

TECHNICAL APPENDIX

THE CEPSTRUM78

Speaking mathematically, the cepstrum is an integral transform with a long history. The use of the cepstrum dates back to Poisson (1823) and Schwarz (1872), who applied cepstra to problems involving potential functions with real parts fixed on the unit circle. Later, Szëgo (1915) and Kolmogorov (1939) used cepstra in the extraction of stable causal systems by factoring power spectra of random processes. However, it is the application of Bogert et al. (1963) in engineering that most coincides with our interest here.

Bogert et al. (1963) considers a time-varying function f(t) which is made up of another function $f_1(t)$ and its additive "echo," $f_1(t-\tau)$, lagged by τ periods. Formally, we have:

$$f(t) = f_1(t) + af_1(t - \tau)$$
(A.1)

The power spectrum of f(t) is

$$\left|F(\boldsymbol{\omega})\right|^{2} = \left|F_{1}(\boldsymbol{\omega})\right|^{2} \left\{1 + a^{2} + 2a\cos(\boldsymbol{\omega}\tau)\right\},\tag{A.2}$$

where $F_1(w)$ is the complex Fourier transform of $f_1(t)$. The "echo", in turn, manifests as a cosine function riding on the envelope of the power spectrum. The period of the cosine function is the reciprocal of the lag τ . In studies involving autocorrelation analysis in hydroacoustic problems, Griffin et al. (1980) has shown that if the spectra of f(t) and its "echo" differ, i.e., they are not perfectly correlated, then the parameter a in Equation (A.2) varies with frequency. Thus, f(t) is better represented by:

$$f(t) = f_1(t) + f_2(t - \tau), \qquad f_1(t) \neq f_2(t)$$
 (A.3)

so that the power spectrum of f(t) is given by:

$$\left|F(\boldsymbol{\omega})\right|^{2} = \left|F_{1}(\boldsymbol{\omega})\right|^{2} + \left|F_{2}(\boldsymbol{\omega})\right|^{2} + 2\left|F_{1}(\boldsymbol{\omega})\right|\left|F_{2}(\boldsymbol{\omega})\right|\cos\left\{\phi_{1}(\boldsymbol{\omega}) - \phi_{2}(\boldsymbol{\omega}) + \boldsymbol{\omega}\tau\right\},\tag{A.4}$$

where $\phi_1(\omega)$ and $\phi_2(\omega)$ are the phase spectra of $f_1(t)$ and $f_2(t)$, respectively.

The modulating cosine is phase-shifted by an amount which is equal to the differences in the phases of functions that composes f(t). Thus, if the function $f_1(t)$ and $f_2(t)$ are not close

⁷⁸ This section relies heavily on the discussion available in Cunnigham and Vilasuso (1994).



copies of one another, the argument of the modulating cosine is not constant, which, in turn, has important consequences as we shall see below.

We proceed under the assumption that $f_1(t) = f_2(t)$ and return to the more general case later. The function f(t) can be represented as the linear convolution of $f_1(t)$ with a train of impulses. Bogert et al. (1963) argue that if the envelope of $|F(\omega)|^2$ could be made optimally white, this would be equivalent to making $|F_1(\omega)|^2$ into a "boxcar" function, whose Fourier transform would be a sinc function at the origin.⁷⁹ In the limit, as the envelope of the power spectrum becomes uniform at all frequencies, the sinc function tends towards a Dirac delta function. The transform of the modulating cosine would be an impulse whose delay is related to the frequency of the modulating cosine which, in turn, is equal to the lag length between $f_1(t)$ and its "echo." Therefore, under ideal conditions and when scaled properly, this resulting series yields a time domain function with a global maximum, or "peak" at the origin, and the local maximum or "peak" indicating the arrival time of the "echo."

In practice, the whitening of the power spectrum is performed by first applying the natural logarithm and then the inverse Fourier transform (IFT). Because it ignores the phase spectrum, and is calculated directly from the log power spectrum, the resultant function is referred to as the power spectrum.

