Chapter 1

INTRODUCTION

> Fracture of components due to fatigue is the most common cause of service failure, ... *E. J. Hearn [19]*

The fatigue of metals has been studied for more than 150 years. During these years extensive research has been performed to quantify fatigue failures. By the 1970's fatigue analysis had become an established engineering tool. Despite all this knowledge, unintended fatigue failures continue to occur.

The design of components that are primarily subjected to constant amplitude cyclic loading, for instance a rotating shaft, is fairly trivial. However, when a structure is subjected to variable or ill-defined loads, the design of the structure could be quite difficult. Obtaining realistic and correct fatigue loads is of primary importance to perform a fatigue analysis on a structure. Wannenburg [39] argues that insufficient knowledge of input loads is the major cause of defective structural designs. Similar comments are also made by Olofsson [27], Rahman [29] and Broek [5].

The design of fatigue loaded structures is usually performed using either design codes or exhaustive durability evaluations. Many vehicle components, or structures, are usually designed through the use of prototypes that are evaluated using laboratory simulations and proving ground durability assessments. Experimental data is subsequently collected and an occurrence histogram is formed from the resulting time histories. The histogram is then scrutinized to establish the worst fatigue loads. The structure (vehicle chassis, suspension arm, etc.) is then analysed using these loads. Another method currently being used to design structures is by the use of design codes. Design codes, for example the SABS road tanker code [36], usually make use of static loads. The loads prescribed by the codes are usually quite high (loads that will not realistically occur in the vehicle during normal usage). The codes also specify an additional factor of safety for the materials used (20% of the ultimate strength). Therefore, the design codes clearly incorporate allowance for fatigue loads.

The above-mentioned practices clearly have a few important drawbacks:

- *Design and development time.* In the current competitive markets, time is of the essence. Manufacturers and designers are pressurized to deliver properly designed products to their customers in the shortest time possible.

- *Cost.* The cost of time-consuming durability analysis can seriously affect the competitive edge of a manufacturer. Furthermore, the cost of an iterative prototype analysis, is currently getting prohibitive. This is especially true for manufacturers that need to deliver new products more frequently. Due to excessive factors of safety, the use of codes sometimes also implies a cost penalty.

- *Unrealistic fatigue loads.* The use of design codes may provide structures that could withstand fatigue loads. This is however not always the case. Olofsson [27] reports that an inspection of Swedish tank vehicles, showed that as much as 46 percent of the aluminum tank vehicles were impaired by cracks caused by fatigue. A design code, with all the safety factors, may therefore not be enough to accommodate fatigue loading.

The objective of this study is to present a methodology that overcomes or alleviates the above-mentioned drawbacks. The Fatigue Equivalent Static Load (FESL) method that is presented is an inexpensive and time efficient process to provide a point of departure in the design of a structure for fatigue loads. More specifically, the methodology yields accurate and realistic fatigue loads which can be used in design.

The FESL method can be summarized as follows:

- Obtaining input data to the vehicle structure.
- Calculating the fatigue damage due to the measured load cases.
- Calculating fatigue equivalent static loads, through the use of finite element models.
- Analysing the durability of the structure, making use of the fatigue equivalent static loads and finite element models.

The structure of the thesis is as follows: *Chapter 2* explains the current theory and practices of fatigue, load determination and methodologies currently being used. *Chapter 3* provides a brief formulation of the Fatigue Equivalent Static Load methodology. *Chapter 4* defines the case-studies used in the thesis. *Chapter 5* discusses the methods employed to determine the input loads to the vehicle structures. *Chapter 6* discusses the fatigue calculations performed on the measurement data. *Chapter 7* defines the finite element structural analyses that were performed. *Chapter 8* presents the assessment of each case-study, making use of the FESL method. *Chapter 9* presents the conclusion of the study.

Chapter 2

CURRENT THEORY AND PRACTICE

2.1 Scope

This chapter aims to present the current theories and practices used by engineers in solving structural fatigue problems. The chapter is sub-divided into four sections. The first three sections explain the 'building blocks' that are necessary to perform a fatigue assessment, while the fourth section presents methodologies to incorporate these 'building blocks' into an engineering solution. Most practices of solving a fatigue damage problem follow these steps:

- *Determination of input loading.* To assess any structure accurately, the loads that act in on it must be determined. This section describes various methods currently being used to determine these loads.

- *Finite element structural analysis.* A load applied to a structure can cause extensive fatigue damage to the structure in question. If, however, the same load is applied to another structure, the fatigue damage could be minimal. To determine whether or not a structure will experience fatigue damage, the structure must be analysed for strength and other structural factors. A component that has a fairly simple geometry, for example a shaft, can structurally be analysed using hand calculations (refer to Shigley [33]). However, for a more complex structure, e.g. a vehicle chassis, the finite element method must be used. This section explains the fundamentals of this method of structural analysis. It should be noted that a fatigue damage prediction can be performed using only strain gauge measurements (without the help of a structural analysis). This type of analysis is however limited to the immediate area around the strain gauge, and therefore has limited value.

- *Fundamentals of fatigue and durability.* The determination of loads acting on a structure and the knowledge of the structural response (strength, natural frequencies, etc.) is not enough to determine whether or not a structure will experience fatigue damage. The information obtained by the two previously mentioned exercises must be processed, using fatigue theories and principles. This section explains the fundamentals of the currently accepted fatigue theories.

- *Integration of measurement and analysis.* This section provides a few examples of strategies and techniques that are currently in use to determine a fatigue damage prediction. These strategies aim to incorporate the various methods previously explained to determine a solution to a structural fatigue problem.

2.2 Determination of Input Loading

The assessment of a vehicle structure requires the determination of the loads that affect the structure during usage. This section discusses the various methods that can be employed to determine these loads.

2.2.1 Time Domain

Fatigue based calculations are almost always done in the time domain, e.g. load or stress/strain data are measured along a time line. Stress versus time,

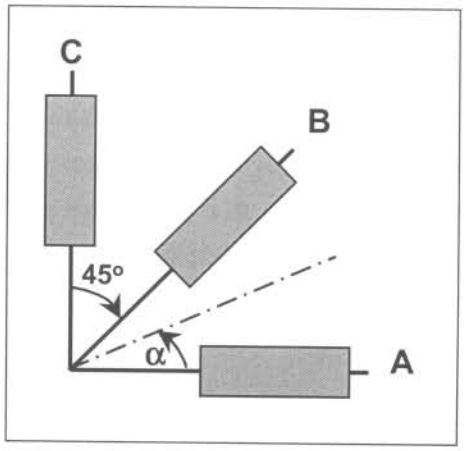


Figure 2.1: Rosette strain gauge configuration

is regarded to be in the time domain. Fatigue data and calculations are usually derived using time as the corresponding measurement unit. The Wöhler or SN curve is a plot depicting measured stress versus time. Linear Elastic Fracture Mechanics also make use of the time variable when using the Paris equation to calculate crack growth. The frequency domain is discussed later on, but all the measurements done in the case-studies make use of time domain data. Due to the importance of measuring time domain data, a few general concepts of this method will be discussed in this section.

Most of the measurements used in this study were strain gauge measurements. The purpose of strain gauge measurement is to determine the stresses and strains in any arbitrary direction on the surface of a component. This method therefore measures plane stress.

If complex loading conditions are expected, where the principle axes of the stresses are unknown, a rosette with three strain gauges is used. The rosette configuration enables the calculation of the unknown axes. Figure 2.1 shows the configuration of the three gauges that form the rosette. The principle stresses are calculated using equations 2.1 and 2.2. The direction of the principle stresses (angle α), is calculated with equation 2.3. It should be noted that if the principle loading directions are known, only two strain gauges are required. For uni-axial loading, only one strain gauge is needed. A strain gauge can be connected to a half bridge configuration to compensate for temperature effects. When applying a strain gauge to a structure, it is vital to take the environmental influences (temperature, moisture etc.) into consideration. Neglecting these aspects, could lead to the failure of strain gauges, resulting in costly re-measurement exercises. The strain gauge measurement technique is, however, usually quite robust if correctly done.

$$\sigma_{1/2} = \frac{E}{2}\left[\frac{\epsilon_a + \epsilon_c}{1-\nu} \pm \frac{1}{1+\nu}\sqrt{2(\epsilon_a - \epsilon_b)^2 + 2(\epsilon_b - \epsilon_c)^2}\right] \tag{2.1}$$

$$\tau_{max} = \frac{E}{2(1+\nu)}\sqrt{2(\epsilon_a - \epsilon_b)^2 + 2(\epsilon_b - \epsilon_c)^2} \tag{2.2}$$

$$\tan 2\alpha = \frac{2\epsilon_b - \epsilon_a - \epsilon_c}{\epsilon_a - \epsilon_c} \tag{2.3}$$

where :

E = Young's modulus

$\sigma_{1/2}$ = Principal stresses

$\epsilon_{a/b/c}$ = Measured strains

ν = Poisson ratio

τ_{max} = Maximum shear stress

α = Angle of reference

The measurement of accelerations, forces and displacements is sometimes necessary. The integration of acceleration to velocity and subsequently, displacement, is a common procedure, but differentiation is usually avoided due to the amplification of the noise present in the data.

