CHAPTER 1

Introduction and literature review

1.1 Introduction

Oscillating machines have become an integral part of modern industry. These machines use mechanisms such as rotating imbalances or pneumatic devices to excite the machine at a particular frequency. Rock-drills, compactors and jackhammers are examples of oscillating machines that are presently used in the industry and are depicted in Figure 1. 1. Because oscillatory motion is vital to the performance of these machines, it is not sensible to attempt to minimize the vibration of these machines with vibration absorbers. On the other hand, these vibrations can cause extensive noise and damage to structures, foundations and humans if proper vibration isolation is not achieved in the linkages between the machine and the surrounding area.

Figure 1. 1 Examples of vibrating machines used in industry today

Operators exposed to vibration when handling portable hand-held vibrating tools can, if exposed for long periods of time, (usually measured in years) be at risk of injuries collectively known as Hand-Arm Vibration Syndrome (HAVS). Raynaud's Phenomena or Vibration White Finger, (VWF) is one example of an HAVS injury, which was described as early as the 1890s. (Raw, 1999)

There are many ways to achieve the isolation of vibrations. Most of these use a soft spring and mass that together have a very low natural frequency. Such isolators lead to large displacements and are usually not suitable for machines that are controlled by an operator. With such large displacements operators are unable to accurately control the machines. There are however a class of isolators that were developed by the helicopter industry to isolate the fuselage from the main rotor's vibration. Examples of such isolators are the IRIS (Improved Rotor Isolation System) (Desjardins & Hooper, 1976 & Desjardins & Hooper, 1980), DAVI (Dynamic Antiresonant Vibration Isolator) (Braun, 1980 & Rita et al., 1976) and LIVE (Liquid Inertia Vibration Eliminator) (Halwes, 1981 and Halwes & Simmons, 1980) systems. These devices are able to achieve good isolation with high stiffness and low mass penalty.

A major problem with the implementation of such isolators on oscillating machines is the fact that the isolator is tuned for a specific isolation frequency and that other frequencies can be amplified. Although oscillating machines usually have a primary excitation frequency, this frequency may easily change by as much as 10% during operation. Furthermore, the vibration at other frequencies can change due to operating conditions such as the hardness of rock. Therefore the isolator has to be able to adapt to these changes in the excitation frequency and noise levels.

1.2 Vibration control for vibrating machines

Vibration on vibrating machines can be reduced in the following ways (Rao, 1995):

1. By controlling the natural frequencies of the system and avoiding resonance under external excitation.
2. By preventing excessive response of the system, even at resonance by introducing a damping or energy-dissipating mechanism.
3. By reducing the transmission of the excitation forces from one part of the machine to another, by the use of vibration isolators.
4. By reducing the response of the system, by the addition of an auxiliary mass neutraliser or vibration absorber.

A more practical explanation of the above is:

1. Make sure that the machine is not excited at its natural frequency.
2. Put enough damping in the system to minimise response at the natural frequency.
3. Install a vibration isolator between the machine and another body.

4. Install a vibration absorber on the machine to minimise the vibration of the machine.

A vibration isolator is the device that we are interested in and will be described in detail in the following section. Although vibration absorbers are not suitable for the purposes of this study, they will also be studied to further explain some basic principles that are also present in vibration isolators.

1.2.1 Vibration absorbers

The basic principle of a vibration absorber is to minimise the response of a machine at a specific frequency. This is usually achieved by adding a secondary mass with a spring to the system. There are three main types of vibration absorbers. All three will be discussed in the following paragraphs.

1.2.1.1 Classic vibration absorber

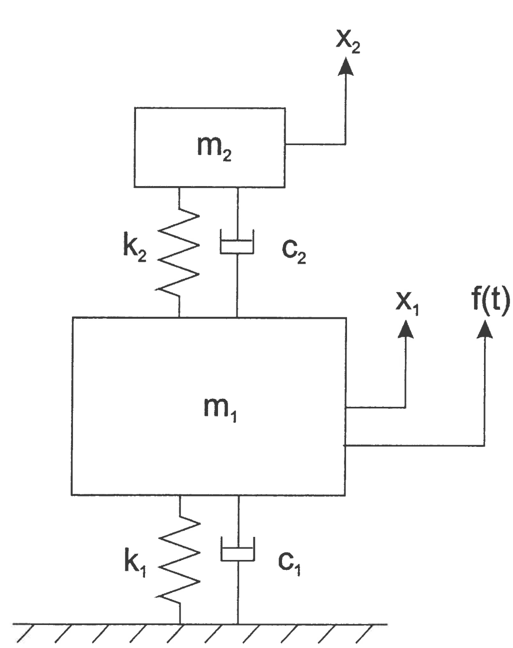


Figure 1. 2 Classic vibration absorber

The classic vibration absorber was patented by Frahm in 1911 (Sun et al., 1994). A vibration absorber consists of a primary system to which a secondary mass is added as shown in Figure 1. 2. The objective of the absorber is to minimise the motion of the primary mass. This is achieved by tuning the natural frequency of the absorber to coincide with the excitation frequency. The result is a two-degree of freedom system with zero response at the tuned frequency if no damping is present. Sometimes the excitation frequency and the natural frequency of the primary system can be the same. In this special case the use of an absorber is very useful as can be seen Figure 1. 3. The use of an absorber is however not limited to this special case and can be used on most structures by tuning the absorber to the excitation frequency.

The equation of the response of the primary mass before and after the addition of an absorber are as follows:

Without absorber:

$$\frac{X_1}{\delta_{st}} = \frac{1}{1-\left(\frac{\omega}{\omega_1}\right)^2} \tag{1.1}$$

With absorber: (Rao, 1995)

$$\frac{X_1}{\delta_{st}} = \frac{1-\left(\frac{\omega}{\omega_2}\right)^2}{\left[1+\frac{k_2}{k_1}-\left(\frac{\omega}{\omega_1}\right)^2\right]\left[1-\left(\frac{\omega}{\omega_2}\right)^2\right]-\frac{k_2}{k_1}} \tag{1.2}$$

With:

X	-	Displacement
δ_{st}	-	Static deflection, displacement due to gravitation
$k_1.k_2$	-	Spring stiffness
m_1,m_2	-	Mass
ω_1, ω_2	-	Circular frequency

Figure 1. 3 gives the response of the primary system before and after the addition of an undamped absorber with a mass ratio of 1/20.

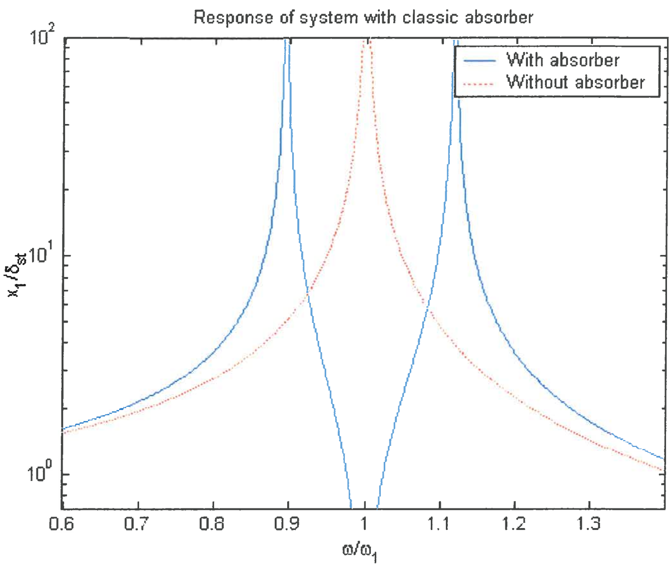


Figure 1. 3 Response of system with and without an absorber

1.2.1.2 Semi-active vibration absorbers

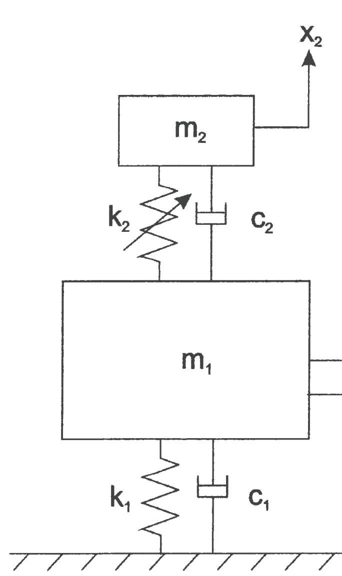


Figure 1. 4 Semi-active vibration absorber

The difference between classic and semi-active vibration absorbers is that where a classic vibration absorber is tuned to a specific frequency, a semi-active vibration absorber can be tuned to different frequencies while in operation. There are many published mechanisms to accomplish this. Viscoelastic materials are often used as the spring element in such a device (Ketema, 1998) and modification to the viscoeleastic material by means of temperature change (Fosdick & Ketema, 1998) result in a semi-active vibration absorber. The most common way of achieving a semi-active absorber is by using a variable stiffness spring as shown in Figure 1. 4. As the stiffness of the spring varies, the natural frequency of the absorber changes and therefore the tuned frequency varies. Different ways to change the stiffness have been implemented based on shape memory alloys (Williams et al., 1999 & Williams et al., 2000), Terfenol-D (Flatau et al., 1998) or by changing the tension in strings (Onoda et al., 1992). Electromechanical systems have also been used (Nagem et al., 1997) to change the stiffness (Tentor &Wicks, 2000) and the damping (Seto & Yamanouch, 1978). The advantage of these absorbers over a classic absorber is that if the primary machine's operating or excitation frequency changes while in operation, the absorber can adapt to such a change and ensure that the minimum response of the primary system is obtained at all times.

Figure 1. 5 show how changing the stiffness of the absorber spring can alter the response of the system. The dotted line is the response of the original system without an absorber.