Let us now go back to case presented in Equations (A.3) and (A.4). If the component functions are not close copies of one another, then the argument of the modulating cosine is not invariant, and, hence, cepstral peaks rapidly degenerate. Therefore, it must be realized that the impulse appears at the appropriate lag of the cepstrum only if the component functions are well correlated.

When we consider the discrete case, cepstra are a class of integral transform whose kernel is a function of the z-transform of a real sequence. The discrete power cepstrum of a data sequence x(nT) with z-transform X(z) is then given by:

$$\tilde{x}(nT) = \frac{1}{2\pi i} \left| \oint_c \log \left| X(z) \right| z^{n-1} dz \right|^2 \tag{A.5}$$

⁷⁹ Note that sinc x = (sinx)/x.



where n= 0, 1, 2, ..., N, enumerates the samples, T is the sampling interval, and C is a closed contour inside the region of convergence of the power series and enclosing the origin. As discussed by Cunningham (1980), this can be extended to the cosine-squared cepstrum by addition of the signum function as follows:

$$\hat{x}(nT) = \tilde{x} \times \text{sgn}\left\{\text{Re}\left[\frac{1}{2\pi i} \oint_{c} \log|X(z)|z^{n-1}dz\right]\right\}$$
(A.6)

The addition of the signum function allows the cepstrum to determine not only the arrival time of the secondary series, but also, and perhaps more importantly, its polarity relative to the original series making its interpretation analogous to the autocovariance function.⁸⁰ Because in real-life applications the real sequences x(nT) are of finite length, the annulus of convergence of X(z) always includes the unit circle, the transforms may be computed by means of the fast Fourier transforms.

Because the cepstrum is essentially the spectrum of a spectrum, the cepstral domain is a time domain. The terminology easily becomes confusing, therefore, Bogert et al. (1963) suggested the following conventions: the term "cepstrum" is an anagram of the word "spectrum." Likewise, periodicities in the cepstrum are discussed in terms of "quefrencies," "gamnitudes," and "repiods," analogous to "frequencies," "gain/amplitudes," and "periods" in the time domain. "Filtering" in the cepstral domain is "liftering" and so on. Further, Bogert et al. (1963) refers to data analysis in the cepstral domain as "alanysis". Finally, the term "saphe," pronounced "safe" and related to "phase," is used to refer to the displacement between the secondary, or lagged series, and the original. Thus, the detection and analysis of cepstral peaks and "phase" shifts of the lagged series is referred to as "saphe cracking."

⁸⁰ The close relationship between the cosine-squared cepstrum and the autocovariance function becomes clear when we write the latter as a function of the z-transform of the same series: $x_{acvf}(\tau) = \frac{1}{2\pi i} \oint_c |X(z)| z^{n-1} dz$. Equivalently, the autocovariance function is the inverse Fourier transform of the power spectrum of a series.



9 Conclusion

Since the February of 2000, the sole objective of the SARB has been to keep the CPIX inflation rate within the target band of 3 percent to 6 percent, using discretionary changes in the Repo rate as its main policy instrument. Given this, this thesis aimed to evaluate inflation targeting regime of South Africa not only in term of welfare cost estimates, but also by comparing mean and volatility of inflation in pre- and post-inflation targeting regimes.

With the welfare cost of inflation lying between 0.70 percent of GDP to 1.33 percent of GDP for the target band of 3 percent and 6 percent, we show that there are considerable welfare gains to be had by reducing the lower level of the target band below the 3 percent mark. In addition, we show that mean and volatility of inflation is higher than it would have actually been had the SARB followed its earlier more eclectic monetary policy regime. We believe this is due to the width of the inflation targeting band. In line with the inflation targeting literature on focal points, we suggest a narrower and possibly lower target band target band could be of immense help in improving the central bank's credibility and causing inflation expectations to converge to a focal point, and hence, bring down the mean and variance of the inflation rate.



