

Analysis of the particle swarm optimization algorithm

by

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Abstract

- Title:** Analysis of the particle swarm optimization algorithm
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Increasing prominence is given to the role of optimization in engineering. The global optimization problem is in particular frequently studied, since this difficult optimization problem is in general intractable. As a result, many a solution technique have been proposed for the global optimization problem, e.g. random searches, evolutionary computation algorithms, taboo searches, fractional programming, etc. This study is concerned with the recently proposed zero-order evolutionary computation algorithm known as the particle swarm optimization algorithm (PSOA). The following issues are addressed:

1. It is remarked that implementation subtleties due to ambiguous notation have resulted in two distinctly different implementations of the PSOA. While the behavior of the respective implementations is markedly different, they only differ in the formulation of the velocity updating rule.

In this thesis, these two implementations are denoted by PSOAF1 and PSOAF2 respectively.

2. It is shown that PSOAF1 is observer independent, but the particle search trajectories collapse to line searches in n -dimensional space.

In turn, for PSOAF2 it is shown that the particle trajectories are space filling in n -dimensional space, but this implementation suffers from observer dependence.

It is also shown that some popular heuristics are possibly of less importance than originally thought; their greatest contribution is to prevent the collapse of particle trajectories to line searches.

3. A novel PSOA formulation, denoted PSOAF1* is then introduced, in which the particle trajectories do not collapse to line searches, while observer independence is preserved. However, the observer independence is only satisfied in a stochastic sense, i.e. the mean objective function value over a large number of runs is independent of the reference frame.

Objectivity and effectiveness of the three different formulations are quantified using a popular unimodal and multimodal test set, of which some of the multimodal functions are decomposable. However, the objective functions are evaluated in both the unrotated, decomposable reference frame, and an arbitrary rotated reference frame.

4. Finally, a practical engineering optimization problem is studied. The PSOA is used to find the optimal shape of a cantilever beam. The objective is to find the minimum vertical displacement at the edge point of the cantilever beam. In order to calculate the objective function the finite element method is used. The meshes needed for the linear elastic finite element analysis are generated using an unstructured remeshing strategy. The remeshing strategy is based on a truss structure analogy.

Opsomming

- Titel:** Analise van die partikel swerm optimeringsalgoritme
- Outeur:** Daniel Nicolas Wilke
- Leier:** Prof. A.A. Groenwold
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- Departement:** Departement Meganiese en Lugvaartkundige Ingenieurswese
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Toenemende belangrikheid word aan die rol van optimering in ingenieurswese gegee. Veral die globale optimeringsprobleem word dikwels bestudeer, aangesien hierdie moeilike optimeringsprobleem in die algemeen onoplosbaar is. Gevolglik is daar voorheen al verskeie oplossingstegnieke voorgestel vir die globale optimeringsprobleem, soos byvoorbeeld lukrake soektogte, evolusionêre berekeningsalgoritmes, taboe soektogte, fraksionele programmering, ens. Hierdie studie is vermoed met die onlangs gepostuleerde nulde-orde evolusionêre berekeningsalgoritme wat bekend staan as die partikel swerm optimeringsalgoritme (PSOA). Die volgende kwessies word bespreek:

1. Daar word opgemerk dat twee verskillende formulerings van die PSOAF bestaan, moontlik as gevolg van onduidelike notasie. Alhoewel die gedrag van die onderskeie implementerings dramaties verskil, verskil hulle slegs ten opsigte van die formulering van die snelheids-opdateringswet.

In hierdie tesis word die onderskeie implementerings as PSOAF1 en PSOAF2 aangedui.

2. Verder word aangetoon dat PSOAF1 waarnemer onafhanklik is, maar dat die partikel bane in n -dimensionele ruimte na lyn soektogte ineenstort.

Om die beurt, word daar vir PSOAF2 aangetoon dat die partikel bane ruimtevullend is in n -dimensionele ruimte, maar hierdie implementering is waarnemer afhanklik.