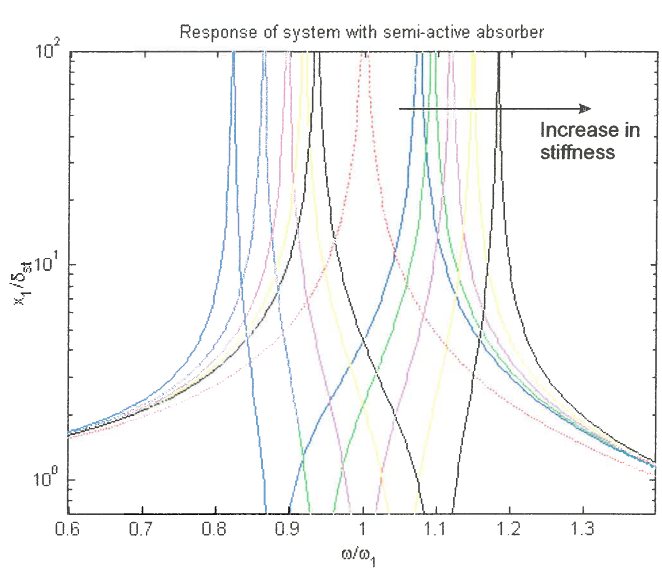


Figure 1. 5　　Response of a semi-active absorber

When a very small mass ratio is used, the use of a semi-active absorber can become crucial because of the small bandwidth of the absorber. With a classic absorber, a small change in excitation frequency can excite a resonance of the system causing serious problems. In such a case a semi-active vibration absorber would be the best option to prevent this from happening.

1.2.1.3 Active vibration absorbers

Where the classic and semi-active vibration absorbers are basically the same mechanism with the one having a variable stiffness spring instead of a normal spring, the active vibration absorber works on a totally different principle. The purpose of the absorber is still the same namely to minimise the response of the primary system, but in this case some type of actuator together with a control system is used to accomplish this.

The control system will sense the response of the primary system with accelerometers or other sensors and will drive the actuator in such a way to minimise that response. The advantage of this type of absorber is that a wide band absorber is achieved, which is effective not only at one frequency, but over a whole frequency band.

It must be noted that there are a lot of differences between the three types of absorbers. Table 1. 1 gives an indication of the differences between the different absorbers.

It can be seen that the active vibration absorbers come at a substantial cost and are only used when it is really needed. Usually classic or semi-active absorbers will be used because of their low cost.

Table 1. 1 Differences between different types of absorbers

Criteria	Classic	Semi-active	Active
Complexity	Low	Medium	Very High
Cost	Low	Medium	Very High
Energy input	None	Low	High
Effectiveness	Low (very small band)	Medium (small band)	High (wide band)

1.2.2 Vibration Isolators

The main difference between a vibration absorber and a vibration isolator is that where a vibration absorber minimises the response of the primary system, a vibration isolator minimises the vibration transmitted from the primary system to another body. Therefore the primary system's response is not altered substantially, but a device is installed between the primary system and a surrounding body that does not transmit the vibrations or forces at certain frequencies.

Again, there are a few different types of isolators that will be discussed in detail.

1.2.2.1 Vibration isolator

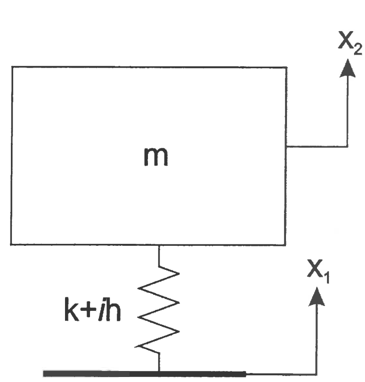


Figure 1. 6 Vibration isolator

The normal passive vibration isolator is simply a spring with low stiffness as shown in Figure 1. 6. Engine mounts are common examples of such isolators (Maw, 1991). The primary system is connected to the ground or to another body with a soft spring. This gives it a very low natural frequency with isolation above $\sqrt{2}$ times the natural frequency.

The response of a mass on a spring with structural damping under an external excitation force is given by:

$$\frac{X}{F_i} = \frac{1}{k - \omega^2 m + ih} \tag{1.3}$$

With:

F_i	-	Applied force
h	-	Hysteresis damping constant

The following substitutions can be made:

$$\omega_n = \sqrt{\frac{k}{m}} \qquad h = k\eta \tag{1.4}$$

With:

η	-	Loss factor

The following equation can be derived for the response of the primary system:

$$\frac{X}{F_i} = \frac{1}{k\left(1 - \left(\frac{\omega}{\omega_n}\right)^2 + i\eta\right)} \tag{1.5}$$

To get the force transmissibility, the force transmitted to the base must be calculated. The only force transmitted to the base is through the spring with structural damping. The following equation gives the force transmitted to the base due to a displacement of the primary mass:

$$F_T = k\left(1 + i\eta\right) X \tag{1.6}$$

By eliminating X in the last two equations the force transmissibility of a vibration isolator is obtained as follows:

$$\frac{F_T}{F_i} = \frac{1+i\eta}{1-\left(\frac{\omega}{\omega_n}\right)^2 + i\eta} \tag{1.7}$$

When this is plotted for different loss factors, the graphs in Figure 1. 7 are obtained.

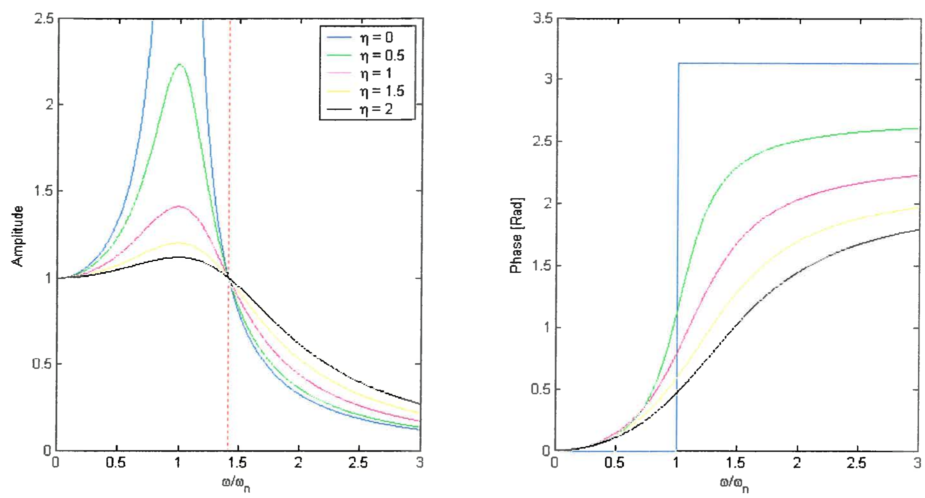


Figure 1. 7 Force transmission response of a vibration isolator

It can be seen that the transmissibility becomes less than one at frequencies above $\sqrt{2}$ times the natural frequency (dotted line). Therefore the isolator is effective above that frequency.

A major disadvantage of this type of isolator is the fact that such a soft spring must be used to make the natural frequency as low as possible. The practical implications of such a soft spring are very large displacements that are usually not permissible. It is furthermore not that easy to make such a soft spring with high lateral stiffness. Usually natural rubber is used to make engine mountings while steel springs are used in vibrating screens. When a very low natural frequency is needed, air springs are usually used because of the very low stiffness that can be obtained.

1.2.2.2 Tuned vibration isolator

The tuned vibration isolator was developed in the helicopter industry in the mid 1960s and employed in the 1970s to isolate the main rotor from the rest of the helicopter. These isolators are anti-resonant vibration isolators that are tuned to a specific frequency that is called the isolation frequency. The major advantages of these isolators over normal isolators are that stiffer springs and smaller masses can be used to achieve better isolation than is possible with normal isolators.

The first two types of isolators were the IRIS (Desjardins & Hooper, 1976 & Desjardins & Hooper, 1980) and the DAVI (Braun, 1980 & Rita et al., 1976 & Flannelly, 1966). The working principle is depicted in Figure 1. 8.

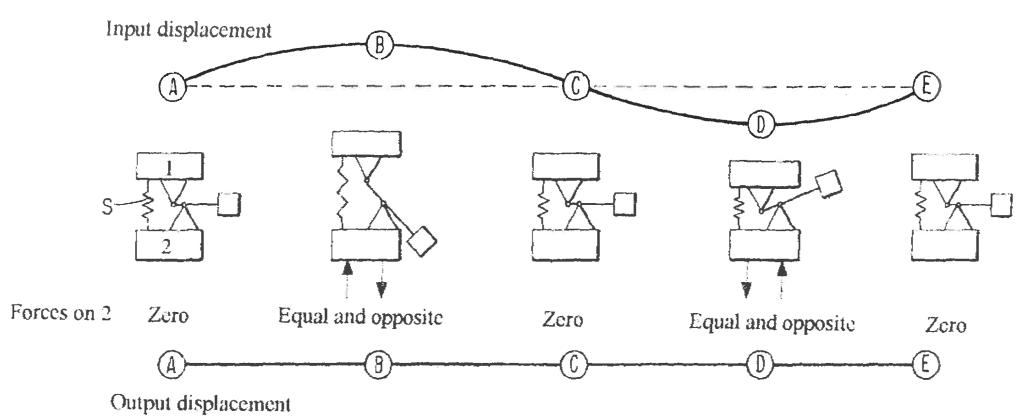


Figure 1. 8 Explanation of force cancellation (Desjardins & Hooper, 1976)

It can be seen that the small mass's motion is amplified with a lever and that the inertia of the mass cancels out the force transmitted through the spring to the base. Therefore the transmitted force is zero at the tuned frequency if no damping is present.

Figure 1. 9 give a schematic model of such an isolator.

