CHAPTER 6

CHAOS THEORY BASED MODELS OF SIMPLE SYSTEMS OF CONGESTION

A modified version of this Chapter was presented at a Southern African Institute for Industrial Engineering Conference, 2004.

6.1 Introduction

When embarking on the use of Chaos Theory in modelling simple Systems of Congestion it is considered prudent to provide a benchmark based on the ***classical M/M/1 queue*** to serve as the necessary introductory backdrop to the investigation:

6.1.1 The classical Poisson arrival system

6.1.1.1 The general modelling approach

Modelling a completely random arrival process traditionally involves using the Poisson distribution (negative exponentially distributed inter-arrival times) as the cornerstone of analysis in generating an ordered sequence of arrival events. This implies that the arrival system is treated as being Markovian.

If arrivals are considered to occur within a temporal sequence of equal time intervals, the cumulative Poisson distribution can adequately generate arrivals with the passage of time.
The Poisson distribution of arrivals is given by

$$P_n = \frac{\lambda^n e^{-\lambda}}{n!} \qquad n=0,1,2\ldots \quad \text{and } \lambda > 0 \qquad (6.1)$$

where n = no. of arrivals in a given time interval
λ = average no. of arrivals in the temporal sequence of time intervals

An example of the generation of a Poisson based arrival process for $\lambda = 8$ over 200 one minute time intervals is shown in Fig. 6.1.1.

The generation of the arrival process is driven by a random number generator. The adequacy of the generation process is demonstrated by the achieved results.

GENERATION OF POISSON ARRIVAL EVENTS

		GENERATED PARAMETERS	
LAMBDA	8	AVERAGE	8.03
		STDEV	2.91

PROBABILITY DISTRUBUTION		
CUMULATIVE	NO. OF ARRIVAL EVENTS	PROBABILITY
0.000335463	0	0.000335463
0.003019164	1	0.002683701
0.013753968	2	0.010734804
0.042380112	3	0.028626144
0.0996324	4	0.057252288
0.191236062	5	0.091603662
0.313374278	6	0.122138215
0.452960809	7	0.139586532
0.592547341	8	0.139586532
0.716624259	9	0.124076917
0.815885793	10	0.099261534
0.888075999	11	0.072190206
0.936202803	12	0.048126804
0.965819298	13	0.029616495
0.98274301	14	0.016923711
0.991768989	15	0.009025979
0.996281979	16	0.00451299
0.998405739	17	0.00212376
0.999349632	18	0.000943893
0.999747061	19	0.000397429
0.999906032	20	0.000158971
0.999966593	21	6.05606E-05
0.999988615	22	2.2022E-05

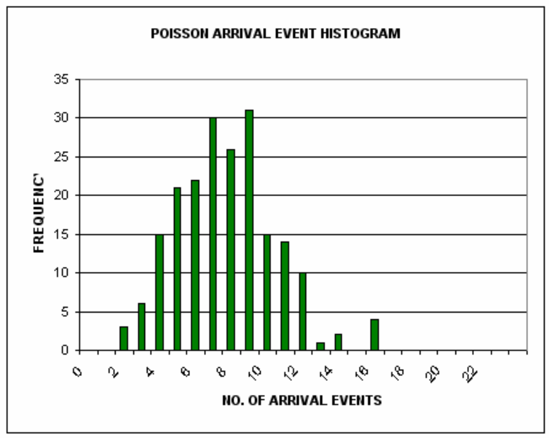
(a)

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	3	3
3	9	6
4	24	15
5	45	21
6	67	22
7	97	30
8	123	26
9	154	31
10	169	15
11	183	14
12	193	10
13	194	1
14	196	2
15	196	0
16	200	4
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

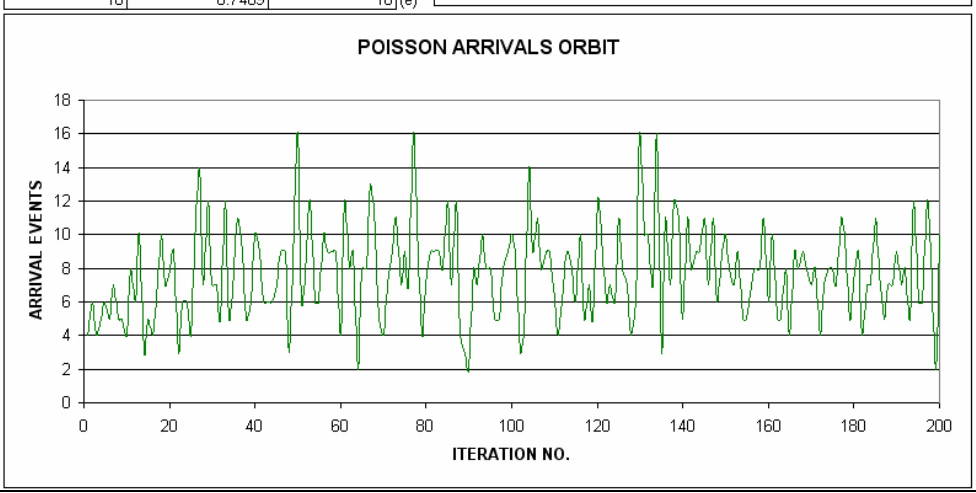
(d)

RANDOM GENERATION		
ITERATION NO.	RAND NO.	ARRIVAL ORBIT
1	0.0571	4
2	0.2663	6
3	0.0483	4
4	0.1391	5
5	0.2272	6
6	0.1425	5
7	0.3459	7
8	0.1163	5
9	0.1401	5
10	0.0728	4
11	0.4712	8
12	0.2148	6
13	0.7215	10
14	0.0363	3
15	0.1081	5
16	0.0573	4
17	0.2419	6
18	0.7409	10

(b)



(e)



(c)

Fig. 6.1.1 GENERATION OF THE ORBIT OF POISSON ARRIVAL EVENTS

6.1.2 The classical exponential service system

6.1.2.1 The general modelling approach

In a similar fashion to the modelling of completely random arrivals (See par. 6.1.1), the modelling of a single completely random service process often involves the Poisson distribution (negative exponentially distributed service times) in generating an ordered sequence of service events. This implies that the service system is treated as being Markovian.

If consecutive service events are considered to occur within a temporal sequence of equal time intervals (synchronously identical to the arrival time intervals) the cumulative Poisson distribution can adequately generate service events with the passage of time.

The Poisson distribution of service events is given by

$$P_n = \frac{\mu^n e^{-\mu}}{n!} \quad n=0,1,2\ldots \text{ and } \mu > 0 \qquad (6.2)$$

where n = no. of service events **offered** in a given time interval

μ = average no. of service events **offered** in the temporal sequence of time intervals

An example of the generation of the service process for $\mu = 10$ over 200 one minute time intervals is shown in Fig. 6.1.2

The generation of the service process is driven by a random number generator. The adequacy of the generation process is demonstrated by the achieved results.

GENERATION OF POISSON SERVICE EVENTS

		GENERATED PARAMETERS	
MU	10	AVERAGE	10.00
		STDEV	3.12

(a)

PROBABILITY DISTRUBUTION		
CUMULATIVE	NO. OF SERVICE EVENTS	PROBABILITY
4.53999E-05	0	4.53999E-05
0.000499399	1	0.000453999
0.002769396	2	0.002269996
0.010336051	3	0.007566655
0.029252688	4	0.018916637
0.067085963	5	0.037833275
0.130141421	6	0.063055458
0.220220647	7	0.090079226
0.332819679	8	0.112599032
0.457929714	9	0.125110036
0.58303975	10	0.125110036
0.696776146	11	0.113736396
0.791556476	12	0.09478033
0.864464423	13	0.072907946
0.916541527	14	0.052077104
0.951259597	15	0.03471807
0.97295839	16	0.021698794
0.985722386	17	0.012763996
0.992813495	18	0.007091109
0.996545658	19	0.003732163
0.998411739	20	0.001866081
0.999300349	21	0.00088861
0.999704263	22	0.000403914

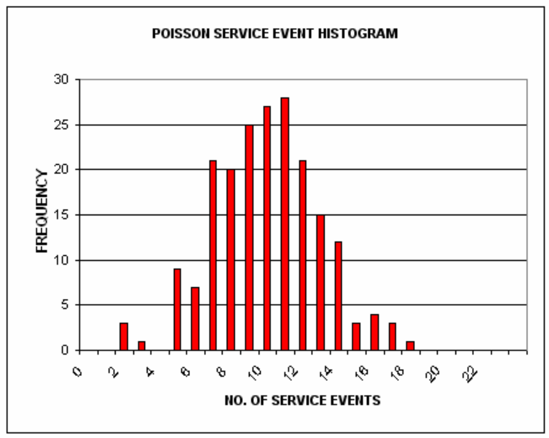
(d)

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	3	3
3	4	1
4	4	0
5	13	9
6	20	7
7	41	21
8	61	20
9	86	25
10	113	27
11	141	28
12	162	21
13	177	15
14	189	12
15	192	3
16	196	4
17	199	3
18	200	1
19	200	0
20	200	0
21	200	0
22	200	0

(b)

RANDOM GENERATION		
ITERATION NO.	RAND NO.	SERVICE ORBIT
1	0.2315	8
2	0.1760	7
3	0.7614	12
4	0.9102	14
5	0.0069	3
6	0.8931	14
7	0.3603	9
8	0.4634	10
9	0.1572	7
10	0.6054	11
11	0.3281	8
12	0.1626	7
13	0.0329	5
14	0.6817	11
15	0.4393	9
16	0.6139	11
17	0.4002	9
18	0.9902	18

(e)



(c)

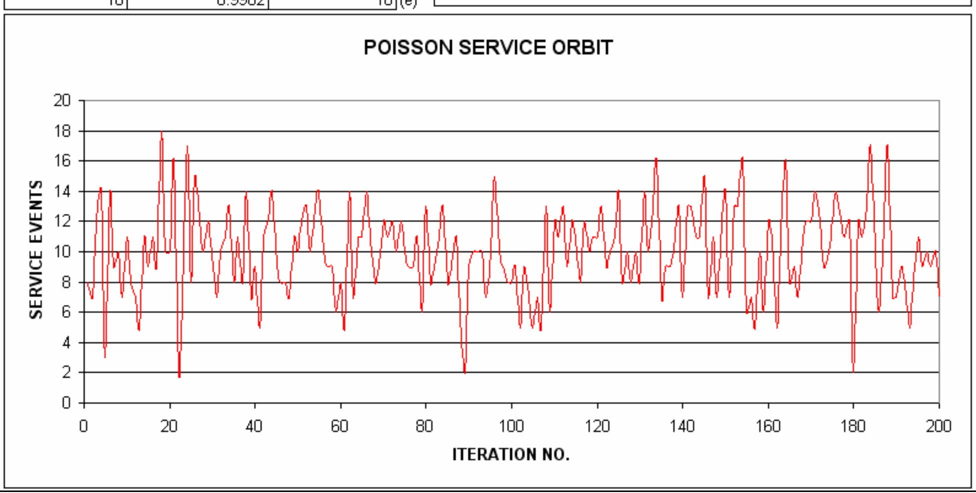


Fig. 6.1.2 GENERATION OF THE ORBIT OF POISSON SERVICE EVENTS

6.1.3 The classical M/M/1 queue

6.1.3.1 The general modelling approach

From the point of view of analyzing Systems of Congestion recent significant developments have addressed approximations and numerical techniques in manipulating steady-state and non steady-state systems. With this in mind and obeying the requirement of **model simplicity and robustness**, the concept of studying ***a temporal sequence of equal time intervals*** plays a central role in modelling the M/M/1 queueing system as it deals with arrival and service events. The novelty of the proposed system model is based on the flow of entities as follows: During a given time interval *(t)* the number of entities in the system at the end of time *t* equals the number of system entities at the beginning of time *t* plus the number of arrival events in time *t* minus the number of service events offered (available) in time *t*, i.e.