Bibliography

- Akinboade, O. A., Siebrits, F. K. and Roussot, E. W. (2004). The Output Costs during Episodes of Disinflation in South Africa, *Ninth Annual conference of Econometric Modelling for Africa*.
- Anderson, R. G. and Robert, H. R. (2001). The Remarkable Stability of Monetary Base Velocity in the United States, 1919-1999, *Working Paper 2001-008A. St. Louis*, Federal Reserve Bank of St. Louis.
- 3. Andrews, D.W.K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation, *Econometrica*, 59(3): 817-858.
- 4. Bae, Y. (2005). The Money Demand Function and Nonlinear Cointegration, Manuscript Columbus: Ohio State University.
- 5. Bailey, M. (1956). The welfare Cost of Inflationary Finance, *Journal of Political Economy* 64(2):93-110.
- Ball, L. and Sheridan, N. (2005). Does Inflation Targeting Matter?, The inflation Targeting Debate, edited by B.S Bernanke and M. Woodford, NBER Studies in Business Cycles, Vol. 32. Chicago: University of Chicago Press.
- 7. Barro, R.J. and Gordon, D.B. (1983). A positive Theory of Monetary Policy in a Natural rate Model, *Journal of Political Economy*, 91(4):589-610.
- 8. Bernanke, B.S., Laubach, T., Mishkin, F.S., and Posen, A.S. (1999). *Inflation targeting: lessons from the International Experience*, Princeton, NJ: Princeton University Press.
- Black, R. Macklem, T. and Poloz, S. (1993). Non-Superneutralities and some benefits of disinflation: A quantitative general equilibrium analysis, *Manuscript* (Research Department, Bank of Canada, Ottawa).



- Blaszkiewicz, M., Konieczny, J., Myslinskaa, A., Radziwil, A. and Wozniak, P. (2003). Some benefits of reducing inflation in transition economies, *Macroeconomies 0303004*, EconWPA.
- 11. Bogert, B.P., Healy, M.J.R. and Tukey, J.W. (1963). The Quefrency Alanysis of Time Series for Echoes: Cepstrum, Pseudo-Autocovariance, Cross –Cepstrum, and Saphe Cracking, *in proceedings of a Symposium on Time Series Analysis*, edited by M. Rosenblatt. New York: John Wiley & Sons, Inc.
- 12. Bonato, L. (1998). Price stability: some costs and benefits in New Zealand, Reserve Bank of New Zealand Discussion Paper No G98/10, Reserve bank of New Zealand.
- 13. Brock, W. (1974). Money and Growth: The case of Long –Run perfect Foresight, International Economic Review, 15(3):750-777.
- 14. Burger, P. and Marinkov, M. (2006). The South African Phillips Curve: How Applicable is the Gordon Model? , *South African Journal of Economics*, 74(2):172-189.
- 15. Burger, P. and Marinkov, M. (2008). Inflation targeting and inflation performance in South Africa, *TIPS Working Paper*.
- Cagan, P. (1956). The Monetary Dynamics of Hyperinflation. In Milton Friedman, Ed.Studies in the Quantity Theory of Money. Chicago: University of Chicago Press.
- 17. Coleman, W.J. II. (1993). Money, Interest, and capital in cash-in- advance economy, *Manuscript*, Department of Economics, Duke University, Durham, NC.
- Cooley, T.F. and Hansen, G.D. (1989). The inflation tax in a real business cycle model, *American Economic Review*, 79(4): 733-748.
- 19. Cooley, T.F. and Hansen, G.D. (1991). The welfare cost of moderate inflations, *Journal of Money, Credit, and Banking*, 23(3): 483-503.