Daar word ook gewys dat sommige gewilde heuristieke moontlik van minder belang is as

wat oorspronklik geag is. Daar word gewys dat hulle grootste bydrae waarskynlik is om partikel baan ineenstorting na lyn soektogte te voorkom.

3. 'n Nuwe PSOA formulering word dan voorgestel, naamlik PSOAF1*. Partikel trajekte stort nie na lyn soektogte ineen nie, terwyl waarnemer onafhanklikheid behou word. Waarne-mer onafhanklikheid word egter slegs in 'n stogastiese sin bevredig, m.a.w. die gemiddelde doelwit funksie waarde is onafhanklik van 'n koördinaatstelsel, gesien oor 'n groot aantal verlope.

Objektiwiteit en effektiwiteit van die drie formulerings word gekwantifiseer deur gebruik te maak van 'n gewilde unimodale en multimodale toets stel, waarvan die meerderheid multimodale funksies skeibaar is. Nietemin word die doelwit funksies geëvalueer in beide die ongeroteerde, skeibare, verwysingsraamwerk en 'n lukraak geroteerde verwysingsraamwerk.

4. Laastens word 'n praktiese ingenieurs optimeringsprobleem bestudeer. Die PSOA word aangewend om die optimale geometrie van 'n kantelbalk te vind. Die doelfunksie wat geminimeer word is die vertikale verplasing by die eindpunt van die kantelbalk. Die doelfunksies word bereken deur gebruik te maak van die eindige element metode. Die mase wat benodig word vir die linieêr elastiese eindige element analyses word gegenereer deur van 'n ongestruktureerde hermasings-strategie gebruik te maak. Die hermasings-strategie is gebaseer op 'n vakwerk struktuur analoog.

Acknowledgments

Some have the pleasure of seeing distinguished men in their lifetime, some have the privilege of meeting them, but I had the honor to undertake a journey alongside them.

- D.N. Wilke

University of Pretoria, 2004.

I would like to dedicate this thesis to
my father and my mother

This section attempts to capture a mere snapshot of my thoughts and experiences over the last two years. This section is written in the form of an informal short story. To anyone who may feel affronted in any way by the writing style or content of this section, I offer my sincere apologies.

Conceit of the Absurd - A sailor's story

This story begins in the year 2002, as I reached my last year of formal enrollment as a sailor. My fellow sailor friends and I used to meet up at a local tap and talk about the adventures to come, and of possible treasures of gold.

As the year progressed I was adamant that I would set sail the following year for either Europe or the middle East, as I considered some lucrative offers of possible gold treasures and adventure in these distant and uncharted lands.

As the year came to an end, the sea tides turned.

I met Captain Groenwold and in the end I decided to trade adventures of Europe, and the middle East for adventures of another kind. The adventures that are about the journey and not the destination.

As the year 2003 dawned, Captain Groenwold and I embarked on an adventure. The adventure started calmly by sailing out of the harbor on a brig, affectionately referred to by the sailors as SORG. At the boat's command stood Captain Groenwold and on the deck stood I, a proud sailor.

We left the harbor and sailed into the open seas, that laid open for traveling and exploration. This was my first time out on the ocean. As the harbor disappeared on the distant horizon I did not quite

know where I was, but I knew I was out there, somewhere. Months past as we encountered some light breezes and stormy clouds here and there.

Then one day Captain Groenwold got word that he was needed in a distant land. We anchored the vessel at the nearest harbor and on board came co-Captain Kok to take command of the brig. Our only contact with Captain Groenwold being the infamous message bottle system.

The vessel sailed further under co-Captain Kok's command, over the calm seas and oceans. Every now and then we would spot land here and there, for me the excitement grew as the frequency of sightings of land increased. Some days the breezes became stronger than others as we set forth towards the land. Not before long by the middle of 2004, the adventure turned into an epic of Gulliver's travels as we finally reached land. The epic started by us nearly shipwrecking the preceding day after I misread the map. Fortunately, co-Captain Kok instantly realized my mistake and recovered our situation. Nevertheless we anchored and set forth our exploration.

