STOCHASTIC RELIABILITY MODELLING FOR COMPLEX SYSTEMS

by

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CONTENTS

CHAPTER 2: A STUDY OF A TWO UNIT PARALLEL SYSTEM WITH ERLANGIAN REPAIR TIME

CHAPTER 3: AN n UNIT SYSTEM OPERATING IN A RANDOM ENVIRONMENT

CHAPTER 4: BUSY PERIOD ANALYSIS OF A TWO UNIT SYSTEM WITH PREVENTITIVE MAINTENANCE AND IMPERFECT SWITCH

CHAPTER 5: RELIABILITY STOCHASTIC OPTIMIZATION WITH BRANCH AND BOUND TECHNIQUE

CHAPTER 6: A TWO UNIT COLD STANDBY SYSTEM WITH NON-INSTANTANEOUS SWITCOVER

CHAPTER 7: A COMPLEX SYSTEM WITH CORRELATED FAILURES

SUMMARY

Two well-known methods of improving the reliability of a system are

(i) provision of redundants units, and
(ii) repair maintenance.

In a redundant system more units are made available for performing the system function when fewer are required exactly. There are two major types of redundancy- parallel and standby. In this thesis we confine to both these redundant systems. A series system is also studied.

Some of the typical assumptions made in the analysis of redundant systems are

(i) the repair times are assumed to be exponential
(ii) the system measures are modeled but not estimated
(iii) the system is available continuously
(iv) environmental factors not affecting the system
(v) the failures take place only in one stage
(vi) the switching device is perfect
(vii) system reliability for given chance constraints
(viii) the time required to transfer a unit from the standby state to the operating stage is negligible (instantaneous switchover)
(ix) the failures and repairs are independent

However, we frequently come across systems where one or more of these assumptions have to be dropped. This is the motivation for the detailed study of the models presented in this thesis.

In this thesis we present several models of redundant systems relaxing one or more of these assumptions simultaneously. More specifically it is a study of stochastic models of redundant repairable systems with ‘rest period’ for the operator, non-instantaneous switchover, imperfect switch, intermittent use, and series system optimization.

The thesis contains seven chapters. Chapter 1 is introductory in nature and contains a brief description of the mathematical techniques used in the analysis of redundant systems.

In chapter 2, a two unit system with Erlangian repair time is studied by relaxing the assumptions (i) and (ii). The difference- differential equations are formulated for the state probabilities, and the system measures like reliability and the availability are obtained over a long run. The asymptotic interval estimation is studied for these system measures. The model has been illustrated numerically.

In chapter 3, an n unit system operating intermittently, and in a random environment is studied, by relaxing the assumptions (iii) and (iv). In an intermittently used system, the mean number of disappointments is one of the important measures, which has been obtained for this system in the steady state.

In chapter 4, the assumption (v) and (vi) are relaxed. In most of the models studied earlier in reliability analysis is the study of system measures like reliability and availability. In this chapter, profit analysis of a single unit system with three possible modes of the failure of the unit is studied .This chapter consists of two models: in model 1, the unit goes under repair (if a repairman is available) the moment it fails partially, whereas in model 2 the unit goes under repair at complete failure. The repairman appears in, and disappears from, the system randomly. A comparison between these two models has been studied, after calculating numerically the profit and the MTSF.

Contrary to the previous chapters, stochastic optimization is studied using the Branch and Bound technique in chapter 5 (relaxing the assumption (vii)). In this chapter, an n unit system operating in a random environment is considered. The environment determines the number of units required for the satisfactory performance of the system. Assuming that a unit in standby can fail and that the environment is described by a Markov process, we obtained expressions for the distribution and the moments of the time to the first disappointment, and the expected number of disappointments over an arbitrary interval (0, t].

In chapter 6, the assumption (viii) is relaxed. The reliability, availability and the busy period analysis is studied with the assumption of the non-instantaneous switchover (the time taken from standby state to the operating state is non-negligible random variable). It is also assumed that the unit has three possible failure modes (normal, partial and total failure). Numerical example illustrated the results obtained.

The assumption (ix) is relaxed in chapter 7, and a two-unit cold standby system with the provision of rest for the operating unit is studied. Also, the failure and repair times of each unit assumed to be correlated by taking their joint density as bivariate exponential. The system is observed at suitable regenerative epochs to obtain various reliability characteristics of interest, such as the distribution of time to system failure and its mean, and the steady-state probabilities of the system being in up or down states or under repair. Earlier results are verified as particular cases. Numerical example illustrated the results obtained.