

# Appendix A

## The extended Dixon-Szegö test set

Problems **G1** and **G2** (Griewank G1 and G2 functions, respectively) [48].

OBJECTIVE FUNCTION:

$$f(x) = \sum_{i=1}^n x_i^2/d - \prod_{i=1}^n \cos(x_i/\sqrt{i}) + 1.$$

For G1,  $n = 2$  and  $d = 200$ ; for G2,  $n = 10$  and  $d = 4000$ .

SEARCH DOMAIN FOR G1:

$$D = \{(x_1, x_2) \in R^2 : -100.0 \leq x_i \leq 100.0, i = 1, 2\}.$$

SEARCH DOMAIN FOR G2:

$$D = \{(x_1, x_2, \dots, x_{10}) \in R^{10} : -600.0 \leq x_i \leq 600.0, i = 1, 2, \dots, 10\}.$$

SOLUTION:

$$x^* = (0.0, \dots, 0.0) \quad f^* = 0.0.$$

**Problem GP** (Goldstein-Price) [42]. ODL.B6-28

OBJECTIVE FUNCTION:

$$f(x) = [1 + (x_1 + x_2 + 1)^2 \cdot (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)].$$

SEARCH DOMAIN:

$$D = \{(x_1, x_2) \in R^2 : -2.0 \leq x_i \leq 2.0, i = 1, 2\}.$$

SOLUTION:

$$x^* = (0.0, -1.0) \quad f^* = 3.0.$$

**Problem C6** (Six-hump Camelback) [49]. ODL.B2-7

OBJECTIVE FUNCTION:

$$f(x) = (4 - 2.1x_1^2 + \frac{1}{3}x_1^4)x_1^2 + x_1x_2 + (-4 + 4x_2^2)x_2^2$$

SEARCH DOMAIN:

$$D = \{x_1 \in R^1 : -3.0 \leq x_1 \leq 3.0\}$$

$$D = \{x_2 \in R^1 : -2.0 \leq x_2 \leq 2.0\}$$

SOLUTION:

$$\mathbf{x}_1^* = (0.0898, -0.7126) \quad \mathbf{x}_2^* = (-0.0898, 0.7126) \quad f^* = -1.0316285$$

**Problem SH** (Shubert function, Levi no. 4) [49].

OBJECTIVE FUNCTION:

$$f(\mathbf{x}) = \left\{ \sum_{i=1}^5 i \cos[(i+1)x_1 + i] \right\} \left\{ \sum_{i=1}^5 i \cos[(i+1)x_2 + i] \right\}$$

SEARCH DOMAIN:

$$D = \{(x_1, x_2) \in R^2 : -10.0 \leq x_i \leq 10.0, \quad i = 1, 2\}$$

SOLUTION:

$$\mathbf{x}_1^* = (5.48289, -1.426531) \quad f^* = -186.73091$$

**Problem RA** (Rastrigin) [50]

OBJECTIVE FUNCTION:

$$f(\mathbf{x}) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$$

SEARCH DOMAIN:

$$D = \{(x_1, x_2) \in R^2 : -1.0 \leq x_i \leq 1.0, \quad i = 1, 2\}.$$

SOLUTION:

$$\mathbf{x}^* = (0.0, 0.0) \quad f^* = -2.0$$

**Problem BR** (Branin) [51]

OBJECTIVE FUNCTION:

$$f(\mathbf{x}) = \left( x_2 - \frac{5.1}{4\pi^2} x_1^2 + \frac{5}{\pi} x_1 - 6 \right)^2 + 10 \left( 1 - \frac{1}{8\pi} \right) \cos(x_1) + 10$$

SEARCH DOMAIN:

$$D = \{x_1 \in R^1 : -5.0 \leq x_1 \leq 10.0\}$$

$$D = \{x_2 \in R^1 : 0.0 \leq x_2 \leq 15.0\}$$

SOLUTION:

$$\mathbf{x}_1^* \approx (3.142, 2.275) \quad f^* \approx 0.398$$

**Problem H3, H6** (Hartman 3, 6) [42]

OBJECTIVE FUNCTION:

$$f(\mathbf{x}) = - \sum_{i=1}^m c_i \exp \left( - \sum_{j=1}^n a_{ij} (x_j - p_{ij})^2 \right),$$

where  $\mathbf{x} = (x_1, \dots, x_n)$ , and

H3 :  $m = 4, n = 3$ 

$i$	$a_{ij}$			$c_i$	$p_{ij}$		
1	3.0	10.0	30.0	1.0	0.3689	0.1170	0.2673
2	0.1	10.0	35.0	1.2	0.4699	0.4387	0.7470
3	3.0	10.0	30.0	3.0	0.1091	0.8732	0.5547
4	0.1	10.0	35.0	3.2	0.03815	0.5743	0.8828

 H6 :  $m = 4, n = 6$ 

$i$	$a_{ij}$						$c_i$
1	10.0	3.0	17.0	3.5	1.7	8.0	1.0
2	0.05	10.0	17.0	0.1	8.0	14.0	1.2
3	3.0	3.5	1.7	10.0	17.0	8.0	3.0
4	17.0	8.0	0.05	10.0	0.1	14.0	3.2

$i$	$p_{ij}$					
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

SEARCH DOMAIN:

$$D = \{(x_1, \dots, x_n) \in R^n : 0.0 \leq x_i \leq 1.0, i = 1, \dots, n\}$$

SOLUTIONS:

H3:

$$x^* = (0.11461478, 0.55564892, 0.85254688), f^* = -3.8627821.$$

H6:

$$x^* = (0.20168955, 0.15000963, 0.47687211, 0.27533377, 0.31165102, 0.65730111),$$

$$f^* = -3.322368.$$

Problem S5, S7, S10 (Shekel 5, 7, 10) (SQRIN) [42]

OBJECTIVE FUNCTION:

$$f(x) = - \sum_{i=1}^m \frac{1}{(x - a_i)^T (x - a_i) + c_i},$$

where:

$i$	$a_i$				$c_i$
1	4.0	4.0	4.0	4.0	0.1
2	1.0	1.0	1.0	1.0	0.2
3	8.0	8.0	8.0	8.0	0.2
4	6.0	6.0	6.0	6.0	0.4
5	3.0	7.0	3.0	7.0	0.4
6	2.0	9.0	2.0	9.0	0.6
7	5.0	5.0	3.0	3.0	0.3
8	8.0	1.0	8.0	1.0	0.7
9	6.0	2.0	6.0	2.0	0.5
10	7.0	3.6	7.0	3.6	0.5

SEARCH DOMAIN:

$$D = \{(x_1, \dots, x_4) \in R^4 : 0.0 \leq x_i \leq 10.0, i = 1, \dots, 4\}.$$

SOLUTIONS:

S5:

$$x^* = (4.00003727, 4.00013375, 4.00003730, 4.00013346) \quad f^* = -10.153200.$$

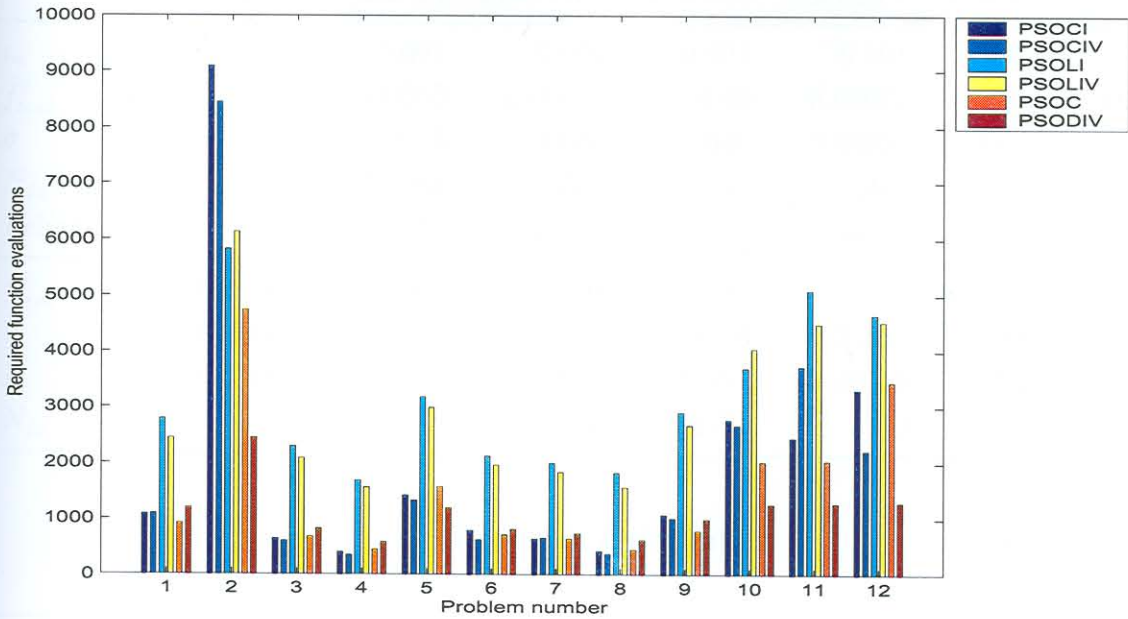
S7:

$$x^* = (4.00057280, 4.00069020, 3.99948997, 3.99960620) \quad f^* = -10.402941.$$

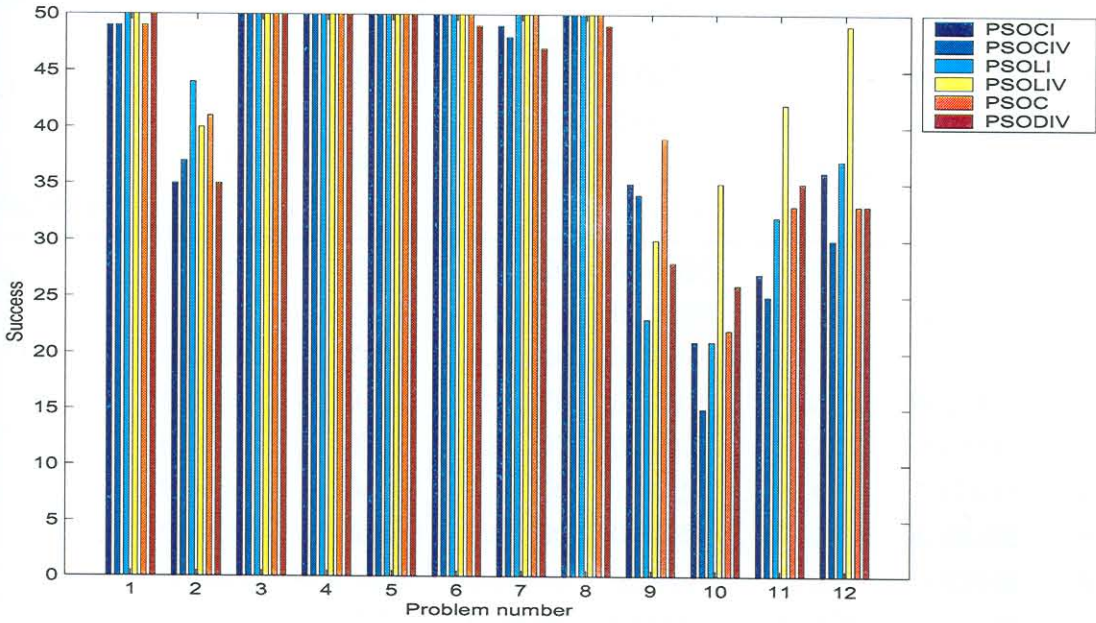
S10:

$$x^* = (4.00074671, 4.00059326, 3.99966290, 3.99950981) \quad f^* = -10.536410.$$

