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**AN OPTIMIZATION APPROACH TO THE
DETERMINATION OF MANIPULATOR
WORKSPACES**

by

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the degree of

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To my parents

ABSTRACT

AN OPTIMIZATION APPROACH TO THE DETERMINATION OF MANIPULATOR WORKSPACES

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Degree: Master of Engineering

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The main objective of this study is to propose and develop a *general* numerical technique by means of which the workspaces of mechanical manipulators may be determined with relative *ease*. The emphasis is on parallel or so-called Stewart platforms.

Stewart platforms have many advantages over the traditional serial manipulators. These advantages include high accuracy, high stiffness, high load-to-weight ratio and most importantly, low cost. According to the literature, it is strongly felt that the use of parallel manipulators in many robotic tasks is so necessary that they will become indispensable in the near future.

In spite of the advantages of these mechanisms, the use of Stewart platforms is still mainly in an experimental stage. This is because there seems to be a lack of rational synthesis tools for the design of practically useful platform manipulators. In particular, the problems of the forward kinematics and workspace determination remain to be satisfactorily solved. This study addresses the latter problem. It is believed that if the workspace is understood, and its characterization properly done, then many design problems will easily be solved.

In this study a novel optimization approach to solving the workspace problem is introduced. An attempt is made to demonstrate that this approach is *general* in the sense that it is applicable to different kinds of manipulators, and may also easily be implemented to determine *various types* of accessible workspaces.

In particular, the generality of the method is illustrated by the fact that the optimization approach was successfully implemented for a redundantly controlled planar serial manipulator, a planar Stewart

platform as well as a spatial 6–3 Stewart platform. The optimization method is also successful in assisting in the characterization of the workspace by, for example, identifying interior curves connecting bifurcation points. This is of great potential importance with regard to the control of a manipulator within its workspace.

The description of the behavior of the planar manipulators, led to a new notation for labeling the workspaces. This notation arises in a natural way from the optimization approach, is generally applicable and easy to understand. Using this notation, the complete workspace may be described in terms of the behavior of the manipulator.

Of great practical importance is the treatment of dexterity requirements imposed on a Stewart platform. The optimization approach successfully determines different specified dextrous workspaces of the planar Stewart platform. An example of a dextrous workspace of the 6–3 Stewart platform was also successfully mapped. This is very significant, because as far as the author is aware, such a mapping has not previously been performed for the spatial case.

It is hoped that this study will lie the foundation for the development of a general and rational synthesis design tool for parallel manipulators. Further research that will be addressed in the near future, and stems from the work done here, is the determination of the feasible workspace for parallel manipulators subject to various additional prescribed mechanical constraints.

This study has important potential impact for the manufacturing industry of South Africa and other developing countries. The implementation of this technology lies in retrofitting existing non-CNC milling equipment to increase their capability at a lower cost than that of the alternative of purchasing traditional 5-axis machining centers.

SAMEVATTING

'n OPTIMERINGS-BENADERING TOT DIE BEPALING VAN MANIPULEERDER- WERKRUIMTES

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Sleutelwoorde: bereikbare werkruimte, bruikbare werkruimte, serie-manipuleerder, parallel-manipuleerder, Stewart platform, optimalisering, bifurkasie, werkruimte-karakterisering

Die doelwit van hierdie studie is om 'n *algemene* numeriese metode te ontwikkel, waarmee die werkruimtes van meganiese manipuleerders redelik *maklik* bepaal kan word. Die klem val op parallel- of sogenaamde Stewart platforms.

In vergelyking met die tradisionele serie-manipuleerders, beskik Stewart platforms oor bepaalde voordele. Hierdie voordele sluit onder andere in: hoë akkuraatheid, hoë styfheid, hoë las-tot-gewig verhouding en, die belangrikste voordeel, lae koste. Vanuit die literatuur is dit duidelik dat die potesiële gebruik van Stewart platforms so' belangrik is, dat hulle binne die afsienbare toekoms onvervangbaar sal wees.

Ten spyte van die voordele, is die gebruik van Stewart platforms hoofsaaklik nog in die eksperimentele fase. Dit wil voorkom asof daar 'n gebrek is aan rasionale sintese-metodes waarmee prakties-bruikbare parallel-manipuleerders ontwerp kan word. Onder andere is die afleiding van die voorwaarste kinematiese vergelykings, asook die bepaling van die werkruimtes van hierdie meganismes, huidiglik nog probleem-areas. Laasgenoemde probleem word in hierdie studie aangespreek. As die werkruimte deeglik omskryf, en dienooreenkomstig gekarakteriseer word, sal baie ontwerp-probleme betreklik maklik opgelos kan word.

'n Nuwe optimaliseringsmetode word voorgestel waarmee die werkruimtes van verskillende manipuleerders bepaal word. Hierdeur word aangetoon dat die metode *algemeen* toepasbaar is, en ook maklik implementeer kan word om *verskillende tipes* bereikbare werkruimtes mee te bepaal.

In besonder word die algemene toepasbaarheid van die metode aangetoon deur die suksesvolle bepaling van die werkruimtes van 'n oortollig-beheerde serie-manipuleerder, 'n vlak Stewart platform en 'n ruimtelike 6–3 Stewart platform. Met behulp van die optimerings-metode, kon die werkruimtes gekarakteriseer word deur, byvoorbeeld, die identifisering van interne krommes wat bifurkasie-punte verbind. Dit is van groot potensiële belang met betrekking tot die beheer van 'n manipuleerder binne die spesifieke werkruimte.

Die beskrywing van die gedrag van die vlak manipuleerders het gelei tot 'n nuwe metode om die werkruimtes te anoteer. Hierdie nuwe notasie, wat voortspruit uit die toepassing van die optimerings-metode, is algemeen bruikbaar en maklik verstaanbaar. Deur gebruik te maak van hierdie notasie, kan die algehele werkruimte beskryf word in terme van die fisiese gedrag van die manipuleerder.

Die afdwing van sekere bruikbaarheids-voorskrifte vir die Stewart platform is noodsaaklik vanuit 'n praktiese oogpunt. Verskillende bruikbare werkruimtes van die vlak Stewart platform is bepaal deur die optimeringsmetode te gebruik. Verder is daar 'n spesifieke voorbeeld van 'n bruikbare werkruimte van die 6–3 Stewart platform bereken. So ver die skrywer kon vasstel, is dit die eerste keer dat 'n bruikbare werkruimte vir hierdie ruimtelike manipuleerder bepaal is.

Daar word gehoop dat hierdie studie die grondslag sal lê vir die ontwikkeling van 'n algemene ontwerp-metodiek vir parallel-manipuleerders. As voortsetting van die werk wat hier gedoen is, word dit beoog om bereikbare werkruimtes van parallel-platforms, onderhewig aan verskeie ander voorgeskrewe meganiese begrensings, te bepaal.

Die vervaardigingssektor van die Suid-Afrikaanse nywerheid en van ander ontwikkelende lande, kan moontlik voordeel trek uit hierdie navorsing. Bestaande freesmasjiene, wat nie gerekenariseer is nie, kan opgradeer word deur die aanhegting van 'n Stewart platform. Sodoende kan die vermoëns van hierdie freesmasjiene uitgebrei word, en word die hoë kapitale uitgawe om 'n 5-as gerekenariseerde freesmasjiene aan te koop, uitgeskakel.

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Praise the Lord!

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