CHAPTER TWO

A MULTI-DIMENSIONAL MEASURE OF POVERTY USING THE FUZZY APPROACH

A modified version of this chapter was published in ***Studies for Economics and Econometrics*****, 2005.**

2.1 INTRODUCTION

One of the laws of thought of Aristotle was the “Law of Excluded Middle” which excludes the possibility of having a logic value other than “true” or “false”. Heraclitus raised the point that things cannot be true and not true simultaneously. Plato laid the origins of what became later a fuzzy logic, indicating that there is a third region between true and false. Many years later Lukasiewicz described a third valued logic as “possible”. The above discussion is highlighted by Gutierrez (2002). Unfortunately none of this logic could satisfactorily describe concepts as “tall”, “fat” or “poor”. In 1965 the notion of infinite value logic was introduced by Zadeh. The basic premise is that the key elements in human thinking are not numbers, but labels of a fuzzy set. In the classical mathematical sense, the “class of rich people” or “the class of poor individuals” do not constitute classes, to be rich or to be poor is of ambiguous status. The transition from membership to non-membership of these classes is gradual. To deal with these types of characteristics, a new concept was introduced. It was called a fuzzy set, which is a class with a continuum of grades of membership.

Fuzzy sets as developed by Zadeh (1965) allow for the treatment of vague concepts such as poverty and are ideal to address the vertical vagueness of poverty and the horizontal vagueness of poverty by allowing every household some degree of deprivation in each dimension of poverty. Fuzzy sets can be used to identify those households that are highly deprived and absolutely poor and those households that are slightly less deprived, that is, households lying on the threshold of poverty.

In South Africa there are many households that can be defined as “poor”, while others can be defined as “not poor”, based on some attribute or some set of attributes. According to the traditional approach, the set of poor households is a crisp set, that is, a household either belongs to the set of poor households, or it does not, depending on some critical level, for example, the poverty line. There are no partially poor households. The fuzzy set approach has two critical levels instead of one minimum level, below which a household absolutely belongs to the set of poor and a maximum level, above which a person absolutely does not belong to the set of poor persons. If a household falls between these two levels then that household partially belongs to the set of poor households. Fuzzy sets allow for more than one dimension of poverty to be used in measuring the poverty status of a household, since the measurement yardstick is simply the degree of membership of the set of poor in each dimension. The overall membership function acts as a deprivation indicator showing each household's overall deprivation relative to its surroundings.

There are several definitions for the membership function in the literature. Cerioli and Zani (1990) proposed the first definition. They indicated that there should be a minimum critical level below which a household should be considered absolutely poor and a maximum critical level above which a household should be considered absolutely not poor. If a household's deprivation were to fall between these two levels, the membership function would be a linear function between the minimum critical level and the maximum critical level.

Cheli and Lemmi (1995) criticised two aspects of the definition proposed by Cerioli and Zani (1990). The first is that deciding on the minimum and maximum critical levels is very arbitrary and is open to the same criticism as the traditional approach to poverty measurements. To overcome this criticism, they proposed that the critical levels coincide with the minimum and maximum values of categories in each dimension. The second criticism is that the linear approach could give too much importance to some rare category in a dimension, leading to an over or underestimation of actual poverty. In this method the proposal is that the poverty rating of each category in every dimension be determined by the number of individuals experiencing the same level of deprivation; their approach was therefore called the Totally Fuzzy and Relative Approach.

Cheli (1995) states that poverty "is certainly not a discrete attribute characterized in terms of presence or absence, but rather a vague predicate that manifests itself in different shades and degrees". Cerioli and Zani (1990) proposed a multidimensional measure of poverty using fuzzy set theory, liable to assume a variety of shades and degrees. Cheli and Lemmi (1995) improved the fuzzy concept method by deriving deprivation indices directly from the distribution function of the attributes measured.

The aim of this chapter is to adopt the Totally Fuzzy and Relative Approach to develop a cross-provincial multidimensional measure of poverty for the Republic of South Africa. In Section 2.2 the basic concepts relating to the logic of fuzzy sets are defined; and the Totally Fuzzy and Relative Approach is applied to a multidimensional analysis of poverty, specifying the individual and collective poverty indices according to a given set of attributes. The membership function is discussed in Section 2.3. In Section 2.4 the data used in the analysis is defined, namely, the Republic of South Africa Census 2001 and Republic of South Africa Census 1996. The set of composite indicators on the basis of both individual and household data is discussed. This section also contains the main results of the analysis, the construction of uni-dimensional poverty ratios for each attribute and the multi-dimensional poverty measure for each province for the years 1996 and 2001. Finally, Section 2.5 contains the conclusions.

2.2 METHODOLOGY

2.2.1 The Ordinary Set Principle

Given a set of X of elements $x \in X$, any subset B of X will be defined as follows:

$$x \in B \quad \Leftrightarrow \quad f_B(x) = 1$$

$$x \notin B \quad \Leftrightarrow \quad f_B(x) = 0$$

where

$f_B(x)$ is the membership function of the set B.