- 20. Cunningham, S.R., (1980). Application of Cepstral Techniques to Rayleigh Wave Multipathing, U.S. national Technical Information Service (NTIS), ENSCO DCS-STR-80-45.
- Cunningham, S.R., and Vilasuso, J. (1994). Comparing U.S. GNP Volatility across Exchange Rate Regimes: An Application of Saphe Cracking, *Journal of Macroeconomics*, 16(3): 445-459.
- 22. De Gregorio, J. (1993). Inflation, taxation, and long-run growth, *Journal of Monetary Economics*, 31(3): 271-289.
- 23. Demertzis, M. and Viegi, N. (2008). Inflation Targets as Focal Points, *International Journal* of Central Banking, 4(1): 55-87
- Demertzis, M. and Viegi, N. (2007). Inflation Targeting a Framework for Communication, DNB Working Papers 149, Netherlands Central Bank, Research Department.
- 25. Demertzis, M. and Viegi, N. (2006). Aiming for the Bull's Eye: Uncertainty and Inertia in Monetary Policy, *Computing in Economics and Finance 2006 150*, Society for Computational Economics.
- 26. Dolado, J.J., Gonzalez-Paramo, J.M. and Vinals, J. (1999). A cost-Benefit analysis of Going from Low to Price Stability in Spain, In M. Feldstein, ed., The costs and Benefits of Price stability Chicago, University of Chicago Press.
- 27. Dotsey, M. and Ireland, P. N. (1996). The Welfare Cost of Inflation in General Equilibrium, *Journal of Monetary Economics*, 37(1): 29-47.
- 28. Engle, R.F. & Granger, C.W. (1987). Cointegration and error correction: representation, estimating and testing, *Econometrica*, 55(2):251-276.
- Fang, W-S., Miller, S. M., and Lee, C-S., (2009). Inflation Targeting Evaluation: Short-run Costs and Long-run Irrelevance, *Working Papers No. 2009-14*, Department of Economics, University of Connecticut.



- 30. Feldstein, M. (1999). Capital Income Taxes and the Benefit of Price Stability, In M. Feldstein, ed., The Cost and Benefit of Price stability, Chicago, University of Chicago Press.
- 31. Feldstein, M. (1997). The Costs and Benefits of Going from Low Inflation to Price Stability. In Christina D. Romer and David H. Romer, Eds. Reducing Inflation: Motivation and Strategy. Chicago: University of Chicago Press.
- Feldstein, M., Green, J. and Sheshindki, E. (1978). Inflation and Taxes in a Growing Economy with Debt and Equity Finance, *Journal of Political Economy*, 86(2), Part 2: Research in Taxation. pp. S53-S70.
- 33. Fischer, S. (1981). Towards an Understanding of the Costs of Inflation: II. *Carnegie-Rochester Conference Series on Public Policy*, 15(1): 5-41.
- 34. Fisher, M. and Seater, J. (1993). Long-Run Neutrality and Superneutrality in an ARIMA Framework, *American Economic Review*, 83(3): 402-415.
- 35. Friedman, M. (1969). The optimum quantity of money, In the Optimum Quantity of Money and Other essays. Chicago: Aldine Publishing Company.
- 36. Gillmax, M. (1993). The Welfare Cost of Inflation in a Cash -in -Advance model with Costy Credits, *Journal of Monetary Economics*, 31(1):97-115.
- 37. Goldfeld, S. M. (1976). The case of missing money, *Brookings Papers on Economic Activity*, 7(3): 683-740.
- Gomme, P. (1993). Money and Growth revisited: Measuring the Costs of Inflation in an Endogenous Growth Model, *Journal of Monetary Economics*, 32(1): 51-77.
- Gonçalves, C. E. S., and Carvalho, A. (2008). Inflation Targeting and the Sacrifice Ratio, Revista Brasileira de Economia, 62(2):177-188.



