# APPENDIX A

## A.1 Design values for SFRC

The values in Table A-1 are determined from third-point beam bending tests. The beams were cast and tested in accordance to the procedure of the Japanese Concrete Institute (1983).

Table A-1: Design values for Residual flexural strength ratio of SFRC (Bekaert, 2001).

| $R_e$ values | | | |
|---|---|---|---|
| Dosage (kg/m³) | RC-80/60-BN(§) | RC-65/60-BN | RL-45/50-BN |
| 15 | 42 | 38 | - |
| 20 | 52 | 47 | 38 |
| 25 | 60 | 56 | 45 |
| 30 | 68 | 63 | 52 |
| 35 | 75 | 69 | 58 |
| 40 | 80 | 75 | 63 |
| 45 | 86 | 80 | 68 |
| 50 | 90 | 85 | 72 |
| 55 | 95 | 89 | 77 |
| 60 | 99 | 93 | 80 |
| 65 | 102 | 97 | 84 |

(§) Notation for hooked-end, collated steel fibres, aspect ratio (length/diameter) of 80 mm, length of 60 mm and made of low carbon bright steel.

## A.2 Interior load-carrying capacity using Meyerhof formula

In the traditional method of designing the SFRC ground slabs, the Meyerhof (1962) formulae are used. The strength term is modified to take the post-cracking strength of the SFRC into account. The interior load-carrying capacity for the SFRC ground slab presented in chapter 3 can be calculated using the following input values:

$K$ = 1.9 MPa / mm (determined from plate-bearing test).

E = 28 GPa (determined from cylinder and beam bending test).

$\mu$ = 0.2 (estimated).

$f_{ct}$ = 6.7 MPa (determined from beam-bending test).

$f_{e,3}$ = 2.3 MPa (determined from beam bending test).

$$R_{e,3} = \frac{2.3}{6.7}.100 = 34 \text{ percent.}$$

Depth of the slab = 125 mm

Steel fibre content = 15 kg/m$^3$ (RC-80/60-BN)

The size of loading plate = 100 x 100 mm (The equivalent radius for the loading plate is 56.4).

Calculate the radius of relative stiffness:

$$L_r = \left( \frac{28 \text{ x } 10^3 \text{ x} (125)^3}{12\left(1-(0.2)^2\right) \text{x} 1.9} \right)^{0.25} = \underline{223.6} \text{ mm}$$

Calculate the limit moment of resistance:

$$M_o = \left(1 + \frac{34}{100}\right) \text{x } 6.7 \frac{1000 \text{ x } (125)^2}{6} \text{x } 10^{-6} = 23.4 \text{ kN.m/ m width}$$

Calculate the interior load-carrying capacity of the SFRC ground slab:

$$P_i = 6 \text{ x } 23.4 \text{ x} \left(1 + \frac{2 \text{ x } 56.4}{223.6}\right) = \underline{211} \text{ kN}$$

The $R_{e,3}$ value obtained from Table A-1 is 42 percent (see the arrow), which is higher than the actual value obtained experimentally by conducting a third-point bending test. If the design $R_{e,3}$ (Table A-1) is used, the load-carrying capacity of the slab is increase to 224 kN (0.6 percent higher).

# APPENDIX B

**Loading and geometry**

$h := 100mm$ (Depth)

$b := 100 \cdot mm$ (Width)

$L := 750mm$

$fsh := \frac{6}{5}$ (Form factor for shear)

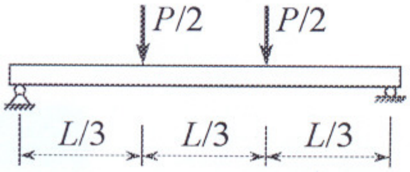


**Figure B-1: Test set up for the beam.**

**Adopted stress - strain response**

$E := 25.4 \cdot \text{GPa}$

$\mu := 0.2$

$G := \frac{E}{2 \cdot (1 + \mu)}$

$G = 10.58333\,\text{GPa}$

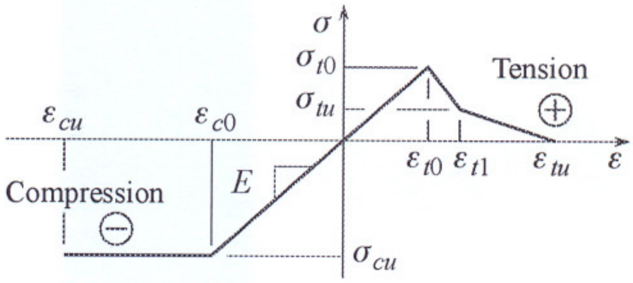


**Figure B-2: Schematic diagram for the stress-strain response**

$\sigma_{t0} := 2.8 \cdot \text{MPa}$ $\quad \varepsilon_{t0} := \frac{\sigma_{t0}}{E}$ $\quad \varepsilon_{t0} = 1.10236 \times 10^{-4}$

$\sigma_u := 1.0 \cdot \text{MPa}$ $\quad \varepsilon_{t1} := 7 \times 10^{-4}$ $\quad \varepsilon tu := 0.1$

$\varepsilon_{c0} := -0.0014$ $\quad \sigma_{cu} := \varepsilon_{c0} \cdot E$ $\quad \sigma_{cu} = -35.56\,\text{MPa}$

$\varepsilon_{cu} := -0.4$ $\quad \lambda := \frac{\sigma_u - \sigma_{t0}}{\varepsilon_{t1} - \varepsilon_{t0}}$ $\quad \lambda = -3.05207\,\text{GPa}$

$\Psi := \frac{\sigma_u}{\varepsilon_{t1} - \varepsilon tu}$ $\quad \Psi = -0.01007\,\text{GPa}$

**Tensile stress-strain function**

$$f_{ct}(\varepsilon_t) := (\varepsilon_t \geq 0) \cdot (\varepsilon_t \leq \varepsilon_{t0}) \cdot \frac{\sigma_{t0}}{\varepsilon_{t0}} \cdot \varepsilon_t \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t0}) \cdot (\varepsilon_t \leq \varepsilon_{t1}) \cdot [\lambda \cdot (\varepsilon_t - \varepsilon_{t0}) + \sigma_{t0}] \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t1}) \cdot (\varepsilon_t \leq \varepsilon tu) \cdot \Psi \cdot (\varepsilon_t - \varepsilon tu)$$

$\varepsilon_t := 0, 0.00001 .. \varepsilon tu$

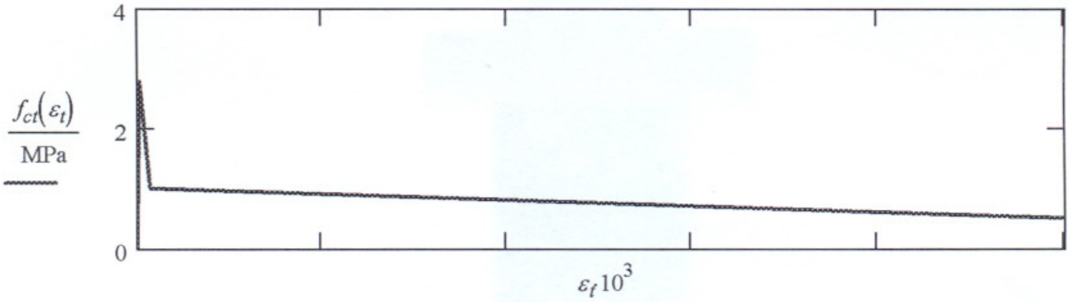


**Figure B-3: Assumed tensile stress-strain response.**

**Compressive stress-strain function**

$$f_{cc}(\varepsilon_c) := (\varepsilon_c \geq \varepsilon_{c0}) \cdot (\varepsilon_c < 0) \cdot E \cdot \varepsilon_c \ldots$$
$$+ (\varepsilon_c \geq \varepsilon_{cu}) \cdot (\varepsilon_c < \varepsilon_{c0}) \cdot \sigma_{cu}$$

$$\varepsilon_c := \varepsilon_{cu}, \frac{199 \cdot \varepsilon_{cu}}{200} .. 0$$

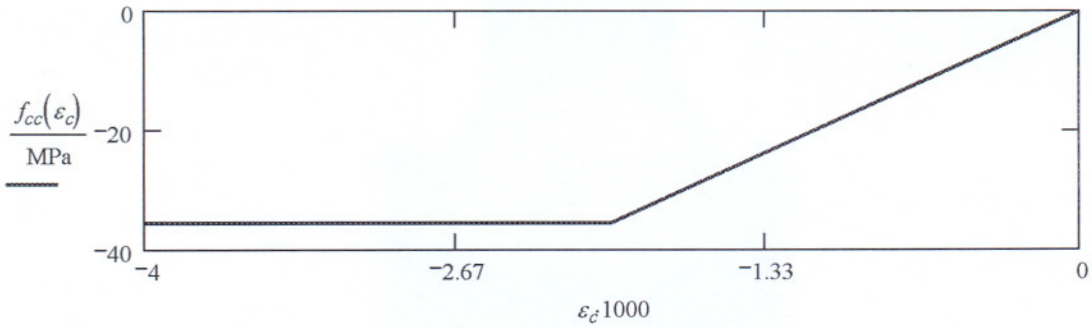


**Figure B-4: Assumed compressive stress-strain response**

**Calculating moment-curvature response**

$$\phi(\varepsilon_{bot}, a) := \frac{\varepsilon_{bot}}{h - a}$$

$$\varepsilon_{c.top}(\varepsilon_{c.bot}, a) := \frac{-a}{h - a} \cdot \varepsilon_{c.bot}$$

$$F_{cc}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c)\, d\varepsilon_c$$

$$F_{ct}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\, d\varepsilon_c$$

$$Mc(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$$Mct(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$$\varepsilon_{bot} := \varepsilon_{t0}$$

$$a0 := \frac{h}{2} \qquad F_{cc}(\varepsilon_{bot}, a0) = -7\,\text{kN}$$

$$F_{ct}(\varepsilon_{bot}, a0) = 7\,\text{kN}$$

$$\varepsilon_{bot} = 1.10236 \times 10^{-4}$$

$$Mc(\varepsilon_{bot}, a0) = 0.23333\,\text{m kN}$$

$$Mct(\varepsilon_{bot}, a0) = 0.23333\,\text{m kN}$$

$$Mext(\varepsilon_{c.bot}, a) := Mc(\varepsilon_{c.bot}, a) + Mct(\varepsilon_{c.bot}, a)$$

*Given* $F_{cc}(\varepsilon_{bot}, a) + F_{ct}(\varepsilon_{bot}, a) = 0 \cdot \text{kN}$

$Solve(\varepsilon_{bot}, a) := Find(a)$

$i := 1 .. 20$

$\varepsilon b_i :=$

| |
|---|
| 0.00001 |
| 0.00004 |
| 0.00007 |
| 0.00008 |
| 0.0001 |
| $\varepsilon_{t0}$ |
| 0.0002 |
| 0.0003 |
| 0.00035 |
| $\varepsilon_{t1}$ |
| 0.001 |
| 0.002 |
| 0.003 |
| 0.007 |
| 0.009 |
| 0.01 |
| 0.03 |
| 0.06 |
| 0.08 |
| $\varepsilon tu$ |

$a_i := Solve(\varepsilon b_i, a0)$ $\quad \varepsilon t_i := \varepsilon_{c.top}(\varepsilon b_i, a_i)$ $\quad \phi_i := \phi(\varepsilon b_i, a_i)$

$M_i := Mext(\varepsilon b_i, a_i)$ $\quad \sigma b_i := f_{ct}(\varepsilon b_i)$

| $\varepsilon b_i =$ | $a_i =$ (mm) | $\varepsilon t_i \cdot 10^3 =$ |
|---|---|---|
| $1 \cdot 10^{-5}$ | 50 | -0.01 |
| $4 \cdot 10^{-5}$ | 50 | -0.04 |
| $7 \cdot 10^{-5}$ | 50 | -0.07 |
| $8 \cdot 10^{-5}$ | 50 | -0.08 |
| $1 \cdot 10^{-4}$ | 50 | -0.1 |
| $1.10236 \cdot 10^{-4}$ | 50 | -0.11024 |
| $2 \cdot 10^{-4}$ | 46.8078 | -0.17599 |
| $3 \cdot 10^{-4}$ | 42.62231 | -0.22285 |
| $3.5 \cdot 10^{-4}$ | 40.78348 | -0.24105 |
| $7 \cdot 10^{-4}$ | 31.15894 | -0.31683 |
| $1 \cdot 10^{-3}$ | 26.04069 | -0.35209 |
| $2 \cdot 10^{-3}$ | 18.35165 | -0.44953 |
| $3 \cdot 10^{-3}$ | 14.9798 | -0.52857 |
| $7 \cdot 10^{-3}$ | 9.81758 | -0.76204 |
| $9 \cdot 10^{-3}$ | 8.65163 | -0.85239 |
| 0.01 | 8.20232 | -0.89352 |
| 0.03 | 4.57493 | -1.43828 |
| 0.06 | 3.07768 | -1.90525 |
| 0.08 | 2.52844 | -2.07522 |
| 0.1 | 2.08689 | -2.13137 |

**Calculated moment-curvature response**

$M_i =$ (m · kN)

| |
|---|
| 0.04233 |
| 0.16933 |
| 0.29633 |
| 0.33867 |
| 0.42333 |
| 0.46667 |
| 0.66769 |
| 0.73533 |
| 0.74633 |
| 0.66 |
| 0.56739 |
| 0.4873 |
| 0.47057 |
| 0.45537 |
| 0.45082 |
| 0.44833 |
| 0.39244 |
| 0.29766 |
| 0.23291 |
| 0.16755 |

$\phi_i =$ ($m^{-1}$)

| |
|---|
| $2 \cdot 10^{-4}$ |
| $8 \cdot 10^{-4}$ |
| $1.4 \cdot 10^{-3}$ |
| $1.6 \cdot 10^{-3}$ |
| $2 \cdot 10^{-3}$ |
| $2.20472 \cdot 10^{-3}$ |
| $3.75995 \cdot 10^{-3}$ |
| $5.22851 \cdot 10^{-3}$ |
| $5.91051 \cdot 10^{-3}$ |
| 0.01017 |
| 0.01352 |
| 0.0245 |
| 0.03529 |
| 0.07762 |
| 0.09852 |
| 0.10894 |
| 0.31438 |
| 0.61905 |
| 0.82075 |
| 1.02131 |

**Measured moment-curvature response**

$k := 1..19$

$\phi c_k :=$

| |
|---|
| $0 m^{-1}$ |
| $0.0014 m^{-1}$ |
| $0.0017 m^{-1}$ |
| $0.0028 m^{-1}$ |
| $0.0034 m^{-1}$ |
| $0.0041 m^{-1}$ |
| $0.0055 m^{-1}$ |
| $0.0076 m^{-1}$ |
| $0.011 m^{-1}$ |
| $0.014 m^{-1}$ |
| $0.017 m^{-1}$ |
| $0.022 m^{-1}$ |
| $0.028 m^{-1}$ |
| $0.034 m^{-1}$ |
| $0.042 m^{-1}$ |
| $0.075 m^{-1}$ |
| $0.1 m^{-1}$ |
| $0.12 m^{-1}$ |
| $0.14 m^{-1}$ |