Define a population A of n households, $A = \{a_1, a_2, \ldots, a_n\}$. The traditional approach to the measurement of poverty holds that any household a_i is classified as poor or not poor according to the following criterion:

$a_i \in B$ if $y_i < z$

$a_i \notin B$ if $y_i \geq z$

where

B represents the set of poor,

y_i is the income observed of the i^{th} household, and

z is the poverty line.

2.2.2 The Fuzzy Set Principle

In classical set theory, an element is either wholly included or wholly excluded, with nothing in between, for example, a day can either belong to a month or not belong to a month. Fuzzy set theory allows an element to partially belong to a set. Fuzzy sets can be viewed as generalizations of classical sets, in that they are classes within which the transition from membership to non-membership takes place gradually.

Given a set of X of elements $x \in X$, any fuzzy subset B of X will be defined as follows:

$$B = \{x, f_B(x)\}$$

where

$f_B(x): X \rightarrow [0,1]$ is called the membership function (m.f.) of the fuzzy set B.

The value indicates the degree of membership of x to A.

Thus,

$$f_B(x) = \begin{cases} 0 & \text{if} \quad x \notin B \\ 1 & \text{if} \quad x \in B \end{cases} \qquad (2.1)$$

where

$$0 < f_B(x) < 1,$$

then x partially belongs to B and its degree of membership of B increases in ratio to the proximity of $f_B(x)$ to 1 (Cheli 1995).

Suppose that for each household, there is a vector of k attributes, $(X_1, X_2, \ldots, X_k)$.

In a population A of n households, $A = \{a_1, a_2, \ldots, a_n\}$, the subset of poor households B includes any household $a_i \in B$ which presents some degree of poverty in at least one of the k attributes of X.

The degree of membership of fuzzy set B of the i^{th} household, $(i=1, 2, \ldots, n)$, in respect of the j^{th} attribute, $(j= 1, 2, \ldots, m)$, is defined as follows:

$$\mu_B(X_j(a_i)) = x_{ij} \qquad 0 \leq x_{ij} \leq 1 \qquad (2.2)$$

Following the above definition,

$x_{ij} = 1$ when the i^{th} household does not possess the j^{th} attribute,

$x_{ij} = 0$ when the i^{th} household possesses the j^{th} attribute, and

$0 \leq x_{ij} \leq 1$ when the i^{th} household possesses the j^{th} attribute with an intensity belonging to the open interval (0,1).

The i^{th} family's membership function of fuzzy subset B of the poor can thus be defined as follows (Cerioli and Zani 1990):

$$f(x_i) = \frac{\sum_{j=1}^{k} \mu(x_{ij}) w_j}{\sum_{j=1}^{k} w_j} \qquad (i = 1, 2, \ldots, n) \qquad (2.3)$$

where

$w_1, w_2, \ldots, w_k$ represent a generic system of weights,

$f(x_i)$ is an individual Index of Global Poverty (IGP), and

$\mu(x_{ij})$ measures the specific deprivation for Item j.

The theory of fuzzy sets was introduced by Zadeh (1965) on the basis of the idea that certain classes of objects may not be defined by precise criteria of membership, in other words, cases where one is unable to determine which elements belong to a given set and which do not.

Let there be a set X and let x be any element of X. A fuzzy subset A of X is defined as the set of the couples $A = \{x, \mu_A(x)\}$ for all $x \in X$ where μ_A is an application of set X to the closed interval [0, 1], which is called the membership function of fuzzy subset A. In other words a fuzzy set or subset A of X is characterized by a membership function which will link any point of X with a real number in the interval [0, 1], the value of the membership function denoting the degree of membership of the element x to set A.

$$\mu_A(x) = \begin{cases} 1 & \text{if x belongs to subset A} \\ 0 & \text{if x does not belong to subset A} \end{cases} \qquad (2.4)$$

If A is a fuzzy subset, then the membership function can be written as

$\mu_A(x) = 0$ if x does not belong to subset A

$\mu_A(x) = 1$ if x completely belongs to subset A

$0 < \mu_A(x) < 1$ if x belongs partially to subset A

The closer to 1 the value of the membership function, the greater the degree of membership of x to A. This simple idea may easily be applied to the concept of poverty. In certain cases households are in such a state of deprivation that they certainly should be considered poor, while in others the level of welfare is such that they certainly should not be classified as poor. There are, however, also instances where it is not clear whether a given household is poor or not. This is especially true when one takes a multidimensional approach to poverty measurement, because according to some criteria one would certainly define the given households as poor, whereas, according to other criteria, one should not regard these households as poor. Such a fuzzy approach to the study of poverty has taken various forms in the literature.

The Totally Fuzzy Approach takes a whole series of variables that are supposed to measure a particular aspect of poverty into account. In the analysis of poverty there are several qualitative variables that may take more than two values. In such cases, the first step is to assume that one may rearrange these values in increasing order, where higher values denote a higher risk of poverty.

