk to public safety, and can ultimately challenge a country's ability to control the use of force. It is therefore very important to provide proper attention to determining the required stock levels during military inventory management.

This dissertation aims, therefore, to develop a reliable decision support tool that can assist with inventory management in the military. To achieve this, a mixed multi-objective mathematical model is used that attempts to minimise cost, shortages, and stock while incorporating demand uncertainty by means of probability distributions and fuzzy numbers. The model considers three different scenarios, and determines the minimum required stock level and the best order quantity for three different stock categories, for a single ammunition item.

The model is converted into its crisp, non-fuzzy, and deterministic counterpart first by transforming the fuzzy constraints into their crisp versions and then deriving the deterministic model of the crisp recourse stochastic model. The corresponding crisp, deterministic model is then solved using exact branch-and-bound embedded in the LINGO 10.0 optimi-

sation software package and the reliability of the solutions in different scenarios is tested by means of discrete event simulation.

The reliability of the model is then compared with the reliabilities of the well known (r, Q) and (s, S) inventory models in the literature. The comparison indicates that the mixed model proposed in this dissertation is more reliable in extreme scenarios than the (r, Q) and (s, S) inventory models in the literature.

A sensitivity analysis is then performed and results indicate that the model yields reliable solutions with a reliability that varies between 74.54% and 100%, depending on the scenario investigated. The lower reliability is during the high demand scenario, this is caused by the ability of the inventory model to prioritise different scenarios based on their estimated possibility to ensure that stock levels are not unneccessary escalated for highly improbable events.

It can be concluded that the proposed mixed multi-objective mathematical model that aims to minimise inventory cost, surplus stock, and shortages is a reliable inventory decision support model for the uncertain military environment.

Keywords: inventory management; fuzzy-stochastic programming; military stock levels; goal programming; military supply chain; discrete event simulation.

Acknowledgements

"Trust in the Lord with all your heart, and lean not on your own understanding.
In all your ways acknowledge Him, and He shall direct your paths."
— Proverbs 3:5-6 —

I also wish to express my sincerest gratitude to my supervisor, Prof. J. W. Joubert, and co-supervisor, Prof. M.K. Luhandjula, for their support and guidance.

I am also grateful to the CSIR for their financial support that enabled me to complete this dissertation, as well as all my colleagues at the CSIR for their motivation and support.

I wish to thank my husband, Richardt, for his love, encouragement, patience and motivation.

Finally, I wish to thank my family and friends for their love and support.

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List of Abbreviations

EOQ	Economic Order Quantity
DVGP	Defuzzification Value Goal Programming
DVM	Defuzzification Value Model
DVMOP	Defuzzification Value Multi-Objective Programming
FIFO	First-In-First-Out
LP	Linear Programming
OR	Operations Research
SCOR	Supply Chain Operations Reference

Chapter 1

Introduction

The military environment has existed for many centuries. Many research studies have been conducted in the field of military decision support over the years, and new studies are constantly emerging.

1.1 Military decision support

One of the earliest contributions to the field of military decision support came from Sun Tzu in the closing years of the sixth century *BC*. In his book *The Art of War*, he attempted to formulate a rational basis for the planning and conduct of military operations [17].

The focus on the field of military strategic and tactical decision-making has expanded ever since this first contribution. Various decision support techniques have been developed and used in the military environment over the years.

One decision support technique commonly used in the military environment is scenario planning. According to Bradfield et al. [5], scenario techniques are rooted in the military, and have been used by military strategists throughout history — generally in the form of war game simulations. The scenario planning method was founded in the 1950s by Herman Kahn, who is often referred to as the father of modern day scenario planning [5].

Discussing the scenario planning technique, Phelps et al. [43] comment that the idea of scenario planning is to plan for the future while minimising unpleasant surprises and broadening thinking about different possibilities. The result of scenario planning is a set of scenarios that describe potential future developments for a specified period that can be used during planning and decision-making. They add that scenario planning is not a forecasting or prediction technique, but rather an exploration in which possible descriptions of the future are developed that force the decision-maker to face uncertainty by providing different outlooks of the future. Scenario planning is a valuable technique as uncertainty presents unique difficulties in decision-making causing decision-makers to be faced with doubtfull situations, requiring an analysis of multiple outcomes in differing states of nature.

A civil defence exercise is an example of an area where scenario planning is commonly used to test and improve the suitability of equipment and systems to respond to certain situations [5].

Although various research efforts have been conducted in the field of military decision support, few of these particularly focus on inventory planning and management. A great deal of available literature in the above-mentioned area — when uncertainty is in the state of affairs — focuses on the commercial environment. There is a lack of available research efforts in non-commercial environments such as the military or humanitarian organisations. These environments are typically characterised by improbable events that can have huge impacts on operations and inventory levels. Since not only costs but also human lives are at stake, there is a great need to focus on inventory planning in these environments to enable quicker response and improved product availability.

1.2 Inventory management in the military

An important area in military inventory management is the planning of stock levels. Determining military stock levels can be a daunting task, given that the future cannot be known with precision. The military is a typical environment where improbable events such as war can have massive impacts on operations. The availability of the correct amount of stock can enhance the responsiveness, efficiency, and preparedness of the military. This ultimately can result in saving human lives.

On the other hand, too much surplus stock — especially surplus ammunition — can result in huge monetary losses through stock degradation, damage, and high stock obsolescence rates. Surplus ammunition also poses a risk to public safety, and can ultimately challenge a country's ability to control the use of force [4].

The problem of excessive ammunition stockpiles is encountered throughout the world, while many countries still struggle to reduce their ammunition surpluses. According to Bevan [4], at the time of his publication the surplus ammunition of Ukraine was estimated to be approximately 2 500 000 tonnes, whereas the surplus ammunition of Iraq was estimated to be approximately 400 000 tonnes, despite the destruction of approximately 200 000 tonnes by the United States. These are merely two examples of countries that struggle with the common problem of properly managing and planning stock levels in an environment where uncertainty play a pivotal role. They highlight the relevance and importance of a structured and proper planning approach to military stockpile management.

1.3 Problem statement

This dissertation focuses on developing and solving an inventory optimisation model that can be used to support inventory management in the turbulent military environment. Furthermore, the reliability of the solution achieved by the corresponding optimisation model must be verified. To achieve this the following research question should be answered:

> *What would be a reliable decision support model to minimise inventory cost and excessive surplus stock, while ensuring demand satisfaction, for a single military item amidst uncertain demand resulting from extreme events?*

1.4 Research design

To deal with the above-mentioned research question, this dissertation focuses on the development of a multi-objective mathematical model that can be used to assist in inventory planning in an uncertain environment characterised by low probability events having a huge impact. The model minimises the cost and stock level while maximising demand satisfaction. The model is applied to the military environment to help in planning stock levels for a single item while taking demand uncertainty into account. Once the model is solved, its reliability is tested by generating various scenarios and analysing the effect that each scenario can potentially have on inventory levels.

Uncertain parameters in the model — namely, the demand for different scenarios — are represented by probability distributions and fuzzy numbers. It is important to understand that probability distributions and fuzzy numbers are not the same. Probability distributions are used to deal with the uncertainty in the occurrence of events, whereas fuzzy quantities are used to deal with informational or intrinsic imprecision, but both quantities can be used during optimisation.

A combination of fuzzy and stochastic programming was chosen, since a probability distribution to express future demand can be determined for training and other routine operations, but the demand for unlikely operations and extreme events cannot be determined due to the uncertainty associated with these events. Therefore fuzzy programming will be used to obtain a rough estimation of what can typically be expected in these unlikely and extreme cases.

A multiple time period inventory model is used in this dissertation, as it is assumed that shortages and surplus stocks are transferred from one period to the next.

1.5 Research methodology

Rajgopal [44] presents the following steps for dealing with an Operations Research (OR) problem. These steps, that are in line with Simon's phases of a decision making process [56], are used in an essential manner in this dissertation.

Orientation A large part of the orientation of the research project has been completed through previous work in a military environment. An analysis of an actual military supply chain has been conducted to identify challenges and opportunities for improvement. The orientation of the research work also included a preliminary literature review to determine whether this problem had been encountered elsewhere and, if so, what methods were used to solve it and other similar problems. The preliminary literature review indicated that military stockpile management is a worldwide challenge and a clear need for a structured approach to military stockpile planning was identified, resulting in the initiation of this study.

Problem statement To define the research problem, this project aims to address some of the challenges experienced in military inventory management. A background study is done to establish the methods currently used in military inventory management, as well as the challenges associated with each method.

Data collection Due to the sensitive nature of military stockpiles, data collection will focus on gathering information from various sources, such as publications by military experts, academic and research publications, military publications, and relevant websites and available databases. The collected data will be used to obtain a clear understanding of the military environment, of typical stock types, and of the management of stock in such an environment, and to identify typical scenarios experienced by most militaries worldwide.

Model formulation The military is a turbulent environment typically characterised by a lack of useful historical information about extreme events such as war. Conversely, abundant information is available on the historical use for training and for very probable operations. Therefore both fuzziness and randomness are incorporated into the inventory model.

Solution The typical techniques that can be used to solve this type of mathematical model are identified through a comprehensive review of the literature. The potential solution methods are analysed and evaluated, and the most appropriate method chosen. During this step, the solution algorithm is described and used to determine the level of inventory for a single item in the military.

Validation and output analysis The military and its operations can commonly be affected by extreme events such as war. Therefore the reliability of the mathematical model is evaluated using simulation. A military supply chain is simulated, and the effects of potential scenarios on the ability of the supply chain to meet the demand for the chosen military item are investigated and analysed.

Implementation and monitoring Even though the implementation of this decision support tool does not form part of the scope of this dissertation, it can aid in solving many of the challenges associated with inventory management in the military. However, its success depends largely on the military's employees and its implementation will merely provide a starting point for an improved approach to military inventory management, by providing an improved estimation of inventory level requirements.

1.6 Outline of the dissertation

This dissertation is organised as follows. A review of relevant literature in the field of inventory management is presented in Chapter 2 and in Chapter 3 there is focused on methods to incorporate uncertainty during inventory modelling. The multi-objective mixed decision support model for inventory management in the military is presented and discussed in Chapter 4. The solution approach for the model is discussed in Chapter 5. To illustrate the functioning of the model, a numerical example is used to compare the results of the model with standard well known inventory models in the literature in Chapter 6. A sensitivity analysis is also conducted and presented in this chapter. We end in Chapter 7 with some concluding remarks along with lines for further development in the field of inventory management under uncertainty as a result of extreme events.

Chapter 2

Inventory management in the literature

Inventory can be described as the stock of any item or resource that is used in an organisation [7]. There are many reasons for an organisation to hold inventory, but a few main reasons are identified by Simchi-Levi et al. [55]. The first is to provide for unexpected changes in customer demand; the second is to function as a buffer that caters for the presence of uncertainty in some situations; the third is to ensure stock availability during supply lead times; and the fourth is to achieve economies of scale offered by transportation companies.

Although inventory can be useful in improving product availability, it is important to note that it is very expensive to hold, and that excess inventory can have devastating effects on an organisation. It is therefore vital that organisations manage their inventory properly.

Inventory management can be seen as the process of organising and planning the inventory in a supply chain. The aim of inventory management is to optimise three conflicting targets: to minimise inventory and operating costs while achieving an acceptable customer service level [61]. The aim of an effective inventory management policy is not to optimise one of the objectives at the expense of the others, but to find a balance between the three.

2.1 Dimensions of inventory models

Various aspects should be considered during inventory management and modelling. Silver [54] presents a number of dimensions that summarise the most important considerations in inventory management. The dimensions directly relevant to this dissertation are discussed in this section.

2.1.1 Single or multiple items

This dimension considers whether a single item can be used in isolation for calculations, or whether multiple interdependent products should be taken into account as a result of collective budget or space constraints, coordinated control, or substitutability between items.

Even though collective budget and space constraints are applicable to inventory in the military, it is possible to derive item-specific budget and space constraints from the collective constraints. Consequently this dissertation focuses on developing an inventory model for a single ammunition item in the military; and it is assumed that there are no interdependencies between this single item and other items in inventory.

2.1.2 Time duration

In some inventory management situations, the selling season for products is short, and excess stock at the end of the season cannot be used to satisfy the demand of the next season. In such cases a single period model is required. A typical single period problem is the 'newsboy problem', where the newsboy must buy newspapers to earn a maximum profit at the end of the day. Only one purchase is made, and no back-ordering can occur, since the newspaper is regarded as obsolete after that day has passed [26].

When multiple periods need to be considered, a common approach is to use a rolling horizon implementation approach. Here, decisions consider only a relatively small number of future periods, and are made at the start of each period. The decisions are then implemented in the current period, and the problem resolved at the start of the subsequent period [54].

A multiple time period inventory model is used in this dissertation as shortages and surplus stocks in the military are transferred from one period to the next. As a rolling horizon approach is taken, the model only focuses on a small number of future periods.

2.1.3 Number of stocking points

Even though a single stocking point can be considered in some instances, most instances require that multiple stocking points be considered. In many real world cases, stock of a certain item is kept at different stocking locations and then redistributed or trans-shipped to other stocking locations as required.

The aim of this project is to determine the stock level requirement for a single item in the military and the different stock categories are typically stored in the same depot, but in different storage locations within the depot; so the available capacity of only one central depot will be considered as a limit to the quantity stored at any time. The allocation of the stock between different depots may be done at a later stage.

2.1.4 Product type

The product type dimension identifies and considers certain product characteristics. For instance, a product may be perishable, consumable, repairable, or recoverable. The type of product will have a large impact on the choice of inventory management and modelling approach.

The product considered in this project is ammunition. Since ammunition is used by military personnel, it can be classified as a consumable product. Furthermore, ammunition has a limited life-cycle, and is inspected, tested, and sometimes reworked during its life-cycle.

2.1.5 Nature of demand

This dimension focuses on the nature of the demand for the product. The following typical demand types are identified by Silver [54]:

Deterministic demand. This type of demand does not have any variation, and the exact quantity of a particular product required every period is known with certainty.

Deterministic demand that may vary. This type of demand varies with time, but the way in which the demand varies is known with certainty.

Stationary distribution with known parameters. This type of demands follows a probability distribution that is known or estimated from historical data. Two of the most common distributions used to model demand are the normal and gamma probability distributions. For additional information on probability distributions, the reader may refer to Montgomery et al. [37].

Stationary distribution with unknown parameters. This type of demand also follows a certain probability distribution, but the parameters of the distribution are not known and cannot be estimated from historical data.

Unknown stationary distribution. This type of demand follows a certain probability distribution, but it is unknown what probability distribution it is.

Non-stationary probabilistic demand. This type of demand behaves like a random walk that evolves over time, with regular changes in its direction and rate of growth or decline [16].

As training and routine operations are regarded as very likely operations whose usage can easily be measured and tracked on a continuous basis, it is assumed that stationary distributions with known parameters can be derived from historical data for training and routine operations. However, the demand for unlikely operations and extreme events cannot easily be determined due to the high level of uncertainty associated with extreme events and the general low occurrence rate of these events; hence the unavailability of historical data. Therefore fuzzy numbers, rather than probability distributions, will be used to present the estimated demand during these events.

2.1.6 Procurement cost structure

This dimension focuses on the unit value of an item. This value may depend on various factors such as the size of the replenishment, supplier discount, and freight consolidation. For instance, when a full truck load is ordered, the transport cost per item will be lower than when less than a truck load is ordered.

The procurement cost structure used in this dissertation is exclusively composed from the fixed purchase cost per item, and any discounting factors are excluded from the analysis. However, discounting factors may be incorporated in future as the need arises.

2.1.7 Nature of supply process

The nature of the supply process refers to any restrictions or constraints that have been imposed on the inbound processes of the supply chain. Minimum or maximum order size, or replenishment lead times are examples of typical factors considered in this dimension.

Silver [54] identifies three possible forms of lead time. The first form is where the lead time of each replenishment is known; the second is where replenishments arrive after a random time; and the final form is where seasonal factors may affect the time it takes for an order to be fulfilled.

A supplier usually has limited capacity; therefore order size restrictions are taken into account in this dissertation. In addition, lead time is assumed to be a constant and known value.

2.1.8 Shelf-life considerations

The final dimension to be taken into account during inventory management considers the obsolescence, deterioration, or perish ability of stock. Obsolete stock refers to stock that is still in an acceptable physical condition but that can only be sold at a price significantly lower than its original value due to the appearance of new competing products [54]. The deterioration and perish ability of stock should also be considered, since stock with these characteristics may become unusable after a certain time.

As large ammunition stockpiles are kept to cater for unforeseen and extreme events, stock deterioration can be a huge problem. In this dissertation stock deterioration is considered by using a First-In-First-Out (FIFO) inventory policy to ensure that the older ammunition is issued first to meet demand.

2.2 Basic inventory management models in the literature

There are various inventory management models in the literature. In its most basic form, the economic lot size model, developed by Ford W. Harris in 1913 [20], deals with the decision of determining the optimal order policy that minimises the total cost while satisfying all demand in a certain time period [55]. In this problem demand is assumed to be a constant rate of D items per period and lead time is assumed to be zero. The output from this model is the quantity of items to be ordered, Q, each time an order for that item is placed.

This basic economic lot size model is an extremely simplified version of reality, and the assumption of no lead time will clearly not be valid for almost all real-world applications. Therefore this basic economic lot size model was expanded by Wilson [63] into the well known Economic Order Quantity (EOQ) model.

Like the economic lot size model, the basic EOQ model assumes a constant, continuous, and known demand rate, but a constant and known lead time is taken into account. The basic EOQ model aims to determine the quantity to be ordered Q, as well as the level at which reordering should occur r, while attempting to minimise the total inventory cost T^C [7].

The total cost comprises annual purchase cost, annual ordering cost, and annual holding cost and is depicted by equation 2.1, where D represents the annual demand rate, C the cost per unit, H the annual holding cost, and K the fixed cost of placing an order.

$$T^C = DC + \frac{QH}{2} + K\frac{D}{Q} \tag{2.1}$$

The optimal quantity to be ordered can then be derived as shown in equation (2.2).

$$\frac{dT^C}{dQ} = 0 \tag{2.2}$$

This results in equation (2.3) that is used to calculate the optimal order quantity.

$$Q = \sqrt{\frac{2DK}{H}} \tag{2.3}$$

Finally, when $\overline{d}$ represents the average daily demand and L the order lead time, the reorder point of the inventory system can be calculated using equation 2.4.

$$r = \overline{d}L \tag{2.4}$$

Even though the basic EOQ model takes lead time into account, it assumes that the demand rate and lead time are known with certainty. The assumption of conditions of certainty can be useful in some instances, but in most cases it can lead to an inaccurate representation of the actual inventory system, and thus to extremely inaccurate, hence ineffective, solutions. This is even more apparent in the military environment, which is characterised by extreme uncertainties. It is therefore imperative for a non-deterministic approach to be followed while solving the problem to ensure that the uncertainty of demand in the military is depicted accurately and realistically.

The problem of uncertainty is circumvented by another adaptation of the basic EOQ model. In this adapted fixed order quantity model — known as the (r, Q) model with a service level measure [64] — uncertain daily demand is represented with a normal distribution with mean, μ^D, and standard deviation, σ^D. A service level measure is also incorporated to allow the decision maker to specify the minimum service level — the probability of not stocking out during an order cycle — required.

As with the basic EOQ model, let Q be the optimal order quantity, r, the reorder level, K the fixed ordering cost, and L the fixed order lead time. The optimal order quantity is then calculated as follows [7]:

$$Q = \sqrt{\frac{2K\mu^D}{H}}. \tag{2.5}$$

The reorder level of this (r, Q) model with a service level measure is then determined using equation (2.6) where $\sigma^L = \sqrt{\sum_{i=1}^{L} \left(\sigma^D\right)^2}$ and z is the number or standard deviations for a specified service probability [7]. The z value can be obtained from standard tables listing the areas of the standard normal distribution for different service level values.

$$r = \mu^D L + z\sigma^L = \mu^D L + z\sqrt{\sum_{i=1}^{L} \left(\sigma^D\right)^2} \tag{2.6}$$

Another widely used adaptation of the fixed order quantity approach is the (s, S) or *min max* inventory management approach that enables the incorporation of lumpy or erratic demand patterns [8]. This model was developed by Arrow et al. [1] as a result of some shortcomings identified with the normal fixed order quantity approach.

They developed a dynamic inventory model with demand represented by a random variable with a known probability distribution. Their model determines the optimal stock level and reorder point while minimising the present value of the total expected cost over a certain number of time intervals. The total expected cost over all time periods is referred to as the total expected loss and comprises ordering cost, inventory holding cost, and a penalty for shortages.

Later, Scarf [51] used dynamic programming to prove that there is an optimal inventory policy for time periods $t \in \{1, 2, \ldots, T\}$ with minimum and maximum stock levels, s_t and

S_t, such that when the inventory level at the start of period t is less than s_t, the difference between the maximum stock level, S_t, and the current inventory level is produced or ordered. On the other hand, if the current inventory level is more than the minimum level, no items are produced or ordered [64]. However, this model assumes that the per unit shortage cost is known — an assumption that is clearly not valid for the military environment.

A way to circumvent this problem is to use another variation of the (s, S) model — the (s, S) model with a service level measure. The main differences between the (s, S) model with a service level measure and the (r, Q) model with a service level measure, is that the (s, S) model has a minimum stock level, s and a desired maximum stock level S, instead of a reorder point, r and fixed order quantity Q. Whenever the current stock level falls below s an order is placed up S. The minimum stock level is determined the same way as the reorder level in the (r, Q) model as shown in equation (2.7).

$$s = r = \mu^D L + z\sigma^D \sqrt{L} \tag{2.7}$$

The maximum desired inventory level is then calculated as follows [64]:

$$S = r + Q = \mu^D L + z\sigma^D \sqrt{L} + \sqrt{\frac{2K\mu^D}{H}}. \tag{2.8}$$

Even though the (r, Q) and (s, S) inventory management approaches are suitable to most organisations, some shortcomings exist for the application to military inventory management.

The first concern is the issue of different stock categories — For more detail on the different stock categories, the purpose of each and the flow between different stock categories refer to Chapter 4. Since the military typically divides the stockpile of one item into various categories used for different purposes, the models have to be re-solved for every stock category and the interdependence between the different stock categories incorporated by means of adjusting the demand of one stock category with the results of another stock category. This can be time consuming and may lead to unnecessary rework of the model for different stock categories.