$Mc_k :=$

| |
|---|
| $0 m \cdot kN$ |
| $0.344 m \cdot kN$ |
| $0.457 m \cdot kN$ |
| $0.571 m \cdot kN$ |
| $0.657 m \cdot kN$ |
| $0.714 m \cdot kN$ |
| $0.738 m \cdot kN$ |
| $0.686 m \cdot kN$ |
| $0.611 m \cdot kN$ |
| $0.543 m \cdot kN$ |
| $0.514 m \cdot kN$ |
| $0.514 m \cdot kN$ |
| $0.514 m \cdot kN$ |
| $0.49 m \cdot kN$ |
| $0.47 m \cdot kN$ |
| $0.45 m \cdot kN$ |
| $0.45 m \cdot kN$ |
| $0.45 m \cdot kN$ |
| $0.45 m \cdot kN$ |

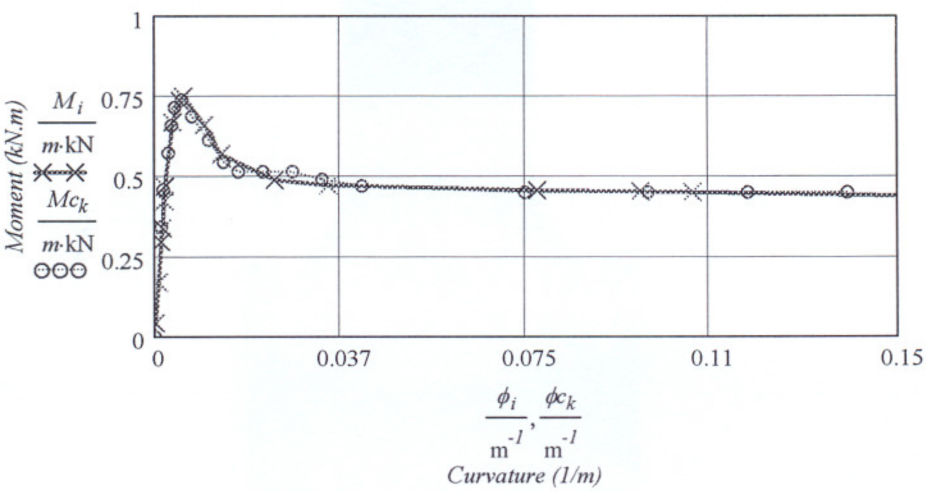


**Figure B-5: Measured and calculated moment-curvature responses**

## Output sStress-strain responses

$$\sigma t_i := f_{cc}\left(\varepsilon_{t_i}\right)$$

$\sigma b_i =$ (MPa)

| |
|---|
| 0.254 |
| 1.016 |
| 1.778 |
| 2.032 |
| 2.54 |
| 2.8 |
| 2.52603 |
| 2.22083 |
| 2.06822 |
| 1 |
| 0.99698 |
| 0.98691 |
| 0.97684 |
| 0.93656 |
| 0.91641 |
| 0.90634 |
| 0.70493 |
| 0.40282 |
| 0.20141 |
| 0 |

$\dfrac{\sigma t_i}{MPa} =$

| |
|---|
| -0.254 |
| -1.016 |
| -1.778 |
| -2.032 |
| -2.54 |
| -2.8 |
| -4.47027 |
| -5.66042 |
| -6.1227 |
| -8.04761 |
| -8.94321 |
| -11.41803 |
| -13.42576 |
| -19.35593 |
| -21.65078 |
| -22.69544 |
| -35.56 |
| -35.56 |
| -35.56 |
| -35.56 |

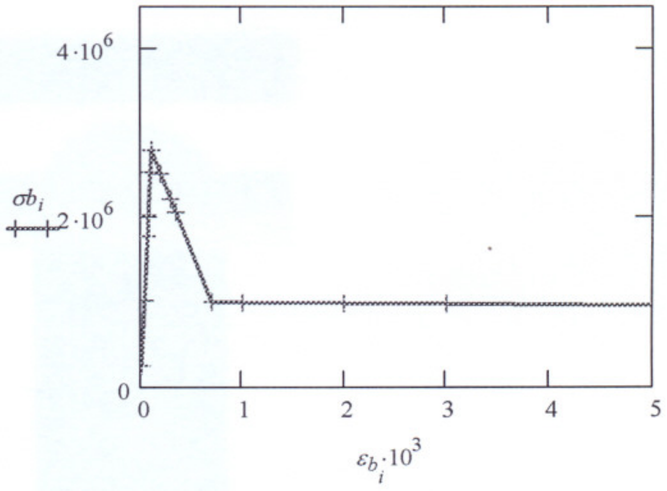


**Figure B-6: Output tensile stress-strain response.**

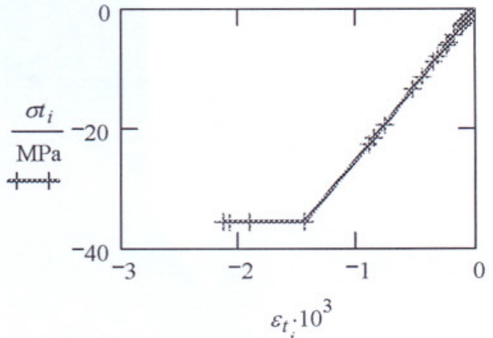


**Figure B-7: Output compressive stress-strain response.**

**Calculating load-deflection response**

$M_{max} := max(M)$

$M_{max} = 0.74633\ m \cdot \text{kN}$

$\phi h := lookup\left(M_{max}, M, \phi\right)_1$ $\phi h = 5.91051 \times 10^{-3}\ \text{m}^{-1}$

$MaxPos := match\left(M_{max}, M\right)_1$ $MaxPos = 9$

$M1 := submatrix(M, 1, MaxPos, 1, 1)$

$\phi_1 := submatrix\left(\phi, 1, MaxPos, 1, 1\right)$

$length(M) = 20$

$M2 := submatrix(M, MaxPos, length(M) - 1, 1, 1)$

$\phi_2 := submatrix\left(\phi, MaxPos, length(M) - 1, 1, 1\right)$

$length(M2) = 11$

$MaxPos2 := match\left(M_{max}, M2\right)_1$

$MaxPos2 = 1$

$Mmin := min(M2)$

$Mmin = 0.23291\ m \cdot \text{kN}$

$MinPos2 := match(Mmin, M2)_1$

$MinPos2 = 11$

$posi := match\left(M2_{MinPos2}, M\right)_1$

$posi = 19$

$length(M2) = 11$

$x := 0m, 0.025m .. 0.25m$

$ms(x,M) := M \cdot \frac{x}{0.25m}$ $\quad$ $\Delta ms(x,M) := max(M2) - ms(x,M)$

$$M1 = \begin{pmatrix} 0.04233 \\ 0.16933 \\ 0.29633 \\ 0.33867 \\ 0.42333 \\ 0.46667 \\ 0.66769 \\ 0.73533 \\ 0.74633 \end{pmatrix} m \cdot \text{kN} \qquad \phi_1 = \begin{pmatrix} 2 \times 10^{-4} \\ 8 \times 10^{-4} \\ 1.4 \times 10^{-3} \\ 1.6 \times 10^{-3} \\ 2 \times 10^{-3} \\ 2.20472 \times 10^{-3} \\ 3.75995 \times 10^{-3} \\ 5.22851 \times 10^{-3} \\ 5.91051 \times 10^{-3} \end{pmatrix} m^{-1} \qquad M2 = \begin{pmatrix} 0.74633 \\ 0.66 \\ 0.56739 \\ 0.4873 \\ 0.47057 \\ 0.45537 \\ 0.45082 \\ 0.44833 \\ 0.39244 \\ 0.29766 \\ 0.23291 \end{pmatrix} m \cdot \text{kN}$$

$\Delta M2 := M_{max} - M2$

$$\phi_2 = \begin{pmatrix} 5.91051 \times 10^{-3} \\ 0.01017 \\ 0.01352 \\ 0.0245 \\ 0.03529 \\ 0.07762 \\ 0.09852 \\ 0.10894 \\ 0.31438 \\ 0.61905 \\ 0.82075 \end{pmatrix} m^{-1}$$

$\Delta M2 =$ (m · kN)

| |
|---|
| 0 |
| 0.08633 |
| 0.17894 |
| 0.25903 |
| 0.27576 |
| 0.29096 |
| 0.29551 |
| 0.298 |
| 0.35389 |
| 0.44867 |
| 0.51342 |

**Deflection due to bending moment**

$$\delta m_i := \begin{cases} \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\,dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(M1, \phi_1, ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\,dx & \text{if } (i \le MaxPos) \\ \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\,dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(\Delta M2, \phi_2, \Delta ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\,dx & \text{if } (MaxPos \le i) \end{cases}$$

**Deflection due to shear force**

$$P_i := M_i \cdot \frac{6}{L} \qquad \delta sh_i := \frac{P_i\,fsh}{G \cdot b \cdot h \cdot 2} \cdot \int_{0mm}^{\frac{L}{3}} 1.\,dx \qquad \delta_i := \delta sh_i + \delta m_i \qquad per_i := \frac{\delta sh_i\,100}{\delta_i}$$

| $P_i =$ (kN) | $\delta m_i =$ (mm) | $\delta sh_i =$ (mm) | $\delta_i =$ (mm) | $per_i =$ |
|---|---|---|---|---|
| 0.33867 | 0.01198 | $4.8\cdot10^{-4}$ | 0.01246 | 3.85259 |
| 1.35467 | 0.04792 | $1.92\cdot10^{-3}$ | 0.04984 | 3.85259 |
| 2.37067 | 0.08385 | $3.36\cdot10^{-3}$ | 0.08721 | 3.85259 |
| 2.70933 | 0.09583 | $3.84\cdot10^{-3}$ | 0.09967 | 3.85259 |
| 3.38667 | 0.11979 | $4.8\cdot10^{-3}$ | 0.12459 | 3.85259 |
| 3.73333 | 0.13205 | $5.29134\cdot10^{-3}$ | 0.13735 | 3.85259 |
| 5.34148 | 0.21772 | $7.5706\cdot10^{-3}$ | 0.22529 | 3.36036 |
| 5.88263 | 0.28739 | $8.33759\cdot10^{-3}$ | 0.29573 | 2.81931 |
| 5.97063 | 0.31661 | $8.4623\cdot10^{-3}$ | 0.32507 | 2.60324 |
| 5.28 | 0.46696 | $7.48346\cdot10^{-3}$ | 0.47445 | 1.5773 |
| 4.53913 | 0.5856 | $6.43341\cdot10^{-3}$ | 0.59203 | 1.08667 |
| 3.8984 | 1.00489 | $5.5253\cdot10^{-3}$ | 1.01042 | 0.54683 |
| 3.76457 | 1.42467 | $5.33561\cdot10^{-3}$ | 1.43 | 0.37312 |
| 3.64293 | 3.07687 | $5.1632\cdot10^{-3}$ | 3.08203 | 0.16753 |
| 3.60654 | 3.89296 | $5.11163\cdot10^{-3}$ | 3.89807 | 0.13113 |
| 3.58665 | 4.29941 | $5.08344\cdot10^{-3}$ | 4.30449 | 0.1181 |
| 3.13952 | 12.3192 | $4.44972\cdot10^{-3}$ | 12.32365 | 0.03611 |
| 2.38128 | 24.21103 | $3.37504\cdot10^{-3}$ | 24.21441 | 0.01394 |
| 1.8633 | 32.08356 | $2.6409\cdot10^{-3}$ | 32.0862 | $8.23063\cdot10^{-3}$ |
| 1.34041 | 40.03073 | $1.89979\cdot10^{-3}$ | 40.03263 | $4.74561\cdot10^{-3}$ |

**Measured load-deflection response**

$j := 1..19$

| $Pa_j :=$ | $\delta a_j :=$ |
|---|---|
| 0kN | 0*mm* |
| 2.6kN | 0.08*mm* |
| 3.5kN | 0.11*mm* |
| 4.5kN | 0.15*mm* |
| 6.0kN | 0.25*mm* |
| 5.7kN | 0.309*mm* |
| 5.5kN | 0.34*mm* |
| 5.0kN | 0.4*mm* |
| 4.5kN | 0.48*mm* |
| 4.0kN | 0.62*mm* |
| 4.0kN | 0.67*mm* |
| 4.0kN | 0.88*mm* |
| 4.0kN | 1.32*mm* |
| 3.8kN | 2*mm* |
| 3.6kN | 2.84*mm* |
| 3.6kN | 3.88*mm* |
| 3.5kN | 4.4*mm* |
| 3.5kN | 4.9*mm* |
| 3.5kN | 6*mm* |

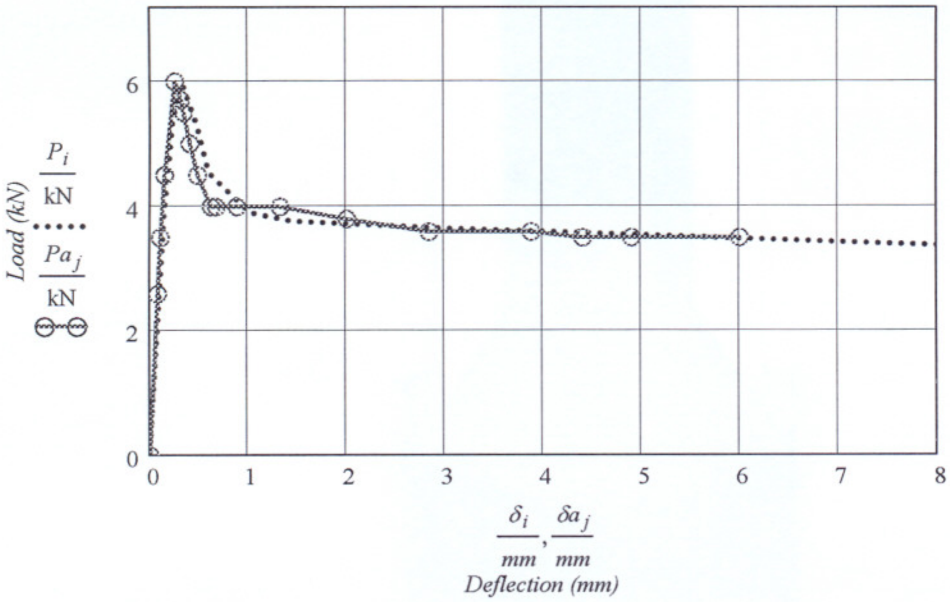


**Figure B-8: Comparison between measured and calculated load-deflection responses.**

# APPENDIX C

## Loading and geometry

$h := 150mm$ (Depth)

$b := 150.mm$ (Width)

$L := 450mm$

$fsh := \frac{6}{5}$ (Form factor for shear)