Let B be the subset of households which are in a situation of deprivation in respect of the attribute j, (j = 1, 2, . . . , k). Let b_j be the set of polytomous variables b_{1j}, b_{2j}. . . , b_{kj} measuring the state of deprivation of the various individuals with respect to attribute j. Let θ_j represent the set of the various states θ_{1j}, θ_{2j}. . . , θ_{kj} that attribute j may take, and let ψ_{ij}, ψ_{2j} . . . , ψ_{kj} represent the scores corresponding to these various states, assuming that $\psi_{1j} < \psi_{2j} \ldots < \psi_{kj}$.

A good illustration of the use of polytomous variables would be that in which individuals are asked to evaluate in subjective terms the physical conditions of the house they live in, the possible answers being "very good", "good", "medium", "bad", "very bad".

The membership function $\mu_{Bj}(i)$ for household i can be defined as follows:

$$\mu_{Bj}(i) = \begin{cases} 0 & \text{if } \psi_{1j} < \psi_{1\min} \\ \dfrac{\psi_{1j} - \psi_{1\min}}{\psi_{1\max} - \psi_{1\min}} & \text{if } \psi_{1\min} < \psi_{1j} < \psi_{1\max} \\ 1 & \text{if } \psi_{1j} > \psi_{1\max} \end{cases} \qquad (2.5)$$

where

$\psi_{1\,min}$ and ψ_{1max} represent the lowest and highest values taken by the scores ψ_{1j}.

In the case where deprivation indicators are continuous variables, for example, income, Cerioli and Zani (1990) defined two threshold values, X_{min} and X_{max} , such that, if the value x taken by the continuous indicator for a given individual is smaller than $X_{min,}$ the household will be defined as poor, whereas, if it is higher than $X_{max,}$ the household should not be considered poor.

Let X_j be the subset of households that are in an unfavourable situation in respect of attribute j, (j = 1, 2,. . . , k). The membership function can be defined as follows:

$$\mu_{Xj}(i) = \begin{cases} 1 & \text{if} \quad 0 < X_{i\,j} < X_{j\,min} \\ \dfrac{X_{jmax} - X_{i\,j}}{X_{jmax} - X_{j\,min}} & \text{if} \quad X_{i\,j} \in \left[X_{j\,min} \;,\; X_{j\,max}\right] \\ 0 & \text{if} \quad X_{1\,j} > X_{j\,max} \end{cases} \qquad (2.6)$$

The Totally Fuzzy and Relative Approach takes a relative approach to poverty according to which one is poor compare to some other households, stressing that when the risk of poverty is very low, then a high proportion of individuals will not be considered poor, as the value taken by the indicator of poverty in the Totally Fuzzy Approach may be too high for those who turn out not to be poor.

Let Bj represent the subset of households who are deprived in respect of indicator j, (j = 1, 2,. . . , k). Let ξ_j be the set of variables ξ_{1j} , ξ_{2j} . . . , ξ_{nj} which measure the state of deprivation of the various n households in respect of indicator j and let F_j be the cumulative distribution of this variable. Let $\xi_{j(m)}$ with (m = 1, 2,…, s) refer to the various values, ordered by increasing risk of poverty, which variable ξ_j may take. Thus $\xi_{j\ (1)}$

represents the lowest risk of poverty and $\xi_{j\,(s)}$ the highest risk of poverty associated with the deprivation attribute j.

The membership function may then be expressed as follows:

$$\mu_{bj}(i) = F_j\,(\xi_{ij}) \tag{2.7}$$

where

$\mu_{bj}(\xi_{j(m-1)})$ denotes the membership function of an individual for which variable ξ_j takes the value m, and

F_j is the distribution function of variable ξ_j.

Another "fuzzy approach" to poverty measurement has recently been suggested by Vero and Werquin (1997). They noted that one of the serious problems one faces when taking a multidimensional approach to poverty measurement, such as the fuzzy approach which has just been described, is that some of the indicators one uses may be highly correlated. To solve this problem, Vero and Werquin (1997) have proposed the following solution.

Let k again be the number of indicators and n the number of individuals. Let f_i represent the proportion of individuals who are at least as poor as individual i when taking into account all the indicators.

