

Chapter 1

1 REVIEW OF STEWART PLATFORMS

1.1 INTRODUCTION

The main objective of this study is to propose and develop a *general* numerical technique by means of which the workspaces of mechanical manipulators may be determined with relative *ease*. The emphasis is on parallel or so-called Stewart platforms.

To give an understanding of the significance and importance of the development of analytical design tools for such mechanical manipulators, this introductory chapter presents a *brief survey* of the field of Stewart platforms. This review will also allow for the later assessment and evaluation of the merits of the method proposed here.

The review, in the form of a literature survey, starts with reference to the *history* and *uses* of Stewart platforms. This is followed by a discussion of the *different designs* and *commercial products* presently available. Because of their significance with respect to the current study, special attention is paid to the discussion of *existing methods* for the determination of manipulator workspaces. With reference to these existing methods the chapter is concluded with a *detailed motivation* for the current study.

1.2 HISTORY OF THE STEWART PLATFORM

In his 1965 article, Stewart [1] “describes a mechanism, which has six degrees of freedom (DOF) controlled in any combination by six motors each having a ground abutment”. He proposed that the mechanism be used for a flight simulator for the training of helicopter pilots.

Although in-parallel devices or parallel-link manipulators are often called Stewart platforms, Stewart was not the original inventor of this type of mechanism. Stewart’s proposed mechanism is only a different configuration of the six linear jack system developed by Gough [2] in 1947. Gough was one of the reviewers of Stewart’s article and in his review he states that he designed a similar tire test machine (see Figure 1.1) in 1949, which was already built and operational in 1954-1955 [3].

Stewart's paper was published together with the communications of the reviewers, as well as the author's reply. Stewart acknowledged the fact that he was not aware of Gough's tire test machine and also states that, although it is similar to his flight simulation mechanism, it was designed using a different approach.

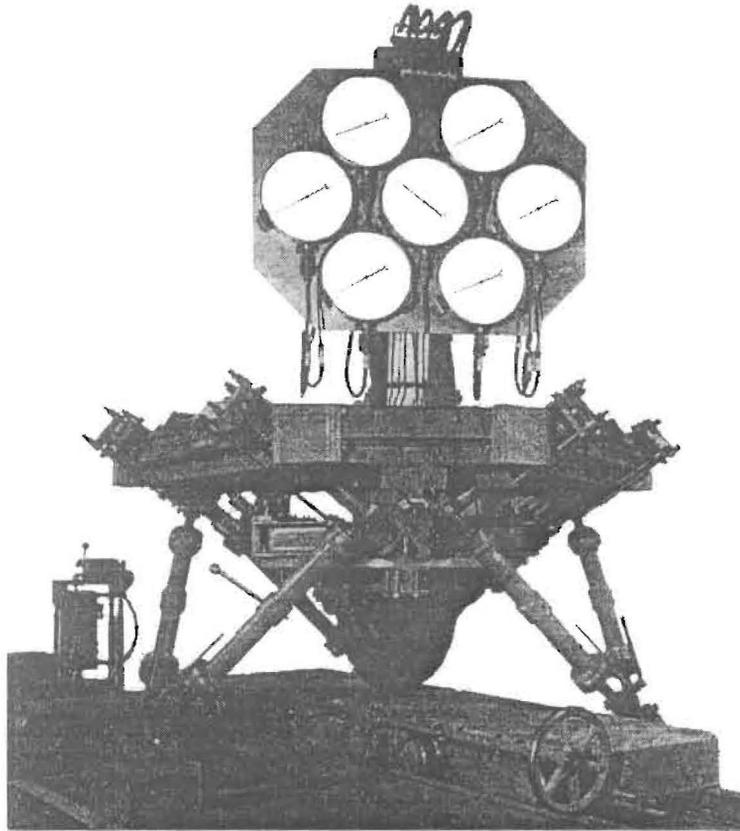


Figure 1.1 Gough's six DOF tire test machine (after [1]).

Ironically, Gough is also not acknowledged as the original inventor of this type of mechanism. According to Merlet [4] parallel manipulators have been known for a long time and the actual invention is attributed to the mathematician Cauchy, who wrote an article on the possible motion and rigidity of an “articulated octahedron” in 1813.

Merlet is of the opinion that although “Stewart platforms” is the name currently associated with these mechanisms, “Gough platforms” would be more appropriate.

Nevertheless, the re-discovery of the parallel manipulator by Stewart in 1965, sparked a flame that is still burning today. A schematic representation of the modern and general so-called 6–6 Stewart platform with prismatic actuators, is shown in Figure 1.2.

Merlet reports that real interest in this type of robot started around 1987, and from that date the number of papers on this subject increased drastically. Because of their inherent advantages of load carrying

capacity and spatial rigidity, Stewart platforms are suitable for a wide range of applications. It is believed that the research and development of parallel devices is currently the most popular topic in the area of robot manipulators [5].

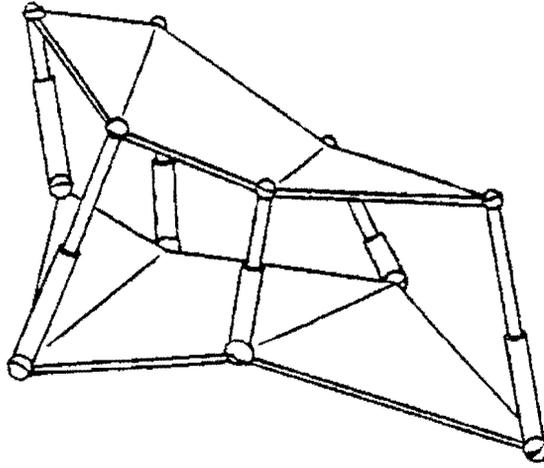


Figure 1.2 Schematic view of the general 6–6 Stewart platform (after [6]).

It is interesting to note that Stewart’s proposal also raised some criticism. In his review on Stewart’s article D. J. Thomas, referring to the possible use of the platform for flight simulation, asks: “Are six degrees of freedom really worthwhile?” In his reply Stewart points out that true flight can only be achieved with six degrees of freedom [1].

1.3 USES OF STEWART PLATFORMS

Although Stewart proposed his mechanism for use as a flight simulator, he also suggested other possible uses for the mechanism, including:

- “a platform held stationary in space mounted on a vessel such as a ship subjected to the random movements of the sea.”
- “a new form of machine tool.”
- “an automatic assembly or transfer machine.”

Some of the reviewers of Stewart’s article also suggested possible uses of platform devices. G. H. Meier [1] stated that the machine tool and medical fields were suitable areas where these platforms may be used. Meier noticed that the inherent stability of the platform, as well as its light working platform,

made it attractive to the machine tool industry as a working table mounted to the platform. With a rotating table mounted to the platform, 360° rotation would be possible.

The second application suggested by Meier, is a stabilizing platform, where a Stewart platform could be used to eliminate rotational motions, and damp linear motions. Looking at a hospital ship for instance, this would mean that a doctor would be able to perform delicate operations as he would be working on a stabilized platform. Meier also mentioned a gun platform aboard a warship as another application.

Another reviewer, J. Tindale, stated that any improvements may be satisfactory from a machine point of view, but to prove their economical viability would require a period of expensive study and development. Tindale included in his review an artistic impression of a possible design of a universal mill, which could machine complicated shapes with simple cutters, as well as an artistic impression of an oil drilling rig, where the platform is supported on a tripod comprising six telescopic legs.

By the time Gough reviewed Stewart's article, his tire test machine was fitted with digitally controlled motor drives attached to the screw jacks, and electronic instrumentation to study tire-to-ground forces and movement.

The development of platforms for flight simulation and amusement park rides, has been ongoing since Stewart's article was published three decades ago. In his comprehensive review [4], Merlet explains that the reason for the main interest in these platforms, is their high nominal load-to-weight ratio. He developed a prototype platform which has a weight of 35kg, and can carry a load of 600kg. The weight of the load is approximately equally distributed on all the links. Another advantage is that the stress in each link is mostly of a traction-compression nature, which is very suitable for linear actuators and therefore contributes to the rigidity of the platforms.

There are other advantages that make Stewart platforms suitable for a wider range of applications. For example, Merlet states that because the position of the moving platform is less sensitive to the errors on the articulated sensors in comparison to serial link robots, Stewart platforms are ideally suited for assembly lineups. As early as 1979 a parallel manipulator was first used in a robotics assembly cell. Merlet also points out that parallel manipulators can be used as six component force sensors. Measuring the traction-compression stress in the links enables one to calculate the resulting force and torque acting on the mobile platform. This capability of parallel manipulators makes it possible for manipulators to be used as assembly units, where the platform comes in contact with its surroundings. Another application where this feature is important, is surface following.

Parallel manipulators can be very small where, for example, the linear actuators range a few micrometers enabling the upper platform to perform motions of a few nanometers. In contrast to this, Merlet also reports on a huge manipulator developed for mining operations.

One of the prototype platforms developed by Merlet is used for ophthalmic surgery and the European Synchrotron Radiation Facility (ESRF) uses a more classical Gough type platform for the manipulation of heavy experimental setups.

Parallel manipulators can also be used in trusses. The mechanism modules are joined to form an articulated truss, which is light and highly redundant. Merlet states that these manipulators may be useful in space applications, once the kinematics and control problems have been solved.

In their article on five DOF parallel mechanisms, Wang and Gosselin [7] explain that the development of these manipulators is of interest, as they can be used for various tasks where mostly serial five DOF mechanisms are presently used. They mention tasks for which axi-symmetric tools are being used (e.g., drilling, welding, riveting, etc.), as well as the positioning and orientation of lasers, mirrors, antennas and spotlights.

1.4 DIFFERENT DESIGNS

Merlet [4] defines a parallel manipulator as a “closed-loop mechanism in which the end-effector is connected to the base by at least two independent kinematic chains.” He also defines a fully-parallel manipulator as “a closed loop-manipulator with an n degrees-of-freedom (DOF) end-effector connected to the base by n independent chains, which have at most two links and are actuated by a unique prismatic or rotary actuator”

According to Merlet, many different designs of parallel manipulators are possible and the scientific literature is very rich on this topic. All have in common their low cost, since most of the components are standard, although the assembly of the manipulator must be done with care.

According to Merlet’s definition, parallel manipulators include all in-parallel devices where the links have rotary or prismatic actuators. Stewart platforms traditionally are parallel manipulators equipped with prismatic actuators.

There are two main categories of parallel manipulators, namely planar and spatial manipulators.

Looking at the first category, a planar parallel manipulator is a special type of closed loop manipulator. Haug et al. [8] investigated the dextrous workspace of an elementary closed loop manipulator with one DOF. Gosselin and Wang [9] considered the singularity loci of a two DOF planar closed loop manipulator consisting of a five bar chain. Bajpai and Roth [10] determined the workspace of the same five bar chain, with a third additional DOF, as the basis of a manipulator mounted on a rotary joint.

The closed loop manipulators mentioned thus far have revolute actuators, and according to Merlet's definition, these mechanisms are not parallel manipulators as they are connected to the ground via two linearly *dependent* kinematic chains.

Gosselin and Wang [9] also consider a planar three DOF parallel manipulator with three revolute actuators. Seeing that this closed loop manipulator has two independent kinematic chains connecting the moving platform with the base, it can be classified as a parallel manipulator (see Figure 1.3).

