

**MAXIMUM NET POWER OUTPUT FROM AN INTEGRATED DESIGN OF A  
SMALL-SCALE OPEN AND DIRECT SOLAR THERMAL BRAYTON CYCLE**

by

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# Abstract

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**Title:** Maximum net power output from an integrated design of a small-scale open and direct solar thermal Brayton cycle

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The geometry of the receiver and recuperator in a small-scale open and direct recuperative solar thermal Brayton cycle can be optimised in such a way that the system produces maximum net power output. The purpose of this work was to apply the second law of thermodynamics and entropy generation minimisation to optimise these geometries using an optimisation method. The dynamic trajectory optimisation method was used and off-the-shelf micro-turbines and a range of parabolic dish concentrator diameters were considered. A modified cavity receiver was used in the analysis with an assumed cavity wall construction method of either a circular tube or a rectangular channel. A maximum temperature constraint of 1 200 K was set for the receiver surface temperature. A counterflow plate-type recuperator was considered and the recuperator length was constrained to the length of the radius of the concentrator. Systems producing a steady-state net power output of 2 – 100 kW were analysed. The effect of various conditions, such as wind, receiver inclination and concentrator rim angle on the maximum net power output, and optimum geometry of the system were investigated. Forty-five different micro-turbines and seven concentrator diameters between 6 and 18 metres were considered. Results show the optimum geometries, optimum operating conditions and minimum entropy generation as a function of the system mass flow rate. The optimum receiver tube diameter was relatively large when compared with the receiver size. The optimum counterflow plate-type recuperator channel aspect ratio is a linear function of the optimum system mass flow rate for a constant recuperator height. The optimum recuperator length and optimum  $NTU$  are small at small system mass flow rates but increase as the system mass flow rate increases until the length constraint is reached. For the optimised systems with maximum net power output, the solar receiver is the main contributor to the total rate of minimum entropy generation. The contributions from the recuperator, compressor and turbine are next in line. Results show that the irreversibilities were spread throughout the system in such a way that the minimum internal irreversibility rate was almost three times the minimum external irreversibility rate for all optimum system geometries and for different concentrator diameters. For a specific environment and parameters, there exists

an optimum receiver and recuperator geometry so that the system can produce maximum net power output.

# Acknowledgement

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**My Creator,  
who did not come to judge, but to save.**

# Table of contents

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Abstract	ii
Acknowledgement	iv
Table of contents	v
List of figures	viii
List of tables	xi
Nomenclature	xii
<b>Chapter 1: Introduction</b>	<b>1</b>
1.1 Historical background	1
1.2 Problem	3
1.3 Purpose of the study	4
1.4 Layout of dissertation	5
<b>Chapter 2: Literature survey</b>	<b>6</b>
2.1 Introduction	6
2.2 Solar thermal power systems	6
2.2.1 Background	6
2.2.2 Power cycles available for solar thermal application	7
2.2.2.1 The Rankine cycle	7
2.2.2.2 Stirling engines	7
2.2.2.3 The Brayton cycle	9
2.2.3 Comparison of solar thermal power cycles	14
2.2.4 Comparison of working fluids and intermediate fluids	16
2.2.5 Conclusion	17
2.3 Solar collectors (concentrators and receivers)	18
2.3.1 Background	18
2.3.2 Concentration ratio and different types of concentrations	19
2.3.3 Rim angle, tracking and solar irradiation	24
2.3.4 Losses and efficiency	26
2.3.5 Solar receivers	27
2.4 Heat exchangers in general and the recuperator	31
2.5 Summary of literature in Sections 2.2 to 2.4	32
2.6 The second law of thermodynamics	32
2.6.1 Background	32
2.6.2 Exergy	34
2.6.2.1 Closed-system exergy balance	34
2.6.2.2 Exergy balance for control volumes	35
2.6.2.3 Exergetic efficiency	36
2.6.3 Entropy	36