The deprivation indicator $m_p(i)$ for individual i will then be defined as:

$$m_p(i) = \frac{\ln\left(\frac{1}{f_i}\right)}{\sum_{i=1}^{n} \ln\left(\frac{1}{f_i}\right)} \tag{2.8}$$

The membership function $\mu_p(i)$ for individual i is then expressed as follows:

$$\mu_p(i) = \frac{m_p(i) - \mathrm{Min}\,[m_p(i)]}{\mathrm{Max}\,[m_p(i)] - \mathrm{Min}\,[m_p(i)]} \tag{2.9}$$

In the TFR method proposed by Cheli and Lemmi (1995), $\mu(x_{ij})$ is defined in terms of the distribution function F(.) of x_j as follows:

$$\mu(x_{ij}) = \begin{cases} F(x_{ij}) & \text{if } j \text{ increases as } X_j \text{ increases,} \\ 1 - F(x_{ij}) & \text{if } j \text{ increases as } X_j \text{ decreases.} \end{cases} \tag{2.10}$$

The normalized form is given by

$$\mu(x_{ij}) = \begin{cases} 0 & \text{if } x_{ij} = x_j^{(1)} \\ \mu(x_j^{(k-1)}) + \dfrac{F(X_j^{(k)}) - F(X_j^{(k-1)})}{1 - F(X_j^{(1)})} & \text{if } x_{ij} = x_j^{(k)},\ (k > 1) \end{cases} \tag{2.11}$$

where

$x_j^{(1)}, x_j^{(2)}, \ldots, x_j^{(m)}$, are the categories of the variable X_j, arranged in increasing order in respect of risk of poverty, and

F(x) is the distribution function of X_j.

The categories have been arranged in increasing order, so that $x_j^{(1)}$ denotes minimum risk and $x_j^{(m)}$ denotes maximum risk.

This ensures that the value of the membership function equal to zero is always associated with the category corresponding to the lowest risk of poverty and the value of the membership function equal to one is associated with the category corresponding to the highest risk of poverty.

The importance of an indicator for the measurement of poverty depends on how representative it is of the community's lifestyle, therefore the weights w_j are defined as a decreasing function of the proportion of the deprived.

Define the weights, w_j, as follows:

$$w_j = -\ln\left\{\left(\frac{1}{n}\right)\sum_{j=1}^{k}\mu(x_{ij})\right\} \qquad (2.12)$$

where

$\frac{1}{n}\sum_{j=1}^{k}\mu(x_{ij})$ represents the fuzzy proportion of the poor in respect of X_j.

By taking the natural logarithm, excessive importance is not given to elite goods. So, for example, the lack of a widespread commodity such as a car is definitely more important than the lack of a yacht.

Cerioli and Zani (1990) suggested that an overall index of poverty, P, for the entire population can be calculated by taking the arithmetic mean of the individual poverty indices, as follows:

$$P = \frac{1}{n}\sum_{i=1}^{k} f(x_{i.}) \qquad (2.13)$$

where P can be interpreted as the proportion of individuals that belong to the fuzzy subset of the poor (a fuzzy generalization of the headcount ratio of the poor). In the special case when $f(x_{i.})$ only assumes values (0, 1), that is, when B is not a fuzzy subset, P coincides with the head count ratio of the poor.

2.3 MEMBERSHIP FUNCTION

The measurement of poverty and deprivation is multidimensional. South Africa and many other countries continue to use only the monetary dimension (income or expenditure) to measure poverty and deprivation. The difficulty arises because many of the attributes or dimensions of poverty are categorical variables defined as "Yes" or "No". In this illustration the attributes "access to water" and "energy for cooking" are used from a sample of the Statistics South Africa Labour Force Survey 2003 dataset.

Table 2.3.1 shows the number of households that have access to running water and use electricity for cooking. There are 1 956 households that do not have access to electricity and water, 335 households that have electricity but no water, and 1 462 households that have water but no electricity.

Table 2.3.1: Contingency table for water and electricity

Electricity	Running water		Total
	Yes	No	
Yes	3 734	335	4 069
No	1 462	1 956	3 418
Total	5 196	2 291	7 487

The binary variables are not convenient for many statistical calculations. It is difficult to combine several attributes to arrive at a single index for poverty.

This study recognizes that any household is subject to several attributes or dimensions of deprivation and that, within an attribute, there are several grades or shades of deprivation. A household with running water inside the dwelling is slightly better off than a household with water in the yard. Similarly, a household with a tap 200 metres away is slightly worse off than a household with a tap in the yard, and a household with no access to water is seriously deprived. The different levels of deprivation that a household can experience for an attribute can be represented by the fuzzy membership function. Table 2.3.2 shows an example of the membership function.

Table 2.3.2: Membership function for attributes assessment and water

Main water supply	Membership Function
Piped water in dwelling	0
Piped water inside yard	0.1
Piped water on community stand less than 200m away	0.2
Piped water on community stand more than 200m away	0.3
Borehole	0.4
Spring	0.5
Rain water tank	0.6
Dam	0.7
River/stream	0.8
Water vendor	0.9
Other	1

Applying the fuzzy membership function to the attributes “access to water” and “energy for cooking”, the frequency set out in table 2.3.3 is obtained.

