

Chapter 2

2 OVERVIEW OF ELECTROMAGNETIC THEORY FOR GROUND, IN-LOOP TDEM DATA

"There is a theory which states that if ever anybody discovers exactly what the Universe is for and why it is here, it will instantly disappear and be replaced by something even more bizarre and inexplicable. There is another theory which states that this has already happened."

(from "The Hitchhiker's guide to the Galaxy", Douglas Adams)

2.1 Introduction

It is the inherent complexity contained in the electromagnetic method that on the one hand forces us to simplify it almost beyond recognition in order to apply it practically; and on the other hand compels us to keep on searching for the "truth" (or the most complete set of information that can be obtained from EM measurements). This continued search is conducted in many, painstakingly slow, very small steps – sometimes referred to as *research*.

All EM methods (except the magneto-telluric method) utilise active sources, or transmitters, in order to generate measurable fields. This is in contrast to passive methods such as the potential field methods (e.g. the gravity and magnetic techniques) where the earth's natural occurring magnetic field is measured. Passive source method instruments only consist of receivers and the data acquired by different instruments are all reduced to the earth's total field with no or minimal computational effort. Active source instruments, on the other hand, require the complete specifications of the source and receiver to be incorporated into the processing and interpretational procedures. Unless stated otherwise, algorithms derived in this study are only suitable for systems conforming to the same basic geometrical set-up and operating parameters as described in the following paragraphs.

2.2 System geometry and operating specifications

The theory developed in this study is directly applicable to central-loop ground TDEM systems with step-current excitation, e.g. Geonics EM37 or EM47 and interpretation

strategy allows for both 1D and 3D model considerations. The late time mathematical approximations are implemented for most procedures.

2.3 Analytical TDEM responses for four common models

Analytical responses for TDEM data are calculated by solving Maxwell's equations. The differential forms of the equations are valid at points in space only, but comparatively easy to calculate, while the integral forms are valid at boundaries of units, more difficult to calculate and used primarily to generate boundary conditions.

Table 2-1 Maxwell's equations (Compiled from Ward and Hohmann (1988) and Kaufman and Keller (1983)).

Maxwell's equations in integral form:	Maxwell's equations in differential form:
$\int_S \mathbf{D} \cdot d\mathbf{S} = q$ $\int_S \mathbf{B} \cdot d\mathbf{S} = 0$ $\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$ $\Phi_B = \int_S \mathbf{B} \cdot d\mathbf{S} = \text{Flux through surface}$ $\oint \mathbf{H} \cdot d\mathbf{l} = \mathbf{I} + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{S}$	$\nabla \cdot \mathbf{D} = \rho$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}$

Maxwell's equations are uncoupled differential equations that need to be coupled using the constitutive equations that contain all the electrical and magnetic properties of the medium through which EM propagation occurs (i.e. the earth in geophysics). The constitutive equations are given by Ward and Hohmann (1988) as:

$$\mathbf{D} = \underline{\underline{\epsilon}} \cdot \mathbf{E}$$

$$\mathbf{B} = \underline{\underline{\mu}} \cdot \mathbf{H}$$

$$\mathbf{j} = \underline{\underline{\sigma}} \cdot \mathbf{E}.$$

\mathbf{E} : electric field intensity [V/m]

\mathbf{D} : dielectric displacement [C/m²]

\mathbf{I} : current [A]

ρ : electric charge density [C/m³]

$\sigma, \underline{\underline{\sigma}}$: conductivity [S/m]

$\epsilon, \underline{\underline{\epsilon}}$: dielectric permittivity [F/m]

\mathbf{B} : magnetic induction [Tesla]

\mathbf{H} : magnetic field intensity [A/m]

\mathbf{j} : current density [A/m²]

q : charge [Coulomb]

$\mu, \underline{\underline{\mu}}$: magnetic permeability [H/m]

In these equations the dielectric permittivity, magnetic permeability and electric conductivity should all be regarded as tensor functions of angular frequency, position, time, temperature, pressure and magnetic/electric field strength. However, in order to derive analytical solutions some assumptions regarding these parameters are necessary.

ASSUMPTIONS:

- All media are linear, isotropic and homogeneous and possess physical properties, which are independent of time, temperature and pressure (implying scalar presentation of physical properties rather than tensors).
- The magnetic permeability of all media is assumed to be equal to that of free space, i.e. $\mu = \mu_0$.

