3. Cyclic Bending Theoretical and Experimental History

3.1 Introduction

The mechanism for energy dissipation by means of plastically deforming metals is commonly used as a means of irreversibly absorbing kinetic energy, in one or another form of motion, and converting it to heat. Many of these devices designed to absorb impact energy are common structural elements such as bars, tubes, wires and frames. The performance prediction of these devices as deforming structures involves taking into account large geometrical changes, strain hardening, strain rate sensitivity and modes of deformation. Oversimplifying these assumptions can lead to inaccurate predictions of performance. In this chapter past research conducted in this field is investigated.

Devices making use of plastic bending and unbending of metal elements to provide a resistive force are of great interest to industry. Many areas can benefit from the accurate magnitude prediction of the resulting resistive force produced when a metal element is plastically deformed.

3.1.1 Cyclically Loaded Devices

The SELDA system has already been implemented and used in industry by Seltrust Engineering [5]. To the authors knowledge no clear performance prediction is available, and the application specification as stated by the manufacturer, is a force range or envelope, which will be delivered by the systems. There are three different sizes of deceleration systems; small, medium and large, which are available to cover a small range of forces. The assumption is that the specified performance envelope covers the variations in performance of the deceleration systems, under quasi-static and dynamic conditions.

Rosslee developed two analytical equations predicting the performance of cyclic bending energy absorbers, in his master's thesis [3]. These models were meant for quasi-static application. Rosslee experimentally verified them for a variety of geometries. The predictions and the experiments showed a very good correlation (refer: 4). In Rosslee's paper the dynamic application of this system as an energy absorber was suggested, provided testing was done to verify a prediction model.

Sheet metal bending as a function of thickness has been studied in the light of air bending by De Vin [22]. The bending performance is important when predicting forces required to press out certain patterns, accounting also for spring back and bend radii. A plane strain bending model was developed on the basis of a moment curvature model. The model was used to simulate the bending process for materials with different work hardening behaviour.

A metal strip being pulled over a roller was considered by Alexander [23], and a theoretical analytical model was developed to predict the difference in magnitude of tip forces required to pull the strip over the roller while conforming to the arc. According to Alexander the device delivered nearly constant resistive force and this was increased with each unit of width and thickness (refer: section 4.). These findings were under quasi-static conditions.

Johnson and Mamalis [4] developed an approximate theory for the force required to bend a metallic strip over a roller while forcing it to conform to the roller radius. It was suggested that this method of energy dissipation should be used to absorb the kinetic energy of a moving conveyance in order to protect the occupants. The magnitude of the force was also a function of width and thickness of the strip that was bent and unbent over the radius (refer: section 4.3). This mechanism was applied to a moving mass of 23 tonnes, but the equation does not make use of strain rate sensitivity.

During sheet metal forming on a double action press, drawbeads on the blank holder supply the restraining force, which controls the flow of metal into the die. As the metal is drawn through the drawbead, the restraining force has two components, bending deformation and friction. In an investigation performed by H.D. Nine, these two components were separated by means of a simulated drawbead and the use of rollers to eliminate friction, isolating the bending component [24]. It was found that the strain rate hardening should be modelled for steel and not for aluminium. Strain as a function of position was measured on the strip by means of grids accurately drawn on the undeformed strip prior to bending. The changes in spacing of the gridlines were closely monitored during the process. The distance needed for the strip to conform to the following arc radius when leaving the roller it had just passed over, was repeatedly recorded as being between 2-3 times that of the strip thickness (refer: section 4.). The experiments were done and verified against a statically calculated value, which compared well, within 5%. The effect of cyclic strain softening was neglected, which was compensated for by the lack of the strain rate hardening component.

3.2 Conclusion

The preceding theory has been scrutinised and summarised, and combined into a theoretically functional form to estimate the performance of the cyclic bending retardation systems. Four analytical equations have been collected and modified to accomplish the task. These have been automated into MATLAB code and allowed to deliver predictions of resistive performance. The results were gathered and compared (refer: section 7.).