For months we set sail and anchored to explore various places, each an exotic place in it's own right. On one of the stops we picked up Captain Groenwold after his return from the distant lands. The adventure continued, I was constantly fascinated by each place, and not before long it dawned. What seemed to be vast and distant lands was actually one big island. We attempted to map what we could of this beautiful island but as our supplies where running low, we had to set forth the journey back home.

By the beginning of 2005, the adventure came to an end as our ship sailed into the harbor. As I disembarked our ship and touched home soil I realized "*I embarked on the right adventure*".

Formally I would like to express my sincere gratitude towards the following persons:

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Contents

Abstract	ii
Opsomming	iv
Acknowledgments	vi
List of Figures	xiii
List of Tables	xiv
1 Introduction	1
1.1 Global optimization	1
1.2 Motivation	2
1.3 Objectives	2
1.4 Approach	2
1.5 Thesis overview	3
2 Problem formulation and background	5
2.1 Global optimization problem formulation	5
2.2 Basic formulation of the PSOA	5
2.3 Brief history of PSOA	6
3 Diversity in the PSOA	7
3.1 Introduction	7
3.2 Notes on the PSOA formulation	8
3.3 Implementation subtleties: Formulation 1 (PSOAF1)	9
3.3.1 Investigation of the limit behavior of PSOAF1	10
3.4 Implementation subtleties: Formulation 2 (PSOAF2)	12
3.4.1 Investigation of the limit behavior of PSOAF2	13

<i>CONTENTS</i>	ix
3.5 Numerical experiments	15
3.6 Discussion of numerical results	16
3.7 Notes on some heuristics of the PSOA	18
3.7.1 Local best neighborhood	18
3.7.2 Non-zero initial velocities	19
3.7.3 Maximum velocity restriction	19
3.7.4 Minimum velocity restriction	22
3.7.5 Position restriction	23
3.7.6 Crazyiness	23
3.7.7 Increasing social awareness	24
3.7.8 Inertia factor	24
3.7.9 Using a single random number	25
3.8 On tuning of PSOA parameters (finding universal optimal parameter values)	25
3.9 Closure	26
4 Objectivity of the PSOA	27
4.1 Introduction	27
4.2 Notes on the investigation	28
4.3 Formulation 1 (PSOAF1)	28
4.3.1 PSOAF1: Investigation of the instantaneous search domain	29
4.4 Formulation 2 (PSOAF2)	31
4.4.1 PSOAF2: Investigation of the instantaneous search domain	31
4.5 Novel Formulation: PSOAF1*	33
4.5.1 PSOAF1*: Investigation of the instantaneous search domain	34
4.6 Numerical experiments	36
4.7 Discussion of Results	37
4.8 Comments on PSOAF1*	42
4.8.1 On invariance	42
4.8.2 Implementational issues of PSOAF1*	43
4.8.3 Alternatives to PSOAF1*	43
4.9 Closure	44
5 Shape optimization problem	45
5.1 Introduction	45
5.2 Problem formulation	46
5.2.1 Accommodation of constraints	46

<i>CONTENTS</i>	x
5.3 Mesh generation	47
5.3.1 Mesh generator based on a truss structure analogy	47
5.4 Numerical results for the cantilever beam problem	47
5.5 Closure	53
6 Conclusions and recommendations	54
6.1 Conclusions	54
6.2 Recommendations	55
Bibliography	60