The second concern is that these models assume that everything ordered are delivered after the lead time. This is not necessarily the case for the military, as a part of the ordered stock can be delivered and the rest kept in storage at the supplier. Even though, this problem can partially be circumvented by deciding after the results has been obtained how many to request for delivery and how many to keep in stock at the supplier.

The third concern is that these models use known probability distributions to represent demand. This may be suitable for very likely scenarios in the military, but due to the lack of available historical data it is often impossible to determine a valid probability distribution for demand during extreme events. This can be circumvented by using fuzzy numbers to represent demand during extreme events and defuzzifying these fuzzy numbers into single scalars — see Chapter 3 for various defuzzification techniques. These scalar values can then be used as the mean demand with no deviation. However, this reduces the (r, Q) model to the basic EOQ model with constant and known demand.

The final concern is that the models may not be reliable enough for inventory management in the military. This concern is confirmed in Chapter 6 where the reliability of these models are investigated and compared with the inventory model presented in this dissertation.

In light of the above mentioned concerns, it was decided that an alternative approach must be used to solve the problem of inventory management in the military.

2.3 Concluding remarks

Inventory management is a well established field in the literature and there are many aspects that should be taken into account during inventory management. These aspects will also be considered during the modelling and solution process to ensure that a useful inventory decision support model is delivered.

There are some existing basic inventory models available in the literature, however these do not take uncertainty adequately into account and therefore alternatives should explored. Certain adaptations of the basic inventory models — where demand uncertainty is incorporated — are also discussed and some shortcomings of these adapted models for inventory management in the military raised in this chapter.

Due to the shortcomings of existing models and the presence of uncertainty in the military, alternative approaches to inventory management under uncertainty are investigated in the following chapter and a more suitable approach identified.

Chapter 3

Alternative approaches to inventory management under uncertainty

The initial work of Arrow et al. [1] was extended by Dvoretzky et al. [10, 11]. In two detailed papers they present two versions of a general inventory model with uncertain demand. Like Arrow et al. [1], their first paper focuses on inventory management where demand is represented by a random variable following a known probability distribution [10]. However, their second paper introduced the idea of inventory management where the probability distribution of demand is not known [11].

3.1 Development of non-deterministic inventory models

Ever since the publication of the pioneering work of Arrow et al. [1] and Dvoretzky et al. [10, 11], various research efforts focusing on inventory management under uncertainty have been conducted and published.

A technique often used in the literature to address the complexity of inventory management under uncertainty is to assume that the uncertain parameter is random and that its probabilistic distribution is known with certainty. In such cases stochastic programming is used to incorporate uncertainty.

Stochastic programming approaches include recourse techniques and chance-constrained methods.

When using recourse models in stochastic programming, recourse variables are introduced into the stochastic model to account for any infeasibilities that may arise due to a particular realisation of uncertainty. These recourse variables are also random variables whose distributions are determined based on the realisation of the uncertain component — for example demand — in the model. The aim of a recourse model is usually to minimise the sum of the total original costs as well as the total expected recourse cost [25].

In a chance-constrained model a reliability level α is identified by the decision-maker. The probability of satisfying a constraint containing a random variable is then measured and forced to be at least α [25]. For example, if the decision-maker prefers a reliability level of 95%, the probability of a constraint — containing a random variable — being satisfied should be 95% or more.

According to Kall and Wallace [25], chance-constrained model are usually mathematically more complex than recourse models since chance-constraints deal more explicitly with the whole distribution whereas recourse models transform the randomness of the model into the expectation of the random variable's distribution. However — depending on the problem — both techniques can be quite useful to incorporate uncertainty into modelling.

There is abundant literature that addresses uncertainty in inventory planning by incorporating probabilistic distributions. An example is presented by Ryu and Lee [50], who develop two dual-sourcing inventory models, one with lead-time reduction and one without, with stochastic lead time and constant unit demand. The reorder point, the optimal order quantity, and the order split proportion between the two suppliers are determined for each of the two models and compared. Another example of a stochastic inventory model is provided by Maiti et al. [33]. They consider price-dependent demand and probabilistic lead time in their formulation, where shortages and advance payments are allowed, in order to determine the optimal order quantity and selling price while minimising the total cost.

Another modelling approach that uses probability distributions to represent the uncertain parameter is stochastic dynamic programming. According to Winston and Venkataramanan [64] dynamic programming is a technique that can be used to obtain a solution to a problem by working backward from the end of the problem to the beginning. This allows the decision maker to break a large problem into a series of smaller problems. Stochastic dynamic programming is often used to solve multi-period inventory management problems where a period's demand is a random variable whose value is only realised after the inventory decision during that period is made. The aim of this type of model is usually to minimise the total cost of to maximise the expected reward of a specified number of time periods.

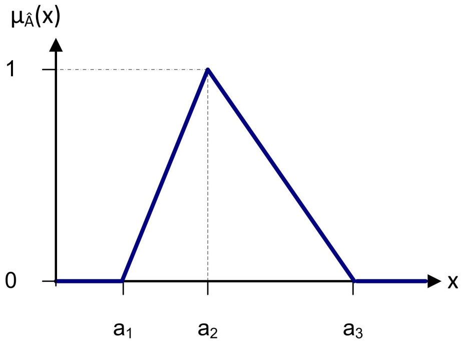
It is unfortunately very often true that you do not know what the underlying distribution of the uncertain parameter is. A mathematical programming approach that is frequently taken to bypass the problem of unknown probability distributions is fuzzy programming.

Fuzzy programming finds its roots in fuzzy set theory, which originated in 1965 [65]. Fuzzy set theory stipulates a way in which imprecise and ambiguous quantities can be incorporated into decision-making. A fuzzy set is characterised by a membership function that assigns a grade of membership ranging between zero and one to each element in the set [65].

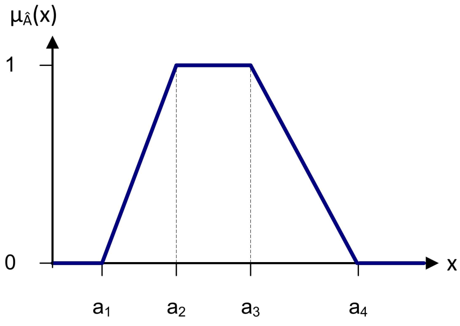
In fuzzy programming, uncertain parameters are described with fuzzy sets. There are many different types of fuzzy sets, and the manner in which quantities are described will determine the type and shape of membership function being used. Consider a fuzzy variable $\widehat{A}$ used to describe a value as about a_2 but between a_1 and a_3, in which case it can be depicted by a triangular membership function, as shown in Figure 3.1(a).

The membership function of $\widehat{A}$, $\mu_{\widehat{A}(x)}$, can then be defined as (3.1) [27].

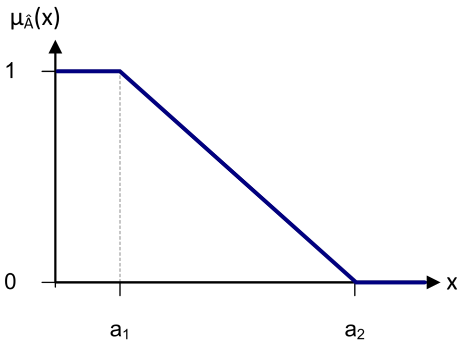
$$\mu_{\widehat{A}}(x) = \begin{cases} 0, \quad x < a_1 \\ \frac{x-a_1}{a_2-a_1}, \quad a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, \quad a_2 \leq x \leq a_3 \\ 0, \quad x > a_3 \end{cases} \tag{3.1}$$



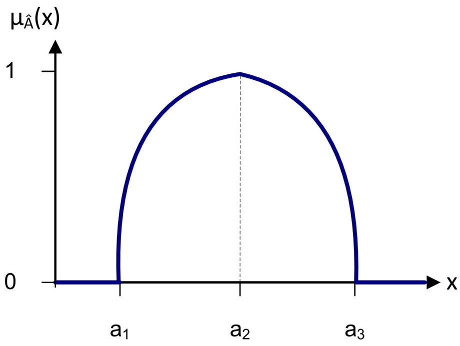
(a) Triangular



(b) Trapezoidal



(c) Linear



(d) Parabolic

Figure 3.1: Some membership function shapes identified in the literature

There are many other shapes of membership functions used in the literature. Three more of them - trapezoidal, linear, and parabolic are shown in Figures 3.1(b), 3.1(c) and 3.1(d) respectively.

Establishing the type and shape of the membership function is a critical aspect of fuzzy programming. Ross [46] identifies some of the most straightforward methods available in the literature, discussed below, to assign membership functions to fuzzy variables.

Intuition When using intuition to establish membership functions, the shape and values of the membership function are derived from the intelligence and understanding of humans.

Inference When using the inference technique, a given body of knowledge and facts is used to deduce suitable membership functions for fuzzy variables.

Rank ordering With the rank ordering method of membership value assignment, the preferences of a person or group of people are used to assign a membership function to a fuzzy variable.

Neural networks The neural network method for determining membership functions attempts to create an intelligent program that simulates the working network of neurons in a human brain. This program then determines the most appropriate membership function based on a given set of data.

Genetic algorithms With the genetic algorithm method, a suitable membership function is derived from a given data set using a program that attempts to simulate certain concepts of biological evolution or 'survival of the fittest'.

Inductive reasoning Inductive reasoning is typically used where a general membership function has to be derived from the data of a specific observation. This is useful for instances where data is available for one observation but where a generic membership function is required that will be suitable to describe set of independent observations.

Ever since the use of fuzzy set theory to model uncertainty was discovered, a lot of research in fuzzy inventory modelling has been done. One example is the work of Roy and Maiti [47], who presented a fuzzy EOQ model with limited storage capacity, demand that is related to the unit price, and a setup cost that varies depending on the quantity ordered. They introduced fuzziness into storage space availability and into the objective function that aims to minimise the total expenditure.

Later they expanded their model into a multi-objective, multi-item inventory model for deteriorating items with stock-dependent demand [48]. The objectives of the model are to maximise the profit and minimise the total wastage cost in an environment where the profit goal, wastage cost, and available storage area are fuzzy in nature. They represent these fuzzy parameters in the model with linear functions while assuming that there are no shortages, an infinite replenishment rate, stock-dependent demand, and a constant deterioration rate.

Mandal et al. [35] present a multi-item, multi-objective inventory model with limited storage space that is similar to that of Roy and Maiti [48]. However, they expand the model to permit shortages that are fully backlogged. Storage space, the number of orders, and the total production cost are represented by fuzzy sets in the model; and set-up, shortage, and holding costs are also expressed as fuzzy numbers.

Another inventory model with fuzzy demand is presented by Kao and Hsu [26]. They adopt a method for ranking fuzzy numbers to find the optimal order quantity, in terms of cost, for a single-period problem where only one procurement can be made in a time period.

One of the most recent contributions to the field of fuzzy inventory modelling is presented by Handfield et al. [19], who develop a (Q, r) inventory model in which the optimal order quantity Q and the reorder point r are determined. They use fuzzy sets to represent various sources of uncertainty in the supply chain which include demand, supplier yield, lead time, and penalty cost. They incorporate human risk attitude into the formulation by including a weight factor that varies depending on the decision-maker's attitude towards risk. Incorporating the risk attitude of the decision-maker into the formulation is a new and valuable expansion in the field of fuzzy programming, as it provides the decision-maker with the option to choose between optimistic, pessimistic, or neutral approaches when making a final decision under uncertainty.

In some instances it may be possible to have both fuzzy and random variables in a mathematical model; such models are often referred to as mixed models. A particular model can be regarded as a mixed model if it expresses both randomness and fuzziness at the same time.

An example of a mixed inventory model in the literature is presented by Ishii and Konno [23], who adapt the classic 'newsboy problem' by introducing a fuzzy shortage cost into the formulation. Their model aims to maximise the total expected profit that can be achieved by selling newspapers amidst random daily demand and fuzzy shortage cost.

Another example is presented by Ouyang and Yao [39], whose mixed inventory model with random lead time and fuzzy annual demand attempts to minimise the total fuzzy expected annual inventory cost. They consider two cases: in the first, annual demand is treated as the triangular fuzzy number, and in the second a statistical method is used to create a confidence interval for annual demand which is then used to establish its associated fuzzy number.

In some cases the parameters of the random variable's probability distribution are vague, resulting in fuzzy random variables. This concept is often referred to as fuzzy-stochastic optimisation; there are many examples in the literature where researchers have focused on inventory problems with fuzzy random variables.

In one such example, Dutta et al. [9] present a single-period inventory model where demand is represented by a fuzzy random variable involving imprecise probabilities, and conclude that the implementation of a fuzzy random variable as demand provides more realistic information where the variable values are imprecise.

Another attempt to develop an inventory model in a fuzzy-stochastic environment is presented by Chang et al. [6]. Their mixture inventory model with variable lead time takes back-ordering and lost sales into account. Lead time demand is estimated with a fuzzy random variable. The objective of the model is to minimise the total cost while determining the optimal order quantity and lead time.

Most of the fuzzy, stochastic, and mixed inventory models in the literature aim to find the most cost-effective inventory management solution, and customer satisfaction is usually enforced by means of penalties for shortages. However, in some environments a monetary value cannot be assigned to shortages. The military can be seen as such an environment, since it is not known what the consequences of ammunition shortages will be during extreme events such as war. Therefore an alternative approach is to minimise potential shortages without assigning monetary values to these shortages.

A way in which shortages can be managed without assigning a specific monetary value to them is to add another objective to the mathematical model. This will result in a multi-objective model that aims to minimise cost while maximising demand satisfaction or minimising total shortages. The major advantage of this approach is that multi-objective optimisation provides the opportunity to address trade-offs between conflicting objectives and to ensure that the most appropriate balance between conflicting goals can be achieved.

Goal programming is one of the most commonly used model formulation approaches for multi-objective problems. When using goal programming, multiple objectives can be combined into a single objective function to allow the optimisation of a single objective, by incorporating certain goals. The work of Roy and Maiti [48] is an example in the literature where goal programming is used to address multi-objectivity in fuzzy inventory modelling. Goal programming is chosen as the preferred approach to incorporate the multi-objectivity of the military environment into the inventory model proposed in this dissertation.

Since the military is an environment typically characterised by many uncertainties and a lack, in some instances, of accurate and useful historical information, mixed multi-objective programming seems to be the most appropriate modelling technique for it. The main argument in favour of using a mixture of fuzzy and stochastic optimisation to address uncertainty in the military environment is the ability of fuzzy set theory to provide an appropriate framework in which the imprecision in modelling uncertainty can be treated through subject expert estimation in the absence of valid historical data [19]. However, in some instances valid historical demand data may be available, and in these cases probabilistic distributions will be incorporated. It is important to note that, due to the incorporation of uncertainty, fuzzy and stochastic optimisation are processes that seek a good solution without being able to prove that it is the best solution [27]. The solution obtained from the inventory model under uncertainty will not necessarily be optimal, but merely a good estimation of the best solution.

3.2 Solution approaches to fuzzy, stochastic, and mixed programming problems

Fuzzy, stochastic, and mixed mathematical models are generally hard to solve, and some form of transformation is required to solve them by means of exact solution methods. In this section, various approaches to derive the deterministic counterparts of fuzzy, stochastic, and mixed mathematical models and other solution approaches will be considered to ensure that an appropriate solution approach is used for the military inventory model proposed in this dissertation.

3.2.1 Stochastic programming problems

A popular approach to solving stochastic mathematical models is to transform the stochastic model with embedded uncertainty into its deterministic counterpart, and then apply solutions procedures to solve the deterministic counterpart model.

One approach where the deterministic counterpart for a stochastic model is derived is presented by Nemirovski and Shapiro [38]. They use a scenario approach to approximate uncertainty in a chance-constrained stochastic programming problem by sampling, using Monte Carlo techniques. The major drawback of the scenario approach is that the sample size becomes impractically large and difficult to apply, even with medium-sized problems.

When dealing with recourse stochastic models, a regular approach is to use a method of decomposition to derive their deterministic counterparts, as recourse stochastic programming problems have very specific structures that can be exploited by using the decomposition technique [49].

Dual decomposition, a specific subset of decomposition, is commonly applied to stochastic programming problems. The main premise behind dual decomposition methods is to depict the probability distributions of random variables by a number of potential realisations and their associated probabilities, and to use the values and probabilities of these realisations during computations. An example in the literature where dual decomposition is applied to a stochastic recourse problem is presented by Bean and Joubert [2]. Their model aims to minimise the total cost of holding and relocating empty containers in a specified network with random demand and supply. Another example is presented by Schütz et al. [53], who develop a model for a supply chain design problem amidst demand uncertainty. Their model aims to minimise the total investment and operating cost of a multi-commodity supply chain.

According to Ruszczyński [49], using decomposition methods has two fundamental advantages. The first is the ability of decomposition methods to replace large and difficult stochastic models with a group of smaller and easier ones. The second is that the smaller models, derived from the bigger one, are usually standard linear, non-linear or quadratic models which have very simple deterministic counterparts that can be solved using standard optimisation software packages.

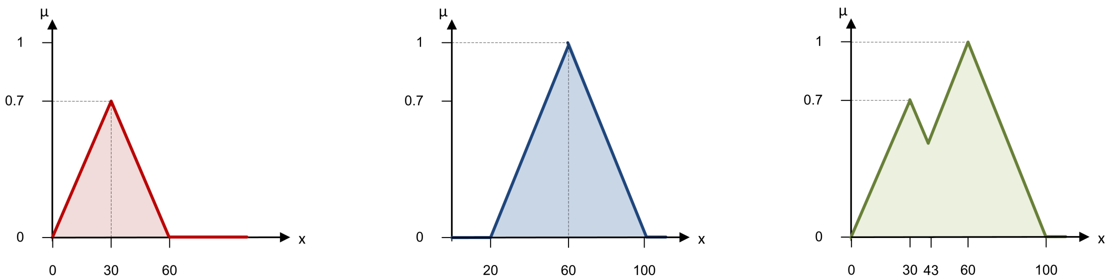
Another approach in the literature to solve stochastic inventory problems is the use of heuristic algorithms, which are approximate techniques that can determine a good feasible solution rather than the optimal solution provided by exact solution methods. Heuristic algorithms will not be used in this dissertation; the interested reader is referred to the work of Maiti et al. [33] and Fisher et al. [13] for examples of stochastic inventory problems solved with heuristic algorithms.

3.2.2 Fuzzy programming problems

A general approach to solving a model with fuzzy variables is to derive a deterministic counterpart model from the fuzzy version, and apply exact solution techniques to the deterministic counterpart model. One method for deriving the deterministic counterpart model is defuzzification.

The defuzzification solution method implies that the fuzzy variable is converted into a single scalar quantity, using some sort of defuzzification method. Once a fuzzy quantity has been defuzzified into a single scalar quantity, the resulting scalar quantity can be considered as an estimation of the initial fuzzy variable. This resulting scalar quantity is used as input into the model, which can then be solved as a deterministic mathematical model using standard optimisation software.

There are many different methods in the literature that can be used to defuzzify a fuzzy quantity. Ross [46] gives an overview of some of the most recently used defuzzication methods. To illustrate how some of the defuzzification methods works, assume that we have two triangular fuzzy numbers. Fuzzy numbers $\widehat{A}$ and $\widehat{B}$ are depicted by Figures 3.2(a) and 3.2(b) respectively. The combination of $\widehat{A}$ and $\widehat{B}$ is also considered to be a fuzzy number, $\widehat{C}$, and is depicted by Figure 3.2(c).



(a) Fuzzy number $\widehat{A}$ (b) Fuzzy number $\widehat{B}$ (c) Fuzzy number $\widehat{C}$

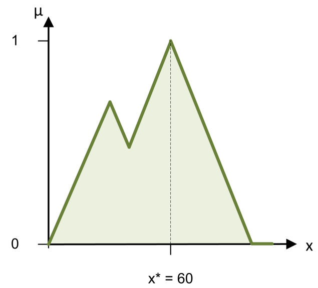
Figure 3.2: Fuzzy numbers used in the defuzzification example.

Using the standard triangular membership function, as shown in (3.1) in Section 3.1, the membership functions of $\widehat{A}$ and $\widehat{B}$ can be defined as (3.2) and (3.3) respectively.

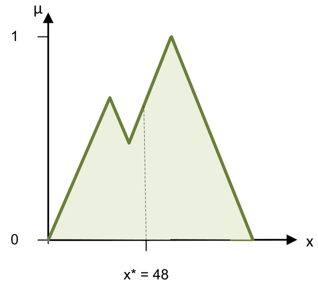
$$\mu_{\widehat{A}}(x) = \begin{cases} \frac{x}{30}, & 0 \leq x \leq 30 \\ \frac{60-x}{30}, & 30 \leq x \leq 60 \\ 0, & x > 60 \end{cases} \tag{3.2}$$

$$\mu_{\widehat{B}}(x) = \begin{cases} 0, & x < 20 \\ \frac{x-20}{40}, & 20 \leq x \leq 60 \\ \frac{100-x}{40}, & 60 \leq x \leq 100 \\ 0, & x > 100 \end{cases} \tag{3.3}$$

Height method This method is also known as the maximum membership principle, and merely chooses the value with the highest membership as the equivalent scalar quantity [46]. The highest membership value in the example, $x^* = 60$, is at a membership value of 1.0.

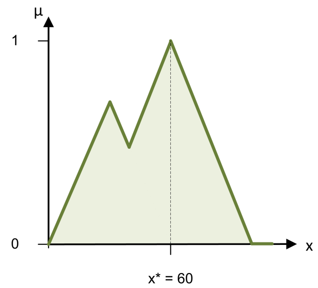


Weighted average method The weighted average method of defuzzification determines the equivalent scalar quantity of a number of symmetric membership functions. The equivalent quantity is determined by weighting each membership function with its maximum value, and determining the average of all the maximum values [46]. The defuzzified value for the two membership functions in the example is calculated as follows:

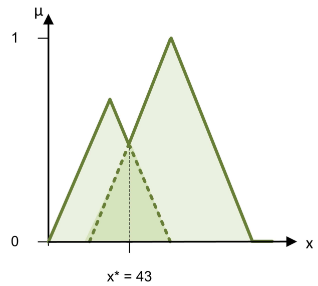


$$x^* = \frac{0.7(30) + 1.0(60)}{1.7} \approx 48.$$

Mean maximum membership This method is similar to the height method, but when the maximum value of the membership function is a plateau of values, this method takes the middle of the plateau as the equivalent scalar quantity of the fuzzy variable [46]. In the example there is only one maximum membership value; therefore the defuzzified value is the same as in the Height Method, $x^* = 70$.



