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Appendix A

Auto-Scaling Test Problems

The test problems described here are from the book of Hock and Schittkowski 1981, and used in Chapter 8 to test the automatic scaling theory, before being applied to the vehicle suspension optimisation problem.

A.1 Hock 2

Objective function:

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \quad (\text{A.1})$$

Constraints:

$$\begin{aligned} -2 &\leq x_1 \leq 2 \\ 1.5 &\leq x_2 \leq 3 \end{aligned} \quad (\text{A.2})$$

Starting point:

$$\begin{aligned} \mathbf{x}_0 &= [-2 \ 1] \\ f(\mathbf{x}_0) &= 909 \end{aligned} \quad (\text{A.3})$$

Optimum:

$$\begin{aligned} \mathbf{x}^* &= [1.22 \ 1.5] \\ f(\mathbf{x}^*) &= 0.05042 \ 61879 \end{aligned} \quad (\text{A.4})$$

A.2 Hock 13

Objective function:

$$f(\mathbf{x}) = (x_1 - 2)^2 + x_2^2 \quad (\text{A.5})$$



Constraints:

$$\begin{aligned} g(\mathbf{x}) &= x_2 + (1 - x_1)^3 \leq 0 \\ -2 &\leq x_1 \leq 2 \\ -2 &\leq x_2 \leq 2 \end{aligned} \tag{A.6}$$

Starting point:

$$\begin{aligned} \mathbf{x}_0 &= [-2 \ -2] \\ f(\mathbf{x}_0) &= 20 \end{aligned} \tag{A.7}$$

Optimum:

$$\begin{aligned} \mathbf{x}^* &= [1 \ 0] \\ f(\mathbf{x}^*) &= 1 \end{aligned} \tag{A.8}$$

A.3 Hock 15

Objective function:

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{A.9}$$

Constraints:

$$\begin{aligned} g(\mathbf{x}) &= 1 - x_1x_2 \leq 0 \\ g(\mathbf{x}) &= -x_1 - x_2^2 \leq 0 \\ -2 &\leq x_1 \leq 0.5 \\ 1 &\leq x_2 \leq 2.5 \end{aligned} \tag{A.10}$$

Starting point:

$$\begin{aligned} \mathbf{x}_0 &= [-2 \ 1] \\ f(\mathbf{x}_0) &= 909 \end{aligned} \tag{A.11}$$

Optimum:

$$\begin{aligned} \mathbf{x}^* &= [0.5 \ 2] \\ f(\mathbf{x}^*) &= 306.5 \end{aligned} \tag{A.12}$$

A.4 Hock 17

Objective function:

$$f(\mathbf{x}) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2 \tag{A.13}$$



Constraints:

$$\begin{aligned}g(\mathbf{x}) &= x_1 - x_2^2 \leq 0 \\g(\mathbf{x}) &= x_2 - x_1^2 \leq 0 \\-0.5 &\leq x_1 \leq 0.5 \\-1 &\leq x_2 \leq 1\end{aligned}\tag{A.14}$$

Starting point:

$$\begin{aligned}\mathbf{x}_0 &= [-2 \ 1] \\f(\mathbf{x}_0) &= 909\end{aligned}\tag{A.15}$$

Optimum:

$$\begin{aligned}\mathbf{x}^* &= [0 \ 0] \\f(\mathbf{x}^*) &= 1\end{aligned}\tag{A.16}$$

Appendix B

Vehicle Model Files

Table B.1: Vehicle mass and inertia properties

Body	Mass [kg]	I_{xx}	I_{yy}	I_{zz}
body front	682	909	0	0
body rear	894	952	0	0
tyres	31.5	1.2	1.2	2.0
front axle	166	22.3	0.13	22.3
rear axle	166	22.3	0.13	22.3
steer link	3	0.4	0.4	0

- all other links have 0 mass properties.

List of Tables

3.1	MSC.ADAMS vehicle model's degrees of freedom	25
3.2	Land Rover 110 test points	28
4.1	Ride Comfort Optimisation Results	60
6.1	Summary of Results for Optimisation Objectives	95
6.2	Summary of optimum damper factors and gas volumes	96
8.1	Results for the standard Dynamic-Q and Auto-Scaling Dynamic-Q methods	117
9.1	Summary of Results for Optimisation Objectives	134
9.2	Summary of optimum damper factors and gas volumes	134
9.3	ADAMS data for 4 test points along pareto optimal front	135
B.1	Vehicle mass and inertia properties	153

List of Figures

1.1	4S ₄ Suspension Unit	5
2.1	Simplified illustration on how Dynamic-Q progresses with optimisation iterations	18
2.2	Finite difference gradient approximation methods	20
3.1	Modelling of the full vehicle in MSC.ADAMS, front suspension	26
3.2	Modelling of the full vehicle in MSC.ADAMS, rear suspension	27
3.3	Test vehicle indicating measurement positions	29
3.4	Discrete bumps, 15 km/h, validation of MSC.ADAMS model's vertical dynamics	30
3.5	Double lane change, 65 km/h, validation of MSC.ADAMS model's handling dynamics	31
3.6	Vehicle characterisation steering input	34
3.7	Tyre's lateral force vs. slip angle characteristics for different vertical loads	35
3.8	Vehicle yaw acceleration response to different steering rate inputs	35
3.9	Definition of driver model parameters	36
3.10	Magic Formula coefficient quadratic fit through equivalent peak values	40
3.11	Magic Formula fit of yaw acceleration gain through the actual data	41
3.12	Determination of curvature coefficients	41
3.13	Magic Formula fits to original vehicle steering behaviour	42
3.14	Correlation of Magic Formula driver model to vehicle test at an entry speed of 63 km/h	44