2.6.4 Second law optimisation and examples of entropy generation minimisation (EGM) for individual components and elemental features	37
2.6.4.1 Background	38
2.6.4.2 Applications	39
2.6.5 Solar radiation and the second law of thermodynamics	43
2.6.5.1 Background	43
2.6.5.2 The exergy of sunlight	44
2.6.6 Exergy analysis for a system as a whole	48
2.6.7 Entropy generation rate equations useful in solar thermal power cycles	48
2.7 Useful information, guidelines and points to ponder	49
2.8 Comments and literature review	51
<b>Chapter 3: Problem formulation</b>	<b>52</b>
3.1 Introduction	52
3.2 Different cases	52
3.2.1 Case 1	53
3.2.2 Case 2	53
3.2.3 Case 3	54
3.2.4 Case 4	55
3.3 Entropy generation in the solar thermal Brayton cycle components	55
3.3.1 Solar collector (receiver)	56
3.3.2 Recuperator	56
3.3.3 Compressor and turbine	57
3.3.4 Solar heat exchanger (for indirect systems)	57
3.3.5 Radiator (used in a closed system)	57
3.4 Exergy analysis for the system	58
3.5 Description of the physical model	60
3.5.1 Geometry variables	60
3.5.1.1 Geometry of the receiver	60
3.5.1.2 Geometry of the recuperator	63
3.5.2 Parameters	67
3.6 Temperatures and pressures in terms of geometry variables	69
3.7 The objective function	73
3.8 Constraints	74
3.9 Constants / assumptions	78
3.10 Summary	79
<b>Chapter 4: Numerical method</b>	<b>80</b>
4.1 Introduction	80
4.2 Optimisation algorithm	80
4.3 Gradients	81
4.4 Structure of the program	82
4.5 Summary	84



<b>Chapter 5: Results</b>	<b>85</b>
5.1 Introduction	85
5.2 Validation	86
5.2.1 First validation	86
5.2.1.1 Thermal efficiency - no recuperator ( $\eta_c$ and $\eta_t = 1$ )	86
5.2.1.2 Thermal efficiency - with recuperator ( $\eta_{reg} < 1$ and $\eta_c$ and $\eta_t = 1$ )	87
5.2.1.3 Thermal efficiency - with recuperator and isentropic efficiencies ( $\eta_c, \eta_t, \eta_{reg} < 1$ )	89
5.2.2 Second validation	91
5.2.3 Conclusion of the validation	94
5.3 Results of the full analysis	95
5.3.1 Introduction	95
5.3.2 Optimum geometry for maximum net power output	96
5.3.3 Maximum net power output with optimum operating conditions and system properties	107
5.3.4 Comparison of second and first law results	118
5.3.5 The effect of the changing of a constant on the maximum net power output, optimum geometry and optimum operating conditions of the system	119
5.3.6 Future work	130
<b>Chapter 6: Conclusion</b>	<b>132</b>
<b>References</b>	<b>135</b>
<b>Appendix A: COLLECTOR</b>	<b>A.1</b>
<b>Appendix B: ENTROPY GENERATION RATE TABLE</b>	<b>B.1</b>
<b>Appendix C: MATLAB CODE</b>	<b>C.1</b>
<b>Appendix D: GARRETT MICRO-TURBINES</b>	<b>D.1</b>