No. in system at the end of ($t+\Delta t$)= [No. in system at the beginning of *t*] +
[No. of arrival events in Δt] –
[No. of service events offered in Δt] (6.3)

The model calculates the average number in the system during the interval (Δt) as follows:

If the number of service events offered in Δt exceeds the sum of the number in the system at *t* plus the number of arrival events in Δt, the average number of units in the system during Δt is given by:

[(No. at *t* + No. of arrival events in Δt)/2]**X**
[(No. at *t* + No. of arrival events in Δt) /
(No. of service events in Δt)] (6.4)

If the sum of the number in the system at *t* plus the number of arrival events in Δt exceeds the number of service events offered in Δt the average number of units in the system during Δt is given by:

[(No. at t + No. of arrival events in Δt)+
(No. at t +No. of arrival events in Δt - No. of service events offered in Δt)]/2

(6.5)

A model of the events which take place within a time interval is an example of a highly simplified model of a deterministic **instantaneous** replenishment inventory system which allows shortages to occur during the time interval i.e. when some service events are analogously on offer but not used within the interval as a result of insufficient arrivals.

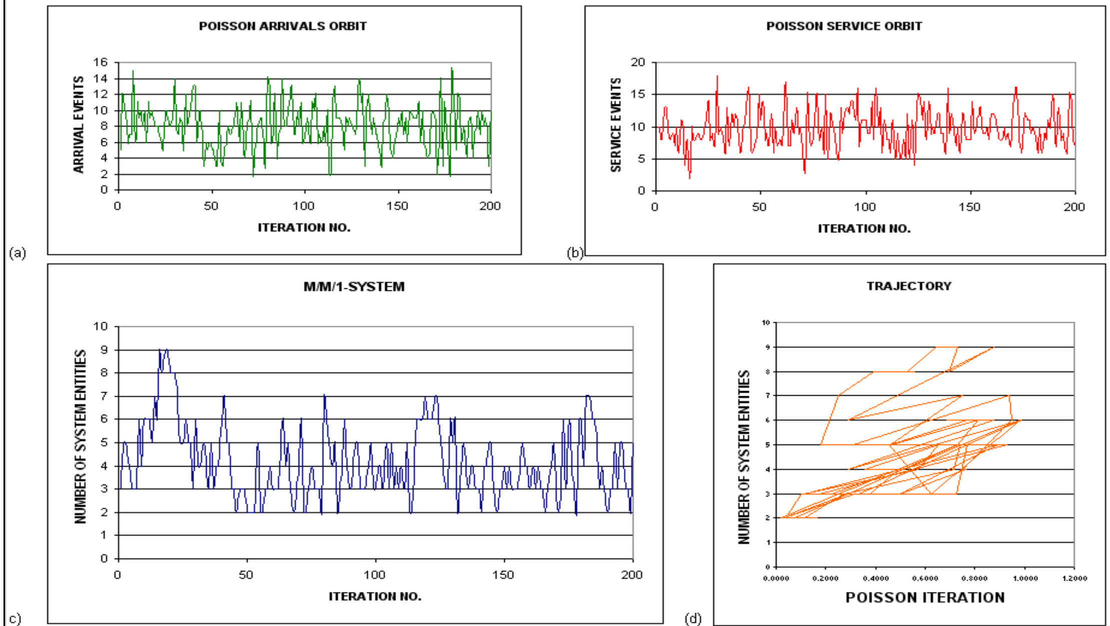
One may speculate that such an elementary model does not meet the requirement of mathematical elegance, or that an attempt is being made to approach the modelling problem pragmatically to avoid immersion into higher mathematics. At this juncture of the modelling process one should await the results which follow, results which are based on further development of the system modelling approach before prematurely judging the merit of the model.

The resulting orbit of number of entities in the system which is obtained by merging the arrival and service processes used in sections 6.1.1 and 6.1.2 does not deliver the required theoretical mean number in the system for the temporal sequence of time intervals. To compensate for this state of affairs the data stream of system entities must be manipulated by means of a ***designer equation***(Appendix B) The designer equation is a necessary adjunct to equations (6.3) and (6.4) to shape the data stream of system entities to reflect reality of system operation modelled via passing reference to interevent times (arrival and service).

The generation of the system state with the passage of time is driven by random number generation and is shown in Fig. 6.1.3. The adequacy of the generation process, which includes the use of a designer equation, is demonstrated by achieved results.

The model can now be used in spreadsheet form for the analysis of steady state and transient operation of an M/M/1 queue. Consequently it may also serve as a ***touchstone*** in evaluating the use of the Chaos based models which follow. One should however not lose sight of the fact that the Poisson/exponential assumption is a mathematical concept and that no real process can be expected to constantly be in agreement with it. It is however heartening to know that use of it as a benchmark will lead to a conservative evaluation of alternative modelling methods.

POISSON ORBIT GENERATION FOR A CLASSICAL M/M/1 SYSTEM



LAMBDA	ARRIVALS		MU	SERVICE			SYSTEM	
8	AVERAGE	8.04	10	AVERAGE	9.99		AVERAGE	4.00
	STDEV	2.79		STDEV	3.11		STDEV	1.41

(e)

PROBABILITY DISTRUBUTION			PROBABILITY DISTRUBUTION		
CUMULATIVE	NO. OF ARRIVAL EVENTS	PROBABILITY	CUMULATIVE	NO. OF SERVICE EVENTS	PROBABILITY
0.000335463	0	0.000335463	4.53999E-05	0	4.53999E-05
0.003019164	1	0.002683701	0.000499399	1	0.000453999
0.013753968	2	0.010734804	0.002769396	2	0.002269996
0.042380112	3	0.028626144	0.010336051	3	0.007566655
0.0996324	4	0.057252288	0.029252688	4	0.018916637
0.191236062	5	0.091603662	0.067085963	5	0.037833275
0.313374278	6	0.122138215	0.130141421	6	0.063055458
0.452960809	7	0.139586532	0.220220647	7	0.090079226
0.592547341	8	0.139586532	0.332819679	8	0.112599032
0.716624259	9	0.124076917	0.457929714	9	0.125110036
0.815885793	10	0.099261534	0.58303975	10	0.125110036
0.888075999	11	0.072190206	0.696776146	11	0.113736396
0.936202803	12	0.048126804	0.791556476	12	0.09478033
0.965819298	13	0.029616495	0.864464423	13	0.072907946
0.98274301	14	0.016923711	0.916541527	14	0.052077104
0.991768989	15	0.009025979	0.951259597	15	0.03471807
0.996281979	16	0.00451299	0.97295839	16	0.021698794
0.998405739	17	0.00212376	0.985722386	17	0.012763996
0.999349632	18	0.000943893	0.992813495	18	0.007091109
0.999747061	19	0.000397429	0.996545658	19	0.003732163
0.999906032	20	0.000158971	0.998411739	20	0.001866081
0.999966593	21	6.05606E-05	0.999300349	21	0.00088861
0.999988615	22	2.2022E-05	0.999704263	22	0.000403914

ITERATION NO.	RAND NO.	ARRIVAL ORBIT	RAND NO.	SERVICE ORBIT	Pop I	F+C-E	Pop J
1	0.1736	5	0.5284	10	0	-5	3
2	0.9290	12	0.4618	10	0	2	5
3	0.7421	10	0.2750	8	2	4	5
4	0.7172	10	0.8555	13	4	1	4
5	0.2748	6	0.8225	13	1	-6	3
6	0.3972	7	0.5112	10	0	-3	3
7	0.3354	7	0.3028	8	0	-1	3
8	0.9879	15	0.4291	9	0	6	6
9	0.2920	6	0.1716	7	6	5	4
10	0.8775	11	0.4910	10	5	6	6
11	0.6879	9	0.1154	6	6	9	6
12	0.7697	10	0.6783	11	9	8	6
13	0.4582	8	0.4420	9	8	7	5
14	0.7478	10	0.0230	4	7	13	7
15	0.2921	6	0.2619	8	13	11	6
16	0.8741	11	0.0022	2	11	20	9

Fig. 6.1.3 GENERATION OF THE ORBIT OF A CLASSICAL POISSON M/M/1 SYSTEM

6.2 Introduction to Chaos generation

Having established the classical M/M/1 queue as the benchmark for the general use of chaos based models the research may progress to create the relevant method of analysis for a chaos driven single channel queue with an average arrival rate of $\lambda=8$ and an average service rate of $\mu=10$. The initial research efforts are based on:

- Verhulst logistic mapping
- Weibull based mapping
- Trigonometric mapping

Fig. 1.1 serves as an example which displays the nature of iterative mapping of the Verhulst type.

6.2.1 The Verhulst generated arrival system

6.2.1.1 The general modelling approach

In attempting to emulate arrival events of an M/M/1 system by using the Verhulst logistic generation method it is necessary to at least achieve "Poissonness" (Grosh [4]) by:

- selecting an appropriate logistic parameter to ensure that "chaotic" randomness is generated, and
- creating an emulated mean and standard deviation which are related as in a Poisson distribution.

At this juncture it must be emphasized that the use of a **designer equation** (Appendix A) becomes mandatory to fashion the data stream of generated arrivals effectively.

An example of the temporal sequence of the number of arrival events in equal time intervals for an average arrival rate of $\lambda=8$ as generated by a Verhulst logistic model over 200 one minute time intervals is shown in Fig. 6.2.1. The adequacy of the generation process is demonstrated by the achieved results.

GENERATION OF VERHULST ARRIVAL EVENTS

LAMBDA	8		
		GENERATED PARAMETERS	
		AVERAGE	7.99
		STDEV	2.87

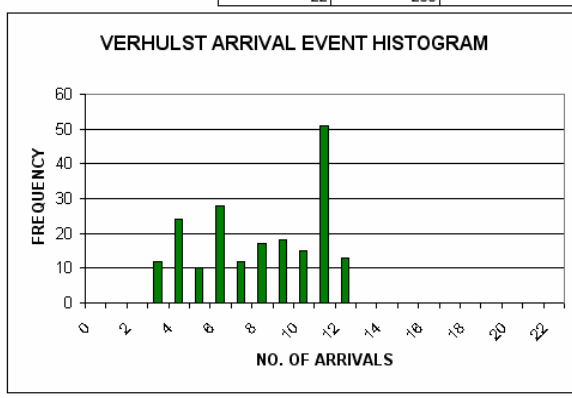
(a)

VERHULST GENERATION			
ITERATION NO	x	F(x)	ARRIVAL ORBIT
1	0.400	0.936	11
2	0.936	0.2336256	4
3	0.234	0.698274248	9
4	0.698	0.821680558	10
5	0.822	0.571434313	8
6	0.571	0.955098842	11
7	0.955	0.167251673	4
8	0.167	0.543186347	7
9	0.543	0.967726264	12
10	0.968	0.121805355	3
11	0.122	0.417178361	6
12	0.417	0.948248247	11
13	0.948	0.191386686	4
14	0.191	0.603555507	8
15	0.604	0.933177402	11
16	0.933	0.24319362	5
17	0.243	0.717796885	9
18	0.718	0.790001616	10
19	0.790	0.647006345	8
20	0.647	0.890717625	11
21	0.891	0.379624978	6
22	0.380	0.918488431	11
23	0.918	0.29198299	5
24	0.292	0.806242802	10
25	0.806	0.609239849	8
26	0.609	0.928459956	11
27	0.928	0.259046058	5
28	0.259	0.748570672	9
29	0.749	0.734029223	9
30	0.734	0.761398259	10
31	0.761	0.708516706	9
32	0.709	0.805431056	10
33	0.805	0.611176294	8
34	0.611	0.926795343	11
35	0.927	0.264598366	5
36	0.265	0.758885675	10
37	0.759	0.713615008	9
38	0.714	0.79703765	10
39	0.797	0.630897674	8
40	0.631	0.908176616	11
41	0.908	0.325228217	5
42	0.325	0.855873813	10

(c

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	12	12
4	36	24
5	46	10
6	74	28
7	86	12
8	103	17
9	121	18
10	136	15
11	187	51
12	200	13
13	200	0
14	200	0
15	200	0
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