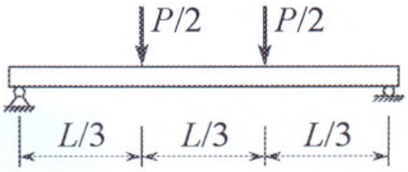


**Figure C-1: Test set up for the beam.**

## First estimation of the stress-strain response

$E := 28 \cdot GPa$

$\mu := 0.2$

$G := \frac{E}{2 \cdot (1 + \mu)}$

$G = 11.66667 \, GPa$

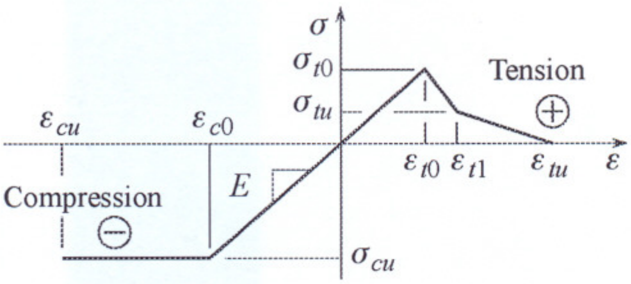


**Figure C-2: Schematic diagram for the stress-strain response.**

$\sigma_{t0} := 4.5 \cdot MPa$ $\quad \varepsilon_{t0} := \frac{\sigma_{t0}}{E}$ $\quad \varepsilon_{t0} = 1.60714 \times 10^{-4}$

$\sigma_u := 1.9 \cdot MPa$ $\quad \varepsilon_{t1} := 11.60714 \cdot 10^{-4}$ $\quad \varepsilon tu := 0.1$

$\varepsilon_{c0} := -0.0016$ $\quad \sigma_{cu} := \varepsilon_{c0} \cdot E$ $\quad \sigma_{cu} = -44.8 \, MPa$

$\varepsilon_{cu} := -0.4$

$\lambda := \frac{\sigma_u - \sigma_{t0}}{\varepsilon_{t1} - \varepsilon_{t0}}$ $\quad \lambda = -2.6 \, GPa$ (The slope of the middle part of the tensile σ-ε response)

$\Psi := \frac{\sigma_u}{\varepsilon_{t1} - \varepsilon tu}$ $\quad \Psi = -0.01922 \, GPa$ (The slope of the last part of the tensile σ-ε response)

## Tensile stress-strain function

$$f_{ct}(\varepsilon_t) := (\varepsilon_t \geq 0)\cdot(\varepsilon_t \leq \varepsilon_{t0})\cdot\frac{\sigma_{t0}}{\varepsilon_{t0}}\cdot\varepsilon_t \ldots$$
$$+(\varepsilon_t > \varepsilon_{t0})\cdot(\varepsilon_t \leq \varepsilon_{t1})\cdot\left[\lambda\cdot(\varepsilon_t - \varepsilon_{t0}) + \sigma_{t0}\right]\ldots$$
$$+(\varepsilon_t > \varepsilon_{t1})\cdot(\varepsilon_t \leq \varepsilon tu)\cdot\Psi\cdot(\varepsilon_t - \varepsilon tu)$$

$$\varepsilon_t := 0, 0.00001 .. \varepsilon tu$$

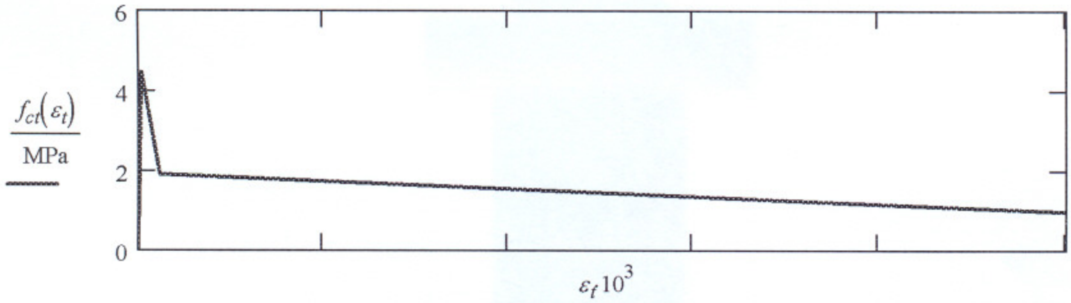


**Figure C-3: Assumed tensile stress-strain response.**

## Compressive stress-strain function

$$f_{cc}(\varepsilon_c) := (\varepsilon_c \geq \varepsilon_{c0})\cdot(\varepsilon_c < 0)\cdot E\cdot\varepsilon_c \ldots$$
$$+(\varepsilon_c \geq \varepsilon_{cu})\cdot(\varepsilon_c < \varepsilon_{c0})\cdot\sigma_{cu}$$

$$\varepsilon_c := \varepsilon_{cu}, \frac{199\cdot\varepsilon_{cu}}{200} .. 0$$

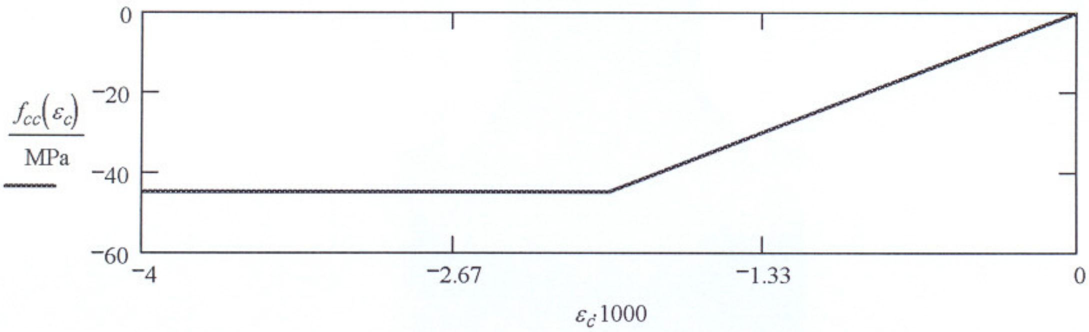


**Figure C-4: Assumed compressive stress-strain response.**

## Calculating moment-curvature response

Refer to Figure 3.2 in section3.2.2

$$\phi(\varepsilon_{bot},a) := \frac{\varepsilon_{bot}}{h-a}$$

$$\varepsilon_{c.top}(\varepsilon_{c.bot},a) := \frac{-a}{h-a}\cdot\varepsilon_{c.bot}$$

$$F_{cc}(\varepsilon_{c.bot},a) := \frac{(h-a)\cdot b}{\varepsilon_{c.bot}}\cdot\int_{\varepsilon_{c.top}(\varepsilon_{c.bot},a)}^{0} f_{cc}(\varepsilon_c)\,d\varepsilon_c$$

$$F_{ct}(\varepsilon_{c.bot},a) := \frac{(h-a)\cdot b}{\varepsilon_{c.bot}}\cdot\int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\,d\varepsilon_c$$

$$Mc(\varepsilon_{c.bot},a) := \frac{(h-a)^2\cdot b}{\varepsilon_{c.bot}^{\;2}}\cdot\int_{\varepsilon_{c.top}(\varepsilon_{c.bot},a)}^{0} f_{cc}(\varepsilon_c)\cdot\varepsilon_c\,d\varepsilon_c$$

$$Mct(\varepsilon_{c.bot},a) := \frac{(h-a)^2\cdot b}{\varepsilon_{c.bot}^{\;2}}\cdot\int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\cdot\varepsilon_c\,d\varepsilon_c$$

$\varepsilon_{bot} := \varepsilon_{t0}$

$a0 := \frac{h}{2}$ $\quad F_{cc}(\varepsilon_{bot},a0) = -25.3125\,\text{kN}$

$F_{ct}(\varepsilon_{bot},a0) = 25.3125\,\text{kN}$

$\varepsilon_{bot} = 1.60714\times10^{-4}$

$Mc(\varepsilon_{bot},a0) = 1.26563\,\text{m kN}$

$Mct(\varepsilon_{bot},a0) = 1.26563\,\text{m kN}$

$$Mext(\varepsilon_{c.bot},a) := Mc(\varepsilon_{c.bot},a) + Mct(\varepsilon_{c.bot},a)$$

Given $F_{cc}(\varepsilon_{bot}, a) + F_{ct}(\varepsilon_{bot}, a) = 0 \cdot \text{kN}$

$$Solve(\varepsilon_{bot}, a) := Find(a)$$

$i := 1..20$

$\varepsilon_{b_i} :=$

| |
|---|
| 0.00001 |
| 0.00004 |
| 0.00007 |
| 0.00008 |
| 0.0001 |
| $\varepsilon_{t0}$ |
| 0.0002 |
| 0.0004 |
| 0.0005 |
| 0.0006 |
| 0.0008 |
| 0.0009 |
| $\varepsilon_{t1}$ |
| 0.002 |
| 0.0025 |
| 0.003 |
| 0.007 |
| 0.009 |
| 0.07 |
| $\varepsilon tu$ |

$a_i := Solve(\varepsilon_{b_i}, a0)$ $\quad \varepsilon_{t_i} := \varepsilon_{c.top}(\varepsilon_{b_i}, a_i)$ $\quad \phi_i := \phi(\varepsilon_{b_i}, a_i)$

$M_i := Mext(\varepsilon_{b_i}, a_i)$ $\quad \sigma b_i := f_{ct}(\varepsilon_{b_i})$

| $\varepsilon_{b_i} =$ | $a_i =$ (mm) | $\varepsilon_{t_i} \cdot 10^3 =$ |
|---|---|---|
| $1 \cdot 10^{-5}$ | 75 | -0.01 |
| $4 \cdot 10^{-5}$ | 75 | -0.04 |
| $7 \cdot 10^{-5}$ | 75 | -0.07 |
| $8 \cdot 10^{-5}$ | 75 | -0.08 |
| $1 \cdot 10^{-4}$ | 75 | -0.1 |
| $1.60714 \cdot 10^{-4}$ | 75 | -0.16071 |
| $2 \cdot 10^{-4}$ | 74.19224 | -0.19574 |
| $4 \cdot 10^{-4}$ | 65.7458 | -0.31213 |
| $5 \cdot 10^{-4}$ | 62.01461 | -0.35241 |
| $6 \cdot 10^{-4}$ | 58.73579 | -0.38615 |
| $8 \cdot 10^{-4}$ | 53.20499 | -0.43973 |
| $9 \cdot 10^{-4}$ | 50.82289 | -0.4612 |
| $1.16071 \cdot 10^{-3}$ | 45.4371 | -0.50438 |
| $2 \cdot 10^{-3}$ | 34.90281 | -0.60649 |
| $2.5 \cdot 10^{-3}$ | 31.31002 | -0.65949 |
| $3 \cdot 10^{-3}$ | 28.65076 | -0.70831 |
| $7 \cdot 10^{-3}$ | 18.94111 | -1.01167 |
| $9 \cdot 10^{-3}$ | 16.72799 | -1.12966 |
| 0.07 | 5.73385 | -2.78214 |
| 0.1 | 4.33583 | -2.97659 |

| $\phi_i =$ ($\frac{1}{m}$) | $M_i =$ ($m \cdot kN$) |
|---|---|
| $1.33333 \cdot 10^{-4}$ | 0.1575 |
| $5.33333 \cdot 10^{-4}$ | 0.63 |
| $9.33333 \cdot 10^{-4}$ | 1.1025 |
| $1.06667 \cdot 10^{-3}$ | 1.26 |
| $1.33333 \cdot 10^{-3}$ | 1.575 |
| $2.14286 \cdot 10^{-3}$ | 2.53125 |
| $2.63825 \cdot 10^{-3}$ | 3.02218 |
| $4.74754 \cdot 10^{-3}$ | 3.99707 |
| $5.68276 \cdot 10^{-3}$ | 4.15126 |
| $6.57432 \cdot 10^{-3}$ | 4.21365 |
| $8.26489 \cdot 10^{-3}$ | 4.17754 |
| $9.07468 \cdot 10^{-3}$ | 4.10633 |
| 0.0111 | 3.81496 |
| 0.01738 | 3.24251 |
| 0.02106 | 3.12254 |
| 0.02472 | 3.05446 |
| 0.05341 | 2.90317 |
| 0.06753 | 2.87106 |
| 0.48521 | 1.69905 |
| 0.68651 | 1.07699 |

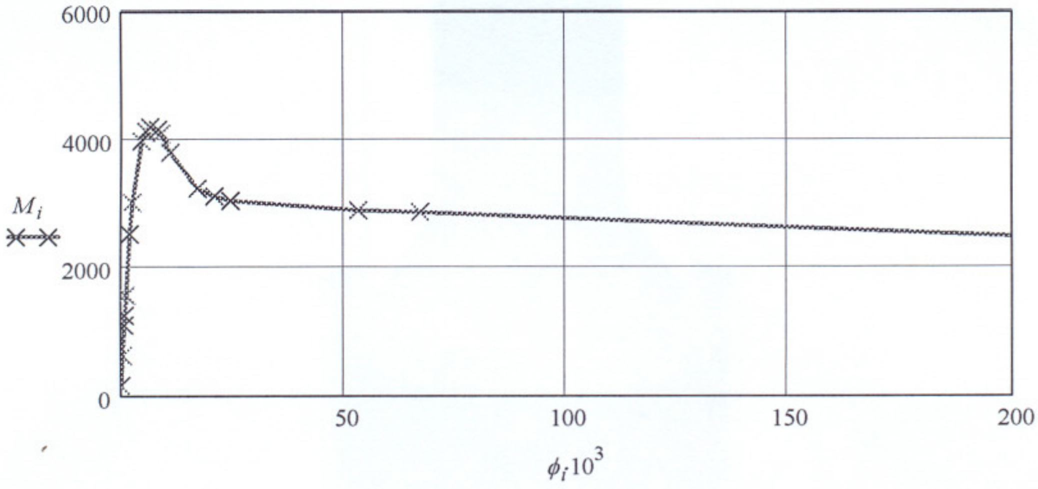


**Figure C-5: Calculated moment-curvature response.**

## Output stress-strain responses

$$\sigma t_i := f_{cc}\left(\varepsilon_{t_i}\right)$$

$\sigma b_i =$ (MPa)

| |
|---|
| 0.28 |
| 1.12 |
| 1.96 |
| 2.24 |
| 2.8 |
| 4.5 |
| 4.39786 |
| 3.87786 |
| 3.61786 |
| 3.35786 |
| 2.83786 |
| 2.57786 |
| 1.9 |
| 1.88387 |
| 1.87425 |
| 1.86464 |
| 1.78775 |
| 1.7493 |
| 0.57669 |
| 0 |

$\frac{\sigma t_i}{MPa} =$

| |
|---|
| -0.28 |
| -1.12 |
| -1.96 |
| -2.24 |
| -2.8 |
| -4.5 |
| -5.48066 |
| -8.73966 |
| -9.8676 |
| -10.81214 |
| -12.31253 |
| -12.91364 |
| -14.12265 |
| -16.9818 |
| -18.46576 |
| -19.83255 |
| -28.32664 |
| -31.63045 |
| -44.8 |
| -44.8 |