First (or last) of maxima This defuzzification method uses the overall union of all the membership functions to determine the smallest value (or biggest value) in the union that has the biggest membership [46]. As there is only one point in the example with the biggest membership, both the first and last of maxima method produces the same result, $x^* = 43$.



Another variant of defuzzification is discussed by Heilpern [21] who uses the notion of the expected interval and expected value of a fuzzy number. Since expectation is normally associated with randomness and not fuzziness, the expected value and expected interval presented by Heilpern [21] will be referred to as the defuzzification value and defuzzification interval respectively.

According to Heilpern [21] the defuzzification interval $D^I(\widehat{A})$ and the defuzzification value, $D^V(\widehat{A})$, of fuzzy number $\widehat{A}$ is determined as

$$D^I(\widehat{A}) = \left[D^l, D^u\right] \tag{3.4}$$

$$D^V(\widehat{A}) = \frac{\left(D^l + D^u\right)}{2}, \tag{3.5}$$

where A^l and A^u are the lower and upper defuzzification values of the fuzzy number. He comments that the defuzzification interval and defuzzification value, depicted by equations (3.4) and (3.5), can be transformed into a simpler notation, as shown in equations (3.6) and (3.7), when using a trapezoidal fuzzy number with parameters (a_1, a_2, a_3, a_4).

$$D^I(\widehat{A}) = \left[\frac{(a_1 + a_2)}{2}, \frac{(a_3 + a_4)}{2}\right] \tag{3.6}$$

$$D^V(\widehat{A}) = \frac{(a_1 + a_2 + a_3 + a_4)}{4} \tag{3.7}$$

An example in the literature where the defuzzification solution approach is applied is presented by Giannoccaro et al. [15]. They propose a method for managing inventory in the supply chain, based on the EOQ model, with additional backorders, that is extended over a multi-echelon supply chain. They represent holding cost, back-ordering cost, and market demand as triangular fuzzy numbers that are defuzzified into their crisp counterparts, and then used as inputs into the deterministic inventory model. They comment that the fuzzy approach they used allows for uncertainty to be properly modelled and a deterministic model to be used, provided that defuzzified values are used instead of deterministic ones.

Even though their comment is valid, defuzzifying the fuzzy parameters and using their crisp counterparts as inputs into the optimisation model is not necessarily the best approach to use when dealing with fuzzy information. This is because the defuzzification of fuzzy variables into crisp, or non-fuzzy, values and using these crisp values in an optimisation model implies that not all the information contained in the fuzzy variable is taken into account during optimisation. However, defuzzification is a valuable technique to use when a crisp, non-fuzzy, value for the fuzzy objective function of the model, resulting from optimisation with fuzzy quantities, is required.

Another exact solution approach to fuzzy optimisation models is the α-level cut technique. The α-level cut of a fuzzy number $\widehat{A}$ results in a non-fuzzy interval, and is denoted as $\widehat{A}_\alpha$ with lower and upper interval values of $[a_1^\alpha, a_2^\alpha]$ [19] as depicted by Figure 3.2.2. Calculations with a fuzzy number can then be performed in the model using the α-level interval for different $\alpha \in (0, 1]$ values.

A more recent solution approach is the use of an expected value operator to derive the deterministic counterpart of a fuzzy mathematical model. The concept of an expected value operator of a fuzzy variable was introduced by Liu and Liu [30], however for the sake of clarity there will be referred to the defuzzification value instead of the expected value of a fuzzy quantity. According to Liu [29], the underlying philosophy of a fuzzy Defuzzification Value Model (DVM) is based on choosing the decisions that will maximise the defuzzified return of the model. A general form of a fuzzy DVM is presented in (3.8) with fuzzy vector $\widehat{\zeta}$, decision vector x, return function $f(x, \widehat{\zeta})$, and fuzzy constraint functions $g_j(s, \widehat{\zeta})$ for $j \in \{1, \ldots, p\}$ [29] with a defuzzification value of $A\left[g_j(s, \widehat{\zeta})\right]$.

$$\begin{aligned}
&max \quad D\left[f(x, \widehat{\zeta})\right] \\
&s.t. \\
&D\left[g_j(s, \widehat{\zeta})\right] \leq 0 \quad \forall j \quad \in \{1, \ldots, p\}
\end{aligned} \tag{3.8}$$

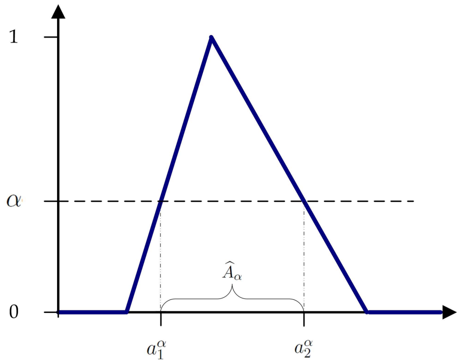


Figure 3.3: α-level of a triangular fuzzy number.

An extension of the single objective DVM is Defuzzification Value Multi-Objective Programming (DVMOP). The general form of the DVMOP is similar to the single objective DVM shown in (3.8), with the exception that it has multiple return functions. The objective then becomes

$$max \quad D\big[f_1(x,\widehat{\zeta})\big], D\big[f_2(x,\widehat{\zeta})\big], \ldots, D\big[f_m(x,\widehat{\zeta})\big]$$

[29].

When using goal programming to address the multi-objectivity of the DVMOP, an Defuzzification Value Goal Programming (DVGP) approach is used to take goal targets and priorities into account in the formulation. The general form of the DVMOP, presented by [29], includes a pre-emptive priority factor expressing the relative importance of each goal, P_j, weighting factors for positive and negative deviations from goal i with priority j, u_{ij} and v_{ij}, and positive and negative deviations from goals, d_i^+ and d_i^-. The general form of the DVMOP is shown in (3.11).

$$min \quad \sum_{j=1}^{l} P_j \sum_{i=1}^{m} (u_{ij} d_i^+ v_{ij} d_i^-) \tag{3.9}$$

$$s.t.$$

$$D\big[f_i(x,\widehat{\zeta})\big] + d_i^- - d_i^+ = b_i \quad \forall \quad i \in \{1,\ldots,m\} \tag{3.10}$$

$$D\big[g_j(s,\widehat{\zeta})\big] \leq 0 \quad \forall \quad j \in \{1,\ldots,p\}$$

$$d_i^+,\ d_i^- \geq 0 \quad \forall \quad i \in \{1,\ldots,m\} \tag{3.11}$$

Once the model is rewritten in the form of DVM, DVMOP, and DVGP problems, the defuzzification values of equations can be derived using principles presented by [29], and the model can be solved exactly by means of standard optimisation packages such as LINGO©.

An approach where Linear Programming (LP) models with fuzzy parameters and crisp decision variables can be transformed into crisp counterpart LP models is presented by Jiménez et al. [24]. They incorporate the notion of the degree of feasibility of the decision vector into the optimisation model, which allows the decision-maker to choose the degree of feasibility required. Later Peidro et al. [40] adopted this approach for tactical supply chain planning under uncertainty and commented that this approach led to improved service levels, while avoiding excessive increases in computational efficiency.

When using this approach, LP models with fuzzy parameters are transformed into crisp LP models using the feasibility degree, the defuzzification intervals of the fuzzy parameters in constraints (3.4), and the defuzzification values of fuzzy parameters in the objective function (3.5). The resulting crisp LP model can then be solved using standard optimisation software packages. To illustrate this approach, consider a sample optimisation problem (3.12) with fuzzy parameters $\widehat{c}$, $\widehat{a}_i$, $\widehat{b}$ and decision variable x [40].

$$\begin{aligned} &min \quad z = \widehat{c}x \\ &s.t. \\ &\widehat{a}_i x \geq \widehat{b}_i \quad \forall \quad i \in \{1, \ldots, m\} \end{aligned} \tag{3.12}$$

This model can be transformed into its crisp counterpart (3.13) by defining a feasibility degree, $\alpha \in [0, 1]$, representing the degree to which all constraints are fulfilled [40].

$$\begin{aligned} &min \quad z = D^V\left[\widehat{c}\right]x \\ &s.t. \\ &\left[(1-\alpha)D^{u_{a_i}} + \alpha D^{l_{a_i}}\right]x \geq \left[\alpha D^{u_{b_i}} + (1-\alpha)D^{l_{b_i}}\right] \quad \forall \quad i \in \{1, \ldots, m\} \end{aligned} \tag{3.13}$$

If the $\geq$ symbol in (3.12) is replaced with $=$ and $\leq$ symbols, the resulting crisp constraints are depicted by (3.14) and (3.15) respectively [40].

$$\begin{aligned} &\left[\left(1-\frac{\alpha}{2}\right)D^{l_{a_i}} + \frac{\alpha}{2}D^{u_{a_i}}\right]x \leq \left[\frac{\alpha}{2}D^{l_{b_i}} + \left(1-\frac{\alpha}{2}\right)D^{u_{b_i}}\right] \quad \forall \quad i \in \{1, \ldots, m\} \\ &\left[\left(1-\frac{\alpha}{2}\right)D^{u_{a_i}} + \frac{\alpha}{2}D^{l_{a_i}}\right]x \geq \left[\frac{\alpha}{2}D^{u_{b_i}} + \left(1-\frac{\alpha}{2}\right)D^{l_{b_i}}\right] \quad \forall \quad i \in \{1, \ldots, m\} \end{aligned} \tag{3.14}$$

$$\left[(1-\alpha)D^{l_{a_i}} + \alpha D^{u_{a_i}}\right]x \geq \left[\alpha D^{l_{b_i}} + (1-\alpha)D^{u_{b_i}}\right] \quad \forall \quad i \in \{1, \ldots, m\} \tag{3.15}$$

Even though there are various attempts in the literature to solve fuzzy mathematical models using exact solution methods, many researchers have focused on using heuristic algorithms to solve these types of problems. Some examples where heuristic algorithms have been used to solve fuzzy inventory models are the work done by Wee et al. [60], Maiti and Maiti [34], and Gen et al. [14].

3.2.3 Mixed programming problems

The general strategy towards mixed optimisation problems is to defuzzify the fuzzy values and derandomise the random values to convert the model into its deterministic counterpart model. Van Hop [59] identified two common methods to achieve this. The first is where defuzzification and derandomisation are performed in a sequential manner, and in the second both actions are performed simultaneously.

The most commonly adopted approach to sequential defuzzification and derandomisation is presented by Luhandjula [32], who addresses fuzziness by discretising fuzzy values in the model using α-level sets and randomness using standard stochastic programming techniques, in particular chance-constrained programming [59]. The most commonly used approach adopted for the simultaneous defuzzification and derandomisation of mixed optimisation models is the work presented by Liu and Liu [31], who derive the deterministic counterpart model by calculating the expected value of fuzzy random variables.

There are many different approaches to sequentially and simultaneously defuzzify and derandomise fuzzy and random values in optimisation models. However, the approach adopted in a particular instance will depend to a large extent on the environment being modelled, the model itself, and computational requirements for the optimisation model.

Once the deterministic counterpart model has been obtained, adopting either the sequential or the simultaneous defuzzification and derandomisation approach, the model can be solved using standard optimisation software packages or heuristics. It is important to note that when dealing with fuzzy and stochastic data, the idea of optimality becomes a rather elusive concept. However, results obtained from mixed optimisation models can still prove to be very useful contributions to inventory management and decision-making.

Once the model is solved, its reliability should be tested to investigate the ability of the supply chain to meet demand during a variety of scenarios, given the inventory levels and order quantities obtained from the model. This is a very important aspect of the model execution process, as the potential consequences of the adjustments suggested by the model are investigated and the model solution quality verified.

3.3 Testing the reliability of the solution

Simulation is a powerful tool that can be used during reliability testing, as it provides the ability to study complex systems in great detail [18]. Kelton et al. [28] define simulation as the process of designing and creating a computerised model of a real or proposed system, for the main purpose of conducting numerical experiments to obtain a better understanding of the behaviour of that system for a given set of conditions. When using simulation to test the robustness and reliability of a supply chain, the supply chain is simulated and the potential effects of some events are investigated.

An example in the literature where simulation is used to test the reliability of a supply chain is presented by Petrovic et al. [42]. They developed a simulation model that provides a dynamic view of the supply chain in their problem, and used that simulation model to assess the potential impact that the solutions of their fuzzy model may have on the supply chain. Another application of simulation to test the potential effect of some events on a supply chain is presented by Hung et al. [22]. They simulate the supply chain network of a pharmaceutical company, and assess the potential effects that some events could have on customer service level, the probability of stocking out, and the average inventory level in the supply chain.

Another approach to test supply chain performance using simulation is presented by Persson and Araldi [41]. They base their supply chain simulation model on a generic supply chain structure provided in the Supply Chain Operations Reference (SCOR) model [57], a process reference model developed and maintained by the Supply Chain Council. The SCOR model provides companies with a generic process modelling framework that integrates benchmarking, best practices, and supply chain performance metrics [57]. Since the process mapping facility of the SCOR model is similar to the process modelling done in discrete-event simulation, it is possible to merge the two tools into a comprehensive supply chain analysis tool [41].

Due to the generic supply chain framework provided by the SCOR model, and the compatibility of discrete event simulation with the SCOR model, it was decided to combine these two tools to test the reliability of the inventory model proposed in this dissertation. However, a military supply chain is significantly different from a commercial supply chain, and the basic SCOR model needs to be adapted to ensure that a more suitable military end-to-end supply chain can be defined.

This is done by Bean et al. [3] who present an augmented SCOR model for the military. In addition, Schmitz et al. [52] propose an end-to-end military supply chain incorporating various aspects of the augmented SCOR model of Bean et al. [3] and life cycle management. It was decided to base the simulation model, that is used to test the reliability of the inventory model proposed in this dissertation, on the work of Schmitz et al. [52] and Bean et al. [3]. The end-to-end supply chain processes used as the basis for the simulation, as adapted from Schmitz et al. [52], are shown in Figure 3.4.

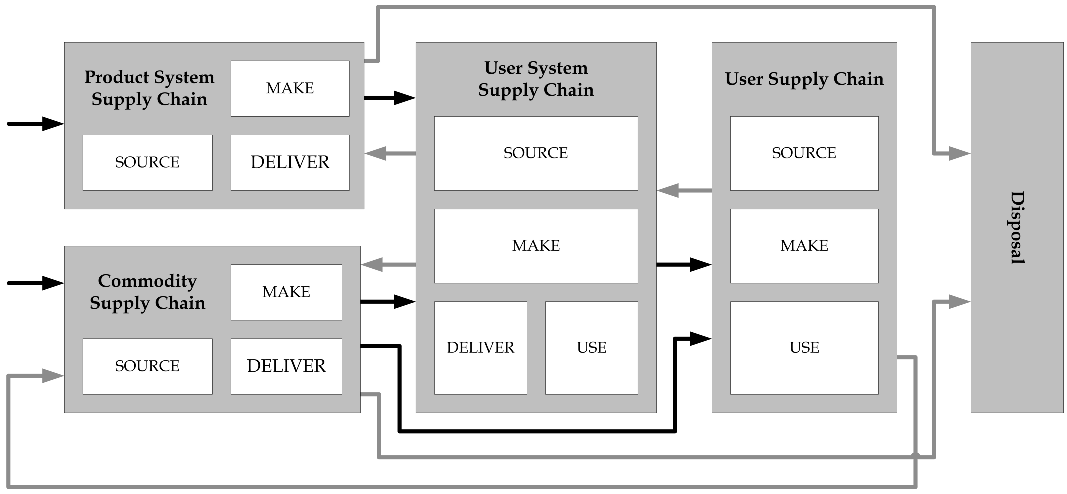


Figure 3.4: End-to-end supply chain processes used in simulation model.

The end-to-end military supply chain comprises the following five smaller supply chains [52]:

Product system supply chain focuses on the flow of product systems in the military, where the term 'product system' refers to the main equipment used to achieve a certain objective [12]. It is important to note that a product system cannot achieve the stated objectives on its own, as it is merely a piece of advanced equipment that must be integrated with the required commodities and personnel before it can be used to achieve the objectives. Examples of product systems include ships, tanks, and aircraft.

Commodity supply chain focuses on the flow of commodities, such as ammunition and medicine, in the military.

User system supply chain focuses on the supply chain in the military, where product systems and commodities are combined (sometimes referred to as commissioned), into user systems. Engelbrecht [12] comments that a user system is a purposeful system that can achieve its objective or output, since it is a product system integrated with the required commodities and personnel.

User supply chain focuses on the main user in the military supply chain. The flow of user systems and commodities to the user is described in this supply chain. Note that ammunition and user systems can also be used in the User system supply chain.

Disposal chain refers to the disposal chain of the military, where defective or obsolete product systems and commodities are removed from the supply chain.

Each of the smaller supply chains embedded in the entire end-to-end supply chain comprises various SCOR processes, adapted from the augmented SCOR model [3]. The various SCOR processes used in the supply chain simulation are the following:

SOURCE Activities and processes associated with the acquisition of items from suppliers inside or outside the military.

MAKE Activities associated with the production, assembly, and maintenance of items.

DELIVER Activities and processes associated with the delivery of items to 'customers'.

USE Activities and processes associated with the use of items in the military by the user.

The above mentioned supply chains and their associated SCOR processes are combined into the military supply chain simulation to investigate the ability of the supply chain, given the results of the inventory model, to provide the correct quantities of ammunition when required by the users.

When dealing with fuzzy quantities, uncertain variables are described by membership functions instead of probability distributions, implying that the possibility distributions must be transformed into probability distributions before they can be used during simulation. To address this problem Giannoccaro et al. [15] based on work done by Mizumoto and Tanaka [36] assumed that, for every uncertain value for which information on probability is unavailable, the most reasonable approach is to assume that the possible values are equally probable. Therefore uniform probability distribution functions are defined over the membership function domain of each fuzzy quantity in the simulation model.

3.4 Concluding remarks

There are many ways to incorporate uncertainty into inventory models, but two of the most common methods is to use stochastic or fuzzy programming. Various researches have also focused on combining these two methods into one mixed model and thereby exploiting the advantages of both techniques at once.

The most appropriate method identified for the military inventory model presented in the next chapter, is to combine stochastic and fuzzy programming into a multi-objective mixed inventory model. Goal programming will be used to take the multi-objectivity of the model into account. The stochastic component of the model will be addressed using recourse stochastic programming and dual decomposition, whereas the fuzzy component will be addressed using the approach of Jiménez et al. [24] to transform fuzzy constraints into crisp counterparts.

The derived deterministic counterpart model will then be solved using a standard optimisation software package, LINGO 10.0, and the results used as inputs into a simulation model of a military supply chain that will be used to test the reliability of the model and perform necessary sensitivity analyses.

We acknowledge that the solutions obtained from the multi-objective mixed inventory model cannot be seen as optimal to the problem, even though optimisation is used. However, the solutions will provide valuable inputs into the process of inventory management decision-making.

Chapter 4

Inventory model formulation

Funding available to a country's military may be severely limited. The model therefore aims to keep the total expenditure on purchases during a specified period as low as possible. As mentioned before, it is critical to have enough stock to meet demand without unnecessary escalating stock levels. The model therefore aims to satisfy the demand for ammunition in a way that minimises both shortages and surplus stock levels.

In some instances not all ordered items are delivered to the military: some may be kept in stock at the supplier and made available when required. A fee may be charged for each item that the supplier keeps in stock, so an appropriate balance should be achieved between the cost of holding an item in stock at military depots and the cost of keeping it in stock at the supplier.

A country's ammunition stockpile can typically be divided into various stock categories [62]. The categories directly relevant to the military are:

Training stock, required to support routine training of military forces.

Operational stock, required to support routine military operations, such as UN peacekeeping operations.

War reserve stock, reserved for less likely events such as peace enforcement operations as well as for high impact, low probability events such as war. War reserve stock can be described as the safety stock in the supply chain that can be used in the case of unforeseen events.

Stock awaiting disposal, in excess to the military requirement due to safety concerns, obsolescence, or damages.

For the purposes of this model three stock categories are used. Category A encompasses the stock used to train new forces and to sustain very likely operations, such as peacekeeping operations; category B defines the stock used exclusively to satisfy demand during less likely operations, such as peace enforcement operations; and category C is the stock reserved for use during unlikely or extreme events, such as war. As the focus of the dissertation is on usable stock, stock awaiting disposal will not be considered.

To avoid unnecessary ageing and degradation of stock, a FIFO approach is adapted for the flow of stock between the different stock categories. It is assumed that stock category A is replenished from stock category B, which in turn is replenished from stock category C. Finally, new orders are placed with the supplier to replenish stock category C. This ensures that older stock is used first.

The purpose of the mathematical model proposed in this chapter is to determine the stock level requirements amidst demand uncertainty due to extreme events, while attempting to minimise the total cost, total shortages, and minimum inventory level required over three specified scenarios derived from Chapters 6 to 8 of the Charter of the United Nations [58].

1. *Training and peace-keeping* – It is assumed that the military is involved in its routine training activities as well as a UN peace-keeping operation. Stock category A is used to meet the ammunition demand during this scenario since training and a very likely operation is taking place. The demand in this scenario is assumed to be a random variable.

2. *Peace enforcement* – In addition to training and a UN peace-keeping operation, it is assumed that a UN peace enforcement operation is also taking place. Stock categories A and B are used to meet ammunition demand during this scenario since training, a very likely, and a less likely operation are taking place. The demand for training and peace-keeping is assumed to be a random variable whereas the demand for peace enforcement is assumed to be a fuzzy quantity.

3. *War* – It is assumed that all the military's resources are used in the war taking place, therefore no additional training, peace-keeping or peace enforcement operations are taking place. Stock category C is used to meet ammunition demand during this scenario as an unlikely event is taking place. The demand during this scenario is assumed to be a fuzzy quantity.

The model determines the overall minimum inventory level for each stock category, the quantity to order for the replenishment of each stock category during time period t, and the amount that should be delivered by the supplier during each time period. Orders may be placed during any period, and the time between placing an order for each stock category and the earliest possible delivery date of that order is assumed to be constant. However, when the total available quantity in stock for each stock category is the same as or less than the minimum inventory level for a certain period, a new order must be placed during that period.

The membership functions for the fuzzy parameters in the model will in most instances be based on the intuition of experts in the military. When using a triangular membership function, experts are merely required to estimate the values that belong to the fuzzy parameter's domain by specifying the upper and lower limits, a_1 and a_3, and the value that best represents this fuzzy parameter, a_2 [15]. A trapezoidal membership function allows the expert to specify a range of values that best represent the fuzzy parameter, instead of only one value; and since a triangular membership function is merely a special case of a trapezoidal membership function, trapezoidal membership functions are chosen as the most appropriate membership functions for this particular inventory model.