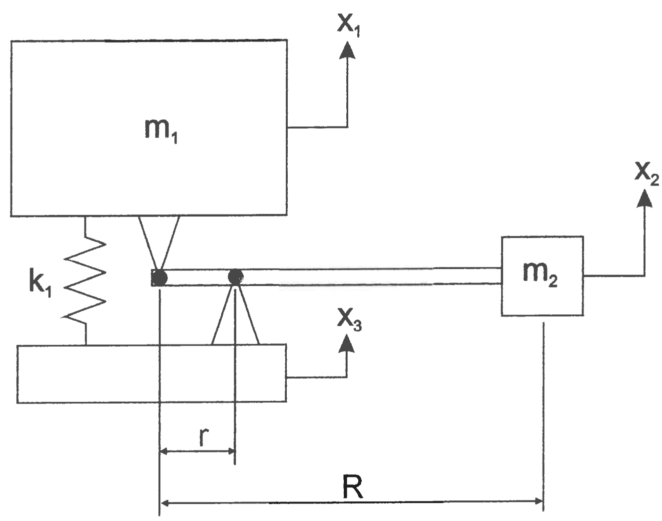


Figure 1. 9 **Tuned vibration isolator**

The transmissibility of the isolator is given by (Flannelly, 1966):

$$T = \frac{k_1 - \omega^2 \left[\frac{I}{r^2} + m_2 \frac{R}{r} \left(\frac{R}{r} - 1 \right) \right]}{k_1 - \omega^2 \left[m_1 + \frac{I}{r^2} + m_2 \left(\frac{R}{r} - 1 \right)^2 \right]} \tag{1.8}$$

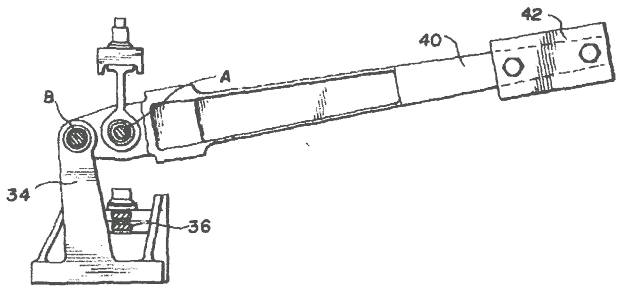
and the isolation frequency by:

$$\omega_a = \sqrt{\frac{k_1}{\frac{I}{r^2} + m_2 \frac{R}{r} \left(\frac{R}{r} - 1 \right)}} \tag{1.9}$$

With:

I	-	Moment of inertia about centre of gravity of m_2
r, R	-	Lever length
ω_a	-	Isolation frequency

A more practical version of the IRIS isolator is given in Figure 1. 10 that was used for floor isolation.



Component description	
A	Airframe pivot
B	Floor pivot
34	Anchor pad
36	Spring
40	Arm
42	Weight

Figure 1. 10 IRIS for helicopter passenger floor isolator (Desjardins & Hooper, 1980)

The transmissibility graph is given in Figure 1. 11. The natural or resonance frequency and the isolation frequency are the prominent features on the graph. The position, form and amplitude of these peaks are influenced by various parameters of the isolator and system.

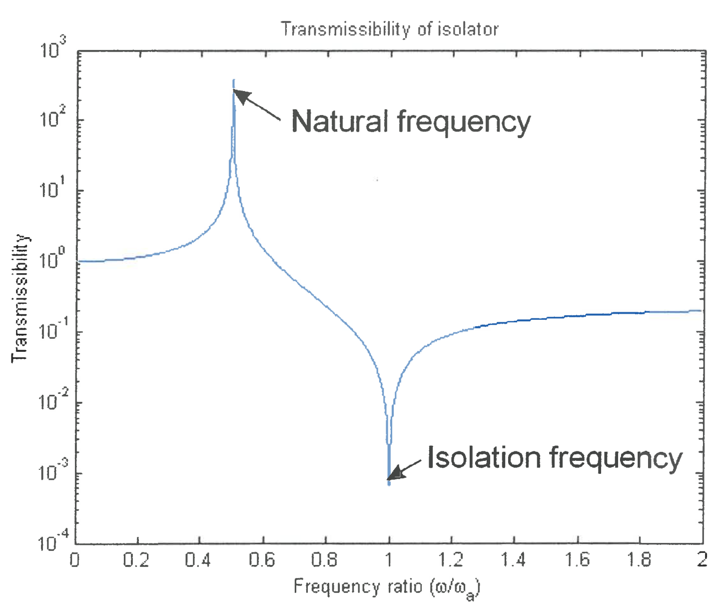


Figure 1. 11 Response of tuned vibration isolator for a typical system

These tuned vibration isolators can now be compared to the normal vibration isolators. For the comparison, systems with the same natural frequency and same amount of damping were used and the result can be seen in Figure 1. 12.

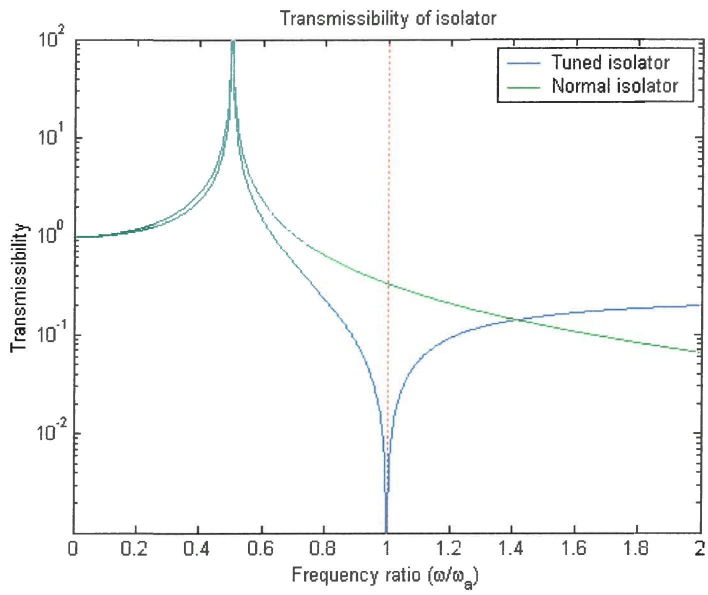
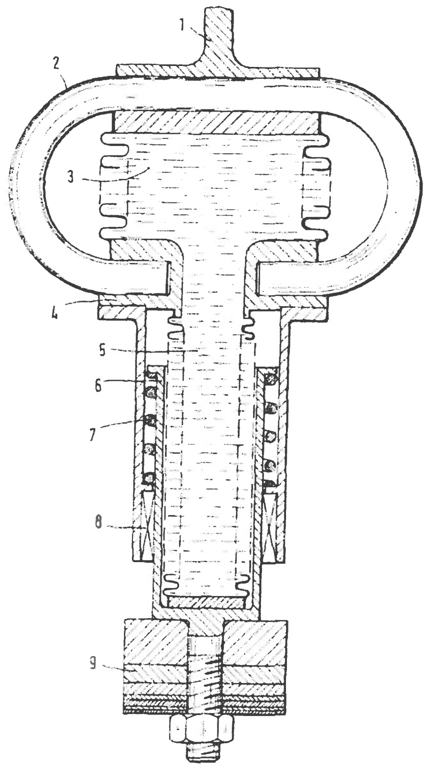


Figure 1. 12 Comparison between a tuned and normal vibration isolator

From the graph it can be seen that at the isolation frequency the normal isolator can only achieve a transmissibility of 0.33 compared to the theoretical 0 of the tuned isolator. It is only at frequencies above a frequency ratio of $\sqrt{2}$ that the normal isolator performs better than the tuned isolator.

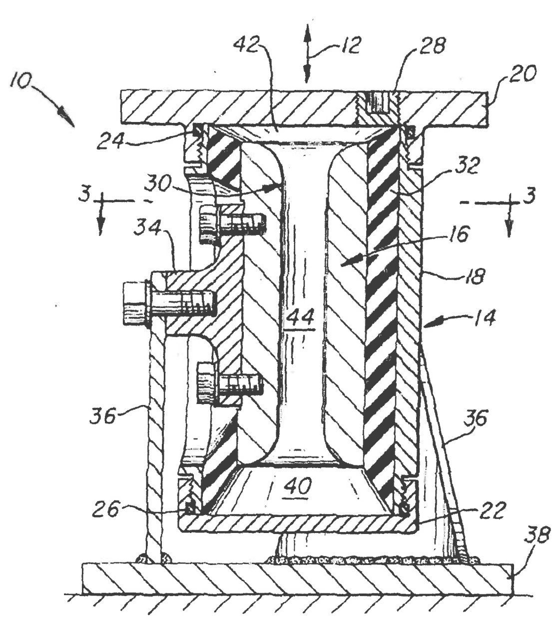
All the tuned vibration isolators use this principle, but use different ways to amplify the motion of the small mass. One very popular way to increase the effective mass was with hydraulic amplification. Figure 1. 13 show an isolator design based on this principle.

The concept of hydraulic amplification resulted in the design of the LIVE (Halwes, 1981 & Halwes & Simmons, 1980) system. This tuned vibration isolator is depicted in Figure 1. 14.



Description of elements	
1	Input force attachment
2	Glass fibre spring
3	Primary fluid chamber
4	Support
5	Secondary fluid chamber
6	Bushing
7	Spring
8	Bearing
9	Inertia mass

Figure 1. 13 MBB vibration absorber (Braun, 1980)



Component description	
10	Vibration isolator
12	Force direction
14	Outer housing
16	Inner housing
18	Central section
20	End section
30	Tuning cylinder
32	Elastomeric spring
38	Plate
40	Chamber
42	Chamber
44	Passage

Figure 1. 14 LIVE system internal design (Halwes et al., 1980)

It used hydraulic amplification and the fluid in the port as the small mass. This makes the design very compact and simple and is why it was also used on helicopters for the main rotor isolation. This type of tuned vibration isolator was selected to be used in this study. The theory will be derived in the next chapter.

Hydraulic engine mountings are another common form of isolators. In principle, they are the same as the LIVE isolator, except that very high damping is used to get wide band isolation (Flower, 1985 & Yu et al., 2000).

1.2.2.3 Semi-active vibration isolators

The same difference between passive and semi-active vibration absorbers exists between passive and semi-active vibration isolators namely that the spring is replaced by a variable stiffness spring to be able to change the natural frequency. It is not logical to use a semi-active vibration isolator as the lower the stiffness, the better the isolation at higher frequencies.

Here follows an example why semi-active vibration isolators are used in practise. While the machine is running at operating speed, the lowest stiffness gives the lowest transmission of forces to the ground. The problem comes in when the machine is shut down or started from standstill. Because the natural frequency is at a low frequency, the machine has to go through this natural frequency to reach its operating frequency. If the machine cannot accelerate through this natural frequency fast enough, it can cause serious problems. With very large machines it will cost too much to install a larger motor to increase the acceleration of the machine and therefore a semi-active vibration isolator offers an advantage. Instead of accelerating the machine through the natural frequency at high speed, the natural frequency is shifted past the excitation frequency at high speed. This can provide a more cost effective solution to the problem than to install a larger motor. Another option can also be to increase the damping in the system for the time which the machine is going through the natural frequency.

With tuned vibration isolators, the objective of the semi-active tuned vibration isolator is not to move the natural frequency of the isolator, as was the case with absorbers and normal isolators, but to move the isolation or anti-resonant frequency of the isolator. This can be achieved in a number of ways.