The TDEM solutions are derived by calculating the frequency domain electromagnetic (FDEM) responses for equivalent models and then applying Fourier transforms to these results. In order to solve the FDEM equations analytically some further assumptions are necessary.

ADDITIONAL ASSUMPTIONS:

- There are no free electric charges or current in the medium, i.e.

$$\rho = 0; \quad \nabla \cdot \mathbf{D} = \nabla \cdot \mathbf{E} = 0.$$

- We assume a harmonic time varying field, i.e.

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t} \text{ and } \mathbf{B} = \mathbf{B}_0 e^{-i\omega t}.$$

- Displacement currents are much smaller than induction currents, i.e.

$$\frac{\partial \mathbf{D}}{\partial t} \ll \mathbf{j} \text{ and } \nabla \times \mathbf{H} = \sigma \mathbf{E} \text{ implying a wavenumber of}$$

$k^2 = i\omega\mu\sigma$ in the solution of the wave (Helmholz) equation.

The receiver and transmitter are in the same plane of observation ($z=0$), also indicating the air-surface boundary in the case of a half space or layered earth.

Incorporating these assumptions into calculations the analytical solutions for a number of different geological models can be found. For the purposes of this study there are four cases deserving special attention, namely:

- Conductive half space

- Thin, conductive sheet (S-layer)
- Finite conductor in a resistive host rock (half space)
- Finite conductor in a conductive host rock (half space).

2.3.1 TDEM central-loop response over a conductive half space

The quasi-static approximation of EM-field propagation in a half space can be described as a diffusion process (Nabighian, 1979). When doing a TDEM survey the current in the transmitter loop is switched off very abruptly, causing a sudden change in the associated magnetic field (large $\partial\mathbf{B}/\partial t$). Following from Maxwell's third equation the electric field (\mathbf{E}) associated with this sudden change in magnetic field will cause a current to flow in the conductive half space ($\mathbf{j} = \sigma\mathbf{E}$). This current will mimic the primary transmitter loop geometry but will only exist in that form for a fraction of a second as it is not confined to a specific path or circuit and has no source (battery) to maintain current flow. The induced current will therefore immediately start to dissipate, generating a new magnetic field that changes with time and consequently induce new currents in the half space. This behaviour was discussed extensively by Nabighian (1979), leading to the smoke-ring concept of EM propagation in a homogeneous half space. In short, a number of EM induced currents exist in the half space after the primary field is terminated and the maximum current density retains a transmitter loop-like shape expanding with time. Contouring the electric field at different points in time clearly show the maxima of the electric field (most dominant currents) migrating outward and downward into the half space at an angle of approximately 30° . Nabighian (1979) also introduced the mathematical alternative of a single equivalent current filament that will produce, with a high degree of accuracy, the same EM fields measured on the surface of the earth, as would the whole system of real currents. This equivalent current filament moves outward and downward at an angle of approximately 47° . A comparison of these two approaches is shown in figure 2.1. The advantage of using the single equivalent current filament is the dramatic simplification in mathematical description that allows the calculation of parameters such as velocity and depth with time as well as relationships of these to half space conductivity.