4. Relevant Applied Theory

4.1 Introduction

The theoretical approach used to solve the presented problem did not encompass the development of new models. It was based on the manipulation of existing models and verifying whether the dynamic aspect applied to them delivered a prediction that coincided with the experimental readings gathered, as well as the FEA results. When a model was found to deliver comparable answers, the range of accuracy had to be evaluated.

In this dissertation, four analytical equations have been gathered, based on work from various authors (refer: section 3). The quasi-static prediction models have been automated into mathematical code (MATLAB), supplemented with the dynamic strain rate effect equations applicable to high strain rate situations, and allowed to predict the response of specific roller strip systems (refer: section 5).

In this chapter, the theory gathered and the application thereof is discussed.

4.2 Important Physical Considerations

4.2.1 Baushinger Effect

When a material is loaded into the plastic regime, unloaded and reloaded in the opposite direction, it is generally observed that the yielding during reloading occurs at a stress level lower than the initial load. This direction dependant yield behaviour is known as Baushinger effect, as discovered by J. Baushinger, a civil engineer in 1881 (refer: Figure 12). There is a dynamic aspect, which affects the degree to which the Baushinger behaviour influences the material characteristics [25]. The materials that were tested by Thakur [25] in the dynamic Baushinger effect experiments were exposed to significantly higher strain rates than the application considered in this study, but the same tendencies were expected. It was assumed that the Baushinger effect is a significantly smaller contributing factor to the dynamic effects of the systems performance.

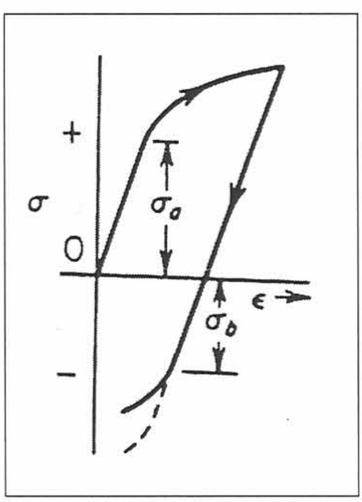

Figure 12 Baushinger effect.

The Baushinger effect is commonly ignored in plastic theory [26]. This was the case in the development of the analytical equations considered in this study, which have been sourced and used as performance predictors [3]. The yield stress was assumed to be the same in both directions of bending.

The roller radius and the thickness of the strip, as well as the velocity of the mass that is retarded, defines the strain rate of the current application. There are checks stipulated in the design protocol, specified by the author for the prevention of situations where the ratio of the thickness of the strip relative to the size of the roller radius are too large. This would prevent situations where the bending angle is too sharp, i.e. when a thick strip bends around a small roller, creating high surface strains.

4.2.2 Material Elasticity or Plasticity Component

During his experiments, H.D. Nine found that the total energy absorbed by a strip which was pulled through a series of rollers, was far greater than the amount of energy that the strip could store elastically [24]. This would then support the assumption that elasticity effects could be neglected in the cyclic plastic bending energy absorber, given that the ratio of roller radius to strip thickness is such that a large percentage of the strip cross section is plastically deformed [3].

4.2.3 Material Strain Hardening

There are two reasons why the material strain hardening effect has a small influence on the performance of the system considered in this study. The first reason is that strain hardening decreases with the increase of strain rate [27]. The second reason is that mild steel, as used in the cyclic plastic bending energy absorber, has moderate strain hardening characteristics, and the influence of this on the results would be negligible [28].

For these reasons, the four authors who developed the theory which was used as the prediction models in this study, ignored the material strain hardening effects.

4.2.4 Material Strain Rate Sensitivity

It has been identified that materials which have a tendency to be strain rate sensitive under high deformation rate conditions, should be modelled accordingly. The dynamic yield stress of a material has been known to increase by 300% from the static yield stress, at high rates of deformation (refer: Figure 13). In an investigation conducted by Perrone, reference was made to the well-known Cowper Symonds model for strain rate sensitive materials [29], [8]. This model was developed for the situation involving the impact loading of cantilever beams, which exhibit strain hardening or strain rate sensitivity.