The line segment moving platform in Figure 1.3 can be replaced by a triangular shaped moving platform. Kumar [11] characterizes the workspace of such a planar three DOF parallel manipulator (see Figure 1.4)

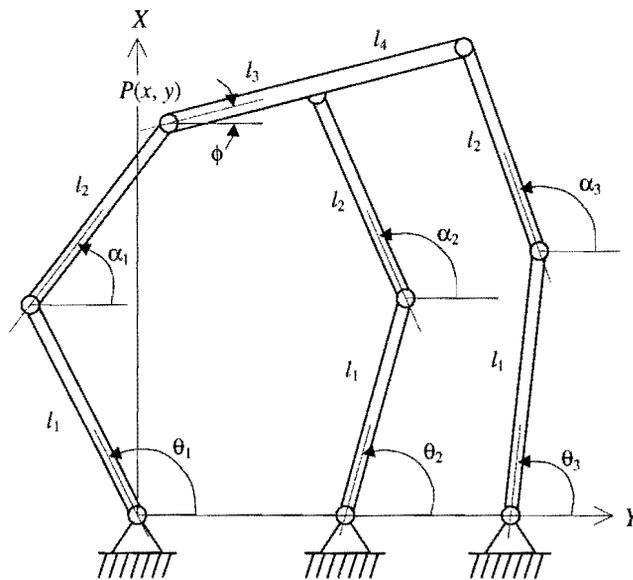


Figure 1.3 Planar three DOF parallel manipulator with revolute actuators (after [9]).

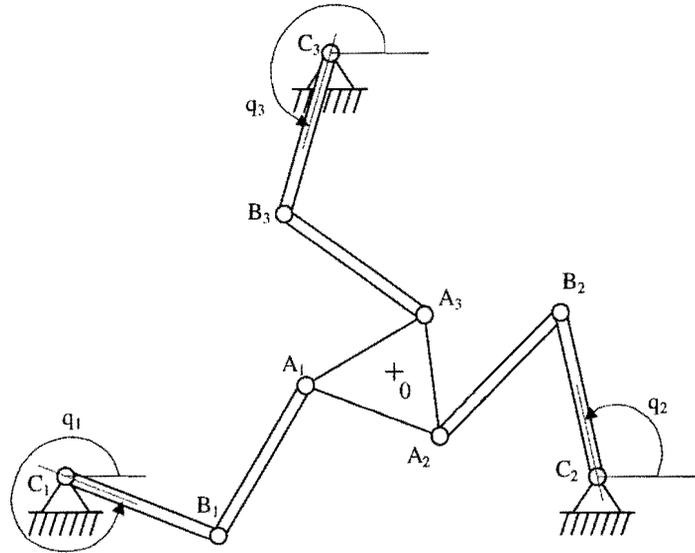


Figure 1.4 Three DOF parallel manipulator with a triangular moving platform and revolute actuators (after [11]).

The literature is also very rich as far as different designs of planar Stewart platforms (planar parallel manipulators with linear actuators) are concerned. Haug et al. [12] determine the workspace of a planar three DOF Stewart platform. This Stewart platform is a line segment platform with two coincident joints (see Figure 1.5).

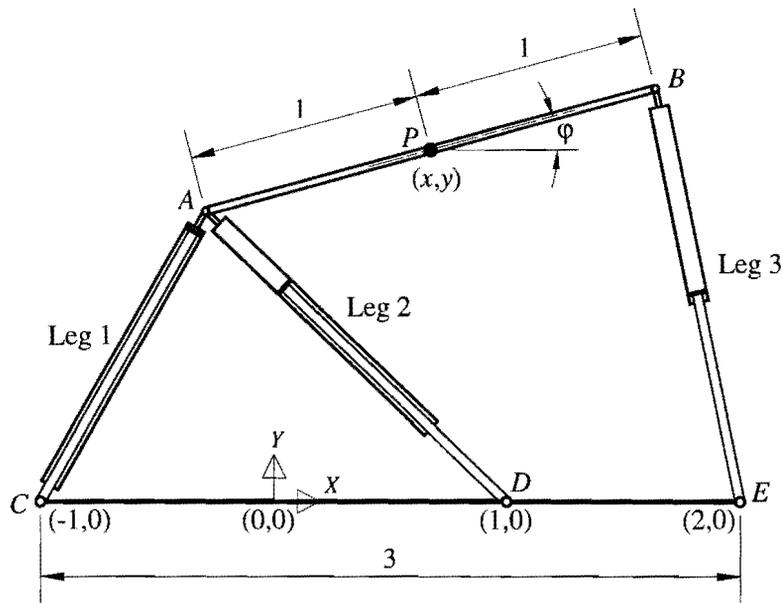


Figure 1.5 Three DOF planar Stewart platform (after [12]).

Some researchers also consider planar three DOF Stewart platforms with triangular moving platforms. Lee et al. [13] found that the optimum design, as far as stability is concerned, of a planar Stewart platform, is an equilateral moving platform (see Figure 1.6).

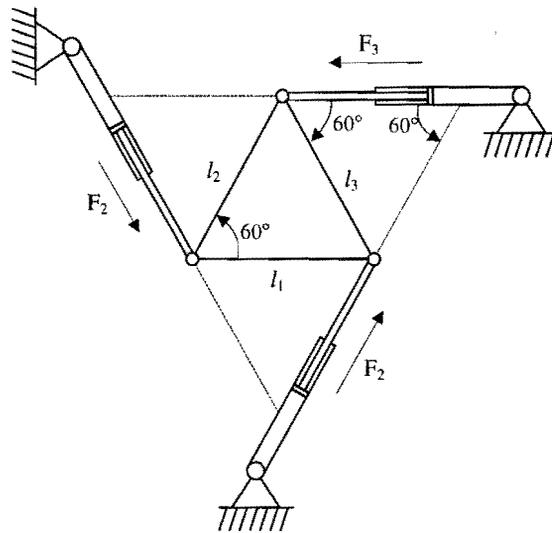


Figure 1.6 Planar three DOF Stewart platform with equilateral moving platform (after [13]).

In a later study Merlet et al. [14] determine the different workspaces of a planar Stewart platform with a triangular moving platform. They describe the mechanism in terms of the number and type of kinematic chains, with which the moving platform is connected to the ground. Both planar Stewart platforms shown in Figure 1.6 and Figure 1.7 are 3-RPR (Revolute-Prismatic-Revolute) parallel manipulators. In each manipulator, the mobile platform is connected to the base via three identical chains consisting of a revolute joint attached to the ground followed by an actuated prismatic joint which is connected to the platform by a revolute joint

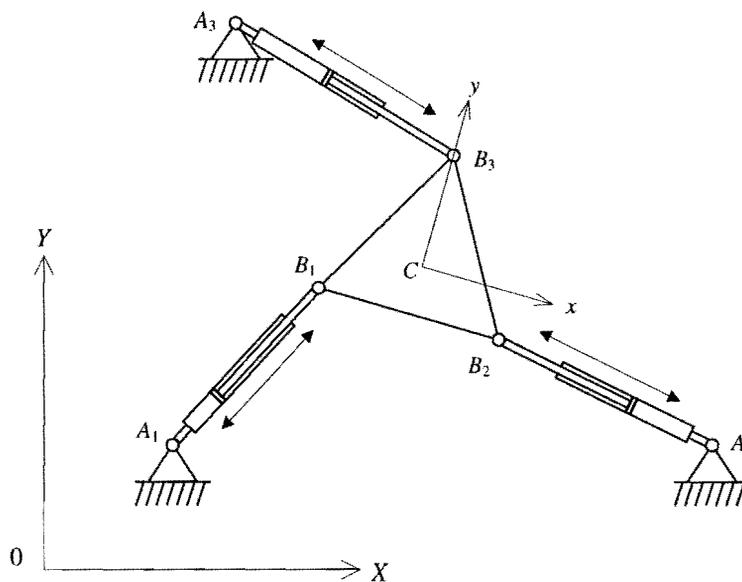


Figure 1.7 The three RPR parallel manipulator (after [14]).

The second category of parallel manipulators is spatial mechanisms. Depending on the design, these mechanisms can have three to six DOF. For instance, Wang and Gosselin [7] explain that three DOF and four DOF devices are sometimes used in flight simulation. Five DOF mechanisms are also available and used for tasks where axi-symmetrical tools are being used.

As pointed out by Stewart in his original paper, there are many possible designs for providing six DOF. One of the obvious designs is a three axis gimbal superimposed on a three axis linear slide system. Stewart [1] rejected this option, because he wanted to achieve the most simple and cohesive design with the highest capabilities in a wide range of applications.

The original mechanism proposed by Stewart [1] comprises out of a triangular plane, called the platform, of which each of the three corners is connected through a three-axis joint (spherical joint or ball-and-socket joint) to one off the three legs. Each leg is connected to the ground by a two-axis joint (universal joint). Three additional actuators are connected to the three legs. Each additional actuator has one end connected via a rotary joint to the outer-cylinder end of each leg. The other end of each additional actuator is connected to the foundation or base via a universal joint. (see Figure 1.8).

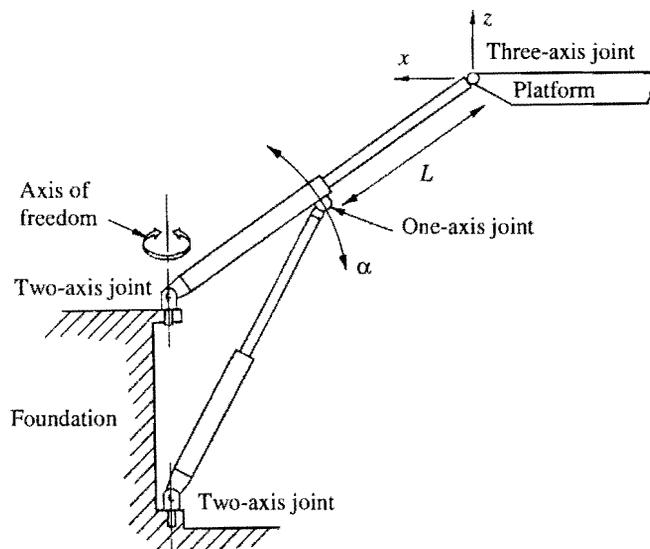


Figure 1.8 Stewart's original platform: General arrangement of single leg system (after [1]).

The two jack foundation connections have one common axis and the remaining axes are parallel to each other. The common axis is not controlled within the single leg system but the plane containing the leg can rotate about it, thereby permitting a three-axis motion on the platform support joint.

The tire test machine of Gough [3], also uses six actuators, but they are arranged differently. Each actuator is attached separately to the upper platform (see Figure 1.1). Gough uses the same universal

joint system to attach the actuators to the platform and the base, as described in his communications on Stewart's original paper [1].

Also included in the communications on Stewart's article, are comments by Murdoch and Meier who mention the preferred arrangement that would result from the use of the "linear coordinate leg system". This is a similar arrangement to the one Gough used for his tire test machine, where the actuator-foundation, and actuator-platform connecting points are co-planar.

A general parallel manipulator has the actuator connection points in any position on the fixed and moving bodies, i.e. the actuator connection points are not restricted to be co-planar [4] (see Figure 1.2).

Spatial parallel manipulators are usually labeled according to the number of connecting points on the base and moving platforms, keeping in mind that the name "Stewart platform" refers to a parallel manipulator equipped with linear prismatic actuators. Stewart's original platform can accordingly be labeled as a *special* 6-3 "Stewart platform", as there are six foundation connecting points and three moving platform connecting points. The word *special* is necessary when referring to this platform, as the three additional actuators are connected to the three actuator legs, instead of the moving platform.

Meier and Murdoch's [1] proposed configuration is a 6-3 Stewart platform, i.e. it has six actuator legs, the bottom ends of which are connected to the six base vertices, and the top ends of the actuators are connected in pairs to the three moving platform vertices.

Gough's tire test machine is accordingly labeled as a 3-6 Stewart platform, where the moving platform with six vertices is connected to the three base vertices. Furthermore, it follows that a 6-6 Stewart platform (see Figure 1.2) is where each of the six linear prismatic actuators connects a moving platform vertex with a base vertex.

The configuration of a spatial Stewart platform is not the only important design aspect. Equally important is the type of connections with which the actuators are connected to the moving platforms and base. Spatial parallel manipulators can also be described according to the kinematic chains that connect the fixed and moving bodies. For example: a 6-6 Stewart platform with the six linear actuator legs connected to the base and moving platforms via ball-and-socket (spherical) joints can also be labeled as a 6-6 Stewart platform with six identical SPS (Spherical-Prismatic-Spherical) chains.

