Chapter 4

Global optimization

4.1 Introduction

In this chapter the PSOA variants formulated in Chapter3 are applied to an extended Dixon-Szegö test. Numerical results are presented in Appendix A.

4.2 Problem formulation

Firstly, we formally define the global optimization problem: Consider the unconstrained (or bounds constrained) mathematical programming problem represented by the following: Given a real valued objective function $f(\boldsymbol{x})$ defined on the set $\boldsymbol{x} \in D$ in $\mathbb{R}^n$, find the point $\boldsymbol{x}^*$ and the corresponding function value f^* such that

$$f^* = f(\boldsymbol{x}^*) = \min\{f(\boldsymbol{x}) | \boldsymbol{x} \in D\} \ , \tag{4.1}$$

if $\boldsymbol{x}^*$ exists and is unique. Alternatively, find a low approximation $\tilde{f}$ to f^*.

If the objective function and/or the feasible domain D are non-convex, then there may be many local minima which are not optimal. Hence, from a *mathematical* point of view, problem (4.1) is essentially insolvable, due to a lack of mathematical conditions characterizing the global optimum, as opposed to a strictly convex continuous function, which is characterized by the Karush-Kuhn-Tucker conditions at the minimum.

The problem of globally optimizing a real valued function is inherently intractable (unless hard restrictions are imposed on the objective function) in that no practically useful characterization of the global optimum is available. Indeed the problem of determining an accurate estimate of the global optimum is mathematically ill-posed in the sense that very similar objective functions may have global optima very distant from each other [41]. Nevertheless, the need in practice to find a relative low local minimum has resulted in considerable research over the last decades to develop algorithms that attempt to find such a low minimum. A comprehensive survey of global optimization is presented by Törn and Zilinskas [1].

Acronym	Name	n	ϵ_a
G1	Griewank G1	2	0.001
G2	Griewank G2	10	0.1
GP	Goldstein-Price	2	0.001
C6	Six-hump camelback	2	0.001
SH	Shubert, Levi No. 4	2	0.001
RA	Rastrigin	2	0.001
BR	Branin	2	0.001
H3	Hartman 3	3	0.001
H6	Hartman 6	6	0.001
S5	Shekel 5	5	0.001
H7	Shekel 7	7	0.001
S10	Shekel 10	10	0.001

Table 4.1: The extended Dixon-Szegö test set.

Acronym	Name
PSO-CI	PSOA with constant inertia
PSO-CIV	PSOA with constant inertia and maximum velocity limiting
PSO-LI	PSOA with linear inertia
PSO-LIV	PSOA with linear inertia and maximum velocity limiting
PSO-C	PSOA with constriction factor
PSO-DIV	PSOA with dynamic inertia and maximum velocity limiting

Table 4.2: Acronyms used to denote algorithm variants.

4.3 The extended Dixon-Szegö test set

The set of well known problems (e.g. see [42]) presented in Table 4.1 will be used to obtain comparative numerical results for the variants of the PSOA under consideration. The ϵ_a values given in the table are the allowable errors used as convergence criteria in the *a priori* stopping condition (discussed later). A mathematical description of the test set is detailed in Appendix A.

4.4 Numerical results for the different PSOA variants

In this section numerical results are presented for the extended Dixon-Szegö test set presented in Table 4.1. Acronyms used to denote the different PSOA variations are tabulated in Table 4.2. Each problem is analyzed $n = 50$ times and the group best fitness value f^g_{best} at termination is reported. The best run n is classified in terms of the best fitness value found between all of the runs. However, similar results can be obtained using the least amount of function evaluations N_{fe} as criteria, since the standard deviation $\overline{\sigma}$ compare closely for all of the problems, as defined by:

$$\overline{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} \left(f^g_{best\ i} - \bar{f}^g_{best}\right)^2}{n}} \tag{4.2}$$

with

$$\bar{f}^g_{best} = \frac{\sum_{i=1}^{n} f^g_{best\ i}}{n} \tag{4.3}$$

The reader is cautioned however, when interpreting the best f^g_{best} and average values $\bar{f}^g_{best}$ reported in the tables because they are highly volatile even for a high number of repeated searches due to the stochastic nature of the algorithm.

Since it is not the objective to test the performance of different stopping methods, but rather the algorithm itself, a simple *a priori* stopping criteria is used. This method compares the known solution value with the swarm best value and stops the algorithm when the swarm fitness converges to within a certain prescribed absolute error ϵ_a (see Table 4.1).

The maximum allowable function evaluations is set to 30000. If the *a priori* stopping condition is not satisfied within this period the search is deemed not to have converged. Unless otherwise stated, a swarm of 20 particles is used, with the cognitive and social constants c_1 and c_2 both set to 2 for all but the constriction factor variant. For the constriction factor variant the c_1 and c_2 parameters are set to 2.8 and 1.3 respectively, as recommended by Carlisle and Dozier [20]. For the dynamic inertia and maximum velocity modification parameter values $\alpha = \beta = 0.99$ and $h = 10$ are used. These are arbitrarily chosen as numerical experimentation has indicated that the algorithm is relatively insensitive to these parameter values, as will be shown later. $\gamma = 1.0$ is used for all problems where the maximum velocity limitation is applied.

In the interest of obtaining a robust and versatile algorithm no exhaustive attempt is made to optimize the algorithm parameters for individual problems. *For the sake of brevity, all of the results in the following sections are summarized in Figure A.1.*

4.4.1 Standard PSOA

The numerical results obtained with the original PSOA indicates that this search method converged for only 3 out of the 12 problems, and then only with very poor reliability. Hence, no numerical results will be presented in tabulated form.