The investigations performed by Perrone and Cowper Symonds was adopted as the basis for applying the strain rate sensitivity effects to the cyclic plastic bending retardation system, considered in this dissertation.

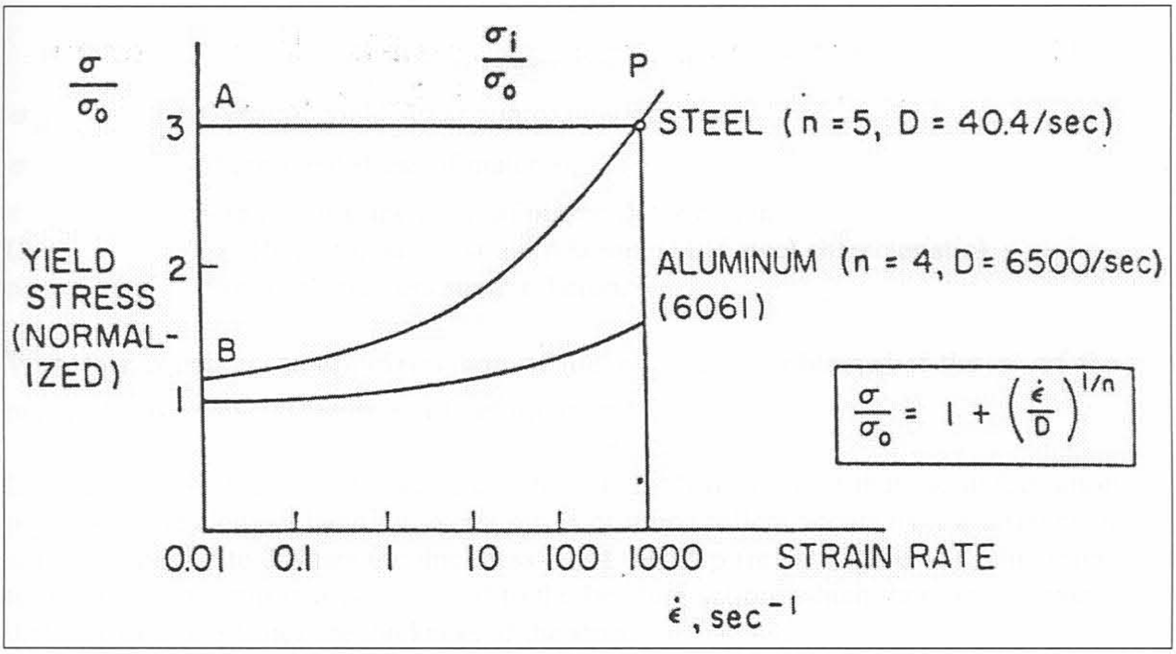

Figure 13 Dynamic yield effect, Perrone.

4.3 Applied Equations

4.3.1 Strain Rate Sensitivity Compensation

The strain rate sensitivity is appended to the prediction equations in the form of the Cowper Symonds formulation [8]. This is combined with a deduction made from the research of H.D. Nine [24] concerning the period in which the straining of the material takes place. When the distance is known, for the strip deformation process at each point of bending to take place, the velocity of the moving material is also known, thus the strain rate at that point can be calculated and the dynamic yield stress at the point of strain deformation can be determined.

The Cowper Symonds formulation is the following:

$$\frac{\sigma_{yd}}{\sigma_{yo}} = 1 + \left(\frac{\varepsilon'}{D}\right)^{\frac{1}{p}}$$

Equation 1 Cowper Symonds. [8]

σ_{yd}	=	Dynamic yield stress approximation.
σ_{yo}	=	Static yield stress of material.
ε'	=	Strain rate at the point of plastic deformation.
D	=	Coefficient in stress strain relation. (Mild steel characteristic)
p	=	Exponent in stress strain relation.

With this equation, an approximation of the σ_{yd} can be obtained if the ε' of the material was known at the point of deformation.