In the case studies presented in this thesis, the measurements of forces were sometimes done with strain gauges. The strains that were measured on the specially designated strain gauges were calibrated with a known force. If the calibration was not possible, manual calculations of the force were done

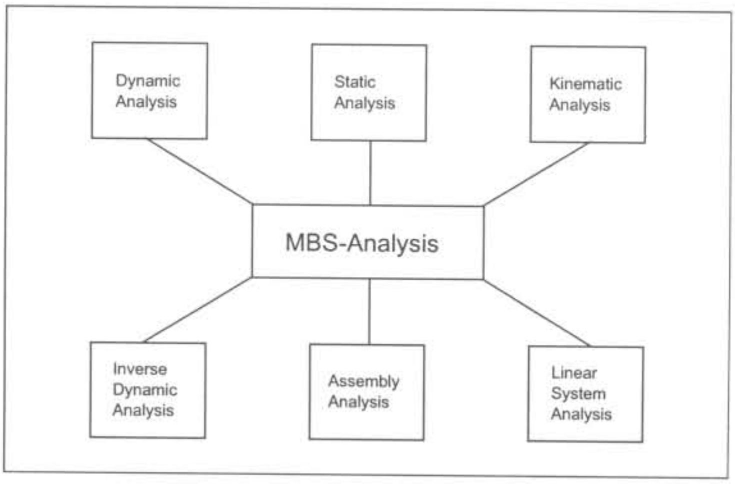


Figure 2.2: MBS: various analyses

to calculate bending or tension stresses. The strain gauges, used for force calculations, must however be positioned where the measured force will have the primary effect on the stress situation.

Forces were sometimes also measured, using a Linear Variable Differential Transformer (LVDT). The displacements of the LVDT were used in conjunction with suspension spring characteristics. The displacements can be differentiated to give velocities. The velocities can be calibrated to forces, using the characteristics of the suspensions shock absorber. More information regarding this subject can be found in [4].

2.2.2 Dynamic Simulation

Presently, the best method to obtain load information for vehicles is to use measurement recording techniques on available prototype vehicles. It would be highly desirable to be able to simulate the vehicle's behaviour over a certain terrain without an actual vehicle. Conle [11] partially solved the problem by using wheel-force recordings as inputs into a DADS vehicle dynamics simulation program. The weak links in the system are tyre models and bushing models. These areas contain components and materials that are highly non-linear. Although progress has been made in these areas, at present the load-time data cannot reliably be derived from full vehicle computer simulations [10].

Gopalakrishnan [16] describes how the load-time history experienced by the vehicle, can be generated using dynamic software codes such as ADAMS. Ideally, the use of a dynamic analysis code makes the measurement process of the prototype vehicles obsolete. A vehicle or its components can thus be designed and analysed without the costly prototype methodology. Gopalakrishnan also reports that the tyre model used in ADAMS needed considerable additional work to predict the suspension loads more accurately.

Dietz [12] reports a procedure wherein a dynamic analysis is used to obtain inputs to a structure. The reported procedure makes use of a finite element method (FEM) and a multi-body system (MBS). The MBS approach makes use of various analysis features (refer to figure 2.2). The MBS method has a great advantage, in obtaining realistic load scenarios, by the use of the finite element method. The determination of time-varying boundary and load conditions can be determined by multi-body simulation. In the case of modelling a locomotive, the dynamic behaviour of the vehicle, the excitation of the track, the vehicle velocity and other operational data, influence these loads. A dynamic load ($y(t)$) can therefore be determined. These loads can be used to calculate fatigue damages of the finite element model (refer to figure 2.3). The finite element model is therefore used in the process of generating loads, as well as using these loads to evaluate the fatigue damage.

Butkunas et al [6] reports the use of a dynamic analysis to establish the inputs to a vehicle. The authors use the measured data of terrain (bump sizes, spacing, etc.) to construct an environment for the use on a dynamic, single degree of freedom system.

2.2.3 Load Spectra

The main influencing parameters on the loading of a vehicle may be defined as usage, structural behaviour (vehicle dynamic properties and design) and operational conditions. Grubisic [17] introduces a technique to determine a

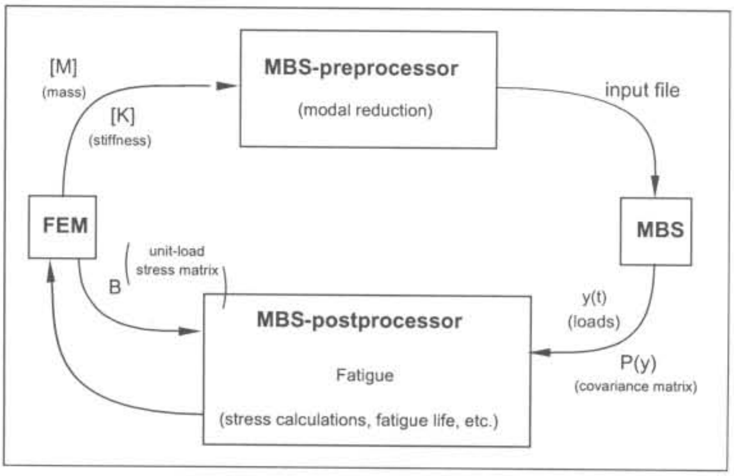


Figure 2.3: FEM-MBS interface

load spectrum of a vehicle for design and testing. The load spectra must fulfil the task of predicting the service life of a vehicle. The main parameters that influence the load spectra are depicted in figure 2.4. Figure 2.5 shows a possible design spectrum. In the figure the characteristics of both loading, as well as material fatigue properties can be represented on a log-log plot of load amplitude versus number of cycles. The loading, is represented as a load spectrum, where-as the material properties are presented as Stress-Life curves. Load spectrum curve (a) may, for arguments sake, represent the loading on an automotive component in off-road conditions (high amplitudes, low number of cycles), where-as curve (b) may represent the same component loading under highway loading conditions (low amplitudes, high number of cycles). The two curves, as well as others in between, represent the scatter in loading on a component. Grubisic concludes that to determine the design spectra for the fatigue evaluation of a vehicle component and assembly the following must be regarded:

- The load spectrum must take into account all possible loading conditions, including extreme values during customer usage which would seldom be achieved.

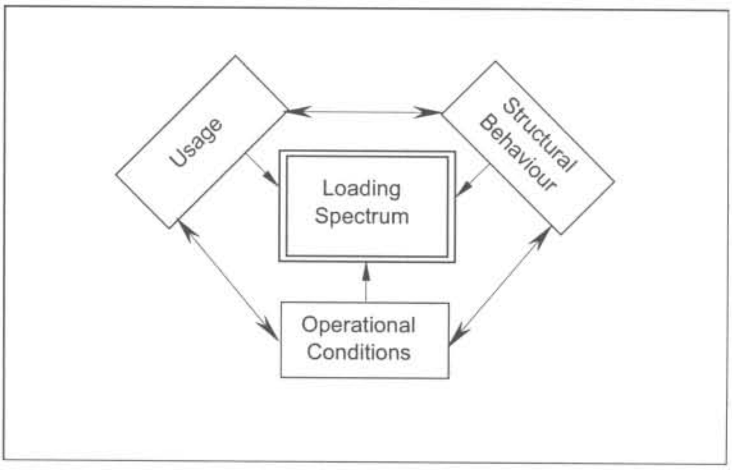


Figure 2.4: Parameters of the loading spectrum

- An appropriate extrapolation of the field measurements can be carried out only if the data for individual loading conditions, originating from the vehicle usage and operational conditions, are separated.

2.2.4 Statistical Domain

Leser et al [23] presents a method to establish fatigue loading histories, using a statistical model. The methods of modelling irregular fatigue loading histories can be divided into two groups, namely counting methods (rainflow etc.) and methods based on correlation theory. The Autoregressive Moving Average (ARMA) model is an example of the statistical correlation theory. The random processes of a ground vehicle travelling on a rough road, are considered to consist of a slowly varying process and a fast varying process. The slow varying process is called the non-stationary mean variation, and the fast varying process is called the stationary random variation. The mean variation is modelled with a Fourier series, while the ARMA model is employed to model the stationary random variation. A concise description of complex loading is achieved, while the overall frequency content and sequential information are statistically preserved.

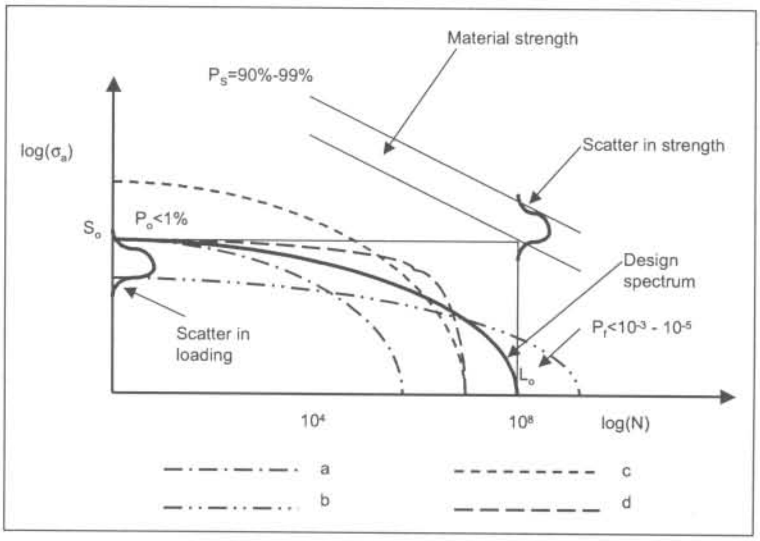


Figure 2.5: Design spectrum of a loading spectrum

2.2.5 Frequency Domain

The frequency domain method of determining the input loads of structure, has become an attractive method on account of the compactness of the data. The time domain stress histories dealt with in engineering can usually not be specified by a formula. The data is represented by a series of values, usually taken at equal time intervals. Time domain data can contain very large amounts of data, that even at this point, where the cost of data space is relatively cheap, can become quite cumbersome to handle. If a structure has to be measured over a long period of time, for instance two years, the use of time domain data can be extremely difficult to store and analyze. Wannenburg [39] employs a method of measuring frequency data in a case study. The frequency domain data is converted to time domain data, to be used in fatigue calculations.