In the IRIS and DAVI systems, it can be achieved by changing parameters like the stiffness or arm length. In the LIVE isolator there are basically three parameters that can be changed to alter the isolation frequency:

1. Stiffness: By changing the stiffness, the natural and isolation frequencies are changed (Hodgson & Duclos, 1991).
2. Port length: The port length can be modified by using a sleeve inside the port and pushing it out (Smith & Stamps, 1995).
3. Port diameter: It is also possible to change the port diameter (Smith & Stamps, 1998). Figure 1. 15 illustrate concepts on how the last two possibilities can be achieved.

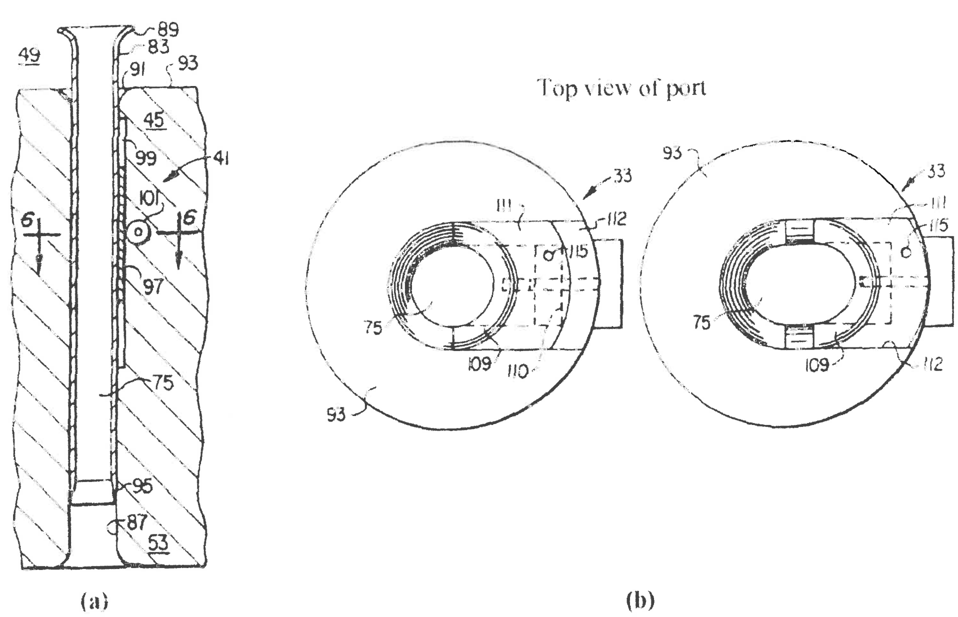


Figure 1. 15 Adjustable (a) port length and (b) area (Smith & Stamps, 1998)

Other more complex methods were also proposed (McKeown et al., 1995), but these concepts will not be dealt with in detail.

1.2.2.4 Active vibration isolator

This type of isolator is basically the same as an active vibration absorber, except that the objective in this case will be to minimise the forces transmitted to an external body. Therefore the response of the external body will be minimised and not the primary body as was the case with the absorber.

At the control side of the isolator, there will usually be a feedforward and a feedback loop. The feedforward loop will sense the vibration of the primary mass so that the actuator can react to that before the external body is influenced by it. The feedback loop senses the external body's vibration and the control system tries to minimise that.

The same table that was given for vibration absorbers is applicable to vibration isolators so that normal engine mountings are by far the most widely used type of isolator and active vibration isolators very rare because of their cost.

1.3 Smart Materials

Smart materials are a group of materials that have the ability to react to changes in environmental conditions like temperature, force and magnetic field. These materials are quite new and are not used widely in the industry. Normally one of the material properties change due to environmental conditions. This has the tendency to let the materials look "smart" or intelligent.

1.3.1 Piezoelectric material

The most common smart material is the piezoelectric material. This material has the ability to change the electric field over its poles when force is applied to it. This ability works in the opposite way as well so that if an electric voltage is supplied over its poles, the material will expand and will work like an actuator. This material is used extensively for accelerometers, sonar and high frequency actuation. A very important property of piezoelectric actuators is that their displacement is extremely small (usually smaller than 5 micron) but that they can operate at extremely high frequencies. This makes them ideal for applications like accelerometers or sonar transducers. A lot of effort has been put into the development of high displacement piezoelectric actuators. The most common way to achieve larger displacements is to put the actuators in stacks (Hooker, 1997). The advantage of stacks is that the force is not sacrificed to achieve large displacements.

Another interesting way to achieve large displacements was the use of inchworms (Herakovic, 1998). These actuators consist of 3 piezo actuators that work together to move like a worm. This enables the actuator to move long distances. Figure 1. 16 illustrates the working principle.

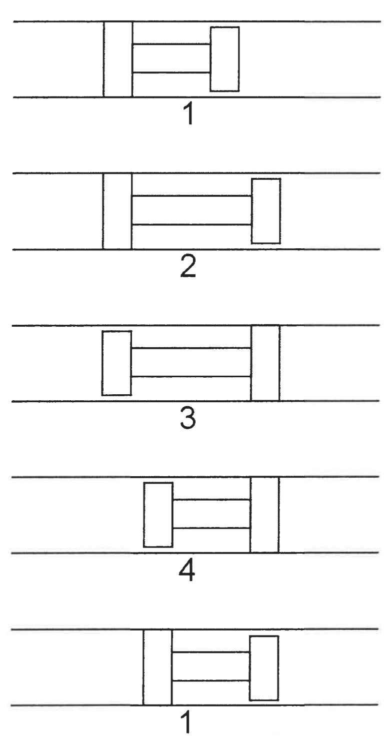


Figure 1. 16 Working principle of an inchworm

Hybrid actuators were developed (Janker, 1998) to improve the displacement of piezo or other small displacement actuators. It consists of a displacement amplification device to improve displacement of the actuator, but at the cost of force. Such an actuator is illustrated schematically in Figure 1. 17.

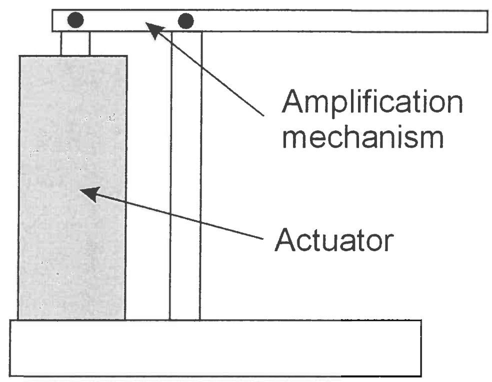


Figure 1. 17 Displacement amplification hybrid actuator

Another interesting mechanism is a locking mechanism to enable a low force actuator to withstand large forces when stationary (Chen, 1999).

It can be seen that piezoelectric actuators are very well suited for high frequency applications and not where large displacements at low frequencies are needed.

1.3.2 Shape memory alloys

Another very common smart material is shape memory alloys (SMA). This material changes its molecular grouping from austenite to martensite above certain temperatures and that results in a change in Young's modulus (Stalmans & Van Humbeeck, 1995 & Duerig et al., 1990). The shape memory effect was first observed in 1932 (Hodgson et al., 1999). If a SMA is deformed when cold, it will return to its original form when heated above its transition temperature (Lin, 1996). This is where the name shape memory alloy comes from, because it seems as if the metal remembered its original form. The forces applied by the metals to return to their original form can be quite significant and allow the SMAs to be used as actuators (Baz et al., 1990 & Meghdari et al., 1993).

Many models have been developed to predict the behaviour of Nitinol, the most common shape memory alloy (Ford et al., 1995, Witting & Cozzarelli, 1994, Bo & Lagoudas, 1994 & Monkman, 2000). The damping of Nitinol has also been characterised (Lin et al., 1993) and many springs have been developed to act as variable stiffness springs due to the material change of the shape memory alloys (Liang & Rogers, 1993). In vibrations, shape memory alloy has been used for altering stiffness in systems (Segalman et al., 1993) or to form part of the system to induce changes (Feng & Li, 1996).

These materials can usually only produce large displacement or large force, and not both. Their availability is also a problem and can usually only be bought in very thin wires that is not always the format needed.

1.3.3 Other materials

Another useful smart material is magnetorheological (MR) elastomers. This is basically an elastomer like natural rubber, but with approximately 30% of very fine iron powder mixed into it (Davis, 1999 & Jolly et al., 1996a & Jolly et al., 1996b & Ginder et al., 2000 & Ginder et al., 1999). The rubber is cured in a magnetic field and this leads to an elastomer that changes stiffness in the presence of a magnetic field. One practical problem with this material is the magnitude of the magnetic field that is required for the change. It is normally not possible to produce such a magnetic field over a large distance, so the physical size of the elastomer must be very small, which limits practical applications.

Wax actuators are another type of compact actuator. The most common example of this is the thermostat of most cars. It consists of a sealed copper cup filled with a type of wax (that expands significantly when heated) and a shaft. When the wax expands it pushes the shaft outwards and acts as an actuator. Problems with these actuators are the heating and especially the cooling of it and also the control of its displacement. In contrast to traditional smart materials, these actuators can produce very large forces and displacements making them very useful. Forces of up to 500N and displacements of 10mm are common with these actuators.

1.4 Variable stiffness springs

Although little has been published about this subject, there are a few attempts at developing variable stiffness springs documented in the literature. The most impressive of these is a spring which can change stiffness by 45 times by separating two beams of a compound leaf spring (Walsh & Lamancusa, 1992). The application of the spring was a semi-active vibration absorber. The mass-spring system that they used is depicted in Figure 1. 18.

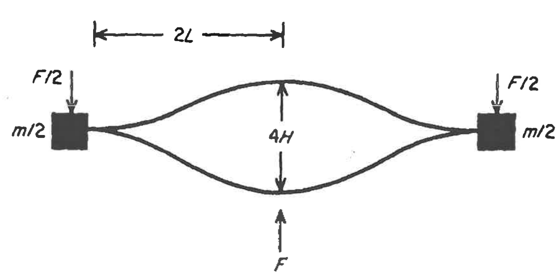


Figure 1. 18 Variable stiffness absorber (Walsh & Lamancusa, 1992)

Although other authors do not see this as practically applicable (Brennan, 2000), it shows that the concept has certain possibilities. The concept was therefore developed further and Figure 1. 19 show different possibilities to create such a semi-active absorber with the use of beams or leaf springs.