# List of figures

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Figure 1.1 A parabolic collector powering a printing press at the 1878 Paris Exposition.	1
Figure 1.2 A solar-powered steam engine in Arizona in the early 1900s.	2
Figure 1.3 Commercially produced point focus concentrators.	2
Figure 2.1 The Rankine cycle.	7
Figure 2.2 The United Stirling Model 4-95 solar Stirling engine.	8
Figure 2.3 The four processes of an ideal Stirling engine cycle.	8
Figure 2.4 The Brayton cycle.	9
Figure 2.5 The thermal efficiency of a two-shaft gas turbine cycle with and without regeneration.	10
Figure 2.6 The regenerative Brayton cycle efficiency compared with the simple cycle efficiency.	11
Figure 2.7 A closed Brayton cycle power plant for use in space.	11
Figure 2.8 A solar sub-atmospheric gas turbine engine for parabolic dish application.	12
Figure 2.9 A solar version of the Garrett Turbine Company's Brayton cycle automotive gas turbine engine.	13
Figure 2.10 A section view of a micro-turbine from the Garrett range.	14
Figure 2.11 Photograph of parabolic dish installed at Shenandoah.	18
Figure 2.12 Different methods of concentration.	19
Figure 2.13 Relationship between concentration ratio and temperature of receiver operation.	21
Figure 2.14 Typical temperatures achievable by concentrating solar collectors.	21
Figure 2.15 Optimum operating temperature change with geometric concentration ratio.	22
Figure 2.16 Temperature reached by solar absorbers using concentration optics.	22
Figure 2.17 The thermal efficiency of a receiver as a function of the fluid temperature and the concentration factor.	23
Figure 2.18 The thermal efficiency of a receiver as a function of the absorber temperature and the concentration factor.	23
Figure 2.19 Expected stagnation temperatures of evacuated solar collectors with concentrators.	24
Figure 2.20 Variation of geometric concentration ratio with rim angle.	24
Figure 2.21 Aperture irradiance for different fixed and tracking aperture configurations for Albuquerque, on June 22.	25
Figure 2.22 Aperture irradiance for different fixed and tracking aperture configurations for Albuquerque on December 22.	26
Figure 2.23 Specular reflectance of selected materials: silver, aluminium and gold.	27
Figure 2.24 A typical cavity receiver.	28
Figure 2.25 Modified cavity receiver.	29
Figure 2.26 Modified cavity receiver.	29
Figure 2.27 Sizing of a collector receiver.	30
Figure 2.28 Heat exchanger with hot stream (1 – 2) and cold stream (3 – 4).	41
Figure 2.29 The entropy increase associated with the constant-energy transformation of monochromatic radiation into blackbody radiation.	45
Figure 2.30 The temperature decrease induced by scattering as a function of the dimensionless frequency.	46
Figure 3.1 Case 1: Direct system, closed cycle.	53



Figure 3.2 Case 2: Direct system, open cycle.	54
Figure 3.3 Case 3: Indirect system, closed cycle.	54
Figure 3.4 Case 4: Indirect system, open cycle.	55
Figure 3.5 Control volume around the open and direct solar thermal Brayton cycle to perform an exergy analysis.	58
Figure 3.6 Geometry of the cavity receiver constructed with a circular tube.	62
Figure 3.7 Cavity constructed with the use of a rectangular channel (plate) and its dimensions.	62
Figure 3.8 Counterflow plate-type recuperator.	64
Figure 3.9 Compressor map for a micro-turbine from Garrett.	68
Figure 4.1 The open and direct solar thermal Brayton cycle with a range of concentrator diameters ( $D$ ) and a range of micro-turbines ( $MT$ ).	84
Figure 5.1 The open and direct solar thermal Brayton cycle with no recuperator.	86
Figure 5.2 The open and direct solar thermal Brayton cycle with recuperator.	87
Figure 5.3 Comparison of the thermal efficiency of the Brayton cycle for different cases of recuperation.	88
Figure 5.4 The thermal efficiency as a function of pressure ratio for different recuperation situations.	90
Figure 5.5 Net power output calculated with the first and second laws of thermodynamics as a function of the pressure ratio.	93
Figure 5.6 Thermal efficiency as a function of the pressure ratio.	94
Figure 5.7 Contribution of the compressor, turbine, recuperator and receiver to the total entropy generation rate.	94
Figure 5.8 Data points for maximum net power output with an optimum geometry for concentrator with $D = 6$ m.	96
Figure 5.9 Maximum net power output at an optimum geometry for a concentrator with $D = 6$ m.	98
Figure 5.10 Maximum net power output at an optimum geometry for a concentrator with $D = 10$ m.	99
Figure 5.11 Maximum net power output for different micro-turbines and their operating ranges for $D = 14$ m.	99
Figure 5.12 All the data points for the range of concentrator diameters and micro-turbines.	100
Figure 5.13 Thermal efficiencies of the optimised systems as a function of concentrator diameter and micro-turbine choice.	100
Figure 5.14 Optimum aspect ratio of the recuperator channels at optimum operating conditions of various micro-turbines.	101
Figure 5.15 Roofline for the maximum net power output for micro-turbine number 27.	102
Figure 5.16 Roofline for the maximum net power output for micro-turbine number 41 with different concentrator diameters.	103
Figure 5.17 Convergence of the optimum recuperator length to its maximum constraint.	104
Figure 5.18 Convergence of the optimum recuperator length for $D = 16$ m with micro-turbine 41.	104
Figure 5.19 Optimum hydraulic diameter of recuperator channels as a function of system mass flow rate for $D = 10, 14$ and $18$ m.	105
Figure 5.20 Optimum number of rectangular tubes between the receiver edge and the receiver aperture as a function of the optimum receiver channel aspect ratio.	105
Figure 5.21 Relationship between the optimum tube diameter and the optimum length for the circular tube receiver.	107
Figure 5.22 Optimum recuperator channel mass flow rate for all data points.	108
Figure 5.23 Optimum $NTU$ for all data points.	109
Figure 5.24 Optimum recuperator channel mass flow rate ( $D = 8, 12$ and $16$ m).	109
Figure 5.25 Optimum $NTU$ for all data points ( $D = 8, 12$ and $16$ m).	110