## A.1 Numerical results for the extended Dixon-Szegö test set



(a) Cost for the different variants



(b) Reliability for the different variants

Figure A.1: Cost and reliability comparison for the different variants



	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		0.00053	0.00054	0.00049	0.00053	0.00054	0.00050
$\bar{\sigma}$		0.00025	0.00031	0.00030	0.00027	0.00032	0.00032
$N_{fe}$ (ave.)		1081	1089	2789	2443	918	1197
<i>Reliability</i>		49/50	49/50	50/50	50/50	49/50	50/50
$f_{best}^g$	0.00000	0.00007	0.00001	0.00002	0.00000	0.00003	0.00000
$x_1$	0.00000	-0.00223	0.00099	0.00626	0.00091	0.00387	0.00023
$x_2$	0.00000	0.01617	-0.00643	-0.00361	0.00119	0.00944	-0.00066
$N_{fe}$		842	2093	4171	2512	831	932

Table A.1: Griewank 1

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.1	0.1	0.1	0.1	0.1	0.1
$\bar{f}_{best}^g$		0.09439	0.09514	0.09550	0.09493	0.09375	0.09501
$\bar{\sigma}$		0.00497	0.00622	0.00726	0.00581	0.00725	0.00943
$N_{fe}$ (ave.)		9084	8450	5826	6131	4743	2451
<i>Reliability</i>		35/50	37/50	44/50	40/50	41/50	35/50
$f_{best}^g$	0.00000	0.07962	0.07326	0.06626	-0.07801	0.06853	0.05242
$x_1$	0.00000	-3.07656	0.06871	-0.07063	-3.12687	-6.25902	-0.08723
$x_2$	0.00000	0.07963	-0.00104	0.00018	4.75444	-4.43530	4.45253
$x_3$	0.00000	-10.9494	5.42670	5.43029	0.03736	5.45368	0.00068
$x_4$	0.00000	0.08763	-12.5407	-0.02418	6.07357	-0.17799	0.00950
$x_5$	0.00000	-0.25870	0.10766	-7.00124	-0.14234	7.17090	0.12157
$x_6$	0.00000	0.19254	-0.04966	0.02699	-7.69208	0.00288	0.00710
$x_7$	0.00000	-8.25078	-0.05076	8.20641	0.05870	0.03646	-0.07244
$x_8$	0.00000	-0.02052	0.00009	0.00001	-0.12609	-0.11542	-0.08604
$x_9$	0.00000	-0.35799	9.39031	0.08981	0.20924	-0.17104	0.01753
$x_{10}$	0.00000	0.3992	-0.0937	-10.046	0.4659	-9.8733	9.30478
$N_{fe}$		4867	11574	11806	4646	8519	2417

Table A.2: Griewank 2

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		3.00049	3.00049	3.00050	3.00050	3.00053	3.00047
$\bar{\sigma}$		0.00032	0.00027	0.00030	0.00030	0.00028	0.00029
$N_{fe}$ (ave.)		641	602	2295	2086	679	824
<i>Reliability</i>		50/50	50/50	50/50	50/50	50/50	50/50
$f_{best}^g$	3.0000	3.00000	3.00006	3.00004	3.00001	3.00001	3.00001
$x_1$	0.0000	-0.00000	-0.00051	-0.00014	-0.00013	0.00020	0.00023
$x_2$	-1.0000	-0.99994	-1.00020	-0.99974	-0.99999	-0.99999	-0.99993
$N_{fe}$		537	715	2280	2105	747	904

Table A.3: Goldstein-Price

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-1.03112	-1.03113	-1.03123	-1.03123	-1.03117	-1.03113
$\bar{\sigma}$		0.00031	0.00028	0.00027	0.00027	0.00029	0.00028
$N_{fe}$ (ave.)		402	348	1687	1566	452	584
<i>Reliability</i>		50/50	50/50	50/50	50/50	50/50	50/50
$f_{best}^g$	-1.03163	-1.03162	-1.03160	-1.03162	-1.03162	-1.03161	-1.03162
$x_1$	$\pm 0.0898$	-0.08906	-0.09086	-0.08921	0.09167	-0.09179	0.09126
$x_2$	$\mp 0.71260$	0.71339	0.71428	0.71347	-0.71272	0.71221	-0.71294
$N_{fe}$		682	218	1796	1621	627	609

Table A.4: Six-hump Camelback

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-186.730	-186.730	-186.730	-186.730	-186.730	-186.730
$\bar{\sigma}$		0.00030	0.00029	0.00029	0.00027	0.00032	0.00032
$N_{fe}$ (ave.)		1422	1332	3184	2990	1572	1197
<i>Reliability</i>		50/50	50/50	50/50	50/50	50/50	50/50
$f_{best}^g$	-186.731	-186.730	-186.730	-186.730	-186.730	-186.730	-186.730
$x_1$	5.48289	-7.70838	-7.70830	-0.80040	-7.70836	-7.08355	4.85802
$x_2$	-1.42653	-0.80027	-0.80037	4.85812	-0.80023	-1.42511	-0.80027
$N_{fe}$		1372	685	3167	3295	1144	1255

Table A.5: Schubert, Levi no. 4

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-1.99952	-1.99947	-1.99951	-1.99942	-1.99949	-1.99947
$\bar{\sigma}$		0.00031	0.00028	0.00031	0.00030	0.00030	0.00029
$N_{fe}$ (ave.)		790	624	2122	1965	713	814
<i>Reliability</i>		50/50	50/50	50/50	50/50	50/50	49/50
$f_{best}^g$	-2.00000	-1.99998	-1.99998	-1.99998	-1.99999	-1.99997	-1.99999
$x_1$	0.00000	0.00034	-0.00024	-0.00036	0.00005	0.00044	-0.00013
$x_2$	0.00000	0.00004	0.00022	-0.00009	-0.00028	-0.00006	0.00027
$N_{fe}$		867	697	1844	1439	948	836

Table A.6: Rastrigin



Optimum solution	PSO variant						
	PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV	
$\epsilon_a$	0.001	0.001	0.001	0.001	0.001	0.001	
$\bar{f}_{best}^g$	0.39845	0.39840	0.39850	0.39848	0.39848	0.39847	
$\bar{\sigma}$	0.00033	0.00034	0.00034	0.00032	0.00033	0.00030	
$N_{fe}$ (ave.)	643	658	1997	1838	645	743	
Reliability	49/50	48/50	50/50	50/50	50/50	47/50	
$f_{best}^g$	0.398	0.39790	0.39791	0.39793	0.39791	0.39789	
$x_1$	3.142	-1.86969	2.17809	2.18510	2.17763	2.18814	2.18117
$x_2$	2.275	9.42601	3.14254	3.13868	3.14319	3.14061	3.14232
$N_{fe}$	381	458	1986	1848	487	708	

Table A.7: Branin

Optimum solution	PSO variant						
	PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV	
$\epsilon_a$	0.001	0.001	0.001	0.001	0.001	0.001	
$\bar{f}_{best}^g$	-3.86218	-3.86218	-3.86220	-3.86216	-3.86218	-3.86221	
$\bar{\sigma}$	0.00028	0.00022	0.00028	0.00027	0.00025	0.00026	
$N_{fe}$ (ave.)	425	374	1827	1567	451	625	
Reliability	50/50	50/50	50/50	50/50	50/50	49/50	
$f_{best}^g$	-3.86278	-3.86270	-3.86267	-3.86275	-3.86277	-3.86272	-3.86274
$x_1$	0.11461	0.11882	0.10160	0.11875	0.11606	0.11021	0.12257
$x_2$	0.55565	0.55419	0.55604	0.55493	0.55549	0.55605	0.55531
$x_3$	0.85255	0.85263	0.85279	0.85240	0.85284	0.85185	0.85262
$N_{fe}$	343	354	1441	852	219	743	

Table A.8: Hartman 3

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-3.32159	-3.32158	-3.32161	-3.32162	-3.32161	-3.32160
$\bar{\sigma}$		0.00016	0.00016	0.00019	0.00020	0.00019	0.00017
$N_{fe}$ (ave.)		1075	1013	2908	2675	787	997
<i>Reliability</i>		35/50	34/50	23/50	30/50	39/50	28/50
$f_{best}^g$	-3.32237	-3.32202	-3.32198	-3.32207	-3.32202	-3.32217	-3.32196
$x_1$	0.20169	0.20192	0.20187	0.19887	0.19938	0.20075	0.20118
$x_2$	0.15001	0.14500	0.15094	0.14956	0.14827	0.15239	0.15542
$x_3$	0.47687	0.47733	0.47345	0.48056	0.47493	0.47399	0.47839
$x_4$	0.27533	0.27666	0.27517	0.27563	0.27617	0.27548	0.27594
$x_5$	0.31165	0.31124	0.31114	0.31242	0.31345	0.31160	0.31159
$x_6$	0.65730	0.65719	0.65420	0.65875	0.65640	0.65618	0.65863
$N_{fe}$		1250	1141	3057	2425	853	1051

Table A.9: Hartman 6

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-10.1524	-10.1525	-10.1524	-10.1524	-10.1524	-10.1525
$\bar{\sigma}$		0.00020	0.00023	0.00020	0.00019	0.00018	0.00023
$N_{fe}$ (ave.)		2772	2672	3700	4046	2020	1262
<i>Reliability</i>		21/50	15/50	21/50	35/50	22/50	26/50
$f_{best}^g$	-10.1532	-10.1529	-10.1529	-10.1529	-10.1528	-10.1527	-10.1530
$x_1$	4.00004	3.99934	4.00039	3.99977	4.00137	3.99991	4.00035
$x_2$	4.00013	4.00098	3.99873	4.00033	4.00113	4.00111	4.00097
$x_3$	4.00004	3.99882	4.00037	4.00150	3.99950	3.99819	4.00060
$x_4$	4.00013	3.99990	4.00075	3.99942	3.99970	4.00014	4.00084
$N_{fe}$		1216	1293	3295	3319	972	1267

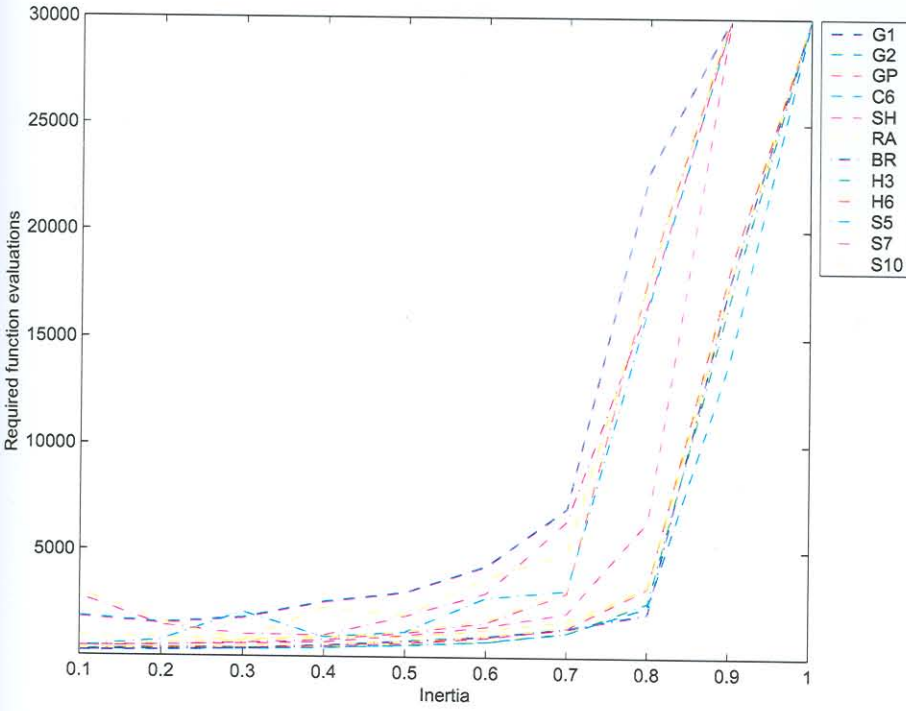
Table A.10: Shekel 5

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-10.4022	-10.4021	-10.4022	-10.4022	-10.4021	-10.4022
$\bar{\sigma}$		0.00024	0.00018	0.00027	0.00024	0.00021	0.00023
$N_{fe}$ (ave.)		2449	3731	5094	4497	2036	1280
<i>Reliability</i>		27/50	25/50	32/50	42/50	33/50	35/50
$f_{best}^g$	-10.4029	-10.4027	-10.4026	-10.4027	-10.4028	-10.4028	-10.4028
$x_1$	4.00057	4.00017	4.00175	4.00076	4.00070	4.00035	4.00033
$x_2$	4.00069	4.00188	4.00107	4.00160	4.00040	4.00045	4.00124
$x_3$	3.99949	4.00023	4.00038	3.99888	3.99945	4.00039	3.99930
$x_4$	3.99961	3.99952	3.99909	3.99910	3.99876	3.99915	4.00010
$N_{fe}$		1808	1432	3279	3156	1297	1849