To avoid unnecessary escalation in stock levels the possibility of a scenario occurring is also considered in the model. Due to the lack of historical data available to determine the probability that each of the scenarios will occur, fuzzy numbers are used instead. Experts can then also define trapezoidal membership functions for each of the possibilities and their defuzzification values can be determined — using equation (3.7) in Chapter 3 — as reasonable deterministic estimations of scenario possibilities.

This model assumes that all the ammunition received from the supplier and other stock categories is in an acceptable condition, therefore it will not be returned as a result of unacceptable quality. In addition this model assumes that the current force structure of a particular military remains constant, hence the number of user systems in use will not change. Consequently, no ammunition is required in this model for the commissioning of additional user systems.

The presence of uncertainty in the model can be dealt with by planning for a recourse option used to remedy shortages or surpluses that may occur during a time period as a result of demand uncertainty. The recourse in this case is to carry shortage or surplus values over from one period to the next and remedy theses values by ordering more or less during the next period to compensate for the previous period's lack or excess.

4.1 Notation

To determine the stock levels, we define some decision variables, utility variables, and parameters. The notation used to represent these variables and parameters in the model formulation are shown and discussed in this section. To distinguish between the random and fuzzy components of the model, random quantities are marked with a tilde, $\widetilde{A}$, and fuzzy quantities with a hat, $\widehat{A}$.

4.1.1 Decision variables

Decision variables are the variables driving the decisions in a mathematical model and can be seen as the primary information required by the decision-maker to make an informed decision.

$S_i \triangleq$ the minimum stock level at which reordering should occur for stock category $i \in \boldsymbol{I} = \{1, 2, 3\}$ where

$$i \triangleq \begin{cases} 1 & \text{stock category A} \\ 2 & \text{stock category B} \\ 3 & \text{stock category C.} \end{cases}$$

$Q_{i,t} \triangleq$ the order quantity of stock category $i \in \boldsymbol{I}$ during time period $t \in \boldsymbol{T} = \{0, 1, \ldots, T\}$.

$D_{i,t} \triangleq$ the quantity of stock category $i \in \boldsymbol{I}$ delivered as replenishment during time period $t \in \boldsymbol{T}$.

4.1.2 Utility variables

Utility variables are not directly required by the decision-maker to make informed decisions, but they are essential to the functionality of the model.

The $\{\widetilde{\xi}_{t'}\}_{t'=1}^{t}$ symbol is used in some utility variables to model the dependence between the random demand of different periods. For instance, if $t = 3$ then $\{\widetilde{\xi}_{t'}\}_{t'=1}^{t}$ will signify the demand during period 1 through 3, such that $\{\widetilde{\xi}_{t'}\}_{t'=1}^{t} = \{\widetilde{\xi}_{t'}\}_{t'=1}^{3} = \{\widetilde{\xi}_1, \widetilde{\xi}_2, \widetilde{\xi}_3\}$. A distinction between T and T_1 is made in the model, with $T \in \{0, 1, \ldots, T\}$ and $T_1 \in \{1, 2, \ldots, T\}$.

$O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}_{t'=1}^{t}\right) \triangleq$ the total number of stock category A items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$ as a function of random training and peace-keeping demand $\widetilde{\xi}_t$.

$O^B_{s,t} \triangleq$ the total number of stock category B items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$.

$O^C_{s,t} \triangleq$ the total number of stock category C items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$.

$U^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right) \triangleq$ the total number of stock category A items, in scenario $s \in \boldsymbol{S}$, in excess at the end of time period $t \in \boldsymbol{T}$ as a function of random demand $\widetilde{\xi}_t$.

$U^B_{s,t} \triangleq$ the total number of stock category B items, in scenario $s \in \boldsymbol{S}$, that are in surplus at the end of time period $t \in \boldsymbol{T}$.

$U^C_{s,t} \triangleq$ the total number of stock category C items, in scenario $s \in \boldsymbol{S}$, that are in surplus at the end of time period $t \in \boldsymbol{T}$.

$N_t \triangleq$ the quantity that is available at the supplier, but not yet delivered, as replenishment of stock category C at the end of time period $t \in \boldsymbol{T}$.

$x_{i,t} \triangleq$ a binary variable that determines if a replenishment order for stock category $i \in \boldsymbol{I}$ is placed during time period $t \in \boldsymbol{T}$.

$d^c \triangleq$ the amount by which the actual cost exceeds its goal.

$d^d \triangleq$ the amount by which demand satisfaction falls short of its goal.

$d^s \triangleq$ the amount by which the specified minimum stock exceeds its goal.

4.1.3 Parameters

Parameters are known values that should be taken into account in the model.

$\widetilde{\xi}_t \triangleq$ random vector describing the demand for training and peacekeeping operations during time period $t \in \boldsymbol{T}$.

$\widehat{\eta} \triangleq$ a trapezoidal fuzzy number with parameters $(\eta_1, \eta_2, \eta_3, \eta_4)$ describing the demand in one time period during a peace enforcement operation.

$\widehat{\delta} \triangleq$ a trapezoidal fuzzy number with parameters $(\delta_1, \delta_2, \delta_3, \delta_4)$ describing the demand in one time period during war.

$\rho^p \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho^p_1, \rho^p_2, \rho^p_3, \rho^p_4)$, describing the possibility that training and peace-keeping may be taking place during a time period.

$\rho^e \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho^e_1, \rho^e_2, \rho^e_3, \rho^e_4)$, describing the possibility that a peace enforcement operation may be taking place during a time period.

$\rho^w \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho^w_1, \rho^w_2, \rho^w_3, \rho^w_4)$, describing the possibility that the country may be at war during a time period.

$Q^O_{i,t} \triangleq$ the initial quantities of orders for stock category $i \in \boldsymbol{I}$, due to orders placed during previous time periods, to be received during time period $t \in \boldsymbol{T}$.

$L_i \triangleq$ the lead time between the placing of an order for stock category $i \in \boldsymbol{I}$ and its delivery.

$F_i \triangleq$ the fixed cost of placing a replenishment order for stock category $i \in \boldsymbol{I}$.

$M_i \triangleq$ the maximum replenishment order size of stock category $i \in \boldsymbol{I}$ that is allowed.

$H \triangleq$	the cost of holding one item in inventory for one time period.
$A^C \triangleq$	the maximum surplus stock quantity of a particular item that can be stored in all the available depots.
$C \triangleq$	the purchasing cost per item at the supplier.
$V \triangleq$	the cost per item for keeping the item in stock at the supplier.
$\alpha^R \triangleq$	factor that allows for the inflation of stock levels if the region is considered to be unstable.
$\alpha^W \triangleq$	factor that allows for the inflation of stock levels if the world is considered to be unstable – from a military point of view.
$\alpha^D \triangleq$	factor that allows for the inflation of stock levels to deter potential threats.
$g^c \triangleq$	the target value for total cost.
$g^d \triangleq$	the target value for total demand satisfaction.
$g^s \triangleq$	the target value for the total minimum stock level.

4.2 Stochastic program with recourse and fuzzy parameters

The objective of the model (4.1) minimises the total deficiency in reaching the three goals.

$$\min \Bigg\{ d^c + d^d + d^s + \epsilon \Bigg[\sum_{t \in \boldsymbol{T_1}} \Bigg[H \sum_{s \in \boldsymbol{S}} \Big(E_{\tilde{\xi}_t} \Big[U^A_{s,t}\Big(\{\tilde{\xi}_{t'}\}^t_{t'=1} \Big) \Big] + U^B_{s,t} + U^C_{s,t} \Big) + \sum_{i \in \boldsymbol{I}} F_i x_{i,t} \tag{4.1}$$

$$+ V N_t + C Q_{3,t} \Bigg] + \sum_{t \in \boldsymbol{T_1}} \sum_{s \in \boldsymbol{S}} \Big(E_{\tilde{\xi}_t} \Big[O^A_{s,t}\Big(\{\tilde{\xi}_{t'}\}^t_{t'=1} \Big) \Big] + O^B_{s,t} + O^C_{s,t} \Big) + \sum_{i \in \boldsymbol{I}} S_i \Bigg] \Bigg\}$$

To ensure that an efficient solution[1] can be achieved, a small portion of the three goals of the model is added to the basic objective function [45], ϵ is therefore defined as a sufficiently small number.

The goals of the *stochastic model with recourse* are incorporated as constraints. The first goal, (4.2), is to minimise the total cost, and has the following components:

1. The cost of keeping the surplus stock category A, B and C items in stock over the entire time horizon, $H\Big(E_{\tilde{\xi}_t}\Big[U^A_{s,t}\Big(\{\tilde{\xi}_{t'}\}^t_{t'=1} \Big) \Big] + U^B_{s,t} + U^C_{s,t} \Big)$, where the total expected value number of stock category A items in surplus over the time horizon is denoted by $E_{\tilde{\xi}_t}\Big[U^A_{s,t}\Big(\{\tilde{\xi}_{t'}\}^t_{t'=1} \Big) \Big]$;

2. The total cost of placing the replenishment orders for each stock category, $F_i x_{i,t}$;

3. The total cost of the fee paid to the supplier for holding stock at its facilities over the time horizon, VN_t; and

4. The total purchase cost for new items ordered from the supplier, $CQ_{3,t}$.

The second goal, (4.3), is incorporated to minimise the total number of shortages over the specified time horizon, and consists of the expected number of shortages for stock

[1] Rardin [45] states that a feasible solution is efficient if no other feasible solutions score at least equally well in all the goals of the model and better in one of the model goals.

category A, $E_{\widetilde{\xi}_t}\left[O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)\right]$ and the total number of shortages for stock categories B and C, $O^B_{s,t}$ and $O^C_{s,t}$, respectively.

The third goal, (4.4), minimises the total minimum stock level, S_i, for each of the three scenarios.

$$\sum_{t\in \boldsymbol{T_1}}\left[H\sum_{s\in \boldsymbol{S}}\left(E_{\widetilde{\xi}_t}\left[U^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)\right]+U^B_{s,t}+U^C_{s,t}\right)+\sum_{i\in \boldsymbol{I}}F_i x_{i,t}+VN_t+CQ_{3,t}\right]-d^c\leq g^c \tag{4.2}$$

$$\sum_{t\in \boldsymbol{T_1}}\sum_{s\in \boldsymbol{S}}\left(E_{\widetilde{\xi}_t}\left[O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)\right]+O^B_{s,t}+O^C_{s,t}\right)-d^d\leq g^d \tag{4.3}$$

$$\sum_{i\in \boldsymbol{I}}S_i-d^s\leq g^s \tag{4.4}$$

Each of these constraints determines the total deviations in reaching the goals, which are essential considerations in the objective function of the model.

To assign the shortage and surplus values for each stock category, equations (4.5) through (4.9) are used. In all instances, the shortage and surplus values are calculated by subtracting the number of items moved out of a stock category during a time period from the shortage or surplus at the start of that time period, and adding the number of items delivered as replenishment of that stock category during the same time period.

For the peacekeeping and peace enforcement scenarios, equation (4.5) determines the surplus and shortage quantities for stock category A, $U^A_{1,t}$ and $O^A_{1,t}$, at the end of each time period by subtracting the uncertain demand during a time period, $\widetilde{\xi}_t$, from the initial shortage and surplus values, $U^A_{1,t-1}$ and $O^A_{1,t-1}$, and adding the quantity delivered during that time period, $D_{1,t}$. The shortage and surplus values for stock category A during each time period are dependent on the realisations of the uncertain demand during previous time periods, hence the use of the notation $\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)$ for these shortage and surplus values.

$$U^A_{s,t-1}\left(\{\widetilde{\xi}_{t'}\}^{t-1}_{t'=1}\right)-O^A_{s,t-1}\left(\{\widetilde{\xi}_{t'}\}^{t-1}_{t'=1}\right)+D_{1,t}-\widetilde{\xi}_t=U^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)-O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)$$
$$\forall \quad t\in \boldsymbol{T_1}=\{1,\ldots,T\},\ s\in\{1,2\} \tag{4.5}$$

Shortage and surplus quantities for stock category B for the peacekeeping scenario are assigned using equation (4.6). The quantity of items moved out of stock are determined as the quantity of items ordered from stock category A for each time period.

During the peace enforcement scenario the fuzzy demand for a peace enforcement operation, $\widehat{\eta}$, is also subtracted as part of the number of items moved out of stock category B, as shown in equation (4.7).

$$U^B_{1,t-1}-O^B_{1,t-1}+D_{2,t}-Q_{1,t}=U^B_{1,t}-O^B_{1,t}\quad\forall\quad t\in \boldsymbol{T_1} \tag{4.6}$$

$$U^B_{2,t-1}-O^B_{2,t-1}+D_{2,t}-\widehat{\eta}-Q_{1,t}\ =\ U^B_{2,t}-O^B_{2,t}\quad\forall\quad t\in \boldsymbol{T_1} \tag{4.7}$$

Stock category C shortage and surplus quantities are assigned in equations (4.8) and (4.9). For the peacekeeping and peace enforcement scenarios, the quantity of items moved out of stock category C are determined as the quantity of items ordered from stock category B during each time period. In the war scenario only stock category C is taken into account, as it is assumed that all available resources are reassigned to focus on the war. The initial ($t = 0$) quantity in stock for constraint (4.9) is recalculated as the sum of the initial quantities for stock categories A, B, and C. The number of items moved out is set to the fuzzy demand during war, $\widehat{\delta}$.

$$U^C_{s,t-1} - O^C_{s,t-1} + D_{3,t} - Q_{2,t} = U^C_{s,t} - O^C_{s,t} \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \{1, 2\} \tag{4.8}$$

$$U^C_{3,t-1} - O^C_{3,t-1} + D_{3,t} - \widehat{\delta} \ = \ U^C_{3,t} - O^C_{3,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{4.9}$$

Equations (4.10) and (4.11) are used to determine the number of items delivered as replenishment of stock categories A and B. The replenishment before the lead time has elapsed is determined as the initial outstanding orders, $Q^O_{i,t}$, whereas the number of items delivered is calculated as the ordered quantities after the lead time has elapsed, $Q_{i,t-L_i}$.

$$D_{i,t} = Q^O_{i,t} \quad \forall \quad i \in \{1, 2\},\ t \in \{1, \ldots, L_i\} \tag{4.10}$$

$$D_{i,t} = Q_{i,t-L_i} \quad \forall \quad i \in \{1, 2\}, t \in \{L_i + 1, \ldots, T\} \tag{4.11}$$

To determine the quantity to keep in storage at the supplier and the number of stock category C items to deliver during a time period, constraints (4.12) and (4.13) are incorporated. The quantity to keep in stock at the supplier, N_t, is calculated by adding either the initial orders outstanding or the quantity ordered from the supplier, depending on the time period, and subtracting the quantity delivered as replenishment of stock category C, $I_{3,t}$.

$$N_t = N_{t-1} + Q^O_{3,t} - D_{3,t} \quad \forall \quad t \in \{1, \ldots, L_3\} \tag{4.12}$$

$$N_t = N_{t-1} + Q_{3,t-L_3} - D_{3,t} \quad \forall \quad t \in \{L_3 + 1, \ldots, T\} \tag{4.13}$$

Equations (4.14) through (4.16) are incorporated to allocate values to the minimum stock level for each category of stock.

The minimum stock level for stock category A should be greater than the uncertain demand over the lead time. This is valid for all the scenarios where this stock category is used. The minimum stock level for stock category A is therefore assigned as shown in equation (4.14).

For the peacekeeping scenario the minimum stock level for stock category B should be greater than the quantity ordered from this stock category as replenishment of stock category A over the lead time. This also holds for the peace enforcement scenario, but in this scenario the minimum stock level for stock category B should also be greater than the estimated demand for a peace enforcement operation over the lead time. To ensure that both these scenario specific requirements are taken into account when determining the minimum stock level for stock category B, each requirement are weighed with the possibility that its associated scenario will occur — As shown in equation (4.15).

When considering the peacekeeping and peace enforcement scenarios, the minimum stock level for stock category C should be greater than the quantity ordered over the lead time from this stock category as replenishment of stock category B. For the war scenario only stock category C is considered, therefore the minimum stock level for this scenario should be greater than the estimated demand for war over the lead time. Both these scenario specific requirements are considered when determining the minimum stock level for stock category C, by weighing each scenario specific requirement with the possibility that its associated scenario will occur.

Inflation factors are also included to allow the user to inflate stock category C levels when instabilities in the region or world are detected, or when it is required to deter potential threats and enemies.

$$S_1 \geq \left(\rho^p + \rho^e\right) \sum_{j=t}^{t+L_1-1} \widetilde{\xi}_j \quad \forall \quad t \in \{1, \ldots, T - L_1 + 1\} \tag{4.14}$$

$$S_2 \geq \rho^p \left(\sum_{j=t}^{t+L_2-1} Q_{1,j} \right) + \rho^e \left(L_2 \widehat{\eta} + \sum_{j=t}^{t+L_2-1} Q_{1,j} \right) \quad \forall \quad t \in \{1, \ldots, T - L_2 + 1\} \tag{4.15}$$

$$S_3 \geq \alpha^R \alpha^W \alpha^D \left(\left(\rho^p + \rho^e\right) \sum_{j=t}^{t+L_3-1} Q_{2,j} + \rho^w \left(L_3 \widehat{\delta} \right) \right) \quad \forall \quad t \in \{1, \ldots, T - L_3 + 1\} \tag{4.16}$$

Equation (4.17) is used to ensure that the quantity ordered falls below the maximum replenishment order size, M_i, for each of the three stock categories.

$$Q_{i,t} \leq M_i x_{i,t} \quad \forall \quad t \in \boldsymbol{T_1}, \; i \in \boldsymbol{I} \tag{4.17}$$

Equations (4.18) through (4.20) are used to determine if an order is placed during a time period. These constraints ensure that a replenishment order is placed whenever the current stock level for each stock category falls below the minimum stock level for that stock category.

$$S_1 - U^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right) + O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right) \leq Q_{1,t} \quad \forall \quad t \in \boldsymbol{T_1}, \; s \in \{1,2\} \tag{4.18}$$

$$S_2 - U^B_{s,t} + O^B_{s,t} \leq Q_{2,t} \quad \forall \quad t \in \boldsymbol{T_1}, \; s \in \{1,2\} \tag{4.19}$$

$$S_3 - U^C_{s,t} + O^C_{s,t} \leq Q_{3,t} \quad \forall \quad t \in \boldsymbol{T_1}, \; s \in \boldsymbol{S} \tag{4.20}$$

To ensure that the available depot storage capacity for a particular item is not exceeded at any time, constraint (4.21) is incorporated. This constraint implies that the total surplus stock of all three stock categories should be less than the maximum storage capacity, for the item in question, during each time period.

$$U^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right) + U^B_{s,t} + U^C_{s,t} \leq A^C \quad \forall \quad t \in \boldsymbol{T_1}, \; s \in \boldsymbol{S} \tag{4.21}$$

To ensure that certain variables are positive, some variables are binary, and some variables are integer values, constraints (4.22) through (4.28) are included in the model.

$$U_{s,t}^{A}\left(\{\widetilde{\xi}_{t'}\}_{t'=1}^{t}\right), O_{s,t}^{A}\left(\{\widetilde{\xi}_{t'}\}_{t'=1}^{t}\right) \geq 0 \quad \forall \quad t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{4.22}$$

$$U_{s,t}^{B}, O_{s,t}^{B}, U_{s,t}^{C}, O_{s,t}^{C} \geq 0 \quad \forall \quad t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{4.23}$$

$$x_{i,t} \in \{0,1\} \quad \forall \quad i \in \boldsymbol{I},\ t \in \boldsymbol{T} \tag{4.24}$$

$$Q_{i,t}, D_{i,t} \geq 0 \text{ and integer} \quad \forall \quad i \in \boldsymbol{I},\ t \in \boldsymbol{T} \tag{4.25}$$

$$S_i \geq 0 \text{ and integer} \quad \forall \quad i \in \boldsymbol{I} \tag{4.26}$$

$$N_t \geq 0 \text{ and integer} \quad \forall \quad t \in \boldsymbol{T} \tag{4.27}$$

$$d^c, d^d, d^s \geq 0 \tag{4.28}$$

4.3 Conclusion

In this chapter a mathematical model for determining the stock level of a single item in the military is proposed. The model should provide the decision-maker with a structured and reliable way in which the stock levels for items in the military are determined.

The model considers three different scenarios — training and peacekeeping, peace enforcement and war — and determines the minimum required stock level for three different stock categories of a single item. The main aims of the model are to minimise the total cost, while minimising the total number of shortages as well as the stock level required for each stock type over a specified time period.

The value of the model lies in its ability to determine the required amount of serviceable ammunition to be kept in stock by a military force. This model is designed as a tool for inventory management, and should be used in conjunction with a proper life-cycle management tool to aid the decision-maker with inventory management decisions.

Chapter 5

Inventory model solution

In this chapter we discuss how to deal with the mixed inventory model presented in Chapter 4. Fuzzy and stochastic constraints are transformed into their crisp counterparts using the approach of Jiménez et al. [24] and the recourse approach [25].

5.1 Transformation of fuzzy constraints

The first step is to transform the fuzzy constraints into their crisp counterparts. To illustrate how this is achieved, refer to fuzzy inventory constraint (4.7) in Chapter 4, repeated here in (5.1).

$$U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \widehat{\eta} - Q_{1,t} \quad = \quad U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{5.1}$$

Using the approach proposed by Jiménez et al. [24], constraint (5.1) can be rewritten as

$$\begin{aligned} U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1 - \frac{\alpha}{2}\right) D^{l_\eta} + \frac{\alpha}{2} D^{u_\eta}\right] - Q_{1,t} \quad &\geq \quad U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \\ U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1 - \frac{\alpha}{2}\right) D^{u_\eta} + \frac{\alpha}{2} D^{l_\eta}\right] - Q_{1,t} \quad &\leq \quad U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1}. \end{aligned}$$

Recall from Section 3.2.2 in Chapter 3 that

$$D^{l_A} = \frac{a_1 + a_2}{2} \text{ and } D^{u_A} = \frac{a_3 + a_4}{2}.$$

Therefore the crisp version of fuzzy constraint (5.1) is obtained as shown in equations (5.2) and (5.3).

$$U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1-\frac{\alpha}{2}\right)\frac{\eta_1+\eta_2}{2} + \frac{\alpha}{2}\left(\frac{\eta_3+\eta_4}{2}\right)\right] - Q_{1,t} \;\geq\; U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{5.2}$$

$$U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1-\frac{\alpha}{2}\right)\frac{\eta_3+\eta_4}{2} + \frac{\alpha}{2}\left(\frac{\eta_1+\eta_2}{2}\right)\right] - Q_{1,t} \;\leq\; U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{5.3}$$

A similar approach is used to convert the other fuzzy constraints in the model into their crisp counterparts. Once this is achieved, the resulting model can be seen as a crisp stochastic model with recourse that can be transformed into its deterministic form and solved using standard optimisation software packages.