A very important point regarding frequency domain data, is that frequency has no fundamental effect on fatigue life. If a steel cantilever beam is moved up and down between certain limits at a frequency of $10 Hz$ and it breaks after 10 000 cycles, a similar beam excited at $1 Hz$, will also fail after 10 000 cycles. Time domain data can be stored in the frequency domain by using the

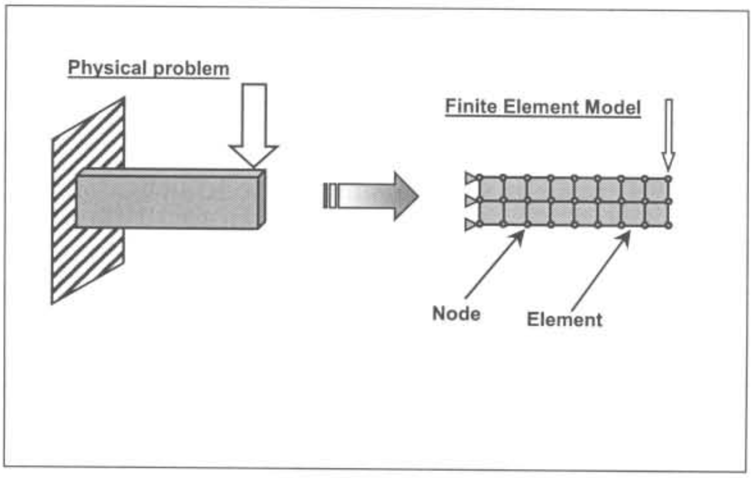


Figure 2.6: Fundamentals of the finite element method

Power Spectral Density (PSD) plot. The PSD format represents the averaged statistical information for each frequency contained in the original time domain data. Using the Fast Fourier Transform (FFT) the ranges ($\Delta\sigma$) are averaged to energies, usually in MPa^2/Hz units, and the number of cycles (n_i) to frequency (Hz). It should therefore be possible to reconstruct a time history from a PSD plot and to carry out a fatigue life prediction. Sherrat [32] explains the method of doing the reconstruction of time domain data, using Inverse Fast Fourier Transform methods. Sherrat mentions the general approach used by Dirlik [13]. The aim of Dirlik's thesis was to produce a general formula predicting the rainflow range distribution from characteristics of the PSD. Random phase angles, in conjunction with statistical methods, were used to create the Dirlik formula. According to Sherrat [32] the agreement between the rainflow ranges, computed by the Dirlik formula, using a PSD plot, and the rainflow ranges computed from time domain data, is excellent. The frequency domain can also be used in the measurement process of vehicles during durability assessments.

2.2.6 Closure

This section discussed the various methods that can be employed to obtain measurements of loads that act on a vehicle structure. The following section discusses the fundamentals of the finite element method (FEM).

2.3 Finite Element Structural Analysis

Finite element procedures are at present widely used in engineering analysis. The procedures are employed extensively in the analysis of structures, heat transfer and fluidflow. Finite element procedures can be employed in virtually every field of engineering analysis. This thesis makes use of the structural capabilities of the finite element method.

2.3.1 Linear Static finite element analysis

The development of the finite element methods for the solution of practical engineering problems began with the advent of the digital computer. The essence of a finite element solution of an engineering problem is that a set of governing algebraic equations is established and solved. By the use of the digital computer, the finite element process could be rendered practical and given general applicability.

The basis of the finite element method for the analysis of solid structures can be summarized in the following steps: The solid structure, representing the real life article, is subdivided in small parts, called *elements*. The elements are assembled through the interconnection at a finite number of points on each element. These points are called *nodes*. The assembly of the elements and nodes is often referred to as a mesh. Within each element we assume a simple general solution to the governing equations. The solution of each element equation is a function of unknown solution values at the nodes. By subdividing the structure in this manner, one can formulate equations for each separate finite element which are then combined to obtain the solution of the whole physical system [7] (refer to figure 2.6). Equation 2.4 indicates the fundamentals of the finite element method. The stiffness matrix, $[K]$, contains the physical geometrical and material properties of the structure. Vector $\{d\}$ contains the displacements of the structure, while the $\{f\}$ vector is the external load that is applied to the model. The displacements (for each node) are solved for a given mathematical model, contained in $[K]$, and the applied loads in $\{f\}$.

$$[K]\{d\} = \{f\} \tag{2.4}$$

where :

$$\begin{aligned} [K] &= \text{Stiffness matrix} \\ \{d\} &= \text{Node displacement vector} \\ \{f\} &= \text{Applied nodal load vector} \end{aligned}$$

Figure 2.7 summarizes the process of finite element analysis. The physical problem typically involves an actual structure subjected to certain loads. The idealization of the physical problem to a mathematical model requires certain assumptions that together lead to differential equations governing the mathematical model. The finite element analysis solves this mathematical model. The finite element analysis will solve only the selected (or assumed) mathematical model [2]. Only physical information contained in the mathematical model can be solved. The analysis can therefore be used only to obtain insight into the physical problem considered, because it is impossible to reproduce, even in the most refined mathematical model, all the information present in the real life situation. The choice of an appropriate mathematical model, governed by boundary conditions, geometry, and above all, loads, is therefore of critical importance. Rahman [29] proposes a methodology for the finite element analysis in a systematic fashion with a view to identifying and controlling any error or uncertainty that may be encountered during the analysis process.

The use of non-linear finite element methods does exist, but is seldom used

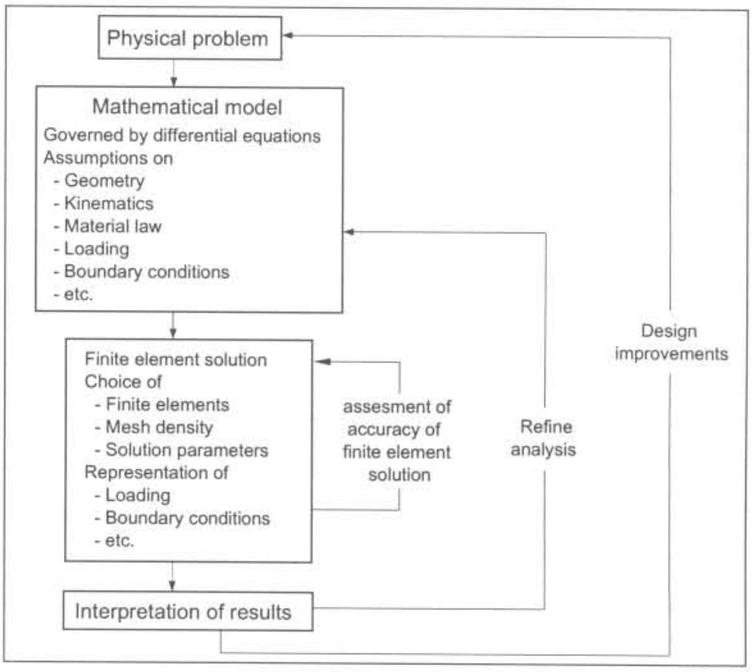


Figure 2.7: The process of finite element analysis

for fatigue analysis. Khatib-Shahidi et al [21] does however make use of a non-linear static analysis. The resulting stresses encountered in the non-linear analysis, according to the paper, are however lower than the yield stress of the material. The most frequently used structural finite element analysis is the linear static analysis. The linear static analysis assumes a linear material behaviour when a load is applied to the structure. A static load, which does not change with time, is applied to the structure in this analysis. In the analysis of structures that would fail by fatigue, the nominal stresses are mostly much lower than the yield strength of the material (otherwise other failure modes would supersede the fatigue mode). The linear static finite element analyses are therefore mostly sufficient.

2.3.2 Linear Dynamic finite element analysis

When a structure is subjected to a load that varies with time, its corresponding response will also vary with time. In a linear static analysis, the loads are assumed to be static, and therefore the response is static and proportional to the structure stiffness and applied loads. If a load is applied at a frequency lower than one third of the frequency of the lowest natural frequency of the structure, the analysis can be done assuming static conditions [7].

However, when the applied loading varies rapidly, the solution techniques must take inertial effects due to damping and material mass in consideration. Several different procedures exist that solve these dynamic analyses. Equation 2.5 is the general formulation of the solution that must be solved during a dynamic analysis. This equation is the set of differential equations of motion in matrix form for the dynamic response of any given structure modelled with a finite number of degrees-of-freedom.

$$\underbrace{[M]\{\ddot{D}\}}_{inertial} + \underbrace{[C]\{\dot{D}\}}_{damping} + \underbrace{[K]\{d\}}_{stiffness} = \{f_t\} \tag{2.5}$$

where :

$$\begin{aligned}
[M] &= \text{Mass matrix} \\
[C] &= \text{Damping matrix} \\
[K] &= \text{Stiffness matrix} \\
\{\ddot{D}\} &= \text{Node acceleration vector} \\
\{\dot{D}\} &= \text{Node velocity vector} \\
\{d\} &= \text{Node displacement vector} \\
\{f_t\} &= \text{Applied time varying nodal load vector}
\end{aligned}$$

Three basic methods exist with which a structure can be dynamically analysed:

- *Eigenvalue Analysis*: The eigenvalue analysis calculates the natural frequencies of the structure. The eigenvalue problem derives from equation 2.5 after zeroing the damping coefficients and applied forces. The corresponding mode shapes for each frequency can subsequently be calculated.