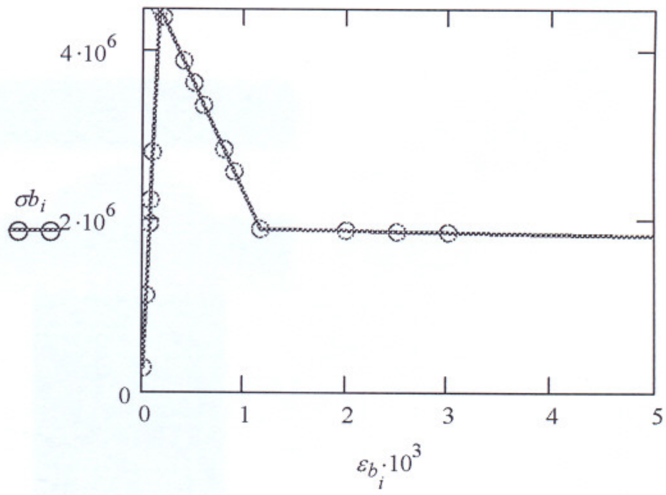


**Figure C-6: Output tensile stress-strain response.**

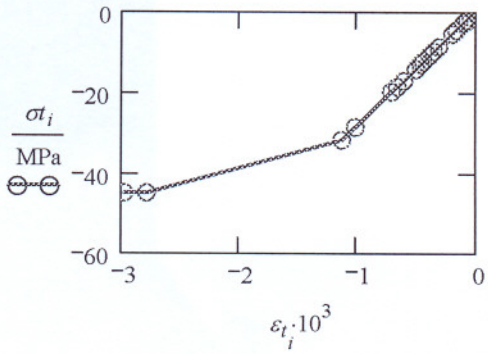


**Figure C-7: Output compressive stress-strain response.**

## Calculating load-deflection response

Refer to Figure 3.4 and Figure 3.5 in section 3.2.3

$M_{max} := max(M)$

$M_{max} = 4.21365\, m \cdot \mathrm{kN}$

$\phi h := lookup\left(M_{max}, M, \phi\right)_1$ $\phi h = 6.57432 \times 10^{-3}\, \frac{1}{\mathrm{m}}$

$MaxPos := match\left(M_{max}, M\right)_1$ $MaxPos = 10$

$M1 := submatrix(M, 1, MaxPos, 1, 1)$

$\phi_1 := submatrix\left(\phi, 1, MaxPos, 1, 1\right)$

$length(M) = 20$

$M2 := submatrix(M, MaxPos, length(M) - 1, 1, 1)$

$\phi_2 := submatrix\left(\phi, MaxPos, length(M) - 1, 1, 1\right)$

$length(M2) = 10$

$MaxPos2 := match\left(M_{max}, M2\right)_1$

$MaxPos2 = 1$

$Mmin := min(M2)$

$Mmin = 1.69905\, m \cdot \mathrm{kN}$

$MinPos2 := match(Mmin, M2)_1$

$MinPos2 = 10$

$posi := match\left(M2_{MinPos2}, M\right)_1$

$posi = 19$

$length(M2) = 10$

$x := 0m, 0.015 \cdot m .. 0.15m$

$ms(x,M) := M \cdot \frac{x}{0.15m}$    $\Delta ms(x,M) := max(M2) - ms(x,M)$

$M1 =$ (m · kN)

| |
|---|
| 0.1575 |
| 0.63 |
| 1.1025 |
| 1.26 |
| 1.575 |
| 2.53125 |
| 3.02218 |
| 3.99707 |
| 4.15126 |
| 4.21365 |

$\phi_1 =$ (1/m)

| |
|---|
| 1.33333·10^-4 |
| 5.33333·10^-4 |
| 9.33333·10^-4 |
| 1.06667·10^-3 |
| 1.33333·10^-3 |
| 2.14286·10^-3 |
| 2.63825·10^-3 |
| 4.74754·10^-3 |
| 5.68276·10^-3 |
| 6.57432·10^-3 |

$$M2 = \begin{pmatrix} 4.21365 \\ 4.17754 \\ 4.10633 \\ 3.81496 \\ 3.24251 \\ 3.12254 \\ 3.05446 \\ 2.90317 \\ 2.87106 \\ 1.69905 \end{pmatrix} m \cdot kN$$

$\Delta M2 := M_{max} - M2$

$$\phi_2 = \begin{pmatrix} 6.57432 \times 10^{-3} \\ 8.26489 \times 10^{-3} \\ 9.07468 \times 10^{-3} \\ 0.0111 \\ 0.01738 \\ 0.02106 \\ 0.02472 \\ 0.05341 \\ 0.06753 \\ 0.48521 \end{pmatrix} m^{-1}$$

$\Delta M2 =$ (m · kN)

| |
|---|
| 0 |
| 0.03612 |
| 0.10733 |
| 0.39869 |
| 0.97114 |
| 1.09112 |
| 1.15919 |
| 1.31048 |
| 1.34259 |
| 2.51461 |

## Deflection due to bending moment

$$\delta m_i := \begin{cases} \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1,\phi_1,ms\left(x,M_i\right)\right)\cdot x\,dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(M1,\phi_1,ms\left(\frac{L}{3},M_i\right)\right)\cdot x\,dx & \text{if } (i \le MaxPos) \\ \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1,\phi_1,ms\left(x,M_i\right)\right)\cdot x\,dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(\Delta M2,\phi_2,\Delta ms\left(\frac{L}{3},M_i\right)\right)\cdot x\,dx & \text{if } (MaxPos \le i) \end{cases}$$

## Deflection due to shear force

$$P_i := M_i \cdot \frac{6}{L} \qquad \delta v_i := \frac{P_i\,fsh}{G\cdot b\cdot h\cdot 2}\cdot\int_{0mm}^{\frac{L}{3}} 1.\,dx \qquad \delta_i := \delta v_i + \delta m_i$$

| $P_i =$ (kN) | $\delta m_i =$ (mm) | $\delta v_i =$ (mm) | $\delta_i =$ (mm) |
|---|---|---|---|
| 2.1 | $2.875\cdot10^{-3}$ | $7.2\cdot10^{-4}$ | $3.595\cdot10^{-3}$ |
| 8.4 | 0.0115 | $2.88\cdot10^{-3}$ | 0.01438 |
| 14.7 | 0.02013 | $5.04\cdot10^{-3}$ | 0.02516 |
| 16.8 | 0.023 | $5.76\cdot10^{-3}$ | 0.02876 |
| 21 | 0.02875 | $7.2\cdot10^{-3}$ | 0.03595 |
| 33.75 | 0.04621 | 0.01157 | 0.05778 |
| 40.29573 | 0.05643 | 0.01382 | 0.07024 |
| 53.29426 | 0.09585 | 0.01827 | 0.11412 |
| 55.35015 | 0.11114 | 0.01898 | 0.13012 |
| 56.18204 | 0.12479 | 0.01926 | 0.14405 |
| 55.7005 | 0.1479 | 0.0191 | 0.167 |
| 54.75101 | 0.15818 | 0.01877 | 0.17695 |
| 50.86612 | 0.18301 | 0.01744 | 0.20045 |
| 43.23349 | 0.26539 | 0.01482 | 0.28021 |
| 41.63382 | 0.31626 | 0.01427 | 0.33053 |
| 40.72619 | 0.36721 | 0.01396 | 0.38117 |
| 38.70896 | 0.76961 | 0.01327 | 0.78288 |
| 38.28087 | 0.96796 | 0.01312 | 0.98108 |
| 22.65397 | 6.83411 | $7.76708\cdot10^{-3}$ | 6.84188 |
| 14.35984 | 9.94767 | $4.92337\cdot10^{-3}$ | 9.9526 |

## Measured load-deflection responses

| $P1_i :=$ | $\delta 1_i :=$ | $P2_i :=$ | $\delta 2_i :=$ | $P3_i :=$ | $\delta 3_i :=$ |
|---|---|---|---|---|---|
| 0kN | $0 \cdot mm$ | $0 \cdot kN$ | 0 | $0 \cdot kN$ | $0 \cdot mm$ |
| $5.65 \cdot kN$ | $0.0195 \cdot mm$ | $5.90 \cdot kN$ | $0.0073 \cdot mm$ | $5.77 \cdot kN$ | $0.0098 \cdot mm$ |
| $10.54 \cdot kN$ | $0.027 \cdot mm$ | $10.55 \cdot kN$ | $0.0171 \cdot mm$ | $10.78 \cdot kN$ | $0.0098 \cdot mm$ |
| $15.17 \cdot kN$ | $0.029 \cdot mm$ | $15.90 \cdot kN$ | $0.022 \cdot mm$ | $20.90 \cdot kN$ | $0.02 \cdot mm$ |
| $20.10 \cdot kN$ | $0.032 \cdot mm$ | $20.40 \cdot kN$ | $0.022 \cdot mm$ | $29.70 \cdot kN$ | $0.029 \cdot mm$ |
| $25.31 \cdot kN$ | $0.039 \cdot mm$ | $25.40 \cdot kN$ | $0.024 \cdot mm$ | $36.00 \cdot kN$ | $0.037 \cdot mm$ |
| $35.56 \cdot kN$ | $0.051 \cdot mm$ | $29.60 \cdot kN$ | $0.029 \cdot mm$ | $41.17 \cdot kN$ | $0.044 \cdot mm$ |
| $40.93 \cdot kN$ | $0.059 \cdot mm$ | $36.60 \cdot kN$ | $0.032 \cdot mm$ | $46.70 \cdot kN$ | $0.049 \cdot mm$ |
| $45.90 \cdot kN$ | $0.066 \cdot mm$ | $45.50 \cdot kN$ | $0.041 \cdot mm$ | $53.99 \cdot kN$ | $0.068 \cdot mm$ |
| $50.21 \cdot kN$ | $0.071 \cdot mm$ | $48.15 \cdot kN$ | $0.05 \cdot mm$ | $20.30 \cdot kN$ | $2.13 \cdot mm$ |
| $51.60 \cdot kN$ | $0.09 \cdot mm$ | $17.00 \cdot kN$ | $1.46 \cdot mm$ | $21.60 \cdot kN$ | $2.6 \cdot mm$ |
| $18.10 \cdot kN$ | $2.29 \cdot mm$ | $17.60 \cdot kN$ | $1.5 \cdot mm$ | $21.00 \cdot kN$ | $2.74 \cdot mm$ |
| $17.10 \cdot kN$ | $2.38 \cdot mm$ | $18.10 \cdot kN$ | $1.52 \cdot mm$ | $20.30 \cdot kN$ | $2.9 \cdot mm$ |
| $17.10 \cdot kN$ | $2.75 \cdot mm$ | $18.10 \cdot kN$ | $1.53 \cdot mm$ | $20.30 \cdot kN$ | $3.13 \cdot mm$ |
| $16.80 \cdot kN$ | $2.8 \cdot mm$ | $18.10 \cdot kN$ | $1.55 \cdot mm$ | $18.35 \cdot kN$ | $3.3 \cdot mm$ |
| $16.80 \cdot kN$ | $3.01 \cdot mm$ | $18.50 \cdot kN$ | $1.67 \cdot mm$ | $18.35 \cdot kN$ | $3.5 \cdot mm$ |
| $16.40 \cdot kN$ | $3.2 \cdot mm$ | $18.70 \cdot kN$ | $1.71 \cdot mm$ | $17.60 \cdot kN$ | $3.7 \cdot mm$ |
| $16.40 \cdot kN$ | $3.35 \cdot mm$ | $18.50 \cdot kN$ | $2.00 \cdot mm$ | $17.40 \cdot kN$ | $3.8 \cdot mm$ |
| $16.40 \cdot kN$ | $3.52 \cdot mm$ | $18.10 \cdot kN$ | $2.15 \cdot mm$ | $16.30 \cdot kN$ | $3.9 \cdot mm$ |
| $15.79 \cdot kN$ | $4.58 \cdot mm$ | $15.67 \cdot kN$ | $4.11 \cdot mm$ | $15.80 \cdot kN$ | $4.02 \cdot mm$ |

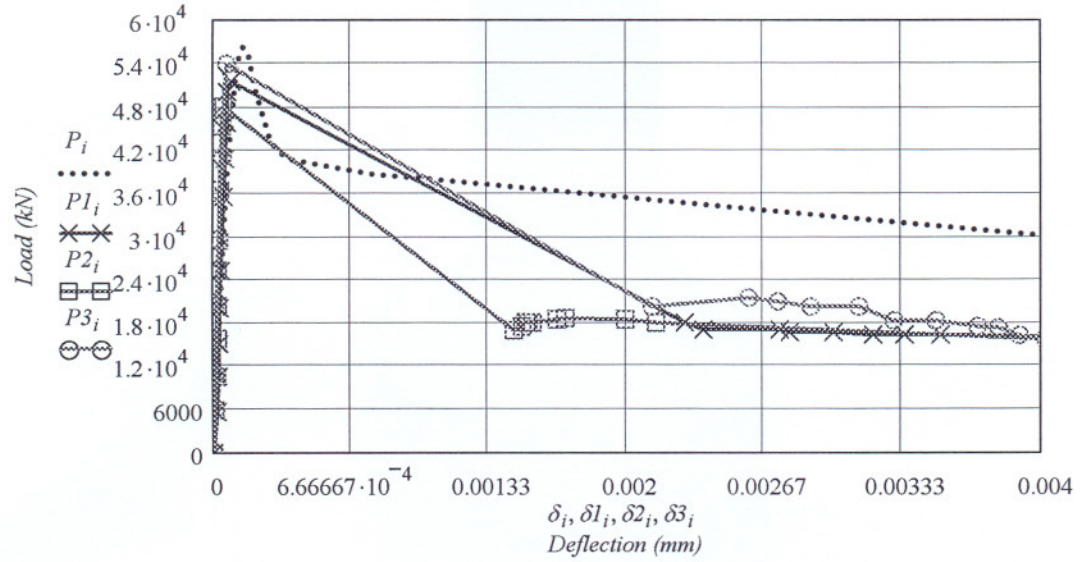


**Figure C-8: Comparison between measured and calculated load-deflection responses.**

## Loading and geometry

$h := 150mm$ (Depth)

$b := 150.mm$ (Width)

$L := 450mm$

$fsh := \frac{6}{5}$ (Form factor for shear)