5.2 Deriving the dual decomposition structure

To transform the stochastic program with recourse into its deterministic version, a multi-stage fixed recourse approach is used to take the stochastic demand into account. In this case the chosen recourse option is to accumulate shortage and surplus values from one period to the next to ensure that enough stock is ordered during a period to cover the shortages incurred during the previous period or that less stock is ordered due to the surplus carried over from the previous period.

To illustrate how the *dual decomposition structure* is derived refer to the minimum shortage constraint (4.3) in Chapter 4, repeated here in (5.4).

$$\sum_{t\in\boldsymbol{T_1}}\sum_{s\in\boldsymbol{S}}\left(E_{\widetilde{\xi}_t}\left[O^A_{s,t}\left(\{\widetilde{\xi}_{t'}\}^t_{t'=1}\right)\right] + O^B_{s,t} + O^C_{s,t}\right) - d^d \leq g^d \tag{5.4}$$

Random demand for training and peacekeeping during time period t is rewritten as $\xi_t^{\beta_t}$, denoting the β_t^{th} realisation of the random training and peacekeeping demand, which has a finite discrete distribution $\left\{(\xi_t^{\beta_t}, p_t^{\beta_t}), \beta_t \in \boldsymbol{\beta} = \{1,\ldots,R\}\right\}$.

In addition, the realisations of the random variables during the different time periods applicable to each expression are rewritten as $\{\xi_{t'}^{\beta_{t'}}\}^t_{t'=1}$. For instance, the shortages during scenario 1 at time 2 would be depicted as $O^A_{1,2}\left(\xi_1^{\beta_1}, \xi_2^{\beta_2}\right)$. The resulting *dual decomposition structure* for constraint (5.4) is depicted by equation (5.5).

$$\sum_{t\in\boldsymbol{T_1}}\sum_{s\in\boldsymbol{S}}\left[\sum_{\{\beta_{t'}\}^t_{t'=1}}\left(\prod_{t'=1}^{t} p_t^{\beta_{t'}}\right)\left[O^A_{s,t}\left(\{\xi_{t'}^{\beta_{t'}}\}^t_{t'=1}\right)\right] + O^B_{s,t} + O^C_{s,t}\right] - d^d \leq g^d \tag{5.5}$$

If the number of time periods in the model is set to $T = 2$, the full notation of the dual decomposition structure depicted in equation (5.5) is shown in equation (5.6). In this constraint the surplus values for the different scenarios are added and all possible realisations of time periods 1 and 2 taken into account.

$$\sum_{\beta_1 \in \boldsymbol{\beta}} p_1^{\beta_1} \Big[O_{1,1}^A(\xi_1^{\beta_1}) + O_{2,1}^A(\xi_1^{\beta_1}) + O_{3,1}^A(\xi_1^{\beta_1}) \Big] + \sum_{\beta_1 \in \boldsymbol{\beta}} \sum_{\beta_2 \in \boldsymbol{\beta}} p_1^{\beta_1} p_2^{\beta_2} \Big[O_{1,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2})$$
$$+ O_{2,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2}) + O_{3,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2}) \Big] + O_{1,1}^B + O_{1,1}^C + O_{2,1}^B + O_{2,1}^C + O_{3,1}^B + O_{3,1}^C$$
$$+ O_{1,2}^B + O_{1,2}^C + O_{2,2}^B + O_{2,2}^C + O_{3,2}^B + O_{3,2}^C - d^d \leq g^d \tag{5.6}$$

Consider the minimising shortages constraint for Scenario 1 depicted by (5.6). The total number of shortages for stock category A during time period 1 is dependent on the realisation of random demand for each of the scenarios during time period 1, i.e. $O_{1,1}^A(\xi_1^{\beta_1}), O_{2,1}^A(\xi_1^{\beta_1})$ and $O_{3,1}^A(\xi_1^{\beta_1})$, whereas the total number of shortages for stock category A during time period 2 is dependent on the realisation of random demand for each of the scenarios during time periods 1 and 2, i.e. $O_{1,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2}), O_{2,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2})$ and $O_{3,2}^A(\xi_1^{\beta_1}, \xi_2^{\beta_2})$. In addition, the probabilities of the realisation of random demand during time period 1 and 2 are depicted by $p_1^{\beta_1}$ and $p_2^{\beta_2}$, respectively.

The resulting full crisp dual decomposition structure of the fuzzy stochastic multi-objective inventory model is presented in Appendix A. The resulting deterministic inventory model can then be solved using exact branch-and-bound embedded in the LINGO 10.0 optimisation software package. Once the solution is obtained, its reliability is tested using the Arena 12.00.00 discrete event simulation software package.

5.3 Testing the model's reliability

In the military, commodity availability is easily affected — and product system availability fairly unaffected — by short-term demand uncertainty in the supply chain. As product systems are used to expand the capability of a military's forces in the long term, new product systems will typically not be purchased as a result of short-term demand uncertainty. Therefore, the focus of the simulation model presented in this chapter is on measuring the ability of an end-to-end supply chain to provide the correct quantity of commodities required by the military at the time required.

The simulation model that is used to test the reliability of the model solutions allows the user to determine how the supply chain will react in different scenarios. Various scenarios can be taken into account; but for the purposes of this dissertation, the three main scenarios presented in Chapter 4 will be considered.

The military supply chain simulation model is based on the augmented Supply Chain Operations Reference (SCOR) model for the military proposed by Bean et al. [3] and the end-to-end military supply chain structure proposed by Schmitz et al. [52]. Section 3.3 in Chapter 3 describes the various supply chains in the end-to-end military supply chain structure; and from Figure 3.4, the flow of ammunition throughout the end-to-end military supply chain can be summarised as follows:

- The *user system supply chain* orders ammunition for use by the supply chain and product systems, with their required ammunition, from the *commodity* and *product system supply chains.*
- A user system is then commissioned in the *user system supply chain* by integrating a product system with its required ammunition. The integration of personnel with

product systems is excluded from the simulation, because the focus of the simulation is on commodity availability, and it is assumed that the required personnel will be available at all times.

- The resulting user system is either used in the *user system supply chain* or sent to meet the demand of the *user supply chain*.
- Additional ammunition required for use in the *user supply chain* is ordered from the *commodity supply chain*, whereas all required user systems are ordered from the *user system supply chain*.
- If there are excess user systems in the *user supply chain*, they are returned to the *user system supply chain* where the excess user systems are decommissioned into product systems and ammunition.
- The resulting product systems and ammunition are returned to the *product system and commodity supply chains* respectively.
- When too much ammunition is ordered for use by the *user system* and *user supply chains*, the excess amount is returned to the *commodity supply chain*.
- Any returned items are inspected, and all defective items are sent to the *disposal chain*.

Even though the reverse supply chain is not explicitly taken into account in the inventory model presented in Chapter 4 because the model assumes that all items received by the supply chains are in an acceptable condition - it is taken into account in the military supply chain simulation. It is argued that the reverse supply chain forms part of the end-to-end military supply chain, and will have an effect on the ability of the supply chain to provide the required commodities. Therefore it needs to be taken into account when testing the reliability of the inventory model.

A detailed description of each subset supply chain in the simulation model is provided in the remainder of this section. The reader is referred to Appendix B for various flowcharts depicting the processes, decisions and flows used in the simulation model for each of these subsets.

5.3.1 Product system supply chain

The first supply chain, the *product system supply chain*, is partially modelled in this simulation model. For this supply chain a partial SOURCE process, a partial MAKE process, and a DELIVER process are modelled. The design, development, and production of products systems are not included directly in the simulation, as it focuses on testing the ability of the supply chain to provide the required ammunition.

SOURCE Product system

The SOURCE process of the *product system supply chain* is not completely modelled, as the focus is on testing the reliability of the inventory model in different scenarios. However, provision is made for the acquisition of new product systems as they are required.

The SOURCE process is initiated when the available quantity of product systems is insufficient to meet the need of the user system supply chain. Once initiated, it is decided whether additional product systems should be ordered. If the decision is to place a new order for additional product systems, they are requested and received after a certain waiting time. The product systems are verified, and if it is determined that a product system is in an acceptable condition, it is transferred to storage; otherwise it is returned to the supplier.

MAKE Product system

The MAKE process for product systems focuses on the inspection and maintenance of returned product systems from the *user system supply chain*. This process is initiated when product systems are no longer required, where after the product system is decommissioned and returned. Once returned, the product system is inspected, and if it is found that it should be disposed of, it is sent to the *disposal chain*. Product systems not identified for disposal are maintained and transferred back into storage.

DELIVER Product system

The DELIVER product system process focuses on preparing the required product systems for delivery and delivering them to the *user system supply chain*.

The DELIVER process is initiated when a request for product systems is received from the *user system supply chain*. Either the full number of product systems required, or the available number of product systems in storage, depending on availability, is received and prepared for delivery. The prepared product systems are shipped to the *user system supply chain*. Finally, an order for the amount of ammunition required to commission a user system is placed with the *commodity supply chain*.

5.3.2 Commodity supply chain

The second supply chain modelled as part of the simulation is the *commodity supply chain*, in particular the sourcing, assembling, maintaining, and delivery of ammunition. As the focus of this simulation is on the availability of ammunition in the supply chain, the SOURCE process is modelled in detail, based on the inventory model presented in Chapter 4. The MAKE process for ammunition focuses on the assembly of items and on the maintenance of items returned from the *user system* and *user supply chains*. The DELIVER process focuses on delivering commodities to the *user system* and *user supply chains* based on their requirements.

SOURCE Commodities

This process is divided into three sub-processes, focusing on the three stock types. In the stock category A sub-process, the stock level is reviewed after a specified time period, and if the current stock level is the same as or below the minimum stock level, as determined by the model presented in Chapter 4, a replenishment order is placed. The order is replenished for stock categories B and C, depending on availability. After a certain waiting time, the order is delivered, inspected, and transferred to storage until it is required by another supply chain.

The stock category B sub-process of SOURCE is similar to the stock category A sub-process. However, stock category B replenishments can only be ordered from stock category C.

The stock category C sub-process determines whether an order for new stock should be placed with the supplier. If the supplier has stock in storage, the replenishment is ordered from available stock, otherwise a new order is placed at the supplier. After a certain waiting time, the order is received from the supplier and verified, and acceptable ammunition is transferred to storage. Unacceptable ammunition is returned to the supplier.

MAKE Commodities

The MAKE process of the *commodity supply chain* can be initiated by four events. The first event is when returned ammunition is received from the *user system supply chain*. The ammunition is verified, and if it is found to be defective it is sent to the *disposal supply chain*; otherwise it is maintained and returned to storage as stock category A.

The remaining three events that can initiate the MAKE process are when an order for ammunition is received from the *user system supply chain*, when an order for ammunition is received from the *user supply chain*, or when an order for ammunition arising from a product system commissioning requirement is received from the *product system supply chain*.

In each of these cases the required amount of ammunition is requested from stock category A, B, and C, depending on stock availability, when the order is received. The requested ammunition is then assembled and released to the DELIVER process of the *commodity supply chain*.

When the required quantity of stock for the specific ammunition type is available when requested, the supply chain is regarded as reliable; and when the requested quantity is unavailable, the supply chain is regarded as unreliable. The reliability of the model is calculated during this process, as shortages in stock in the *commodity supply chain* can have an adverse effect on the reliability of the entire end-to-end supply chain.

The *days-of-supply* measure, also calculated during this process, indicates how long the supply chain is able to supply the specific ammunition type to areas where it is required before the first shortage is experienced after a scenario started. For instance, a *days-of-supply* measure of 10 implies that the supply chain will be able to fulfill the requirements for that ammunition type for 10 days before experiencing its first stock-out.

The *days-of-supply* counter increases every time an order is met without any shortages. However, as soon as any shortage is experienced, the counter is terminated.

DELIVER Commodities

The DELIVER commodity process is initiated when ammunition in the MAKE process is released for delivery. Once the process is initiated the required ammunition is picked, packaged, loaded and delivered to the appropriate supply chain.

5.3.3 User system supply chain

The third supply chain, represented as a subset of the simulation, is the *user system supply chain*. The SOURCE process of the supply chain focuses on ordering product systems from the *product system supply chain* and ammunition from the *commodity supply chain*, as well as receiving and verifying deliveries of these orders. The MAKE process focuses on combining the product systems and ammunition into a user system, and the DELIVER process covers the delivery of user systems to the *user supply chain*. Finally, the USE process covers the delivery of product systems for use in the *user system supply chain*.

SOURCE User system

The SOURCE process is initiated for the *user system supply chain* when a product system or ammunition is received from the *product system supply chain* or the *commodity supply chain*. The received item is verified, and if it is in an acceptable condition, the item is released to the MAKE process. If it is found that the item is in an unacceptable condition, the item is returned to the relevant supply chain.

Ammunition received for use in the *user system supply chain* is placed into storage until required, instead of being released to the MAKE process.

MAKE User system

In this process the product systems and the ammunition required to commission these products systems are integrated into user systems. These user systems are either released for delivery to the *user supply chain* or released for use in the *user system supply chain*.

DELIVER User system

The DELIVER process initiates when the user system is released for delivery from the MAKE process. The user system is received and prepared for delivery, where after it is shipped to the *user supply chain*.

USE User system

The USE process covers the use of user systems and ammunition in the *user system supply chain*, and determines when and how much ammunition to order from the *commodity supply chain* to replenish used ammunition.

This process also determines the new user system requirement of the *user system supply chain*. If additional user systems are required, the additional requirement is sent to the *product system supply chain*. If the number of user systems in use is more than required, the excess number is decommissioned, and the resulting excess product systems and ammunition returned to the appropriate supply chains.

5.3.4 User supply chain

The final subset of the simulation is the SOURCE process of the user supply chain. Since the main focus of the simulation is to test the reliability of the results provided by the inventory model, based on ammunition availability in the supply chain, the SOURCE process is the only aspect directly taken into account in the simulation. The SOURCE process covers the ordering and receiving of ammunition from the commodity supply chain and user systems from the user system supply chain.

SOURCE User

The SOURCE process of the *user supply chain* is initiated on a regular basis to determine how much ammunition was used during a certain time period. A replenishment order for the used ammunition is placed with the *commodity supply chain*. When the ammunition is received it is verified, and the ammunition in an acceptable condition is transferred to storage. Ammunition in an unacceptable condition is returned to the *commodity supply chain*.

The requirements for user systems in the *user supply chain* are tracked on an annual basis. If additional user systems are required, an order is placed with the *user system supply chain*. The additional user systems are inspected and verified upon delivery, and unacceptable user systems are returned. User systems in an acceptable condition are released for use by the *user supply chain*.

5.4 Concluding remarks

This chapter focuses on the approach used to derive the deterministic equivalent of the inventory model presented in Chapter 4. The fuzziness in the model is addressed by adopting the approach proposed by Jiménez et al. [24] to derive the crisp equivalents of fuzzy constraints. The randomness of the model is addressed by using the dual decomposition to derive the deterministic equivalent of the random model. The resulting deterministic equivalent model will be solved exactly in LINGO 10.0 optimisation software.

In addition, a simulation model of a military supply chain is presented that can be used to test the reliability of the inventory model. The simulation, based on the military SCOR model and the end-to-end military supply chain presented by Bean et al. [3] and Schmitz et al. [52] respectively, determines the percentage of time the supply chain is able to meet the demand for a single ammunition type while using the stock levels and order quantities obtained from the inventory model.

Chapter 6

Numerical example

This chapter considers a numerical example of an ammunition type, with quarterly time periods. The purpose of this example is to illustrate the functioning of the inventory model presented in Chapter 4. The chosen ammunition type for this example is small arms ammunition, which is used in large quantities and has a relatively low purchase cost per unit.

6.1 Data collection and assumptions

The assumptions made and the values chosen as inputs into the inventory management model and supply chain simulation are discussed in this section. Even though care was taken to ensure that these values are as realistic as possible it should be noted that for security reasons we could not obtain values that are representative of the actual stock situation in any particular military.

6.1.1 Inventory model inputs

The lead time L_i for all three stock categories i is assumed to be 1 quarter (three months), and the maximum order size M_i for all three stock categories is set to 400 000 units per order. It is assumed that there are no outstanding deliveries at the time of initiating the model; therefore all Q_{it}^{O} values are set to 0. In addition, it is assumed that the fixed cost associated with placing an order for stock category A and B replenishments is R10. However, the fixed cost of placing a replenishment order for stock category C with the supplier, is assumed to be R1 000 per order. The initial stock quantities are assumed to be 20 000 units, 80 000 units and 150 000 units for stock category A, B and C respectively.

The probability distribution for quarterly training and peacekeeping demand is assumed to be a normal distribution, N(15 000, 3 000) with a mean μ of 15 000 units and a standard deviation σ of 3 000 units. This distribution is discretised into five bins, each with a value and probability, as shown in Figure 6.1. These values and probabilities are used as inputs into the model.

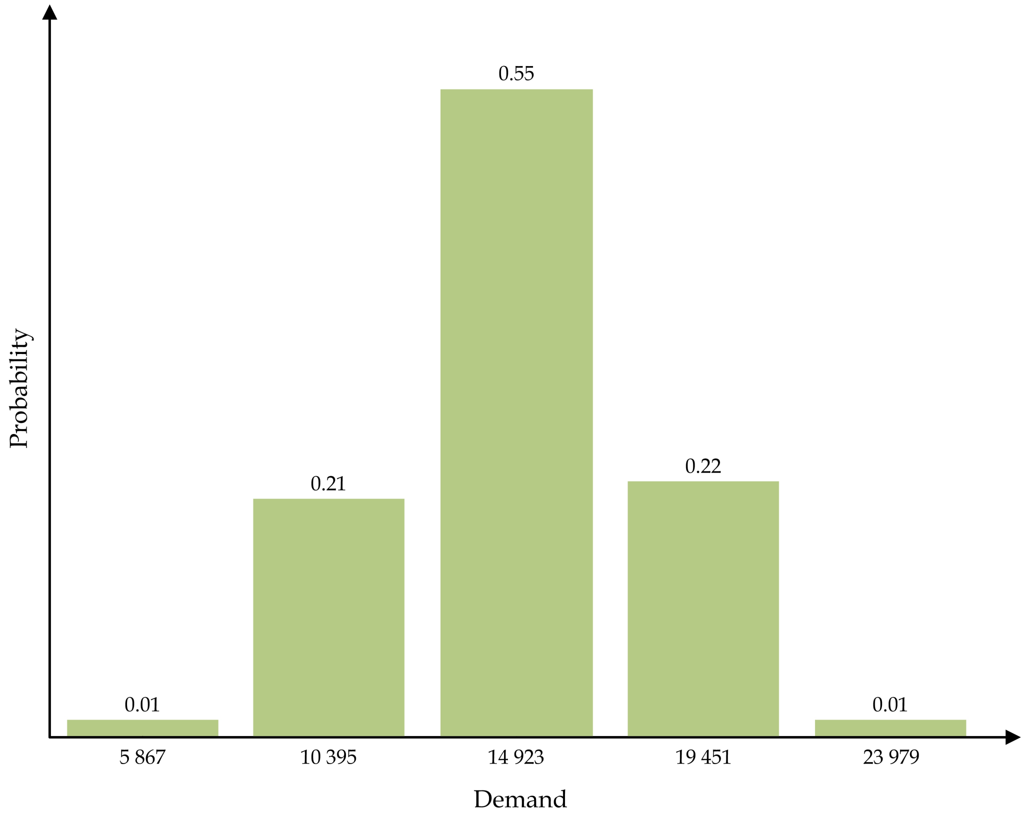


Figure 6.1: Discretised probability distribution for training and peacekeeping demand.

The quarterly demands during a peace enforcement operation, $\widehat{\eta}$, and war, $\widehat{\delta}$, are depicted by trapezoidal fuzzy numbers with the parameters shown in Figures 6.2(a) and 6.2(b) respectively.

In addition, possibilities that each of the three scenarios may occur are depicted by trapezoidal fuzzy numbers, shown in Table 6.1.

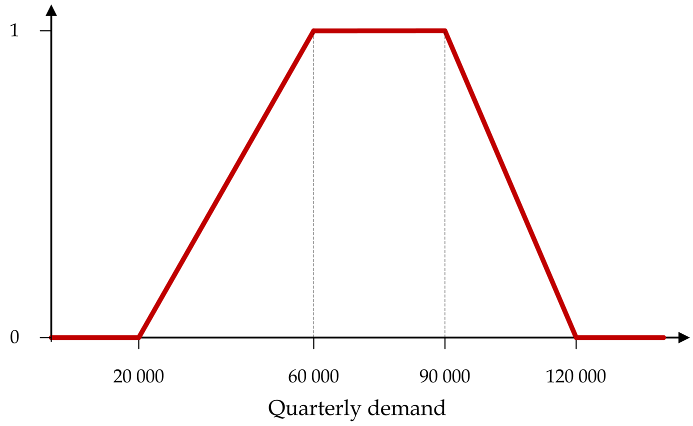
Table 6.1: Possibilities for each scenario.

Scenario	**Symbol**	**Fuzzy values**
Training and peacekeeping	$\widehat{\rho^p}$	$(0.80, 0.85, 0.95, 1.00)$
Peace enforcement	$\widehat{\rho^e}$	$(0.00, 0.05, 0.20, 0.30)$
War	$\widehat{\rho^w}$	$(0.00, 0.02, 0.07, 0.10)$

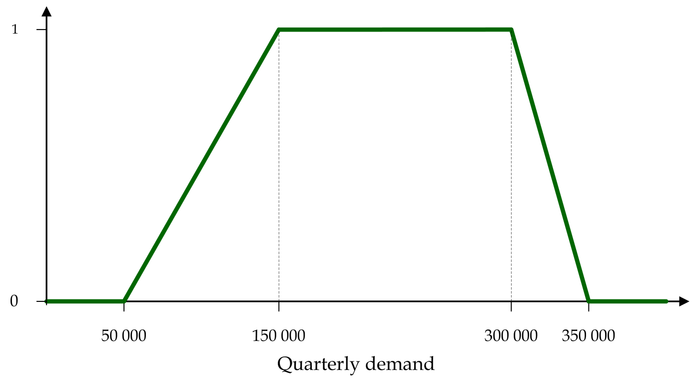
Finally, the remaining parameters required for the inventory model and the chosen values are shown in Table 6.2.