46	
4.1	Definition of spring characteristics for various gas volumes . . . 50
4.2	Definition of damper characteristics for various damper scale factors 50
4.3	Vehicle roll angle, double lane change at 80 km/h for the two variable design space 54
4.4	Vehicle Ride comfort, Belgian paving at 60 km/h for the two variable design space 54
4.5	Handling optimisation, 2 design variables 56
4.6	Handling optimisation, 4 design variables 56
4.7	Ride comfort optimisation, 2 design variables 58
4.8	Dynamic-Q ffd ride comfort, 4 design variables, 10 % move limit 59
4.9	Ride comfort optimisation, 4 design variables 59
4.10	Tyre hop investigation. Vertical tyre accelerations for SQP optimised suspension compared to baseline vehicle. 62
5.1	Definition of $4S_4$ spring characteristics for various gas volumes 66
5.2	Definition of $4S_4$ damper characteristics for various damper scale factors 67
5.3	Level of inherent numerical noise in objective function and inequality constraints, for change in front damper design variable x_1 , for full vehicle MSC.ADAMS model 71
5.4	Level of inherent numerical noise in objective function and inequality constraints, for change in front damper design variable x_1 , when considering the simplified MATLAB model 72
5.5	Top view of vehicle in handling manoeuvre 73
5.6	Rear view of vehicle indicating body roll 73
5.7	Simple pitch-plane vehicle model 75
5.8	Validation of 1st peak roll angle over design space, for double lane change. 77
5.9	Validation of RMS roll velocity over design space, for double lane change. 78



5.10	Model validation of ride comfort for differing front and rear gas volumes	78
5.11	Model validation of ride comfort for differing front and rear damper scale factors	79
5.12	Model validation of ride comfort for differing front and rear damper scale factors: effect on rear tyre hop	79
6.1	Handling optimisation convergence histories for full MSC.ADAMS model, and using the simplified MATLAB model for gradient information, 2 design variables	85
6.2	Handling optimisation convergence history using the full MSC.ADAMS model for gradient information, 4 design variables	86
6.3	Handling optimisation convergence histories using the simplified MATLAB model for gradient information, 4 design variables	87
6.4	Implementing tyre hop as a constraint in ride comfort optimisation	89
6.5	Observing tyre hop value while performing ride comfort optimisation	90
6.6	Comparison of the optimisation histories for the MSC.ADAMS gradient and simple MATLAB model gradient methods for 2 design variable ride comfort optimisation	91
6.7	Ride comfort optimisation convergence history for using only the simple MATLAB based model, for objective function value, gradients, and tyre hop information.	92
6.8	Ride Comfort optimisation convergence history for 4 design variables using the full MSC.ADAMS model for gradient information, starting at the optimum from two design variables	93
6.9	Ride Comfort optimisation convergence history for 4 design variables using the simple matlab model for gradient information, starting at the optimum from two design variables	94
7.1	Definition of damper characteristics with quadratic approximation to the baseline rear Land-Rover damper	99
7.2	Handling 14 design variable optimisation convergence history.	101

7.3 Change in objective function value with respect to design variable x_2 , when the other design variables are in the middle of the design space 102

7.4 Normalised change in objective function value with respect to design variable x_2 , when the other design variables are in the middle of the design space 103

7.5 Illustration of the effect of ellipticity and sphericity on the convergence to the optimum 104

7.6 Ride comfort optimisation convergence history illustrating first 20 iterations 105

7.7 Ride comfort optimisation convergence history with rescaled design variables 106

8.1 Illustration of the effect of ellipticity and sphericity on the convergence to the optimum 110

8.2 Comparison of standard Dynamic-Q convergence to optimum and Dynamic-Q with automatic scaling, for test 1 115

8.3 Comparison of standard Dynamic-Q convergence to optimum with Dynamic-Q with automatic scaling, for test problem 2 . . 116

8.4 Comparison of convergence histories for standard Dynamic-Q and for the implementation of the automatic scaling, for 14 design variable handling 120

8.5 Optimisation convergence history for handling, starting at the found minimum 120

8.6 Comparison of optimisation convergence histories for standard Dynamic-Q and for the implementation of the automatic scaling (14 design variable ride comfort) 122

8.7 Optimisation convergence history for starting at minimum, using automatic scaling 122

8.8 Optimum damper characteristics for handling compared to the baseline rear damper 123

8.9 Optimum damper characteristics for ride comfort 123



9.1 Combined convergence history, first handling optimisation, then ride comfort. 127

9.2 Combined optimisation convergence history, maximum of handling and ride comfort objectives, 2 design variables. 129

9.3 Combined optimisation convergence history, maximum of handling and ride comfort objectives, 4 design variables. 130

9.4 Investigation of convergence history to pareto front for different weights of the objective function for handling (h) and ride comfort (r), compared to random feasible points of design space 131

9.5 Pareto front plot including the change in the design variables along the pareto front 132