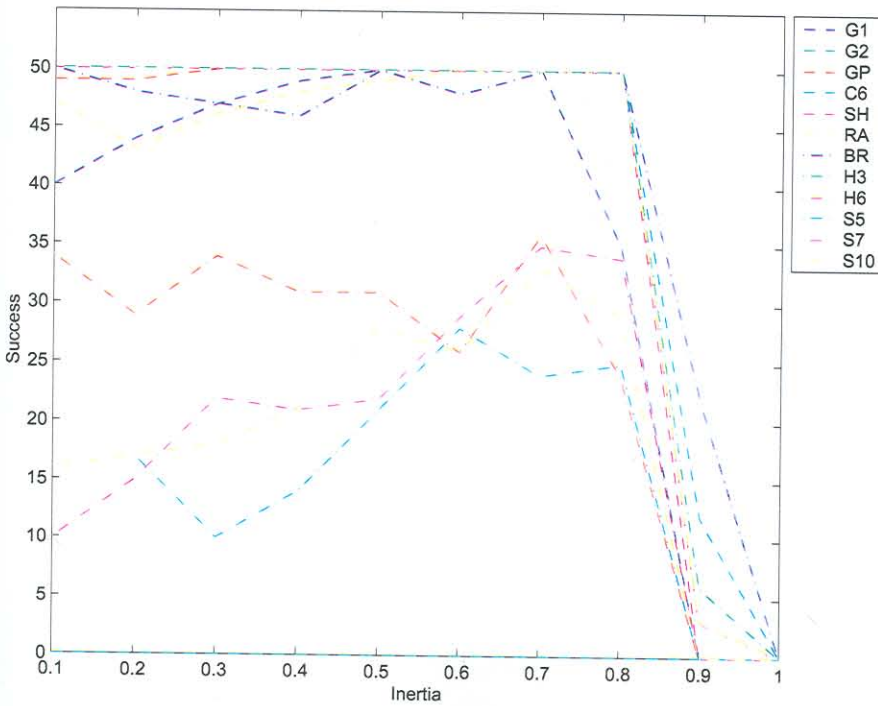
Table A.11: Shekel 7

	Optimum solution	PSO variant					
		PSO-CI	PSO-CIV	PSO-LI	PSO-LIV	PSO-C	PSO-DIV
$\epsilon_a$		0.001	0.001	0.001	0.001	0.001	0.001
$\bar{f}_{best}^g$		-10.5357	-10.5356	-10.5357	-10.5357	-10.5356	-10.5357
$\bar{\sigma}$		0.00025	0.00018	0.00022	0.00023	0.00023	0.00024
$N_{fe}$ (ave.)		3317	2221	4654	4532	3451	1296
<i>Reliability</i>		36/50	30/50	37/50	49/50	33/50	33/50
$f_{best}^g$	-10.5364	-10.5363	-10.5360	-10.5362	-10.5363	-10.5361	-10.5362
$x_1$	4.00075	4.00120	4.00137	3.99971	4.00052	4.00173	4.00160
$x_2$	4.00059	4.00097	4.00119	4.00149	4.00022	4.00018	4.00021
$x_3$	3.99966	3.99907	3.99814	3.99958	3.99987	3.99875	4.00019
$x_4$	3.99951	3.99984	3.99915	3.99921	3.99886	3.99896	3.99961
$N_{fe}$		1392	1730	3373	2857	1212	1155

Table A.12: Shekel 10



(a) Cost as a function of  $w$



(b) Reliability as a function of  $w$

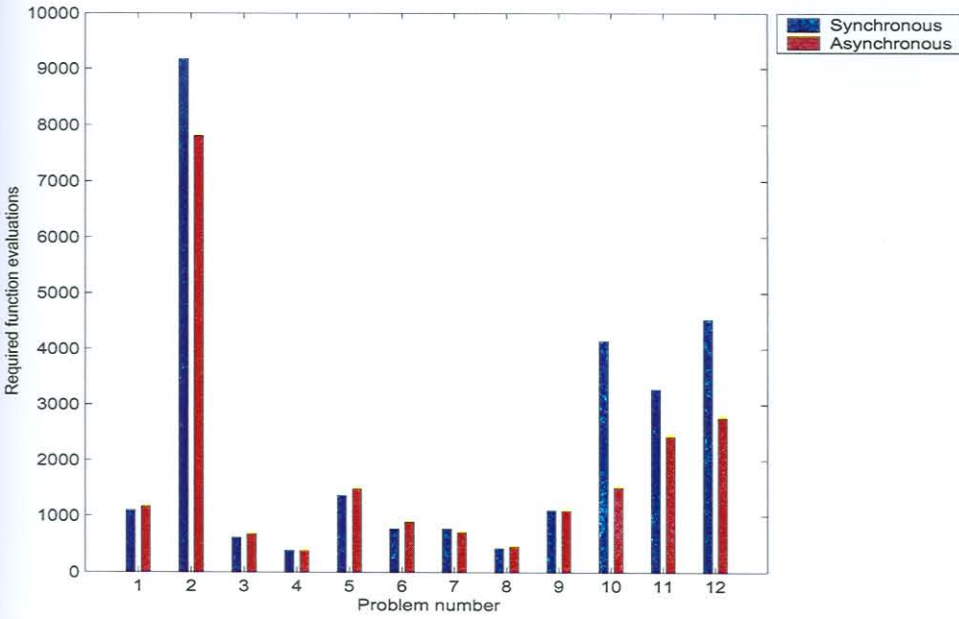
Figure A.2: PSOA-CI variant: Cost and reliability as a function of the inertia weight  $w$



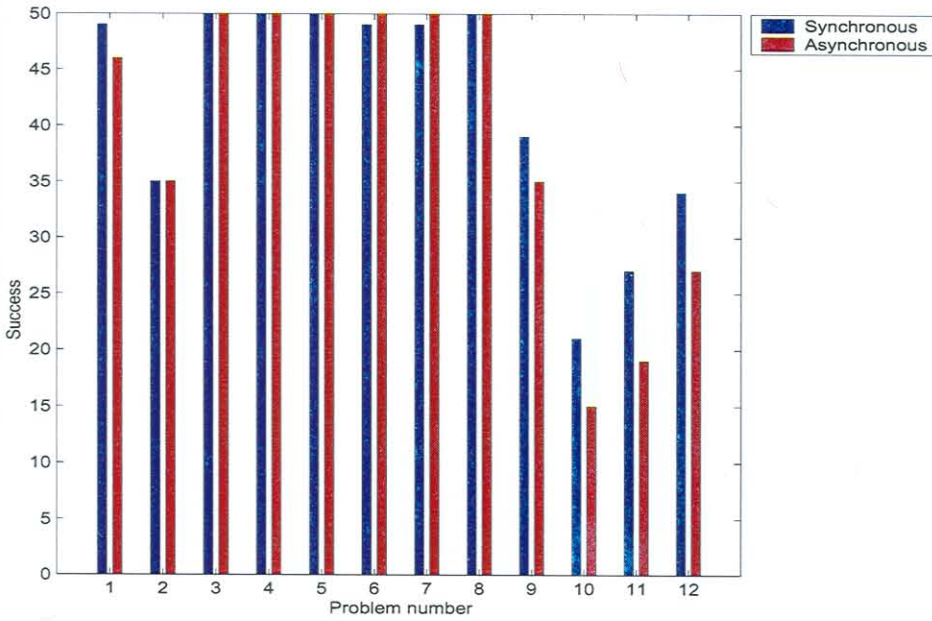
## A.2 Asynchronous vs. synchronous PSOA



Figure A.3. PSOA-G1 variant: Cost and reliability comparison between asynchronous and synchronous variants

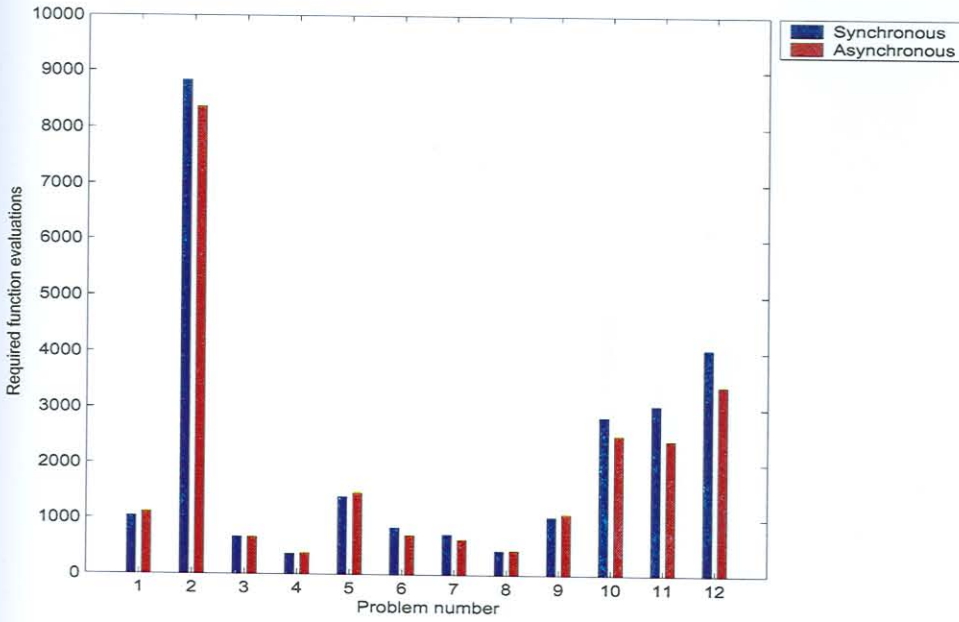


(a) Cost

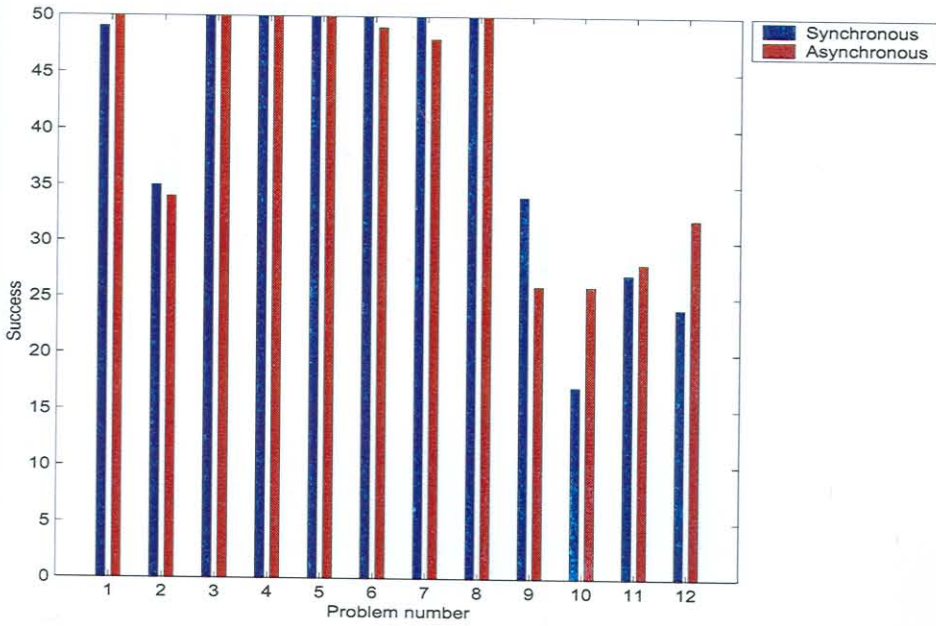


(b) Reliability

Figure A.3: PSOA-CI variant: Cost and reliability comparison between asynchronous and synchronous variants

