

Chapter 12

Conclusions

“Reasoning draws a conclusion, but does not make the conclusion certain, unless the mind discovers it by the path of experience.” – Roger Bacon

This chapter summarises the research of this thesis. Section 12.1 summarises the findings and contributions of the thesis. In addition, possible future related research are proposed in Section 12.2.

12.1 Summary of Conclusions

The main objective of this thesis was to develop and analyse a PSO-based MOA for DMOO. However, in order to determine whether the algorithm efficiently solves DMOOPs, benchmark functions are required that are representative of typical real-world problems. Furthermore, performance measures are required to quantify the performance of the algorithm.

Chapter 3 provided a comprehensive overview of benchmark functions used for DMOO. The overview highlighted three limitations of current DMOOPs, namely: all benchmark functions have a POS that is defined by linear functions and all decision variables have the same POS, none of the DMOOPs have an isolated POF, and none of the DMOOPs have a deceptive POF. In order to address these shortcomings of currently used DMOOPs, new DMOOPs were proposed that have POSs defined by non-linear functions and where each

decision variable has its own POS. In addition, approaches to adapt current DMOOPs' POF to become either deceptive or isolated were presented.

A comprehensive overview of the performance measures used to quantify the performance of DMOO algorithms was provided in Chapter 4. Furthermore, the following issues of currently used performance measures were discussed:

- The effect of outliers on the value of distance based performance measures;
- Misleading HV values when the shape of the POF changes from convex to non-convex over time, since a higher HV value is obtained by algorithms that lose track of the true POF than algorithms that are tracking the POF;
- The effect of boundary constraint violations on the HV values of an algorithm; and
- How the choice of performance measures may produce misleading results when comparing various DMOAs' performance.

The original VEPSO algorithm was introduced in Chapter 7. VEPSO is a multi-swarm PSO-based algorithm, where each sub-swarm solves only one objective function that was assigned to the specific sub-swarm. Therefore, VEPSO is easy to implement and can easily be extended to solve additional objective functions. Extensions proposed to the original VEPSO algorithm were also discussed in Chapter 7, namely storing the non-dominated solutions found so far by VEPSO in an archive, managing particles that move outside the bounds of the search space and various approaches to share knowledge between the various sub-swarms.

The PSO-based MOA proposed in this thesis, DVEPSO, was introduced in Chapter 9. The required adaptations to the original VEPSO algorithm for DMOO were highlighted. These changes include detecting whether a change occurred in the environment and responding to a change in an appropriate manner. The search process of DVEPSO is guided by local and global guides. Three new approaches to update local and global guides were proposed. Furthermore, an empirical study was conducted to determine whether the new approaches that use Pareto-dominance information outperforms the original VEPSO guide update approach that does not use Pareto-dominance information. The benchmark functions and performance measures used in the empirical study were selected according to the findings of Chapters 3 and 4. A new approach was introduced to analyse the obtained data, namely calculating the number of statistical significant wins

or losses for each function, each combination of frequency (τ_t) and severity (n_t) of change and each performance measure. These results were then analysed for each DMOO type, each n_t - τ_t combination and each performance measure. The results indicated that the approaches that use Pareto-dominance information outperformed the original VEPSO guide update approach.

Chapter 10 investigated the effect of various parameters on the performance of DVEPSO. These parameters included approaches to manage boundary constraint violations, approaches to share knowledge between the different sub-swarms and responses to a change in the environment applied to either the particles of the sub-swarms or the non-dominated solutions in the archive. The results of the empirical study indicated that:

- The best performing approach to manage boundary constraint violations was clamping, where any particle that violates a specific boundary of the search space is placed on or close to the violated boundary.
- The knowledge sharing approach that produced the best performance was using a random topology for the sub-swarms and selecting the global guide of another sub-swarm with tournament selection.
- The response applied to particles of the sub-swarms after a change in the environment occurred that performed the best, was re-initialising 30% of the particles of only the sub-swarm(s) whose objective function changed.
- The best performing response applied to the archive was to remove all solutions from the archive after a change in the environment occurred.

In order to determine whether DVEPSO solves DMOOPs efficiently, DVEPSO was compared against four other state-of-the-art DMOO algorithms. An empirical study was conducted and the results indicated that the PSO-based algorithms (DMOPSO and DVEPSO) completely outperformed the DMOEAs (DNSGA-II-A, DNSGA-II-B and dCOEA). The DNSGA-II algorithms obtained the best performance for DMOOPs with either a deceptive or isolated POF. Furthermore, DVEPSO was the only algorithm that efficiently solved DIMP2, a Type I DMOOP where each decision variable has its own rate of change. dCOEA found solutions, but did not converge towards *POF* of DIMP2. However, the other three DMOAs found on average only one solution or no solutions

at all. However, the results indicated that DVEPSO performed poorly when solving DMOOPs with a discontinuous POF. Taking all of the results into consideration, it can be concluded that PSO-based DMOAs efficiently solve DMOOPs of various types, and with various frequencies and severities of change.

12.2 Future Research

The following related research is suggested for future investigation:

- DVEPSO is easy to implement and to extend for additional objectives. However, a scalability study is required to determine to which extent DVEPSO is scalable with regards to both the number of decision variables and the number of objectives. However, when increasing the number of objectives, it is important to note that the usage of Pareto-dominance to evaluate the quality of solutions is ineffective, since many solutions will be non-dominated with regards to the other found solutions. Therefore, research in many-objective optimisation is emerging [130].
- The empirical studies discussed in this thesis investigated the influence of various parameters on the performance of DVEPSO. The knowledge gained from these studies should be used to develop a self-adapting DVEPSO algorithm, eliminating the need to optimise parameters to solve new DMOOPs.
- The results of the empirical studies indicated that DVEPSO struggles to solve DMOOPs with a discontinuous POF, i.e. where the POF consists of various separated continuous parts. Various local search approaches should be incorporated into DVEPSO and investigated to determine whether the local search algorithm improves DVEPSO's performance when solving DMOOPs with discontinuous POFs.
- The empirical study of Chapter 9 investigated various approaches to update local and global guides. In addition, various guide selection approaches should be investigated to determine whether certain guide selection approaches lead to an improved diversity or spread of the found non-dominated solutions.
- Incorporating dynamic PSOs, for example the quantum PSO or charged PSO, into DVEPSO when solving DMOOPs where the POS changes over time.
- Evaluating the performance of DVEPSO in noisy environments.

- Developing performance measures for DMOO that are not vulnerable to the issues discussed in Chapter 4 and that do not require prior knowledge about the true POF.