According to Griffis and Duffy [15], it should be noted that in an SPS serial chain, the prismatic joint (P) has an extra DOF, a rotation about the line joining the centers of its S pairs. This extra DOF in each leg

does not affect the gross motion of the top platform. Either one of the S pairs could be replaced by a Hooke joint. However, this may not be desirable from a design standpoint.

As described earlier, Stewart uses 1-axis rotary, 2-axes universal and three axes spherical joints in his original platform. With a 6–3 or 3–6 configuration, the actuators are connected to the base with either a spherical or a universal joint, allowing rotation about respectively three or two axes. “Special” ball-and-socket joints are to be used to connect the top ends of the actuators in pairs to the moving platform.

Due to the design problems that arise from using pairs of concentric spherical joints, Lin et al. [16] emphasize that it is very important to eliminate, as far as possible, the use of concentric spherical joints.

Fichter [17] proposes that the ends of the legs be mounted on gimbals (Hooke joints), because if it is designed properly, a gimbal gives a much greater range of motion than a ball-and-socket joint. The platform gimbal is doubled to make the two adjacent legs coincident. The platform gimbal also has a third axis perpendicular to the platform plane, which makes it equivalent to a double ball joint. The base gimbal Fichter uses, has its first revolute axis inclined to the base plate to increase the useful range of motion of the joint (see Figure 1.9).

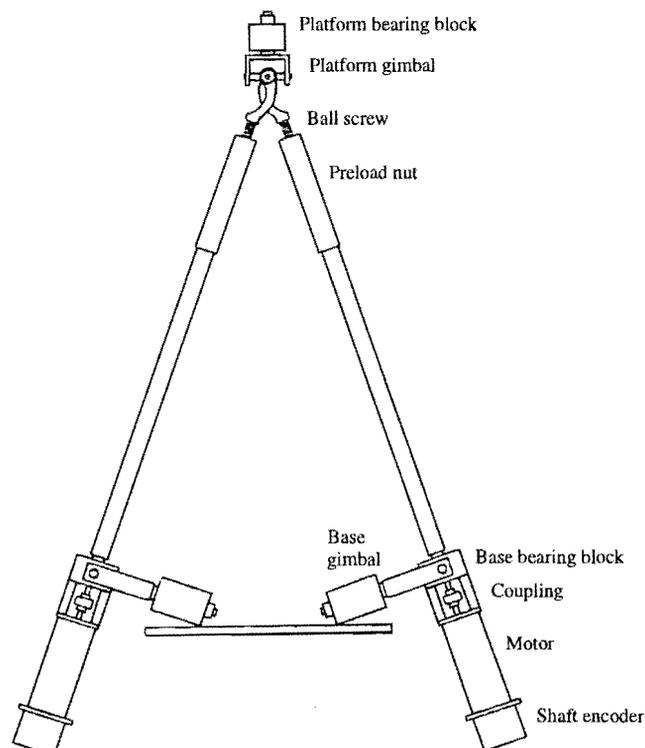


Figure 1.9 The leg triangle of the Stewart Platform built at Oregon State University (after [17]).

Liu et al. [18] have done the kinematic analyses of a 6–3 Stewart platform of the form shown in Figure 1.10. This Stewart platform has been built (see Figure 1.11) and is operational in the Arlington *Automation & Robotics Research Institute (ARRI)* of the University of Texas. As can be seen it uses the gimbal arrangement proposed by Fichter [17] as actuator connections (see Figure 1.12).

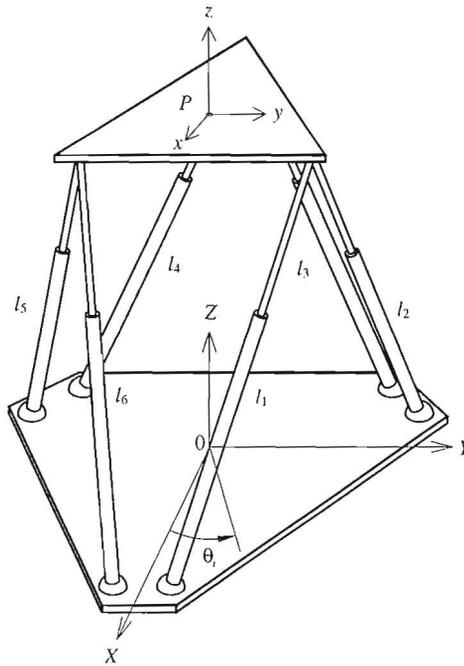


Figure 1.10 The 6-3 Stewart platform (after [18]).

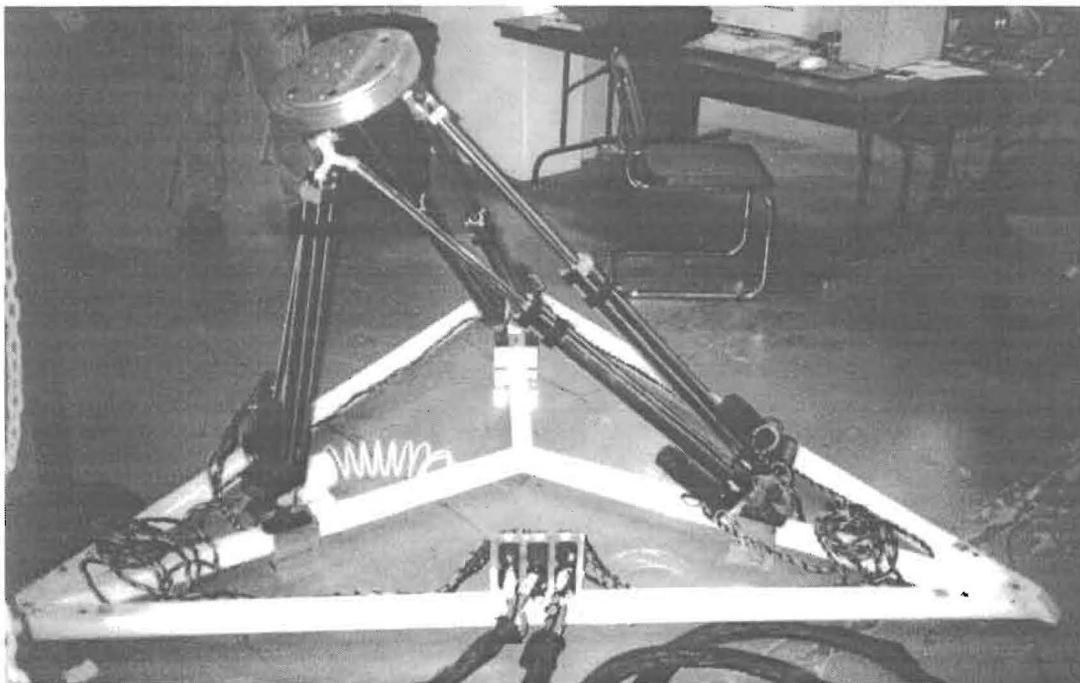


Figure 1.11 Photograph of the ARRI-Stewart platform.

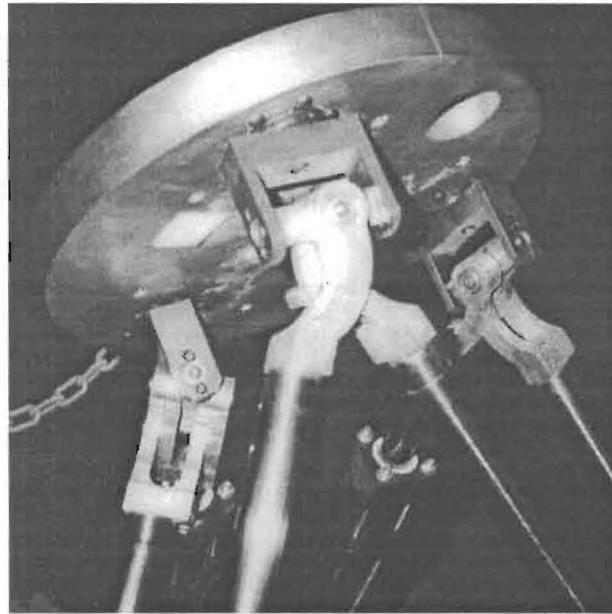


Figure 1.12 Photograph of the ARRI-upper platform gimbals.

In his review Merlet [4] describes the configuration of a 6–6 Stewart platform which has universal joints connecting the prismatic actuators to the base, and ball-and-socket joints connecting them to the moving platform.

Fichter [17] does the kinematic and dynamic analysis of a general Stewart platform, which he considers as two solid bodies connected to each other via six linear actuators (see Figure 1.13). The connections at both ends of each actuator are ball-and-socket joints.

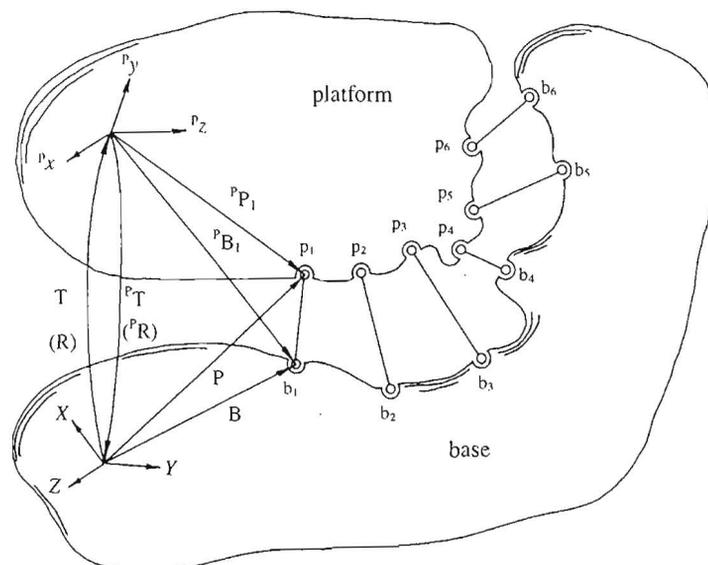


Figure 1.13 A schematical of the generalized Stewart platform (after [17]).

Similar to planar parallel mechanisms, spatial parallel mechanisms can also be designed using rotary actuators. Arai et al. [19] describe a 6–6 parallel manipulator with rotary actuators, as a parallel manipulator having six linkage chains, each consisting of three joints and two links. The first joint (joint 1) is an actuated rotary joint connecting link 1 with the foundation. Joint 2 is a Hooke / universal joint connecting links 1 and 2. Joint 3 is a spherical joint connecting link 2 with the moving platform (see Figure 1.14).

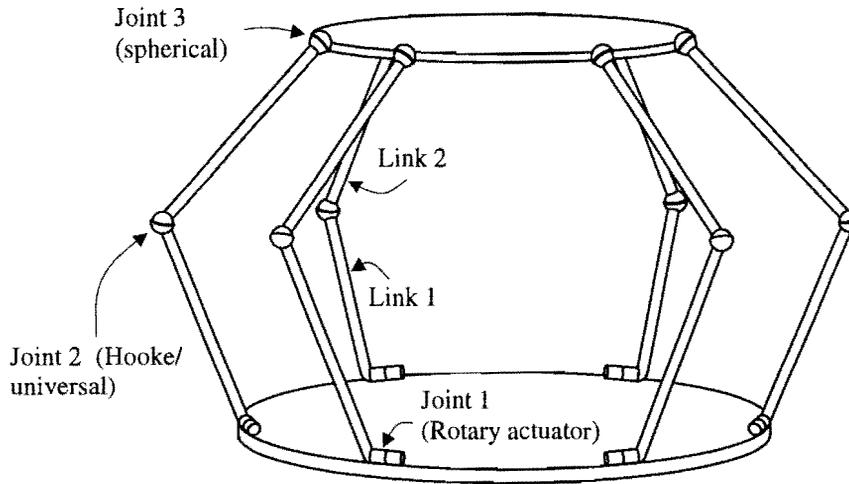


Figure 1.14 Rotary actuated 6-6 parallel manipulator (after [19]).