List of Figures

3.1	The position vector \mathbf{x}_{k+1}^i , partitioned into a deterministic contribution ($\mathbf{x}_k^i + w\mathbf{v}_k^i$) and a stochastic contribution ($\mathbf{v}_k^i \in \mathcal{X}_k^i$).	8
3.2	Partitioning the position vector \mathbf{x}_{k+1}^i into a deterministic contribution ($\mathbf{x}_k^i + w\mathbf{v}_k^i$), and a stochastic contribution ($\mathbf{v}_k^i \in \mathcal{P}_k^i$), for $c_1 = c_2 = 2$	10
3.3	Partitioning the position vector \mathbf{x}_{k+1}^i into a deterministic contribution ($\mathbf{x}_k^i + w\mathbf{v}_k^i$), and a stochastic contribution ($\mathbf{v}_k^i \in \mathcal{L}_k^i$), for $c_1 = c_2 = 2$	10
3.4	PSOAF1: Position vectors \mathbf{x}_{k+1}^i generated for 25 consecutive iterations. The best particle position \mathbf{p}_k^i and the best global position \mathbf{p}_k^g are kept constant.	11
3.5	PSOAF1: The average angle θ between $(\mathbf{p}_k^i - \mathbf{x}_k^i)$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i)$ versus iteration number, for a dimensionality of a) $n = 3$, and b) $n = 30$. For the 1000 iterations, the best particle position \mathbf{p}_k^i and the best global position \mathbf{p}_k^g are stationary.	12
3.6	PSOAF2: Position vectors \mathbf{x}_{k+1}^i generated over 25 iterations without updating the particle best position vector \mathbf{p}_k^i and the global best position vector \mathbf{p}_k^g . No restriction is imposed on the velocity vector.	14
3.7	PSOAF2: The average angle θ between $(\mathbf{p}_k^i - \mathbf{x}_k^i)$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i)$ versus iteration number, for a dimensionality of a) $n = 3$, and b) $n = 30$. For the 1000 iterations, the best particle position \mathbf{p}_k^i and the best global position \mathbf{p}_k^g are stationary.	14
3.8	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Rosenbrock test function (f_0).	16
3.9	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Quadric test function (f_1).	16
3.10	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Ackley test function (f_2).	17
3.11	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Rastrigin test function (f_3).	17
3.12	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Griewank test function (f_4).	17
3.13	Average convergence history for Ackley's test function, for a) PSOAF1, and b) PSOAF2.	19

3.14	The velocity restriction implemented by a) restricting the component values of the velocity vector a) restricting the magnitude of the velocity vector.	20
3.15	Velocity restriction on a) the components, and b) the magnitude of velocity. Depicted is the average angle θ between $(\mathbf{p}_k^i - \mathbf{x}_k^i)$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i)$ versus iteration number, for a dimensionality of $n = 30$. For the 1000 iterations, the best particle position \mathbf{p}_k^i and the best global position \mathbf{p}_k^g are stationary.	21
3.16	Average function value for the Griewank test function (f_4) after 1000 iterations, averaged over 100 runs.	21
3.17	Average function value after 2×10^5 function evaluations (10000 iterations) over 100 runs on the Griewank test function (f_4).	23
4.1	Partitioning the position vector \mathbf{x}_{k+1}^i into a deterministic contribution $(\mathbf{x}_k^i + w\mathbf{v}_k^i)$, and a stochastic contribution $(\mathbf{v}_k^i \in \mathcal{P}_k^i)$, for $c_1 = c_2 = 2$	29
4.2	Partitioning the position vector \mathbf{x}_{k+1}^i into a deterministic contribution $(\mathbf{x}_k^i + w\mathbf{v}_k^i)$, and a stochastic contribution $(\mathbf{v}_k^i \in \mathcal{L}_k^i)$, for $c_1 = c_2 = 2$	29
4.3	PSOAF1: Scatter plot of 10^4 possible stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations, with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [-\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [0 \ 2]$. Each point represents the end point of a stochastic vector with origin at $[0 \ 0]$	30
4.4	PSOAF1: Scatter plot of 10^4 possible stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations, with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [2 \ 0]$	31
4.5	PSOAF2: Scatter plot of 10^4 possible stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [-\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [0 \ 2]$	32
4.6	PSOAF2: Scatter plot of 10^4 possible stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [2 \ 0]$	33
4.7	PSOAF1*: Scatter plot of 10^4 possible stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations, with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [-\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [0 \ 2]$	35
4.8	PSOAF1*: Scatter plot of 10^4 instances of the stochastic vectors \mathbf{v}_k^i , generated using Monte Carlo simulations, with a) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}}]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [\sqrt{2} \sqrt{2}]$ and b) $(\mathbf{p}_k^i - \mathbf{x}_k^i) = [1 \ 0]$ and $(\mathbf{p}_k^g - \mathbf{x}_k^i) = [2 \ 0]$	35
4.9	Average function value obtained with a) PSOAF1 and PSOAF1*, and b) PSOAF2 after 2×10^5 function evaluations (10000 iterations) averaged over 100 runs on the rotated and unrotated Rosenbrock test function f_0	38
4.10	Average function value obtained with a) PSOAF1 and PSOAF1*, and b) PSOAF2 after 2×10^5 function evaluations (10000 iterations) averaged over 100 runs on the rotated and unrotated Quadric test function f_1	38