- *Frequency Response Analysis*: The frequency response analysis calculates the steady state response of a structure that is subjected to harmonic forces at a given frequency. A harmonic load with a frequency equal to the natural frequency will produce infinite displacement responses if no damping is specified. Determining the amount of damping in a structure is a very difficult process. Most structures are however lightly damped, and can be simplified by neglecting the damping, bearing in mind the natural frequencies.

- *Transient Response Analysis*: If the input loading function is not harmonic, but an arbitrary dependent function, a transient response dynamic analysis must be performed. One method to solve a random load is to use a *direct integration method*. Equation 2.5 is solved at discrete time intervals (Δt) apart. The direct integration technique is based on the assumption that displacements, velocities and accelerations within each time interval vary. A second approach to the transient response analysis is the *modal superposition* method. The basis of this approach is an assumption that superposition of the mode shapes corresponding to the lower natural frequencies adequately represents the dynamic response of the structure. The complete response is found by the summation of correct fractions of the low frequency mode shapes.

The linear dynamic analysis for a random input signal is an expensive and difficult finite element method, due to the fact that hundreds of static solutions must be solved. Most literature concerning random load vectors, prefers the use of a static finite element solution. The aim of this thesis is to explain the use of a linear static solution for a random time varying load.

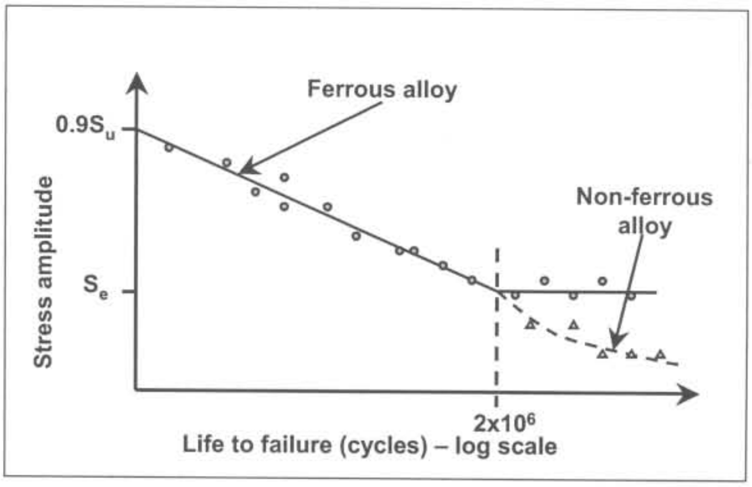


Figure 2.8: The Wöhler or SN curve

2.3.3 Closure

This section explained the fundamentals of the finite element method. Measurements, finite element analyses and fatigue calculations form the building blocks of a *fatigue assessment* of a structure. The combination of these tools enables the engineer to asses a vehicle for fatigue loads. The following section will discuss fundamentals of fatigue and durability calculations.

2.4 Fundamentals of Fatigue and Durability

The load and structural information acquired in accordance with the previous two sections must be analysed, using fatigue principles, to determine the appropriate fatigue damage. This section explains the fundamentals of these theories.

2.4.1 Fatigue analysis

Stress-life

The stress-life (SN) method was the first attempt to understand and quantify metal fatigue. August Wöhler conducted experiments from approximately 1850 to 1875 to establish a safe alternating stress below which failure would not occur. It has since been the standard design method for the past 150 years. The stress-life approach to fatigue life prediction is still widely used in design applications where the applied stress is primarily within the elastic range of the material and the fatigue life of the component is long. The stress-life method does however not work very well for applications where a low-cycle fatigue life is experienced.

The fundamentals of the stress-life approach can be summed up in the Wöhler or SN diagram (refer to [1]). The Wöhler diagram plots stress amplitudes σ_a (alternating stress) versus cycles to fatigue (N). The data for the Wöhler diagram is generated using laboratory tests. Due to the extensive work that has been done in the past 100 years, fatigue data relevant to the stress-life approach is fairly common.

Certain materials, mostly ferrous steels, have a fatigue limit stress (S_e), which is a stress amplitude level below which the material has an 'infinite' life-time. Most nonferrous alloys, for example aluminium, have a SN line with a continuous slope. For *both* these SN curves, a pseudo-endurance limit or fatigue stress is defined at a chosen number of cycles (for example 2×10^6 cycles). It should be noted that certain empirical relationships between the fatigue properties and the monotonic tension and hardness properties have been determined for steel. The fatigue limit stress (S_e) can be related to the ultimate strength (S_u) in the following way: $S_e = 0.5S_u$. A designer can therefore determine the fatigue material properties (of ferrous steel) without doing expensive laboratory tests.

Instead of the graphic approach, a power relationship can be used to estimate the fatigue life of a material (refer to equation 2.6).

$$\sigma_a = C\sigma_f(N)^b \tag{2.6}$$

where :

σ_a = amplitude stress
C = constant relating to the SN – curve (fatigue ductility coefficient)
σ_f = fracture stress
N = number of cycles to failure
b = Basquin's fatigue strenght exponent

In equation 2.6 the constant C represents the value where the SN curve intersects the y-axis of the Wöhler diagram. The exponent b is the gradient of the the SN curve. These two values can be determined using the empirical relationships previously mentioned, or through data derived from test specimens in a laboratory. The constant C is sometimes replaced by the true fracture stress (σ_f). The true fracture stress is an *estimate* of the stress amplitude at either 1 or $\frac{1}{4}$ cycles. The following relationships and definitions are used when discussing alternating stresses. Also refer to figure 2.9.

$$\begin{aligned}
\Delta\sigma &= \sigma_{max} - \sigma_{min} &&= \textit{stress range} \\
\sigma_a &= \tfrac{1}{2}(\sigma_{max} - \sigma_{min}) &&= \textit{stress amplitude} \\
\sigma_m &= \tfrac{1}{2}(\sigma_{max} + \sigma_{min}) &&= \textit{mean stress}
\end{aligned}$$

Strain-life

The strain-life method was developed in the 1950's, during work that was done to establish the quantified relationships between plastic strain and fatigue life. Early research showed that fatigue damage is dependent on deformation or strain. When a specimen is subjected to low load levels, the stresses and strains are linearly related. However, in the low cycle fatigue domain (cycles $< 10^3$), the cyclic stress-strain response and the material behaviour are best modelled

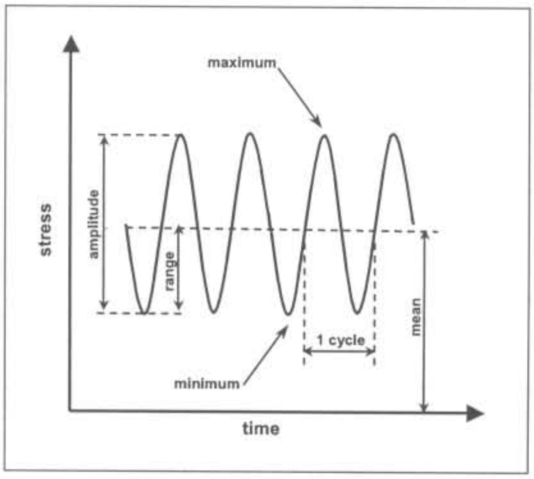


Figure 2.9: Terminology for alternating stress

using strain-life techniques. At long fatigue lives, where the plastic strain is negligible, the stress-life and strain-life approaches are essentially the same. Bannantine [1] explains the fundamentals of the strain-life approach. The strain-life equation (refer to equation 2.7) is used to predict fatigue life, using four empirical constants. A graphical representation of equation 2.7 illustrates how the strain-life methodology approaches the stress-life approach at low amplitude loading conditions (refer to figure 2.10).

$$\frac{\Delta\epsilon_p}{2} = \underbrace{\frac{\sigma'_f}{E}(2N_f)^b}_{elastic} + \underbrace{\epsilon'_f(2N_f)^c}_{plastic} \tag{2.7}$$

where :

σ'_f = fatigue strenght coefficient

$\Delta\epsilon_p/2$ = plastic strain amplitude

$2N_f$ = number of reversals to failure

b = Basquin's fatigue strenght exponent

E = Young's elasticity modulus

ϵ'_f = fatigue ductility coefficient

c = fatigue ductility exponent

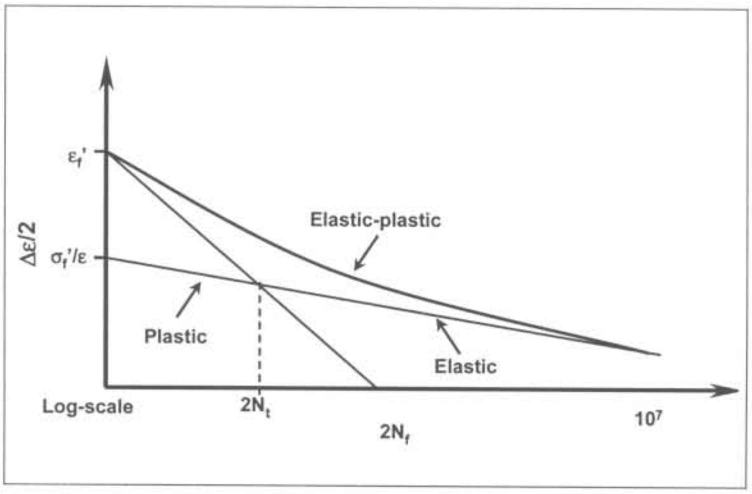


Figure 2.10: Graphical representation of the strain-life equation

Neuber's rule can be used in the strain-life approach to determine the notch root stresses and strains (local stresses and strains) at stress concentrations, for instance a hole in a plate (refer to equation 2.8). Conle et. al [10] make use of the stress-strain behaviour of a material to measure fatigue life. Conle reports that the uni-axial parameter and equivalent stress parameters does not work well under multi-axial conditions. The use of strain-life procedures is therefore more adequate for complex multi-axial loading conditions.