Figure 5.26 Geometry optimised system data points ( $MT = 41$ , $D = 16$ m) (a - d).	111
Figure 5.27 Maximum receiver surface temperature of all the optimised data points (all micro-turbines and each of its operating conditions).	112
Figure 5.28 Optimum pressure drop in receiver and recuperator channel for micro-turbine 41 and $D = 16$ m.	114
Figure 5.29 Optimum friction factor in receiver and recuperator for $D = 16$ m.	114
Figure 5.30 Optimum friction factor for all data points.	115
Figure 5.31 Linear relationship between optimum recuperator efficiency and channel hydraulic diameter.	116
Figure 5.32 Linear relationship between optimum recuperator channel mass flow rate and optimum system mass flow rate.	116
Figure 5.33 Minimum internal system irreversibility rate as a function of minimum external system irreversibility rate for maximum system net power output.	117
Figure 5.34 $C_W$ as a function of the system mass flow rate.	118
Figure 5.35 Comparison of net power output calculated for two optimised systems using the first and second laws of thermodynamics.	119
Figure 5.36 Change in maximum net power output for system using $MT = 13$ and $D = 8$ m.	121
Figure 5.37 Change in maximum net power output for system using $MT = 32$ and $D = 12$ m.	121
Figure 5.38 Change in optimum receiver tube diameter due to changes in constants for $D = 12$ m and $MT = 32$ .	122
Figure 5.39 Change in optimum recuperator channel aspect ratio due to changes in constants for $D = 12$ m with $MT = 32$ .	122
Figure 5.40 Optimum recuperator channel mass flow rate with changes in constants for $D = 8$ m with $MT = 13$ .	123
Figure 5.41 Optimum recuperator channel mass flow rate with changes in constants for $D = 12$ m with $MT = 32$ .	124
Figure 5.42 The optimum recuperator $NTU$ with specific scenarios $D = 8$ m and micro-turbine 13 for changed constants.	125
Figure 5.43 The optimum recuperator $NTU$ with specific scenarios for $D = 12$ m and micro-turbine 32 for changed constants.	125
Figure 5.44 The effect of different conditions on the optimum performance of $MT = 41$ and $D = 16$ m.	126
Figure 5.45 Maximum net power output and minimum irreversibility rates for $D = 16$ m with $MT = 41$ .	127
Figure 5.46 Validation of the optimum receiver tube diameter.	128
Figure 5.47 Validation of the optimum receiver tube length.	128
Figure 5.48 Validation of the optimum recuperator channel aspect ratio.	129
Figure 5.49 Validation of the optimum recuperator channel hydraulic diameter.	129
Figure 5.50 Validation of the optimum recuperator length.	130
Figure A.1 Receiver-sizing algorithm.	A.1
Figure A.2 Definition of the rim angle.	A.2
Figure A.3 Reflection of non-parallel rays from a parabolic mirror.	A.3
Figure A.4 Relation between net absorbed heat rate and the aperture diameter for a range of concentrator diameters according to the function 'collector'.	A.6

