Appendix A

Poisson Processes

In reduced-form (hazard rate) models, the fundamental modelling tool is the Poisson process, and this appendix reviews some important aspects of Poisson processes. The definitions and concepts in this appendix are taken from (and covered in much more detail in) Rogers and Williams (1994).

A *homogeneous* Poisson counting process $\{N_t\}_{t\geq 0}$ is a non-decreasing process with right-continuous paths and values in Z^+ such that

1. $N_0 = 0$;
2. for any $0 \leq s_1 \leq t_1 \leq s_2 \leq t_2 \leq \ldots \leq s_n \leq t_n$, the random variables $X_i \equiv N(t_i) - N(s_i)$ are independent, and the distribution of each X_i depends only on the length $t_i - s_i$;
3. $\forall t \geq 0, N_t - N_{t-}$ is either 0 or 1.

   The definition of the Poisson process uniquely determines its distribution to within a single parameter λ called the rate of the process. When $\lambda = 1$, we speak of a standard Poisson process. Here are other key properties, in which the positive parameter λ appears explicitly.

4. the process $\widetilde{N}_t \equiv N_t - \lambda t$ is a martingale;
5. the inter-event times $T_n - T_{n-1}$ are independent with common exponential

(λ) distribution:

$$P[(T_n - T_{n-1}) > t] = e^{-\lambda t} \; \forall t \geq 0 \tag{A.1}$$

Here

$$T_n \equiv \inf\{t \geq 0 \mid N_t = n\} \tag{A.2}$$

6. for any $s \leq t, N_t - N_s \sim P(\lambda(t-s))$, the Poisson distribution with mean λ:

$$P[N_t - N_s = k] = \frac{1}{k!}\lambda^k (t-s)^k \exp\{-(t-s)\lambda\}, k \in Z^+ \tag{A.3}$$

The simple (homogeneous) Poisson process can be generalized as follows. N is called an *inhomogeneous* Poisson process with deterministic intensity function $\lambda(t)$, if the increments $N(t) - N(s)$are independent and for s, t we have

$$P[N_t - N_s = k] = \frac{1}{k!}\left(\int_s^t \lambda(u)du\right)^k \exp\left\{-\int_s^t \lambda(u)du\right\} \tag{A.4}$$

The only difference to property (6) above is that $\lambda(t-s)$ has been replaced by the integral of $\lambda(u)$ over the respective time span.

Appendix B

Summary of Credit Risk Model Features

In chapters 4 and 5, we provided an overview of structural and reduced-form models of default. The purpose of this chapter is to summarize the features of the models reviewed in chapters 4 and 5.

Credit Risk Model	Merton (1974)	LS (1995)	JLT (1997)	DS#1 (1999)
Default Process	Default occurs when firm value falls below debt value.	Default occurs when firm value falls below a stochastic boundary.	Default occurs when a firm transitions into the lowest level.	Only model hazard rate of default.
Default Probability	Determined by firm value growth and volatility.	Determined by the growth, volatility &, correlation of firm value and boundary.	Determined by a Markov Process in the firm's credit ratings.	
Recovery Process	Assumed to be value of firm at time of default.	Assumed to be a constant fraction of face value, received at maturity.	Assumed to be a constant fraction of face value, received at maturity.	Exogenously given fractional loss of market value.
Risk-Free Rate Process	Constant interest rates.	Vasicek model.	None given.	Use any interest rate model to arrive at risk-adjusted short rate.

Credit Risk Model	Merton (1974)	LS (1995)	JLT (1997)	DS#1 (1999)
Correlation Modelling	None	Between interest rates, and firm value processes.	None	None
Model Category	Structural Continuous (Closed Form) Equilibrium	Structural Continuous (Closed Form with recursion) Equilibrium	Reduced-Form Continuous Arbitrage-Free	Reduced-Form Continuous and Discrete Arbitrage-Free

Credit Risk Model	DS#2 (1999)	DS#3 (1999)
Default Process	Model mean loss rate directly.	Model default probability and loss percentage separately.
Default Probability		Estimate historically by bond class.
Recovery Process		Model using historic recovery rates.
Risk-Free Rate Process	None given.	Cox-Ingersoll-Ross for interest rates and credit spreads.

Credit Risk Model	DS#2 (1999)	DS#3 (1999)
Correlation Modelling	Between mean loss rate and interest rates.	Between interest rates and credit spreads.
Model Category	Reduced-Form Continuous Arbitrage-Free	Reduced-Form Continuous Arbitrage-Free

Appendix C

Strengths and Drawbacks of Credit Risk Models

In chapters 4 and 5, we provided an overview of structural and reduced-form models of default. The purpose of this chapter is to summarize the strengths and drawbacks of the models reviewed in chapters 4 and 5.

Model	Advantages	Disadvantages
Merton (1974)	Simple to implement.	(a) Requires inputs related to firm value. (b) Default occurs only at the maturity of debt. (c) Information in the history of defaults and credit rating changes cannot be used.
Longstaff and Schwartz (LS) (1995)	(a) Simple to implement. (b) Allows for stochastic term structure and correlation between defaults and interest rates.	(a) Requires inputs related to firm value. (b) Information in the history of defaults and credit rating changes cannot be used.
Jarrow, Lando and Turnbull (JLT) (1997)	(a) Simple to implement. (b) Can exactly match the existing prices of default-risky bond to infer risk-neutral probabilities of defaults and credit rating changes. (c) Uses the information in the history of defaults and credit rating changes.	(a) Correlation not allowed between default probabilities and the level of interest rates. (b) Credit spreads change only when credit rating changes.

Model	Advantages	Disadvantages
Duffie and Singleton (DS) (1999)	(a) Allows correlation between default probabilities and the level of interest rates. (b) Recovery ratio can be random and depend on the pre-default value of the security. (c) Any interest rate model can be accommodated and existing valuation results for risk-free term structure models can be readily used.	(a) Information in the credit history of defaults and rating changes cannot be used.

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