$$K_t = \sqrt{K_\sigma K_\epsilon} \tag{2.8}$$

where :

K_t = theoretical stress concentration factor

K_σ = local stress concentration factor

K_ϵ = local strain concentration factor

Multi-axial fatigue theory is still the subject of ongoing research. The

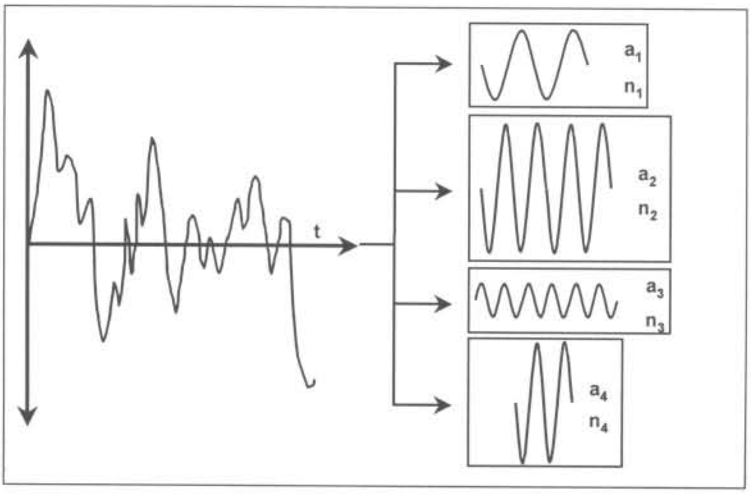


Figure 2.11: Primary objective of cycle counting

strain-life approach is mostly applicable to the low cycle fatigue domain. Vehicle structures are however designed to withstand many millions of load cycles. For these reasons, the stress-life approach was the preferable choice for durability analyses in this thesis.

Fracture Mechanics

Fracture mechanics approaches are used to estimate the crack propagation life due to fatigue loads.

These methods require that an initial crack size be known or assumed. Some components with imperfections, such as welding porosities, inclusions etc., have an initial crack size that can be determined. The assumed initial crack size may be determined also by the requirements of the designer. The linear elastic fracture mechanics approach (LEFM) is the most commonly used method in this field.

The stress intensity factor, K, is used to relate the local stress magnitude at the crack tip, using remote nominal stresses. Refer to equation 2.9. This factor depends on loading, crack size, crack shape and geometric boundaries.

$$K = f(g)\sigma_n\sqrt{\pi a} \tag{2.9}$$

where :

σ_n = remote nominal stress applied to the component

a = crack length

$f(g)$ = correction factor (depends on geometry, loading and crack shape)

The LEFM approach aims to calculate the crack growth in a component. The most current applications of the LEFM concepts describe this crack growth in the region where the crack growth is stable. The Paris equation (eq. 2.10) is the most widely accepted curve fit for this region.

$$\frac{da}{dN} = C(\Delta K)^m \tag{2.10}$$

where :

C = material constant

m = material constant

ΔK = $K_{max} - K_{min}$ (stress intensity range)

Multi-axial loading

Engineering components are often subjected to complex loading conditions. A component, for example a vehicle's rear axle, is subjected to a combination of bending and torsion. Complex stress states, stress states in which the three principal stresses are non-proportional and/or whose directions change during a loading cycle, very often occur at geometric discontinuities, such as notches. However, metal fatigue due to this phenomenon is still the subject of ongoing basic research [1]. Although there has been no consensus yet about the best approach among the various methods proposed, the need to use multi-axial fatigue methods for non-proportional loading conditions has been recognized by the significant improvement in fatigue life prediction accuracy these analyses yield over traditional uniaxial methods [8]. Many different parameters have been suggested to correlate loading and fatigue life of multi-axial stressed components. All of these parameters can be subsumed under the term *critical plane approach*. The critical plane approach models initiation and growth of small fatigue cracks by analyzing different potential crack directions at one structural location, selecting the most damaging direction as crack initiation plane [14]. Chu [8] describes a procedure to solve multi-axial fatigue problems using (a) a three-dimensional cyclic stress-strain model, (b) the critical plane approach, (c) a bi-axial damage criterion for better fatigue damage evaluation and (d) a multi-axial Neuber equivalencing technique used to estimate multi-axial stress and strain history of plastically deformed notch areas.

Due to the limited amount of experience at present, Dreßler et. al. [14] therefore proposes a procedure that not necessarily predicts fatigue life very accurately, but rather uses numerical procedures to evaluate different design alternatives. Due to the fact that multi-axial fatigue is still the subject of ongoing research, and has not yet been resolved, the testing of components for this type of loading is still essential.

2.4.2 Cycle counting and Damage accumulation

Cycle counting

To predict the fatigue life of a component that is subjected to a *random* load, the variable time history must be reduced from a complex history to a number of events that can be compared to available *amplitude* history (refer to figure 2.11). This process of reducing the complex load history is termed *cycle counting* [1]. The proper way of counting the loads is the rainflow cycle counting method. Matsuishi and Endo first proposed it in 1969 [25]. A number of variations of the rainflow cycle counting method has been proposed and it has subsequently been standardized by the ASTM, SAE and AFNOR [14]. The

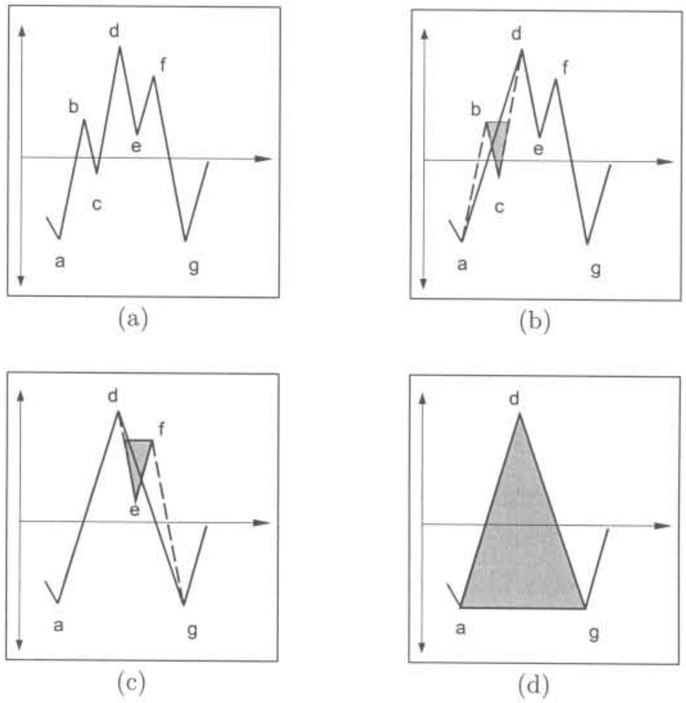


Figure 2.12: Rainflow counting: ASTM standard

ASTM standard for rainflow counting (also called range-pair-range counting) is shown in figure 2.12. The recommended practice is also called the four point method. It should be noted that various cycle counting techniques are equally well regarded.

The Markov matrix is used to store the data that is obtained from the rainflow matrix [12]. The Markov matrix stores the total number of hysteresis loops (or cycles) for all the rain flow ranges (refer to figure 2.13). In this figure, the cycle counting method has counted four cycles that go *from* stress level 70 *to* stress level 10. Dreßler et al [14] also describes methods to do rainflow counting on multi-axial loading processes, as well as rainflow reconstruction. Olagnon [26] discusses the practical implementation of the statistical properties of the rainflow counting method established by Rychlik. Sherrat [32] describes various methods when using the rainflow counting method with frequency domain data. This method makes use of computer modelling of the inverse

FROM \ TO	10	20	30	40	50	60	70	80	90
10		2							
20		3	7				6		
30					5				55
40			2						
50					75				
60								91	
70	4					7			
80							47		4
90				4					

Figure 2.13: An example of a Markov matrix

Fourier transform first, and then the establishment of a theoretical link between the Power Spectral Density diagrams and the rainflow ranges. However, this method is still a topic of research. Dietz [12] makes use of another method to do cycle counting. His method maps an equivalent stochastic load signal to an equivalent harmonic process, using *cumulative frequency distributions*. The rainflow counting (or range-pair-range) method described in figure 2.12 is used in this thesis to analyze the measured data.

Damage accumulation

The data contained in the Markov matrix can be used to calculate the damage that the loads cause. There are two distinctly different approaches used when dealing with cumulative fatigue damage during the initiation period and the propagation period. During the crack propagation period, the damage can be related to a measurable crack length. However, during the initiation stages, the damage of the component can be detected only in a highly controlled laboratory environment (slip bands, micro-cracks etc.). Because of the difficulty of measuring the damage during the initiation stage, the damage summing methods are *empirical* in nature [1]. The linear damage rule was first proposed by Palmgren in 1924 and then further developed by Miner in 1945. The linear damage rule is usually referred to as Miner's rule in the literature.