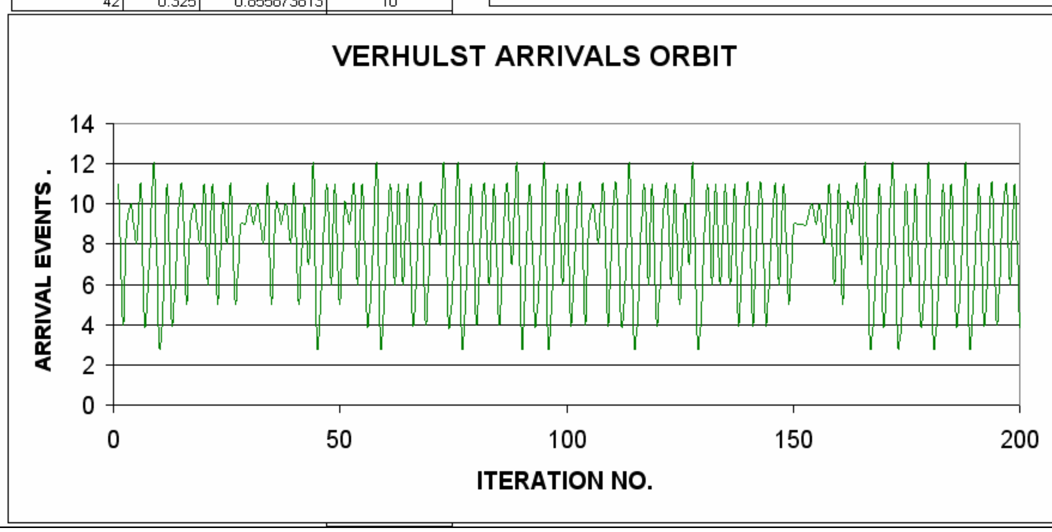


Fig. 6.2.1 GENERATION OF THE ORBIT OF VERHULST ARRIVAL EVENTS

6.2. The Verhulst generated service system

6.2.2.1 The general modelling approach

In attempting to emulate service events of an M/M/1 system by using the Verhulst generation method it is necessary as in the case of arrival events to at least achieve "Poissonness" (Grosh [4]) by:

- selecting an appropriate logistic parameter to ensure that "chaotic" randomness is generated, and
- creating an emulated mean and standard deviation which are related as in a Poisson distribution.

An example of the temporal sequence of the number of service events in 200 one minute equal time intervals for an average service rate of μ=10 as generated by a Verhulst logistic model is shown in Fig. 6.2.2. The adequacy of the generation process is demonstrated by the achieved results.

GENERATION OF VERHULST SERVICE EVENTS

		GENERATED PARAMETERS	
MU	10	AVERAGE	10.04
		STDEV	3.18

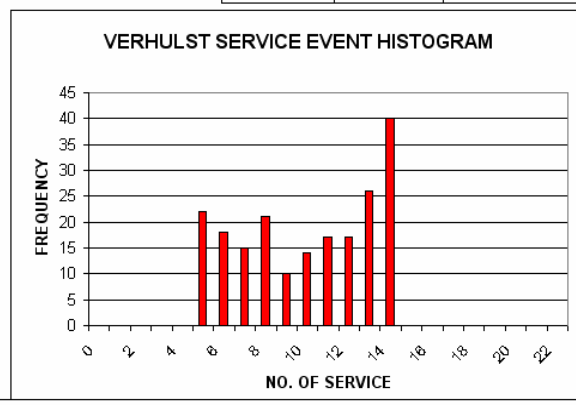
(a)

VERHULST GENERATION			
ITERATION NO	x	F(x)	SERVICE ORBIT
1	0.400	0.936	14
2	0.936	0.2336256	6
3	0.234	0.698274248	11
4	0.698	0.821680558	12
5	0.822	0.571434313	10
6	0.571	0.955098842	14
7	0.955	0.167251673	5
8	0.167	0.543186347	9
9	0.543	0.967726264	14
10	0.968	0.121805355	5
11	0.122	0.417178361	8
12	0.417	0.948248247	14
13	0.948	0.191386686	6
14	0.191	0.603555507	10
15	0.604	0.933177402	14
16	0.933	0.24319362	6
17	0.243	0.717796885	11
18	0.718	0.790001616	12
19	0.790	0.647006345	11
20	0.647	0.890717625	13
21	0.891	0.379624978	8
22	0.380	0.918488431	13
23	0.918	0.29198299	7
24	0.292	0.806242802	12
25	0.806	0.609239849	10
26	0.609	0.928459956	14
27	0.928	0.259046058	6
28	0.259	0.748570672	12
29	0.749	0.734029223	11
30	0.734	0.761398259	12
31	0.761	0.708516706	11
32	0.709	0.805431056	12
33	0.805	0.611176294	10
34	0.611	0.926795343	14
35	0.927	0.264598366	6
36	0.265	0.758885675	12
37	0.759	0.713615008	11
38	0.714	0.79703765	12
39	0.797	0.630897674	10
40	0.631	0.908176616	13
41	0.908	0.325228217	7
42	0.325	0.855873813	13

(c)

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0
5	22	22
6	40	18
7	55	15
8	76	21
9	86	10
10	100	14
11	117	17
12	134	17
13	160	26
14	200	40
15	200	0
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

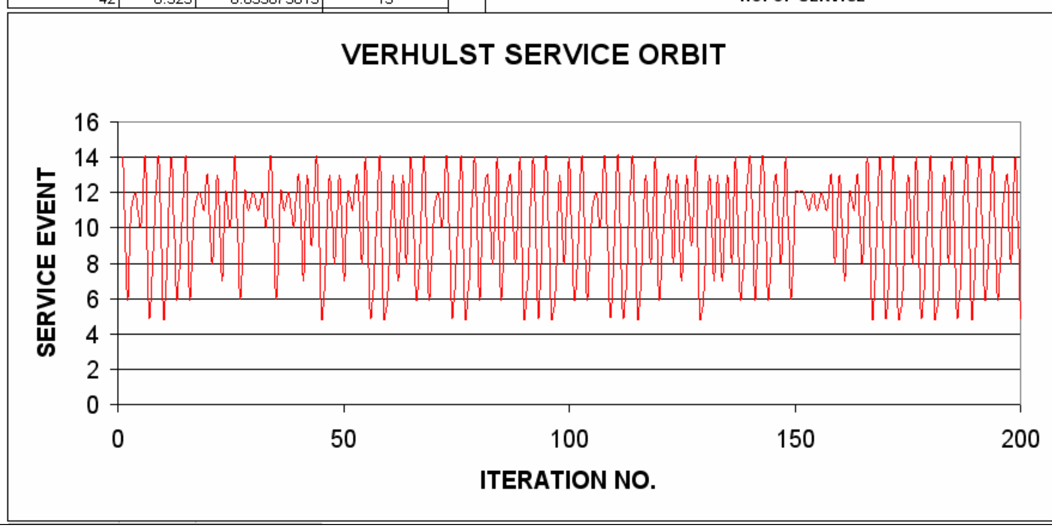


Fig. 6.2.2 GENERATION OF THE ORBIT OF VERHULST SERVICE EVENTS

6.2.3 The Verhulst generated single channel queue

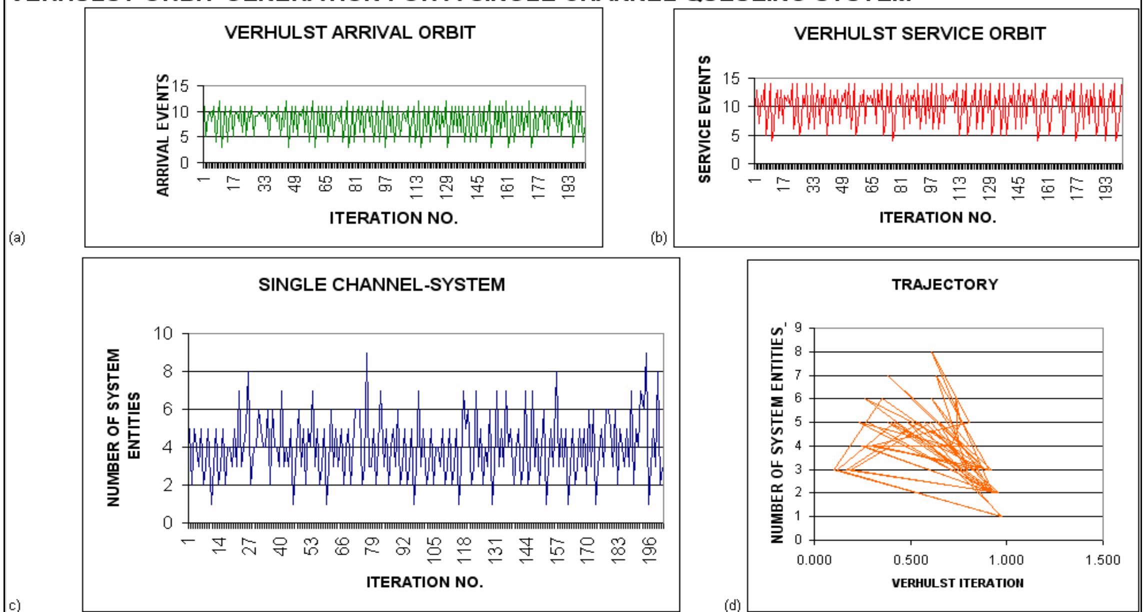
6.2.3.1 The general modelling approach

If as at the outset of this chapter considering the use of chaos generation methods to model a single channel queueing system by means of approximations and numerical techniques is heeded, and robustness and simplicity of modelling is to be achieved, the concept of studying a temporal sequence of equal time intervals which accommodate arrival and service events is justified.

As in the case of the classical M/M/1 queue analysis of par. 6.1.3.1 the Verhulst system model makes use of the highly simplified model described in equation (6.3) which also requires manipulation of the generated data stream by ***designer equations***.

The generation of the system state with the passage of time is driven by chaos iterative generation and is shown in Fig 6.2.3. The adequacy of the generation process, which includes the use of a designer equation, is demonstrated by the achieved results.

VERHULST ORBIT GENERATION FOR A SINGLE CHANNEL QUEUEING SYSTEM



LAMBDA	ARRIVALS		MU	SERVICE	
8	AVERAGE	8.04	10	AVERAGE	10.02
	STDEV	2.81		STDEV	3.14

SYSTEM	
AVERAGE	3.98
STDEV	1.73

(e)