During the investigation of drawbead forces, H.D. Nine showed that the deformation process of the element bending over a series of inline rollers occurs over a distance of approximately 2 to 3 times the thickness (t) of the strip (refer: Figure 14). This refers to the length of strip that is subjected to the bending action, which then occurs over a distance of 2 to 3 times the thickness of the strip.

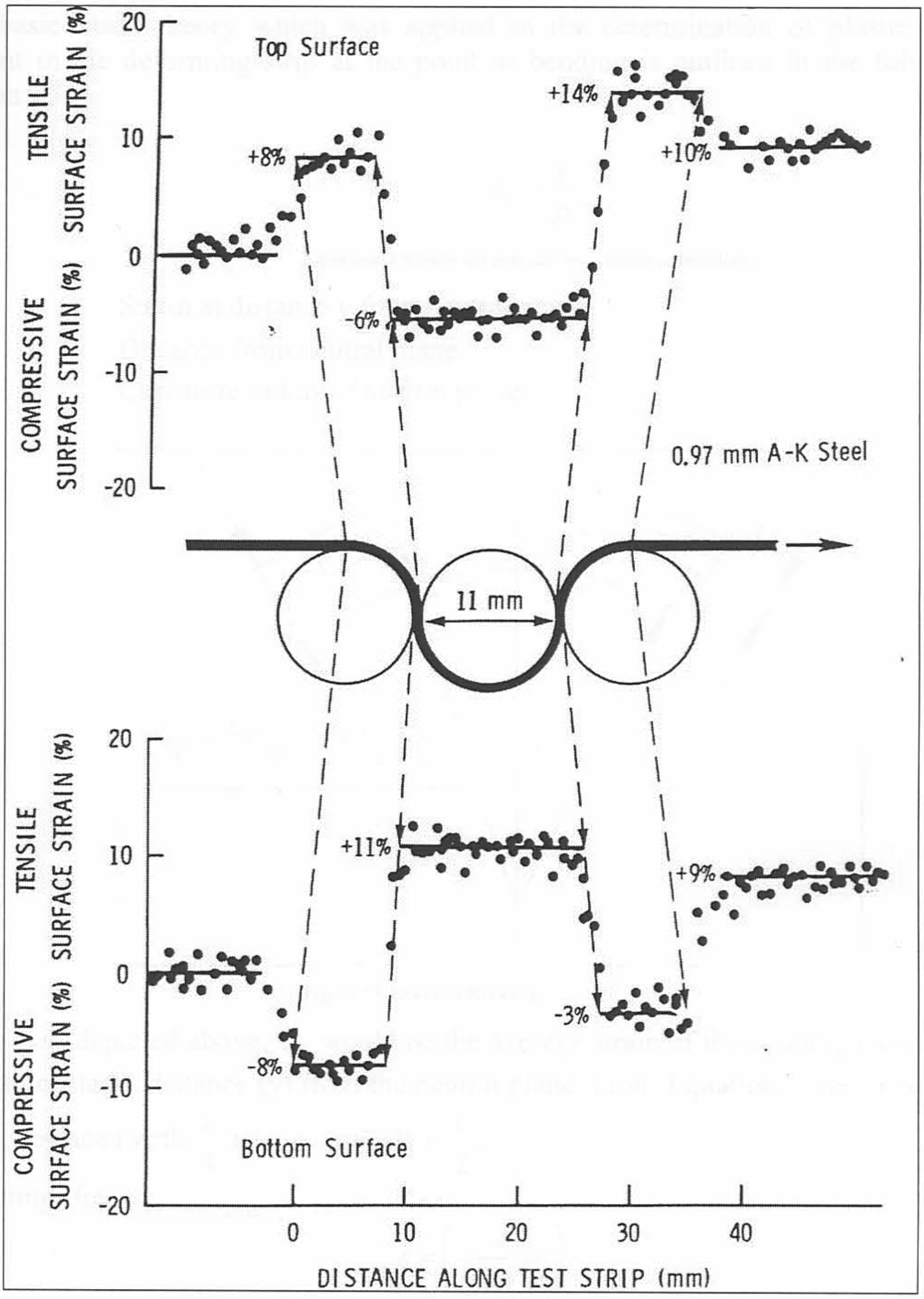

Figure 14 Surface strain relative to position, H.D. Nine.