Arai et al. also point out that these rotary actuated parallel mechanisms have the advantage of good dynamic characteristics, which stems from the fact that the actuators are fixed in the basement, resulting in extremely lightweight movable elements.

Wang and Gosselin [7] studied the kinematics and singularity representation of spatial five DOF parallel mechanisms, with both prismatic and revolute actuators. Both mechanisms consist of six kinematic chains, five of which have the same topology.

Wang and Gosselin first considered the mechanism with the revolute actuators. Each of the five actuated legs consists of an actuated revolute joint connecting the first moving link to the base, a Hooke / universal joint connecting a second moving link to the first link and a spherical joint attaching the moving platform to the second moving link (see Figure 1.15). The five DOF spatial Stewart platform has a Hooke / universal joint connecting each of the five prismatic actuators to the base, and a spherical joint connecting the moving platform to each actuator.

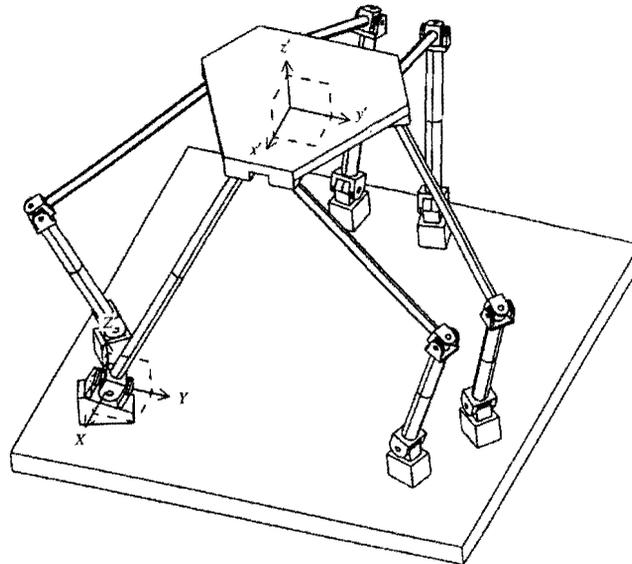


Figure 1.15 The spatial five DOF parallel mechanism with revolute actuators (after [7]).

The sixth kinematic chain of both mechanisms connects the moving platform with the base, and is not actuated. It consists of a Hooke \ universal joint attached to the base, a moving link, and a spherical joint attached to the platform (see Figure 1.15). It follows that the sixth kinematic chain constrains the motion of the platform to five DOF. According to Wang and Gosselin , the five DOF mechanism could also be built using only five legs, i.e. by removing one of the five identical legs, and actuating the first joint of the special leg. Apparently, these two arrangements lead to similar kinematic equations.

Having considered spatial parallel manipulators with revolute links, it is interesting to consider the new parallel manipulator proposed by Arai et al. [19]. The proposed new manipulator is a combination of a standard 6–6 Stewart platform with prismatic actuators, and a 6–6 parallel manipulator with revolute actuators (see Figure 1.16). It has a base plate and an “intermediate platform” connected to each other via six firmly fixed linear actuators, i.e. the position and orientation of the “intermediate platform” cannot change. A movable link is connected via a ball joint to the actuated part of each of the six linear actuators. The other ends of the six movable links are connected via spherical joints to the moving upper platform. As soon as the linear actuators move, the movable links move and the moving upper platform changes its position and orientation. The proposed mechanism is designed to achieve high speed and good accuracy capabilities.

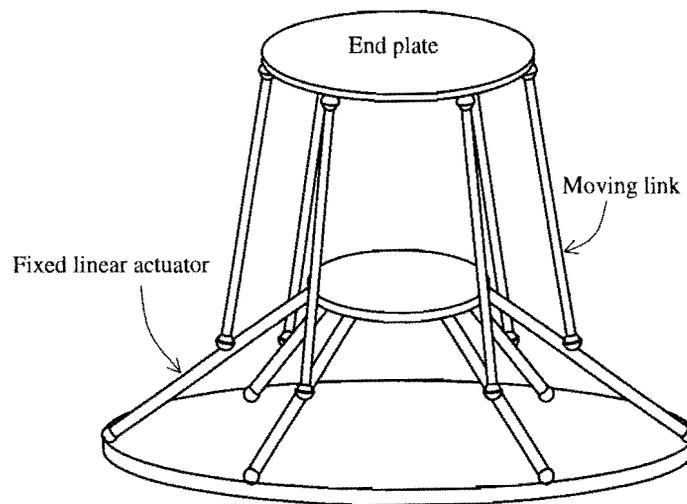


Figure 1.16 New parallel manipulator with fixed linear actuators (after [19]).

Similar to the parallel manipulator with the fixed linear actuators, is the Hexaglide developed at the *Institute of Machine Tools of the Swiss Federal Institute of Technology* in Zurich. According to Honneger et al. [20], who studied the control of this parallel manipulator, the Hexaglide is intended to be used as a high speed milling machine.

The Hexaglide also has six movable links with one end of each link connected to a linear motor. The motors are distributed on three linear rails mounted parallel to each other. The other ends of the moving links are connected in pairs to the triangular moving platform. One could describe the Hexaglide as a special 6–3 parallel manipulator (see Figure 1.17).

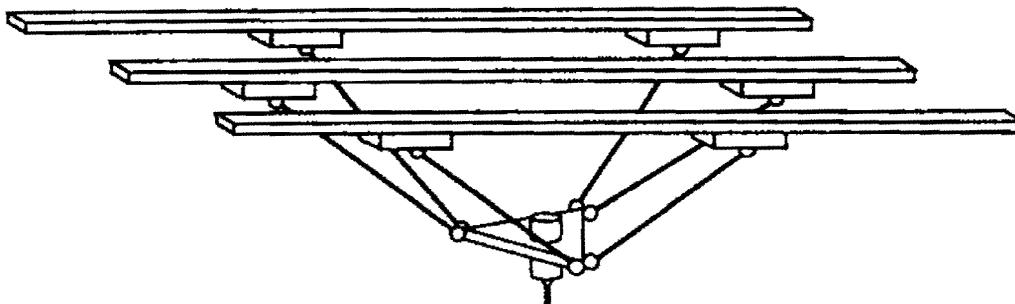


Figure 1.17 The six DOF Hexaglide (after [20])

Another design which is similar to the Hexaglide and the parallel manipulator with the fixed linear actuators, is the three translation DOF parallel manipulator designed by Hervé [21]. Three pairs of parallel links connect the fixed orientation platform with the three linear motors. The three motors run

along three radiating rails. Changing the position of the three motors, causes the moving platform only to translate, as the parallel links ensure the fixed orientation of the moving platform.

An interesting design found in the literature is the reconfigurable platform manipulator designed by Ji and Song [22]. According to them, the mechanical structure of a parallel manipulator is very suitable for reconfiguration, since parallel manipulators are actuated in parallel. The limited workspace of platform manipulators further points to the need for a modular, reconfigurable platform with adjustable configurations, so that the platform will have a wide range of applications.

As part of the investigation into a reconfigurable platform manipulator, an experimental platform was built consisting of a base and mobile plate and six prismatic actuated legs. The base connections are normal two axes Hooke / universal joints, and the moving platform connections are Hooke joints with an additional revolute joint, providing three axes rotation. The base and platform plates of the experimental platform have reconfiguration hole patterns used to connect the actuator leg units to the plates.

The objective of Ji and Song is to develop an inventory of standardized leg modules and customized mobile platforms and base plates, so that the parallel manipulator can be custom-configured, portable and easy to repair.

1.5 AVAILABLE COMMERCIAL PRODUCTS

The research and development of in-parallel devices have enjoyed much attention over the last three decades [5]. However, in spite of this Ji [23] reports as recently as 1996, that the use of platform manipulators is still mainly in an experimental stage. He attributes this to the lack of rational synthesis tools for the design of practical platform manipulators.

Many proposed prototypes, developed by researchers from around the globe, are to be found in the literature on parallel manipulators.

The authors of the *Hexapod* [24] brochure published by Geodetic Technology in the USA, state that the “cost of computing strut lengths has until recently been too high for most (practical) applications. Where the cost of processing could be justified, some interesting hexapod-based machines, such as the flight simulator, were made.”

An example of a company that has been involved in the flight simulator is *FRASCA International, Inc.*. They have been in business since 1958, and have developed a wide range of flight simulators, which include Stewart platform mounted simulators.

As far as other applications for parallel manipulators are concerned, it is reported in the *Hexapod* brochure that, “now that the cost of computing strut lengths has fallen dramatically, many companies are offering hexapod-based machines”. This is happening in spite of the problems mentioned by Ji [23].

In Merlet’s [4] review he states that *Marconi* designed the first commercial parallel manipulator in 1985. This is the six DOF *Gadfly*, designed to be used for the assembly of electronic components. In 1986 *Marconi* designed the *Tetrabot*, a huge hybrid serial-parallel manipulator.

Another company mentioned by Merlet [4] is *Demaurex* who sells the *Delta*, which is a 3-4 DOF manipulator used for very fast pick-and-place tasks involving light loads. Another product is the *Hexa*, which was still under development in 1994. The *Hexa* is a six DOF manipulator based on a design similar to that of the *Delta*. Merlet also mentions a new product, the *SmartTee* which is being developed by *Hughes*.

In his analysis of the design parameters in parallel manipulators, Ji [23] mentions the *NIST Robocrane* capable of manipulating tools and devices to perform a variety of tasks such as cutting, shaping and finishing, excavating and grading.

Ji also list the *Octahedral Hexapod* machine tool developed by *Ingersoll Milling Machines Co.*, which can perform machining operations on large workpieces such as engine blocks. This unique mill consists of a moving tool assembly guided over the workpiece by six ball screw actuators forming a 6–6 Stewart platform as shown in Figure 1.18.

It is reported that modern milling centers such as the *Octahedral Hexapod* machine tool, are capable of 1 m/s velocities over a volume of 1 cubic meter with a resolution of 1 million increments per second. Spindle speeds can be as high as 24000 revolutions per minute.

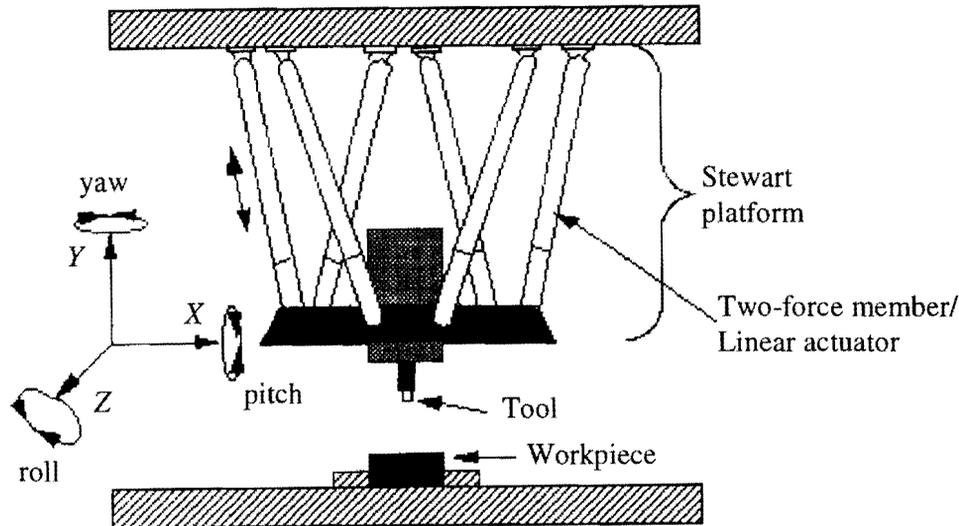


Figure 1.18 Schematic of a Hexapod six-axis machining center.