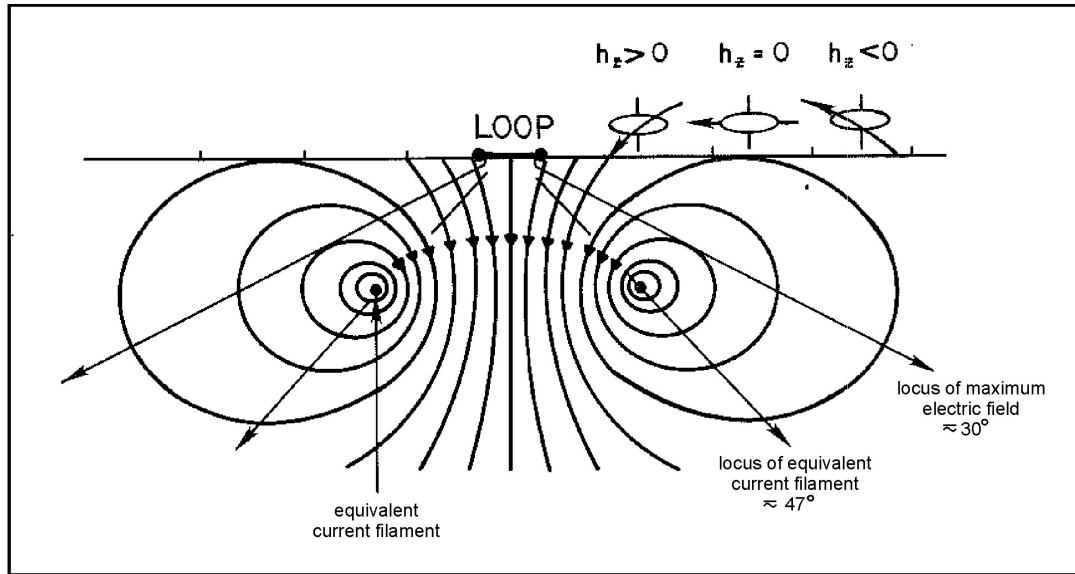


Figure 2-1 The use of equivalent current filament concept in understanding the behaviour of TEM fields over a conducting half space (after Nabighian and Macnae, 1991).

The quasi-stationary (derivative of) magnetic field transient response of a central-loop, step-current system over a conductive earth is given by Kaufman and Keller (1985) as

$$\frac{\partial H_z}{\partial t} = -\frac{I}{\mu_0 \sigma a^3} \left[3 \operatorname{erf}(u) - \frac{2}{\pi^{1/2}} u (3 + 2u^2) e^{-u^2} \right] \quad (2.1)$$

where

$$\operatorname{erf}(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-x^2} dx$$

$$u = \frac{\pi \sqrt{2} a}{\tau}$$

$$\tau = 2\pi \left(\frac{2t}{\mu_0 \sigma} \right)^{1/2}$$

a = radius of the transmitter loop[m]

t = time [s],

with a late time asymptotic approximation of

$$\frac{\partial H_z}{\partial t} = (\operatorname{emf}) \cdot (\text{effective receiver area}) = -\frac{I a^2 (\sigma \mu_0)^{3/2}}{20\pi^{1/2}} t^{-5/2}. \quad (2.2)$$

In the “late time” (which can be achieved through either large values of t , or small values of σ), it can be seen from equation 2.2 that the vertical component of the measured

electromotive force (emf) is directly proportional to $t^{-5/2}$ for an ideal step function excitation. This behaviour is described as a power-law decay with time and manifests as a straight line with a slope of $m = -5/2$ on a graph of $\log(\text{emf})$ versus $\log(t)$. The horizontal component of the emf can be shown to exhibit the same behaviour (Kaufman and Keller, 1983); the only difference being that it shows a time-decay proportional to t^{-3} in the late time. In terms of the smoke-ring analogy, the “late time” for profiling commences when the equivalent current filament is so far away that any receiver position can be approximated to be at the centre of the smoke-ring. With the central-loop system (as used in this study) this is always the case and it is easy to see why the late time approximations are particularly useful for this survey geometry.

2.3.2 Thin, conductive sheet (S-layer)

Parameters used in the conductive sheet model are shown in figure 2.2.

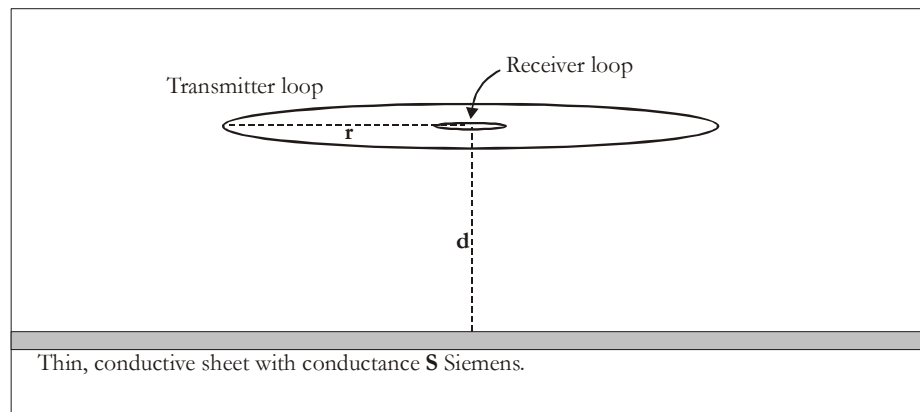


Figure 2-2 Conductive sheet parameters.