# List of tables

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Table 2.1 Solar thermal power cycles compared.	15
Table 2.2 Comparison of fluids used in solar thermal power cycles.	16
Table 2.3 Advantages and disadvantages of having a direct or indirect cycle/working fluid.	17
Table 2.4 A summary of entropy generation literature.	49
Table 4.1 Geometric variables used in numerical optimisation with description.	80
Table 4.2 Settings used for the optimisation algorithm (LFOPC).	81
Table 5.1 Assumptions for second validation.	92
Table 5.2 Constants used for the analysis.	95
Table 5.3 Results showing optimised geometry variables and maximum net power output for $MT = 4$ and $D = 6$ m using a circular tube as receiver construction method.	97
Table 5.4 Results showing optimised geometry variables and maximum net power output for $MT = 4$ and $D = 6$ m using a rectangular channel as receiver construction method.	97
Table 5.5 Each constant is changed to a new value to see the effect of the changing of one constant.	120
Table B.1 Entropy generation rate equations from the literature.	B.1
Table D.1 Data for the Garrett micro-turbines (Garrett, 2009).	D.1

# Nomenclature

Alphabetic Symbols:		Units
$a, a$	Longer side of rectangle (channel width)	m
$A$	Area	$m^2$
$b$	Shorter side of rectangle (channel height)	m
$c$	Heat exchanger capacity ratio	-
$c$	Specific heat ( $c_p$ or $c_v$ )	J/kgK
$C$	Concentration factor	-
$CR$	Concentration ratio	-
$C_w$	Optimum ratio of minimum internal to external irreversibility rate	-
$d, d$	Cavity receiver aperture diameter	m
$D$	Parabolic dish concentrator diameter	m
$D, D$	Cavity receiver diameter	m
$D$	Tube / rectangular channel diameter ( $D_{rec}, D_{h,rec}, D_{h,reg}$ )	m
$e$	Specific exergy	J/kg
$\hat{e}$	Specific energy	J/kg
$e_p$	Parabolic concentrator error	rad
$E$	Exergy	J
$\dot{E}$	Exergy rate	W
$f$	Darcy friction factor	-
$F$	Heat removal factor	-
$F_D$	External drag force	N
$g$	Gravitational constant	$m/s^2$
$GF$	Gradient vector of the objective function	-
$Gr$	Grashof number	-
$h$	Convection heat transfer coefficient	$W/m^2K$
$h$	Planck's constant (solar radiation)	-
$h$	Small increment	m
$h$	Specific enthalpy	J/kg

$H$	Recuperator height	m
$I$	Irradiance	W/m <sup>2</sup>
$\dot{i}$	Irreversibility rate	W
$k$	Boltzmann's constant (solar radiation)	-
$k$	Gas constant ( $c_p/c_v$ )	-
$k$	Thermal conductivity	W/mK
$K$	Constant	-
$KE$	Kinetic energy	J
$L$	Length	m
$L$	Length of channel / tube ( $L_{rec}, L_{reg}$ )	m
$\dot{m}$	Mass flow rate	kg/s
$MT$	Micro-turbine model number (Appendix D)	-
$n$	Number of recuperator flow channels	-
$N$	Number of fins	-
$N_D$	Optimum number of tube diameters between receiver edge and aperture edge	-
$N_R$	Optimum number of rectangular channels between receiver edge and aperture edge	-
$NTU$	Number of transfer units	-
$Nu$	Nusselt number	-
$P, p, p$	Pressure	Pa
$PE$	Potential energy	J
Pr	Prandtl number	-
$q$	Specific heat	J/kg
$Q, Q$	Heat energy	J
$\dot{Q}$	Heat energy rate	W
$\dot{Q}_0$	Rate of solar energy lost due to convection and radiation	W
$\dot{Q}^*$	Rate of solar energy intercepted by the cavity receiver aperture	W
$\dot{Q}_{net}$	Net solar energy rate available for receiver fluid	W
$r$	Pressure ratio ( $P_2/P_1$ )	-
$R$	Gas constant	J/kgK