Table 2.3.3: Membership function for water and cooking

Cooking	Water									Total
	0	**0.1**	**0.2**	**0.3**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	
0	2 411	1 307	15	270	25	8	22	8	1	4 067
0.14	0	0	1	0	0	1	0	0	0	2
0.29	34	51	6	22	4	3	13	2	0	135
0.43	89	627	9	341	12	19	95	15	2	1 209
0.57	43	138	0	39	1	5	5	4	1	236

0.71	53	383	13	680	39	108	465	16	3	1 760
0.86	1	6	0	20	2	1	11	2	2	45
1	0	9	0	24	0	0	0	0	0	33
Total	2 631	2 521	44	1396	83	145	611	47	9	7 487

In table 2.3.4 the membership function is calculated for the attribute "toilet facility". The different categories are valued in order from least deprived, that is, Sewer, Septic Tanks, Chemical, Pit Latrine with Vent, Pit Latrine without Vent, Bucket and None. The membership functions are calculated for the methods proposed by Cerioli and Zani (1990), Cheli and Lemmi (1995) and Vero and Werquin (1997).

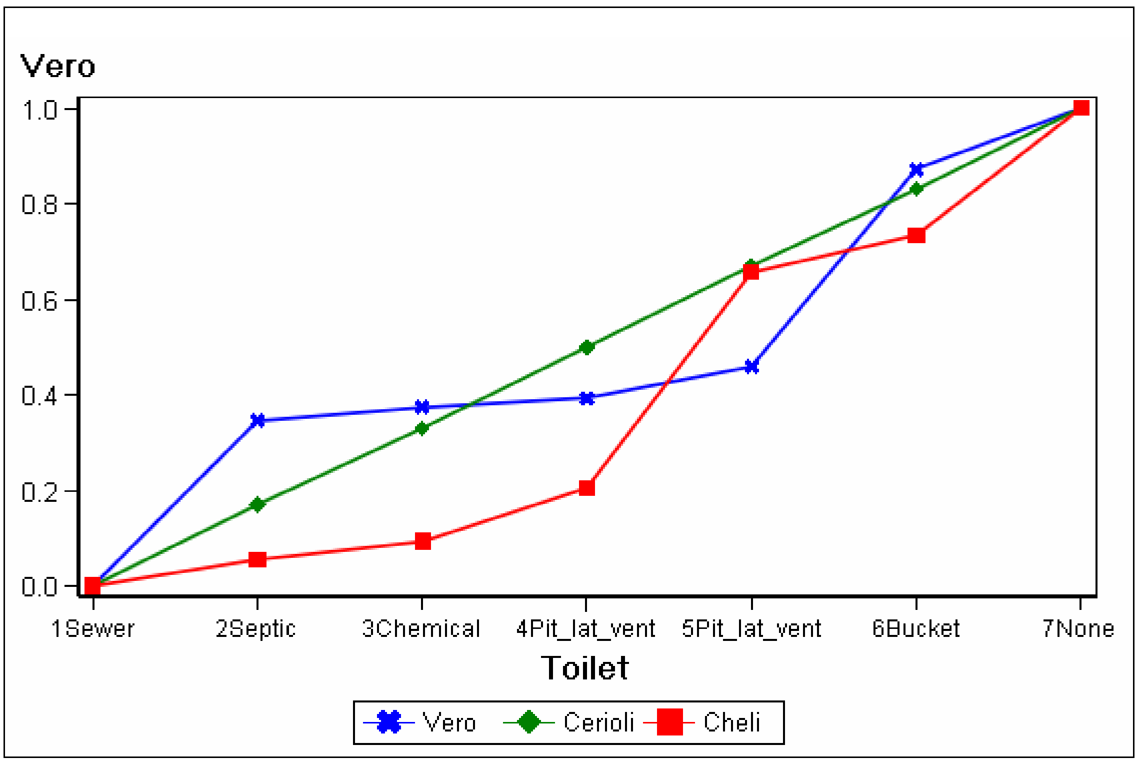
Table 2.3.4: Membership function for three attribute methods

Toilet	Vero	Cerioli	Cheli
Sewer	0.00	0.00	0.00
Septic	0.35	0.17	0.05
Chemical	0.37	0.33	0.09
Pit Latrine with Vent	0.39	0.50	0.20
Pit Latrine without Vent	0.46	0.67	0.66
Bucket	0.87	0.83	0.73
None	1.00	1.00	1.00

The various membership functions that were calculated in table 2.3.4 are shown in figure 2.3.1. The different categories of toilet facilities are shown on the X axis and the membership function is shown on the Y axis. The membership proposed by Cerioli and Zani (1990) is a straight line and calculated independently of the positions of the household. Cheli and Lemmi (1995) believe that if the majority of the households possess an attribute, then any household without this attribute is severely deprived. The membership function for the deprived household is largely, very close to one. One the other hand if the majority of the households do not possess an attribute then any household without this attribute is not severely deprived. The membership function for the deprived household is small, that is, closer to zero. The Cheli and Lemmi membership function is determined once the frequency in each category is known, in other words, the membership function is relative to the frequency.

The Vero approach was introduced to accommodate highly correlated indicators by logarithmically calculating the membership function for two attributes and obtaining the results shown in figure 2.3.1.