The equivalent current smoke-ring behaviour for an S-layer (originally solved by Maxwell) is described by Nabighian and Macnae (1991). With the primary field termination currents are induced in the S-layer, once again representing the geometry of the transmitter loop. As time passes the equivalent current filament retains its original size and intensity and only appears to migrate downwards with a velocity related to the conductance of the S-layer as $v = 2/S\mu_0 \text{ ms}^{-1}$ (figure 2.3). This is an exact solution for velocity (v).

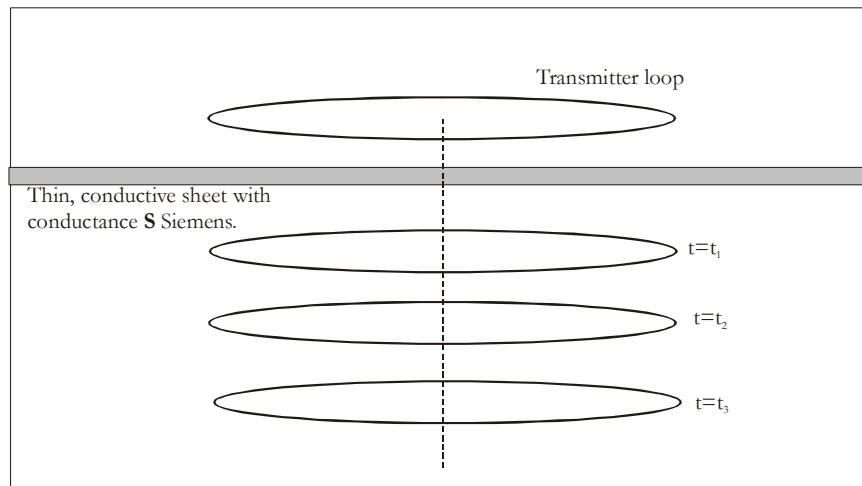


Figure 2-3 Equivalent current filaments (images) for a conducting thin sheet at various times after current interruption in the transmitter loop (after Nabighian and Macnae, 1991).

The emf induced in a loop which is located in the horizontal plane, with a small dipole source located at its centre, is given by Kaufman and Keller (1983) as:

$$emf = -\frac{M}{Sr^2} \frac{3(\tau_s + 2h_0)}{[1 + (\tau_s + 2h_0)^2]^{\frac{3}{2}}} \quad (2.3)$$

where

M = magnetic dipole moment of the transmitter (small) loop

$$\tau_s = \frac{2t}{\mu_0 S r}$$

r = radius of the large receiver loop [m]

t = time [s]

S = conductance of the thin plate [S]

$$h_0 = \frac{d}{r}$$

d = vertical distance between the loops and the conductive plate.

The quasi-stationary (derivative of) magnetic field transient response of a central-loop, step current system over a thin conductive sheet can be calculated from equation 2.3 by applying the principle of reciprocity (i.e. in any system the same emf will still be measured if the transmitter and receiver geometries are interchanged). Making the late time approximation

of $\frac{2t}{\mu_0 S} + 2d \gg r$ (where r now represents the small loop radius after reciprocity) and

simplifying equation 2.3 we have

$$emf_{late\ time} = -\frac{3M}{16Sr^2} \frac{1}{(\tau + d)^4} \quad (2.4)$$

$$\frac{\partial H_z}{\partial t} = \frac{emf_{late\ time}}{\text{effective Rx area}} = \frac{emf_{late\ time}}{n_r \pi r^2} = -\frac{3M}{16S\pi} \frac{1}{(\tau + d)^4} \quad (2.5)$$

where

M = magnetic dipole moment of the transmitter (large) loop

$$\tau = \frac{t}{\mu_0 S}$$

n_r = number of turns in receiver loop

r = radius of the small receiver loop[m].

This is a very good approximation for the chosen system geometry as the radius of the receiver loop is always in the order of one meter and the late time condition is always satisfied under field conditions. The “late time” approximation of the S-layer response is therefore directly proportional to t^{-4} . This behaviour manifests as a straight line with a slope of $m = -4$ on a graph of $\log(emf)$ versus $\log(t)$.