4.4.2 Constant inertia weight variant

Numerical results are obtained for this method with the inertia weight w varied between 0.1 and 1.0 at 0.1 increments (Figure A.2). Reliability is defined as the amount of times out of the total number of searches (n) the algorithm converges to the optimum value within an allowable error value ϵ_a. From Figures A.2(a) and A.2(b) it can be seen that the optimum value for w, when considering the average cost (Figure A.2(a)) and the optimum reliability (Figure A.2(b)) for the test set lies in the region of $w = 0.5$ to 0.7. Reliability decreases outside this region, more pronounced for the higher values of w than for the lower values. The average cost also increases rapidly above $w = 0.7$. It is interesting to note that for $w = 1$, which reduces the velocity rule (2.2) to the original implementation of the PSOA by Kennedy and Eberhart, none of the runs on any of the problems in the set converged to the required error values.

With the above in mind tabulated results are presented with $w = 0.6$. Very good results are obtained for most of the problems in the set.

The enforcement of a maximum velocity limitation marginally improves the average required function evaluations for most of the problems with the exception of the Shekel 7 problem (Table A.11). This enforcement also worsens the reliability for the Shekel 5 and Shekel 10 problems (Tables A.10, A.12).

4.4.3 Linearly decreasing inertia weight variant

For these results the the inertia weight w is scaled linearly between 0.8 and 0.4 during the first 4000 function evaluations of the search. This variation yields improvements in terms of average cost over the PSO-CIV variation for the Shekel group of problems (Tables A.10, A.11, A.12), with the reliability remaining more or less the same. For most of the other problems however, the average cost increases, indicating that the optimum rate of inertia reduction could be problem dependent.

With a maximum velocity limitation both the average cost and the reliability are improved for all of the problems, with the exception of Shekel 5 (Table A.10), where only the reliability is improved.

4.4.4 Constriction factor variant

Numerical experimentation and work done by others [20] indicate that the maximum allowable velocity modification does not contribute to an increased efficiency for this variant if

bounds constraints are enforced as in our case. Numerical results are very impressive notwithstanding, with a vast increase in convergence rates over the fixed and linearly decreasing inertia variants. For some of the more difficult higher dimensional problems however, the algorithm reliability decreases noticeably (Tables A.10, A.11, A.12).

4.4.5 Dynamically decreasing inertia weight and maximum velocity variant

This method is marginally slower in converging than the constriction factor method for most of the lower dimensional problems (Tables A.1, A.3, A.4, A.6, A.7, A.8, A.9) but for the more difficult higher dimensional problems (Tables A.2, A.10, A.11, A.12) a reduced cost, and in some cases improved reliability compared to constriction is obtained. By using this method of adjusting the inertia parameter w, the inertia modification regains equal footing with the constriction factor method.

4.4.6 Synchronous vs. asynchronous particle swarm algorithm

Finally, the synchronous and asynchronous particle swarm algorithm variants are compared for all of the problems in the test set with all the PSOA variants. From the numerical results presented in Tables A.13, A.14, A.15, (summarized in Figures A.3-A.8), the synchronous method is shown to be less costly and more reliable for all of the problems in the set, supporting the findings of Carlisle and Dozier [20].

4.5 Fitness history

For the difficult G2 problem, a typical history plot for all of the variants is presented in Figure 4.1. From this figure it can clearly be seen that the constriction and dynamic variants are superior to the other variants when considering convergence rates. In terms of reliability however, the linear inertia reduction variant outperforms the constriction and dynamic methods by a small margin.

If we use a points system with equal marks awarded for both the best average number of function evaluations and reliability, and pick the best out of the six variants for each problem the PSOA-CI, PSOA-CIV and PSOA-DIV variants rank highest.

4.6 Summary

From the numerical results presented in this chapter is obvious that the constant-inertia, constriction, and the dynamic inertia/velocity reduction variants are the main contenders when both reliability and cost are considered. Although the linear inertia reduction variation

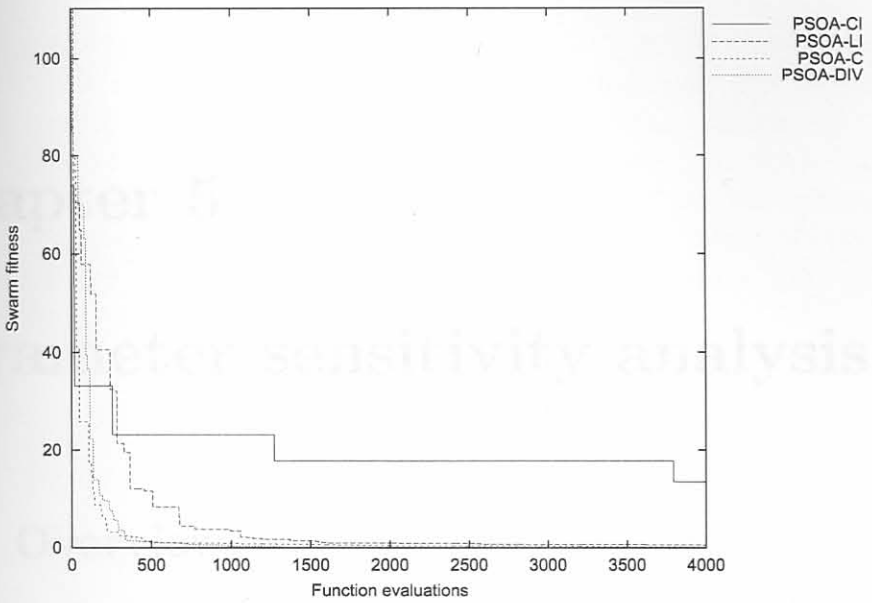

Figure 4.1: PSOA variants comparison: Swarm fitness history

on the PSOA delivers high reliability, this advantage is offset by the high cost for the more difficult problems.