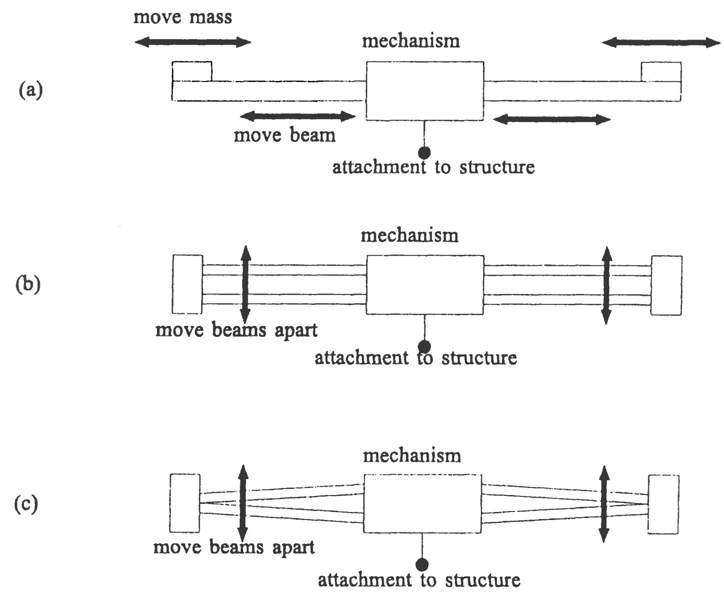


Figure 1. 19 Different ways of configuring a beam as a semi-active vibration absorber (Brennan, 2000)

Another concept is a mass suspended between two air springs of which the stiffness can be changed by altering the pressure inside the air springs (Longbottom & Rider, 1987). This concept has the ability to achieve very low stiffness due to the use of air springs. This concept was later improved with a simple method of automatically adjusting the stiffness (Brennan et al., 1996).

Changing the number of active coils in a coil spring is also a way of making a variable stiffness spring (Franchek et al., 1995) and also the heating of a temperature dependant viscoelastic element (Smith, 1991). Another concept is to change the angle at which the coil springs are orientated to change the effective stiffness as depicted in Figure 1. 20 (Ribakov & Gluck, 1998).

Electro magnetism can be used to influence the stiffness of a spring. Figure 1. 21 show such a variable stiffness spring that has primary coil springs that is de-stiffened by an electro-magnetic negative stiffness (Von Flotow et al., 1994).

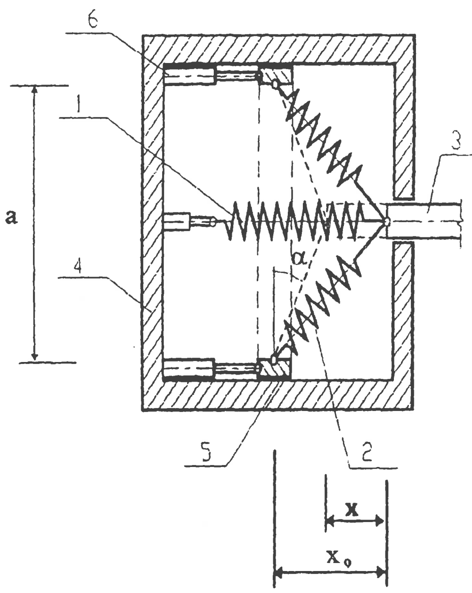


Figure 1. 20 Variable stiffness coils spring assembly (Ribakov & Gluck, 1998)

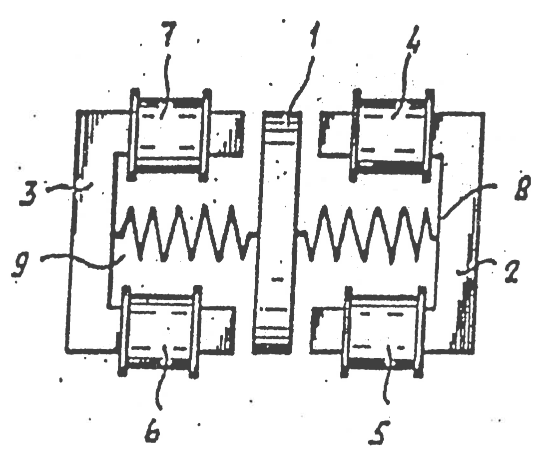


Figure 1. 21 Electro-magnetic variable stiffness spring (Von Flotow et al., 1994)

As mentioned in the previous section, shape memory alloys have the ability to change their Young's modulus when heated. Therefore a variable stiffness spring can be made if such shape memory alloy wire is turned into a coil spring. Figure 1. 22 shows a concept of a variable stiffness spring that utilizes shape memory alloy coil springs to achieve the variation in stiffness (Siler & Demoret, 1996).

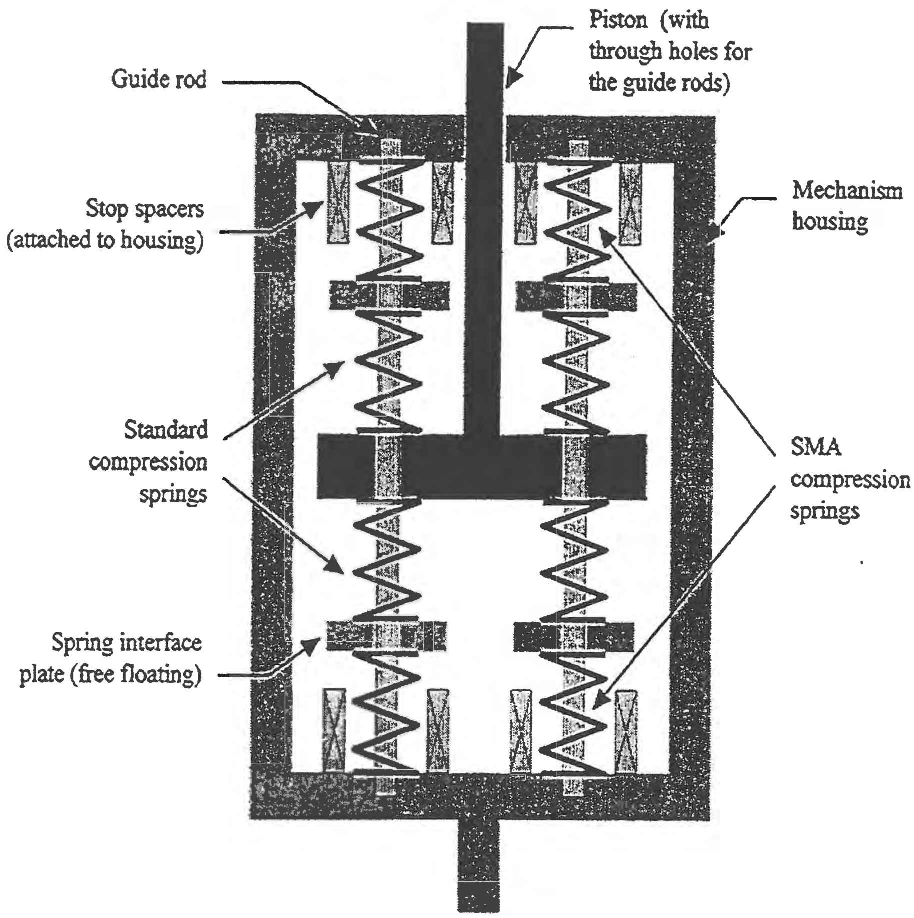


Figure 1. 22 Shape memory alloy variable stiffness spring (Siler & Demoret, 1996)

One last interesting concept used packs of cylindrical panels to create a variable stiffness spring (Mashinostroehiya, 1987). Most of these devices were developed for vibration absorbers and were designed appropriately. In vibration isolators the requirements for the stiffness element is different concerning lateral stiffness, damping and stroke and therefore it is necessary to develop a new variable stiffness spring for application in vibration isolators.

1.5 Objectives

The LIVE concept gives the possibility of a very compact and practical isolator. The hydraulic amplification mechanism makes it possible to modify parameters of the isolator to create a semi-active tuned vibration isolator. Smart materials are a new class of materials that is not used in many applications up to now. These materials are seen as materials with a lot of potential for actuators and sensors, but more suitable applications are needed to develop these materials. The aim of this study is to develop a tunable LIVE isolator by changing the stiffness and damping of the isolator. The change in stiffness shifts the isolation frequency of the isolator, while the variable damping control the amplification at the natural frequency. This will be dealt with in more detail in Chapter 2.

The specific objectives of the study are to:

- Develop a variable stiffness spring that can be incorporated in a LIVE isolator to change the isolation frequency of the isolator. It is preferable to make use of smart materials to obtain the change in stiffness.
- Design and build a tunable LIVE isolator, incorporating the variable stiffness spring and a variable damping mechanism.
- Characterise the isolator and determine the effectiveness of the stiffness and damping changes.
- Develop a control system to demonstrate that it is practically possible to tune the isolator automatically to the excitation conditions.

1.6 Conclusions

This chapter gave an overview of the background of absorbers, isolators, smart materials and variable stiffness springs. The differences between absorbers and isolators were addressed as well as all the different types of absorbers and isolators. A literature study on smart materials was done and the relevant concepts were mentioned and explained. Variable stiffness springs were also addressed with a detailed literature study. Explanations of the different concepts encountered were provided. The objectives for the project were listed.

CHAPTER 2

Mathematical model of isolator

2.1 Introduction

It is important to derive a mathematical model of the isolator before the design phase to be able to predict the performance of the isolator. In this chapter a one-dimensional model of the isolator will be derived to obtain equations for the natural and isolation frequency as well as the transmissibility of the isolator.

The derivations in sections 2.2.1 to 2.2.5 have been done by Du Plooy, 1999 and are given because of the relevance to this work. The derivation was done for an isolator that was attached to the ground on the one side and to a vibrating machine on the other. The transmissibility equation obtained is still applicable if both ends are free to move (acceleration transmissibility), as long as the direction is kept the same. If the direction is turned around, the equations change slightly. Therefore the derivation of the equation for this case is done in section 2.2.7. The derivation of the damped isolation and natural frequency in section 2.2.6 has not been published previously and was done as part of this study.