2.3.3 Finite conductor in a resistive host rock (half space)

The significance of having a confined target surrounded by an insulator is that there is always some stage of time (“late time”) when the current distribution in the conductor becomes invariant with time and the decay becomes exponential at a rate determined by the shape, size, and conductivity of the body (McNeill, 1980). An intuitive feeling for the general behaviour of isolated (confined) conductors can be developed from examining the simplified case of a conducting sphere assumed to be in a region of uniform magnetic field that is suddenly terminated. Immediately after termination of the primary current (early time) the secondary currents flow on the surface of the sphere and are independent of its conductivity (figure 2.4a). From then on the currents diffuse radially inwards, similar to the smoke-ring propagation found in a half space. This stage of changing current distribution is referred to as intermediate time (figure 2.4c). The late time commences when the inductance and resistance associated with each current ring, have stabilized and from this time onwards both the currents and their associated external magnetic field commence to decay exponentially with a time-constant given by

$$\tau = \frac{\sigma \mu a^2}{\pi^2} \quad (2.6)$$

σ = conductivity of sphere [S]

a = radius of sphere [m],

so that

$$emf_{sphere \text{ late time}} = A e^{-t/\tau} \quad (2.7)$$

A = constant containing geometrical information.

Emf measurements taken at this stage of “late time” will manifest as a straight line when plotted on a semi-log graph of $\ln(emf)$ versus time.

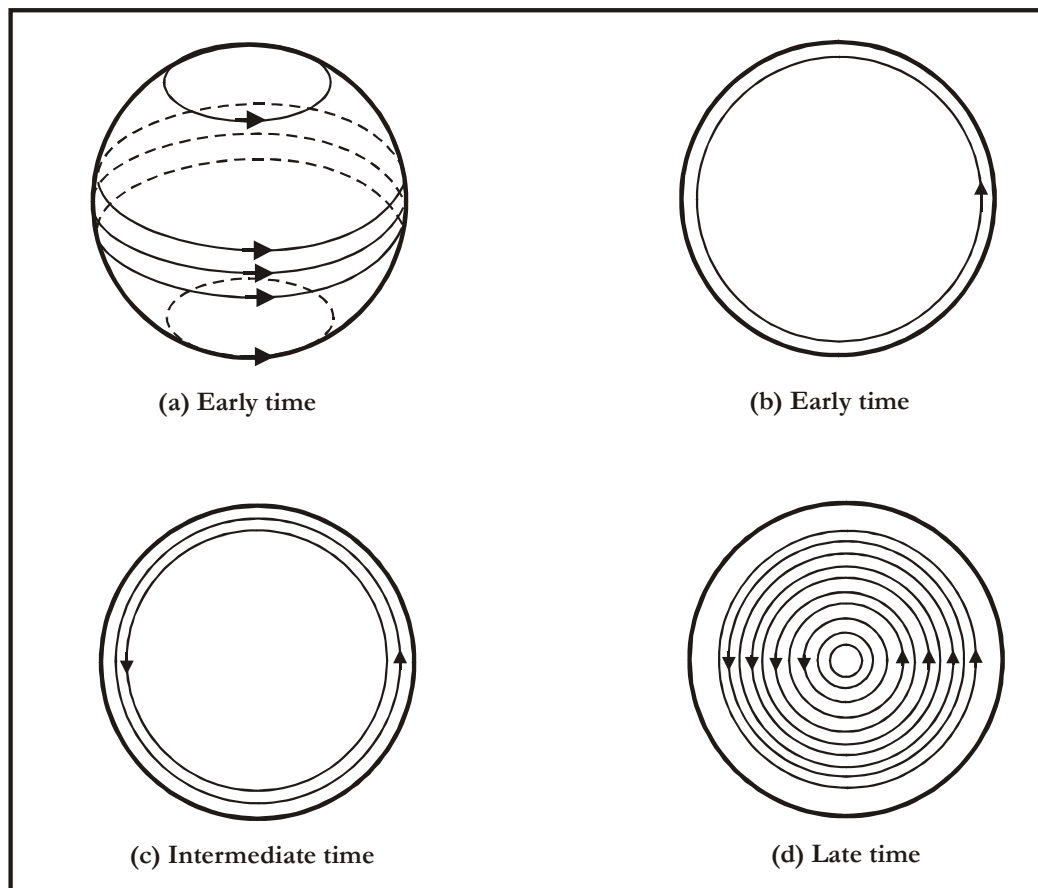


Figure 2-4 Sphere currents at various times (a – surface currents; b-d – equatorial/plane currents) (after McNeill, 1980).