4.11	Average function value obtained with a) PSOAF1 and PSOAF1*, and b) PSOAF2 after 2×10^5 function evaluations (10000 iterations) averaged over 100 runs on the rotated and unrotated Ackley test function f_2	39
4.12	Average function value obtained with a) PSOAF1 and PSOAF1*, and b) PSOAF2 after 2×10^5 function evaluations (10000 iterations) averaged over 100 runs on the rotated and unrotated Rastrigin test function f_3	39
4.13	Average function value obtained with a) PSOAF1 and PSOAF1*, and b) PSOAF2 after 2×10^5 function evaluations (10000 iterations) averaged over 100 runs on the rotated and unrotated Griewank test function f_3	40
4.14	Mean function value history plot averaged over 100 runs on the rotated and unrotated Quadric test function f_3 with PSOAF1 (with $w = 0.8$), PSOAF2 (with $w = 0.4$) and PSOAF1* (with $w = 0.5$ and $\alpha = 3$).	42
4.15	Scatter plot of 10^4 possible stochastic vectors ν_k^i , generated using Monte Carlo simulations, with $(p_k^i - x_k^i) = [-1 \ 0]$ and $(p_k^i - x_k^i) = [2 \ 0]$ using a) identical rotation matrices $Q_{1k}^i = Q_{2k}^i$ and b) independent rotation matrices $Q_{1k}^i \neq Q_{2k}^i$	44
5.1	Initial structure and definition for the cantilever beam.	48
5.2	Mean function value history plot averaged over 10 runs for the cantilever beam shape optimization problem for PSOAF1 (with $w = 0.7$), PSOAF2 (with $w = 0.5$) and PSOAF1* (with $w = 0.6$ and $\alpha = 5$).	49
5.3	Cantilever beam: results obtained after 100 iterations with PSOAF1 for a) the worst run (run 6), and b) the best run (run 7).	50
5.4	Cantilever beam: results obtained after 500 iterations with PSOAF1 for a) the worst run (run 6), and b) the best run (run 7).	50
5.5	Cantilever beam: results obtained after 1000 iterations with PSOAF1 for a) the worst run (run 6), and b) the best run (run 7).	50
5.6	Cantilever beam: results obtained after 100 iterations with PSOAF2 for a) the worst run (run 7), and b) the best run (run 6).	51
5.7	Cantilever beam: results obtained after 500 iterations with PSOAF2 for a) the worst run (run 7), and b) the best run (run 6).	51
5.8	Cantilever beam: results obtained after 1000 iterations with PSOAF2 for a) the worst run (run 7), and b) the best run (run 6).	51
5.9	Cantilever beam: results obtained after 100 iterations with PSOAF1* for a) the worst run (run 3), and b) the best run (run 4).	52
5.10	Cantilever beam: results obtained after 500 iterations with PSOAF1* for a) the worst run (run 3), and b) the best run (run 4).	52
5.11	Cantilever beam: results obtained after 1000 iterations with PSOAF1* for a) the worst run (run 3), and b) the best run (run 4).	52

List of Tables

3.1	Test function parameters.	16
3.2	Constant inertia factor at which the best average objective function value is obtained.	18
3.3	Effect of probability of craziness P_{cr} on $f_5(\mathbf{x})$	24
4.1	Test function parameters	37
4.2	Constant inertia factor at which the best average objective function value is obtained for the unrotated test functions. The accompanying average objective function value for rotated test functions is also presented.	41
5.1	Optimal results for the cantilever beam after only 1000 iterations.	49