The damage fraction, D_i, is defined as the fraction of life used up by an events or a series of events (refer to equation 2.11). The number of cycles counted by the rainflow counting method is n_i, at a stress level S. The number of fatigue life in cycles, N_i, is obtained from the Wöhler diagram (figure 2.8) at stress level S.

$$D_i = \frac{n_i}{N_i} \tag{2.11}$$

where :

D_i = damage fraction

n_i = number of load cycles at stress level S

N_i = number cycles to failure for stress level S

The total damage, D, is the sum of all the damage fractions (refer to equation 2.12). Failure of the component is assumed to occur when the summation of damage fractions equals one (1). Considerable test data has been generated in an attempt to verify Miner's rule. Most of the results tend to fall between 0.5 and 2.0. In most cases the average value is close to Miner's proposed value of one. Non-linear damage theories have been proposed which attempt to overcome the shortcomings of Miner's rule. Equation 2.13 shows an example of a nonlinear equation. However, nonlinear damage rules do not give significantly more reliable predictions. For most situations, where there is a pseudo-random load history, Miner's rule is adequate.

$$D = \sum_{i=1}^{n} \left(\frac{n_i}{N_i}\right) \tag{2.12}$$

$$D_i = \left(\frac{n_i}{N_i}\right)^P \tag{2.13}$$

Various damages, calculated for various measured loads using equation 2.12, can be added to give a more realistic damage regarding the total life cycle of a vehicle. This is an important concept due to the cost involved in the measurement of loads on a vehicle. Various articles have been published that address this issue. Beamgard et al [3] reports a method of establishing durability test objectives which accurately reflect field usage. Three basic inputs are required for this procedure:

1. Field Usage Data: This input defines the distribution of field usage in terms of cargo and passenger loading, mileage over various types of terrain, etc. This data is obtained from interviews of vehicle owners.

2. Field Fatigue Damage Data: This data is obtained from a fatigue damage analysis of road-load-strain data acquired during a large sampling of public roads.

3. Durability Fatigue Damage Data: This input defines the fatigue damage incurred in the proving ground durability events.

Inputs (1) and (2) are used to calculate the damage incurred by each customer. This data is used to determine the distribution of the customer damage. Input (3) is consequently used to match the customer damage distribution with an equivalent durability damage. It should be noted that the inputs obtained for this analysis took extensive research that was done over a three year period. This approach is therefore mostly applicable to large corporations with a large number of vehicles (and customers) that can be evaluated. Slavik and Wannenburg [34] also employs a similar method to evaluated vehicle failures due to fatigue. The article reports interesting procedures, using statistical methods, to predict the failure of components of vehicles. The same three basic inputs mentioned by Beamgard are used in the analysis. The obtained input data is also converted to damage values that represent the various roads and users. These damage values are subsequently manipulated using statistical methods and Monte Carlo simulations to obtain realistic damage values that can be used. Wannenburg and Slavik used the damage values to predict failures in the field, while Beamgard used the damage calculations to calibrate durability

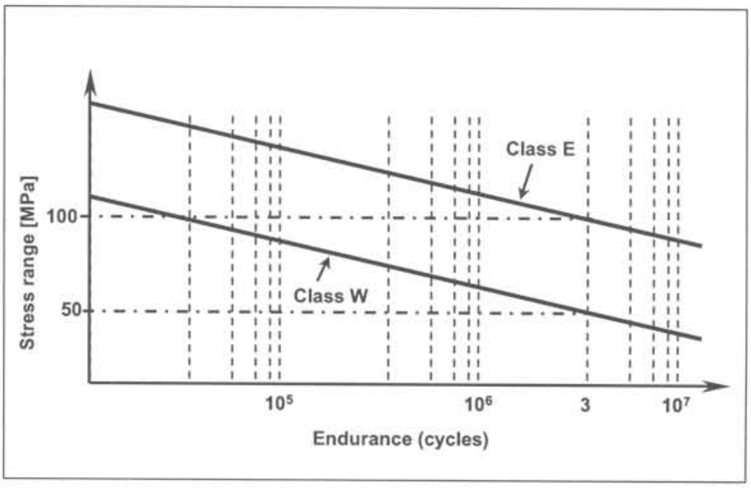


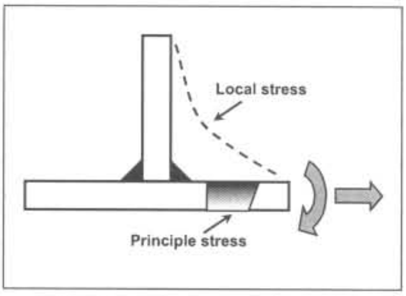
Figure 2.14: BS fatigue SN curves for welds

test tracks as well as to identify potential areas where components must be redesigned.

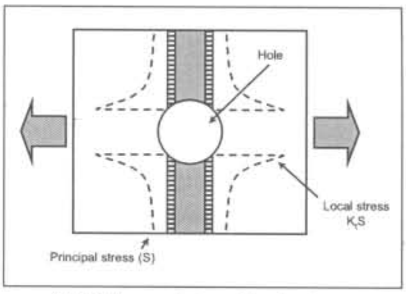
2.4.3 Welds

Welded joints

The fatigue life of a welded joint is almost always lower than the fatigue life of the parent material. The fatigue evaluation of welded joints in vehicles is therefore of utmost importance. Many codes exist that address the fatigue design of components (see references [40], [15] and [9]). The fatigue codes describe different classes of welds. The class of weld determines the fatigue life of the welded joint. For example, according to the BS steel-code [40], a weld in class W would have a fatigue life of 3×10^6 cycles if a load with a stress range of 50 MPa is applied. A weld in class E could however withstand a stress range of 100 MPa resulting in the same fatigue life (refer to figure 2.14). Gurney [18] delivered various publications regarding the fatigue design of welds. The use of fatigue design codes is consequently explained, making use of Gurney's article [18]. The fatigue design codes make use of Stress-Life (SN) diagram. The fatigue weld SN-diagram looks, and is, very similar to the Wöhler curve



(a) Explanation of nominal stress



(b) Stress intensity factor

Figure 2.15: Weld stress interpretation

introduced in the stress-life methodology. The reader should however note that theory of these curves is closely tied in with fracture mechanics. The assumption is made that the whole of the fatigue life of a welded joint consists of the propagation of a pre-existing small defect. The ECCS code [15] considers all of the weld classes to have slope (m) of -3. The BS-code does however have 2 classes with less steep slopes, but essentially all the other joint classes also have a SN-slope of -3. This is the same as m exponent used in the Paris equation (2.10) to calculate fatigue crack growth (refer to figure 2.14).

The stress range stresses referred to in figure 2.14 are to be considered as nominal stresses. Thus, the combined effect of bending, shear, etc. should be considered. The design stresses quoted can therefore be regarded as the nominal stresses adjacent to the detail under considerations, usually the parent material at the weld toe. The codes make the assumption that, in an as-welded structure, high tensile residual stresses are liable to exist as points of fatigue crack formation. However, stresses at weld joints that are situated in regions of geometric stress concentrations, must be factored to take the higher stresses into account. If a nominal stress (S) is applied to a T-joint with a hole in, the factored nominal stress, K_tS, must be considered using the stress curves (refer to figure 2.15(b)).

Jones et al [20] discusses the method of using finite element models to analyze the fatigue failures of welds on a structure. The paper makes use of the British Standards codes [40], combined with the above mentioned procedures.

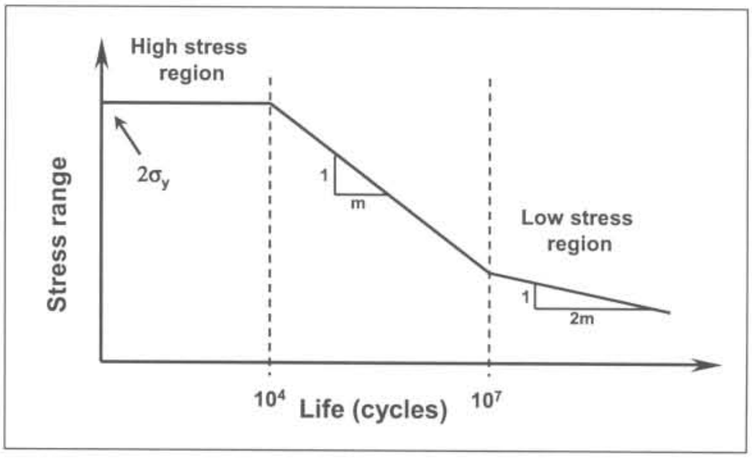


Figure 2.16: A hot-spot SN curve

Stephens [37] also makes use of the above mentioned method. However, Stephens' paper also makes use of the United Kingdom Department of Energy's research on offshore structural welds (UKOSRP) [30]. The research uses the 'hot-spot' stresses on and near the weld to define an additional fatigue life region (refer to figure 2.16 [38]).