6.1.2 Supply chain simulation inputs

The minimum stock levels for each stock category, obtained from the results of the inventory model, are used as inputs into the supply chain simulation. In addition, it is assumed



(a) Peace enforcement



(b) War

Figure 6.2: Quarterly fuzzy demand for peace enforcement and war.

that the lead times and initial stock quantities for each stock category are the same as in the mathematical model.

Total demand, or supply chain usage, during training and peacekeeping is the same as in the inventory model. In addition, the demand during peace enforcement and war is assumed to be uniform distributions over the ranges of fuzzy numbers $\widehat{\eta}$ and $\widehat{\delta}$ respectively, as it is assumed that the possible values — the fuzzy numbers' ranges — are equally probable. However, the total usage in each scenario is equally divided between the *user system* and *user supply chain* usage. The resulting demand for each of the two supply chains during the three scenarios is shown in Table 6.3.

Additional input values used for the military supply chain simulation are depicted in Tables 6.4 through 6.6.

It is assumed that there is a product system associated with the ammunition type in the example that requires 5 000 units of the ammunition type when it is commissioned, and that there are 50 such product systems in the military. The annual user system and user supply chain demands for user systems are assumed to be normal distributions N(20, 5). However, the initial number of user systems in use at the start of the simulation is set to zero. In addition, in this example it is also assumed that all the received ammunition and product systems are in an acceptable condition, and that returned commodities and product systems are in an acceptable condition 90% of the time; thus 10% of the time they are disposed.

Table 6.2: Arbitrary values used for parameters in the numerical example.

Symbol	Description	Value
H	Holding cost per item per quarter	R0.50
C	Purchase cost per item	R5.00
V	Fee for keeping stock at the supplier	R1.00
A^C	Maximum quantity to be kept in stock at all the military's depots	3 000 000
g_1	Total cost goal over 5 quarters	R2 500 000
g_2	Goal for number of shortages over 5 quarters	0
g_3	Preferred total stock level over 5 quarters	150 000
α^R	Inflation factor for the stability of the region	1
α^W	Inflation factor for the stability of the world	1
α^D	Inflation factor for deterrence purposes	1
α	Feasibility measure	0.90

Table 6.3: Demand during different scenarios.

Scenario	User system supply chain	User supply chain
Training and peacekeeping	N(7 500, 1 500)	N(7 500, 1 500)
Peace enforcement	U(10 000, 60 000)	U(10 000, 60 000)
War	U(25 000, 175 000)	U(25 000, 175 000)

Table 6.4: Values for product system supply chain.

Description	Value
Lead time for product system delivery from supplier	20 quarters
Time to receive and verify product systems	5 days
Time to inspect and test returned product systems	5 days
Time to maintain and test product systems	30 days
Time to prepare product system for delivery	1 day
Shipping time from product system to user system supply chain	1 day

Table 6.5: Values for commodity supply chain.

Description	Value
Time to receive and verify ammunition	1 day
Time to assemble ammunition	0 hours (no assembly)
Time to receive ammunition from MAKE for delivery	1 hour
Time to pick ammunition	4 hours
Time to package ammunition	4 hours
Time to load ammunition for shipment	4 hours
Time to inspect and test returned ammunition	1 day
Time to maintain returned ammunition	2 days
Shipping time from commodity to user system supply chain	1 day
Shipping time from commodity to user supply chain	1 day

Table 6.6: Values for user system supply chain.

Description	Value
Time to receive and verify ammunition	1 day
Time to receive and verify product systems	5 days
Time to assemble user system	30 days
Time to receive user system from MAKE for delivery	1 hour
Shipping time from user system to user supply chain	1 day

6.2 Results and findings

The inventory model is solved exactly and the results are used as input into the military supply chain simulation to test the reliability of the inventory model. Since this is a numerical example, the reader may be more interested in the outcome of the sensitivity analysis than in the actual model results. Therefore, major results and findings are discussed in this section; for a more comprehensive list of the additional inventory model results please refer to Appendix C.

6.2.1 Inventory model results

The minimum required stock level for stock category A is very low, as the demand for training and peacekeeping operations is very low. The minimum stock level requirement for stock category B is higher because peace enforcement, which is very ammunition intensive, is covered with this stock category. The minimum stock level for stock category C is very high, as this stock category is used to meet demand during war which is extremely ammunition intensive.

The overall minimum stock levels determined by the inventory model are shown in Table 6.7. The values for stock category B and stock category C are lower than expected, due to the low possibility of peace enforcement and the even lower possibility for war used to manage the influence of each scenario on stock levels. Whenever the threat or possibility of a scenario increases, the associated stock levels will also increase.

Table 6.7: Minimum stock level for each stock category.

Stock type	**Minimum stock level**
Stock category A	24 879 units
Stock category B	43 484 units
Stock category C	120 488 units

6.2.2 Inventory model reliability

The reliability of the inventory model results is tested using the military supply chain simulation discussed in Chapter 5. For the simulation it is assumed that there are 90 days in each quarter. The minimum stock levels specified by the model results are used as the reorder point and the order quantities in the model results are used as the quantity to be ordered each time a replenishment order is placed in the supply chain. Therefore, the order quantities chosen for the simulation are the quantities ordered most frequently over the five quarters, for each stock category, during each scenario. These values are shown in Table 6.8.

Table 6.8: Order quantities used for each stock type during each scenario.

Stock type	**Order quantity**
Stock category A	28 858 units
Stock category B	102 283 units
Stock category C	201 250 units

The military supply chain is simulated in Arena 12.00.00 simulation software and executed for 10 iterations of five quarters. The results obtained from the simulation model are the average over the 10 iterations.

The Reliability and *days-of-supply* values for each of the three scenarios are shown in Table C.4. The reliability of the model is the same in scenarios 1 and 2, and the lower during scenario 3, which can be attributed to the higher usage during war. The reliability of the supply chain in the numerical example varies between 84.10% and 100.00%, which implies that at worst the supply chain will be able to supply the required commodities in the supply chain 84.10% of the time. In addition, the estimated number of days that will elapse before the first ammunition shortage is experienced during war - for the ammunition type in question - is at least 112 days. The *days-of-supply* measure for scenarios 1 and 2 were terminated at 450 days due the simulation duration of 450 days; so no shortages were experienced at any time during scenarios 1 and 2.

Table 6.9: Reliability and *days-of-supply* value for each scenario.

Scenario	Description	Reliability	Days of Supply
1	Training and peacekeeping	100.000%	450
2	Peace enforcement	100.000%	450
3	War	84.10%	112

Based on the results obtained from the numerical example, the model appears to be extremely reliable for training, peacekeeping, and peace enforcement operations and very reliable for war. However, the possibility that one of the scenarios may occur needs to be considered, since certain unlikely scenarios may cause unnecessary escalation of stock levels and costs. Therefore it may be argued that it is acceptable for less likely scenarios, such as war, to have a lower reliability, even though 84.10% is still considered to be reliable.

To investigate how the mixed model presented in this dissertation compares with existing inventory models in the literature, a comparison between the mixed model, the (r, Q) fixed order quantity inventory model, and the (s, S) inventory model is presented in the following section.

6.3 Comparison with (r, Q) and (s, S) inventory models

The results of the numerical example indicate that the model presented in this dissertation can be extremely reliable and useful for inventory management in the military. However, to confirm that this model is indeed an improved approach from existing inventory models in the literature this numerical example is resolved for two standard inventory models used to determine inventory management plans under uncertainty. The first model is the (r, Q) fixed order quantity model with a service level measure and the second model is the (s, S) inventory model with a service level measure, both models are presented in Chapter 2.

The example of the mixed model presented in this dissertation incorporates a feasibility degree of 0.9, therefore is was decided to use a service level measure of 90% for the (r, Q) and (s, S) models to allow for comparison. The input values are more or less the same as those of the mixed model example. The mean demand and demand deviation for stock category A are the same, however due to the fuzzy numbers used to represent demand for stock categories B and C, the Defuzzification Value — calculated using equation 3.7 in Chapter 3 — of these fuzzy numbers are used as the mean demand for these stock categories. The quarterly demand deviation for these two stock categories are then assumed to be zero. All the input values shown in Table 6.10 are converted into daily equivalents and used as inputs into the (r, Q) and (s, S) models.

For the (r, Q) model, the optimal order quantity for stock category A, Q^A, is calculated using equation (2.5) and the reorder level for the same stock category, r^A, is determined using equation (2.6). The resulting stock category A order quantity is then added to

Table 6.10: Arbitrary values used for parameters in the (r, Q) and (s, S) models.

Symbol	**Description**	**Value**
h	Holding cost per item per quarter	R0.50
C	Purchase cost per item	R5.00
L	Constant lead time per order for all stock categories	90 days
K^A	Fixed ordering cost for stock category A	R10.00
K^B	Fixed ordering cost for stock category A	R10.00
K^C	Fixed ordering cost for stock category A	R1 000.00
μ^A	Mean quarterly demand for stock category A	15 000 items
μ^B	Mean quarterly demand for stock category B	72 500 items
μ^C	Mean quarterly demand for stock category C	212 500 items
σ^A	Mean quarterly demand for stock category A	3 000 items
σ^B	Mean quarterly demand for stock category B	0 items
σ^C	Mean quarterly demand for stock category C	0 items

the quarterly demand of stock category B, as it is assumed that only one order is placed per quarter — due to the 90 day lead time associated with an order. The optimal order quantity and the reorder point for stock category B, Q^B and r^B, are then also determined using equations (2.5) and (2.6). Finally, the optimal order quantity of stock category B is then added to the quarterly demand of stock category C and the order quantity and reorder point for category C, Q^C and r^C, determined using the same equations as before.

The same approach is taken for the (s, S) model, but the Q and r values for the different stock categories are used as inputs into equations (2.7) and (2.8) to determine the minimum and maximum stock levels for each of the stock categories. The results of the two models are shown in Tables 6.11 and 6.12.

Table 6.11: Results for (r, Q) model example.

		r	Q
Stock category	**A**	18 110	775
	B	73 275	1 713
	C	214 213	29 273

The reliabilities of these two models are then determined using the same approach as before and compared with the reliability of the mixed model. The results are shown in Figure 6.3.

It is clear that all three models are 100% reliable for the training and peace-keeping

Table 6.12: Results for (s, S) model example.

		s	S
Stock	**A**	18 110	18 885
category	**B**	73 275	74 988
	C	214 213	243 486

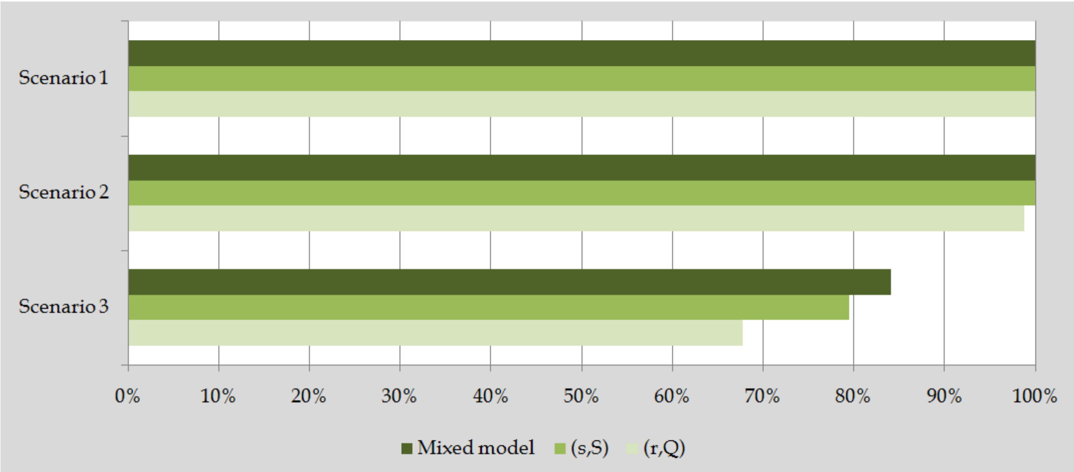


Figure 6.3: Reliability of different models during different scenarios.

scenario, this is due to the low demand during this scenario. The (r, Q) model's reliability reduces to 98.80% during the second scenario — peace enforcement — due to the higher demand associated with this scenario. However, the (s, S) model's reliability still compares favourably with the mixed model during this second scenario at 100.00%. The tiebreaker is the third scenario — war — where both the (r, Q) and (s, S) models' reliability reduces to 67.73% and 79.50% respectively. The mixed model's reliability also reduces during this scenario but to a lesser extend. The reliability of 84.10% for this model proves that the mixed model presented in this dissertation is indeed more reliable during extreme scenarios than the (r, Q) and (s, S) models.

Even though the mixed inventory model is more reliable than existing (r, Q) and (s, S) inventory models, investigating the sensitivity of the model to various input parameters is very important to obtain a complete understanding of the functionality and limitations of the model. So a sensitivity analysis is presented in the following section.

6.4 Sensitivity analysis

Since the model deals with uncertainty, the idea of optimality diminishes. Therefore it is important to understand the sensitivity of the model to varying feasibility degrees, α, to allow the decision-maker to have a better understanding of the impact of the feasibility degree on the final results.

The first aspect under consideration is the impact of different feasibility degrees on the model's reliability. The results from this analysis are presented in Figure 6.4. The results indicate that there is an upward trend in the reliability of the model with increasing feasibility degrees. This was expected, since higher feasibility degrees effectively consider larger ammunition usage during peace enforcement operations and war than do lower feasibility degrees.

It is interesting to note that the reliability for scenario 2 initially increases and then stabilises with rising feasibility degrees. The highest reliability point is reached with a feasibility degree of 0.3, indicating that a feasibility degree of 0.3 is sufficient for maximum reliability in this scenario. Using a feasibility degree of more that 0.3 will result in unnecessary cost escalations without any increase in benefit for the peace enforcement scenario.

The effect on reliability is much more pertinent in scenario 3. However, the reliability for scenario 3 does not exceed 85.37%. This is due to the low possibility of a war occurring, and hence the lower priority to meet the ammunition demand during war.

It is important to note that even though war may not currently have a very high priority, due to the low likelihood that war will happen in the near future, the possibility that war may occur can increase in the future. This will result in a higher reliability for war when this model is resolved on a rolling horizon basis.

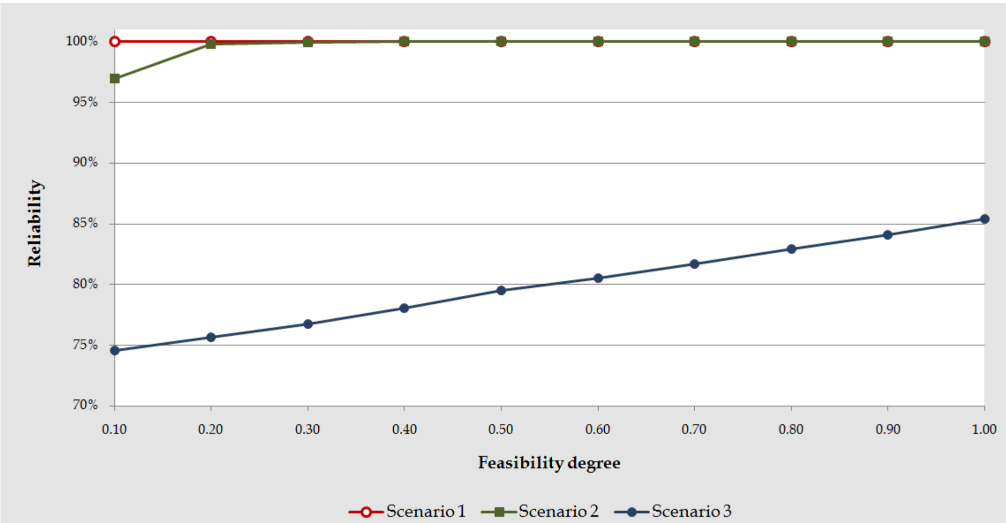


Figure 6.4: Feasibility degree impact on reliability.

The second aspect under investigation is the *days-of-supply* measure and the impact that the feasibility degree may have on this measure. The results of this particular analysis are shown in Figure 6.5. The measure remains constant for scenarios 1 and 3, irrespective of the feasibility degree. This is because the supply chain does not run out of stock during the five quarters in scenario 1 — training and peacekeeping operation — due to the low ammunition usage in this particular scenario. In all instances a shortage is experienced by day 112 of scenario 3. This is due to the lower stock levels caused by the ‘prioritising’ of scenarios based on the possibility of a scenario occurring, as explained in the previous paragraph.

During scenario 2 the *days-of-supply* measure initially increases for increasing feasibility degrees, due to the higher stock levels associated with higher feasibility degrees. However, this value stabilises from feasibility degrees of 0.4 and above. This indicates that a feasibility degree of 0.4 is sufficient to obtain the highest *days-of-supply* measure for scenario 2. Using a higher feasibility degree than 0.4 will increase the cost significantly, as a higher feasibility degree results in higher inventory levels without any noticeable benefit for scenario 2.

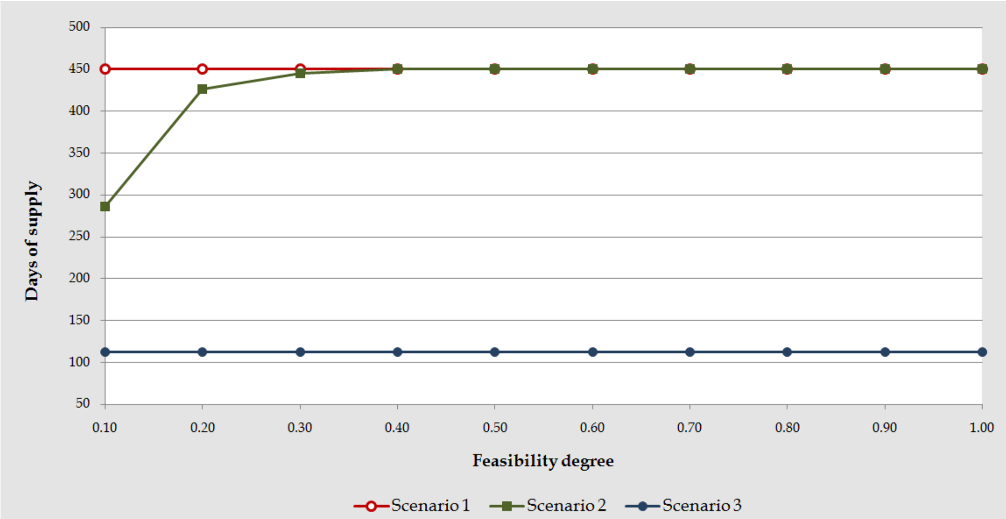


Figure 6.5: Feasibility degree impact on *days-of-supply* measure.

The third aspect under consideration is the impact of the feasibility degree on the minimum required stock level for each of the three stock categories. The results, shown in Figure 6.6, indicate that the minimum stock level increases as the feasibility degree increases for stock categories B and C. This is due to the larger usage taken into account for higher feasibility degrees. The stock level for stock category C — which meets the demand during war — is constantly higher than that of stock category B — which meets demand during peace enforcement. This is expected, as the usage during war is much higher than the usage during peace enforcement, thus driving the stock level for category C higher.

The minimum stock level for stock category A is completely unaffected by varying feasibility degrees, because this stock category is used to meet the stochastic demand during training and peacekeeping operations, which are not affected by the feasibility degree.

The final three aspects investigated during the sensitivity analysis are the impact of the feasibility degrees on the realisation of the goal constraints. These results are shown in Figures 6.7, 6.8, and 6.9. The goals for each of these three aspects were R 2 500 000, 0 units, and 150 000 units respectively.

The total cost and the total stock values increase as the feasibility degree improves. This is due to the higher usage taken into account for peace enforcement operations and war, with increasing feasibility degrees.

The total number of shortages experienced remains constant at first for feasibility degrees between 0.05 and 0.30, due to sufficient initial stock to meet the estimated demand during the initial time periods, and high enough re-order levels and order quantities to meet demand during the last time periods without experiencing shortages. Finally, the number of shortages increases with feasibility degrees of 0.35 to 1.00, indicating that as a higher feasibility is desired, the minimum stock levels and order quantities are not sufficient to meet the higher demand at all times.

The sensitivity of the (r, Q) and (s, S) models' reliabilities to different service levels are also investigated. The reliabilities of both models during the three scenarios remains

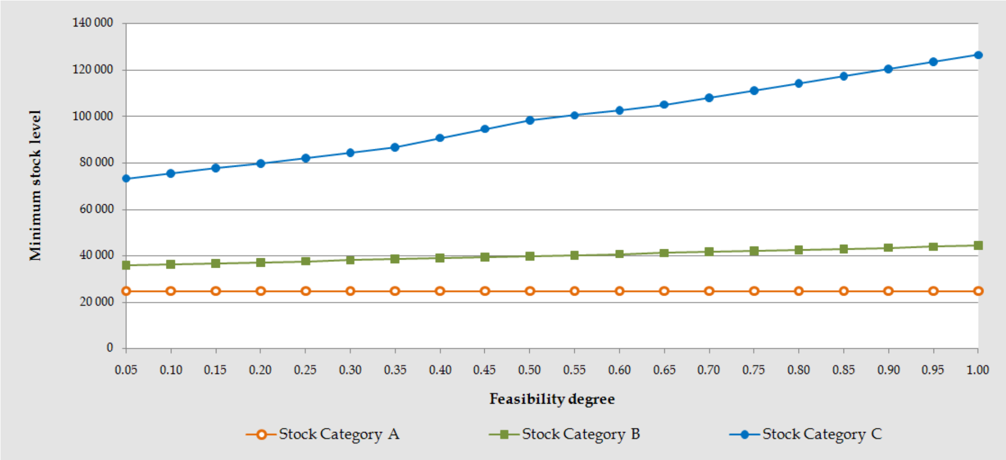


Figure 6.6: Feasibility degree impact on minimum stock level.