ITERATION NO.	CHAOS GENERATION		ARRIVALS ORBIT	CHAOS GENERATION		SERVICES ORBIT	BEGIN POP	BE+AR-SE	END POP
1	0.400	0.936	11	0.800	0.632	10	0	1	5
2	0.936	0.2336256	5	0.632	0.9186752	13	1	-7	2
3	0.234	0.698274248	9	0.919	0.295108754	7	0	2	5
4	0.698	0.821680558	10	0.295	0.82167733	12	2	0	4
5	0.822	0.571434313	8	0.822	0.578768597	10	0	-2	3
6	0.571	0.955098842	11	0.579	0.962992257	14	0	-3	5
7	0.955	0.167251673	4	0.963	0.14077077	5	0	-1	2
8	0.167	0.543186347	7	0.141	0.477769724	9	0	-2	3
9	0.543	0.967726264	12	0.478	0.985547969	14	0	-2	5
10	0.968	0.121805355	3	0.986	0.056260523	4	0	-1	1
11	0.122	0.417178361	6	0.056	0.209726341	6	0	0	3
12	0.417	0.948248247	11	0.210	0.654677751	11	0	0	5
13	0.948	0.191386686	4	0.655	0.892995433	13	0	-9	2
14	0.191	0.603555507	8	0.893	0.377440628	8	0	0	3
15	0.604	0.933177402	11	0.377	0.928167841	13	0	-2	5
16	0.933	0.24319362	5	0.928	0.263355585	7	0	-2	2
17	0.243	0.717796885	9	0.263	0.766297712	12	0	-3	4
18	0.718	0.790001616	10	0.766	0.707387839	11	0	-1	4
19	0.790	0.647006345	8	0.707	0.817611624	12	0	-4	3
20	0.647	0.890717625	11	0.818	0.589035284	10	0	1	5
21	0.891	0.379624978	6	0.589	0.956187237	14	1	-7	3
22	0.380	0.918488431	11	0.956	0.165478158	6	0	5	7
23	0.918	0.29198299	5	0.165	0.545475793	9	5	1	3
24	0.292	0.806242802	10	0.545	0.979331212	14	1	-3	4
25	0.806	0.609239849	8	0.979	0.079954279	5	0	3	5
26	0.609	0.928459956	11	0.080	0.29056829	7	3	7	8
27	0.928	0.259046058	5	0.291	0.814246517	12	7	0	2
28	0.259	0.748570672	9	0.814	0.597434051	10	0	-1	4
29	0.749	0.734029223	9	0.597	0.950001093	14	0	-5	4
30	0.734	0.761398259	10	0.950	0.187621115	6	0	4	6
31	0.761	0.708516706	9	0.188	0.602056758	10	4	3	5
32	0.709	0.805431056	10	0.602	0.946358452	14	3	-1	4
33	0.805	0.611176294	8	0.946	0.200518322	6	0	2	4
34	0.611	0.926795343	11	0.201	0.633227363	10	2	3	6
35	0.927	0.264598366	5	0.633	0.917389355	13	3	-5	2
36	0.265	0.758885675	10	0.917	0.299355198	7	0	3	6
37	0.759	0.713615008	9	0.299	0.82847957	12	3	0	4
38	0.714	0.79703765	10	0.828	0.56129963	10	0	0	4
39	0.797	0.630897674	8	0.561	0.972657304	14	0	-6	3
40	0.631	0.908176616	11	0.973	0.105050539	5	0	6	7
41	0.908	0.325228217	5	0.105	0.371358946	8	6	3	3
42	0.325	0.855873813	10	0.371	0.922133343	13	3	0	4
43	0.856	0.481079935	7	0.922	0.283623592	7	0	0	3
44	0.481	0.973603921	12	0.284	0.802565938	12	0	0	5
45	0.974	0.10022737	3	0.803	0.62589272	10	0	-7	1

Fig. 6.2.3 GENERATION OF THE ORBIT OF A VERHULST SINGLE CHANNEL QUEUEING SYSTEM

6.2.4 Benchmarking the Verhulst generated single channel queue model

Comparison of the Poisson M/M/1 and Verhulst methods of generating system dynamics as depicted in Figs. 6.1.3 and 6.2.3 respectively results in

- achieving equivalence of mean and standard deviation values for the arrival and service processes,
- achieving graphical plausibility of system orbit likeness i.e. applying the TLAR criterion ("that looks about right") in comparing the two system entity orbits.

No quantitative justification for "Poissonness" other than the foregoing parameter determination and application of the TLAR plausibility criterion has been carried out.

As a further matter of interest the Verhulst methods of generating system dynamics over 200 one minute intervals are shown in Fig. 6.2.4 for a **<u>general service distribution</u>** queueing system for $\lambda = 8, \mu = 10$ and $\sigma = 0.010$. The average number of entities in the system is given by:

$$L = L_q + \rho \tag{6.6}$$

for $$L_q = \frac{\lambda^2\sigma^2 + \rho^2}{2(1-\rho)} \tag{6.7}$$

and $$\rho = \frac{\lambda}{\mu} \tag{6.8}$$

where:

L = the average number of entities in the system

L_q =the average number of entities in the queue

ρ =the traffic intensity

λ =the average number of arrivals entering the system per unit time

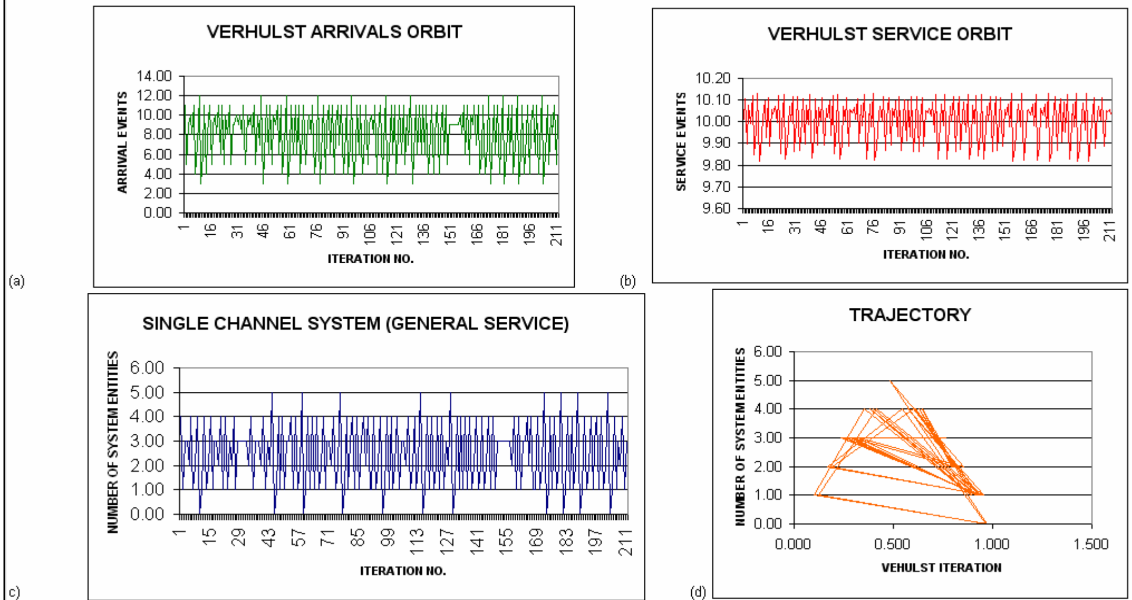
σ^2 =the variance of the service time

μ =the average number of services offered per unit time

The results indicate

- achieving equivalence of mean and standard deviation values for the arrival and service processes,
- achieving graphical plausibility of system orbit likeness i.e. applying the TLAR criterion in comparing the two system entity orbits.

VERHULST-ORBIT GENERATION FOR A SINGLE CHANNEL QUEUEING SYSTEM (GENERAL SERVICE)



LAMBDA	ARRIVALS		MU	SERVICE		SYSTEM	
8	AVERAGE	8.00	10	AVERAGE	10.00	AVERAGE	2.42
	STDEV	2.82		VARIANCE	0.010	STDEV	1.42

(e)

ITERATION NO.	CHAOS GENERATION		ARRIVALS ORBIT	CHAOS GENERATION		SERVICES ORBIT	BEGIN POP	BE+AR-SE	END POP
1	0.400	0.936	11.00	0.800	0.632	10.01	0	0.99	4.00
2	0.936	0.234	5.00	0.632	0.919	10.11	0.99	-4.12	1.00
3	0.234	0.698	9.00	0.919	0.295	9.90	0.00	-0.90	3.00
4	0.698	0.822	10.00	0.295	0.822	10.07	0.00	-0.07	3.00
5	0.822	0.571	8.00	0.822	0.579	9.99	0.00	-1.99	2.00
6	0.571	0.955	11.00	0.579	0.963	10.12	0.00	0.88	4.00
7	0.955	0.167	4.00	0.963	0.141	9.85	0.88	-4.97	1.00
8	0.167	0.543	7.00	0.141	0.478	9.96	0.00	-2.96	2.00
9	0.543	0.968	12.00	0.478	0.986	10.13	0.00	1.87	4.00
10	0.968	0.122	3.00	0.986	0.056	9.82	1.87	-4.95	0.00
11	0.122	0.417	6.00	0.056	0.210	9.87	0.00	-3.87	1.00
12	0.417	0.948	11.00	0.210	0.655	10.02	0.00	0.98	4.00
13	0.948	0.191	4.00	0.655	0.893	10.10	0.98	-5.12	1.00
14	0.191	0.604	8.00	0.893	0.377	9.93	0.00	-1.93	2.00
15	0.604	0.933	11.00	0.377	0.928	10.11	0.00	0.89	4.00
16	0.933	0.243	5.00	0.928	0.263	9.89	0.89	-4.00	1.00
17	0.243	0.718	9.00	0.263	0.766	10.06	0.00	-1.06	3.00
18	0.718	0.790	10.00	0.766	0.707	10.04	0.00	-0.04	3.00
19	0.790	0.647	8.00	0.707	0.818	10.07	0.00	-2.07	2.00
20	0.647	0.891	11.00	0.818	0.589	10.00	0.00	1.00	4.00
21	0.891	0.380	6.00	0.589	0.956	10.12	1.00	-3.12	1.00
22	0.380	0.918	11.00	0.956	0.165	9.86	0.00	1.14	4.00
23	0.918	0.292	5.00	0.165	0.545	9.98	1.14	-3.84	1.00
24	0.292	0.806	10.00	0.545	0.979	10.13	0.00	-0.13	3.00
25	0.806	0.609	8.00	0.979	0.080	9.83	0.00	-1.83	2.00
26	0.609	0.928	11.00	0.080	0.291	9.90	0.00	1.10	4.00
27	0.928	0.259	5.00	0.291	0.814	10.07	1.10	-3.97	1.00
28	0.259	0.749	9.00	0.814	0.597	10.00	0.00	-1.00	3.00
29	0.749	0.734	9.00	0.597	0.950	10.12	0.00	-1.12	3.00
30	0.734	0.761	10.00	0.950	0.188	9.86	0.00	0.14	3.00
31	0.761	0.709	9.00	0.188	0.602	10.00	0.14	-0.86	3.00
32	0.709	0.805	10.00	0.602	0.946	10.12	0.00	-0.12	3.00
33	0.805	0.611	8.00	0.946	0.201	9.87	0.00	-1.87	2.00
34	0.611	0.927	11.00	0.201	0.633	10.01	0.00	0.99	4.00
35	0.927	0.265	5.00	0.633	0.917	10.11	0.99	-4.12	1.00
36	0.265	0.759	10.00	0.917	0.299	9.90	0.00	0.10	3.00
37	0.759	0.714	9.00	0.299	0.828	10.08	0.10	-0.98	3.00
38	0.714	0.797	10.00	0.828	0.561	9.99	0.00	0.01	3.00
39	0.797	0.631	8.00	0.561	0.973	10.12	0.01	-2.11	2.00
40	0.631	0.908	11.00	0.973	0.105	9.84	0.00	1.16	4.00
41	0.908	0.325	5.00	0.105	0.371	9.92	1.16	-3.76	1.00
42	0.325	0.856	10.00	0.371	0.922	10.11	0.00	-0.11	3.00
43	0.856	0.481	7.00	0.922	0.284	9.89	0.00	-2.89	2.00
44	0.481	0.974	12.00	0.284	0.803	10.07	0.00	1.93	5.00
45	0.974	0.100	3.00	0.803	0.626	10.01	1.93	-5.08	0.00

Fig. 6.2.4 GENERATION OF THE ORBIT OF A VERHULST QUEUEING SYSTEM (GENERAL SERVICE DISTRUBUTION)

6.2.5 Extending the Verhulst generated single channel queue model to deal with variable traffic intensity

Having achieved a degree of likeness greater than a scant semblance between the classical M/M/1 and Verhulst queueing system one may embark on extending the Verhulst model to include a range of traffic intensities which may prove to be beneficial in analysing the transient (dynamic) and steady state operation of a single channel queue.

Consequently the Verhulst queueing system has been extended to include a range of average arrival rates ($0.2 \leq \lambda < 1$) for an average service rate μ=10.

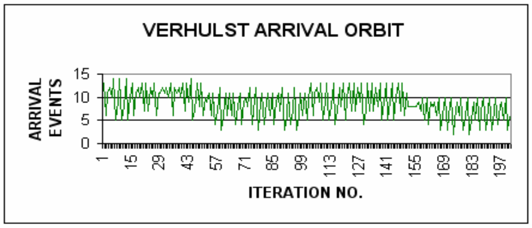
Traffic intensity $\rho = \frac{\lambda}{10}$

An example of a Verhulst generated single channel queue for a chronological sequence of values of λ of 9.8; 8.0; 9.5; and 7.0 over 200 one minute intervals is shown in Fig. 6.2.5.