Spot welds

Rui and Borsos [31] describes a method for life prediction of multi-spot-welded structures. The fatigue strength of spot welds in a multi-spot-welded structures is one of the key issues of concern for achieving structural durability and optimum design in the vehicle industry. Three failure mechanism criteria have been proposed to predict spot weld failures: (a) load range criteria, (b) strain range criteria and (c) stress intensity factor (K) criteria. Although the load range criteria is the easiest method to use, the method requires adequate experimental data. The other two criteria can however not be used in conjunction with the *simplified* finite element methods. A spot weld can be modelled in detail, using solid elements to connect the two plates that are spot welded, though this method is impractical if thousands of spot welds occur on a vehicle. The accepted method of modelling a spot weld is the use of a

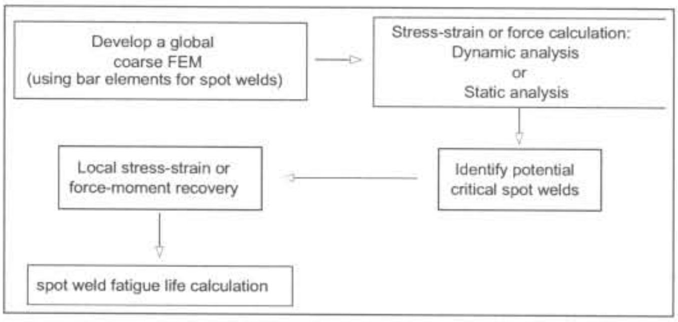


Figure 2.17: A flow chart of spot-weld fatigue life prediction

bar element[1]. Fatigue life calculations can however not be done on the bar element itself. Rather, the 'global' stresses or strains, typically two times the diameter of the spot weld, away from the centre of the spot weld is used. The general methodology of analyzing a multi-spot-welded structure is shown in figure 2.17.

Another method to address the problem of life prediction of shear spot welds is through the use of fracture mechanics. Smith and Cooper [35] noted that a spot weld could be considered to be a circular solid surrounded by a deep circumferential crack, which, when loaded in a combination of Mode I and Mode II, would grow a branch crack in the direction of maximum local Mode I. The forces (and moments) that the bar element experiences are used to calculate the structural or nominal stresses in the weld nugget. These stresses can then be used to calculate the fatigue strength of the spot-weld [24].

2.4.4 Closure

This section discussed the fundamentals of fatigue calculations. The previous three sections presented the building blocks or tools of a fatigue assessment. The combination of measurements, FE analysis and fatigue calculations, enables an engineer to asses a structure for fatigue loads. The following section discusses various strategies developed, using the previously mentioned tools, to asses structures subjected to fatigue loads.

[1]a bar element is structural finite element use to represent a beam

2.5 Integration of Measurements and Analysis

This section provides a few examples of techniques that have been developed to determine a fatigue damage prediction. These strategies incorporate the various methods explained in the previous sections to determine a solution to a structural fatigue problem.

2.5.1 Remote Parameter Analysis

Poutney and Dakin [28] describes a method to integrate finite element analysis and simulation or road test data for durability life prediction. The method was termed the Remote Parameter Analysis (RPA) method. Loads that are applied to automotive components are usually very difficult to determine. The heart of the RPA method is the ability to back-calculate all the free-body component forces, by making use of remotely measured parameters, usually strain. The Remote Parameter Analysis provides the following solution to the determination of input loading:

1. Develop a free body diagram of the component.

2. Build a finite element model. Constraints and unit loads are applied to the model in accordance with the free body diagram determined in step 1.

3. Determine the so-called 'Load2Gauge' matrix at stress concentration areas. This matrix shows direct relationship between the applied unit loads and the resulting stresses/strains on the finite element model.

4. The finite element model is now verified by selecting strain gauge positions on the component (using the finite element results). The component is then tested with known loads, relating to the finite element model, and the stresses/strains are measured. If needed, the finite element model is then modified to achieve a high confidence level.

5. Record in-service strains on a prototype vehicle.

6. The 'Load2Gauge' matrix is used to convert the recorded in-service strains to input loads as a function of time.

7. The most damaging load histories are selected. These loads are then used to calculate stresses and strains, using the 'Load2Gauge' matrix, without performing a finite element analysis. The resulting information is then used to perform durability calculations on the component structure.

Poutney describes a methodology to design a complex, fatigue loaded structure, using the finite element method as well as actual in-service strain measurements. However, the post-processing of the fatigue data (derived from the calculated input loads) could be tedious work, as well as difficult to evaluate.

2.5.2 Body-structure durability analysis

Kuo and Kelkar [22] describes a method developed by Ford engineers to predict structural life of a vehicle before prototypes are built. The method employed by Kuo and Kelkar is very similar to the Remote Parameter Analysis method described by Poutney (also a Ford engineer). The methodology follows the following four steps:

1. Identify relative stress sensitivity, using a finite element model and unit loads.

2. Identify critical load paths.

3. Identify critical road events.

4. Compute fatigue life.

Kuo and Kelkar does however use its method on a full body system, rather than on components taken out of the vehicle. The difficulty of determining the input loading and boundary conditions on a cut-out component is therefore eliminated.

2.5.3 Durability testing

A test spectrum must be different in comparison to the design spectrum due to the demands for a short test time and an economical test procedure. Grubisic [17] also proposed various methods to determine test spectrums. The tests of components should, as far as possible, be accelerated and to some extent simplified (refer to figure 2.4). Curves c and d indicate various load spectrums that can be implemented on a vehicle structure. The following general conditions must be regarded:

- Tests should be accelerated by adjustment of the load spectrum in only the medium and high load levels and omission of non-damaging high-cycle, low-intensity loads, predominantly originating from operational conditions during straight driving over smooth roads. The acceleration by increasing spectrum maximum loads should be avoided.

- It is of decisive importance that the deformations of tested components in a test facility correspond to the deformations under service loading conditions. Adjustments of the test loading must be approved by calibration.

- To meet reliability requirements, several durability tests should be carried out. Using a 'risk-factor', based on a statistical approach, the test requirements could be determined.

2.5.4 Computational fatigue life prediction

Stephens et al [37] illustrates a variable amplitude computational fatigue life prediction method for the use of the ground vehicle industry. This method is especially useful in the prototype iteration/optimization design stage of a vehicle, and can be used with welded or non-welded components. The welded components were evaluated using the hot-spot stress approach developed for off-shore structures (see sub-section 2.4.3). For non-welded notched components, the method incorporates the local notch strain approach. The loads

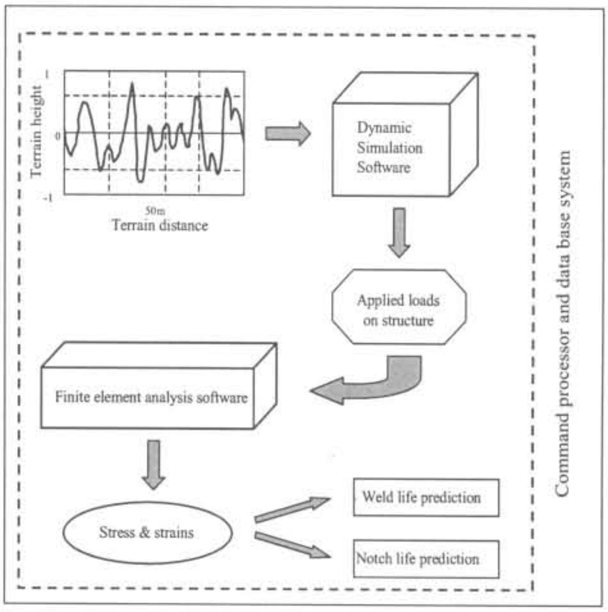


Figure 2.18: Computational fatigue life prediction process

on the vehicle structure were determined using a dynamic simulation software package (DADS). A vehicle terrain profile was selected and used as input to the dynamic simulation software. The resulting forces are therefore calculated using rigid body dynamics and flexible body dynamics. A finite element analysis program (ANSYS) was used to determine stresses and strains using the loads generated by the dynamic simulation software. The resulting stresses and strains are then used to calculated fatigue damage, using either the weld or non-weld methodology. Rainflow counting and linear damage accumulation along with specific material or weld classification properties are incorporated. The interactions between the various calculation phases are managed by a database and command processor that accumulates information, launches application software (like DADS and ANSYS) and stores the resulting data. This software enables the design engineer accurately to control the various processes and resulting data that is generated. Figure 2.18 illustrates the basic principles of the methodology developed by Stephens et al.

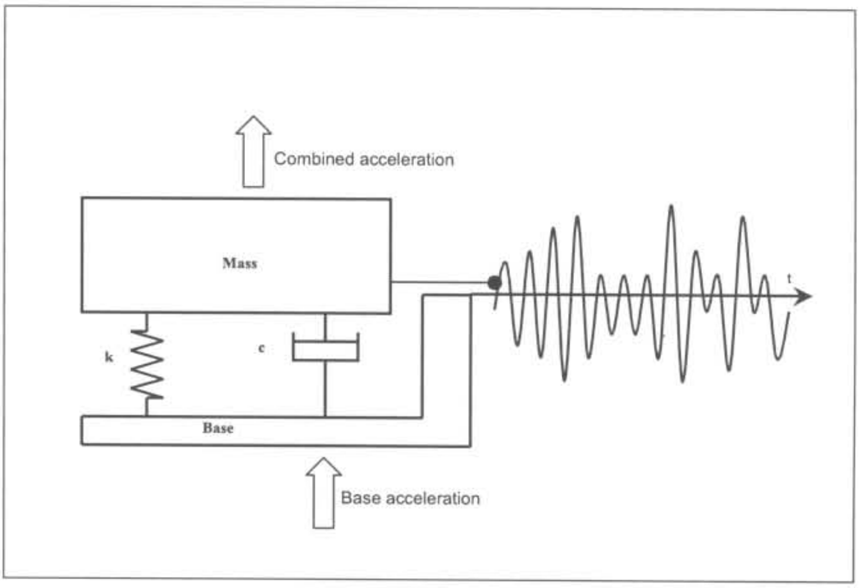


Figure 2.19: Principal sketch of a shock response spectrum

The technique described by Stephens can be very useful during the initial development stages of a vehicle where very little is known about the fatigue loads. The design engineer can acquire a good intuition of the performance of the structure. However, the article does not compare the loads and resulting durability predictions with real life results. As previously mentioned in section 2.2.2, dynamic simulations are still relatively unreliable in determining accurate, realistic random dynamic fatigue loads for ground vehicle structures.