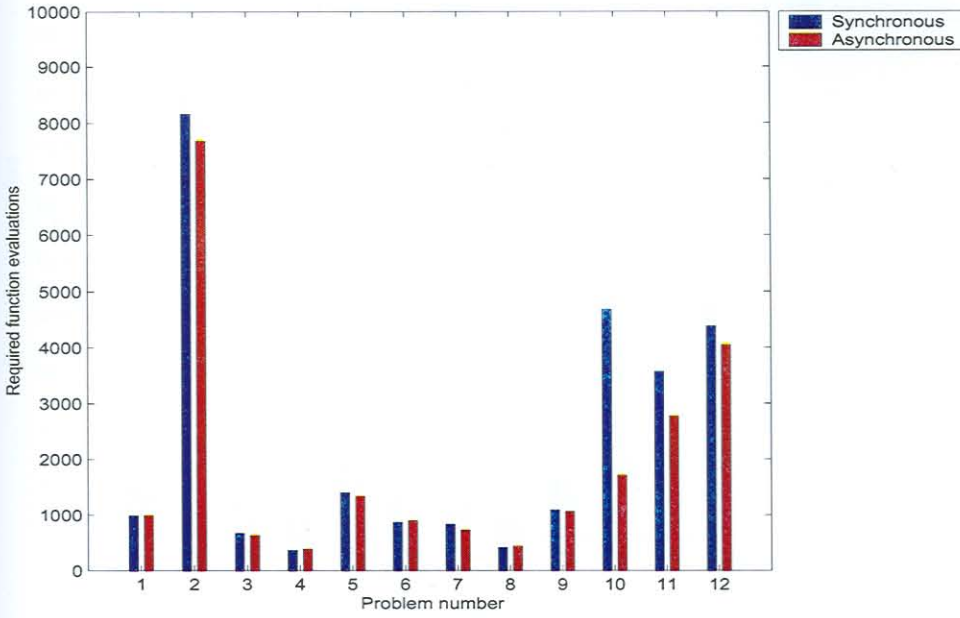


(a) Cost

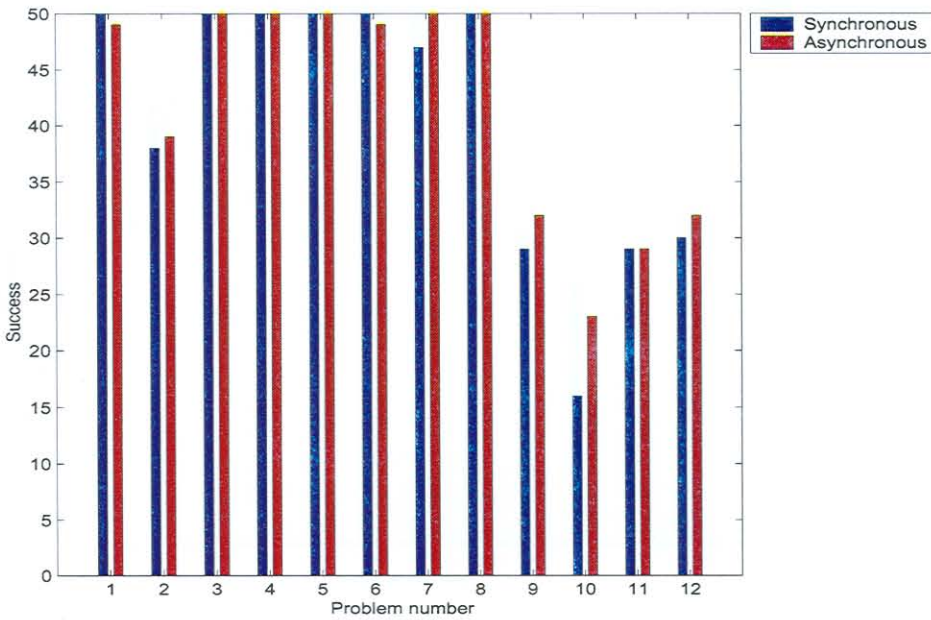


(b) Reliability

Figure A.4: PSOA-CIV variant: Cost and reliability comparison between asynchronous and synchronous variants



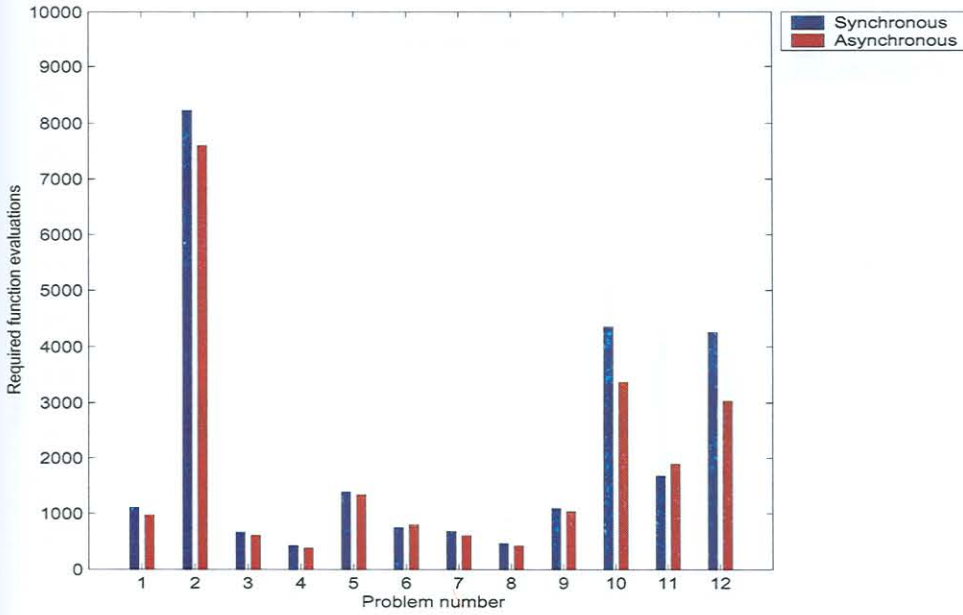
(a) Cost



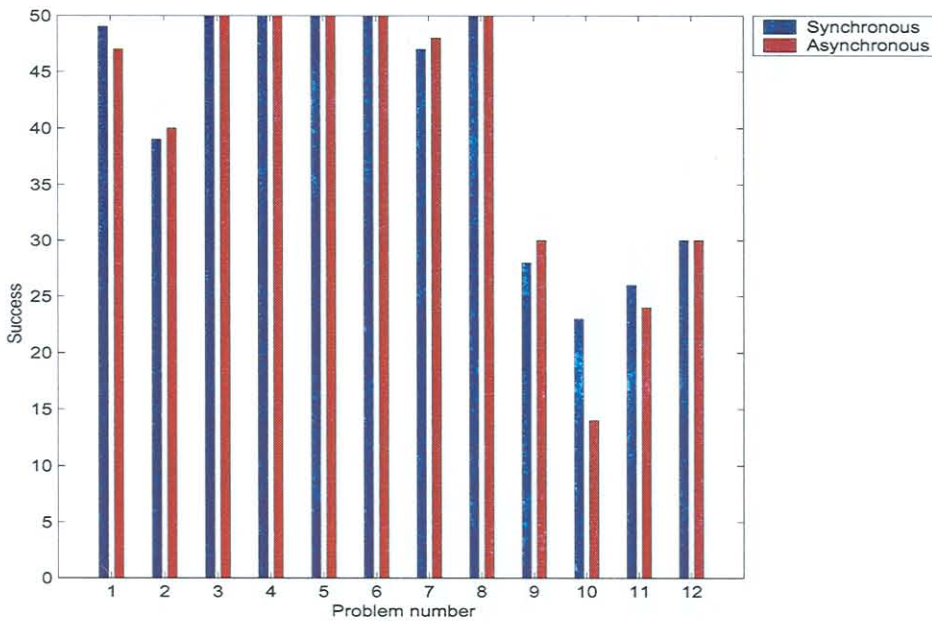
(b) Reliability

Figure A.5: PSOA-LI variant: Cost and reliability comparison between asynchronous and synchronous variants



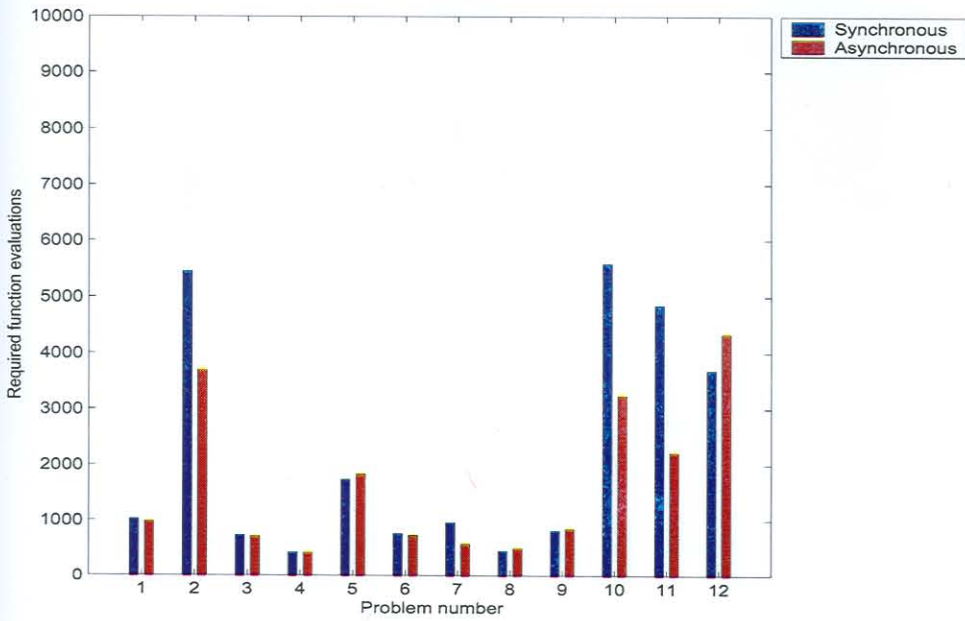


(a) Cost

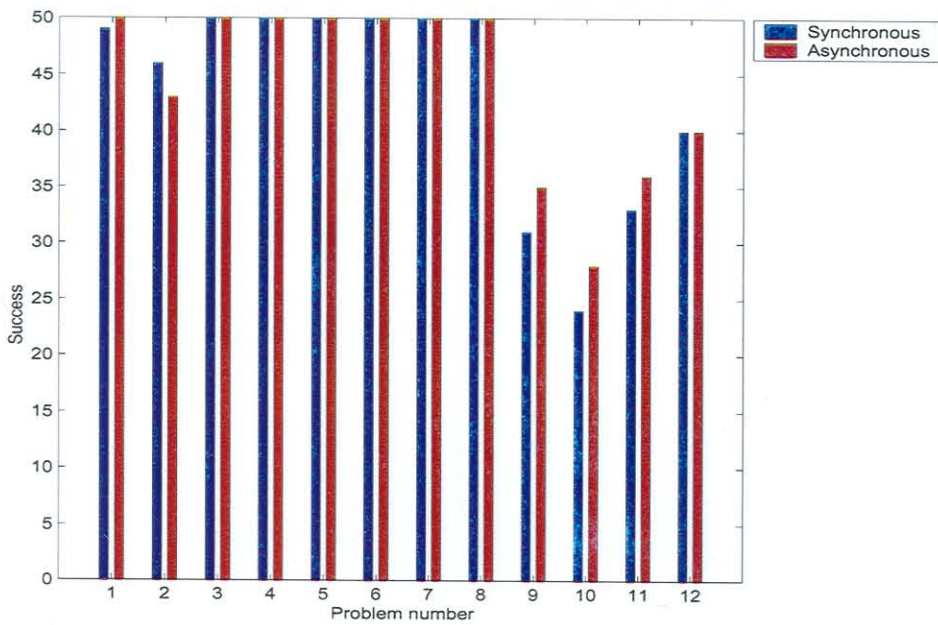


(b) Reliability

Figure A.6: PSOA-LIV variant: Cost and reliability comparison between asynchronous and synchronous variants