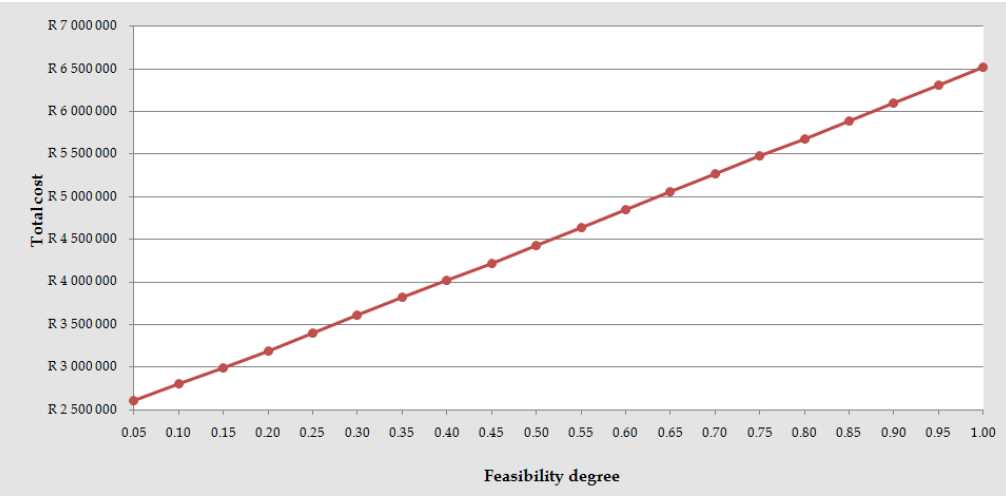


Figure 6.7: Feasibility degree impact on cost.

constant, irrespective of the service level. This is because the supply chain never runs out out stock in scenario one due to the low demand during this scenario. The demands for extreme events during scenarios two and three — which affected stock categories B and C — were assumed to be constant with no deviations. Their minimum stock levels, order quantities and maximum stock levels were therefore unaffected by the service level measure.

6.5 Concluding remarks

In this chapter, a numerical example was presented to illustrate the functionality of the inventory model proposed in Chapter 4. These results and were compared with the results obtained for the (r, Q) and (s, S) inventory models. The comparison shows that the mixed model proposed in this dissertation is more reliable in extreme scenarios than the (r, Q) and (s, S) inventory models in the literature. The sensitivity of the model was investigated, and it can be concluded that the model is flexible enough to incorporate the preferences

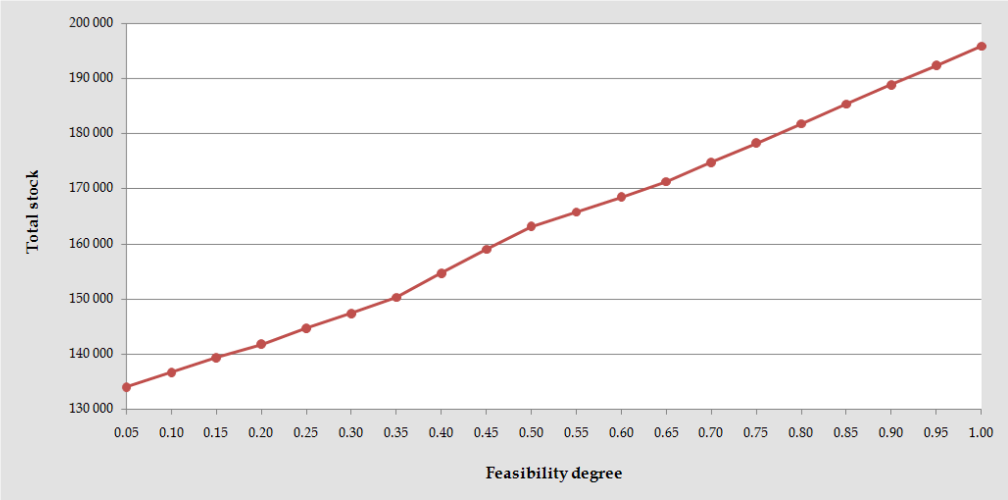


Figure 6.8: Feasibility degree impact on total stock level.

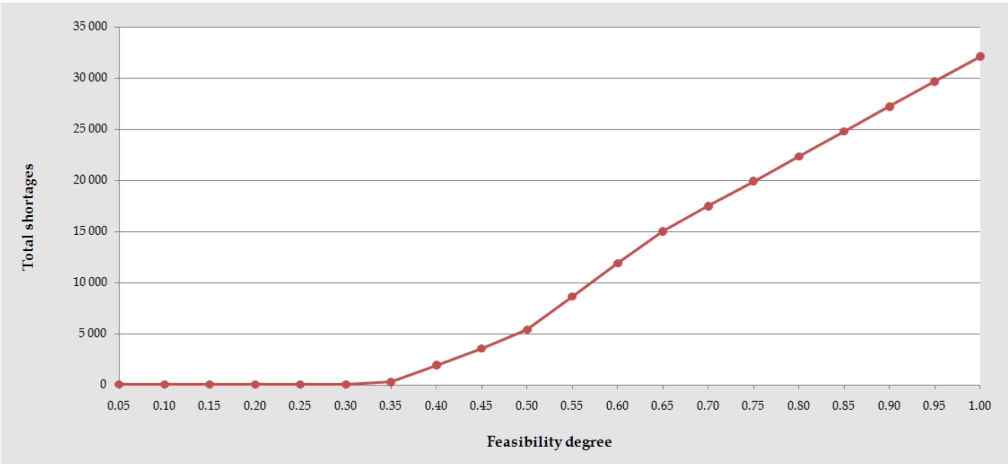


Figure 6.9: Feasibility degree impact on shortages.

of the decision-maker by means of a feasibility degree embedded in the formulation.

It is important to note that this numerical example is presented merely for illustrative purposes, and that the results do not provide an indication of the actual stock situation in a particular military.

Chapter 7

Conclusion and future research directions

Many countries experience the problem of insufficient ammunition stockpiles. The proper planning of ammunition stock levels is a critical aspect for the efficient functioning of any military. Determining the amount of ammunition to keep in stock can very easily become a daunting task due to uncertainty about the future. To make the problem even harder, the military is also typically an environment where improbable events such as war can have massive impacts on operations. The availability of the correct amount of stock is therefore critical as it can enhance the responsiveness, efficiency, and preparedness of the military, and ultimately save human lives. This highlights the need for a structured and proper approach to stockpile planning in the military.

The aim of this dissertation was to develop a reliable decision support model that can be used to assist with military inventory management amidst demand uncertainty. To achieve this, a mathematical inventory model was developed that incorporates demand uncertainty, multiple objectives, and three different stock categories for a single ammunition item in the military. The model takes three different scenarios into account and determines the minimum required stock level for three different stock categories of a single item.

The objectives of the model include the minimisation of the total cost, the total number of shortages, and the total amount of surplus ammunition. Goal programming was used to solve this multi-objective programme. Uncertainty related to unknown demands for different scenarios was represented by probability distributions and fuzzy numbers. A combination of fuzzy and stochastic programming was chosen because a probability distribution can be established to represent future demand for very probable scenarios. On the other hand, the estimated future demand for less likely operations and extreme events cannot be determined, due to the high level of uncertainty associated with these events. Fuzzy programming was chosen for these cases, as it allows one to obtain a rough estimation of what can typically be expected in these unlikely and extreme cases.

The mixed multi-objective model was converted into deterministic terms to allow it to be solved exactly using standard optimisation software. This was achieved first by transforming fuzzy constraints into crisp ones using the approach introduced by Jiménez et al. [24], and second by deriving the deterministic counterpart for the stochastic model by means of dual decomposition.

The model solution was obtained using LINGO 10.00 optimisation software. The model reliability was tested by investigating the ability of a military supply chain to meet demand during a variety of scenarios, given the results obtained from the model. An end-to-end military supply chain was simulated, based on the end-to-end military supply chain proposed by Schmitz et al. [52] and on the adapted SCOR model for the military presented by Bean et al. [3]. The simulation was executed for different scenarios with different demands, and the ability of the simulated supply chain to provide the required amount of ammunition was investigated. In all the instances where the supply chain was able to supply the amount of ammunition required by the user, it was regarded as reliable; conversely it was considered to be unreliable when it could not.

The reliability of this model was then compared with the reliability of the well known (r, Q) and (s, S) inventory models in the literature. Results indicated that the model presented in this dissertation are more reliable in extreme scenarios than the (r, Q) and (s, S) models.

A sensitivity analysis was performed and indicated that the inventory model yields reliable results. Three different scenarios with low, medium, and high demands were investigated. The model's reliability amounted to 100% for the low, between 99.98% and 100.00% for the medium, and between 74.54% and 85.37% for the high demand scenario. The lower, but still reasonably good, reliability for the high demand scenario is due to the ability of the inventory model to prioritise different scenarios, based on its ability to ensure that stock levels are not unneccessary escalated for highly improbable events.

Based on the inventory model proposed in this dissertation, and on the reliability of its results, the research question presented in Chapter 1 can now be successfully answered as follows:

> *A mixed multi-objective mathematical model that aims to minimise inventory cost and surplus stock, while ensuring demand satisfaction, through the minimisation of shortages, is a reliable inventory decision support model for the uncertain military environment.*

Even though the research question of this dissertation was successfully answered, it is important to understand that the resulting inventory model addresses only some of the aspects in the broader field of military inventory management. The implementation of this decision support model will not instantly solve all the challenges associated with inventory management in the military. However, this model will provide a starting point for an improved approach to military inventory management, by providing an improved estimation of the required inventory levels. A practical response plan should be developed by the military to determine how inventory should be acquired, stored, and repositioned to ensure improved responsiveness to improbable events. During implementation it could also be required to integrate the inventory model proposed in this dissertation with a more advanced life-cycle management tool, as the life-cycle of ammunition is only addressed in the model by adapting a FIFO approach.

In addition to the above mentioned implementation considerations, there are many potential research and improvement opportunities to build on the work presented in this dissertation. Alternative solution approaches to address the fuzzy component of the model should be investigated, whereas chance-constraints could be considered as a suitable alternative for the recourse approach to deal with the stochastic component of the model. The possibility of combining recourse stochastic programming and chance-constraints for the stochastic component of the model could also be investigated, keeping in mind the constant trade-off between computational complexity and improved accuracy.

The inventory model could also be expanded to include the reverse aspects of the military supply chain more effectively in an attempt to improve the reliability of the model even further. Moreover, the possibility of adjusting this model to be suitable to other stock types or other environments, such as disaster management, should also be explored. Finally, the model can be expanded to take multiple inventory items into account.

In conclusion, the research conducted for this dissertation focused on addressing an existing problem in the military environment and finding a good solution that can be used for inventory management in the military. To the best of the author's knowledge, it is the first time that a multi-objective inventory model with both fuzzy and stochastic demand that minimises cost, stock levels, and shortages has been described. The application of inventory models in the military is limited, and no literature was discovered that incorporates fuzzy set theory into the field of military stockpile or inventory planning. This dissertation will therefore make a significant contribution to the field of inventory management.

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Appendix A

Deterministic counterpart of the inventory model

Decision variables:

$S_i \triangleq$ the minimum stock level at which reordering should occur for stock category $i \in \boldsymbol{I} = \{1, 2, 3\}$ where

$$i \triangleq \begin{cases} 1 & \text{stock category A} \\ 2 & \text{stock category B} \\ 3 & \text{stock category C.} \end{cases}$$

$Q_{i,t} \triangleq$ the order quantity of stock category $i \in \boldsymbol{I}$ during time period $t \in \boldsymbol{T} = \{0, 1, \ldots, T\}$.

$D_{i,t} \triangleq$ the quantity of stock category $i \in \boldsymbol{I}$ delivered as replenishment during time period $t \in \boldsymbol{T}$.

Utility variables:

$O^A_{s,t}\left(\left\{\xi^{\beta_{t'}}_{t'}\right\}^t_{t'=1}\right) \triangleq$ the total number of stock category A items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$ as a function of the β^{th}_t realisation of random demand $\widetilde{\xi}_t$, where $\beta_t \in \boldsymbol{\beta} = \{1, \ldots, 5\}$.

$O^B_{s,t} \triangleq$ the total number of stock category B items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$.

$O^C_{s,t} \triangleq$ the total number of stock category C items, in scenario $s \in \boldsymbol{S}$, that are short at the end of time period $t \in \boldsymbol{T}$.

$U^A_{s,t}\left(\left\{\xi^{\beta_{t'}}_{t'}\right\}^t_{t'=1}\right) \triangleq$ the total number of stock category A items, in scenario $s \in \boldsymbol{S}$, in excess at the end of time period $t \in \boldsymbol{T}$ as a function of the β^{th}_t realisation of random demand $\widetilde{\xi}_t$, where $\beta_t \in \boldsymbol{\beta}$.

$U^B_{s,t} \triangleq$ the total number of stock category B items, in scenario $s \in \boldsymbol{S}$, that are in surplus at the end of time period $t \in \boldsymbol{T}$.

$U^C_{s,t} \triangleq$ the total number of stock category C items, in scenario $s \in \boldsymbol{S}$, that are in surplus at the end of time period $t \in \boldsymbol{T}$.

$N_t \triangleq$ the quantity that is available at the supplier, but not yet delivered, as replenishment of stock category C at the end of time period $t \in \boldsymbol{T}$.

$x_{i,t} \triangleq$ a binary variable that determines if a replenishment order for stock category $i \in \boldsymbol{I}$ is placed during time period $t \in \boldsymbol{T}$.

$d^c \triangleq$ the amount by which the actual cost exceeds its goal.

$d^d \triangleq$ the amount by which demand satisfaction falls short of its goal.

$d^s \triangleq$ the amount by which the specified minimum stock exceeds its goal.

Parameters:

$\xi_t^{\beta_t} \triangleq$ random vector describing the demand for stock category A during time period t, where $t \in \boldsymbol{T}$.

$\widehat{\eta} \triangleq$ a trapezoidal fuzzy number with parameters $(\eta_1, \eta_2, \eta_3, \eta_4)$ describing the demand in one time period during a peace enforcement operation.

$\widehat{\delta} \triangleq$ a trapezoidal fuzzy number with parameters $(\delta_1, \delta_2, \delta_3, \delta_4)$ describing the demand in one time period during war.

$\rho^p \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho_1^p, \rho_2^p, \rho_3^p, \rho_4^p)$, describing the possibility that training and peace-keeping may be taking place during a time period.

$\rho^e \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho_1^e, \rho_2^e, \rho_3^e, \rho_4^e)$, describing the possibility that a peace enforcement operation may be taking place during a time period.

$\rho^w \triangleq$ the defuzzification value of a trapezoidal fuzzy number with parameters $(\rho_1^w, \rho_2^w, \rho_3^w, \rho_4^w)$, describing the possibility that the country may be at war during a time period.

$Q_{i,t}^O \triangleq$ the initial quantities of orders for stock category i, due to orders placed during previous time periods, to be received during time period t, where $i \in \boldsymbol{I}$ and $t \in \boldsymbol{T}$.

$L_i \triangleq$ the lead time between the placing of an order for stock category i and the receipt of the delivery thereof, where $i \in \boldsymbol{I}$.

$F_i \triangleq$ the fixed cost of placing a replenishment order for stock category i, where $i \in \boldsymbol{I}$.

$M_i \triangleq$ the maximum replenishment order size of stock category i that is allowed, where $i \in \boldsymbol{I}$.

$H \triangleq$ the cost of holding one item in inventory for one time period.

$A^C \triangleq$ the maximum surplus stock quantity of a particular item that can be stored in all the available depots.

$C \triangleq$ the purchasing cost per item at the supplier.

$V \triangleq$ the cost per item for keeping the item in stock at the supplier.

$\alpha^R \triangleq$ an inflation factor for the stability of a region.

$\alpha^W \triangleq$ an inflation factor for the stability of the world.

$\alpha^D \triangleq$ an inflation factor for deterrence purposes.

$g^c \triangleq$ the target value for total cost.

$g^d \triangleq$ the target value for total demand satisfaction.

$g^s \triangleq$ the target value for the total minimum stock level.

$$\min \quad \Bigg\{ d^c + d^d + d^s + \epsilon \Bigg[\sum_{t \in \boldsymbol{T_1}} \Bigg(H \sum_{s \in \boldsymbol{S}} \Bigg[\sum_{\{\beta_t\}} \prod_{t'=1}^{t} p_t^{\beta_{t'}} \Big[U_{s,t}^{A} \Big(\{ \xi_{t'}^{\beta_{t'}} \}_{t'=1}^{t} \Big) \Big] + U_{s,t}^{B} + U_{s,t}^{C} \Bigg] + \sum_{i \in \boldsymbol{I}} F_i x_{i,t} + VN_t + CQ_{3,t} \Bigg) + \sum_{t \in \boldsymbol{T_1}} \sum_{s \in \boldsymbol{S}} \Bigg(\sum_{\{\beta_t\}} \prod_{t'=1}^{t} p_t^{\beta_{t'}} \Big[O_{s,t}^{A} \Big(\{ \xi_{t'}^{\beta_{t'}} \}_{t'=1}^{t} \Big) \Big] + O_{s,t}^{B} + O_{s,t}^{C} \Bigg) + \sum_{i \in \boldsymbol{I}} S_i \Bigg] \Bigg\} \tag{A.1}$$

Subject to:

Goals.

Total cost goal.

$$\sum_{t \in \boldsymbol{T_1}} \Bigg(H \sum_{s \in \boldsymbol{S}} \Bigg[\sum_{\{\beta_t\}} \prod_{t'=1}^{t} p_t^{\beta_{t'}} \Big[U_{s,t}^{A} \Big(\{ \xi_{t'}^{\beta_{t'}} \}_{t'=1}^{t} \Big) \Big] + U_{s,t}^{B} + U_{s,t}^{C} \Bigg] + \sum_{i \in \boldsymbol{I}} F_i x_{i,t} + VN_t + CQ_{3,t} \Bigg) - d^c \leq g^c \tag{A.2}$$

Total number of shortages goal.

$$\sum_{t \in \boldsymbol{T_1}} \sum_{s \in \boldsymbol{S}} \Bigg(\sum_{\{\beta_t\}} \prod_{t'=1}^{t} p_t^{\beta_{t'}} \Big[O_{s,t}^{A} \Big(\{ \xi_{t'}^{\beta_{t'}} \}_{t'=1}^{t} \Big) \Big] + O_{s,t}^{B} + O_{s,t}^{C} \Bigg) - d^d \leq g^d \tag{A.3}$$

Total surplus stock goal.

$$\sum_{i \in \boldsymbol{I}} S_i - d^s \leq g^s \tag{A.4}$$

Assign shortage and surplus values.

Stock category A.

$$U^A_{s,t-1}\Big(\{\xi^{\beta_{t'}}_{t'}\}^{t-1}_{t'=1}\Big) - O^A_{s,t-1}\Big(\{\xi^{\beta_{t'}}_{t'}\}^{t-1}_{t'=1}\Big) + D_{1,t} - \xi^{\beta_t}_t = U^A_{s,t}\Big(\{\xi^{\beta_{t'}}_{t'}\}^{t}_{t'=1}\Big) - O^A_{s,t}\Big(\{\xi^{\beta_{t'}}_{t'}\}^{t}_{t'=1}\Big) \quad \forall \quad t \in \boldsymbol{T_1} = \{1, \ldots, T\},\ \beta_t \in \boldsymbol{\beta},\ s \in \{1,2\} \tag{A.5}$$

Stock category B.

$$U^B_{1,t-1} - O^B_{1,t-1} + D_{2,t} - Q_{1,t} = U^B_{1,t} - O^B_{1,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{A.6}$$

$$U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1 - \frac{\alpha}{2}\right)\frac{\eta_1 + \eta_2}{2} + \frac{\alpha}{2}\left(\frac{\eta_3 + \eta_4}{2}\right)\right] - Q_{1,t} \ \geq\ U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{A.7}$$

$$U^B_{2,t-1} - O^B_{2,t-1} + D_{2,t} - \left[\left(1 - \frac{\alpha}{2}\right)\frac{\eta_3 + \eta_4}{2} + \frac{\alpha}{2}\left(\frac{\eta_1 + \eta_2}{2}\right)\right] - Q_{1,t} \ \leq\ U^B_{2,t} - O^B_{2,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{A.8}$$

Stock category C.

$$U^C_{s,t-1} - O^C_{s,t-1} + D_{3,t} - Q_{2,t} = U^C_{s,t} - O^C_{s,t} \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \{1,2\} \tag{A.9}$$

$$U^C_{3,t-1} - O^C_{3,t-1} + D_{3,t} - \left[\left(1 - \frac{\alpha}{2}\right)\frac{\delta_1 + \delta_2}{2} + \frac{\alpha}{2}\left(\frac{\delta_3 + \delta_4}{2}\right)\right] \ \geq\ U^C_{3,t} - O^C_{3,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{A.10}$$

$$U^C_{3,t-1} - O^C_{3,t-1} + D_{3,t} - \left[\left(1 - \frac{\alpha}{2}\right)\frac{\delta_3 + \delta_4}{2} + \frac{\alpha}{2}\left(\frac{\delta_1 + \delta_2}{2}\right)\right] \ \leq\ U^C_{3,t} - O^C_{3,t} \quad \forall \quad t \in \boldsymbol{T_1} \tag{A.11}$$

Assign initial delivery quantities.

$$D_{i,t} = Q^O_{i,t} \quad \forall \quad i \in \{1,2\},\ t \in \{1, \ldots, L_i\} \tag{A.12}$$

$$D_{i,t} = Q_{i,t-L_i} \quad \forall \quad i \in \{1,2\},\ t \in \{L_i + 1, \ldots, T\} \tag{A.13}$$

Determine quantity to deliver.

$$N_t = N_{t-1} + Q^O_{3,t} - D_{3,t} \quad \forall \quad t \in \{1, \ldots, L_3\} \tag{A.14}$$

$$N_t = N_{t-1} + Q_{3,t-L_3} - D_{3,t} \quad \forall \quad t \in \{L_3 + 1, \ldots, T\} \tag{A.15}$$

Determine minimum stock level.