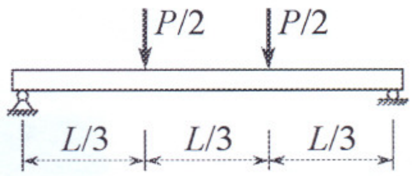


**Figure C-9: Test set up for the beam.**

## Adopted stress-strain response

$E := 28 \cdot \text{GPa}$

$\mu := 0.015$

$G := \frac{E}{2 \cdot (1 + \mu)}$

$G = 13.7931\,\text{GPa}$

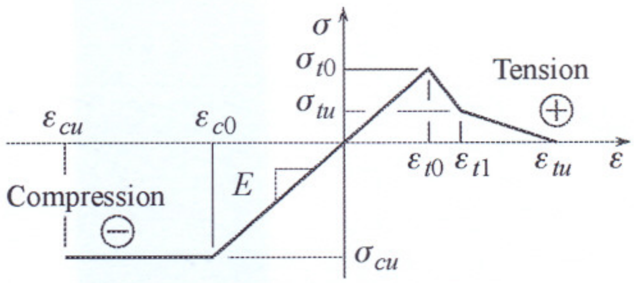


**Figure C-10: Schematic diagram for the stress-strain response.**

$\sigma_{t0} := 4.2 \cdot \text{MPa}$

$\varepsilon_{t0} := \frac{\sigma_{t0}}{E}$ $\qquad$ $\varepsilon_{t0} = 1.5 \times 10^{-4}$

$\sigma_u := 1.1 \cdot \text{MPa}$

$\varepsilon_{t1} := 13 \cdot 10^{-4}$ $\qquad$ $\varepsilon tu := 0.08$

$\varepsilon_{c0} := -0.0016$

$\sigma_{cu} := \varepsilon_{c0} \cdot E$ $\qquad$ $\sigma_{cu} = -44.8\,\text{MPa}$

$\varepsilon_{cu} := -0.4$

$\lambda := \frac{\sigma_u - \sigma_{t0}}{\varepsilon_{t1} - \varepsilon_{t0}}$ $\qquad$ $\lambda = -2.69565\,\text{GPa}$ (The slope of the middle part of the tensile σ-ε response)

$\Psi := \frac{\sigma_u}{\varepsilon_{t1} - \varepsilon tu}$ $\qquad$ $\Psi = -0.01398\,\text{GPa}$ (The slope of the last part of the tensile σ-ε response)

## Tensile stress-strain function

$$f_{ct}(\varepsilon_t) := (\varepsilon_t \geq 0) \cdot (\varepsilon_t \leq \varepsilon_{t0}) \cdot \frac{\sigma_{t0}}{\varepsilon_{t0}} \cdot \varepsilon_t \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t0}) \cdot (\varepsilon_t \leq \varepsilon_{t1}) \cdot \left[\lambda \cdot (\varepsilon_t - \varepsilon_{t0}) + \sigma_{t0}\right] \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t1}) \cdot (\varepsilon_t \leq \varepsilon tu) \cdot \Psi \cdot (\varepsilon_t - \varepsilon tu)$$

$\varepsilon_t := 0, 0.00001 .. \varepsilon tu$

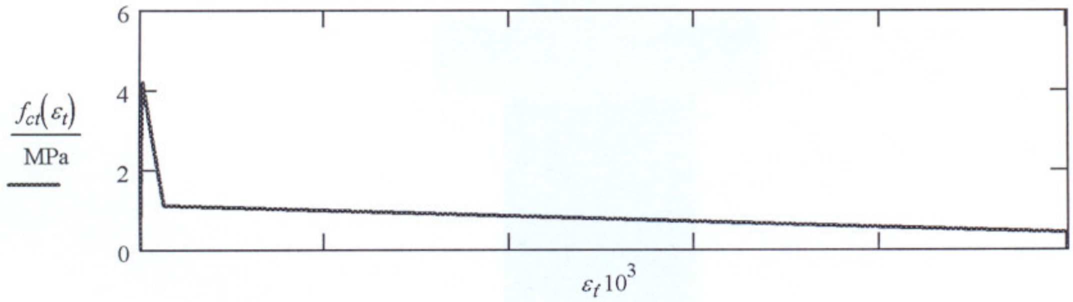


**Figure C-11: Assumed tensile stress-strain response.**

## Compressive stress-strain function

$$f_{cc}(\varepsilon_c) := (\varepsilon_c \geq \varepsilon_{c0}) \cdot (\varepsilon_c < 0) \cdot E \cdot \varepsilon_c \ldots$$
$$+ (\varepsilon_c \geq \varepsilon_{cu}) \cdot (\varepsilon_c < \varepsilon_{c0}) \cdot \sigma_{cu}$$

$$\varepsilon_c := \varepsilon_{cu}, \frac{199 \cdot \varepsilon_{cu}}{200} .. 0$$

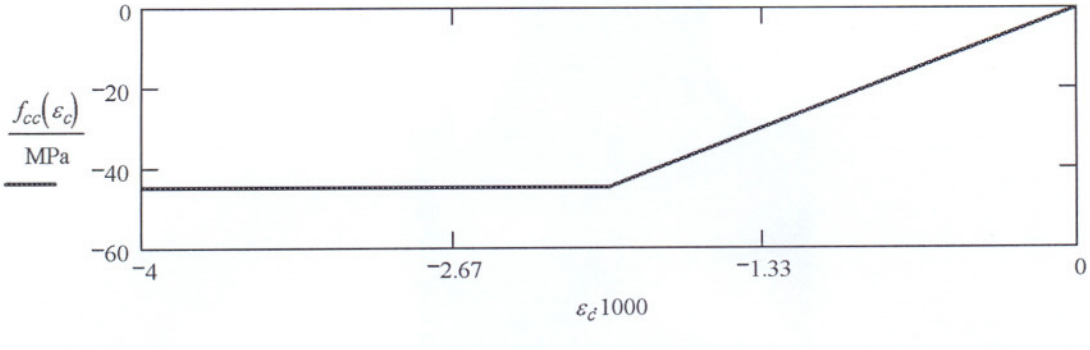


**Figure C-12: Assumed compressive stress-strain response.**

## Calculating moment-curvature response

$$\phi(\varepsilon_{bot}, a) := \frac{\varepsilon_{bot}}{h - a}$$

$$\varepsilon_{c.top}(\varepsilon_{c.bot}, a) := \frac{-a}{h-a} \cdot \varepsilon_{c.bot}$$

$$F_{cc}(\varepsilon_{c.bot}, a) := \frac{(h-a)\cdot b}{\varepsilon_{c.bot}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c)\, d\varepsilon_c$$

$$F_{ct}(\varepsilon_{c.bot}, a) := \frac{(h-a)\cdot b}{\varepsilon_{c.bot}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\, d\varepsilon_c$$

$$Mc(\varepsilon_{c.bot}, a) := \frac{(h-a)^2\cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c)\cdot \varepsilon_c\, d\varepsilon_c$$

$$Mct(\varepsilon_{c.bot}, a) := \frac{(h-a)^2\cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\cdot \varepsilon_c\, d\varepsilon_c$$

$\varepsilon_{bot} := \varepsilon_{t0}$

$a0 := \frac{h}{2}$ $\quad F_{cc}(\varepsilon_{bot}, a0) = -23.625\,\text{kN}$

$F_{ct}(\varepsilon_{bot}, a0) = 23.625\,\text{kN}$

$\varepsilon_{bot} = 1.5 \times 10^{-4}$

$Mc(\varepsilon_{bot}, a0) = 1.18125\,\text{m kN}$

$Mct(\varepsilon_{bot}, a0) = 1.18125\,\text{m kN}$

$$Mext(\varepsilon_{c.bot}, a) := Mc(\varepsilon_{c.bot}, a) + Mct(\varepsilon_{c.bot}, a)$$

*Given* $F_{cc}(\varepsilon_{bot}, a) + F_{ct}(\varepsilon_{bot}, a) = 0 \cdot \text{kN}$

$Solve(\varepsilon_{bot}, a) := Find(a)$

$i := 1..20$

$\varepsilon b_i :=$

| |
|---|
| 0.00001 |
| 0.00004 |
| 0.00007 |
| 0.00008 |
| 0.00009 |
| $\varepsilon_{t0}$ |
| 0.0002 |
| 0.0004 |
| 0.0005 |
| 0.0006 |
| 0.0007 |
| 0.0008 |
| $\varepsilon_{t1}$ |
| 0.002 |
| 0.0025 |
| 0.003 |
| 0.007 |
| 0.009 |
| 0.07 |
| $\varepsilon tu$ |

$a_i := Solve(\varepsilon b_i, a0)$ $\varepsilon t_i := \varepsilon_{c.top}(\varepsilon b_i, a_i)$ $\phi_i := \phi(\varepsilon b_i, a_i)$

$M_i := Mext(\varepsilon b_i, a_i)$ $\sigma b_i := f_{ct}(\varepsilon b_i)$

| $\varepsilon b_i =$ | $a_i =$ (mm) | $\varepsilon t_i \cdot 10^3 =$ |
|---|---|---|
| $1 \cdot 10^{-5}$ | 75 | -0.01 |
| $4 \cdot 10^{-5}$ | 75 | -0.04 |
| $7 \cdot 10^{-5}$ | 75 | -0.07 |
| $8 \cdot 10^{-5}$ | 75 | -0.08 |
| $9 \cdot 10^{-5}$ | 75 | -0.09 |
| $1.5 \cdot 10^{-4}$ | 75 | -0.15 |
| $2 \cdot 10^{-4}$ | 73.66931 | -0.19303 |
| $4 \cdot 10^{-4}$ | 64.58606 | -0.30246 |
| $5 \cdot 10^{-4}$ | 60.7309 | -0.34016 |
| $6 \cdot 10^{-4}$ | 57.35878 | -0.37149 |
| $7 \cdot 10^{-4}$ | 54.36866 | -0.39797 |
| $8 \cdot 10^{-4}$ | 51.68005 | -0.42051 |
| $1.3 \cdot 10^{-3}$ | 41.0663 | -0.49008 |
| $2 \cdot 10^{-3}$ | 32.03271 | -0.54308 |
| $2.5 \cdot 10^{-3}$ | 28.15605 | -0.57771 |
| $3 \cdot 10^{-3}$ | 25.3521 | -0.61017 |
| $7 \cdot 10^{-3}$ | 15.71833 | -0.81938 |
| $9 \cdot 10^{-3}$ | 13.67899 | -0.9031 |
| 0.07 | 3.81283 | -1.82573 |
| 0.08 | 3.37498 | -1.84142 |

| $\phi_i =$ ($\frac{1}{m}$) | $M_i =$ ($m \cdot kN$) |
|---|---|
| $1.33333 \cdot 10^{-4}$ | 0.1575 |
| $5.33333 \cdot 10^{-4}$ | 0.63 |
| $9.33333 \cdot 10^{-4}$ | 1.1025 |
| $1.06667 \cdot 10^{-3}$ | 1.26 |
| $1.2 \cdot 10^{-3}$ | 1.4175 |
| $2 \cdot 10^{-3}$ | 2.3625 |
| $2.62018 \cdot 10^{-3}$ | 2.94361 |
| $4.68308 \cdot 10^{-3}$ | 3.77405 |
| $5.60104 \cdot 10^{-3}$ | 3.8887 |
| $6.4766 \cdot 10^{-3}$ | 3.91902 |
| $7.31978 \cdot 10^{-3}$ | 3.89386 |
| $8.1367 \cdot 10^{-3}$ | 3.82901 |
| 0.01193 | 3.1575 |
| 0.01695 | 2.43118 |
| 0.02052 | 2.20399 |
| 0.02407 | 2.06926 |
| 0.05213 | 1.76063 |
| 0.06602 | 1.71135 |
| 0.47884 | 0.78373 |
| 0.54561 | 0.63011 |

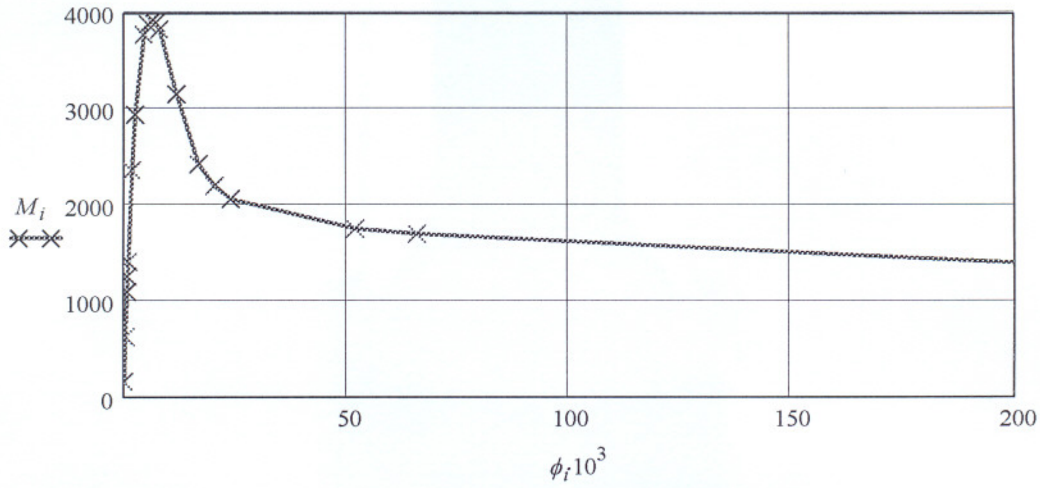


**Figure C-13: Calculated moment-curvature response.**

## Output stress-strain responses

$$\sigma t_i := f_{cc}\left(\varepsilon_{t_i}\right)$$

$\sigma b_i =$ MPa

| |
|---|
| 0.28 |
| 1.12 |
| 1.96 |
| 2.24 |
| 2.52 |
| 4.2 |
| 4.06522 |
| 3.52609 |
| 3.25652 |
| 2.98696 |
| 2.71739 |
| 2.44783 |
| 1.1 |
| 1.09022 |
| 1.08323 |
| 1.07624 |
| 1.02033 |
| 0.99238 |
| 0.13977 |
| 0 |

$\frac{\sigma t_i}{MPa} =$

| |
|---|
| -0.28 |
| -1.12 |
| -1.96 |
| -2.24 |
| -2.52 |
| -4.2 |
| -5.40475 |
| -8.46892 |
| -9.52438 |
| -10.40171 |
| -11.14306 |
| -11.77414 |
| -13.72223 |
| -15.20618 |
| -16.1758 |
| -17.08473 |
| -22.94277 |
| -25.28667 |
| -44.8 |
| -44.8 |