2.5.5 Fatigue assessment through response spectrum methods

Olofsson et al [27] published an exciting paper discussing a methodology to determine fatigue damage caused due to dynamic vibrations. A road vehicle experiences mechanical vibrations due to road surface irregularities. These vibrations may lead to mechanical damage of at least two kinds: (a) the ultimate strength is exceeded as a result of an isolated shock, and (b) fatigue in the material caused by a large number of load cycles. Olofsson proposes that a Shock Response Spectrum (SRS) analysis could be the appropriate method. The SRS is constructed in the following way. The vibration to be described is theoretically applied to a number of single degree of freedom systems. The maximum response for each system is obtained and then defined as the Shock Response for the frequency and damping factor (see figure 2.19). The more likely reason for mechanical damage to a road vehicle is fatigue damage caused by a large number of cycles. Olofsson describes the Fatigue-Damage Response Spectrum (FDRS) analysis to establish a test sequence for the analysis of structures. The FDRS analysis is carried out in the following steps.

1. From measured acceleration data, the response is obtained from objects with different dynamic properties, modelled with single degree of freedom mechanical systems. This can be done by using fast-Fourier transformations (FFT). The result is a number of response time histories, one for each eigenfrequency or damping factor.

2. From each response time history, the resulting fatigue damage is determined. This is accomplished by combining a stress level count with material properties (Wöhler curve) using Miner's law. The result is a damage for each combination of dynamic properties.

3. The Fatigue-Damage Response Spectrum is constructed as a diagram showing the fatigue-damage measure as a function of eigenfrequency. If damping factors are used it will contain one curve for each damping factor (refer to figure 2.20).

2.5.6 Establishment of input loading for fatigue load structures

Wannenburg [39] published a doctoral thesis regarding the establishment of the fatigue loads that affect transport and vehicle structures. The author aims

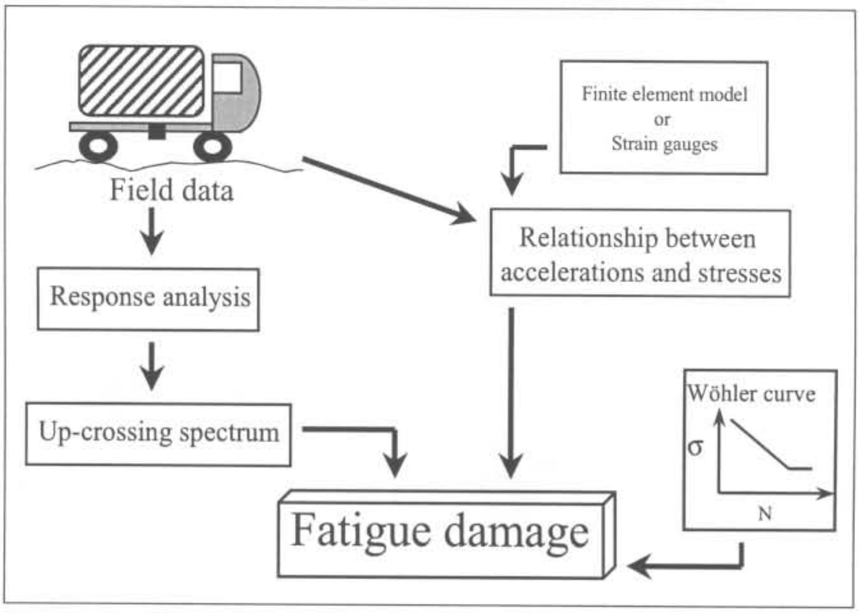


Figure 2.20: Principles of the Fatigue-Damage Response Spectrum method

to develop a Grand Unified Theory (GUT) for fatigue assessment. Wannenburg argues that defective structural design is caused mostly by insufficient knowledge regarding the input of fatigue loads (also refer to Dreßler [14]). Wannenburg's Grand Unified Theory can best be summed up by means of figure 2.21[2]. The left part of the flowchart illustrates the various methods employed by the author to obtain initial input loading. The centre of the flowchart shows the various methodologies developed by the author to process the initial input loads into a well-defined fatigue load. The right part illustrates how the fatigue load can be used for various assessment techniques, for instance design loads, failure prediction and test requirements.

[2]With permission: J Wannenburg

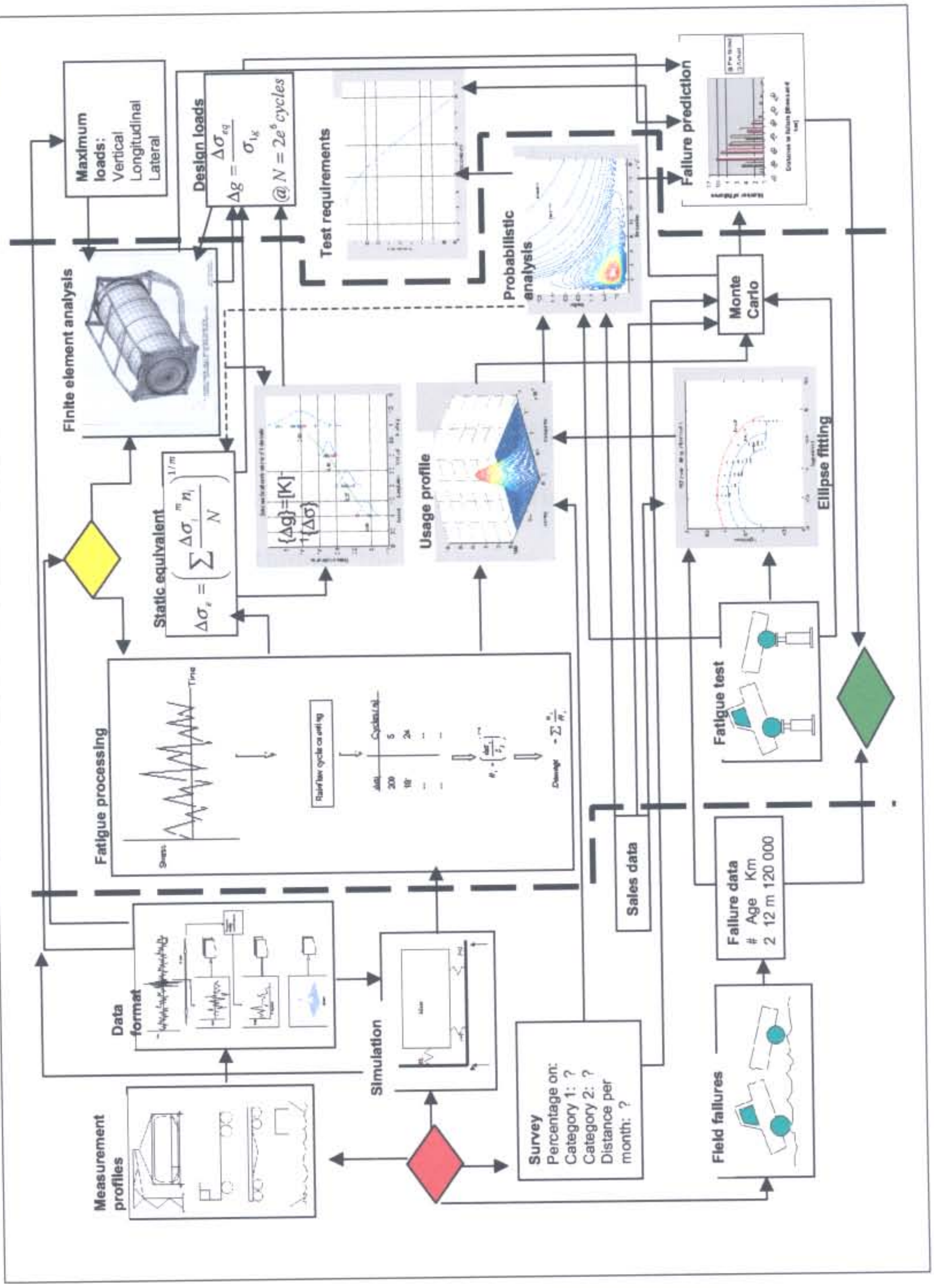


Figure 2.21: Wannenburg's Grand Unified Theory

2.6 Closure

This chapter presented the current theories and practices used by engineers in solving structural fatigue problems. The chapter is sub-divided in sections that follow the various stages of solving a fatigue damage problem. These stages are:

- the determination of input loading.
- the use of finite element structural analysis.
- the fundamentals of fatigue and durability.
- the integration of measurements and various analyses.

This thesis explains, through the use various case-studies, the methodology of the Fatigue Static Equivalent Load (FESL) strategy. The FESL method is very similar to the RPA method that was mentioned previously. The FESL method does have the advantage of simplifying a complex load time history into a *single* load. The following chapter will present a formulation of the Fatigue Equivalent Static Load methodology.