Figure 2.3.1: Fuzzy membership functions



In table 2.3.5 a population, A, of ten households is assumed, $A = \{a_1, a_2, \ldots, a_{10}\}$, the subset of poor households, B, includes any household $a_i \in B$ which presents some degree of poverty in at least one of the ten attributes.

The degree of membership of fuzzy set B of the i^{th} household, (i = 1, 2,…,10), in respect of the j^{th} attribute, (j =1, 2,…, 8), is

$$\mu_B (X_j (a_i)) = x_{ij} , \qquad 0 \le x_{ij} \le 1 \qquad (2.14)$$

Table 2.3.5: Example of fuzzy set multidimensional analysis of poverty

Attribute / Household	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	Poverty ratio per household
1	1	1	1	1	1	1	1	1	1.00
2	1	0	1	0	1	0	0	0	0.09
3	1	0	1	0	1	1	1	0	0.41
4	1	1	1	1	0	0	0	0	0.28
5	1	1	1	0	1	1	0	0	0.27
6	1	0	1	1	1	0	0	0	0.23
7	1	1	1	0	0	1	0	0	0.22
8	1	1	1	1	1	1	0	0	0.41
9	1	0	0	0	1	1	0	0	0.13
10	1	0	0	0	0	0	0	0	0.00
$A_j = \sum_{j=1}^{10} \mu(x_{ij})$	10	5	8	4	7	6	2	1	P=0.3024
$\frac{1}{n} \sum_{j=1}^{k} \mu(x_{ij})$	1.00	0.50	0.80	0.40	0.70	0.60	0.20	0.10	
w_j	0	0.69	0.22	0.92	0.36	0.51	1.61	2.3	

Table 2.3.5 shows that none of the ten households possesses attribute a_1 and therefore the corresponding weight, $w_{i,}$ is equal to zero, indicating that attribute a_1 does not contain useful information about the degree of poverty of the analysed households. Only

one household does not possess attribute a_8 and the corresponding weight, w_8, is equal to 2.3. This indicates the strong social exclusion perceived by the only household not possessing attribute a_8.

Analysing the rows of table 2.3.5, the greatest poverty is attached to the household which does not possess any of the eight attributes, thus a poverty ratio per household of 1. The lowest poverty ratio refers to the household that does not possess only the first attribute, a poverty ratio of zero.

The multidimensional poverty ratio of the population is the arithmetic mean of the individual poverty ratios per household, p = 0.3024.

2.4 ANALYSIS

The data used in this study come from the Republic of South Africa Census 2001 and Census 1996. The following eight attributes, as shown in table 2.4.1, were selected to determine the relative deprivation, degree of social exclusion and the inability for a household to achieve the living standard of the province to which it belongs**.**

Table 2.4.1: Attributes for poverty measurement

Attribute	**Categories**
Formal dwelling	Brick structure, flats, town house, rooms in back yard, traditional dwelling, informal dwelling, caravans and tents.
Energy source for cooking	Electricity, gas, paraffin, coal, wood, solar.
Energy source for heating	Electricity, gas, paraffin, coal, wood, solar.
Energy source for lighting	Electricity, gas, paraffin, candles, solar.
Main water supply	Tap in dwelling, tap in yard and public tap excludes borehole, rain water tank, dam spring and river.
Toilet facilities	Flush toilet, pit latrines and bucket latrine.
Refuse removal	Municipal removal, communal and own refuse dump.
Telephone facilities	Telephone in dwelling, neighbour, work and nearby location.

2.5 RESULTS

The membership functions for each province are calculated from the Republic of South Africa 1996 Census data and are shown in table 2.5.1. The membership function for each attribute is obtained by multiplying the degree of membership for the attribute of every household in the Republic of South Africa. The degree of membership for each attribute is given in Appendix A. Table 2.5.1 shows that the level of deprivation for households in the Eastern Cape province for the attribute lack of electricity for cooking is 66%, while this figure for the Gauteng province is only 19.5%.

Table 2.5.1: Membership function for attributes for Census 1996

Membership function									
Province	EC	FS	GP	KZ	MP	NC	LP	NW	WC
Lack of elect for cooking	0.662	0.435	0.195	0.462	0.534	0.339	0.753	0.519	0.154
Lack of formal dwelling	0.541	0.364	0.267	0.465	0.359	0.209	0.391	0.303	0.199
Lack of elect for heating	0.690	0.472	0.199	0.472	0.527	0.423	0.727	0.534	0.197
Lack of elect for lighting	0.584	0.409	0.197	0.449	0.405	0.273	0.570	0.540	0.126
Lack of tap water	0.584	0.254	0.141	0.451	0.343	0.213	0.492	0.395	0.105
Lack of toilet	0.480	0.356	0.097	0.331	0.319	0.300	0.480	0.339	0.106
Lack of refuse removal	0.394	0.174	0.079	0.303	0.298	0.147	0.461	0.300	0.077
Lack of telephone	0.615	0.385	0.244	0.416	0.423	0.329	0.573	0.458	0.185

The weights for each province are calculated from the Republic of South Africa 1996 Census data and are shown in table 2.5.2. Equation 2.12 is used to calculate the weights. The weight for an attribute is the negative logarithm of the membership function. If the level of deprivation is low, then the corresponding weight is high. Lack of electricity for cooking in the Eastern Cape Province has a weight of 0.412, while the weight for the Western Cape Province is 1.868.