2.3.4 Finite conductor in a conductive host rock (half space)

When a conductive target is located in a conductive host rock the inducing magnetic field, at the location of the target, is no longer a step function but a slowly varying field due to

the diffusion of the smoke-ring currents (Nabighian and Macnae, 1991) and the assumptions stated in 2.3 are not valid anymore. This different nature of the inducing field has two consequences. Firstly, the toroidal vortex (induced) currents will not be as strong as in the case of a conductor in free space, because of the reduced $\partial\mathbf{B}/\partial t$ component (fields are varying slower with time). Secondly, the smoke-rings will also cause galvanic currents to flow in the conductor and lead to charge accumulation on the boundaries between the conductor and the host rock. The smoke-ring electric field will instantaneously be opposed within the target by the field of this electric charge distribution created at the target boundaries. A secondary electric field created by these charges will cause a poloidal current flow that tends to cancel most of the primary electric field (i.e. the field associated with the smoke-rings) inside the conducting target (Nabighian and Macnae, 1991). This cancellation is more effective for short strike length bodies which, therefore, have less poloidal (galvanic) current flow than longer bodies. The strength of these currents depends mainly on the conductivity of the host rock and is only weakly dependent on the much higher conductivity of the target. As such, the secondary magnetic field will decay at a rate governed chiefly by the host rock conductivity. Anomalies due to both poloidal (galvanic) and toroidal (inductive) currents are generally of the same sign and increase the detectability of a given target according to Nabighian and Macnae (1991). However, the combined anomaly will be spatially smeared toward longer wavelengths compared to that of vortex currents alone – leading to erroneous interpretation if modelled using a conductor in free-space approach. Singh (1973) has calculated the TDEM response for a conductive sphere in a conducting infinite space and shown that the late time response can also be presented as an inverse power law that is characteristic of whole- and half space responses (i.e. straight-line behaviour in the late time on a logarithmic graph of emf versus time). In the following graphs, h refers to the magnetic field component, the first subscript 1 to the multi-pole, the second to the spherical component (r , θ , or ϕ), and the third to the orientation of the magnetic dipole (r for radial, θ for transverse). The superscripts v and u refer to the magnetic mode or transverse electric (TE) and electric mode or transverse magnetic (TM) solutions. In free space (i.e. making the quasi-static approximation) only TE solutions are generated with a radial magnetic dipole as source (horizontal loop above sphere). Figures 2.6 to 2.8 show the differences in late time decay behaviour between a conductive sphere in free space ($\sigma_1/\sigma_2=0$) and a conductive sphere in a conducting infinite space for various conductivity contrasts ($\sigma_1/\sigma_2=1/10$ to $1/100$). The secondary magnetic field component $h_{1r}^v(t)$ in figure 2.6, is equivalent to the vertical

component ($h_z(t)$) generated from a vertical magnetic dipole source through abrupt termination of current in a horizontal loop. The response for a sphere in free space ($\sigma_1/\sigma_2=0$) corresponds with solutions obtained by Nabighian (1971) and McNeill (1980).

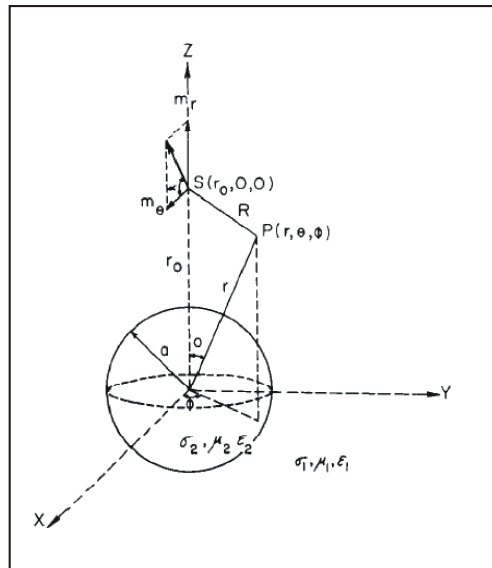


Figure 2-5 A permeable conducting sphere embedded in a conducting infinite space. The dipolar source is located at $S(r_0,0,0)$ outside the sphere (after Singh 1973).

