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MODELLING THE BEHAVIOUR OF STEEL FIBRE REINFORCED CONCRETE PAVEMENTS

W.A. ELSAIGH

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DEDICATION

To my wife and son, Rasha and Mohamed, who sacrificed the most so that I could pursue my interests

ABSTRACT

MODELLING THE BEHAVIOUR OF STEEL FIBRE REINFORCED CONCRETE PAVEMENTS

BY:
W.A. ELSAIGH

Supervisor: Prof. E.P. Kearsley
Co supervisor: Dr. J.M. Robberts
Department: Civil Engineering
University: University of Pretoria
Degree: Ph.D. of Engineering (Transportation Engineering)

Steel Fibre Reinforced Concrete (SFRC) is defined as concrete containing randomly oriented discrete steel fibres. The main incentive of adding steel fibres to concrete is to control crack propagation and crack widening after the concrete matrix has cracked. Control of cracking automatically improves the mechanical properties of the composite material (SFRC). The most significant property of SFRC is its post-cracking strength that can impart the ability to absorb large amounts of energy before collapse.

Ground slabs are structural applications that could benefit from these advantageous features of the SFRC. Many tests on SFRC ground slabs show that the material can offer distinct advantages compared to plain concrete. In concrete road pavements, SFRC is particularly suitable for increasing load-carrying capacity and fatigue resistance. Not surprisingly, recent years have witnessed acceleration in full-scale tests of SFRC and eventually acceptance of its use in concrete pavements. The use of SFRC in pavements has been slowed down by the absence of a reliable theoretical model to analyse and design these pavements.

The analysis of ground slabs has traditionally been based on an elastic analysis assuming un-cracked concrete. Using such a method for SFRC would ignore the post-cracking contribution the SFRC can make to the flexural behaviour of the slab. Despite the growing trend of using methods of analysis based on yield-line theory, which can consider the post-cracking strength of SFRC, these methods were also found to underestimate the load-carrying capacity of SFRC ground slabs. To effectively account for the post-cracking strength of SFRC in the analysis of such slabs requires a method such as the finite element method.

In the present work, non-linear methods are used to model the behaviour of SFRC ground slabs subjected to mechanical load. An analytical method is used to determine a tensile stress-strain response for SFRC. In this method, the post-cracking strength of SFRC is taken into account and hence the material model is sensitive to the element size used. The calculated stress-strain response is utilised in finite element analysis of SFRC beams and ground slabs. A smeared crack approach is used to simulate the behaviour of concrete cracking. The analytical method used to determine the tensile stress-strain response, as well as the finite element model, are evaluated using results from experiments on SFRC beams and ground slabs. The analytical results are found to compare well with the observations. The non-linear methods are further used to study the effect of the material model parameters as well as the support stiffness on load-displacement behaviour of SFRC ground slabs.

The developed finite element model is shown to be more efficient compared to methods based on the yield-line theory. This is because it produces the load-displacement behaviour of the SFRC ground slab up to a reasonable limit and it provides the tensile stresses as well as the extent of cracking of the slab at every point on the load-displacement response. Using the developed finite element model will allow for considerable material saving since smaller slab thickness can be calculated compared to analytical models currently in use.

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LIST OF SYMBOLS

σ = Stress.

σ_{t0} = Cracking strength of a composite material.

σ_m = Stress in the concrete matrix.

σ_f = Stress in the steel fibre.

σ_{ui} = Stress in the cross-section of a single steel fibre.

σ_{fu} = Ultimate fibre stress (fibre fracture stress).

σ_{tu} = Residual stress.

σ_{tu}' = Second stage residual stress.

σcu = Compressive strength of SFRC.

σ_1, σ_2, and σ_3 = Principal stresses.

f_{ct} = Flexural strength.

$\bar{f}_{ct}$ = $1.15 f_{ct}$.

$f_{ct,equ,150}$ = Post-cracking flexural strength calculated at a deflection value of length/300.

$f_{ct,equ,300}$ = Post-cracking flexural strength calculated at a deflection value of length/150.

$f_{eq.2}$ = Post-cracking flexural strength calculated at a deflection value of 0.7 mm.

$f_{eq.3}$ = Post-cracking flexural strength calculated at a deflection value of 2.7 mm.

$f_{e,3}$ = Equivalent flexural strength.

f_d = Design flexural strength for SFRC ($f_{ct} + f_{e,3}$).

f_{cu} = Cube compressive strength.

f_s = Tensile strength of fictitious reinforcement element.

f_{sh} = Factor for shear (equals 6/5 for rectangular section).

$R_{e,3}$ = Residual flexural strength ratio ($f_{ct}/f_{e,3}$).

τ = Shear stress.