Zou et al. [25] list a few companies who are specifically involved in designing parallel manipulators used for manufacturing. One of these is the Italian based company *Comau*, who manufactures the *TRICEPT HP*.

The *TRICEPT HP* has three linear actuators, and the end effector is connected to a three-axis wrist, resulting in a total of six degrees of freedom (DOF). According to the manufacturer, the *TRICEPT HP* is a very rigid manipulator, capable of applying a maximum force of 15 kN and lifting up to 500 kg. The wrist can handle a payload of up to 150 kg. Typical applications listed in the technical brochure of the *TRICEPT HP*, includes: assembling with force, deburring, polishing, wood working, aluminum milling, laser and water-jet cutting, as well as spot and laser welding [26].

Zou et al. [25] also refers to *Giddings and Lewis* in the USA, and *Lapik* in Russia. Both companies have apparently developed their own parallel type CNC machine tools.

Geodetic Technology has patented their *Hexapod 6-3* parallel manipulator (see Figure 1.19) in a number of countries [24]. According to the manufacturer, this machine competes with human adaptability and dexterity. *Geodetic Technology* has been developing hexapod technology since 1988, and it comprises of unique mechanisms, sophisticated control, together with calibration and translation software.

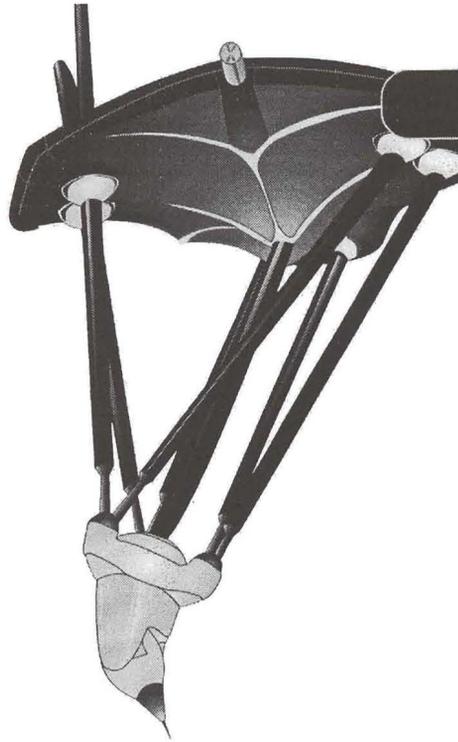


Figure 1.19 The *Hexapod 6-3* Stewart platform type machine tool (after [24]).

Fanuc Robotics is another company that designed a 6-3 hexapod, the *FLEXTOOL*, which has a base and tooling faceplate connected by six servo actuated legs, resulting in six DOF. The *FLEXTOOL* can precisely move locators, clamps and other end-of-faceplate tooling, to multiple positions for welding and joining [27].

In conjunction with the idea of a reconfigurable Stewart platform proposed by Ji and Song [22], the modular design of parallel manipulators is possible thanks to the German company *INA*. *INA* manufactures high precision joint units, which are ready-to-fit and clearance-free [28]. *INA* also designs telescopic arms, which can be mounted onto precision joints, making the *INA* system ideal for the modular construction of Stewart platforms.

In order to develop hexapod technology as an economical alternative to conventional designs, the *Machine Tool Laboratory (WZL)* of *Aachen RWTH* and the *ISW* of the University of Stuttgart are co-operating with machine tool manufacturers in the research project “*DYNAMIL II*” (Dynamic Innovative Lightweight Designs for Production Machines of the Year 2000) [28]. *INA* is also a partner in this project.

INA is also currently demonstrating the functioning of its joints and telescopic arms for hexapod technology in a reliable machine which was designed by the Institute for Machine Tools of the University of Stuttgart and built jointly by INA [28].

1.6 AVAILABLE DESIGN INFORMATION

1.6.1 Introduction

According to Liu et al. [18], there are two fundamental characteristics that set a Stewart platform manipulator apart from other industrial robots: a Stewart platform is a *closed kinematic system* with *parallel links*. These characteristics contribute to the advantages of Stewart platforms.

Wang and Hsieh [29] state that the rigidity and load-carrying capacity of parallel robots are better than those of conventional serial robots. Merlet [4] explains further that the high load / weight ratio is due to the equal distribution of the load on the platform. Each link of a six legged Stewart platform is submitted to only 1/6 of the total weight. Merlet contributes the rigidity of the platform to the stress in the links being mostly traction-compression, which may easily be handled by linear actuators.

According to Merlet [4], another advantage of parallel manipulators is that the position of the end-effector is much less sensitive to the error on the articulated sensors than serial link robots. Geng et al. [30] explain that the reason for the higher accuracy of parallel manipulators is because the positioning error on each actuator is averaged out instead of being accumulated at the end-effector. Merlet [4] also mentions that the high positioning accuracy of these manipulators is also due to their high stiffness, which insures that the deformations of the links are minimal. He emphasizes that the most important advantage of parallel manipulators is their low cost, since mostly standard components can be used. This corresponds to the statement of Geng et al. [30] saying that the mechanical design of the Stewart platform based parallel link manipulator is relative simple. Liu et al. [18] agree, saying that relatively inexpensive commercially available servo actuator technology is used for many Stewart platforms. It is interesting to note that this corresponds to Stewart's [1] original design aims, namely to achieve the most simple and cohesive design with the highest capabilities for a wide range of applications.

The architecture of parallel manipulators is very different from that of serial link manipulators, and Merlet rightfully states that most of the theoretical problems concerning parallel platforms still have to be addressed. According to Merlet, there exists a duality between parallel and serial link manipulators, as easy problems for serial link manipulators are often difficult to solve for parallel manipulators, and vice versa. Merlet mentions the attempts by Waldron and Hunt [31] as well as that of Zamanov and Sotirov [32] to explain this duality.

1.6.2 Kinematic Analysis

Starting in 1987, more and more researchers have become interested in addressing problems concerning parallel manipulators. Merlet [33] points out that, for example, the direct kinematics problem has drawn the attention of many researchers, and is currently still under investigation, as it has not yet been completely solved. Merlet in fact says that Roth, the prestigious researcher of Stanford, has stated that the direct kinematic problem of parallel robots is the “kinematic problem of the century”.

Solving the inverse kinematics, i.e. determining the leg lengths once the position and orientation of the top platform are known, is easy to do. Finding the position and orientation of the top platform with the leg lengths known is, however, far more complicated. Researchers have experimented with different techniques to solve the forward direct kinematics with the objective of obtaining the closed form solution for the general 6–6 parallel manipulator. Merlet [4] states that the closed form forward kinematic solution for some special configurations of spatial manipulators has been found, but as yet, no success has been reported for the general 6–6 parallel manipulator. This is confirmed by Innocenti [34] who also mentions that in spite of their relatively simple arrangement, the kinematic analysis of fully-parallel mechanisms is extremely challenging.

The emphasis currently lies on finding the analytical or closed-form solution of the forward kinematics. A closed-form solution is most desirable since it generally reduces the problem to solving one algebraic equation with only one unknown. Accordingly, the degree of the equation provides the number of platform locations in the complex field, and the locations themselves can be found by determining all roots of the equation.

Geng et al., for example, state that in general for a set of leg lengths, the forward kinematics problem can have no solution, or multiple solutions. They also categorize the different approaches used to solve the forward kinematics into iterative (numerical) methods and direct (closed form) solutions. Merlet [4] also states that in general, the forward kinematic analysis has more than one solution.

Innocenti [34] explains that when performing the direct positional analysis of a mechanism, the displacement values of all actuated kinematic pairs are known. Accordingly, all actuators can be thought of as frozen, and the mechanism itself can be regarded as a structure. The direct kinematics of the mechanism is then equivalent to finding all closure configurations of the structure.

Griffis and Duffy [15] point out that the forward displacement analysis of parallel manipulators is important, as it will provide feedback information, i.e. the position and orientation of the moving platform relative to the base, which can be used in a Cartesian controller. They further state that the use

of a Stewart platform as a force / torque sensor in the field of force / control, will be enhanced by a forward analysis of a Stewart platform. Geng et al. [30] also report that in general, the forward kinematics is used in a position feedback control problem, where the lengths are measured by sensors mounted on the actuators.

In their 1993 article, Liu et al. [18] state that the lack of efficient algorithms for solving the kinematic equations is the main obstacle in realizing the potential of the Stewart platform as an industrial robotic manipulator.

The fact that multiple solutions exist for the direct kinematics of a Stewart platform makes the interpretation of the results an important factor. Liu et al. also state that among the solution sets of the forward kinematics, only some are feasible due to mechanical constraints. They distinguish between “mathematical” solutions and “mechanical” solutions of the forward kinematics problem.

1.6.3 Workspace Analysis

1.6.3.1 General Observations

From the geometry of a parallel manipulator, it follows that it is the limited ranges of the actuator legs that determine the size of the workspace. For Stewart’s [1] original design, he explains that similar movement of all three legs in the XYZ coordinates, results in XYZ motion of the platform, and differential movement results in attitude (rotational) movement of the platform. From this Stewart directly relates the linear displacements of the platform, to the amplitude of movement of the legs, and the angular motions are proportional to the spacing of the three points on the platform, relative to the linear motions of the legs. It follows that the smaller the platform size relative to the stroke of the legs, the larger the angular motions.

The determination of parallel manipulator workspaces has come a long way since Stewart [1] did the motion analysis of his flight simulator parallel mechanism in 1965. However, in a recent article Merlet et al. [14] state that the workspace determination of parallel manipulators remains a challenging problem. The solution of this problem is very important in the design and trajectory planning of parallel manipulators. Kumar [11] also states that the point based definition of the workspace is an important design consideration and of considerable theoretical significance.

According to Gosselin [35], many authors have pointed out that the major drawback of parallel manipulators is their limited workspace. He further states that it is therefore of primary importance to develop efficient tools that will allow the determination of their workspace. Moreover, in the context of design, the workspace determination procedure should be simple enough to be included in an

optimization cycle in which, for example, the size of the workspace may be maximized with regard to design variables.

The successful mapping and characterizing of the workspace also assist in eliminating the mechanically infeasible solutions of the forward kinematics. It is indeed believed that the lack of understanding the workspace is a major obstacle in the development of the Stewart platforms into practically useful industrial robots (see Section 1.7).

Unfortunately the determination of the workspace is one of the dualities of serial and parallel manipulators mentioned in Section 0. This corresponds to Kumar's [11] statement that, compared to the research work on workspaces of serial manipulators, much less work has been reported on parallel manipulators. Kumar also says that although parallel manipulators are increasingly being used in robotics research, there is very little evidence of any general approach to workspace analysis and characterization of the reachable and dextrous workspace (defined in Section 1.6.3.6) of such manipulators

Merlet [4] explains that the problem with representing the three dimensional workspace of a spatial manipulator, is that the workspace cannot be de-coupled in two three dimensional workspaces characterizing the possible translational and orientational motions. Although there is no human readable way to represent the complete workspace, some projections of the full workspace can be drawn. He further reports that some researchers represent the possible translations of a parallel robot in a plane, for a fixed orientation and altitude of the mobile platform. Fichter [17] does this simulating the Stewart platform at different points in the Cartesian space. Merlet recommends the use of a geometric algorithm which can take into account the limited range of the actuators, the mechanical limits of the passive joints and the link interferences.

Another technique used to represent and characterize the workspace, is to consider the possible rotation of the end effector around a fixed point. This method is proposed by Merlet, and apparently it takes into account all the constraints limiting the workspace.

In his review, Merlet [4] also reports that he has solved a corollary problem, namely that of verifying that a straight line trajectory lies fully inside the workspace of a parallel manipulator. It is further stated that Merlet addressed the problem of determining the dimensions of a parallel robot so that the corresponding workspace includes a specified task-space.

The workspace boundaries of various planar and spatial parallel manipulators have been studied in the literature using different methods. Some of the more important and interesting methods proposed to date is discussed in the remainder of Section 1.6.3.