A further explanation of Nine's research would be as follows [24]. The bending element, upon leaving the roller it had just passed over to conform to the next roller in line, would undergo strain deformation for a distance of approximately three times its thickness. With this fact in mind, knowing the velocity at which the strip is moving, and the magnitude of the average strain the strip is experiencing during bending based on beam theory for bending over a roller of specified radius, the strain rate at the point of plastic strain can be calculated. In turn, the dynamic yield stress can be determined.

The basic beam theory which was applied in the determination of plastic strain present in the deforming strip at the point of bending is outlined in the following section: [30]

$$\varepsilon_x = \frac{y}{\rho}$$

Equation 2 Strain magnitude for a beam in bending,

ε_x	=	Strain at distance y from neutral plane.
y	=	Distance from neutral plane.
ρ	=	Curvature radius of neutral plane.

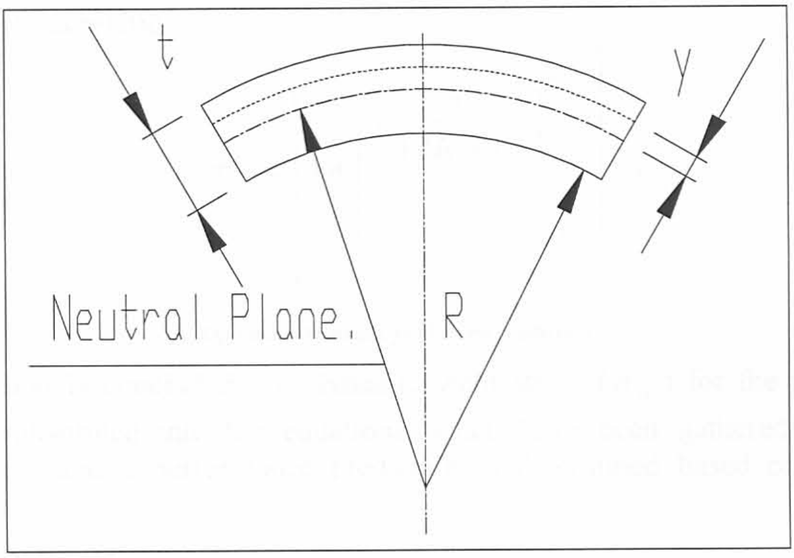

Figure 15 Beam in Bending.

In the case depicted above, ε_x would be the average strain of the bending element, at a point located a distance (y) from the neutral plane. From Equation 2 the value for y can be replaced with $\frac{t}{4}$ and ρ with (R + $\frac{t}{2}$).

This simplifies to

$$\varepsilon = \left(\frac{t}{4R + 2t}\right)$$

Equation 3 Strain as a function of radius and thickness.

Considering Figure 14 of H.D. Nine, the assumption that the plastic strain takes place within 3 times the thickness (t) of the strip, moving at a velocity (v), leads to the following:

$$\text{Duration of strain} = \frac{3 \times t}{v} \qquad \text{(in seconds)}$$

Equation 4 Time duration of strain.

This is true for infinitesimally small time segments, which would reveal constant velocity during the considered interval.

A combination of equations 3 and 4, results in the following:
If

$$\varepsilon' = \left(\frac{t}{4R + 2t}\right) \div \frac{3 \times t}{v}$$

then becomes,

$$\varepsilon' = \left(\frac{v}{12R + 6t}\right)$$

Equation 5 Bending Strain Rate equation.

Since the strain rate (ε') is now quantified for any bending radius magnitude and velocity, Equation 5 can now be combined with Equation 1. This would deliver the dynamic yield stress relation.

$$\sigma_{yd} = \left[1 + \left(\frac{\left(\frac{v}{12R + 6t}\right)}{D}\right)^{\frac{1}{p}}\right] \times \sigma_{yo}$$

Equation 6 Dynamic yield stress prediction.