Table 2.5.2: Weights for attributes for Census 1996

Weights									
Province	EC	FS	GP	KZ	MP	NC	LP	NW	WC
Lack of elect for cooking	0.412	0.831	1.634	0.773	0.627	1.083	0.284	0.656	1.868
Lack of formal dwelling	0.614	1.012	1.321	0.766	1.024	1.563	0.938	1.194	1.614
Lack of elect for heating	0.371	0.752	1.615	0.750	0.640	0.860	0.319	0.627	1.625
Lack of elect for lighting	0.538	0.894	1.623	0.800	0.905	1.299	0.563	0.615	2.074
Lack of tap water	0.538	1.370	1.957	0.796	1.071	1.545	0.709	0.930	2.256
Lack of toilet	0.733	1.031	2.337	1.105	1.144	1.204	0.735	1.083	2.243
Lack of refuse removal	0.930	1.751	2.534	1.195	1.211	1.919	0.774	1.203	2.566
Lack of telephone	0.487	0.956	1.413	0.877	0.861	1.110	0.557	0.781	1.688
Sum of weights	4.623	8.597	14.434	7.063	7.481	10.582	4.879	7.089	15.935

Table 2.5.3 shows the deprivation index for the 9 provinces in the Republic of South Africa calculated on the data from the 1996 census. The Western Cape Province has the smallest deprivation index while the Eastern Cape Province has the largest deprivation index.

Table 2 5.3: Deprivation index for provinces for Census 1996

Deprivation Index									
Province	EC	FS	GP	KZ	MP	NC	LP	NW	WC
Deprivation index	0.542	0.330	0.164	0.408	.383	.260	.515	0.398	0.136

The membership functions for each province are calculated from the Republic of South Africa 2001 Census data and are shown in table 2.5.4. The level of deprivation for households for households in the Eastern Cape Province for the attribute lack of electricity for cooking is 62%. This is a reduction of 4% from 1996 level of deprivation of 66%. The percentages for all the other provinces have also decreased in the year 2001.

Table 2 5.4: Membership function for attributes for Census 2001

Membership function									
Province	EC	FS	GP	KZ	LP	MP	NC	NW	WC
Lack of elect for cooking	0.620	0.398	0.194	0.438	0.702	0.499	0.302	0.444	0.144
Lack of formal dwelling	0.499	0.325	0.258	0.399	0.270	0.295	0.171	0.269	0.183
Lack of elect for heating	0.237	0.198	0.094	0.195	0.319	0.301	0.163	0.189	0.039
Lack of elect for lighting	0.445	0.244	0.184	0.378	0.342	0.305	0.231	0.287	0.101
Lack of tap water	0.584	0.317	0.203	0.470	0.550	0.402	0.232	0.434	0.144
Lack of toilet	0.518	0.386	0.122	0.378	0.576	0.394	0.257	0.411	0.119
Lack of refuse removal	0.345	0.203	0.065	0.260	0.433	0.295	0.130	0.298	0.049
Lack of telephone	0.356	0.296	0.179	0.286	0.327	0.273	0.239	0.299	0.145

The weights for each province are calculated from the Republic of South Africa 1996 Census data and are shown in table 2.5.5. Equation 2.12 was used to calculate the weights The weight for the attribute lack of electricity for cooking for the Eastern Cape Province has increased from 0.412 in 1996 to 0.477 in 2001. It can clearly be seen that as the level of deprivation for an attribute in a province decreases the corresponding weight increases.

Table 2 5.5: Weights for attributes for Census 2001

Weights									
Province	EC	FS	GP	KZ	LP	MP	NC	NW	WC
Lack of elect for cooking	0.477	0.920	1.638	0.826	0.354	0.696	1.197	0.811	1.936
Lack of formal dwelling	0.695	1.125	1.355	0.918	1.308	1.221	1.768	1.312	1.701
Lack of elect for heating	1.438	1.620	2.363	1.634	1.143	1.202	1.815	1.664	3.240
Lack of elect for lighting	0.811	1.411	1.693	0.974	1.073	1.187	1.466	1.248	2.291
Lack of tap water	0.537	1.150	1.593	0.756	0.598	0.912	1.463	0.835	1.938
Lack of toilet	0.657	0.952	2.106	0.972	0.551	0.932	1.358	0.889	2.132
Lack of refuse removal	1.066	1.594	2.727	1.348	0.836	1.219	2.037	1.212	3.018
Lack of telephone	1.031	1.218	1.719	1.252	1.119	1.297	1.432	1.209	1.929
Sum of weights	6.713	9.991	15.194	8.680	6.983	8.666	12.536	9.180	18.186

Table 2.5.6 shows the deprivation index for the 9 provinces in South Africa calculated on the data from the 1996 census and the 2001 census. The Western Cape Province still has the smallest deprivation index while the Eastern Cape Province has the largest deprivation index.