1.6.3.2 The Work of Bajpai and Roth

Bajpai and Roth [10] present and analyze the basic kinematic geometry and workspace properties of the simplest closed-chain manipulator, with revolute joints, that can be used to position a point in space. The planar, revolute jointed, five bar chain considered is shown in Figure 1.20.

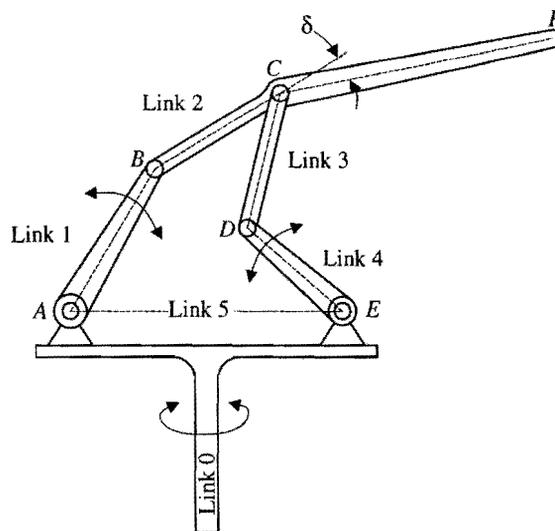


Figure 1.20 Closed loop manipulator with revolute joints (after [10]).

In their analysis of the closed loop manipulator, they emphasize the influence of the link lengths on the reachable workspace. They also comment that unlike the commonly used open-loop manipulators, the entire workspace of a closed-loop manipulator will not in general be freely reachable. Bajpai and Roth derive the conditions for which the complete independent mobility of the driving links of the five bar chain exist. In the end, they are able to suggest a few design rules to be considered when closed loop manipulators are to be built.

1.6.3.3 The Geometric Method of Merlet and Co-workers

The geometric methods introduced by Bajpai and Roth in 1986, are also used by Merlet et al. [14] in a recent article where they consider the workspaces of planar parallel manipulators. The planar three DOF parallel manipulators they consider, are composed of three kinematic chains connecting the mobile platform to the fixed base. It follows that different types of manipulators are obtained depending on the nature of these chains. In particular, they consider the 3-RPR manipulator, where the mobile platform is connected to the base via three identical chains (see Figure 1.7). Each chain consists of a revolute joint

(R) attached to the ground, followed by an actuated prismatic joint (P), which is connected to the platform by a revolute joint (R).

They define a fixed reference frame on the base, and a moving reference frame is attached to the platform with its origin fixed at point C . The position of the moving frame is defined by coordinates of point C in the fixed reference frame, and its orientation is given by the angle θ between one axis of the fixed reference frame, and the corresponding axis of the moving frame. Merlet and his co-workers further define an *annular region* as the region which lies between two concentric circles with different radii. It follows that the circle with the largest radius will be referred to as the external circle, and the smaller circle will be referred to as the internal circle. The internal circle may not exist. The assumption is made that no mechanical interference can occur between the links.

The first “type” of workspace considered, is the “constant orientation workspace”. This workspace is defined as the region, which can be reached by point C when the orientation of the moving platform is kept constant. For any position of C on the boundary of the workspace, at least one of the link lengths should be at its extreme value. If this is not the case, the platform may move in any direction, and therefore C cannot be located on the boundary of the workspace.

In determining the fixed orientation workspace, the region which can be reached by the endpoint of one of the actuator legs B_i is considered. This region is an annular region ϵ_i , centered at A_i , i.e. the base point of leg i . The external circle has a radius corresponding to the maximum actuator leg length, and similarly the internal circle has a radius corresponding to the minimum actuator leg length.

Any specific fixed orientation workspace will therefore be the intersection of the three annular regions of point C . Similarly, if the revolute joints attached to the ground cannot fully rotate, then point C will only be able to reach an angular sector, and the fixed orientation workspace will be the intersection of the three angular sectors.

Examples of fixed orientation workspaces, as determined by Merlet and his co-workers, are shown in Figure 1.21.

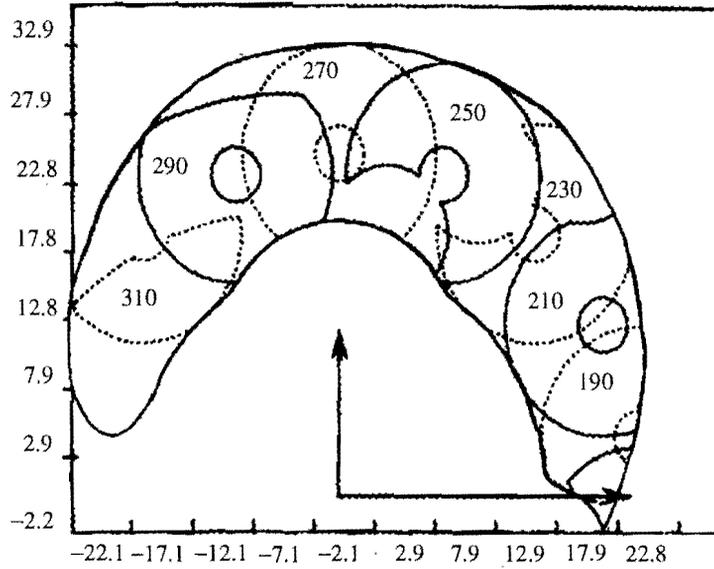


Figure 1.21 Examples of constant orientation workspaces of one of the planar Stewart platform considered by Merlet et al. (after [14]).

They also consider the maximal workspace which is defined as the region the reference point C can reach with at least one orientation. They state that the maximal workspace depends upon the choice of the reference point on the moving platform, and their objective is to geometrically determine the boundary of the maximal workspace. The maximal workspace of another of the manipulators considered by Merlet et al. is shown in Figure 1.22.

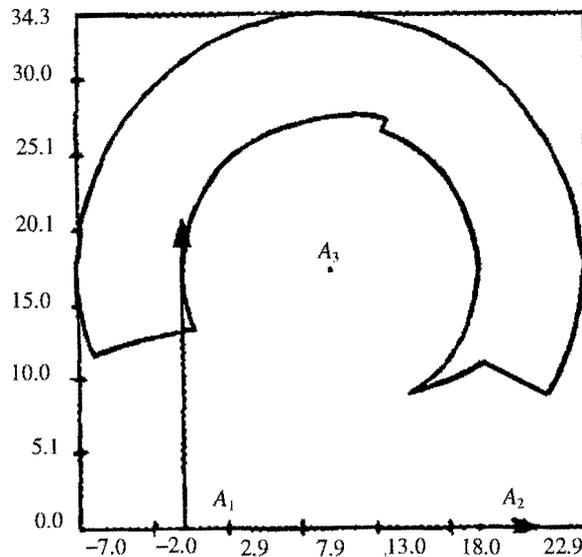


Figure 1.22 An example of the maximal workspace of a planar Stewart platform determined by Merlet et al. (after [14]).

They continue to describe how their geometric algorithms can be applied to determine various other “types” of workspaces. The *inclusive workspace* is defined as the set of all the positions, which can be reached by the reference point, with at least one orientation of the platform in a given interval. This inclusive workspace is obtained from the constant and reachable workspaces. The *total orientation workspace* is also determined, i.e. the region which can be reached by point C with every orientation of the platform in a given range. Finally the *dextrous workspace*, which is a particular case of the total orientation workspace, is also determined. This is the region that can be reached by the reference point with any orientation in a given range.

1.6.3.4 Gosselin’s Method of Spheres

Moving onto spatial manipulators, Gosselin [35] proposed an algorithm for the determination of the workspace of a parallel mechanism, using a method based on the geometric properties of the workspace. Gosselin points out that many of the methods to determine the workspace of a spatial six DOF parallel manipulator, are based on a complete sampling of the Cartesian space. Instead, Gosselin geometrically obtains the workspace, i.e. the region of the three dimensional Cartesian space that can be attained by the manipulator with a given orientation of the platform.

Gosselin states that, if mechanical interference is neglected, the boundary of the workspace is attained whenever at least one of the actuators reaches one of its limits. This corresponds to the work of Merlet et al. [14] on planar manipulators.

For a given fixed orientation of the platform, the portion of the three dimensional Cartesian space attainable by i -th leg is circumscribed by the concentric spheres of radii ρ_i^{\min} and ρ_i^{\max} . It follows that ρ_i^{\min} is the minimum reach of actuator leg i , and ρ_i^{\max} the maximum reach.

For the given fixed orientation the center of the spheres does not coincide with the center of the joint connecting the i -th leg to the base. Considering the local reference frame, which is connected to moving platform, the vector to the platform connection of leg i is subtracted in the local reference frame from the base connection of leg i to find the center of the spheres (see Figure 1.23).

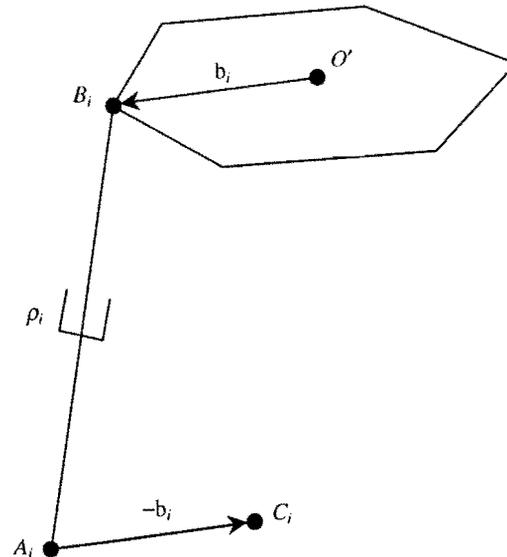


Figure 1.23 Location of the center of the spheres used to compute the workspace (after [35]).

For a given orientation of the platform, the workspace of the parallel manipulator in three-dimensional Cartesian space can be described as the intersection of six regions, each of these regions being the difference of two concentric spheres. Six pairs of concentric circles will be obtained if the intersection of the spheres with a horizontal plane is considered (see Figure 1.24). The intersection of the six annular regions is the section of the workspace contained in the plane.

Gosselin derives an algorithm to find the intersection of the six annular regions, keeping in mind that the boundary of each section will be made up of circular arcs, i.e. portions of circles (see Figure 1.25). Since Gosselin obtains the list of circular arcs, the area of the workspace section can accurately be determined by performing an integration process along the boundary. When the areas of the series of parallel planes are obtained, the volume of the workspace can be obtained by numerically integrating the area of the “slices” containing the workspace sections.

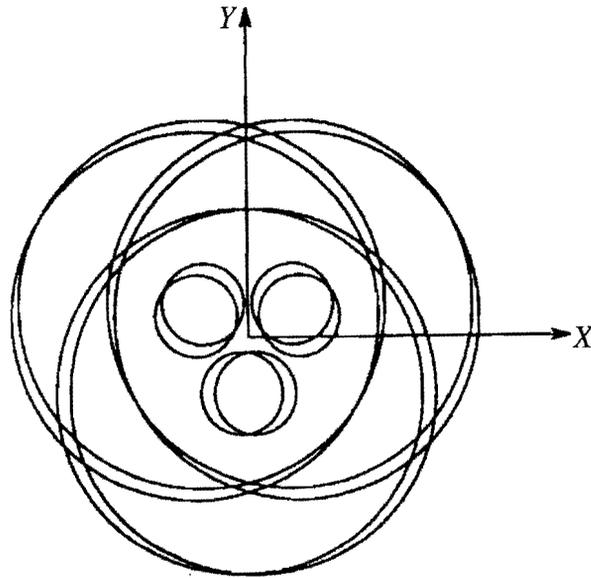
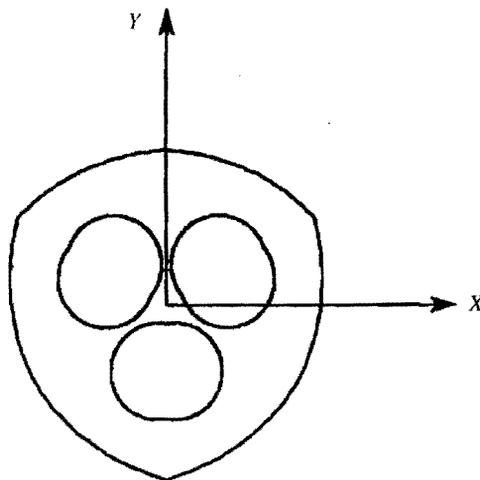


Figure 1.24 An example of the six concentric circles obtained if the intersection of the spheres with a horizontal plane ($z = 512 \text{ mm}$) is considered (after [35]).



$$\text{Area} = 55168 \text{ mm}^2$$

Figure 1.25 Boundary of the workspace for $z = 512 \text{ mm}$ (after [35]).