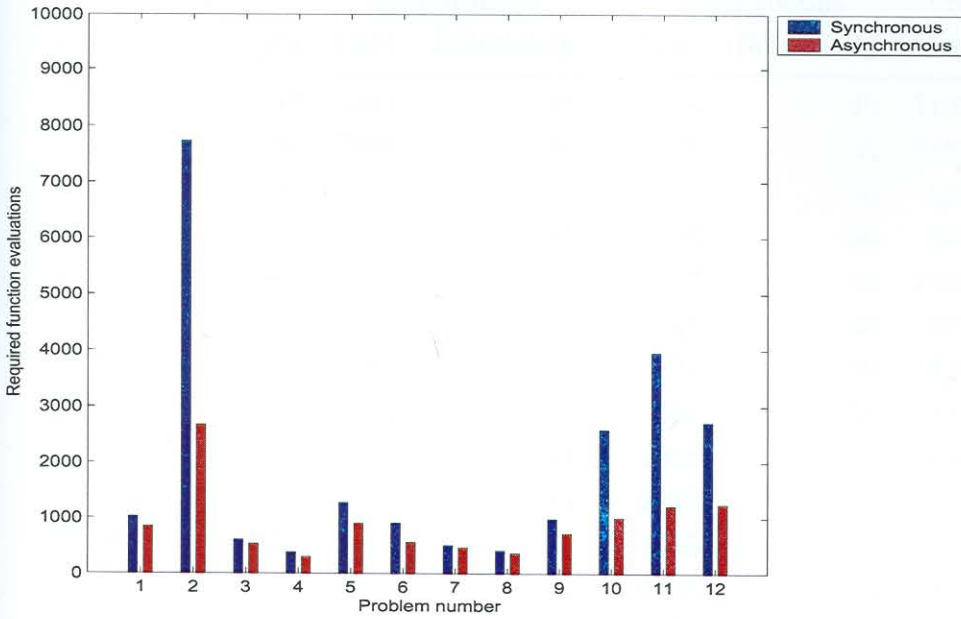


(a) Cost

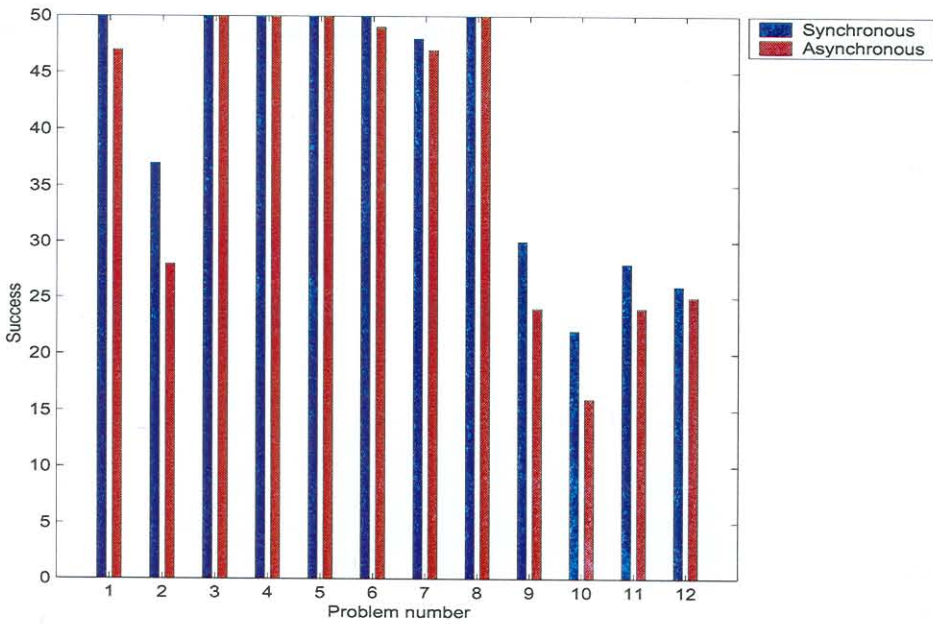


(b) Reliability

Figure A.7: PSOA-C variant: Cost and reliability comparison between asynchronous and synchronous variants



(a) Cost



(b) Reliability

Figure A.8: PSOA-DIV variant: Cost and reliability comparison between asynchronous and synchronous variants

PSO-CI				PSO-CIV			
Asynchronous		Synchronous		Asynchronous		Synchronous	
Cost	Reliability	Cost	Reliability	Cost	Reliability	Cost	Reliability
1097	49	1163	46	1036	49	1108	50
9173	35	7809	35	8831	35	8365	34
611	50	675	50	670	50	666	50
384	50	383	50	366	50	381	50
1366	50	1478	50	1389	50	1460	50
772	49	891	50	844	50	707	49
772	49	705	50	721	50	630	48
423	50	454	50	431	50	436	50
1099	39	1088	35	1035	34	1086	26
4143	21	1503	15	2844	17	2505	26
3278	27	2419	19	3055	27	2417	28
4523	34	2753	27	4056	24	3393	32

Table A.13: PSO-CI and PSO-CIV variants: Cost and reliability comparison for asynchronous and synchronous algorithm variants



PSO-LI				PSO-LIV			
Asynchronous		Synchronous		Asynchronous		Synchronous	
Cost	<i>Reliability</i>	Cost	<i>Reliability</i>	Cost	<i>Reliability</i>	Cost	<i>Reliability</i>
989	50	995	49	1112	49	973	47
8158	38	7682	39	8229	39	7601	40
678	50	635	50	667	50	606	50
367	50	390	50	422	50	377	50
1402	50	1339	50	1389	50	1337	50
870	50	896	49	742	50	796	50
836	47	729	50	674	47	603	48
420	50	443	50	464	50	420	50
1093	29	1061	32	1093	28	1032	30
4684	16	1709	23	4351	23	3373	14
3566	29	2775	29	1678	26	1893	24
4380	30	4047	32	4256	30	3030	30

Table A.14: PSO-LI and PSO-LIV variants: Cost and reliability comparison for asynchronous and synchronous algorithm variants

PSO-C				PSO-DIV			
Asynchronous		Synchronous		Asynchronous		Synchronous	
Cost	<i>Reliability</i>	Cost	<i>Reliability</i>	Cost	<i>Reliability</i>	Cost	<i>Reliability</i>
1021	49	978	50	1026	50	847	47
5452	46	3687	43	7728	37	2674	28
734	50	712	50	607	50	532	50
419	50	415	50	377	50	299	50
1733	50	1835	50	1276	50	905	50
758	50	731	50	909	50	559	49
959	50	574	50	506	48	468	47
449	50	493	50	409	50	362	50
821	31	845	35	986	30	722	24
5587	24	3236	28	2587	22	1008	16
4855	33	2217	36	3955	28	1215	24
3687	40	4335	40	2715	26	1246	25

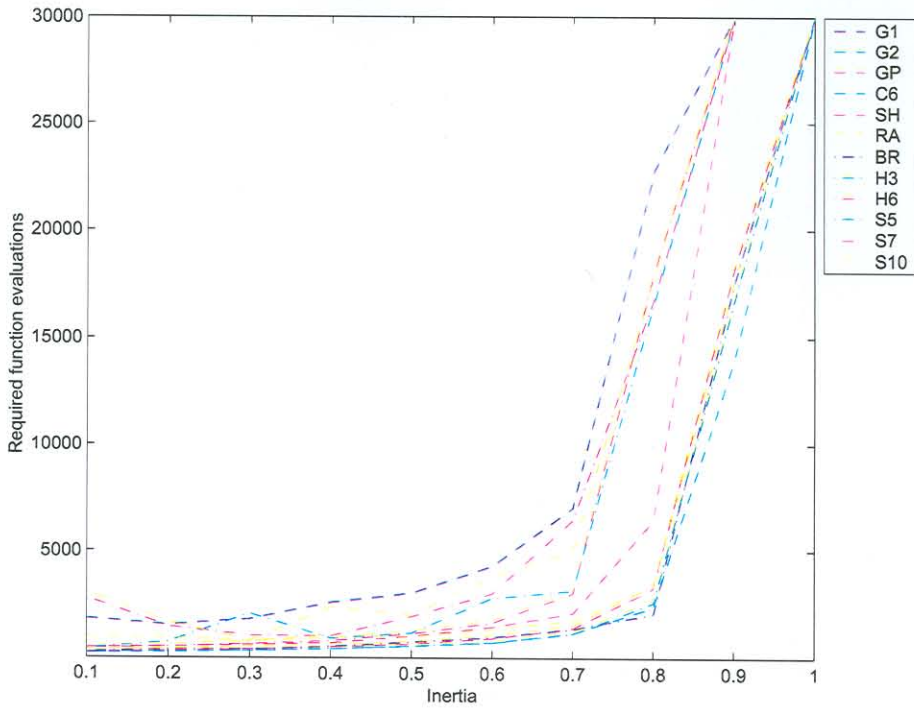
Table A.15: PSO-C and PSO-DIV variants: Cost and reliability comparison for asynchronous and synchronous algorithm variants

### A.3 Numerical results for the parameter sensitivity study

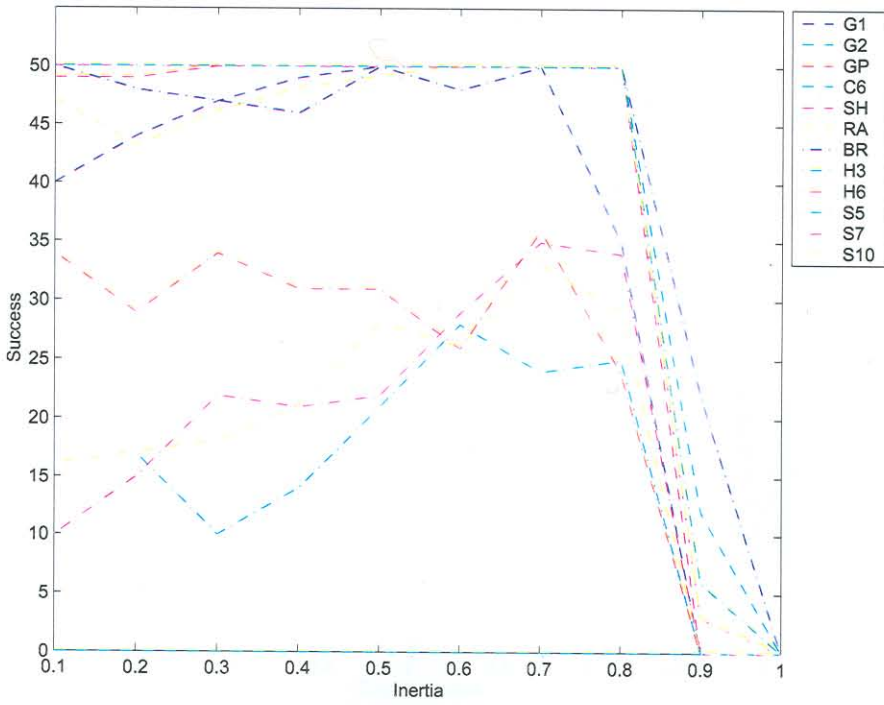


(b) Reliability as a function of the inertia weight  $\omega$

Figure A.9: Constant inertia: Cost and reliability as a function of the inertia weight  $\omega$