Once this relation is concluded, the dynamic yield stress (σ_{yd}) for the prescribed condition, is substituted into the equations which have been gathered from the literature survey, and a performance prediction is determined based on the new material property.

4.3.2 Force Prediction Model 1 [3]

The analytical model presented in this section, has been developed by Rosslee [3]. The model is based on bending moment theory. Consider a metal strip being bent over a roller radius (R) under a pure bending moment (M). The strip has thickness (t) and width (w) and plane strain deformation conditions prevail. If the roller radius is very large, only elastic deformation occurs. As the radius decreases the deformation becomes elastic-plastic and finally almost purely plastic. When nearly the whole section is plastically deformed, the effects of elasticity can be neglected and rigid-perfect-plasticity can be assumed. When the radius becomes too small, tensile and compressive failure of the material at the surfaces of the strip will start to occur.

When a strip is being pulled over a single roller, the strip would be subjected to a bending and tensile force simultaneously. In this formulation it was assumed that:

- The strip shows rigid perfectly plastic behaviour.
- The transverse planes would remain plane during and after deformation, distributing axial strain linearly through the strip cross-section.
- The yield stress is equal in both directions of deformation, tension and compression.
- Plane strain under frictionless roller conditions exist.

Referring to Figure 16, the strip bends to conform to the first roller radius at point 1, and then straightens to leave the curve again at point 2. These points are assumed to be two plastic hinges. These hinges transform energy and dissipate it through heat created by plastic deformation work.

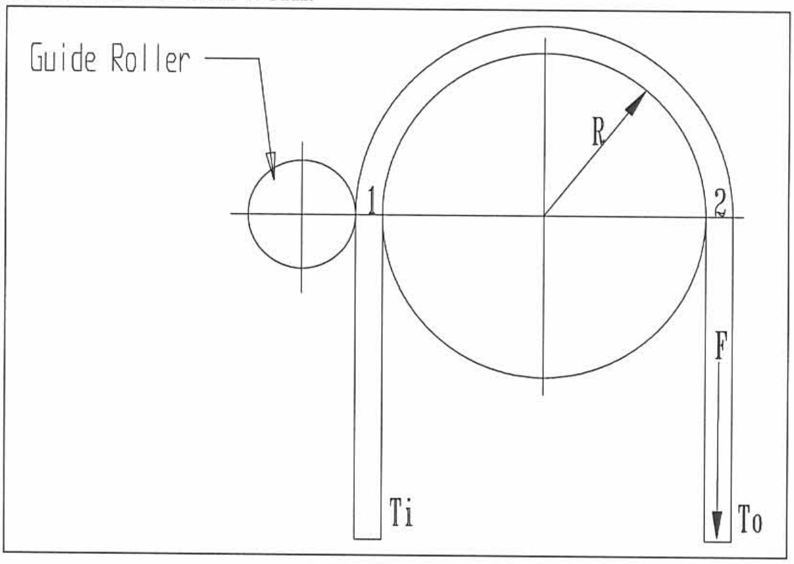

Figure 16 Element conforming to roller radius.

The two hinges are considered similar, only acting in opposite directions. By following basic principles, Rosslee concluded that: [3].

$$F = 2\sigma_y w\left[(2R+t)-\sqrt{(2R+t)^2-t^2}\right]$$

Equation 7 Rosslee bending model.

This expression can now be used as an approximation of the force required to pull a strip of metal over a single roller, given the initial assumptions. From the above formulation notice can be taken that the arc of contact between the roller and the strip

is not represented. The number of times the strip bends and unbends determines the magnitude of the applied force, and not the arc of contact.
Equation 7 can then be expanded with the addition of Equation 6 to produce the first of the dynamic cyclic bending approximation models.

$$F = 2\left(\left[1+\left(\frac{\left(\frac{v}{12R+6t}\right)}{D}\right)^{\frac{1}{p}}\right]\times\sigma_{yo}\right)w\left[(2R+t)-\sqrt{(2R+t)^2-t^2}\right]$$

Equation 8 Force Prediction Model 1.