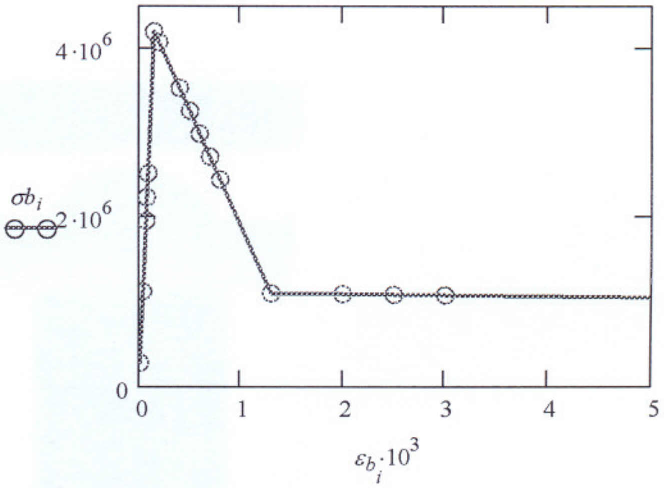


**Figure C-14: Output tensile stress-strain response.**

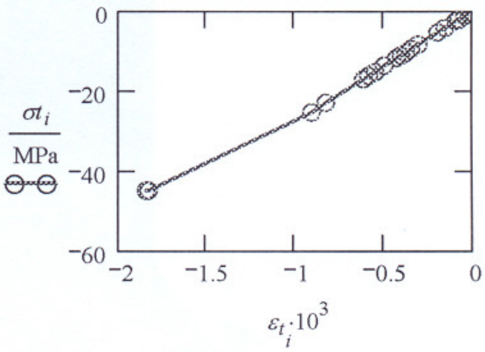


**Figure C-15: Output compressive stress-strain response.**

## Calculating load-deflection response

$M_{max} := max(M)$

$M_{max} = 3.91902\, m \cdot kN$

$\phi h := lookup(M_{max}, M, \phi)_1$ $\phi h = 6.4766 \times 10^{-3} \frac{1}{m}$

$MaxPos := match(M_{max}, M)_1$ $MaxPos = 10$

$M1 := submatrix(M, 1, MaxPos, 1, 1)$

$\phi_1 := submatrix(\phi, 1, MaxPos, 1, 1)$

$length(M) = 20$

$M2 := submatrix(M, MaxPos, length(M) - 1, 1, 1)$

$\phi_2 := submatrix(\phi, MaxPos, length(M) - 1, 1, 1)$

$length(M2) = 10$

$MaxPos2 := match(M_{max}, M2)_1$

$MaxPos2 = 1$

$Mmin := min(M2)$

$Mmin = 0.78373\, m \cdot kN$

$MinPos2 := match(Mmin, M2)_1$

$MinPos2 = 10$

$posi := match(M2_{MinPos2}, M)_1$

$posi = 19$

$length(M2) = 10$

$x := 0m, 0.015 \cdot m .. 0.15m$

$ms(x,M) := M \cdot \frac{x}{0.15m}$ $\Delta ms(x,M) := max(M2) - ms(x,M)$

$M1 =$ (m · kN)

| |
|---|
| 0.1575 |
| 0.63 |
| 1.1025 |
| 1.26 |
| 1.4175 |
| 2.3625 |
| 2.94361 |
| 3.77405 |
| 3.8887 |
| 3.91902 |

$\phi_1 =$ (1/m)

| |
|---|
| $1.33333 \cdot 10^{-4}$ |
| $5.33333 \cdot 10^{-4}$ |
| $9.33333 \cdot 10^{-4}$ |
| $1.06667 \cdot 10^{-3}$ |
| $1.2 \cdot 10^{-3}$ |
| $2 \cdot 10^{-3}$ |
| $2.62018 \cdot 10^{-3}$ |
| $4.68308 \cdot 10^{-3}$ |
| $5.60104 \cdot 10^{-3}$ |
| $6.4766 \cdot 10^{-3}$ |

$$M2 = \begin{pmatrix} 3.91902 \\ 3.89386 \\ 3.82901 \\ 3.1575 \\ 2.43118 \\ 2.20399 \\ 2.06926 \\ 1.76063 \\ 1.71135 \\ 0.78373 \end{pmatrix} m \cdot kN$$

$\Delta M2 := M_{max} - M2$

$$\phi_2 = \begin{pmatrix} 6.4766 \times 10^{-3} \\ 7.31978 \times 10^{-3} \\ 8.1367 \times 10^{-3} \\ 0.01193 \\ 0.01695 \\ 0.02052 \\ 0.02407 \\ 0.05213 \\ 0.06602 \\ 0.47884 \end{pmatrix} m^{-1}$$

$\Delta M2 =$ (m · kN)

| |
|---|
| 0 |
| 0.02516 |
| 0.09001 |
| 0.76152 |
| 1.48784 |
| 1.71504 |
| 1.84977 |
| 2.15839 |
| 2.20767 |
| 3.1353 |

## Deflection due to bending moment

$$\delta m_i := \left| \begin{array}{l} \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(M1, \phi_1, ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \ \ if\ (i \le MaxPos) \\ \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(\Delta M2, \phi_2, \Delta ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \ \ if\ (MaxPos \le i) \end{array} \right.$$

## Deflection due to shear force

$$P_i := M_i \cdot \frac{6}{L} \qquad \delta v_i := \frac{P_i fsh}{G \cdot b \cdot h \cdot 2} \cdot \int_{0mm}^{\frac{L}{3}} 1.\, dx \qquad \delta_i := \delta v_i + \delta m_i \qquad perc_i := \frac{\delta v_i \cdot 100}{\delta_i}$$

| $P_i =$ (kN) | $\delta m_i =$ (mm) | $\delta v_i =$ (mm) | $\delta_i =$ (mm) | $perc_i =$ |
|---|---|---|---|---|
| 2.1 | $2.875 \cdot 10^{-3}$ | $6.09 \cdot 10^{-4}$ | $3.484 \cdot 10^{-3}$ | 17.47991 |
| 8.4 | 0.0115 | $2.436 \cdot 10^{-3}$ | 0.01394 | 17.47991 |
| 14.7 | 0.02013 | $4.263 \cdot 10^{-3}$ | 0.02439 | 17.47991 |
| 16.8 | 0.023 | $4.872 \cdot 10^{-3}$ | 0.02787 | 17.47991 |
| 18.9 | 0.02588 | $5.481 \cdot 10^{-3}$ | 0.03136 | 17.47991 |
| 31.5 | 0.04313 | $9.135 \cdot 10^{-3}$ | 0.05226 | 17.47991 |
| 39.24811 | 0.0558 | 0.01138 | 0.06719 | 16.9412 |
| 50.32062 | 0.09364 | 0.01459 | 0.10823 | 13.48299 |
| 51.84927 | 0.10833 | 0.01504 | 0.12337 | 12.18804 |
| 52.25365 | 0.12123 | 0.01515 | 0.13639 | 11.11087 |
| 51.91817 | 0.13259 | 0.01506 | 0.14764 | 10.19768 |
| 51.05347 | 0.14301 | 0.01481 | 0.15781 | 9.38169 |
| 42.10002 | 0.18854 | 0.01221 | 0.20075 | 6.08172 |
| 32.41574 | 0.25385 | $9.40056 \cdot 10^{-3}$ | 0.26325 | 3.5709 |
| 29.38647 | 0.30253 | $8.52208 \cdot 10^{-3}$ | 0.31105 | 2.73977 |
| 27.59007 | 0.35159 | $8.00112 \cdot 10^{-3}$ | 0.35959 | 2.22505 |
| 23.47509 | 0.74425 | $6.80778 \cdot 10^{-3}$ | 0.75105 | 0.90643 |
| 22.818 | 0.93928 | $6.61722 \cdot 10^{-3}$ | 0.9459 | 0.69957 |
| 10.4497 | 6.73864 | $3.03041 \cdot 10^{-3}$ | 6.74167 | 0.04495 |
| 8.40141 | 7.69906 | $2.43641 \cdot 10^{-3}$ | 7.70149 | 0.03164 |

## Measured load-deflection responses

| $P1_i :=$ | $\delta 1_i :=$ | $P2_i :=$ | $\delta 2_i :=$ | $P3_i :=$ | $\delta 3_i :=$ |
|---|---|---|---|---|---|
| 0kN | $0 \cdot mm$ | $0 \cdot kN$ | 0 | $0 \cdot kN$ | $0 \cdot mm$ |
| $5.65 \cdot kN$ | $0.0195 \cdot mm$ | $5.90 \cdot kN$ | $0.0073 \cdot mm$ | $5.77 \cdot kN$ | $0.0098 \cdot mm$ |
| $10.54 \cdot kN$ | $0.027 \cdot mm$ | $10.55 \cdot kN$ | $0.0171 \cdot mm$ | $10.78 \cdot kN$ | $0.0098 \cdot mm$ |
| $15.17 \cdot kN$ | $0.029 \cdot mm$ | $15.90 \cdot kN$ | $0.022 \cdot mm$ | $20.90 \cdot kN$ | $0.02 \cdot mm$ |
| $20.10 \cdot kN$ | $0.032 \cdot mm$ | $20.40 \cdot kN$ | $0.022 \cdot mm$ | $29.70 \cdot kN$ | $0.029 \cdot mm$ |
| $25.31 \cdot kN$ | $0.039 \cdot mm$ | $25.40 \cdot kN$ | $0.024 \cdot mm$ | $36.00 \cdot kN$ | $0.037 \cdot mm$ |
| $35.56 \cdot kN$ | $0.051 \cdot mm$ | $29.60 \cdot kN$ | $0.029 \cdot mm$ | $41.17 \cdot kN$ | $0.044 \cdot mm$ |
| $40.93 \cdot kN$ | $0.059 \cdot mm$ | $36.60 \cdot kN$ | $0.032 \cdot mm$ | $46.70 \cdot kN$ | $0.049 \cdot mm$ |
| $45.90 \cdot kN$ | $0.066 \cdot mm$ | $45.50 \cdot kN$ | $0.041 \cdot mm$ | $53.99 \cdot kN$ | $0.068 \cdot mm$ |
| $50.21 \cdot kN$ | $0.071 \cdot mm$ | $48.15 \cdot kN$ | $0.05 \cdot mm$ | $20.30 \cdot kN$ | $2.13 \cdot mm$ |
| $51.60 \cdot kN$ | $0.09 \cdot mm$ | $17.00 \cdot kN$ | $1.46 \cdot mm$ | $21.60 \cdot kN$ | $2.6 \cdot mm$ |
| $18.10 \cdot kN$ | $2.29 \cdot mm$ | $17.60 \cdot kN$ | $1.5 \cdot mm$ | $21.00 \cdot kN$ | $2.74 \cdot mm$ |
| $17.10 \cdot kN$ | $2.38 \cdot mm$ | $18.10 \cdot kN$ | $1.52 \cdot mm$ | $20.30 \cdot kN$ | $2.9 \cdot mm$ |
| $17.10 \cdot kN$ | $2.75 \cdot mm$ | $18.10 \cdot kN$ | $1.53 \cdot mm$ | $20.30 \cdot kN$ | $3.13 \cdot mm$ |
| $16.80 \cdot kN$ | $2.8 \cdot mm$ | $18.10 \cdot kN$ | $1.55 \cdot mm$ | $18.35 \cdot kN$ | $3.3 \cdot mm$ |
| $16.80 \cdot kN$ | $3.01 \cdot mm$ | $18.50 \cdot kN$ | $1.67 \cdot mm$ | $18.35 \cdot kN$ | $3.5 \cdot mm$ |
| $16.40 \cdot kN$ | $3.2 \cdot mm$ | $18.70 \cdot kN$ | $1.71 \cdot mm$ | $17.60 \cdot kN$ | $3.7 \cdot mm$ |
| $16.40 \cdot kN$ | $3.35 \cdot mm$ | $18.50 \cdot kN$ | $2.00 \cdot mm$ | $17.40 \cdot kN$ | $3.8 \cdot mm$ |
| $16.40 \cdot kN$ | $3.52 \cdot mm$ | $18.10 \cdot kN$ | $2.15 \cdot mm$ | $16.30 \cdot kN$ | $3.9 \cdot mm$ |
| $15.79 \cdot kN$ | $4.58 \cdot mm$ | $15.67 \cdot kN$ | $4.11 \cdot mm$ | $15.80 \cdot kN$ | $4.02 \cdot mm$ |

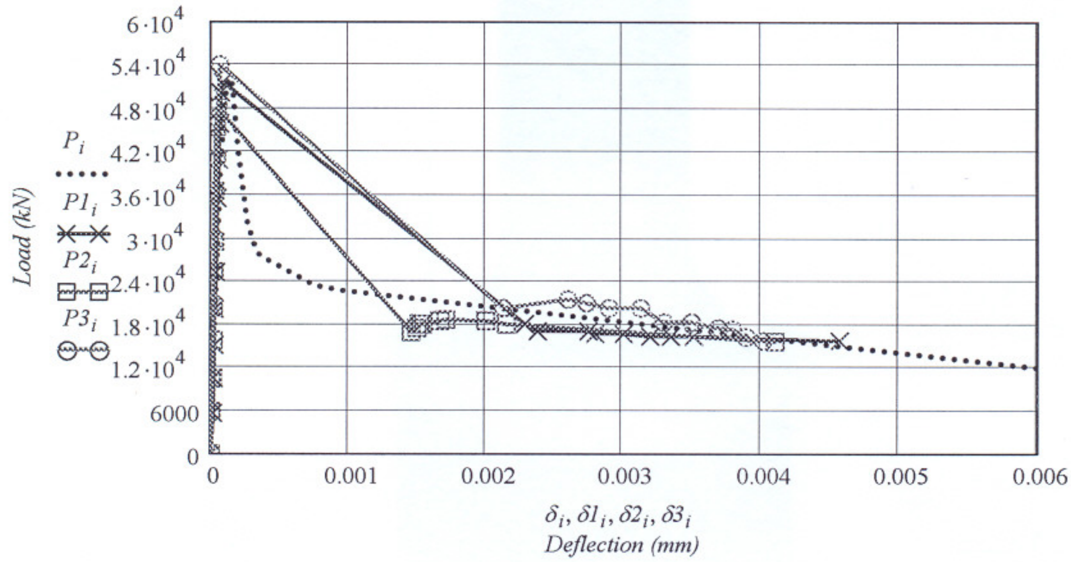


**Figure C-16: Comparison between measured and calculated load-deflection responses.**

The developed cracking subroutine is shown in Figure C-172. The FORTRAN V.6 was used to write the subroutine.

```
      subroutine ucrack(scrack,esoft,ecrush,ecp,dt,dtdl,n,nn,kc,inc,
     * ndi,nshear,shrfac)
      implicit real*8 (a-h,o-z)                                     dp

        StressMax = 4.d0

        CrushingStrain = 1e20

        ElasticMod = 10.0d3

        SoftMod1 = 0.077d4
        StrainLimit = 30d-4
        SoftMod2 = 0.002d4

c     programme
        scrack = StressMax
        esoft = SoftMod1
        ecrush = CrushingStrain

        if (ECP .GT. StrainLimit ) then
        esoft = SoftMod2
c     calculate the yield stress that corresponds to the second softening modulus..
c      Calculate the strain at original yielding
        Sigma1 = StressMax
        Epsilon1 = Sigma1/ElasticMod

c      Calculate the stress at switchover from softening modulus 1 to softening modulus 2
        Epsilon2 = StrainLimit
        Sigma2 = Sigma1 - SoftMod1*(Epsilon2-Epsilon1)

c      calculate the B value in y = Ax + B [Sigma = -SoftMod2*Epsilon + Constant]
      B = Sigma2 + SoftMod2*Epsilon2

c      calculate the strain at which the second softening modulus line crosses the elastic one
      Epsilon3 = B / (ElasticMod + SoftMod2)

c      convert to stress and assign to cracking stress
        Sigma3 = ElasticMod * Epsilon3

        scrack = Sigma3
        endif
```

Figure C-17: The developed subroutine to allow the input of a bilinear softening curve.