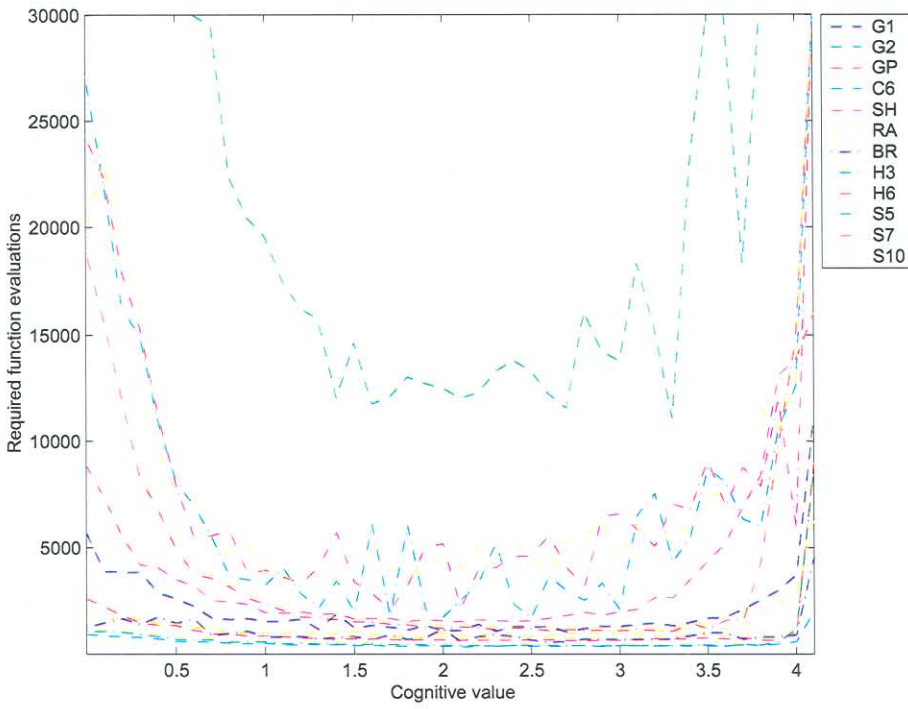


(a) Cost as a function of the inertia weight  $w$

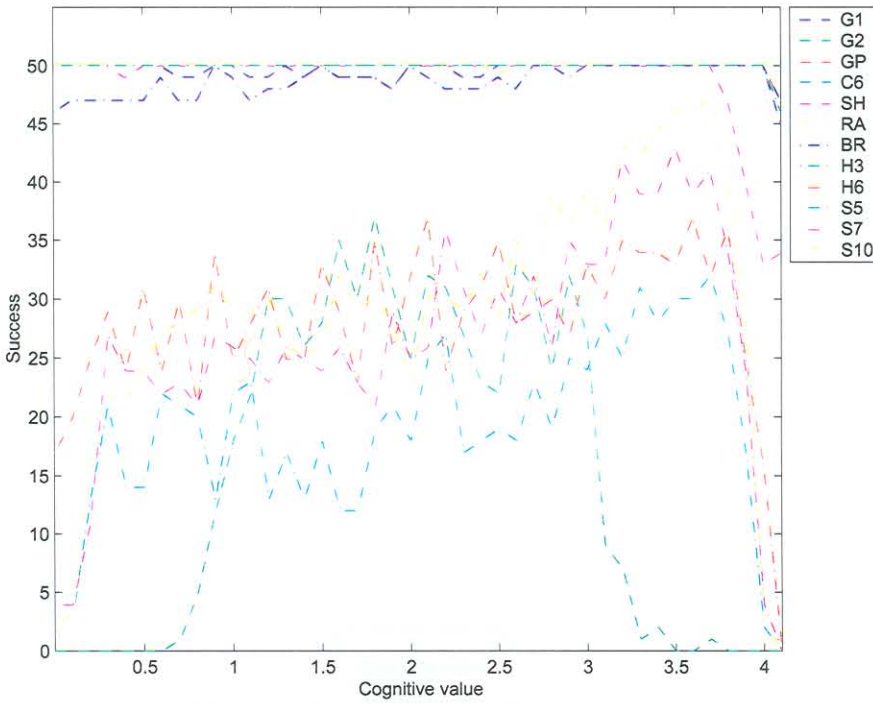


(b) Reliability as a function of the inertia weight  $w$

Figure A.9: Constant inertia: Cost and reliability as a function of the inertia weight  $w$

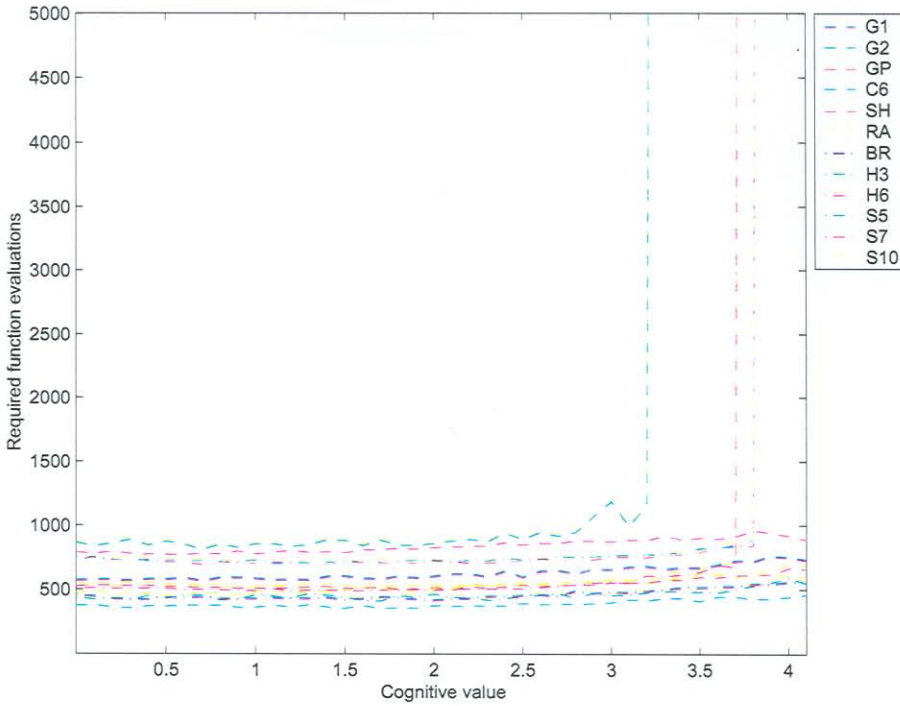


(a) Cost as a function of the cognitive parameter  $c_1$

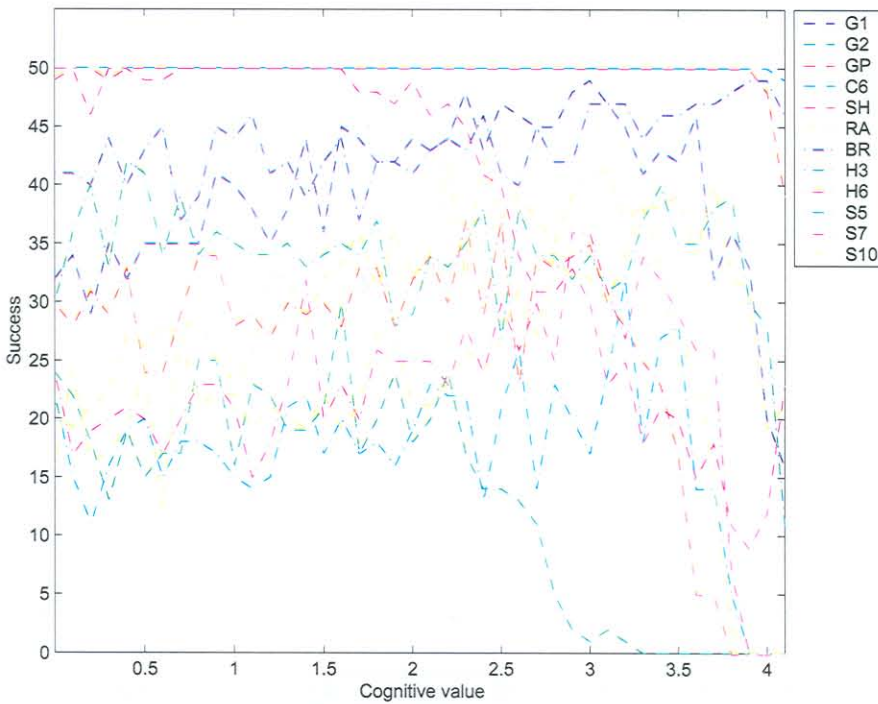


(b) Reliability as a function of the cognitive parameter  $c_1$

Figure A.10: Constriction: Cost and reliability as a function of the cognitive parameter  $c_1$



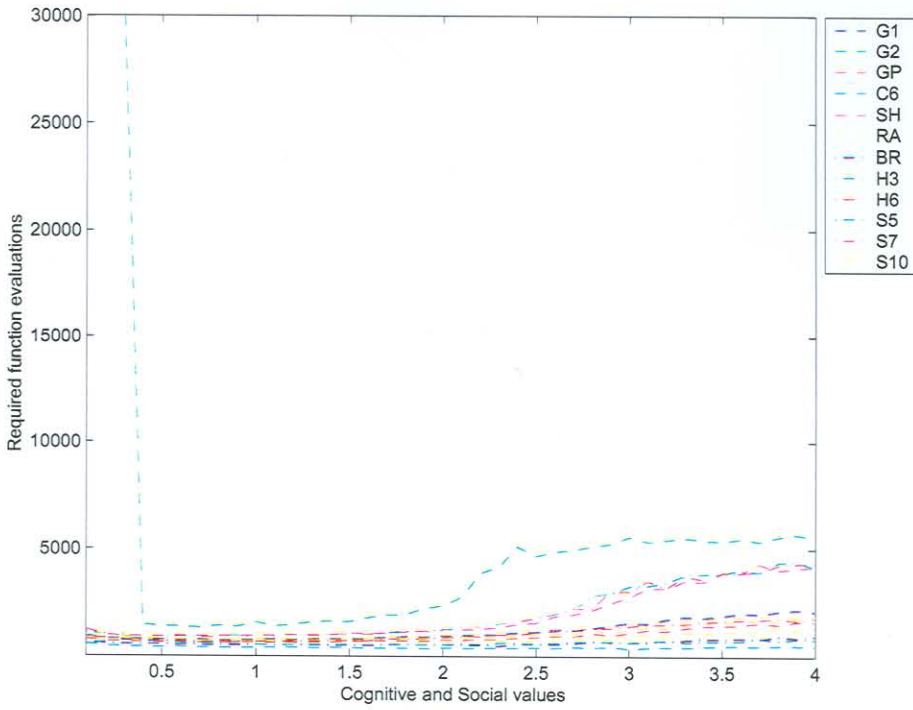
(a) Cost as a function of the cognitive parameter  $c_1$