This is one equation that has been used to deliver predictions of cyclic plastic bending performance when automated into mathematical code, as detailed in section 5.

4.3.3 Force Prediction Model 2 [3]

Rosslee also used deformation energy techniques to model a solution for cyclic plastic bending [3]. Consider a wide strip of metal being pulled over a frictionless roller, as shown in Figure 16. The strip of original width (w) and thickness (t), undergoes plastic bending and elongation as it passes over the roller of radius (R).

It is assumed that the strip bends suddenly as it reaches position 1 in Figure 16, to conform to the roller contour. This curvature is maintained for the duration of contact with no additional deformation occurring, until the strip again straightens suddenly, at position 2. The experiment conducted by H.D. Nine shows this assumption to be fairly accurate (refer: Figure 14) It is further assumed that the ratio of roller radius to strip thickness (R/t) is such that complete plasticity exists at the bending points 1 and 2. Rigid plastic behaviour with infinite plasticity and no strain hardening is assumed. It has also been proven experimentally [24], that if (w) is much greater than (t), then no deformation takes place in the width of the strip. Therefore, plane strain can be assumed. It is further assumed that all the energy absorbed is dissipated in plastic deformation. The energy is absorbed in cyclic plastic straining, as well as elongation and thinning of the strip. The bending positions 1 and 2 are also assumed to be exactly the same thus the combined energy absorbed would merely be twice the amount at point 1.

Keeping all these factors in mind the following has been determined:

Deformation work at a point (W), is calculated by:

$$W = Volume \cdot \int \bar{\sigma} \cdot d\bar{\varepsilon}$$

Equation 9 Strain work integral at one hinge.

then the following equation was deduced from basic principles [3],

$$F = \frac{2 \cdot w \cdot t \cdot \sigma_y \cdot \left[\ln\left(\frac{R+t}{R+\frac{t}{2}} \right) - \ln\left(\frac{R+t}{R+\frac{t}{2}} \right) \right]}{2 \cdot \sqrt{3} - \left[\ln\left(\frac{R+t}{R+\frac{t}{2}} \right) + \ln\left(\frac{R+t}{R+\frac{t}{2}} \right) \right]}$$

Equation 10 Rosslee's deformation energy formulation [3].

The formulation of Equation 10, can be appended to the strain rate dependant dynamic yield stress predictor, Equation 6, which reveals,

$$F = \frac{2 \cdot w \cdot t \cdot \left[\left[1 + \left(\frac{\left(\frac{v}{12R+6t} \right)}{D} \right)^{\frac{1}{p}} \right] \times \sigma_{yo} \right] \cdot \left[\ln\left(\frac{R+t}{R+\frac{t}{2}} \right) - \ln\left(\frac{R+t}{R+\frac{t}{2}} \right) \right]}{2 \cdot \sqrt{3} - \left[\ln\left(\frac{R+t}{R+\frac{t}{2}} \right) + \ln\left(\frac{R+t}{R+\frac{t}{2}} \right) \right]}$$

Equation 11 Force Prediction Model 2.

This force prediction model has also been automated into mathematical code to deliver an indication of performance (refer: section 5.).

4.3.4 Force Prediction Model 3 [4]

Another analytical model has been developed by W. Johnson and A.G. Mamalis, based upon plastic work principles [4]. The formulation predicts that for a wide rectangular strip of width (w) and thickness (t), being bent across the arc of a roller with radius (R), the difference in tension between the taut and slack sides, applied to the ends of the strip to ensure conformation to the roller curvature (refer: Figure 16) would be equal to :

$$(T_o - T_i) \times (R + \frac{t}{2}) = 2M_p$$

Equation 12 Plastic work equation [4].

Where:

T_0	=	Tension on the taut side of the bending element. (N)
T_i	=	Tension on the slack side of the bending element. (N)
M_p	=	Plastic bending moment. (N.m)

The reason for the double representation of the plastic moment in Equation 12 is that firstly, the bending element is plastically deformed to conform to the arc of the roller it is being bent over and then secondly, is bent away from the surface to straighten again.