Table 2 5.6: Deprivation index for provinces for Census 2001

Deprivation Index									
Province	EC	FS	GP	KZ	LP	MP	NC	NW	WC
Deprivation index(1996)	0.542	0.330	0.164	0.408	0.515	0.383	0.260	0.398	0.136
Deprivation index(2001)	0.407	0.281	0.149	0.328	0.388	0.332	0.207	0.309	0.105

2.6 CONCLUSION

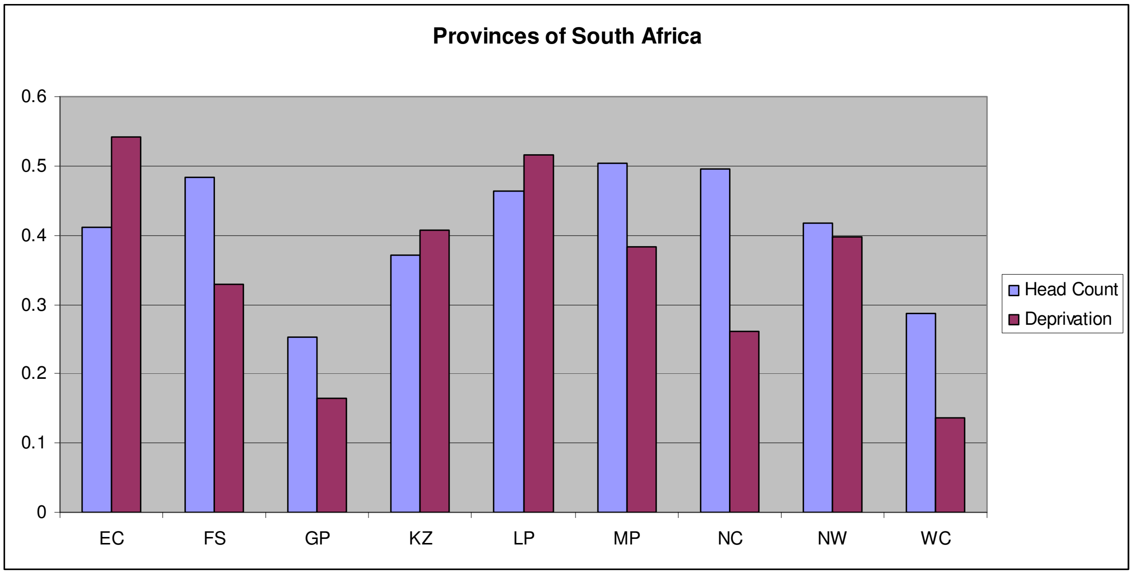
Table 2.6.1 shows the head count ratio and the deprivation index for the nine provinces in the Republic of South Africa. The head count ratio is determined by calculating the proportion of households that receive an income of below R800 per month.

Table 2.6.1: Comparison of head count ratios and poverty ratios

Provinces	EC	FS	GP	KZ	LP	MP	NC	NW	WC
Head Count Ratio 1996	0.412	0.484	0.252	0.372	0.463	0.504	0.496	0.417	0.287
Head Count Ratio 2001	0.391	0.507	0.214	0.358	0.495	0.456	0.475	0.355	0.263
Deprivation index 1996	0.542	0.330	0.164	0.408	0.515	0.383	0.260	0.398	0.136
Deprivation index 2001	0.407	0.281	0.149	0.328	0.388	0.332	0.207	0.309	0.105

In Figure 2.6.1 the headcount ratio for the Eastern Cape is lower than the deprivation index indicating that a large proportion of the community does not have access to basic services. In the Free State, the headcount ratio is higher than the deprivation index. A large proportion of the households have access to basic services while many households are unemployed and cannot pay for the services.

Figure 2.6.1: Head count ratio and deprivation index by province



This chapter has investigated the problem of analysing poverty dynamics according to a multidimensional, fuzzy and relative approach. After discussing the limitations of the traditional approach based on the rigid classification of either being poor or being not poor, the Totally Fuzzy and Relative method for the multidimensional approach to poverty measurement was proposed.

The empirical analysis involved the application of the proposed methodology to the Republic of South Africa Census 1996 and Census 2001 data. The disparities between the head count ratio and the deprivation index could be clearly seen for the different provinces in the Republic of South Africa.

The methodology considered in this chapter represents a powerful tool for a multidimensional analysis of poverty that complements the unidimensional measurement of poverty to devise effective strategies to reduce current poverty and prevent future poverty.