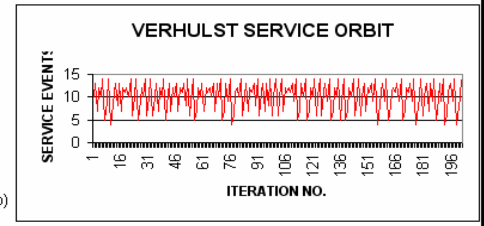
Each of the chronological values of λ are employed for four consecutive epochs of 50 consecutive intervals.

VERHULST ORBIT GENERATION FOR A SINGLE CHANNEL QUEUE
CONSECUTIVE AVERAGE ARRIVAL RATES=9.8; 8.0; 9.5; 7.0
OVERALL AVERAGE SERVICE RATE =10

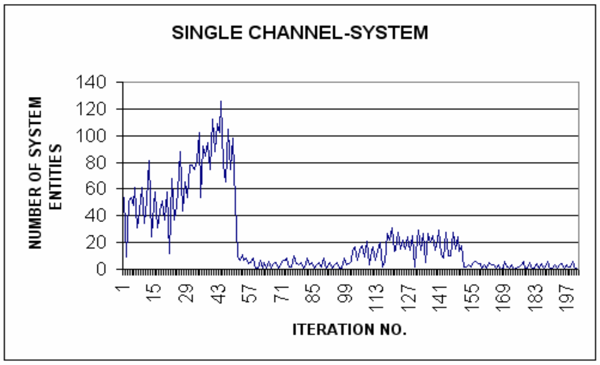
(a)



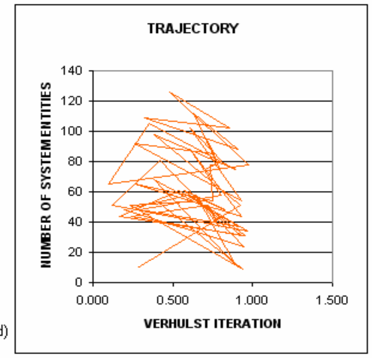
(b)



c)



(d)



(e)

SECTOR	1	2	3	4
ARRIVALS	9.8	8	9.5	7
RHO	0.98	0.8	0.95	0.7

Iteration	RHO	CHAOS GENERATION			ARRIVALS ORBIT	CHAOS GENERATION		SERVICES ORBIT	Pop I	D-I+J	Pop J
1	0.98	0.400	0.936	13.38	13	0.800	0.632	10	0	3	54
2	0.98	0.936	0.233626	5.92	6	0.632	0.9186752	13	3	-4	9
3	0.98	0.234	0.698274	10.86	11	0.9186752	0.295108754	7	0	4	51
4	0.98	0.698	0.821681	12.17	12	0.2951088	0.82167733	12	4	4	54
5	0.98	0.822	0.571434	9.51	10	0.8216773	0.578768597	10	4	4	48
6	0.98	0.571	0.955099	13.58	14	0.5787686	0.962992257	14	4	4	61
7	0.98	0.955	0.167252	5.22	5	0.9629923	0.14077077	5	4	4	31
8	0.98	0.167	0.543186	9.21	9	0.1407708	0.477769724	9	4	4	44
9	0.98	0.543	0.967726	13.72	14	0.4777697	0.985547969	14	4	4	61
10	0.98	0.968	0.121805	4.73	5	0.985548	0.056260523	4	4	5	34
11	0.98	0.122	0.417178	7.87	8	0.0562605	0.209726341	6	5	7	51
12	0.98	0.417	0.948248	13.51	14	0.2097263	0.654677751	11	7	10	81
13	0.98	0.948	0.191387	5.47	5	0.6546778	0.892995433	13	10	2	24
14	0.98	0.191	0.603556	9.85	10	0.8929954	0.377440628	8	2	4	48
15	0.98	0.604	0.933177	13.35	13	0.3774406	0.928167841	13	4	4	58
16	0.98	0.933	0.243194	6.02	6	0.9281678	0.263355585	7	4	3	31
17	0.98	0.243	0.717797	11.06	11	0.2633556	0.766297712	12	3	2	44
18	0.98	0.718	0.790002	11.83	12	0.7662977	0.707387839	11	2	3	51
19	0.98	0.790	0.647006	10.31	10	0.7073878	0.817611624	12	3	1	37
20	0.98	0.647	0.890718	12.90	13	0.8176116	0.589035284	10	1	4	58
21	0.98	0.891	0.379625	7.47	7	0.5890353	0.956187237	14	4	-3	12
22	0.98	0.380	0.918488	13.20	13	0.9561872	0.165478158	6	0	7	68
23	0.98	0.918	0.291983	6.54	7	0.1654782	0.545475793	10	7	4	37
24	0.98	0.292	0.806243	12.00	12	0.5454758	0.979331212	14	4	2	48
25	0.98	0.806	0.60924	9.91	10	0.9793312	0.079954279	5	2	7	58
26	0.98	0.609	0.92846	13.30	13	0.0799543	0.29056829	7	7	13	88
27	0.98	0.928	0.259046	6.19	6	0.2905683	0.814246517	12	13	7	44
28	0.98	0.259	0.748571	11.39	11	0.8142465	0.597434051	10	7	8	65
29	0.98	0.749	0.734029	11.24	11	0.5974341	0.950001093	14	8	5	54
30	0.98	0.734	0.761398	11.53	12	0.9500011	0.187621115	6	5	11	78
31	0.98	0.761	0.708517	10.97	11	0.1876211	0.602056758	10	11	12	78
32	0.98	0.709	0.805431	11.99	12	0.6020568	0.946358452	14	12	10	75
33	0.98	0.805	0.611176	9.93	10	0.9463585	0.200518322	6	10	14	81
34	0.98	0.611	0.926795	13.28	13	0.2005183	0.633227363	10	14	17	102
35	0.98	0.927	0.264598	6.25	6	0.6332274	0.917389355	13	17	10	54
36	0.98	0.265	0.758886	11.50	12	0.9173894	0.299355198	7	10	15	92
37	0.98	0.759	0.713615	11.02	11	0.2993552	0.82847957	12	15	14	85
38	0.98	0.714	0.797038	11.91	12	0.8284796	0.56129963	10	14	16	95
39	0.98	0.797	0.630898	10.14	10	0.5612996	0.972657304	14	16	12	75
40	0.98	0.631	0.908177	13.09	13	0.9726573	0.105050539	5	12	20	112

Fig. 6.2.5 GENERATION OF THE ORBIT OF AN EXTENDED VERHULST SINGLE CHANNEL QUEUE

The system orbit generated for a total of 200 one minute consecutive intervals unambiguously displays how the system behaves dynamically in a natural sense to being subjected to step functions in average arrival rate, albeit that the transitions from one steady state to a following steady state are ephemeral.

The extended model is versatile and amenable to use of many values of traffic intensity which may occur in practical situations. Such traffic intensity values may be selected a priori by external control or by automatically adjusting the arrival and service processes by means of internal system feedback mechanisms.

6.3 Further examples of Chaos generation

The introduction to Chaos generating methods described in par. 6.2 makes mention of other methods of mapping which may be considered as alternatives to Verhulst logistic mapping i.e.

- Weibull based mapping, and
- Trigonometric mapping. (Stewart [38])

The general modelling approach used for the generation of orbits for the two above- mentioned methods of mapping slavishly follows the underlying mathematical regimen employed in par.6.2.

The results which have been achieved are shown in :

- Fig. 6.3.1: Generation of the orbit of Weibull arrival events
- Fig. 6.3.2: Generation of the orbit of Weibull service events
- Fig. 6.3.3: Generation of the orbit of a Weibull single channel queueing system
- Fig. 6.3.4: Generation of the orbit of Sin arrival events
- Fig. 6.3.5: Generation of the orbit of Sin service events
- Fig. 6.3.6: Generation of the orbit of a Sin single channel queueing system.

The orbits shown have all been prepared for an average arrival rate of $\lambda = 8$ and an average service rate of $\mu = 10$.

GENERATION OF WEIBULL ARRIVAL EVENTS

		GENERATED PARAMETERS	
LAMBDA	8	AVERAGE	8.00
		STDEV	2.83

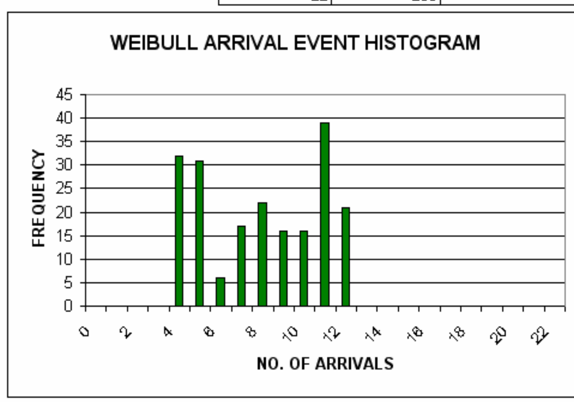
(a)

WEIBULL GENERATION			
ITERATION NO	x	F(x)	ARRIVAL ORBIT
1	0.400	0.65682321	11.00
2	0.657	0.18938712	5.00
3	0.189	0.68906617	11.00
4	0.689	0.15044298	5.00
5	0.150	0.59582709	10.00
6	0.596	0.28034964	6.00
7	0.280	0.77566855	12.00
8	0.776	0.07511082	4.00
9	0.075	0.33170071	7.00
10	0.332	0.75030692	12.00
11	0.750	0.09310860	4.00
12	0.093	0.40327982	8.00
13	0.403	0.65112079	11.00
14	0.651	0.19693552	5.00
15	0.197	0.70326265	11.00
16	0.703	0.13527252	5.00
17	0.135	0.55083657	9.00
18	0.551	0.36074484	7.00
19	0.361	0.71731931	11.00
20	0.717	0.12140092	4.00
21	0.121	0.50576143	9.00
22	0.506	0.44945949	8.00
23	0.449	0.56381140	10.00
24	0.564	0.33655443	7.00
25	0.337	0.74562342	12.00
26	0.746	0.09677699	4.00
27	0.097	0.41730143	8.00
28	0.417	0.62586178	10.00
29	0.626	0.23276804	6.00
30	0.233	0.75310997	12.00
31	0.753	0.09096602	4.00
32	0.091	0.39499689	8.00
33	0.395	0.66535496	11.00
34	0.665	0.17846629	5.00
35	0.178	0.66626762	11.00
36	0.666	0.17732447	5.00
37	0.177	0.66373080	11.00
38	0.664	0.18051086	5.00
39	0.181	0.67073797	11.00
40	0.671	0.17180517	5.00
41	0.172	0.65106326	11.00
42	0.651	0.19701269	5.00

(c)

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	0	0
4	32	32
5	63	31
6	69	6
7	86	17
8	108	22
9	124	16
10	140	16
11	179	39
12	200	21
13	200	0
14	200	0
15	200	0
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