In the subroutine different terms were used to represent parameters in the $\sigma$-$\varepsilon$ response of the SFRC. These terms are:

StressMax = Tensile strength ( $\sigma_{t0}$ ).

CrushingStrain = Crushing strain (Large value to prevent occurrence of crushing).

ElasticMod = Young's modulus for the SFRC.

SoftMod1 = Absolute value of the slope of the middle part of the tensile $\sigma$-$\varepsilon$ response.

StrainLimit = Residual strain ( $\varepsilon_{tu}$ ).

SoftMod2 = Absolute value of the slope of the last part of the tensile $\sigma$-$\varepsilon$ response.

**APPENDIX D**

**APPENDIX D**

**Loading and geometry**

$h := 150mm$ (Depth)

$b := 150.mm$ (Width)

$L := 600mm$

$fsh := \frac{6}{5}$ (Form factor for shear)

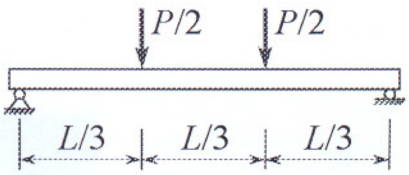


**Figure D-1: Test set up for the beam.**

**Adopted stress-strain response**

$E := 23.0 \cdot \text{GPa}$

$\mu := 0.2$

$G := \frac{E}{2 \cdot (1 + \mu)}$

$G = 9.58333\,\text{GPa}$

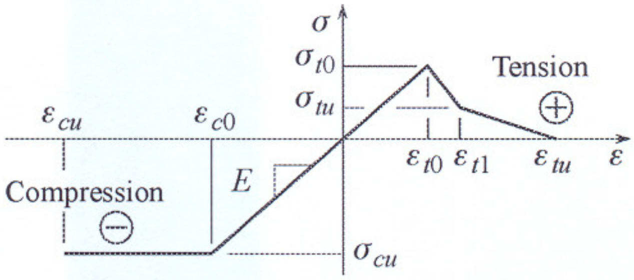


**Figure D-2: Schematic diagram for the stress-strain response.**

$\sigma_{t0} := 1.9 \cdot \text{MPa}$ $\varepsilon_{t0} := \frac{\sigma_{t0}}{E}$ $\varepsilon_{t0} = 8.26087 \times 10^{-5}$

$\sigma_u := 0.9 \cdot \text{MPa}$ $\varepsilon_{t1} := 9 \cdot 10^{-4}$ $\varepsilon tu := 0.1$

$\varepsilon_{c0} := -0.0016$ $\sigma_{cu} := \varepsilon_{c0} \cdot E$ $\sigma_{cu} = -36.8\,\text{MPa}$

$\varepsilon_{cu} := -0.4$

$\lambda := \frac{\sigma_u - \sigma_{t0}}{\varepsilon_{t1} - \varepsilon_{t0}}$ $\lambda = -1.2234\,\text{GPa}$ (The slope of the middle part of the tensile σ-ε response)

$\Psi := \frac{\sigma_u}{\varepsilon_{t1} - \varepsilon tu}$ $\Psi = -9.08174 \times 10^{-3}\,\text{GPa}$ (The slope of the last part of the tensile σ-ε response)

**Tensile stress-strain function**

$$f_{ct}(\varepsilon_t) := (\varepsilon_t \geq 0) \cdot (\varepsilon_t \leq \varepsilon_{t0}) \cdot \frac{\sigma_{t0}}{\varepsilon_{t0}} \cdot \varepsilon_t \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t0}) \cdot (\varepsilon_t \leq \varepsilon_{t1}) \cdot [\lambda \cdot (\varepsilon_t - \varepsilon_{t0}) + \sigma_{t0}] \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t1}) \cdot (\varepsilon_t \leq \varepsilon tu) \cdot \Psi \cdot (\varepsilon_t - \varepsilon tu)$$

$$\varepsilon_t := 0, 0.00001 .. \varepsilon tu$$

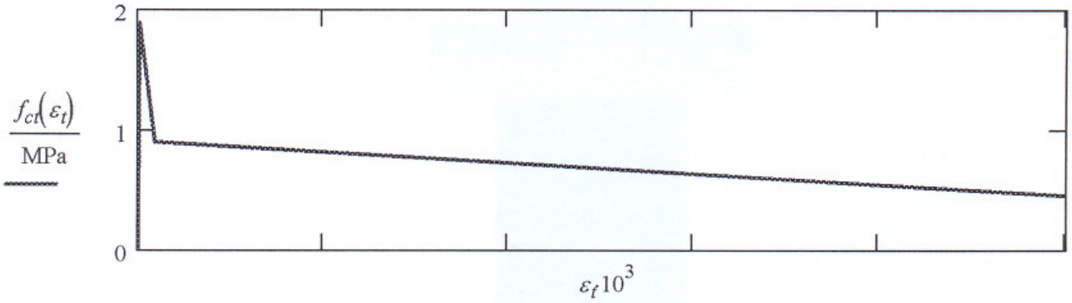


**Figure D-3: Assumed tensile stress-strain response.**

**Compressive stress-strain function**

$$f_{cc}(\varepsilon_c) := (\varepsilon_c \geq \varepsilon_{c0}) \cdot (\varepsilon_c < 0) \cdot E \cdot \varepsilon_c \ldots$$
$$+ (\varepsilon_c \geq \varepsilon_{cu}) \cdot (\varepsilon_c < \varepsilon_{c0}) \cdot \sigma_{cu}$$

$$\varepsilon_c := \varepsilon_{cu}, \frac{199 \cdot \varepsilon_{cu}}{200} .. 0$$

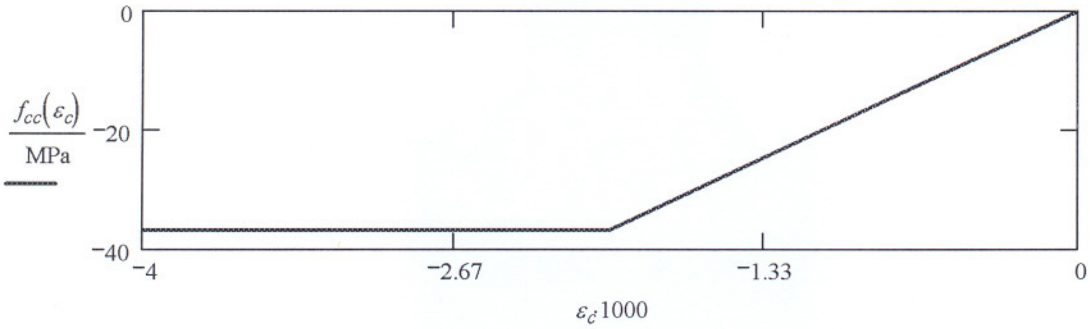


**Figure D-4: Assumed compressive stress-strain response.**

**Calculating moment-curvature response**

$$\phi(\varepsilon_{bot}, a) := \frac{\varepsilon_{bot}}{h - a}$$

$$\varepsilon_{c.top}(\varepsilon_{c.bot}, a) := \frac{-a}{h - a} \cdot \varepsilon_{c.bot}$$

$$F_{cc}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c)\, d\varepsilon_c$$

$$F_{ct}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\, d\varepsilon_c$$

$$Mc(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$$Mct(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{\;2}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$$\varepsilon_{bot} := \varepsilon_{t0}$$

$$a0 := \frac{h}{2} \qquad F_{cc}(\varepsilon_{bot}, a0) = -10.6875\ \text{kN}$$

$$F_{ct}(\varepsilon_{bot}, a0) = 10.6875\ \text{kN}$$

$$\varepsilon_{bot} = 8.26087 \times 10^{-5}$$

$$Mc(\varepsilon_{bot}, a0) = 0.53438\ \text{m kN}$$

$$Mct(\varepsilon_{bot}, a0) = 0.53438\ \text{m kN}$$

$$Mext(\varepsilon_{c.bot}, a) := Mc(\varepsilon_{c.bot}, a) + Mct(\varepsilon_{c.bot}, a)$$

*Given* $F_{cc}(\varepsilon_{bot}, a) + F_{ct}(\varepsilon_{bot}, a) = 0 \cdot \text{kN}$

$Solve(\varepsilon_{bot}, a) := Find(a)$

$i := 1..20$

$\varepsilon b_i :=$

| |
|---|
| 0.00001 |
| 0.00004 |
| 0.00007 |
| 0.00008 |
| $\varepsilon_{t0}$ |
| 0.00009 |
| 0.0002 |
| 0.0004 |
| 0.0005 |
| 0.0006 |
| 0.0007 |
| 0.0008 |
| $\varepsilon_{t1}$ |
| 0.002 |
| 0.0025 |
| 0.003 |
| 0.007 |
| 0.009 |
| 0.08 |
| $\varepsilon tu$ |

$a_i := Solve(\varepsilon b_i, a0)$

$\varepsilon t_i := \varepsilon_{c.top}(\varepsilon b_i, a_i)$

$\phi_i := \phi(\varepsilon b_i, a_i)$

$M_i := Mext(\varepsilon b_i, a_i)$

$\sigma b_i := f_{ct}(\varepsilon b_i)$

**Calculating load-deflection response**

$M_{max} := max(M)$ $\qquad M_{max} = 1.92952\, m \cdot \text{kN}$ $\qquad \phi h := lookup(M_{max}, M, \phi)_1$

$\phi h = 5.05274 \times 10^{-3} \frac{1}{\text{m}}$ $\qquad MaxPos := match(M_{max}, M)_1$ $\qquad MaxPos = 9$

$M1 := submatrix(M, 1, MaxPos, 1, 1)$ $\qquad \phi_1 := submatrix(\phi, 1, MaxPos, 1, 1)$

$length(M) = 20 \quad M2 := submatrix(M, MaxPos, length(M) - 1, 1, 1)$

$\phi_2 := submatrix(\phi, MaxPos, length(M) - 1, 1, 1) \quad length(M2) = 11$

$MaxPos2 := match(M_{max}, M2)_1 \quad MaxPos2 = 1 \quad Mmin := min(M2)$

$Mmin = 0.70978\, m \cdot \mathrm{kN} \quad MinPos2 := match(Mmin, M2)_1 \quad MinPos2 = 11$

$posi := match(M2_{MinPos2}, M)_1 \quad posi = 19 \quad length(M2) = 11$

$x := 0m, 0.02 \cdot m .. 0.2m \quad ms(x, M) := M \cdot \frac{x}{0.2m} \quad \Delta ms(x, M) := max(M2) - ms(x, M)$

$$\Delta M2 := M_{max} - M2$$

**Deflection due to bending moment**

$$\delta m_i := \left| \begin{array}{l} \displaystyle\int_{0m}^{\frac{L}{3}} linterp(M1, \phi_1, ms(x, M_i)) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(M1, \phi_1, ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \quad if\ (i \le MaxPos) \\ \displaystyle\int_{0m}^{\frac{L}{3}} linterp(M1, \phi_1, ms(x, M_i)) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(\Delta M2, \phi_2, \Delta ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \quad if\ (MaxPos \le i) \end{array} \right.$$

**Deflection due to shear force**

$$P_i := M_i \cdot \frac{6}{L}$$

$$\delta v_i := \frac{P_i fsh}{G \cdot b \cdot h \cdot 2} \cdot \int_{0m}^{\frac{L}{3}} 1.\, dx$$

$$\delta_i := \delta v_i + \delta m_i$$

**Measured load-deflection responses**

| $P1_i :=$ | $\delta l_i :=$ |
|---|---|
| 0kN | $0 \cdot mm$ |
| $20.0 \cdot kN$ | $0.16 \cdot mm$ |
| $17.0 \cdot kN$ | $0.25 \cdot mm$ |
| $15.5 \cdot kN$ | $0.5 \cdot mm$ |
| $14.50 \cdot kN$ | $1.0 \cdot mm$ |
| $14.0 \cdot kN$ | $1.5 \cdot mm$ |
| $14.4 \cdot kN$ | $1.6mm$ |
| $14.3 \cdot kN$ | $1.8mm$ |
| $14.5 \cdot kN$ | $1.9mm$ |
| $14.6 \cdot kN$ | $2.2mm$ |
| $14.3 \cdot kN$ | $2.3mm$ |
| $14.2 \cdot kN$ | $2.4mm$ |
| $14.1 \cdot kN$ | $2.5mm$ |
| $13.6 \cdot kN$ | $2.7mm$ |
| $13.5 \cdot kN$ | $2.9mm$ |
| $13.2 \cdot kN$ | $3.0 \cdot mm$ |
| $13.1 \cdot kN$ | $3.2mm$ |
| $12.9 \cdot kN$ | $3.4mm$ |
| $12.5 \cdot kN$ | $3.8mm$ |
| $12.1 \cdot kN$ | $4.0 \cdot mm$ |

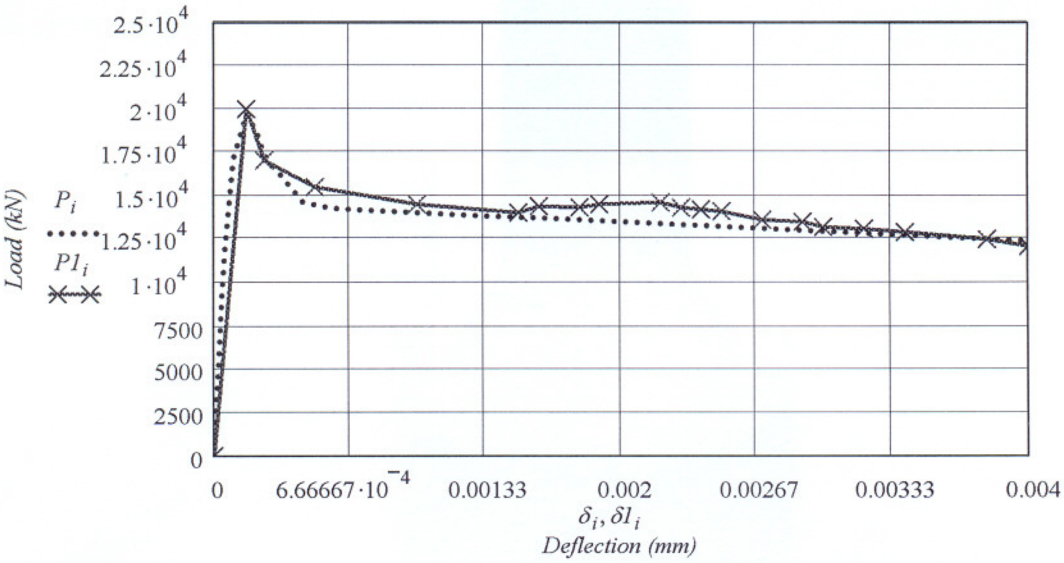


**Figure D-5: Comparison between measured and calculated load-deflection responses.**

**Loading and geometry**

$h := 150mm$ (Depth)