2.2 Mathematical 1-DOF model of isolator:

2.2.1 Equations of motion

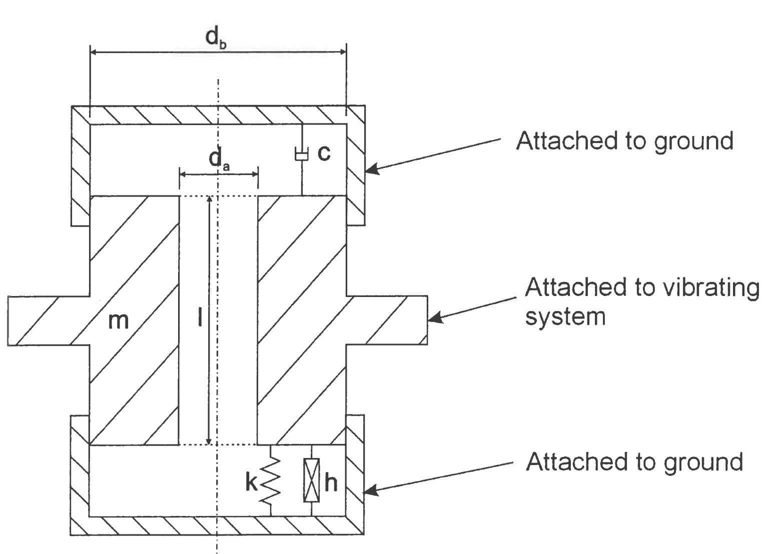


Figure 2. 1 Definition of LIVE system geometry

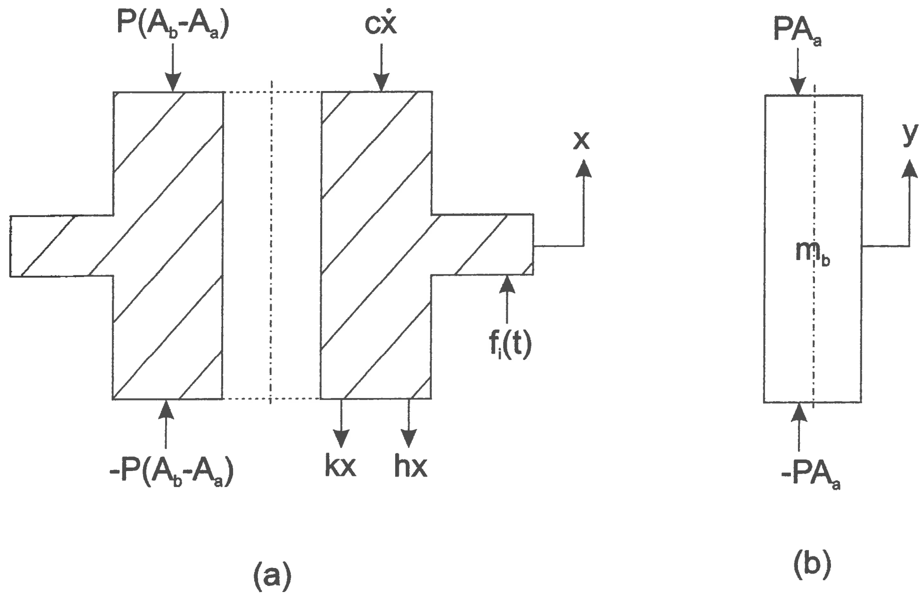


Figure 2. 2 **Forces acting on (a) isolator assembly and (b) liquid column in port**

In the figure above, the following abbreviations are used:

m	-	System mass
m_b	-	Absorber mass
k	-	Stiffness of spring
h	-	Structural damping of spring
c	-	Viscous damping of fluid
l	-	Length of the port
d_a	-	Diameter of the port
d_b	-	Diameter of the reservoir
x	-	Displacement of the port
y	-	Displacement of liquid in port

The relationship between $\dot{x}$ and $\dot{y}$ can be found by applying mass conservation and finding the continuity equation:

$$\dot{y}A_a = -\dot{x}(A_b - A_a)$$
$$\dot{y} = \left(1 - \frac{A_b}{A_a}\right)\dot{x} \tag{2.1}$$

The force balance on the system mass (m) is:

$$m\ddot{x} = -c\dot{x} - kx - ihx - 2P\left(A_b - A_a\right) + f_i\left(t\right) \tag{2.2}$$

The force balance on the absorber mass ($m_b = \rho l A_a$) is:

$$m_b \ddot{y} = -2PA_a \tag{2.3}$$

Substituting the acceleration in equation (2.3) with the conservation of mass found in equation (2.1) gives the equation for the peak pressure:

$$P = -\frac{m_b}{2A_a^2}\left(A_a - A_b\right)\ddot{x} \tag{2.4}$$

Substituting this peak pressure and $h = k\eta$ into equation (2.2) give the following single degree-of-freedom equation of motion for the system:

$$\left[m + m_b\left(1 - \frac{A_b}{A_a}\right)^2\right]\ddot{x} + c\dot{x} + k\left(1 + i\eta\right)x = f_i\left(t\right) \tag{2.5}$$

On inspection it can be seen that this equation is basically the same as a normal single DOF mass, spring and damper system except that the total mass here is a summation of the system mass (m) and the absorber mass (m_b) amplified by the area ratio. The implication of this is that it can be handled as a simple mass, spring and damper system, except that the mass used in the equations must always be the combined mass of the isolator.

From this insight, the undamped natural frequency can be written directly as follow:

$$\omega_n = \sqrt{\frac{k}{m + m_b\left(1 - \frac{A_b}{A_a}\right)^2}} \tag{2.6}$$

2.2.2 Frequency response function

Equation (2.5) can be transformed to the frequency domain by substituting the harmonic response and its derivatives, which result from harmonic excitation:

$$\begin{aligned} x(t) &= Xe^{i\omega t} \\ \dot{x}(t) &= i\omega Xe^{i\omega t} \\ \ddot{x}(t) &= -\omega^2 Xe^{i\omega t} \\ f_i(t) &= F_i e^{i\omega t} \end{aligned} \tag{2.7}$$

By making the substitutions, the following equation for the dynamic stiffness of the isolator can be found:

$$\frac{F_i}{X} = k(1+i\eta) + i\omega c - \omega^2 \left[m + m_b \left(1 - \frac{A_b}{A_a} \right)^2 \right] \tag{2.8}$$

This is graphically represented in Figure 2. 3:

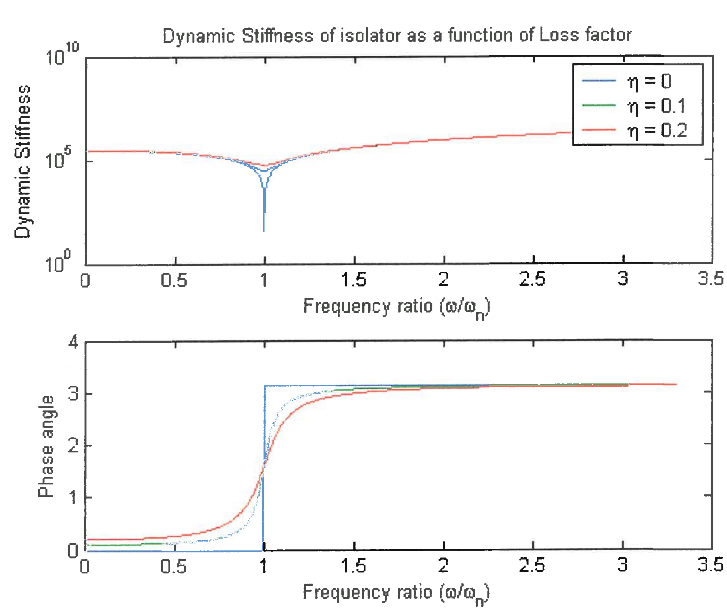


Figure 2. 3 Dynamic stiffness of isolator as function of loss factor (*c*=0)

2.2.3 Transmissibility

The transmissibility is defined as the force amplitude ratio of the force transmitted to the ground (F_0) and the applied force (F_i).

The applied force can be derived from the dynamic stiffness equation (2.8):

$$F_i = \left\{ k\left(1+i\eta\right) + i\omega c - \omega^2 \left[m + m_b \left(1 - \frac{A_b}{A_a}\right)^2 \right] \right\} X \tag{2.9}$$

The force transmitted to the ground is a combination of the force due to the pressure difference, spring and damping.

The force resulting from the pressure difference in the reservoirs is:

$$F_p = 2PA_b \tag{2.10}$$

Substituting the peak pressure found in equation (2.4) into equation (2.10) gives the force due to the pressure difference:

$$F_p = m_b \left(1 - \frac{A_b}{A_a}\right) \frac{A_b}{A_a} \omega^2 X \tag{2.11}$$

Combining this with the spring and damper forces give the reaction force on the ground:

$$F_0 = \left[k\left(1+i\eta\right) + i\omega c + \omega^2 m_b \left(1 - \frac{A_b}{A_a}\right) \frac{A_b}{A_a} \right] X \tag{2.12}$$

The transmissibility is given by eliminating X in equations (2.9) and (2.12):

$$\frac{F_0}{F_i} = \frac{k\left(1+i\eta\right) + i\omega c + \omega^2 m_b \left(1 - \frac{A_b}{A_a}\right) \frac{A_b}{A_a}}{k\left(1+i\eta\right) + i\omega c - \omega^2 \left[m + m_b \left(1 - \frac{A_b}{A_a}\right)^2 \right]} \tag{2.13}$$

The objective of the isolator is to minimize the force transmitted to the ground and therefore minimizing the transmissibility.

2.2.4 Undamped natural and isolation frequency

The undamped natural frequency of the isolator has already been determined previously in equation (2.6) as:

$$\omega_n = \sqrt{\frac{k}{m + m_b\left(1 - \frac{A_b}{A_a}\right)^2}} \tag{2.14}$$

At the undamped isolation frequency, the transmissibility will be equal to zero and therefore the undamped transmitted force (F_0) must also be equal to zero:

$$k + \omega_a^2 m_b \left(1 - \frac{A_b}{A_a}\right)\frac{A_b}{A_a} = 0 \tag{2.15}$$

where ω_a is the undamped isolation frequency.

By rearranging equation (2.15) the following equation for the undamped isolation frequency is obtained:

$$\omega_a = \sqrt{\frac{-k}{m_b\left(1 - \frac{A_b}{A_a}\right)\frac{A_b}{A_a}}} \tag{2.16}$$

In the literature this frequency is often called the anti-resonant frequency. Since anti-resonance for an undamped system is defined as a point of no response to excitation (Maia *et al.*, 1998) and since the isolator discussed here does not aim to do that, the term isolation frequency will be used.