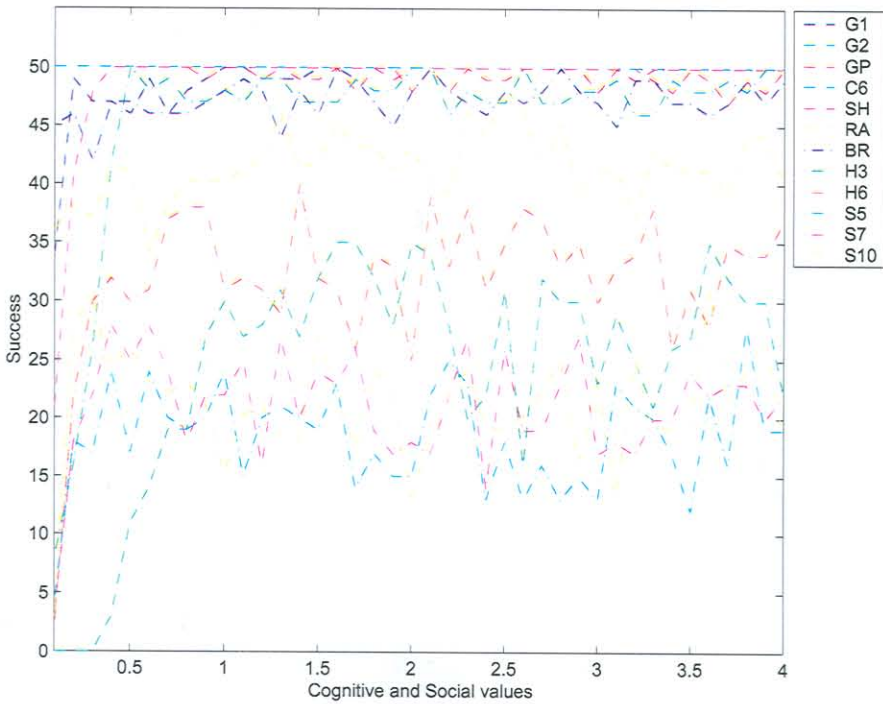
(b) Reliability as a function of the cognitive parameter  $c_1$

Figure A.11: Dynamic inertia reduction: Cost and reliability as a function of the cognitive parameter  $c_1$



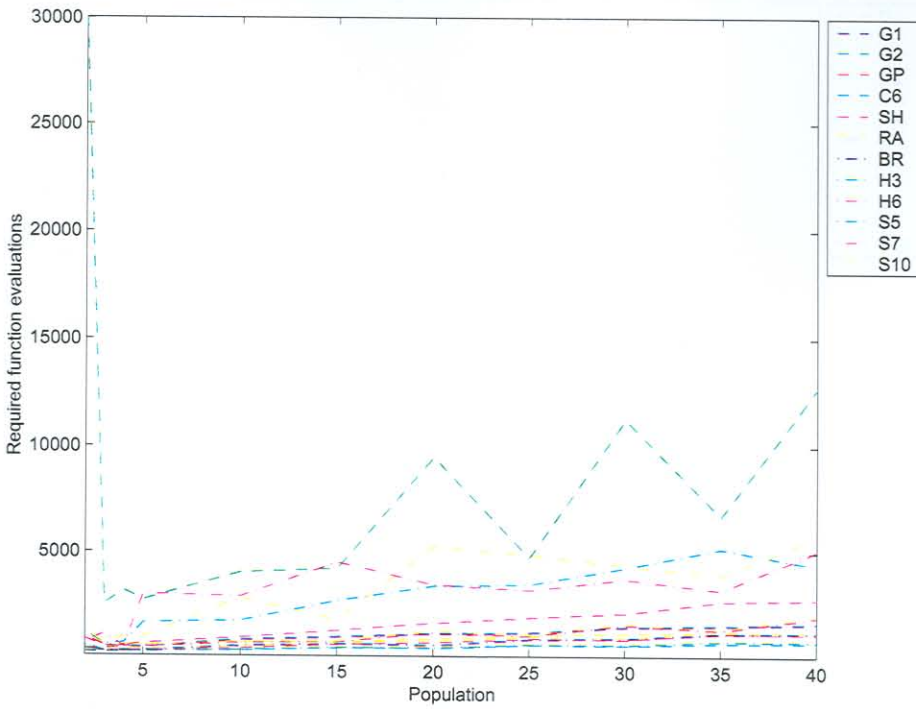


(a) Cost with  $c_1 = c_2$

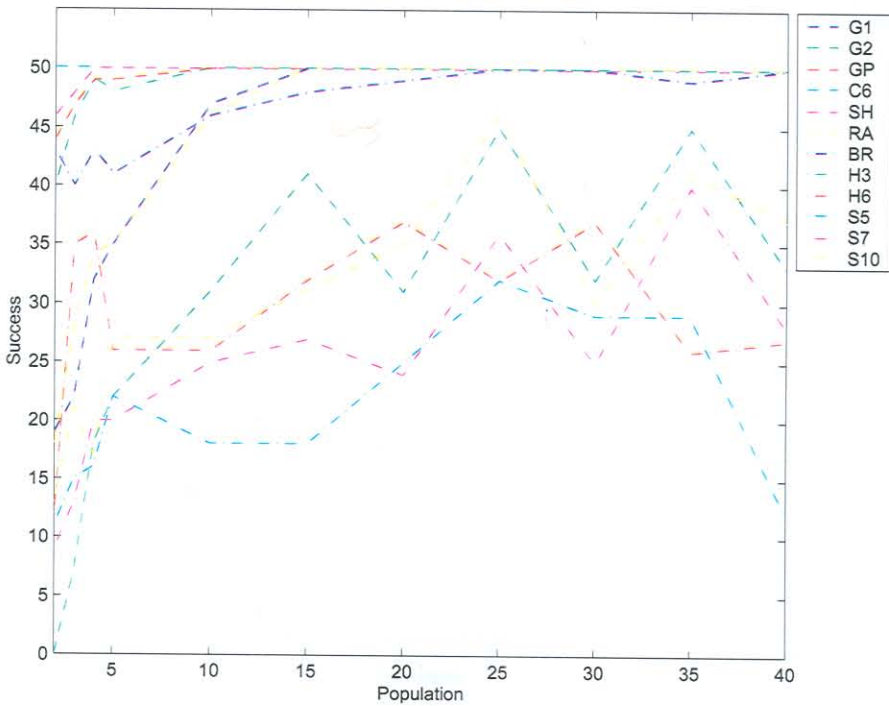


(b) Reliability with  $c_1 = c_2$

Figure A.12: Dynamic inertia reduction: Cost and reliability as a function of cognitive and social parameters with  $c_1 = c_2$

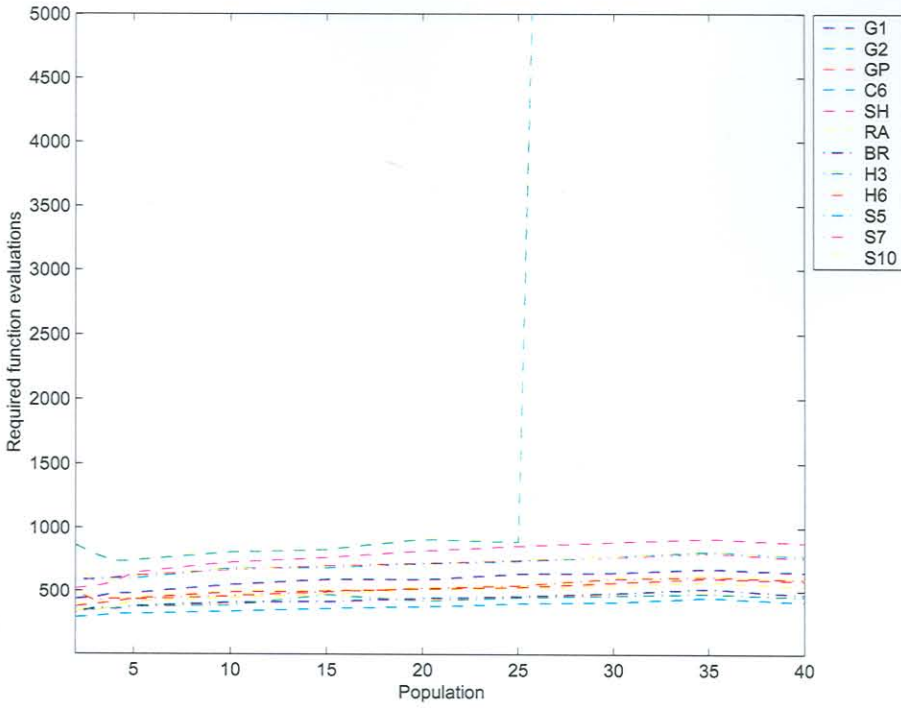


(a) Cost as a function of population size

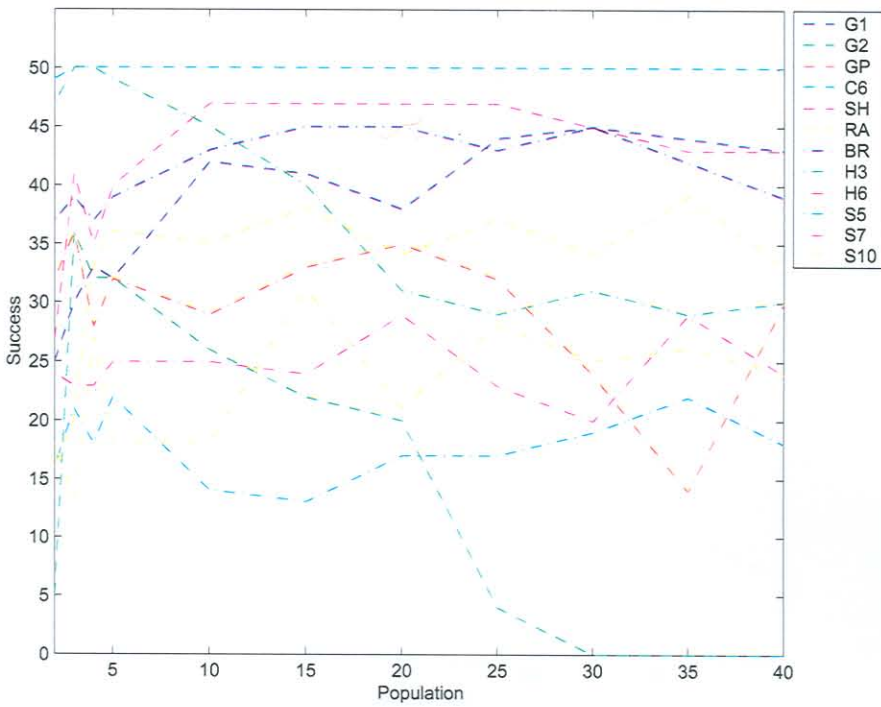


(b) Reliability as a function of population size

Figure A.13: Constriction: Cost and reliability as a function of swarm population size  $p$