$b := 150.mm$ (Width)

$L := 600mm$

$fsh := \frac{6}{5}$ (Form factor for shear)

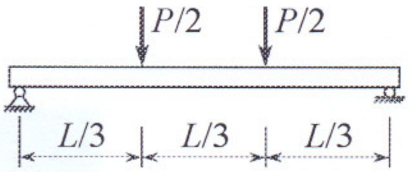


**Figure D-6: Test set up for the beam.**

**Adopted streaa-strain response**

$E := 26.7 \cdot \text{GPa}$

$\mu := 0.2$

$G := \frac{E}{2 \cdot (1+\mu)}$

$G = 11.125\,\text{GPa}$

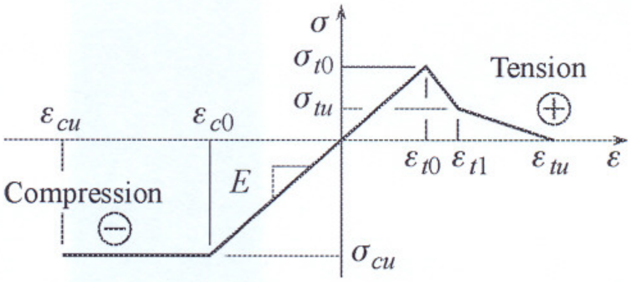


**Figure D-7: Schematic diagram for the stress-strain response.**

$\sigma_{t0} := 1.90 \cdot \text{MPa}$ $\quad \varepsilon_{t0} := \frac{\sigma_{t0}}{E}$ $\quad \varepsilon_{t0} = 7.1161 \times 10^{-5}$

$\sigma_u := 0.6 \cdot \text{MPa}$ $\quad \varepsilon_{t1} := 9 \cdot 10^{-4}$ $\quad \varepsilon tu := 0.1$

$\varepsilon_{c0} := -0.0016$ $\quad \sigma_{cu} := \varepsilon_{c0} \cdot E$ $\quad \sigma_{cu} = -42.72\,\text{MPa}$

$\varepsilon_{cu} := -0.4$

$\lambda := \frac{\sigma_u - \sigma_{t0}}{\varepsilon_{t1} - \varepsilon_{t0}}$ $\quad \lambda = -1.56846\,\text{GPa}$ (The slope of the middle part of the σ-ε response)

$\Psi := \frac{\sigma_u}{\varepsilon_{t1} - \varepsilon tu}$ $\quad \Psi = -6.05449 \times 10^{-3}\,\text{GPa}$ (The slope of the last part of the σ-ε response)

**Tensile stress-strain function**

$$f_{ct}(\varepsilon_t) := (\varepsilon_t \geq 0) \cdot (\varepsilon_t \leq \varepsilon_{t0}) \cdot \frac{\sigma_{t0}}{\varepsilon_{t0}} \cdot \varepsilon_t \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t0}) \cdot (\varepsilon_t \leq \varepsilon_{t1}) \cdot [\lambda \cdot (\varepsilon_t - \varepsilon_{t0}) + \sigma_{t0}] \ldots$$
$$+ (\varepsilon_t > \varepsilon_{t1}) \cdot (\varepsilon_t \leq \varepsilon tu) \cdot \Psi \cdot (\varepsilon_t - \varepsilon tu)$$

$\varepsilon_t := 0, 0.00001 .. \varepsilon tu$

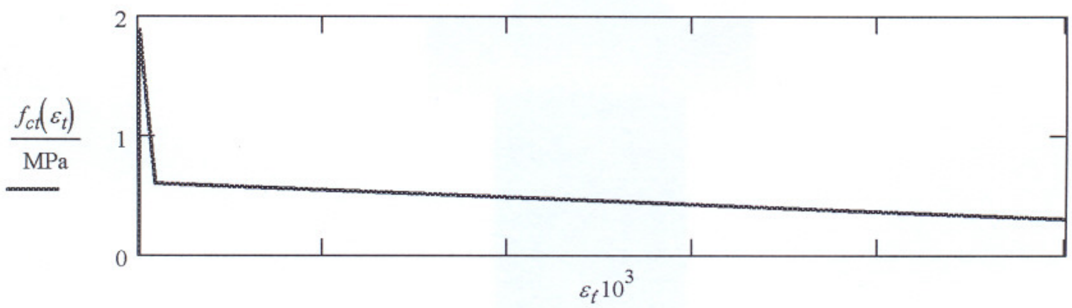


**Figure D-8: Assumed tensile stress-strain response.**

**Compressive stress-strain function**

$$f_{cc}(\varepsilon_c) := (\varepsilon_c \geq \varepsilon_{c0}) \cdot (\varepsilon_c < 0) \cdot E \cdot \varepsilon_c \ldots$$
$$+ (\varepsilon_c \geq \varepsilon_{cu}) \cdot (\varepsilon_c < \varepsilon_{c0}) \cdot \sigma_{cu}$$

$$\varepsilon_c := \varepsilon_{cu}, \frac{199 \cdot \varepsilon_{cu}}{200} .. 0$$

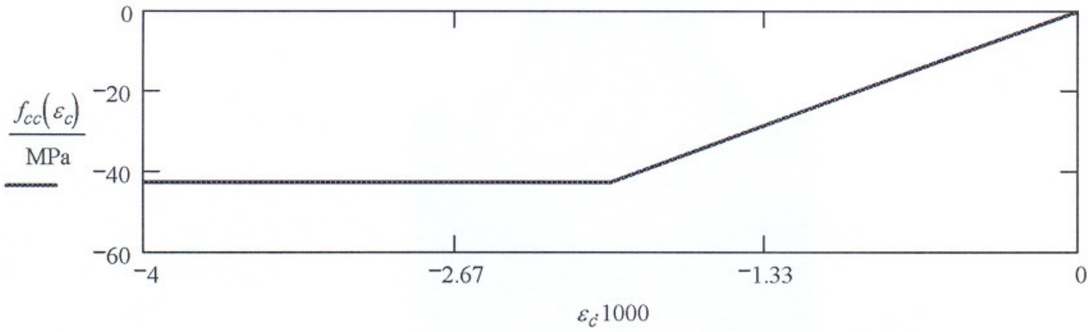


**Figure D-9: Assumed compressive stress-strain response.**

**Calculating moment-curvature response**

$$\phi(\varepsilon_{bot}, a) := \frac{\varepsilon_{bot}}{h - a}$$

$$\varepsilon_{c.top}(\varepsilon_{c.bot}, a) := \frac{-a}{h - a} \cdot \varepsilon_{c.bot}$$

$$F_{cc}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c)\, d\varepsilon_c$$

$$F_{ct}(\varepsilon_{c.bot}, a) := \frac{(h - a) \cdot b}{\varepsilon_{c.bot}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c)\, d\varepsilon_c$$

$$Mc(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{2}} \cdot \int_{\varepsilon_{c.top}(\varepsilon_{c.bot}, a)}^{0} f_{cc}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$$Mct(\varepsilon_{c.bot}, a) := \frac{(h - a)^2 \cdot b}{\varepsilon_{c.bot}^{2}} \cdot \int_{0}^{\varepsilon_{c.bot}} f_{ct}(\varepsilon_c) \cdot \varepsilon_c\, d\varepsilon_c$$

$\varepsilon_{bot} := \varepsilon_{t0}$

$a0 := \frac{h}{2}$ $\quad F_{cc}(\varepsilon_{bot}, a0) = -10.6875\,\text{kN}$

$F_{ct}(\varepsilon_{bot}, a0) = 10.6875\,\text{kN}$

$\varepsilon_{bot} = 7.1161 \times 10^{-5}$

$Mc(\varepsilon_{bot}, a0) = 0.53438\,\text{m kN}$

$Mct(\varepsilon_{bot}, a0) = 0.53438\,\text{m kN}$

$$Mext(\varepsilon_{c.bot}, a) := Mc(\varepsilon_{c.bot}, a) + Mct(\varepsilon_{c.bot}, a)$$

Given $F_{cc}(\varepsilon_{bot}, a) + F_{ct}(\varepsilon_{bot}, a) = 0 \cdot \text{kN}$

$Solve(\varepsilon_{bot}, a) := Find(a)$

$i := 1..20$

$\varepsilon_{b_i} :=$

| |
|---|
| 0.00001 |
| 0.00004 |
| 0.00007 |
| $\varepsilon_{t0}$ |
| 0.00008 |
| 0.00009 |
| 0.0002 |
| 0.0004 |
| 0.0005 |
| 0.0006 |
| 0.0007 |
| 0.0008 |
| $\varepsilon_{t1}$ |
| 0.002 |
| 0.0025 |
| 0.003 |
| 0.007 |
| 0.009 |
| 0.08 |
| $\varepsilon tu$ |

$a_i := Solve(\varepsilon_{b_i}, a0)$

$\varepsilon_{t_i} := \varepsilon_{c.top}(\varepsilon_{b_i}, a_i)$

$\phi_i := \phi(\varepsilon_{b_i}, a_i)$

$M_i := Mext(\varepsilon_{b_i}, a_i)$

$\sigma b_i := f_{ct}(\varepsilon_{b_i})$

**Calculating load-deflection response**

$M_{max} := max(M)$ $M_{max} = 1.90367\, m \cdot \text{kN}$ $\phi h := lookup(M_{max}, M, \phi)_1$ $\phi h = 4.08891 \times 10^{-3} \frac{1}{\text{m}}$

$MaxPos := match(M_{max}, M)_1$ $MaxPos = 8$ $M1 := submatrix(M, 1, MaxPos, 1, 1)$

$\phi_1 := submatrix(\phi, 1, MaxPos, 1, 1)$ $length(M) = 20$ $M2 := submatrix(M, MaxPos, length(M) - 1, 1, 1)$

$\phi_2 := submatrix(\phi, MaxPos, length(M) - 1, 1, 1)$ $\quad length(M2) = 12$ $\quad MaxPos2 := match(M_{max}, M2)_1$

$MaxPos2 = 1$ $\quad Mmin := min(M2)$ $\quad Mmin = 0.47457\, m \cdot \text{kN}$ $\quad MinPos2 := match(Mmin, M2)_1$

$MinPos2 = 12$ $\quad posi := match(M2_{MinPos2}, M)_1$ $\quad posi = 19$ $\quad length(M2) = 12$

$x := 0m, 0.02 \cdot m .. 0.2m$ $\quad ms(x, M) := M \cdot \dfrac{x}{0.2m}$ $\quad \Delta ms(x, M) := max(M2) - ms(x, M)$

$$\Delta M2 := M_{max} - M2$$

**Deflection due to bending moment**

$$\delta m_i := \left| \begin{array}{l} \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(M1, \phi_1, ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \quad \text{if } (i \le MaxPos) \\ \displaystyle\int_{0m}^{\frac{L}{3}} linterp\left(M1, \phi_1, ms\left(x, M_i\right)\right) \cdot x\, dx + \int_{\frac{L}{3}}^{\frac{L}{2}} linterp\left(\Delta M2, \phi_2, \Delta ms\left(\frac{L}{3}, M_i\right)\right) \cdot x\, dx \quad \text{if } (MaxPos \le i) \end{array} \right.$$

**Deflection due to shear force**

$$P_i := M_i \cdot \frac{6}{L}$$

$$\delta v_i := \frac{P_i \cdot fsh}{G \cdot b \cdot h \cdot 2} \cdot \int_{0m}^{\frac{L}{3}} 1.\, dx$$

$$\delta_i := \delta v_i + \delta m_i$$

**Measured load-deflection response**

| $P1_i :=$ | $\delta 1_i :=$ |
|---|---|
| 0kN | $0 \cdot mm$ |
| $19.0 \cdot kN$ | $0.1 \cdot mm$ |
| $10.0 \cdot kN$ | $0.5 \cdot mm$ |
| $10. \cdot kN$ | $1.0 \cdot mm$ |
| $9.50 \cdot kN$ | $2.0 \cdot mm$ |
| $9.30 \cdot kN$ | $2.1 mm$ |
| $9.30 \cdot kN$ | $2.2 mm$ |
| $9.30 \cdot kN$ | $2.4 mm$ |
| $9.30 \cdot kN$ | $2.5 \cdot mm$ |
| $9.30 \cdot kN$ | $2.6 mm$ |
| $9.30 \cdot kN$ | $2.8 mm$ |
| $9.30 \cdot kN$ | $2.9 mm$ |
| $9.30 \cdot kN$ | $3.0 \cdot mm$ |
| $9.30 \cdot kN$ | $3.1 mm$ |
| $9.30 \cdot kN$ | $3.3 mm$ |
| $9.30 \cdot kN$ | $3.4 mm$ |
| $9.30 \cdot kN$ | $3.50 \cdot mm$ |
| $9.2 \cdot kN$ | $3.7 mm$ |
| $9.0 \cdot kN$ | $3.8 mm$ |
| $9.0 \cdot kN$ | $4.0 \cdot mm$ |

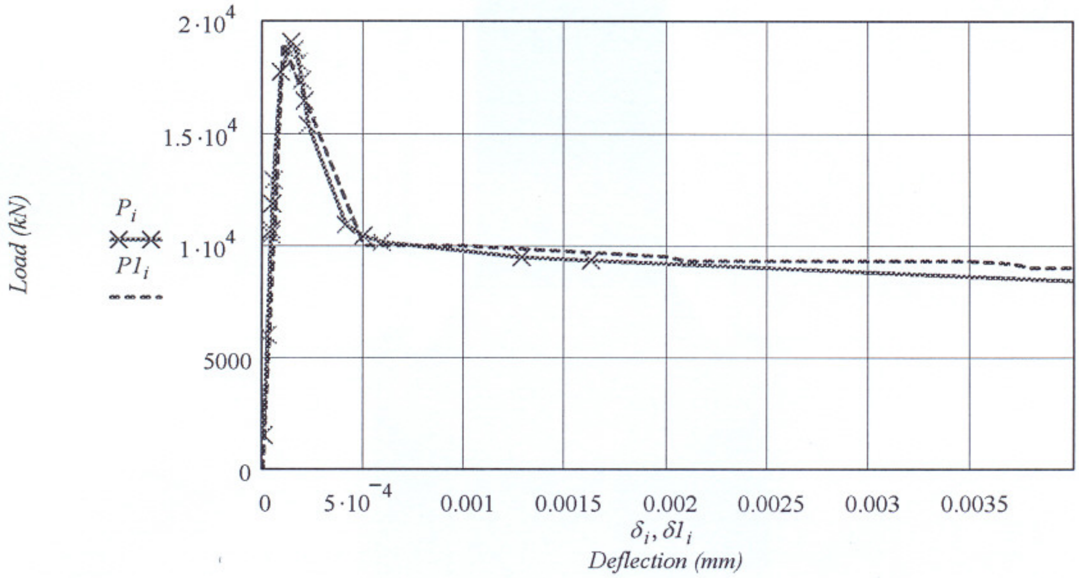


**Figure D-10: Comparison between measured and calculated load-deflection responses.**