Gosselin also demonstrates the ability of his method to determine the workspace of a spatial mechanism. As a representative example, the workspace of the parallel manipulator developed at the French national institute for research in computer science and control (*INRIA*) has been studied [35]. The *INRIA* prototype is a six DOF fully parallel manipulator, and the volume of the workspace was also determined (see Figure 1.26).

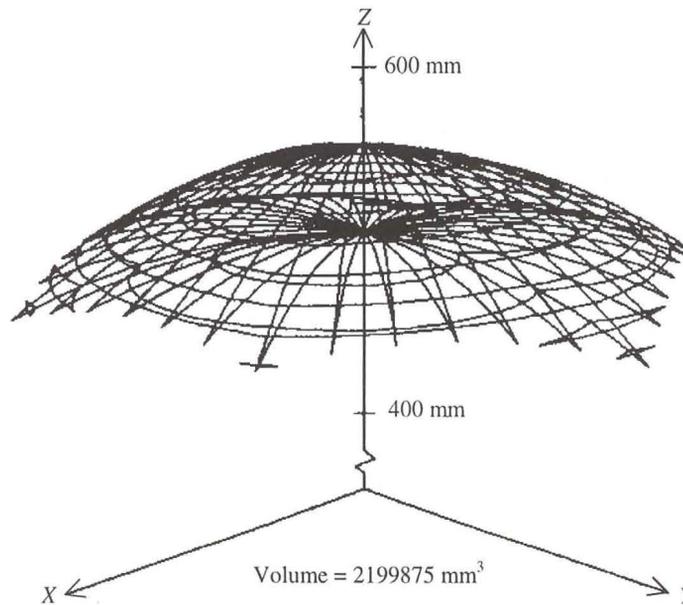


Figure 1.26 Boundary surfaces of the reachable workspace of the *INRIA* manipulator (after [35]).

1.6.3.5 Kumar's Method Based on Screw Theory

In his attempt to characterize the workspaces of parallel manipulators, Kumar [11] defines the *reachable workspace* as the volume or space within which a reference point on the hand or end effector can be made to coincide with any point in space. The reference point on the manipulator end effector is on the workspace boundary when the manipulator is at full extension in any direction. Because the end effector does not possess the ability to translate along the direction of maximal extension, the manipulator is at a positional singularity.

Kumar uses screw theory to express the necessary condition for the end effector to be on the workspace boundary. This condition can be expressed as: the instantaneous joint screws all being reciprocal to a zero pitch screw through the reference point on the end effector along the direction of full extension.

From a geometrical viewpoint, a parallel manipulator is considered to be a collection of n serial manipulators all of which have a common base and end effector. The i -th chain possesses m_i single DOF revolute joints each capable of 360° rotation.

Kumar isolates the i -th serial chain, and denotes its reachable workspace as R_i , i.e. the workspace of the resulting serial manipulator once the other $n - 1$ chains have been removed. The resulting serial manipulator consists of the base, the links in the i -th chain, and the end effector.

Kumar points out that the configuration in which the joint screws are reciprocal to a zero pitch screw passing through the reference point does not necessarily imply that the reference point lies on a boundary. When the reference point of a parallel manipulator is at the workspace boundary, all the twists available to the end effector, (about which the end effector can twist), are reciprocal to a zero pitch wrench whose axis passes through the reference point on the end effector.

In general, there is a set of k possible configurations that satisfy the condition of reciprocity to a zero pitch screw through the reference point. For a serial chain with m joints, $k = 2^{(m-1)}$. The method Kumar uses to generate the workspace boundary, consists in basic terms of finding all of the k possible configurations, and determining which loci of the reference point are on the workspace boundary. There is a special consideration in order to accommodate mechanisms in which all joint screws span a screw system of order less than six (for example a planar mechanism).

The first example considered by Kumar, is a five bar linkage, which is a two DOF parallel manipulator similar to the one analyzed by Bajpai and Roth [10] and shown in Figure 1.20. The manipulator consists of two serial chains, a 2- R (Link 1 in Figure 1.20) and a 3- R chain (Links 3 & 4 in Figure 1.20). For a given position of the reference point P in Figure 1.20, the 2- R chain assumes one of two configurations, and for each configuration, the 3- R has two possible configurations. Hence, there are $2^2 = 4$ possible solutions for the inverse kinematics. The direct kinematics has two solutions.

If a zero pitch wrench is applied at the end point, and swept through 360° in the plane, the workspace boundaries can easily be traced (see Figure 1.27).

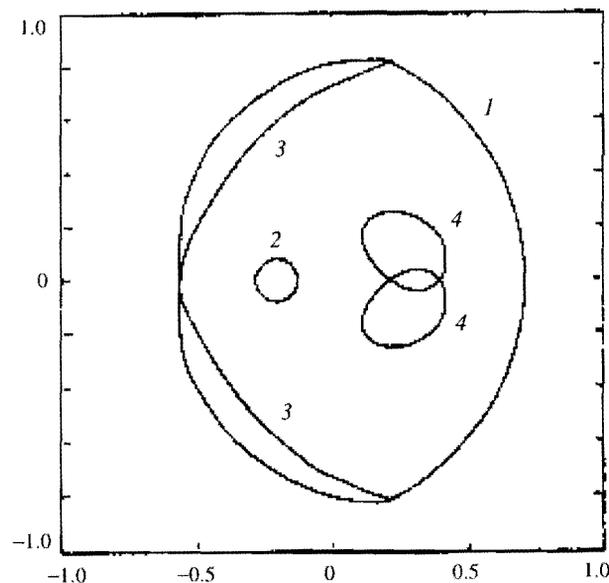


Figure 1.27 The workspace boundaries of the two DOF planar parallel manipulator (after [11]).

The second example considered by Kumar, is a three DOF planar parallel manipulator, where the end effector is constrained by three serial chains each of which possesses three revolute joints as depicted in Figure 1.4.

Having 3-*R* chains, results in $2^3 = 8$ possible solutions for the inverse kinematics. The number of solutions or closures to the direct kinematics, is at most six. The boundary of the workspace of this mechanism consist out of contours generated from analytically derived algebraic expressions, since they are arcs of circles or parts of coupler curves of which explicit closed form expressions exist (see Figure 1.28).

In determining the dextrous workspace of a parallel manipulator, Kumar proves that if the reference point of a parallel manipulator is at the boundary of its dextrous workspace, there exists at least one chain such that a zero pitch screw is reciprocal to all the joint screws in that chain.

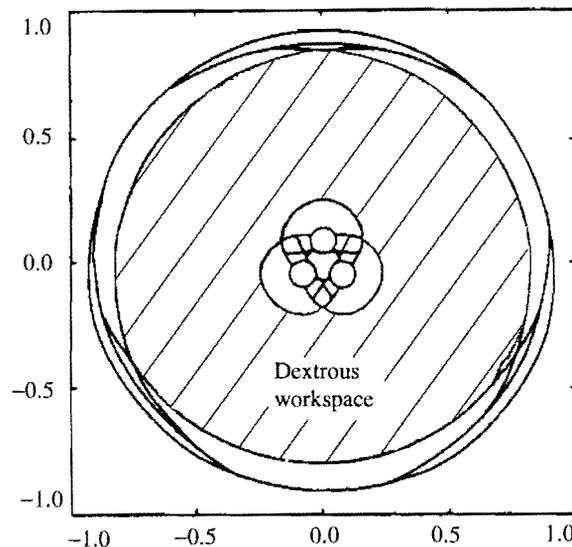


Figure 1.28 Workspace boundary of the planar three DOF parallel manipulator (after [11]).

After the workspace boundary is computed, Kumar identifies the closed regions that are completely bounded by workspace contours. Each closed region is then tested for dexterity, i.e. for a particular region any convenient point Q is tested: Firstly the ability of the manipulator to provide all attitudes of the end-effector with the reference point at Q is tested, and secondly for all attitudes of the end effector, the ability to complete a 360° rotation about any (all) axis (axes) through Q is tested.

As an extension to the definition of the dextrous workspace, Kumar also defines the controllable dextrous workspace. "A point Q belongs to the controllable dextrous workspace if: (a) the reference point of the end effector can reach Q , (b) the manipulator can provide the end effector any (every)

attitude with its reference point at Q , and (c) for any reference attitude of the end effector, the manipulator can produce a complete rotation of the end effector about any (every) axis through the reference point, such that , during the rotation, the end effector does not *lose* or *gain* a degree of freedom.”

1.6.3.6 Haug et al.’s Continuation Method

So far each proposed method successfully determines the workspaces of a certain type of manipulator. Haug et al. [12] suggest a more broadly applicable numerical algorithm for mapping the boundaries of manipulator workspaces.

In simplest terms, the initial step of their numerical method consists of the non-trivial task of finding an initial point on the workspace boundary. Starting from an assembled configuration of the manipulator, a unit vector c in the output-space is selected, and the ray emanating from the starting point along vector c is traced until the workspace boundary is encountered. Once a boundary point is found, they use a continuation method to proceed stepwise along the solution curve. A computer code is implemented in the final stage of their method to map the boundary of the accessible output set, until a closed trajectory is found. The continuation method is extended to accommodate bifurcation points, where more than one solution curve intersect.

A planar redundantly controlled serial manipulator, a planar Stewart platform and a spatial Stewart platform are analyzed by Haug and his co-workers. They determine both the exterior boundary of the accessible output set, as well as the exterior-interior bifurcation point connecting curves. They suggest that these “interior” curves often represent local impediments to motion or controllability and are therefore of practical importance.

As an extension to the work done on determining the reachable workspace boundaries of various manipulators, Haug et al. [8] also analyzed the dextrous workspace of manipulators. Apparently Kumar and Waldron [36] introduced the concept of a dextrous workspace. The dextrous workspace is a space within which every point can be reached by a reference point on the manipulator’s hand (end-effector), with the hand (end-effector) in any desired orientation. The importance of the dextrous workspace lies in the statement of Haug et al. [8] that “methods for characterizing the region in space in which a manipulator can both position and control the orientation of a working body, throughout specified ranges of inputs and outputs are not well developed.”

Haug and his co-workers [8] address the existing need for characterizing the region in space in which a manipulator can both position and control the orientation of a working body, throughout specified ranges

of inputs and outputs. One of the objectives of their research is to define analytical criteria for the boundary of the set of points that can be reached by a given point on the working body. Throughout the set of points, specified ranges of rotation of the working body must be achieved.

It is of practical interest and importance to note that, as stated by Haug et al. [8], the necessary conditions for boundaries of dextrous workspaces are often less complex than associated conditions for the boundaries of workspaces without any dexterity requirement. This is consistent with experience in constrained optimization, where the introduction of additional constraints causes the boundary of the feasible solution set, in the present case the dextrous accessible output set, to become more regular and in many cases easier to compute.

Considering that the determination of workspaces of planar parallel manipulators is already a challenging problem to solve, the situation worsens significantly when spatial manipulators are considered. To handle general parallel robots with arbitrary link geometry, a numerical approach is essential. The numerical approach suggested by Haug et al. [8, 12] is very complicated and not easily implemented.