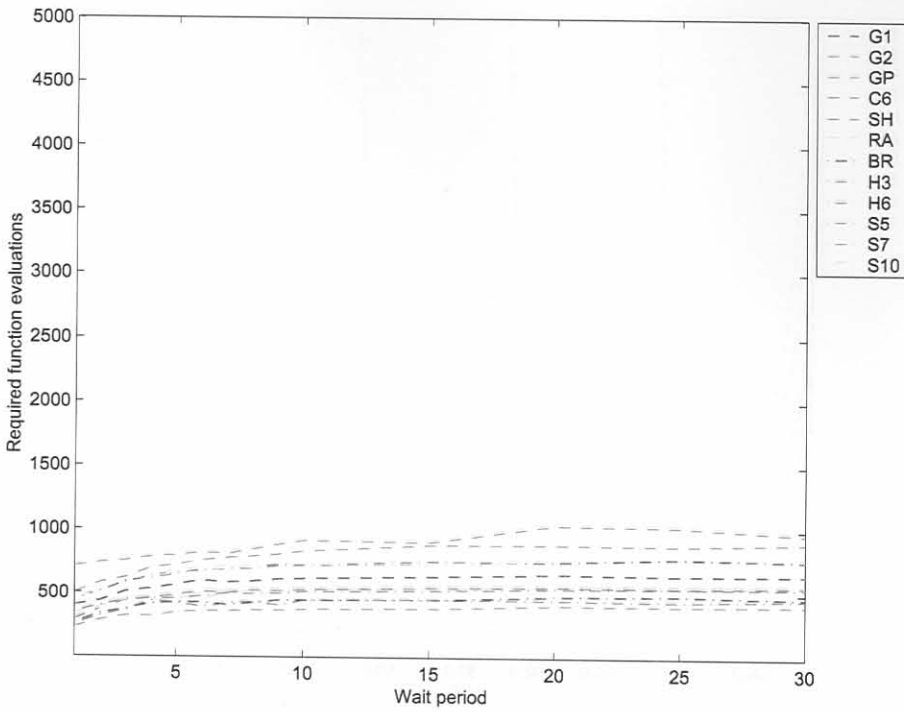


(a) Cost as a function of population size

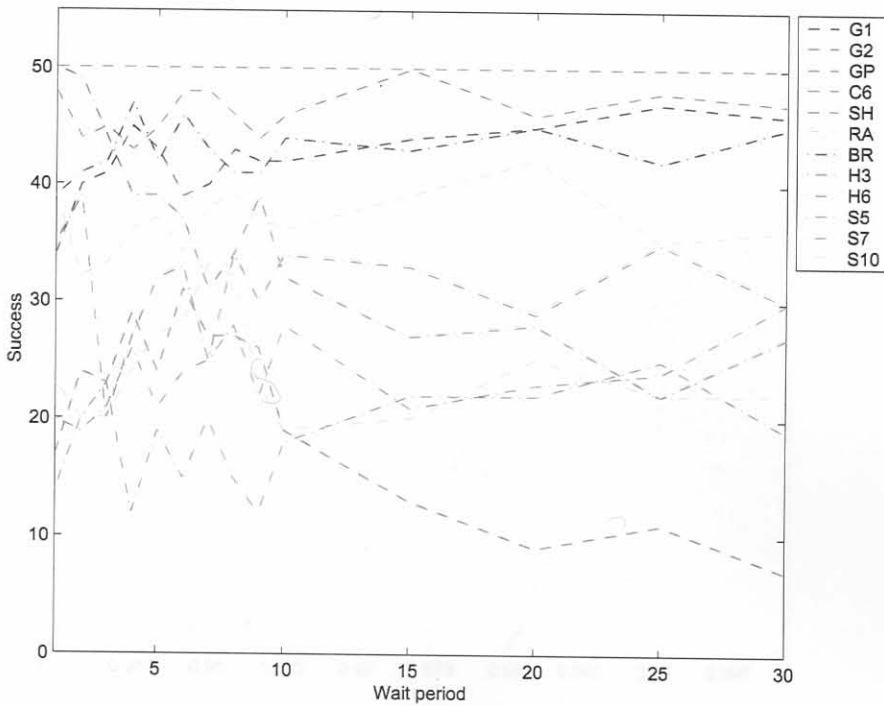


(b) Reliability as a function of population size

Figure A.14: Dynamic inertia reduction: Cost and reliability ratio as a function of swarm population  $p$

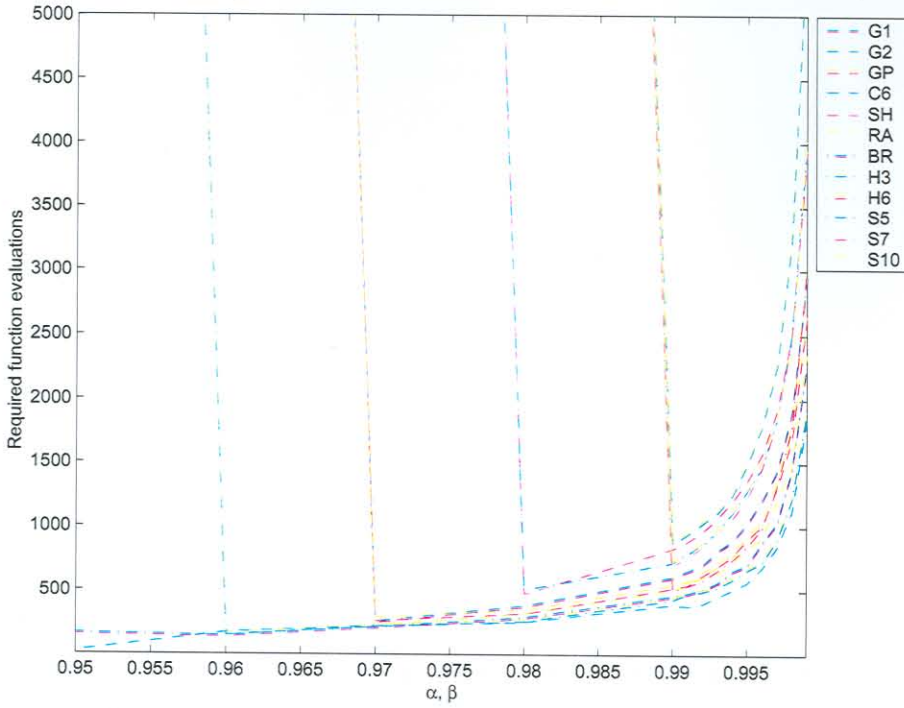


(a) Cost as a function of the dynamic delay period

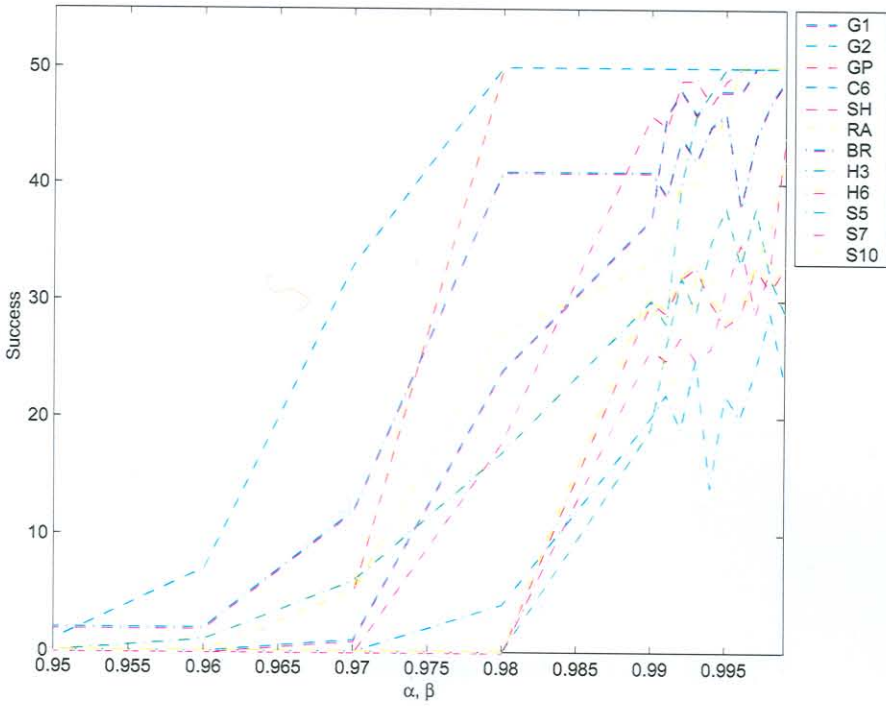


(b) Reliability as a function of the dynamic delay period

Figure A.15: Dynamic inertia reduction: Cost and reliability as a function of the dynamic delay period  $h$



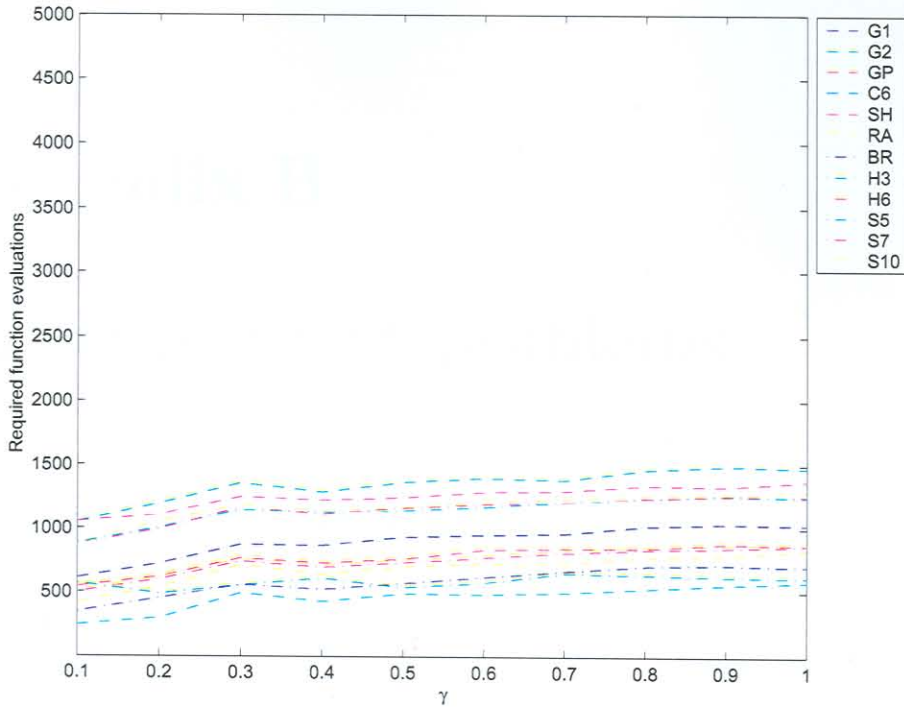
(a) Cost as a function of the reduction parameters  $\alpha$  and  $\beta$  (with  $\alpha = \beta$ )



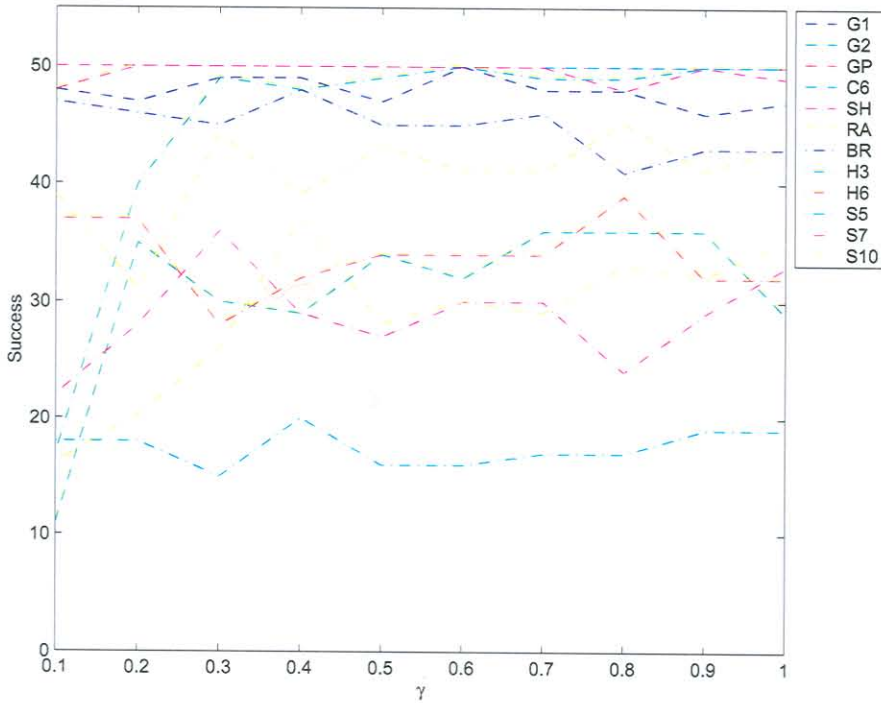
(b) Reliability as a function of the reduction parameters  $\alpha$  and  $\beta$  (with  $\alpha = \beta$ )

Figure A.16: Dynamic inertia reduction: Cost and reliability as a function of the reduction parameters  $\alpha$  and  $\beta$





(a) Cost as a function of the initial velocity fraction  $\gamma$



(b) Reliability as a function of the initial velocity fraction  $\gamma$

Figure A.17: Dynamic inertia reduction: Cost and reliability as a function of the initial velocity fraction  $\gamma$