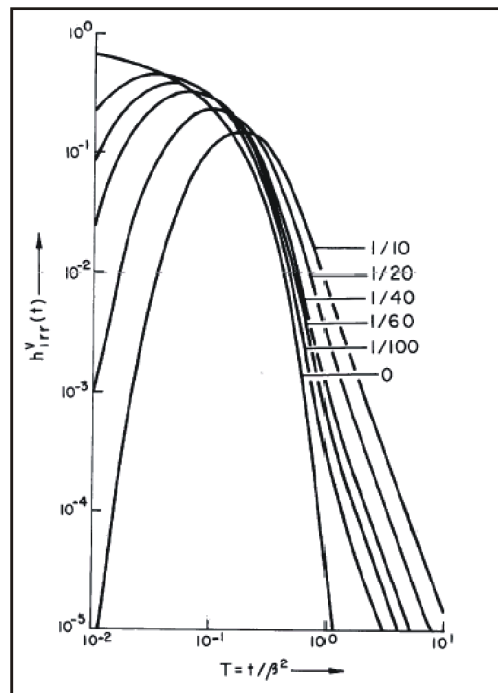


Figure 2-6 Time characteristic of $h^v_{1rr}(t)$ for $r/a=2$, $\mu_2/\mu_1=1$, and variable σ_1/σ_2 (after Singh 1973).

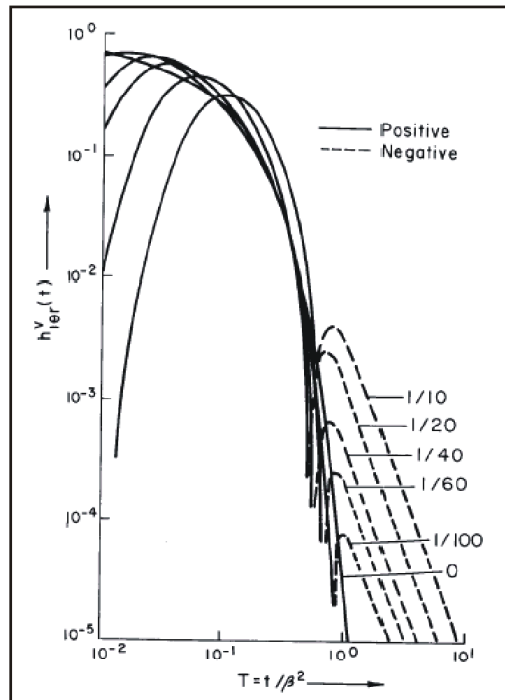


Figure 2-7 Time characteristic of $h^v_{10r}(t)$ for $r/a=2$, $\mu_2/\mu_1=1$, and variable σ_1/σ_2 (after Singh 1973).

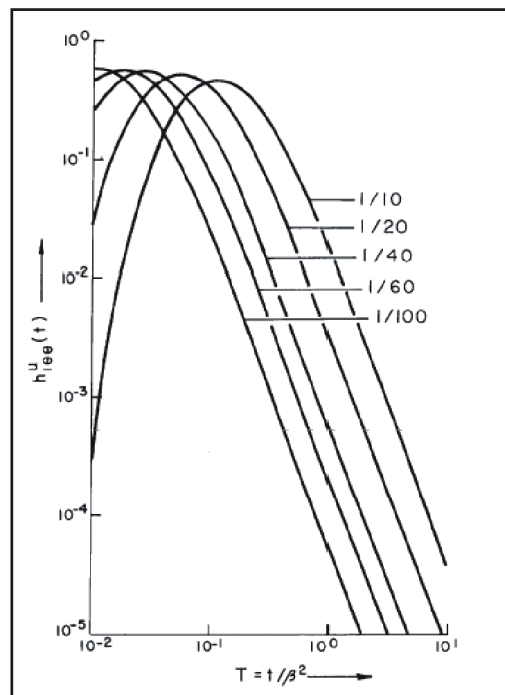


Figure 2-8 Time characteristic of $h^u_{100}(t)$ for $r/a=2$, $\mu_2/\mu_1=1$, and variable σ_1/σ_2 (after Singh 1973).