It is important to note that this frequency is independent of the system mass. This is an extremely attractive property since a changing system mass, as may be the case for many machines, will not effect the isolation frequency. Because of real materials and fluids, the damping will never be zero and therefore it must be ensured that the lowest possible damping is achieved to obtain the best possible isolation at the isolation frequency.

With the undamped natural and isolation frequency known, the frequency ratio can now be defined as follows:

$$G_a = \frac{\omega_n}{\omega_a} = \sqrt{\frac{m_b\left(\frac{A_b}{A_a}-1\right)\frac{A_b}{A_a}}{m + m_b\left(\frac{A_b}{A_a}-1\right)^2}} \tag{2.17}$$

There is a correlation between the frequency ratio and the amount of isolation that can be achieved. Figure 2. 4 illustrate this by giving the minimum transmissibility that can be achieved for a specific frequency ratio.

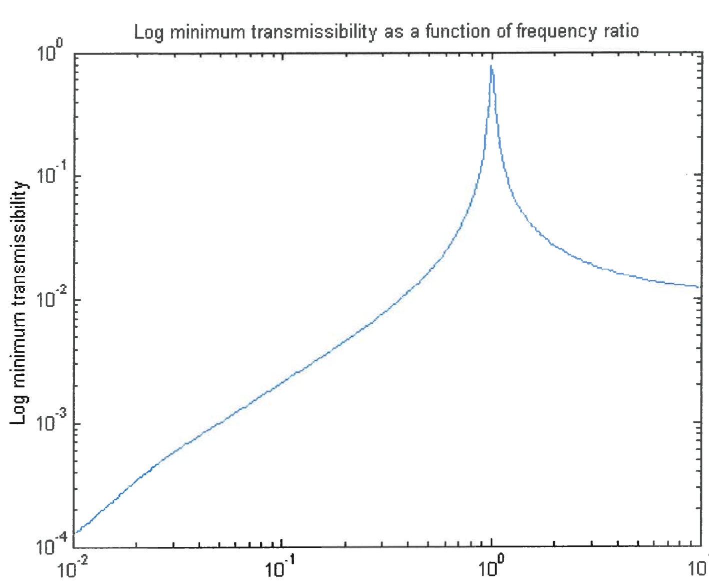


Figure 2. 4 **Log minimum transmissibility as a function of frequency ratio**

It can be seen that a low frequency ratio results in a low transmissibility.

2.2.5 Non-dimensional transmissibility

It is always a good practice to use non-dimensional equations. To derive a non-dimensional transmissibility equation, the following substitutions must be made together with the undamped natural and isolation frequencies:

$$\zeta = \frac{c}{2\left[m + m_b\left(1 - \frac{A_b}{A_a}\right)^2\right]\omega_n} \tag{2.18}$$

$$\eta = \frac{h}{k} \tag{2.19}$$

The non-dimensional transmissibility (T_r) is the given by:

$$T_r = \frac{1 - \left(\frac{\omega}{\omega_a}\right)^2 + i\left(2\zeta\frac{\omega}{\omega_n} + \eta\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + i\left(2\zeta\frac{\omega}{\omega_n} + \eta\right)} \tag{2.20}$$

The absolute transmissibility is:

$$|T_r| = \left\{\frac{\left[1 - \left(\frac{\omega}{\omega_a}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n} + \eta\right]^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n} + \eta\right]^2}\right\}^{\frac{1}{2}} \tag{2.21}$$

The phase angle is:

$$\phi = \tan^{-1}\left\{\frac{2\zeta\frac{\omega}{\omega_n} + \eta}{1 - \left(\frac{\omega}{\omega_a}\right)^2}\right\} - \tan^{-1}\left\{\frac{2\zeta\frac{\omega}{\omega_n} + \eta}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right\} \tag{2.22}$$

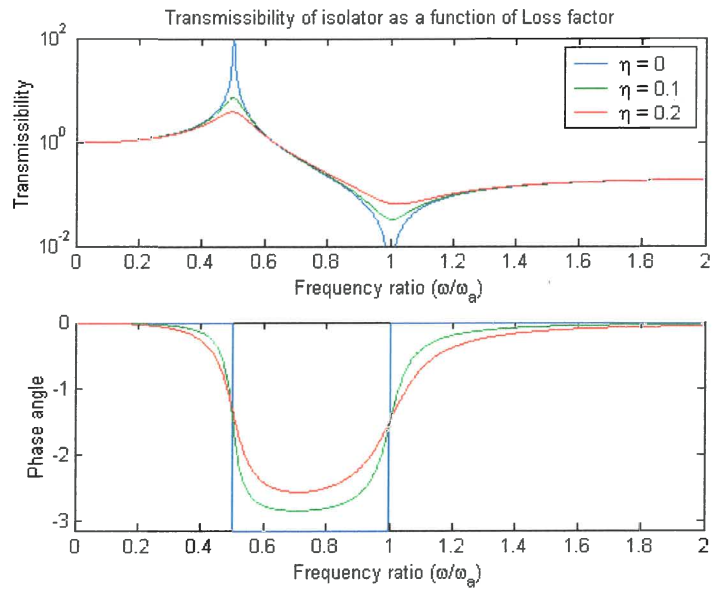


Figure 2. 5 Transmissibility of isolator as function of loss factor (ζ=0)

2.2.6 Damped natural and isolation frequency

There will always be damping in a system. Furthermore, it is the aim of the project to develop a variable stiffness and damping isolator that will have quite a large amount of damping present when set to maximum damping. Therefore it is necessary to determine the equations for the damped natural and isolation frequency.

For this derivation, structural damping will be neglected. The reason for this is that the derivation is not possible with the structural damping included and that it is a fair assumption to combine all damping in the system into one viscous damping constant.

To determine the natural and isolation frequency, the derivative with respect to frequency, of the absolute value of the transmissibility, must be set equal to zero. The frequency values at which this will be true are the natural and isolation frequency. In essence, the frequency of maximum transmissibility is determined and not the damped natural frequency. Because a single degree of freedom system is considered, it is a fair assumption that the frequency of maximum transmissibility and the damped natural frequency will be the same or very close to each other. Therefore the term damped natural frequency will be used to be consistent with the undamped natural frequency used in previous sections.

The transmissibility without structural damping is:

$$|T_r| = \left\{ \frac{\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \right\}^{\frac{1}{2}} \tag{2.23}$$

Setting its derivative equal to zero gives:

$$\frac{\partial}{\partial\omega} \left\{ \frac{\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \right\}^{\frac{1}{2}} = 0 \tag{2.24}$$

$$\frac{\frac{1}{2}\left\{ \frac{\frac{-4\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]\omega}{\omega_a^2} + \frac{4\left(2\zeta\frac{\omega}{\omega_n}\right)\zeta}{\omega_n}}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2} - \frac{\left\{\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2\right\}\left\{\frac{-4\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]\omega}{\omega_n^2} + \frac{4\left(2\zeta\frac{\omega}{\omega_n}\right)\zeta}{\omega_n}\right\}}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2} \right\}}{\left\{ \frac{\left[1-\left(\frac{\omega}{\omega_a}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2} \right\}^{\frac{1}{2}}} = 0$$

(2.25)

Solving this equation gives five roots of which one is zero, two are negative frequencies ($-\omega_{nd}$, $-\omega_{ad}$) and the other two give the damped natural and isolation frequencies that look as follow:

Damped natural and isolation frequency:

$$\omega_{n_d},\omega_{a_d}=\omega_n\sqrt{\frac{-1-\left(\frac{\omega_a}{\omega_n}\right)^2\pm\sqrt{\left(1-\left(\frac{\omega_a}{\omega_n}\right)^2\right)^2+8\left(\frac{\omega_a}{\omega_n}\right)^2\zeta^2\left(1+\left(\frac{\omega_a}{\omega_n}\right)^2\right)}}{4\zeta^2\left(1+\left(\frac{\omega_a}{\omega_n}\right)^2-\frac{1}{2\zeta^2}\right)}} \tag{2.26}$$

It is important to note that the inverse of the frequency ratio features a few times in this equation. The importance of this is that this damped natural and isolation frequency is dependent on the system mass, because the frequency ratio is dependent on it as well as the undamped natural frequency that stands in front of the equation.

2.2.7 Acceleration transmissibility

The force transmissibility was derived in the previous sections. This is applicable as long as one side of the isolator is attached to the ground. If the isolator is implemented between a handle and a vibrating machine, the force transmissibility is no longer applicable. An acceleration or displacement transmissibility is then representative of the system. The force transmissibility is the same as the acceleration transmissibility due to Newton's second law. The difference is that the outside part is no longer attached to ground, but is also moving. If the isolator is used in this way, say for example for isolation of a handle from a machine, the heavier outside part is attached to the handle and the inner part to the vibrating machine. This will result in better isolation due to the heavier mass on the handle. This orientation causes the transmissibility to change slightly and the derivation for the case depicted in Figure 2. 6 will follow.