Stock category A.

$$S_1 \geq \left(\rho^p + \rho^e\right) \sum_{j=t}^{t+L_1-1} \xi_j^{\beta_j} \quad \forall \quad t \in \{1, \ldots, T - L_1 + 1\},\ \beta_t \in \boldsymbol{\beta} \tag{A.16}$$

Stock category B.

$$S_2 \geq \rho^p \left(\sum_{j=t}^{t+L_2-1} Q_{1,j} \right) + \rho^e \left(L_2 \left[\alpha \frac{\eta_3 + \eta_4}{2} + (1-\alpha) \frac{\eta_1 + \eta_2}{2} \right] + \sum_{j=t}^{t+L_2-1} Q_{1,j} \right)$$
$$\forall \quad t \in \{1, \ldots, T - L_2 + 1\} \tag{A.17}$$

Stock category C.

$$S_3 \geq \alpha^R \alpha^W \alpha^D \left(\left(\rho^p + \rho^e\right) \sum_{j=t}^{t+L_3-1} Q_{2,j} + \rho^w L_3 \left[\alpha \frac{\delta_3 + \delta_4}{2} + (1-\alpha) \frac{\delta_1 + \delta_2}{2} \right] \right)$$
$$\forall \quad t \in \{1, \ldots, T - L_3 + 1\} \tag{A.18}$$

Determine if an order is placed during a time period.

$$Q_{i,t} \leq M_i x_{i,t} \quad \forall \quad t \in \boldsymbol{T_1},\ i \in \boldsymbol{I} \tag{A.19}$$

Ensure that an order is placed when inventory level falls below the minimum.

$$S_1 - U_{s,t}^A\left(\{\xi_{t'}^{\beta_{t'}}\}_{t'=1}^t\right) + O_{s,t}^A\left(\{\xi_{t'}^{\beta_{t'}}\}_{t'=1}^t\right) \leq Q_{1,t} \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \{1,2\},\ \beta_t \in \boldsymbol{\beta} \tag{A.20}$$

$$S_2 - U_{s,t}^B + O_{s,t}^B \leq Q_{2,t} \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \{1,2\} \tag{A.21}$$

$$S_3 - U_{s,t}^C + O_{s,t}^C \leq Q_{3,t} \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \boldsymbol{S} \tag{A.22}$$

Ensure that depot capacity is not exceeded.

$$U_{s,t}^A\left(\{\xi_{t'}^{\beta_{t'}}\}_{t'=1}^t\right) + U_{s,t}^B + U_{s,t}^C \leq A^C \quad \forall \quad t \in \boldsymbol{T_1},\ s \in \boldsymbol{S},\ \beta_t \in \boldsymbol{\beta} \tag{A.23}$$

Non-negativity, binary and integer constraints.

$$U_{s,t}^A\left(\{\xi_{t'}^{\beta_{t'}}\}_{t'=1}^t\right), O_{s,t}^A\left(\{\xi_{t'}^{\beta_{t'}}\}_{t'=1}^t\right) \geq 0 \quad \forall \quad t \in \boldsymbol{T},\ s \in \boldsymbol{S},\ \beta_t \in \boldsymbol{\beta} \tag{A.24}$$

$$U_{s,t}^B, O_{s,t}^B, U_{s,t}^C, O_{s,t}^C \geq 0 \quad \forall \quad t \in \boldsymbol{T},\ s \in \boldsymbol{S} \tag{A.25}$$

$$x_{i,t} \in \{0,1\} \quad \forall \quad i \in \boldsymbol{I},\ t \in \boldsymbol{T} \tag{A.26}$$

$$Q_{i,t}, D_{i,t} \geq 0 \text{ and integer} \quad \forall \quad i \in \boldsymbol{I},\ t \in \boldsymbol{T} \tag{A.27}$$

$$S_i \geq 0 \text{ and integer} \quad \forall \quad i \in \boldsymbol{I} \tag{A.28}$$

$$N_t \geq 0 \text{ and integer} \quad \forall \quad t \in \boldsymbol{T} \tag{A.29}$$

$$d^c, d^d, d^s \geq 0 \quad \forall \quad s \in \boldsymbol{S} \tag{A.30}$$

The objective of the model (A.1) minimises the total deficiency in reaching the three goals over the three scenarios where ϵ represents a sufficiently small number. The goals of the model, incorporated as constraints, are to minimise the total cost (A.2), the total number of shortages (A.3), and the minimum inventory level required (A.4) over the specified time horizon. To assign the shortage and surplus values for each stock category, equations (A.5) through (A.11) are used. Equations (A.12) and (A.13) are used to determine the number of items delivered as replenishment of stock categories A and B. To determine the quantities to keep in storage at the supplier and the number of stock category C items to deliver during a time period, constraints (A.14) and (A.15) are incorporated. Equations (A.16) through (A.18) are incorporated to allocate values to the minimum stock level for each category of stock. Equation (A.19) is used to determine the order quantity for the different stock categories. Equations (A.20) through (A.22) are used to determine if an order is placed during a time period. To ensure that the available depot storage capacity for a particular item is not exceeded at any time, constraint (A.23) is incorporated.To ensure that certain variables are positive, that some variables are binary, and that some variables are integer values, constraints (A.24) through (A.30) are included in the model.

Appendix B

Flow charts of military supply chain simulation

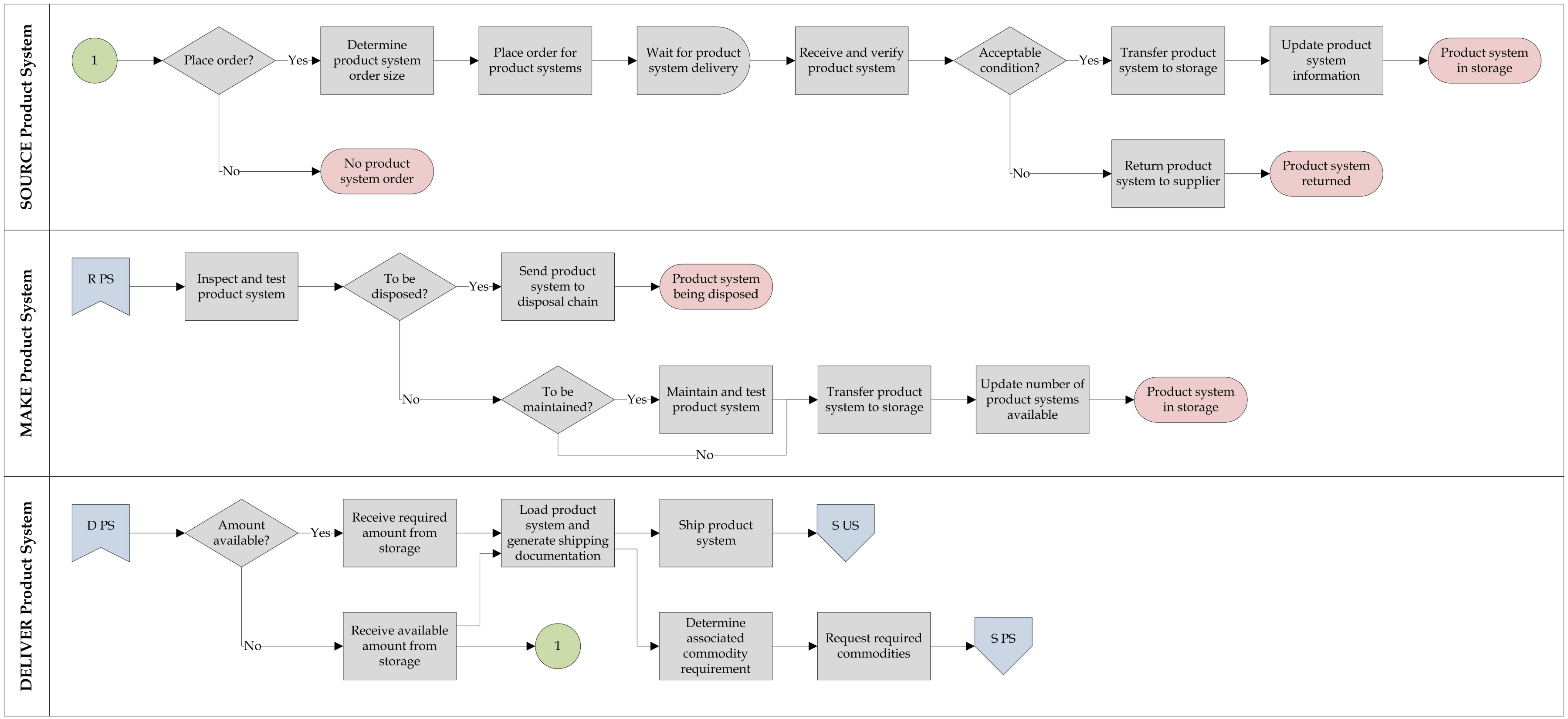

Figure B.1: Product system supply chain.

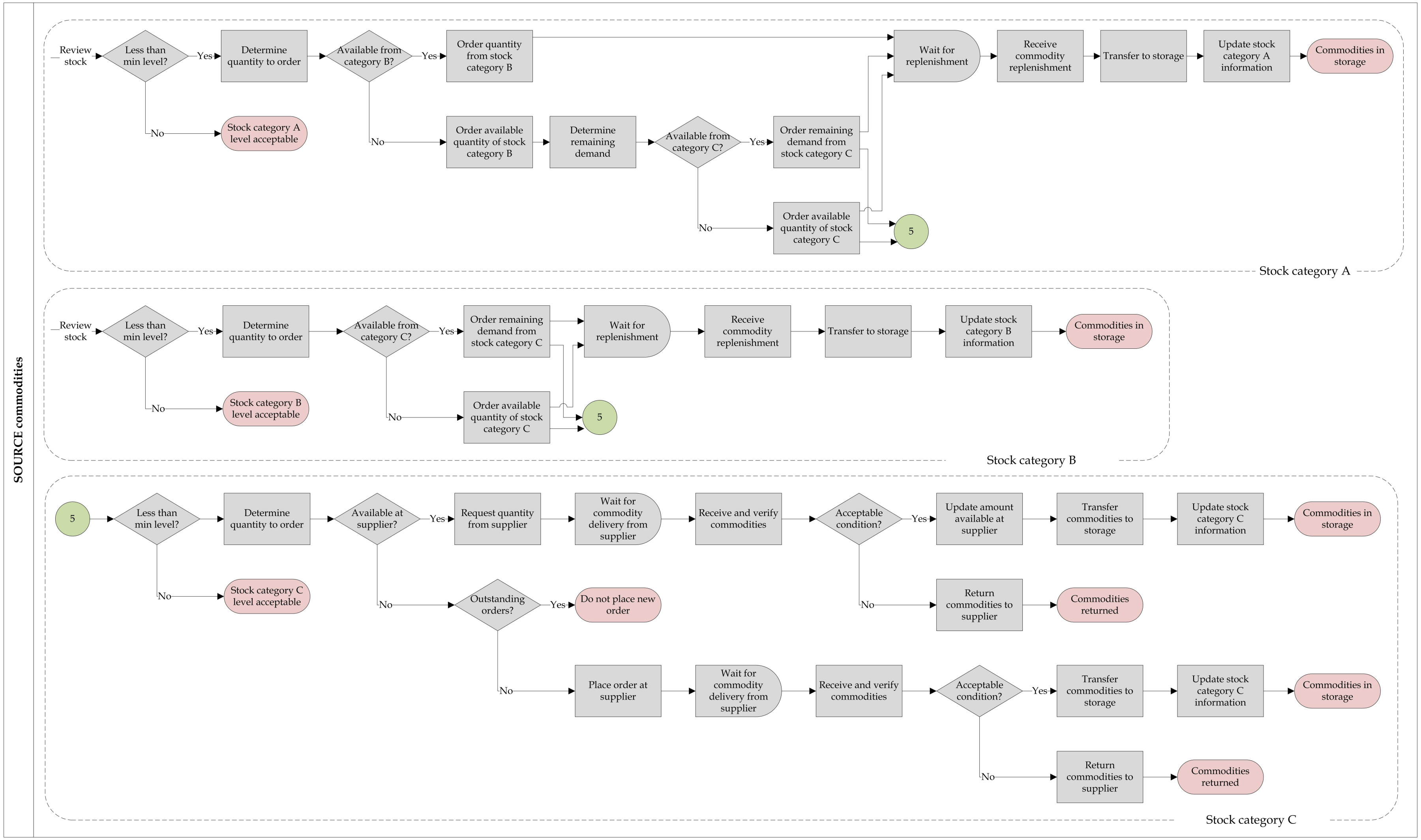

Figure B.2: Commodity supply chain part one.

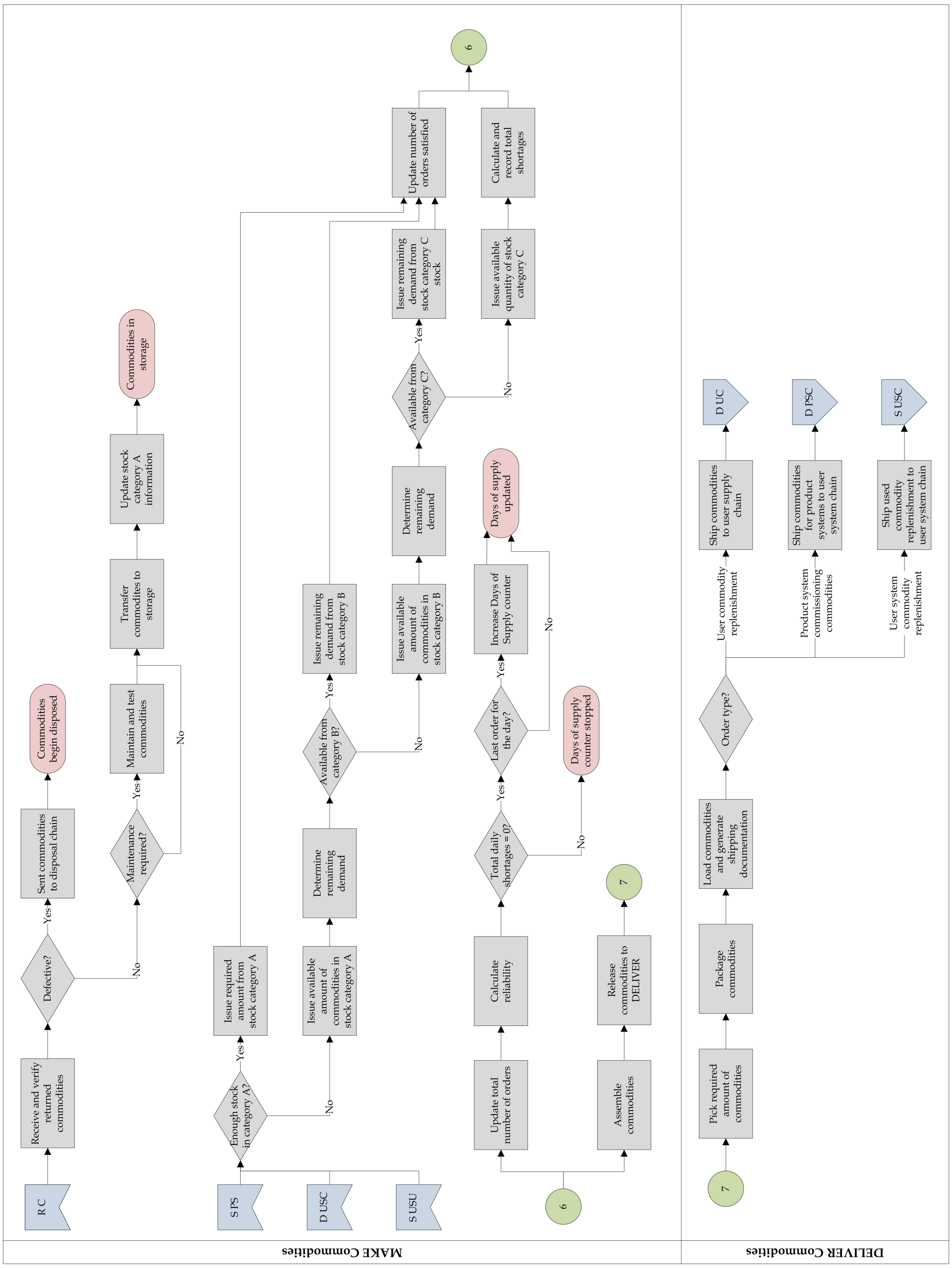

Figure B.3: Commodity supply chain part two.

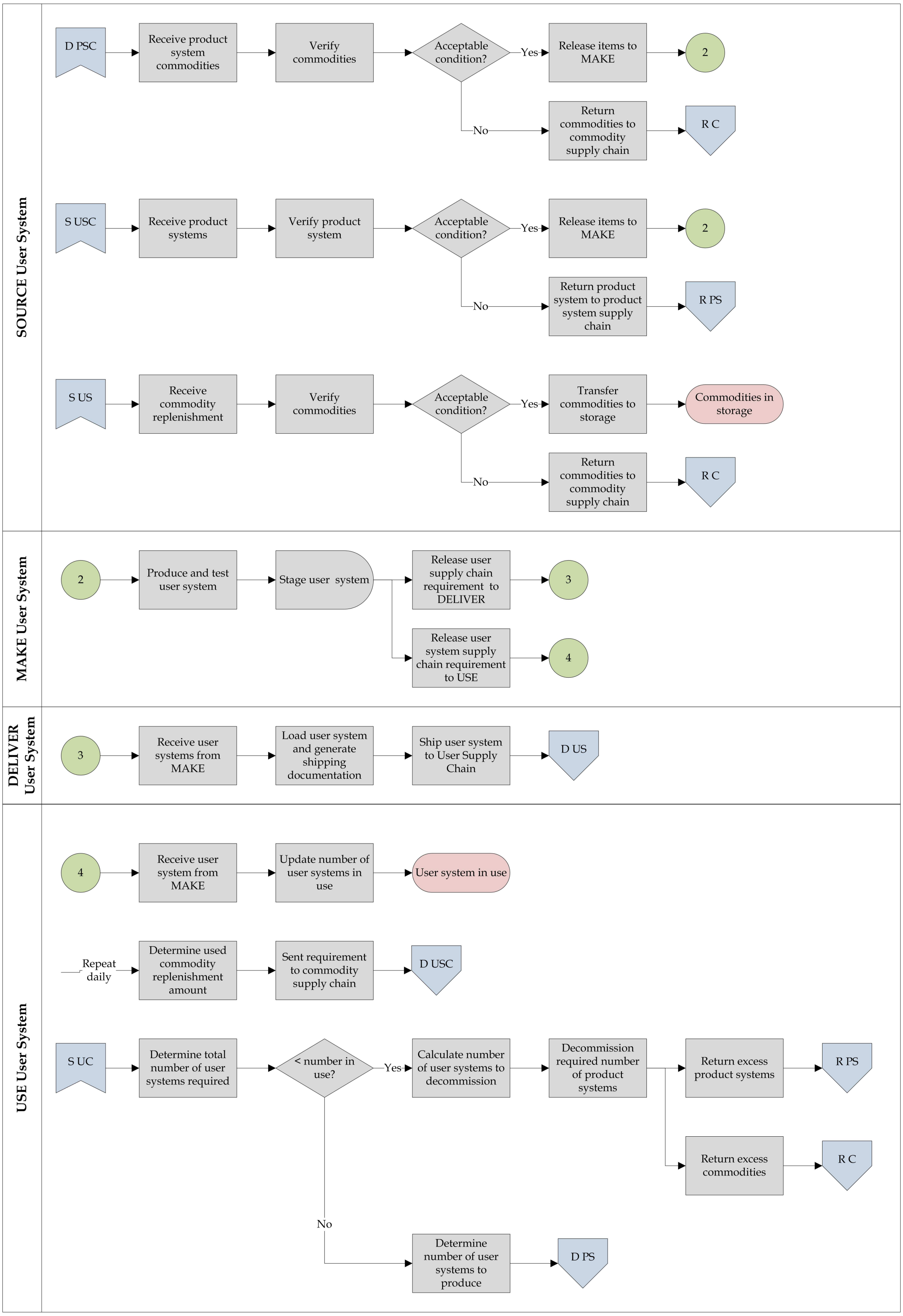

Figure B.4: User system supply chain.

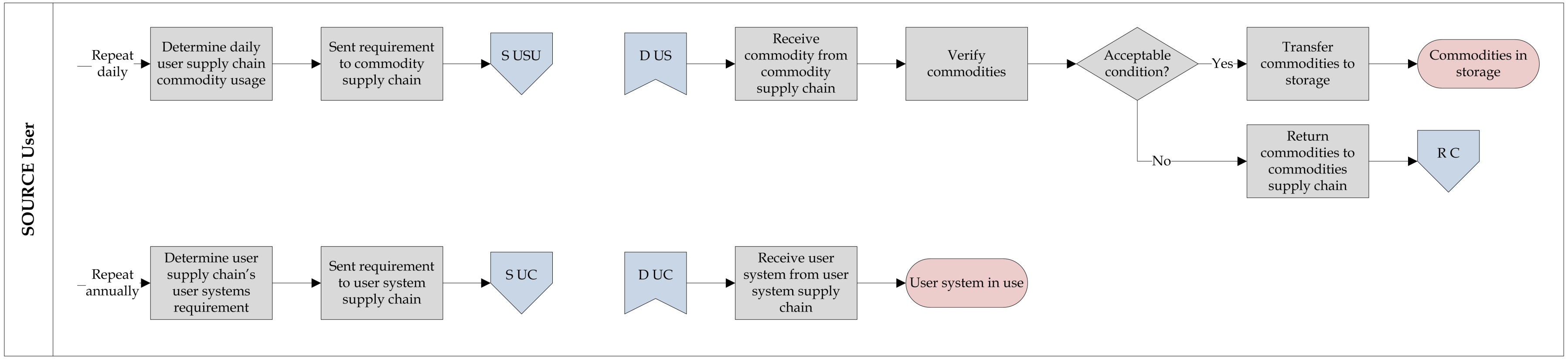

Figure B.5: User supply chain.

Appendix C

Numerical example results

Additional results of the numerical example discussed in Chapter 6 are presented in this appendix. However, it should be noted that these solutions provide no indication of the actual stock situation in any particular military, as they are based on an arbitrary example used to illustrate the functioning of the inventory model presented in Chapter 4.

The inventory model is solved for a duration of five quarters using LINGO 10© optimisation software. A global optimal solution is obtained within 10 minutes and seven seconds on a 2.66 GHz Intel$^{(}R)$ Core$^{(}TM)$2 Duo CPU with 3.48 GB of RAM.

Table C.1: Minimum stock level for every stock category.

Stock category		
A	**B**	**C**
24 879	43 484	120 488

Table C.2: Quantity to order.

		Stock category		
		A	**B**	**C**
Time	**1**	28 858	102 283	152 500
	2	23 979	102 283	201 250
	3	23 979	102 283	201 250
	4	28 858	102 283	201 250
	5	28 858	102 283	120 488

Table C.3: Quantity to deliver.

		Stock category		
		A	**B**	**C**
Time	**1**	0	0	0
	2	28 858	102 283	152 500
	3	23 979	102 283	201 250
	4	23 979	102 283	201 250
	5	28 858	102 283	201 250

Table C.4: Shortage, cost and stock values over five quarters.

	Value
Total cost	R6 097 627
Total number of shortages	27 262 units
Total amount in stock	188 851 units