The curves for $h^v_{1rr}(t)$ reach a maximum at $T > 0$ and then decay at a rate slower than the case $\sigma_1/\sigma_2=0$, whose maximum occurs at $T = 0$. With increasing σ_1/σ_2 (lower contrast between sphere and conducting space implying more conductive space), the peak value of $h^v_{1rr}(t)$ is reduced and delayed in time. Also, in late time the curves exhibit typical inverse

power law decays; straight lines with equal slopes, independent of the actual conductivity values. Figure 2.7 shows the $h_{1\theta r}^v(t)$ component as a function of time, once again comparing free space with conducting space responses. The delayed and reduced peaks with straight-line behaviour in the late time are still present, but the most interesting feature is the sign change that occurs in this response when a conducting space is introduced. The last component of interest is the $h_{1\theta\theta}^u(t)$ in figure 2.8. This component doesn't exist for the free-space case and presents the magnetic field components associated with galvanic currents generated in the sphere. These responses are comparable in magnitude to the fields generated by the vortex currents. The sign changes observed in figure 2.7 can be explained by thinking of the sphere acting as a secondary source of smoke-rings. Smoke-rings (or more probably shells) move out from this source in all directions and the moment they pass the receiver position a sign change is recorded in the measured radial component (figure 2.9).

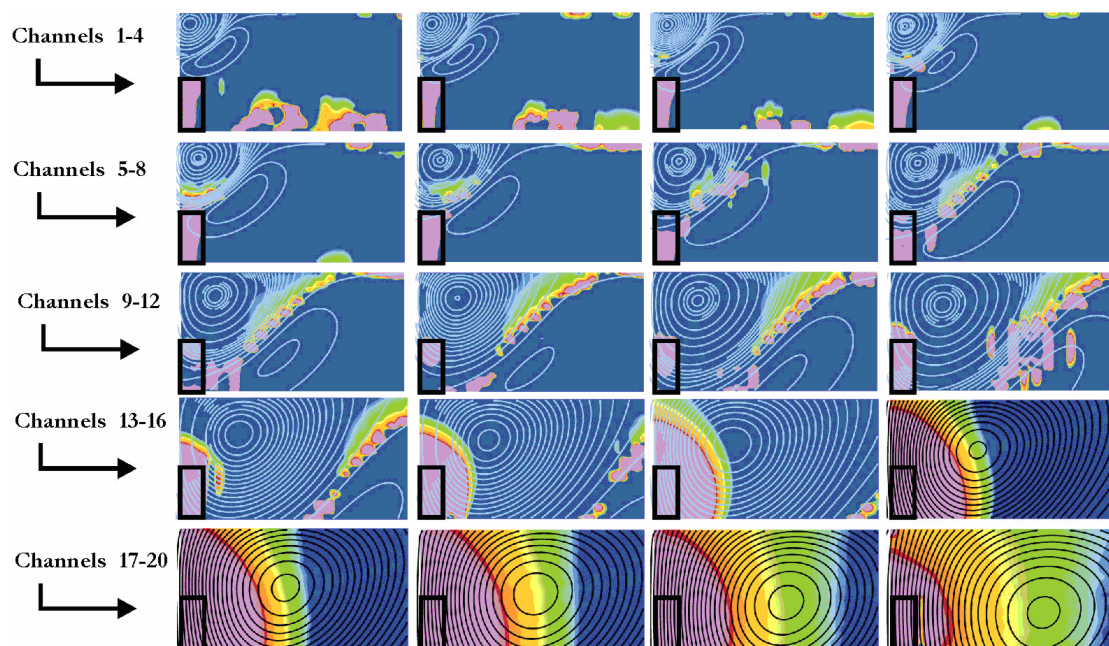


Figure 2-9 “Reflected” smoke-rings. Electric field intensity is presented as sections for 20 consecutive time channels. The cyan and black contour lines present the electric field intensity for a homogeneous 50 Ohm.m half space and indicate the well-known diffusive smoke-ring behaviour. The colour contours indicate electric field intensity of currents for a 5 Ohm.m prism (black rectangle) in a 50 Ohm.m half space as percentage of the half space response. The “reflected” currents are clearly visible in channels 13 to 20.

2.4 Conclusion

It is possible to describe electromagnetic field propagation analytically for only a few simplified earth models. Modelling realistic geological environments imply numerical solution of Maxwell's equations which are very time-consuming and not yet commonly used in industry. Several interpretation techniques and strategies have therefore been developed based on the simplified equations given above and these will be discussed in the following chapters.