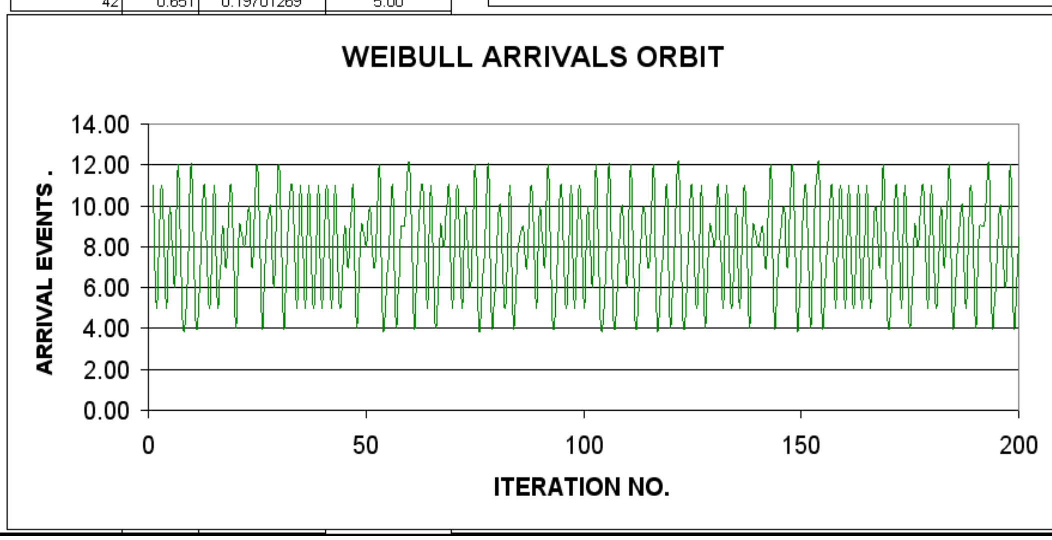


Fig. 6.3.1 GENERATION OF THE ORBIT OF WEIBULL ARRIVAL EVENTS

GENERATION OF WEIBULL SERVICE EVENTS

		GENERATED PARAMETERS	
MU	10	AVERAGE	10.00
		STDEV	3.15

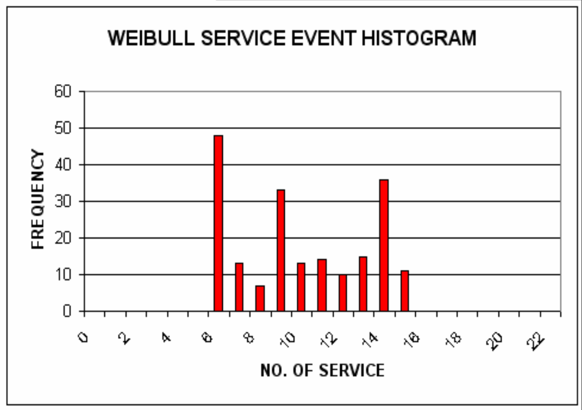
(a)

WEIBULL GENERATION			
ITERATION NO	x	F(x)	SERVICE ORBIT
1	0.800	0.05590191	6.00
2	0.056	0.25718547	8.00
3	0.257	0.78185409	15.00
4	0.782	0.06598148	6.00
5	0.066	0.30111610	9.00
6	0.301	0.77908007	15.00
7	0.779	0.06764601	6.00
8	0.068	0.30826131	9.00
9	0.308	0.77505888	14.00
10	0.775	0.07011954	6.00
11	0.070	0.31881806	9.00
12	0.319	0.76746148	14.00
13	0.767	0.07499337	6.00
14	0.075	0.33939644	9.00
15	0.339	0.74738719	14.00
16	0.747	0.08919508	6.00
17	0.089	0.39752768	10.00
18	0.398	0.66052394	13.00
19	0.661	0.17614791	7.00
20	0.176	0.67456697	13.00
21	0.675	0.15903049	7.00
22	0.159	0.63242650	13.00
23	0.632	0.21415531	8.00
24	0.214	0.74390829	14.00
25	0.744	0.09186038	6.00
26	0.092	0.40810961	10.00
27	0.408	0.64114435	13.00
28	0.641	0.20182387	7.00
29	0.202	0.72511987	14.00
30	0.725	0.10735631	6.00
31	0.107	0.46737089	11.00
32	0.467	0.52203615	11.00
33	0.522	0.40860719	10.00
34	0.409	0.64021325	13.00
35	0.640	0.20311780	7.00
36	0.203	0.72725910	14.00
37	0.727	0.10549589	6.00
38	0.105	0.46046890	11.00
39	0.460	0.53644417	12.00
40	0.536	0.37982589	10.00
41	0.380	0.69087035	13.00
42	0.691	0.14069829	7.00

(c

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0
5	0	0
6	48	48
7	61	13
8	68	7
9	101	33
10	114	13
11	128	14
12	138	10
13	153	15
14	189	36
15	200	11
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

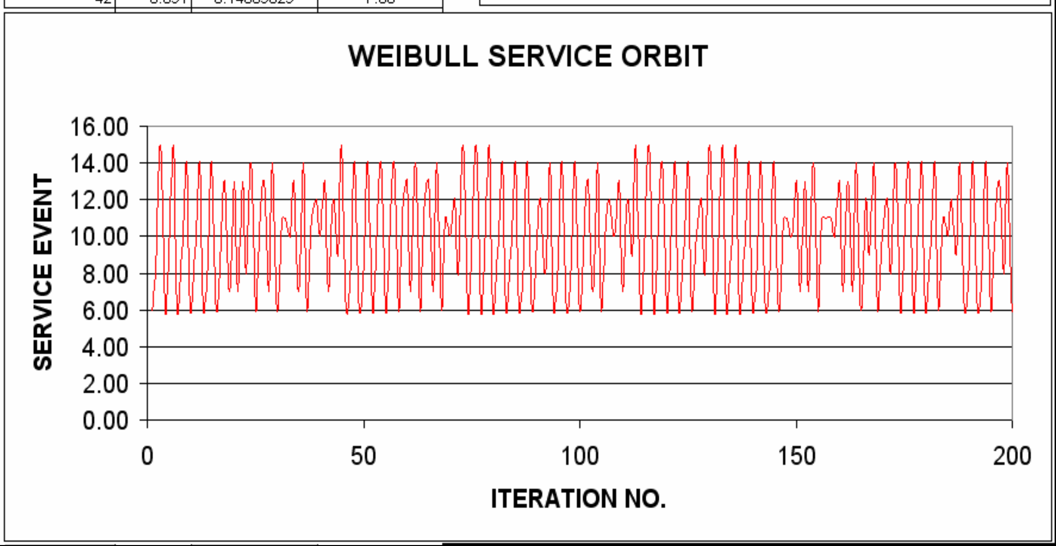
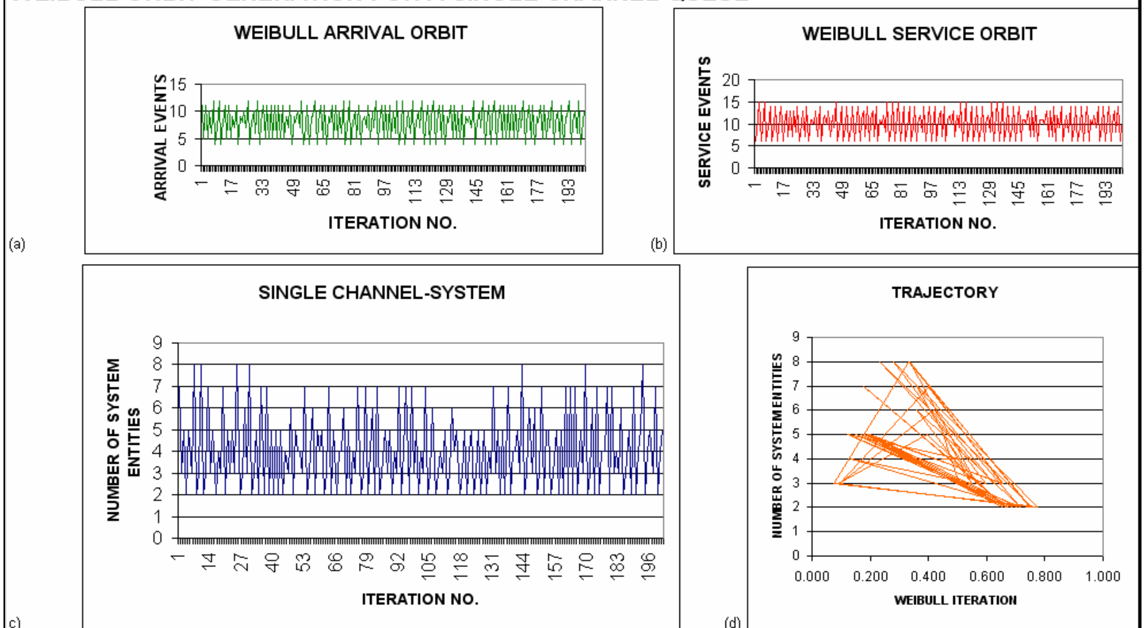


Fig. 6.3.2 GENERATION OF THE ORBIT OF WEIBULL SERVICE EVENTS

WEIBULL ORBIT GENERATION FOR A SINGLE CHANNEL QUEUE



LAMBDA	ARRIVALS		MU	SERVICE		SYSTEM	
8	AVERAGE	8.00	10	AVERAGE	10.00	AVERAGE	**4.01**
	STDEV	2.83		STDEV	3.15	STDEV	**1.86**

(e)

ITERATION NO.	CHAOS GENERATION		ARRIVAL ORBIT	CHAOS GENERATION		SERVICE ORBIT	BEGIN POP	BE+AR-SE	END POP	POP INT
1	0.400	0.65682321	11.00	0.800	0.05590191	6.00	0	5.00	6.96	7
2	0.657	0.18938712	5.00	0.056	0.25718547	8.00	5.00	2.00	3.04	3
3	0.189	0.68906617	11.00	0.257	0.78185409	15.00	2.00	-2.00	4.78	5
4	0.689	0.15044298	5.00	0.782	0.06598148	6.00	0.00	-1.00	2.17	2
5	0.150	0.59582709	10.00	0.066	0.30111610	9.00	0.00	1.00	4.78	5
6	0.596	0.28034964	6.00	0.301	0.77908007	15.00	1.00	-8.00	2.61	3
7	0.280	0.77566855	12.00	0.779	0.06764601	6.00	0.00	6.00	7.83	8
8	0.776	0.07511082	4.00	0.068	0.30826131	9.00	6.00	1.00	2.17	2
9	0.075	0.33170071	7.00	0.308	0.77505888	14.00	1.00	-6.00	3.04	3
10	0.332	0.75030692	12.00	0.775	0.07011954	6.00	0.00	6.00	7.83	8
11	0.750	0.09310860	4.00	0.070	0.31881806	9.00	6.00	1.00	2.17	2
12	0.093	0.40327982	8.00	0.319	0.76746148	14.00	1.00	-5.00	3.48	3
13	0.403	0.65112079	11.00	0.767	0.07499337	6.00	0.00	5.00	6.96	7
14	0.651	0.19693552	5.00	0.075	0.33939644	9.00	5.00	1.00	2.61	3
15	0.197	0.70326265	11.00	0.339	0.74738719	14.00	1.00	-2.00	4.78	5
16	0.703	0.13527252	5.00	0.747	0.08919508	6.00	0.00	-1.00	2.17	2
17	0.135	0.55083657	9.00	0.089	0.39752768	10.00	0.00	-1.00	3.91	4
18	0.551	0.36074484	7.00	0.398	0.66052394	13.00	0.00	-6.00	3.04	3
19	0.361	0.71731931	11.00	0.661	0.17614791	7.00	0.00	4.00	6.52	7
20	0.717	0.12140092	4.00	0.176	0.67456697	13.00	4.00	-5.00	1.74	2
21	0.121	0.50576143	9.00	0.675	0.15903049	7.00	0.00	2.00	4.78	5
22	0.506	0.44945949	8.00	0.159	0.63242650	13.00	2.00	-3.00	3.48	3
23	0.449	0.56381140	10.00	0.632	0.21415531	8.00	0.00	2.00	5.22	5
24	0.564	0.33655443	7.00	0.214	0.74390829	14.00	2.00	-5.00	3.04	3
25	0.337	0.74562342	12.00	0.744	0.09186038	6.00	0.00	6.00	7.83	8
26	0.746	0.09677699	4.00	0.092	0.40810961	10.00	6.00	0.00	1.74	2
27	0.097	0.41730143	8.00	0.408	0.64114435	13.00	0.00	-5.00	3.48	3
28	0.417	0.62586178	10.00	0.641	0.20182387	7.00	0.00	3.00	5.65	6
29	0.626	0.23276804	6.00	0.202	0.72511987	14.00	3.00	-5.00	2.61	3
30	0.233	0.75310997	12.00	0.725	0.10735631	6.00	0.00	6.00	7.83	8
31	0.753	0.09096602	4.00	0.107	0.46737089	11.00	6.00	-1.00	1.74	2
32	0.091	0.39499689	8.00	0.467	0.52203615	11.00	0.00	-3.00	3.48	3
33	0.395	0.66535496	11.00	0.522	0.40860719	10.00	0.00	1.00	5.22	5
34	0.665	0.17846629	5.00	0.409	0.64021325	13.00	1.00	-7.00	2.17	2
35	0.178	0.66626762	11.00	0.640	0.20311780	7.00	0.00	4.00	6.52	7
36	0.666	0.17732447	5.00	0.203	0.72725910	14.00	4.00	-5.00	2.17	2
37	0.177	0.66373080	11.00	0.727	0.10549589	6.00	0.00	5.00	6.96	7
38	0.664	0.18051086	5.00	0.105	0.46046890	11.00	5.00	-1.00	2.17	2
39	0.181	0.67073797	11.00	0.460	0.53644417	12.00	0.00	-1.00	4.78	5
40	0.671	0.17180517	5.00	0.536	0.37982589	10.00	0.00	-5.00	2.17	2
41	0.172	0.65106326	11.00	0.380	0.69087035	13.00	0.00	-2.00	4.78	5
42	0.651	0.19701269	5.00	0.691	0.14069829	7.00	0.00	-2.00	2.17	2
43	0.197	0.70340115	11.00	0.141	0.58010632	12.00	0.00	-1.00	4.78	5
44	0.703	0.13513032	5.00	0.580	0.29810402	9.00	0.00	-4.00	2.17	2
45	0.135	0.55039316	9.00	0.298	0.78049390	15.00	0.00	-6.00	3.91	4