1.6.3.7 The Method of Wang and Hsieh

Wang and Hsieh [29] present what they call “a systematic method for the numerical analysis of the extreme reaches, and reachable workspace of general parallel robots”. They formulate an *optimization problem* in order to find the extreme reach of the manipulator. The distance between the center of the end-effector, and a fixed base point with respect to any given search direction is maximized or minimized depending on the desired objective.

In their article, Wang and Hsieh refer to Haug et al.’s [8, 12] continuation numerical method for determining the workspaces of multi-body mechanical systems. They point out that explicit expressions of the constraint equations and Jacobian matrix of the mechanism is necessary if Haug’s method is to be used. It may be difficult to derive these expressions for general parallel robots having complex coupled kinematic loops and multi-DOF joints.

Wang and Hsieh handle the kinematic constraint equations in an implicit and systematic way, and their method does not require the computation of the constraint Jacobian matrix. Various types of joints are taken into account and the advantages of efficient recursive computational schemes are exploited.

The extreme reach is defined as the extreme distance between a base point and the center point of the end-effector. Wang and Hsieh denote l as a straight line defined by a fixed base point B and a unit vector u . The reachable workspace boundary is intersected by the straight line l as shown in Figure 1.29.

The current position of the end effector is $\mathbf{P}_{oc}(\bar{\mathbf{q}})$, in which $\bar{\mathbf{q}}$ is the joint variable vector. The objective is to find the farthest (or nearest) position with respect to B that the center point C of the end effector coincident with l without violating the kinematic constraints.

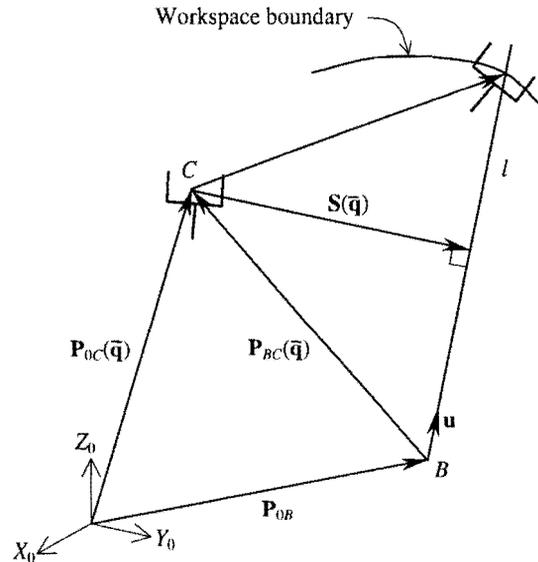


Figure 1.29 Definition of the extreme reach (after [29]).

The perpendicular distance from C to l is given by $S(\bar{\mathbf{q}})$. Wang and Hsieh formulate the problem of finding the extreme reach as follows: “Find the optimum value of $\bar{\mathbf{q}}$ that maximizes (or minimizes) the Euclidian norm of $\mathbf{P}_{BC}(\bar{\mathbf{q}})$, subject to $S(\bar{\mathbf{q}})=0$ and the kinematic constraints of the closed loops of the driving mechanisms.” The search direction \mathbf{u} (see Figure 1.29) is maintained by means of the equality constraint $S(\bar{\mathbf{q}})=0$, ensuring that the perpendicular distance from C to l is zero.

Using this problem formulation, the boundaries of the reachable workspace are determined by slicing the space into equally spaced horizontal planes along the vertical axis. The intersection point of the plane and the vertical axis is used as the base point from which the maximum and minimum reaches are determined (see Figure 1.30).

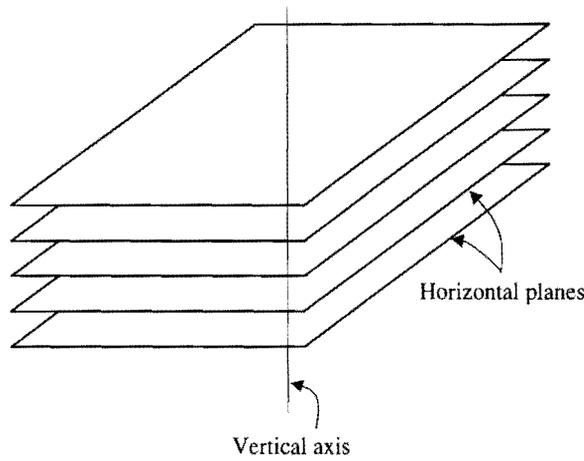


Figure 1.30 Space slicing strategy (after [29]).

The search directions lie on the plane, and are constantly incremented until a closed envelope is determined for each horizontal plane (see Figure 1.31).

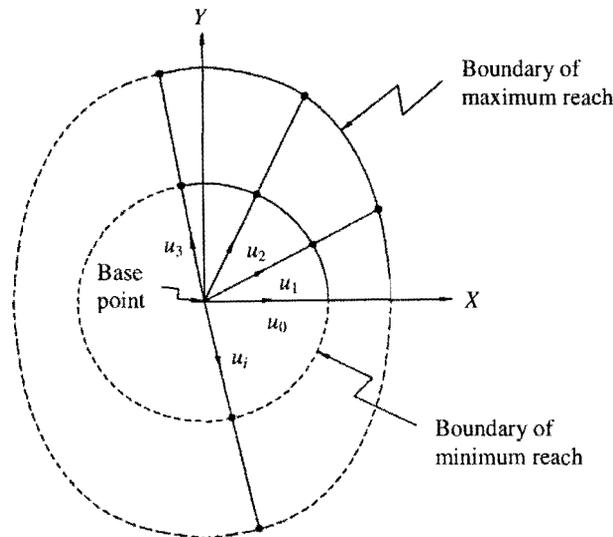


Figure 1.31 Workspace boundaries of a plane (after [29]).

Finally three dimensional curve fitting techniques are used to plot the boundary surfaces of the reachable workspace.

The objective function of the constrained nonlinear optimization problem constructed by Wang and Hsieh is:

$$\text{minimize}_{\bar{q}} \|K\mathbf{u} - \mathbf{P}_{BC}(\bar{q})\|^2$$

Where the constant K depends on whether the maximum or minimum reach has to be found. For the maximum reach, the value associated with K is very large so as to overestimate the maximum reach of the manipulator. The minimum reach is found by setting $K = 0$.

A combined solution procedure is used to solve the above highly non-linear optimization problem. Instead of using an ordinary steepest decent method, Wang and Hsieh uses a combined method which first uses the cyclic coordinate descent (CCD) method to find a good approximation to the solution vector, and then uses a quasi-Newton method to converge to the solution to the desired degree of precision.

Although the proposed method is numerically stable and computationally efficient, multiple local optimal points may exist so that there is no guarantee that the method will converge to the global solution. The reason for the different possible local optimal points is the fact that the kinematic loops may be closed in several different configurations, and each may end up in a different extreme reach. Wang and Hsieh suggest that this complication be overcome by solving the optimization problem for various initial approximations. The best solution among the alternatives may then be picked.

Wang and Hsieh consider, as an example, a platform-type parallel robot which has six revolute joints, six universal joints and six spherical joints (see Figure 1.32). The center points of the universal joints and spherical joints coincide with the vertices of the platforms. The three dimensional workspace boundaries obtained, as well as the working areas at different planes, are shown in Figure 1.33.

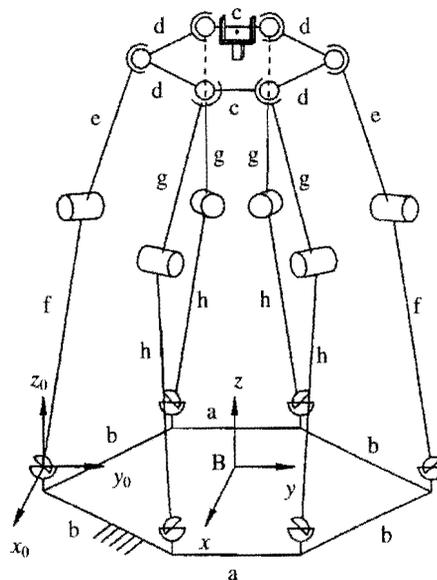


Figure 1.32 A platform type fully parallel robot (after [29]).

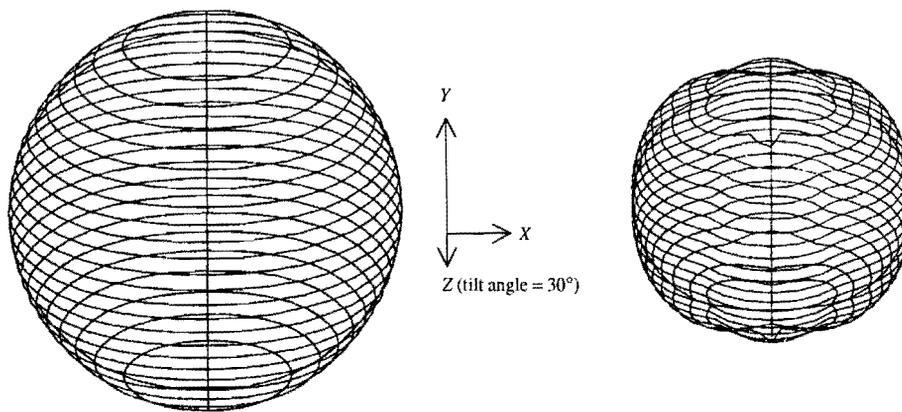
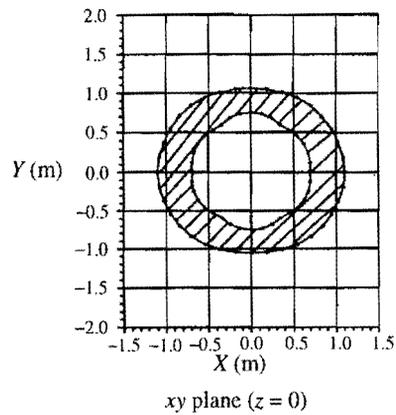


Figure 1.33 Boundary surfaces of the reachable workspace determined by Wang and Hsieh [29].

1.7 MOTIVATION FOR THIS STUDY

Merlet concludes his 1994 review of the development of parallel manipulators by saying: “Parallel manipulators present various advantages which can be useful in many robotic tasks. Although interesting theoretical problems remain to be solved, the current state of the art has enabled prototypes and commercial manipulators to be designed. Although the concept of a parallel manipulator is too recent and too different from the design of most classical manipulators to be widely accepted and frequently chosen by the designers of robotic systems, it is strongly felt that their use in many robotic tasks is so necessary that they will become indispensable in the near future”.

Unfortunately this promise has, as yet, not been fulfilled. In fact, as recently as 1996 Ji [23] states that the use of platform manipulators is still mainly in an experimental stage. The biggest reason for this seems to be the lack of rational synthesis tools for the design of practically useful platform manipulators.

In particular, as is evident from this survey, the problems of the forward kinematics and workspace determination remains to be satisfactorily solved. This study addresses the latter problem. It is believed that if the workspace is understood and its characterization properly done, then many design problems will easily be solved. In particular, workspace characterization will assist in solving design problems relating to, amongst others, the determination of the relative sizes of the platforms; the positioning of the joints; the determination of acceptable ranges of translation and rotation for which the platform is stable; the determination of the ranges of leg displacements, and in assessing the problems caused by leg interference.

The literature reports on a number of different methods for the determination of the manipulator workspaces. Some of the more important methods are summarized in Section 1.6.3. Each approach has its own advantages, but all the proposed methods appear to be complicated in one way or the other, and cannot be generally applied to a wide range of platform devices. The urgent need therefore exists for a general, yet easily implemented methodology for the determination and characterization of manipulator workspaces.

In this study a novel optimization approach to solving the workspace problem is introduced. An attempt is made to demonstrate that this approach is not only general, but may also be easily implemented to determine various types of accessible workspaces.

Finally it should be mentioned that, although the approach presented here was independantly conceived, it has certain features that are philosophically similar to that of the recent optimization method of Wang and Hsieh [29], and which is summarized in Section 1.6.3.7.