The assumptions made are the following:

- The ratio of t/R is assumed to be much smaller than 1.
- Perfect plasticity exists.
- There is no friction of the rolling roller element.

Ignoring the Baushinger effect as previously discussed in section 4.2.1, the bending moment can be considered of the same magnitude in both directions of application. This bending moment magnitude is represented by:

$$M_p = \frac{w \times t^2 \times \sigma_y}{4}$$

Equation 13 Plastic bending equation.

Where:

σ_y = mean yield stress. (Mpa)

After substituting Equation 13 into Equation 12, the following result is obtained.

$$(T_0 - T_i) = \frac{w \times t^2 \times \sigma_y}{2 \times R + t}$$

Equation 14 Combined plastic bending equation [4].

The static force prediction depicted in Equation 14 was expanded with Equation 6 to deliver the deformation rate dependence element of the formulation.

The above substitution process results in the following:

$$F = \frac{w \times t^2 \times \left[1 + \left(\frac{\left(\frac{v}{12R + 6t} \right)}{D} \right)^{\frac{1}{p}} \right] \times \sigma_{yo}}{2 \times R + t}$$

Equation 15 Force prediction model 3.

The above force prediction model, has also been automated in mathematical code and used to generate force resistance estimations of a steel element or strip being drawn over a roller of given dimensions, as is discussed in greater detail in section 5.

4.3.5 Force Prediction Model 4 [23]

The final model predicting the resistance force generated by a metallic strip being drawn between inline rollers (refer: Figure 16) was developed by J.M. Alexander, and then published by W. Johnson and S.R. Reid [23], [4].

The model assumes a plane strain condition, implying that the width of the strip is much larger than the thickness. Further assumptions are that the roller radius is much larger than the thickness of the strip and lastly, that the elastic effects are neglected. With these conditions established, the observation was made by Alexander that the device being tested, provides a nearly constant resistive force and that this force increases with unit width. The relation of the forces applied to the strips ends, referring to Figure 16, is then:

$$\Delta T = \frac{T_o - T_i}{w}$$

Equation 16 Alexander's basic formulation [23].

where :

$$\Delta T = \frac{4 \times \sigma_y}{\sqrt{3}} \times (t - 2k)$$

Equation 17 Tension difference equivalent.

with :

$$k = R\left[\sqrt{1 + t/R} - 1\right]$$

Equation 18 Roller radius and strip thickness ratio.

Combining all the above components delivers the following:

$$T_o - T_i = \frac{4 \times \sigma_y \times w}{\sqrt{3}} \times \left(t - 2 \times R\sqrt{1 + \frac{t}{R}} + 2R\right)$$

Equation 19 Static force prediction equation.

Equation 6 combined with Equation 19, accommodates for the dynamic aspect of the required prediction formula. The units of the value predicted in Equation 20 are, force per unit width.

$$T_o - T_i = \frac{4 \times \left(\left[1 + \left(\frac{\left(\frac{v}{12R + 6t}\right)}{D}\right)^{\frac{1}{p}}\right] \times \sigma_{yo}\right) \times w}{\sqrt{3}} \times \left(t - 2 \times R\sqrt{1 + \frac{t}{R}} + 2R\right)$$

Equation 20 Force prediction model 4.

4.4 Conclusion

In this chapter, the four prediction models used to mathematically predict the dynamic performance of the bending element's resistive force have been developed. The determining factor of the generated force magnitude is the number of bend deformations the strip would encounter during its travel between the rollers. The angle of contact with the roller surface is inconsequential to the resistance force produced since energy is absorbed, and thus resistance is produced, only when bending of the element takes place, upon arrival on the roller surface or departing from it, and not during contact. These models have been based upon previous existing quasi-static models. In the following chapter these models have been automated into mathematical code and allowed to evaluate similar scenarios to those physically tested, in an attempt to produce comparable data.