Fig. 6.3.3 GENERATION OF THE ORBIT OF A WEIBULL SINGLE CHANNEL QUEUEING SYSTEM

SIN ORBIT GENERATION OF ARRIVAL EVENTS

		GENERATED PARAMETERS	
LAMBDA	8	AVERAGE	8.00
		STDEV	2.82

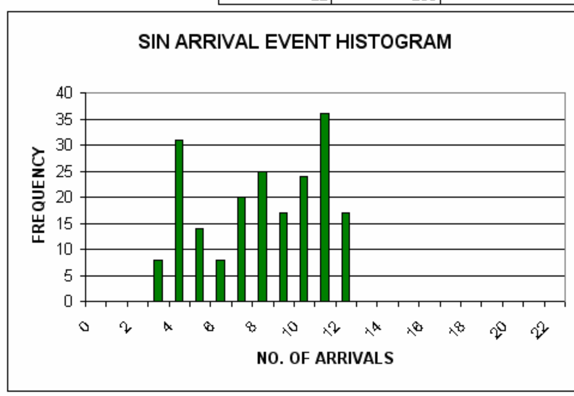
(a)

SIN GENERATION			
ITERATION NO	x	F(x)	ARRIVAL ORBIT
1	0.400	0.951056516	11.00
2	0.951	0.15315533	5.00
3	0.153	0.462800384	7.00
4	0.463	0.993178933	12.00
5	0.980	0.063806157	4.00
6	0.253	0.712856248	9.00
7	0.713	0.784623557	10.00
8	0.785	0.626165179	8.00
9	0.626	0.922472526	11.00
10	0.922	0.241158826	5.00
11	0.241	0.687196417	9.00
12	0.687	0.8319991	10.00
13	0.832	0.503625645	7.00
14	0.504	0.999935131	12.00
15	1.000	0.000611334	3.00
16	0.025	0.077598315	4.00
17	0.279	0.767631276	10.00
18	0.768	0.666875505	9.00
19	0.667	0.865697175	10.00
20	0.866	0.409517182	7.00
21	0.410	0.959869403	11.00
22	0.960	0.125740271	4.00
23	0.126	0.384830996	6.00
24	0.385	0.935256225	11.00
25	0.935	0.201999	5.00
26	0.202	0.59285429	8.00
27	0.593	0.957753394	11.00
28	0.958	0.13233232	4.00
29	0.132	0.40386174	7.00
30	0.404	0.954735433	11.00
31	0.955	0.141724054	4.00
32	0.142	0.430673813	7.00
33	0.431	0.976376351	11.00
34	0.976	0.074147771	4.00
35	0.272	0.754871682	10.00
36	0.755	0.696202232	9.00
37	0.696	0.815972136	10.00
38	0.816	0.546467659	8.00
39	0.546	0.989363471	11.00
40	0.989	0.033409424	4.00
41	0.183	0.543186773	8.00
42	0.543	0.990810223	11.00

(c

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	8	8
4	39	31
5	53	14
6	61	8
7	81	20
8	106	25
9	123	17
10	147	24
11	183	36
12	200	17
13	200	0
14	200	0
15	200	0
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

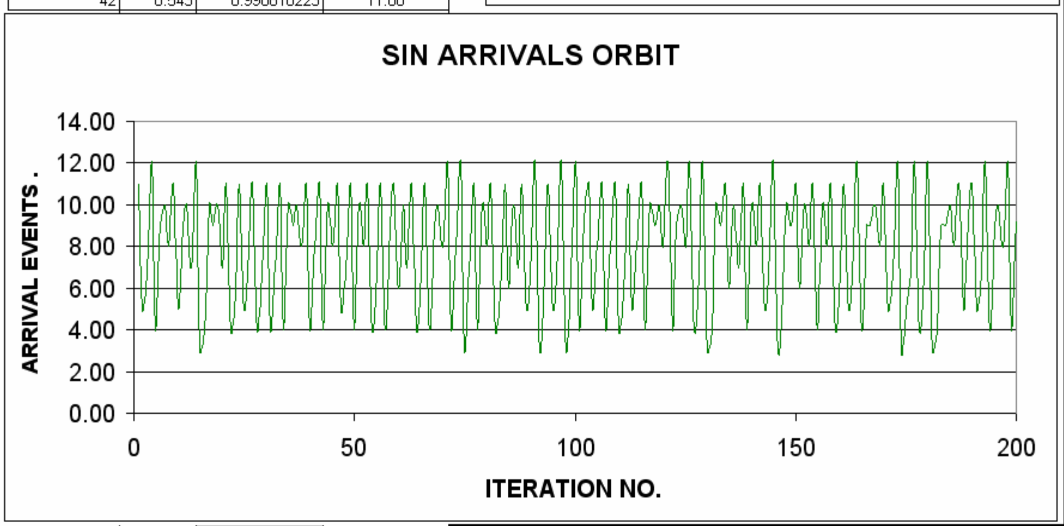


Fig. 6.3.4 GENERATION OF THE ORBIT OF SIN ARRIVAL EVENTS

SIN ORBIT GENERATION OF SERVICE EVENTS

		GENERATED PARAMETERS	
MU	10	AVERAGE	10.00
		STDEV	3.16

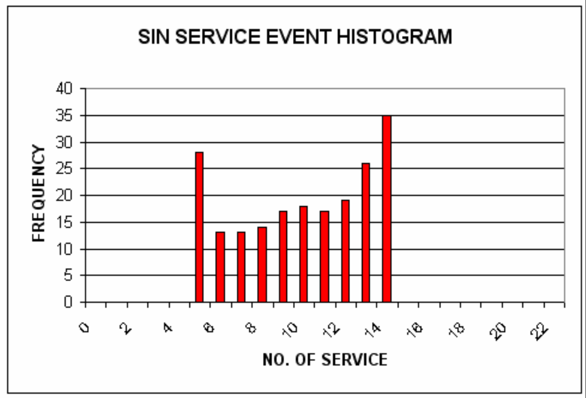
(a)

SIN GENERATION			
ITERATION NO	x	F(x)	SERVICE ORBIT
1	0.800	0.587785252	10.00
2	0.588	0.9622116	14.00
3	0.962	0.118437105	6.00
4	0.118	0.363554951	8.00
5	0.364	0.909525717	13.00
6	0.910	0.280421636	7.00
7	0.280	0.771356904	12.00
8	0.771	0.658108263	11.00
9	0.658	0.879154282	13.00
10	0.879	0.370593576	8.00
11	0.371	0.91849362	13.00
12	0.918	0.253270838	7.00
13	0.253	0.714335298	11.00
14	0.714	0.781734352	12.00
15	0.782	0.63321631	11.00
16	0.633	0.913694928	13.00
17	0.914	0.26782551	7.00
18	0.268	0.745575942	12.00
19	0.746	0.716865955	11.00
20	0.717	0.776751802	12.00
21	0.777	0.645253378	11.00
22	0.645	0.897677095	13.00
23	0.898	0.315949156	8.00
24	0.316	0.837440753	13.00
25	0.837	0.48878363	9.00
26	0.489	0.999379232	14.00
27	0.998	0.005846938	5.00
28	0.076	0.237918983	7.00
29	0.238	0.679766727	11.00
30	0.680	0.844720378	13.00
31	0.845	0.468705985	9.00
32	0.469	0.995171164	14.00
33	0.986	0.045275818	5.00
34	0.213	0.619787345	10.00
35	0.620	0.930022213	13.00
36	0.930	0.218075137	7.00
37	0.218	0.632752965	11.00
38	0.633	0.914285534	13.00
39	0.914	0.266037392	7.00
40	0.266	0.741820553	12.00
41	0.742	0.725041503	11.00
42	0.381	0.931093023	13.00

(c

HISTOGRAM VALUES		
N	CUMULATIVE FREQUENCY	FREQUENCY
0	0	0
1	0	0
2	0	0
3	0	0
4	0	0
5	28	28
6	41	13
7	54	13
8	68	14
9	85	17
10	103	18
11	120	17
12	139	19
13	165	26
14	200	35
15	200	0
16	200	0
17	200	0
18	200	0
19	200	0
20	200	0
21	200	0
22	200	0

(d)



(b)

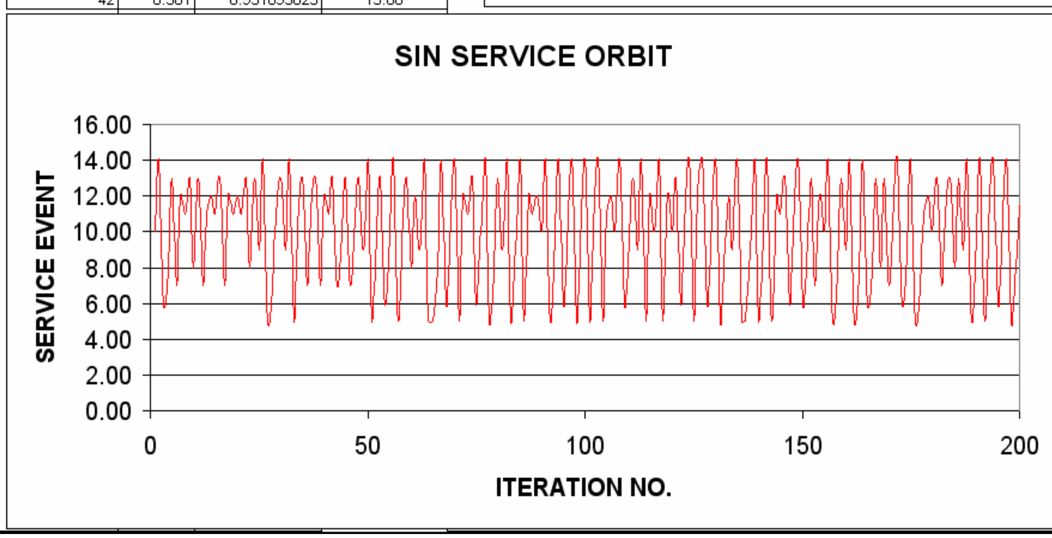
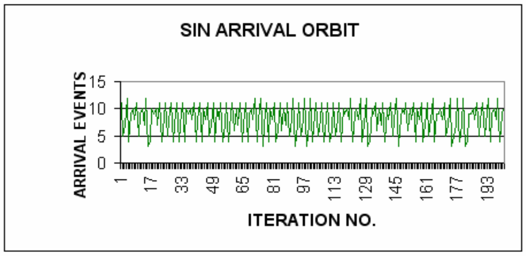
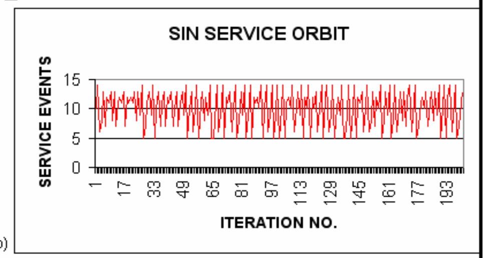