$Re$	Reynolds number	-
$refl$	Mirror surface specular reflectivity	-
$R_f$	Fouling factor in the calculation of overall heat transfer coefficient	-
$s, s$	Specific entropy	J/kgK
$S$	Entropy	J/K
$\dot{S}$	Entropy rate	W/K
$t$	Thickness of recuperator separator plate	m
$t$	Time	s
$T, T$	Temperature	K, °C
$T^*$	The apparent sun's temperature as an exergy source	
$u$	Specific internal energy	J/kg
$U$	Heat coefficient ( $U_L$ )	W/mK
$U$	Internal energy	J
$U$	Overall heat transfer coefficient	W/mK
$\nu$	Frequency (solar radiation)	Hz
$\nu$	Specific volume	m <sup>3</sup> /kg
$V$	Velocity	m/s
$w$	Specific work	J/kg
$w$	Wind factor	-
$W$	Work	J
$\dot{W}$	Power	W
$W_n$	Receiver aperture diameter vector	m
$x$	Discrete least squares approximation constant	-
$X$	Optimisation vector	-
$z$	Height	m

**Greek symbols:**

$\alpha$	Receiver absorptance	-
$\beta$	Inclination of receiver	-
$\epsilon$	Effectiveness	-
$\epsilon$	Exergetic efficiency	-
$\epsilon$	Emissivity	-
$\phi$	Horizontal inclination angle	-
$\phi$	Angle	-

$\eta$	Efficiency	-
$\lambda$	Dimensionless parameter ( $\lambda_{BH}$ )	-
$\lambda$	Wave length (solar radiation)	m
$\mu$	Dynamic viscosity	kg/ms
$\rho$	Density	kg/m <sup>3</sup>
$v$	Volume	m <sup>3</sup>
$\Omega_1$	Solid angle subtended by the solar disc seen from earth	rad
$\Omega_2$	Solid angle for the outgoing radiation	rad
$\psi$	Angle	-

**Subscripts:**

$0, 0$	Surrounding / environment
$0$	Zero pressure (ideal gas) for $c_p$
$1$	State 1
$2$	State 2
$a$	Receiver aperture
$A$	Absorber
$atm$	Atmospheric
$avg$	Average
$b$	Boundary
$b$	Of blackbody radiation, final
$B$	Base
$BH$	Defined by Bahnke and Howard
$c$	Cold stream
$c$	Compressor
$c$	Cross-sectional
$conc$	Concentrator
$conv$	Convection
$cv$	Control volume
$D$	Based on receiver diameter
$D$	Destruction
$e$	Electric
$e$	Outlet
$ext$	External



F	Average of the heat transfer fluid
<i>F</i>	Fuel
<i>FirstLaw</i>	According to the first law of thermodynamics
<i>gen</i>	Generation
<i>h</i>	Hot stream
<i>h</i>	Hydraulic
<i>high</i>	Highest value on island of maximum compressor efficiency
<i>i</i>	Inlet
<i>i</i>	Inner
<i>in, in</i>	In
<i>int</i>	Internal
L	Loss due to convection and conduction
<i>L</i>	Loss
<i>loss</i>	Loss
<i>low</i>	Lowest value on island of maximum compressor efficiency
max	Maximum
min	Minimum
<i>net</i>	Net
<i>o</i>	Out/Outer
<i>opt</i>	Optimum
<i>out, out</i>	Out
<i>p</i>	For constant pressure
<i>P</i>	Product
<i>rad</i>	Radiation
<i>rec</i>	Receiver
<i>reg</i>	Recuperator
<i>rim, rim</i>	Concentrator rim
<i>s</i>	Shaft
<i>s</i>	Surface
<i>sngl</i>	For a single fin
<i>t</i>	Turbine
<i>th, th</i>	Thermal
<i>v</i>	For constant volume
<i>v</i>	Per unit frequency (solar radiation)





$w$	Cavity receiver inner surface
$\infty$	Surrounding area / free-stream
$\Delta P$	Due to fluid friction
$\Delta T$	Due to temperature difference

**Superscripts:**

*	Solar
'	Per unit length
.	Time rate of change
$CH$	Chemical