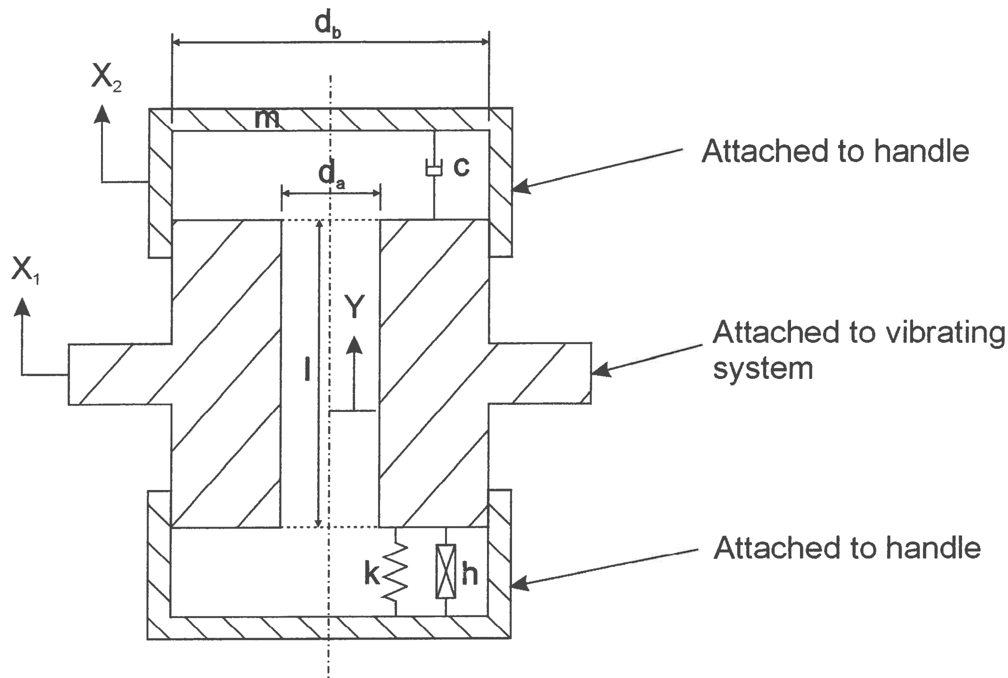


Figure 2. 6 LIVE system implemented on handle

In the above system the displacement X_1 will be given a prescribed displacement. The continuity equation becomes:

$$\dot{y}A_a = -\dot{x}_1(A_b - A_a) + \dot{x}_2 A_b$$
$$\dot{y} = \left(1 - \frac{A_b}{A_a}\right)\dot{x}_1 + \frac{A_b}{A_a}\dot{x}_2 \qquad (2.27)$$

The force balance on the system mass (m) is:

$$m\ddot{x}_2 = -c\left(\dot{x}_1 - \dot{x}_2\right) - k\left(x_1 - x_2\right) - ih\left(x_1 - x_2\right) + 2P\left(A_b\right) \qquad (2.28)$$

The force balance on the absorber mass ($m_b = \rho l A_a$) is:

$$m_b\ddot{y} = -2PA_a \qquad (2.29)$$

Substituting the acceleration in equation (2.29) with the conservation of mass found in equation (2.27) gives the equation for the peak pressure:

$$P = -\frac{m_b}{2A_a}\left[\left(1-\frac{A_b}{A_a}\right)\ddot{x}_1 + \frac{A_b}{A_a}\ddot{x}_2\right] \tag{2.30}$$

Substituting this peak pressure and $h = k\eta$ into equation (2.28) and transforming it to the frequency domain give the following motion for the system:

$$k(1+i\eta)(X_1 - X_2) + i\omega c(X_1 - X_2) + \omega^2 m X_2 + \omega^2 m_b \frac{A_b}{A_a}\left[\left(1-\frac{A_b}{A_a}\right)X_1 + \frac{A_b}{A_a}X_2\right] = 0 \tag{2.31}$$

Rewriting of the equation gives the transmissibility directly as:

$$\frac{X_2}{X_1} = \frac{k(1+i\eta) + i\omega c + \omega^2 m_b \left(1-\frac{A_b}{A_a}\right)\frac{A_b}{A_a}}{k(1+i\eta) + i\omega c - \omega^2\left[m + m_b\left(\frac{A_b}{A_a}\right)^2\right]} \tag{2.32}$$

This is then the displacement or acceleration transmissibility for the isolator for a forced response of the inner part. For this case the undamped natural frequency becomes:

$$\omega_n = \sqrt{\frac{k}{m + m_b\left(\frac{A_b}{A_a}\right)^2}} \tag{2.33}$$

The undamped isolation frequency stay unchanged.

2.3 Reasons for changing stiffness and damping

In this study, it was decided to develop a tunable vibration isolator by means of changing the stiffness and damping of the isolator. At this stage it is necessary to explain why the stiffness and damping of the isolator will be changed and what the effect of the changes will be.

Tuned vibration absorbers were developed for tonal (single frequency) excitation. For the isolator to perform optimal, the tonal excitation frequency and the isolation frequency of the isolator must coincide exactly. This is not as easy as it sounds even if the excitation frequency is constant. To design and build such a LIVE isolator where the practical and designed isolation frequency is exactly the same, is almost impossible. Therefore it is usually necessary in practice to be able to shift the isolation frequency after the isolator has been installed to tune the isolator to exactly the excitation frequency.

If such isolators are implemented on machines where the excitation frequency is not constant, the problem becomes even greater and that is the situation for which this isolator is designed. In practice, machines seldom operate at exactly the same frequency throughout their lifetime. Some machines can change over the years, but others can have constant change over minutes as operating conditions change. In such a case, it is necessary to be able to continuously change the isolation frequency of the isolator while in operation to achieve maximum isolation. Imagine what would happen if the excitation frequency decreases to the extent that it coincides with the natural frequency of the isolator! Therefore it is necessary to be able to change the isolation frequency of the isolator.

As stated in the previous chapter, the isolation frequency of a LIVE isolator can be changed in three ways:

- Change of the stiffness of the isolator
- Change of the port length of the isolator
- Change of the port diameter of the isolator.

The last two options have practical difficulties and have already been patented. Therefore it was decided to use the change of the isolator's stiffness to change the isolation frequency of the isolator. Another advantage of changing the stiffness rather than the port length and port diameter, is the amount of isolation achieved. Changing the port diameter and length results in a change in the absorber's mass (the mass of the fluid in the port), which causes the amount of isolation achieved to change. The stiffness is the only parameter that will not change the absorber mass and that will result in a constant amount of isolation.

Figure 2. 7 shows how the transmissibility graphs change as the stiffness is changed.

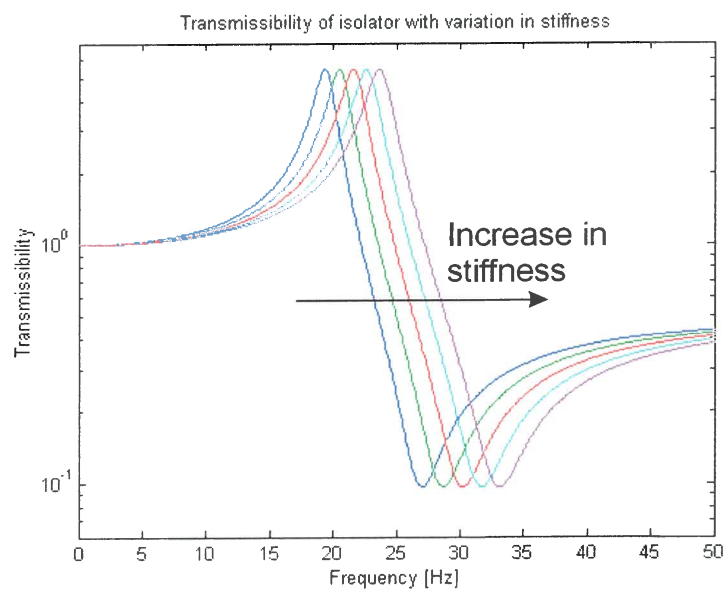


Figure 2. 7 Transmissibility of isolator with variation in stiffness

It can be seen how the isolation frequency increases as the stiffness is increased. The natural frequency also increases with the increase in stiffness. It is therefore clear that the change in stiffness enables the isolator to adapt to a change in excitation frequency.

The reason for changing the damping is that tonal excitation is not encountered in practice. Noise is always present on real machines and differs in amplitude and frequency. Usually a noise floor will be present: a constant amplitude noise over the whole frequency range.

When the transmissibility of a tuned vibration isolator is considered, it is noted that there is a region below the isolation frequency, where the vibrations are amplified and that maximum amplification occurs at the natural frequency. For pure tonal excitation the natural frequency will not have any influence, but the moment a noise floor is present, the noise at the natural frequency will be amplified. If the amplitude of the noise is very small, the effect will be very small, but as the noise levels increase, the amplification of the noise can have a significant influence. The factor that has to be considered is the ratio between the amplitude of excitation at the isolation frequency and at the natural frequency of the isolator. As this ratio decreases, a point will be reached where the noise at the natural frequency will be amplified more than the excitation frequency is decreased by the isolator. Therefore the result will be that one would have been better off without the isolator.

Increasing the damping in the isolator can reduce the response to noise at the natural frequency of the isolator. Figure 2. 8 show the effect of an increase in viscous damping on the transmissibility graph of the isolator.

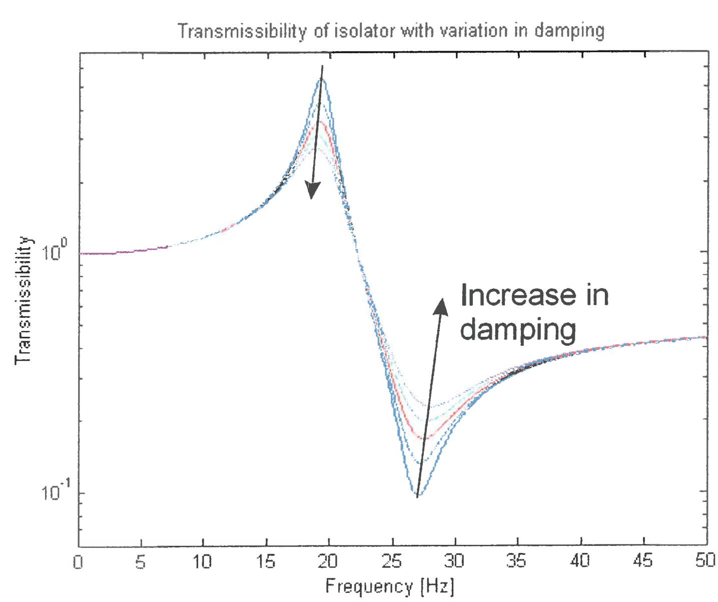


Figure 2. 8 Transmissibility of isolator with variation in damping

As the damping increases, the amplitude of the natural frequency decreases, but at the same time the amplitude at the isolation frequency also increases. For a certain level of noise, there will be an amount of damping that would give the least transmission of vibration over the whole frequency range that would not necessarily be the least damping. Therefore it is also necessary to be able to change the damping of the isolator to be able to obtain best isolation at all times under all excitation conditions.

2.4 Conclusion

In this chapter a one-degree of freedom model was formulated for the LIVE isolator. Equations for the undamped natural frequency and isolation frequency were derived as well as an equation for the transmissibility of the isolator that was also non-dimensionalised. The last equations derived were that of the damped natural frequency and isolation frequency. The last part of the chapter addressed the issue around the reasons for changing the stiffness and damping of the isolator. It was explained how the stiffness change the isolation frequency of the isolator and how the change in damping helps with control of noise amplification.