Fig. 6.3.5 GENERATION OF THE ORBIT OF SIN SERVICE EVENTS

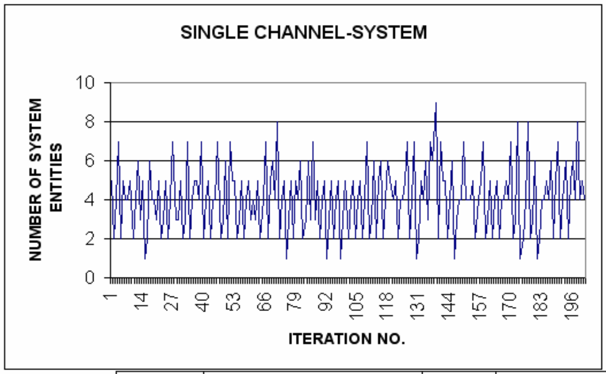
SIN ORBIT GENERATION FOR A SINGLE CHANNEL QUEUE



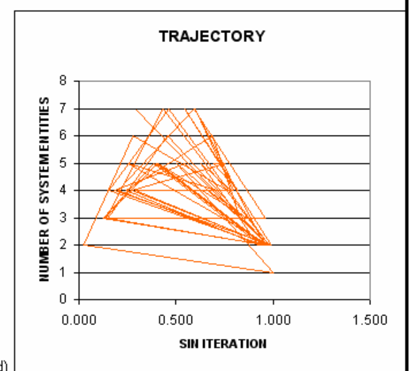
(a)



(b)



c)



(d)

LAMBDA	ARRIVALS		MU	SERVICE		SYSTEM	
8	AVERAGE	8.00	10	AVERAGE	10.00	AVERAGE	4.07
	STDEV	2.82		STDEV	3.16	STDEV	1.75

(e)

ITERATION NO.	CHAOS GENERATION		ARRIVAL ORBIT	CHAOS GENERATION		SERVICE ORBIT	BEGIN POP	BE+AR-SE	END POP	POP INT
1	0.400	0.951056516	11.00	0.800	0.587785252	10.00	0	1.00	5.00	5
2	0.951	0.15315533	5.00	0.588	0.9622116	14.00	1.00	-8.00	2.00	2
3	0.153	0.462800384	7.00	0.962	0.118437105	6.00	0.00	1.00	4.00	4
4	0.463	0.993178933	12.00	0.118	0.363554951	8.00	1.00	5.00	7.00	7
5	0.980	0.063806157	4.00	0.364	0.909525717	13.00	5.00	-4.00	2.00	2
6	0.253	0.712856248	9.00	0.910	0.280421636	7.00	0.00	2.00	5.00	5
7	0.713	0.784623557	10.00	0.280	0.771356904	12.00	2.00	0.00	4.00	4
8	0.785	0.626165179	8.00	0.771	0.658108263	11.00	0.00	-3.00	4.00	4
9	0.626	0.922472526	11.00	0.658	0.879154282	13.00	0.00	-2.00	5.00	5
10	0.922	0.241158826	5.00	0.879	0.370593576	8.00	0.00	-3.00	2.00	2
11	0.241	0.687196417	9.00	0.371	0.91849362	13.00	0.00	-4.00	4.00	4
12	0.687	0.8319991	10.00	0.918	0.253270838	7.00	0.00	3.00	6.00	6
13	0.832	0.503625645	7.00	0.253	0.714335298	11.00	3.00	-1.00	3.00	3
14	0.504	0.999935131	12.00	0.714	0.781734352	12.00	0.00	0.00	5.00	5
15	1.000	0.000611334	3.00	0.782	0.63321631	11.00	0.00	-8.00	1.00	1
16	0.025	0.077598315	4.00	0.633	0.913694928	13.00	0.00	-9.00	2.00	2
17	0.279	0.767631276	10.00	0.914	0.26782551	7.00	0.00	3.00	6.00	6
18	0.768	0.666875505	9.00	0.268	0.745575942	12.00	3.00	0.00	4.00	4
19	0.667	0.865697175	10.00	0.746	0.716865955	11.00	0.00	-1.00	4.00	4
20	0.866	0.409517182	7.00	0.717	0.776751802	12.00	0.00	-5.00	3.00	3
21	0.410	0.959869403	11.00	0.777	0.645253378	11.00	0.00	0.00	5.00	5
22	0.960	0.125740271	4.00	0.645	0.897677095	13.00	0.00	-9.00	2.00	2
23	0.126	0.384830996	6.00	0.898	0.315949156	8.00	0.00	-2.00	3.00	3
24	0.385	0.935256225	11.00	0.316	0.837440753	13.00	0.00	-2.00	5.00	5
25	0.935	0.201999	5.00	0.837	0.48878363	9.00	0.00	-4.00	2.00	2
26	0.202	0.59285429	8.00	0.489	0.999379232	14.00	0.00	-6.00	4.00	4
27	0.593	0.957753394	11.00	0.998	0.005846938	5.00	0.00	6.00	7.00	7
28	0.958	0.13233232	4.00	0.076	0.237918983	7.00	6.00	3.00	3.00	3
29	0.132	0.40386174	7.00	0.238	0.679766727	11.00	3.00	-1.00	3.00	3
30	0.404	0.954735433	11.00	0.680	0.844720378	13.00	0.00	-2.00	5.00	5
31	0.955	0.141724054	4.00	0.845	0.468705985	9.00	0.00	-5.00	2.00	2
32	0.142	0.430673813	7.00	0.469	0.995171164	14.00	0.00	-7.00	3.00	3
33	0.431	0.976376351	11.00	0.986	0.045275818	5.00	0.00	6.00	7.00	7
34	0.976	0.074147771	4.00	0.213	0.619787345	10.00	6.00	0.00	2.00	2
35	0.272	0.754871682	10.00	0.620	0.930022213	13.00	0.00	-3.00	4.00	4
36	0.755	0.696202232	9.00	0.930	0.218075137	7.00	0.00	2.00	5.00	5
37	0.696	0.815972136	10.00	0.218	0.632752965	11.00	2.00	1.00	5.00	5
38	0.816	0.546467659	8.00	0.633	0.914285534	13.00	1.00	-4.00	4.00	4
39	0.546	0.989363471	11.00	0.914	0.266037392	7.00	0.00	4.00	7.00	7
40	0.989	0.033409424	4.00	0.266	0.741820553	12.00	4.00	-4.00	2.00	2
41	0.183	0.543186773	8.00	0.742	0.725041503	11.00	0.00	-3.00	4.00	4
42	0.543	0.990810223	11.00	0.381	0.931093023	13.00	0.00	-2.00	5.00	5
43	0.973	0.085712803	4.00	0.931	0.214790826	7.00	0.00	-3.00	2.00	2
44	0.293	0.795453894	10.00	0.215	0.62472967	10.00	0.00	0.00	4.00	4
45	0.795	0.599279305	8.00	0.625	0.9242042	13.00	0.00	-5.00	4.00	4

Fig. 6.3.6 GENERATION OF THE ORBIT OF A SIN SINGLE CHANNEL QUEUEING SYSTEM

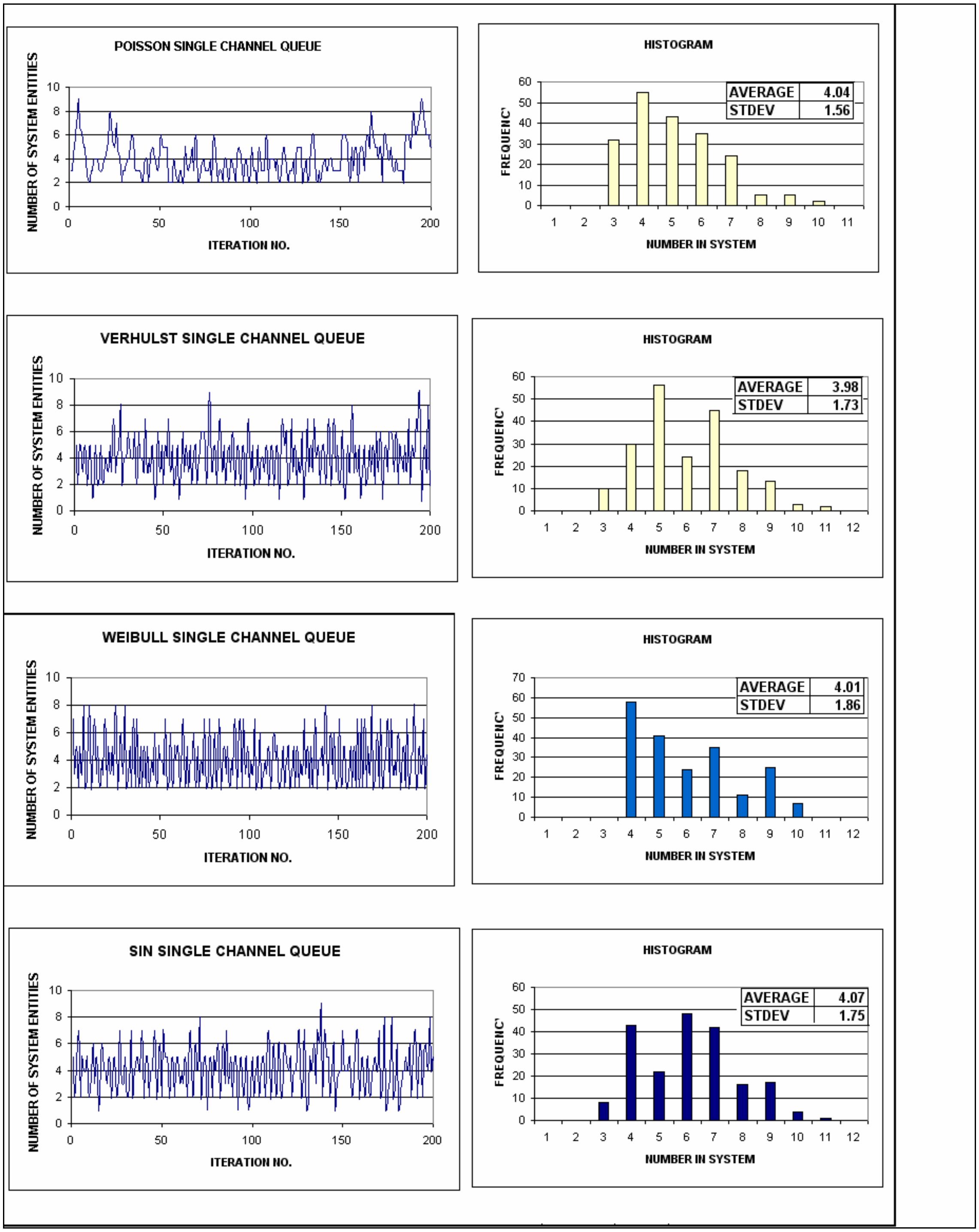


Fig. 6.3.7 PICTORIAL COMPARISON OF ORBITS OF SEVERAL SINGLE CHANNEL QUEUEING MODELS

6.4 Concluding remarks on single channel orbits resulting from a menu of methods of generation.

When viewing the various generated orbits shown in Fig. 6.3.7 one perceives that

- the various numerical values of average and standard deviation are virtually identical,
- one is inclined to believe that a measure of similarity exists in the histograms, and
- one consequently cautiously harbours the suspicion that **further extension and embellishment of the concept of chaos based system orbit generation** to match examples from the plethora of practical complex Systems of Congestion which exist, may be attempted.

The practical complex Systems of Congestion which are to be modelled in the following chapter are of a divergent nature and of necessity at least contain real time feedback rules to support decision making in achieving optimum transient and stable system operation.