τ_u = Average ultimate pullout bond strength.

ε_{c0} = Elastic compressive strain.

ε = Strain.

ε_{t0} = Cracking strain.

ε_{t1} = Residual strain.

ε_{tu} = Ultimate tensile strain.

ε_{cu} = Ultimate compressive strain.

ε_m = Cracking strain of concrete matrix.

ε_{fp} = Strain relating to proportional limit in the stress-strain response of a steel fibre.

ε^{co} = Strain of the material or continuum.

ε^{cr} = Strain of cracked material.

ε_{bot} = Tensile strain at bottom ligament of the beam cross-section.

ε_{top} = Compressive strain at top ligament of the beam cross-section.

ε_R = Strain at proportionality limit of a fictitious reinforcement element.

γ = Shear strain.

γ_m = Maximum shear strain.

γ_c = Shear strain at any point on the shear force-shear strain relationship.

E = Young's modulus for of the composite (SFRC).

E_m = Young's modulus for a concrete matrix.

E_f = Young's modulus for a steel fibre.

$E_{tc} = 0.5\,E$.

E_u = Unloading modulus.

ψ = Slope of first softening part of the tensile stress-strain curve.

λ = Slope of the second softening part of the tensile stress-strain curve.

P = Vertical load.

P_{max} = Maximum load obtained from beam-bending test.

$P_{e,3}$ = Mean load calculated at a deflection of 3 mm.

P_i = Interior load carrying capacity of ground slab.

F = Total force on a beam cross-section.

δ = Deflection of elevated beam or slab.

δ_m = Deflection due to moment.

δ_V = Deflection due to shear.

Δ = Deflection of ground slabs or plates (distinguish from elevated beams or slabs).

M = Moment.

M_m = Maximum moment.

M_u = Moment due to a unit load.

M_L = Moments due to actual load.

M_c = Moment on the descending part of the moment-curvature relationship.

M_0 = Limit moment of resistance of ground slab.

ϕ = Curvature.

ϕ_m = Curvature corresponding to the maximum moment (M_m).

ϕ_c = Curvature on the descending part of the moment-curvature relationship.

V_f = Volume percentage of the steel fibres.

V_m = Volume fraction of the concrete matrix.

V_{eff} = Effective volume fraction of the steel fibre.

V = Shear force.

V_m = Maximum shear force.

V_c = Shear force at any point on the shear force-shear strain relationship.

V_u = Shear force due to a unit load.

V_L = Shear forces due actual load.

A = Area of beam cross-section.

A_m = Area fraction of the concrete matrix.

A_{eff} = Effective area fraction of the steel fibres.

A_s = Surface area of single steel fibre.

A_{fi} = Cross-section area of a single steel fibre.

$A_{150 \times 150}$, $A_{100 \times 100}$ = Area under the softening part of tensile stress-strain response.

η_o = Orientation factor.

η_l = Length efficiency factor.

L_c = Critical fibre length required to develop the ultimate fibre stress.

L_f = Length of a steel fibre.

L = Span of the beam.

L_r = Radius of relative stiffness of ground slab.

β = Material factor depending on the steel fibre type.

ρ = Pressure.

μ = Poisson's ratio.

α = Threshold angle.

r = Radius of loading plate.

R.I. = Reinforcing index.

w_f = Weight fraction of steel fibres.

W_f = Weight percentage of steel fibres.

w = Crack width.

h = Depth of a beam.

b = Width of a beam.

b_0 = Unit width of a ground slab.

d_f = Diameter of a steel fibre.

d_{sf} = Depth of SFRC slab.

d = Depth of a slab.

d_p = Depth of plain concrete slab.

dy , dx = Length and width of differential element.

a = Depth of neutral axis.

y = Variable representing the depth from neutral axis.

S1, S2, and S3 = Slopes on the load-deflection response of a SFRC beam.

X , Y, and Z = Orthogonal directions.

Δ_X, Δ_Y and Δ_Z = Displacement in the X, Y, and Z directions respectively.

θ_X, θ_Y and θ_Z = Rotation in the X, Y and Z directions respectively.

l_b = Width of the fracture process zone.

I_a, I_b , and I_c = Notations used for integration points of a finite element.

I = Second moment.

N_1 and N_2 = Notations used for nodes.

G = Shear modulus.

G_S = Mean shear transfer stiffness in N/mm^3.

G_f = Fracture energy for SFRC.

G_{f0} = Fracture energy for plain concrete.

C2, G5, G6 and G9 = Types of soil according to South African soil classification system.

K = Spring stiffness or modulus of subgrade reaction.

K_1, K_2, and K_3 = Modulus of subgrade reactions- non-linear Pressure-displacement response.