- 40. Griffin, J.N., Jones, R.L., and Cunfer, R. S. (1980). Experimental Procedures for Estimating Depths and Yields of Underwater Explosions Using Recordings of Limited Dynamic Range, *Final Report*, Project T/ 8152/, ENSCO.
- 41. Gupta, R., Kabundi, A., and Modise, M. P. (Forthcoming). Has the SARB Become More Effective Post Inflation Targeting?, *Economic Change and Restructuring*, Forthcoming.
- 42. Gupta, R. and Komen, K. (2008). Time Aggregation and the Contradictions with Causal Relationships: Can Economic Theory Come to the Rescue?, *Studies in Economics and Econometrics*, 33(1):13-24.
- 43. Gupta, R. and Uwilingiye, J. (Forthcoming c). Evaluating the Welfare Cost of Inflation in a Monetary Endogenous Growth General Equilibrium Model: The Case of South Africa, *International Business and Economics Research Journal*, Forthcoming.
- 44. Gupta, R. and Uwilingiye, J. (Forthcoming b). Comparing South African Inflation Volatility across Monetary Policy Regimes: An Application of Saphe Cracking, *Journal of Developing Areas*, Forthcoming.
- 45. Gupta, R. and Uwilingiye, J. (Forthcoming a). Some benefits of Reducing Inflation in South Africa, *International Business and Economics Research Journal*, Forthcoming.
- 46. Gupta, R. and Uwilingiye, J. (2010). Dynamic Time Inconsistency and the SARB, *South African Journal of Economics*, 78(1):76-88.
- 47. Gupta, R. and Uwilingiye, J. (2009b). Time Aggregation, Long-Run Money Demand and Welfare Cost of Inflation, *Studies in Economics and Econometrics*, 33(3):95-109.
- 48. Gupta, R. and Uwilingiye, J. (2009a). Measuring welfare cost of inflation in South Africa: A Reconsideration, *South African Journal of Economic and Management Sciences*, 12(2):137-146.
- 49. Gupta, R. and Uwilingiye, J. (2008). Measuring the Welfare cost of Inflation in South Africa, South African Journal of Economics, 76(1):16-25.
- 50. Hendy, D.F. and Juselius, K. (2000). Explaining Cointegration Analysis: Part II, *Department of Economics*, University of Copenhagen, Denmark.



- Ireland, P.N. (2009). On welfare cost of inflation and the Recent Behaviour of Money Demand, *American Economic Review*, 99(3): 1040-1052.
- 52. Ireland, P. N (1999). Does the Time-consistency Problem Explain the Behaviour of Inflation in the United States, *Journal of Monetary Economics*, 44: 279-291.
- 53. Johansen, S. (1995). Likelihood-based Inference in Cointegrated Vector Autoregressive Models, Oxford: Oxford University Press
- 54. Johansen, S. (1991). Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models, *Econometrica*, 59(6): 1551–1580.
- 55. Johansen, S. (1988). Statistical Analysis of Cointegration Vectors, Journal of Economic Dynamics and Control, 12(2-3): 231-254.
- 56. Johnson, D.R. (2002). The effect of inflation targeting on the behavior of expected inflation: evidence from an 11 country panel, *Journal of Monetary Economics*, 49(8):1521-1538.
- 57. Jones, L.E. and Manuelli, R.E. (1993). Growth and the effects of inflation, *Manuscript* Kellogg School of Management, Northwestern University, Evanston, IL.
- 58. Kolmogorov, A. N. (1939). Sur l'interpolation et extrapolation des suites stationnaires, Compte Rendus Hebdomadaires des Séances de l'Académie des Sciences, 208 :2043-2045.
- 59. Liu, D. G. and Gupta, R. (2007). A Small-Scale DSGE Model for Forecasting the South African Economy, *South African Journal of Economics*, 75(2): 179-193.
- 60. Lucas, R. E. Jr. (2000). Inflation and Welfare, Econometrica, 68(62):247-74.
- 61. Lucas, R. E. Jr. (1981). Discussion of: Stanley Fischer, Towards an Understanding of the Costs of Inflation II, *Carnegie-Rochester Conference Series on Public Policy*, 15(1): 43-52.
- 62. Lucas, R.E. jr. and Stockey, N. L. (1983). Optimal Fiscal and Monetary Policy in an Economy Without capital, *Journal of Monetary economics*, 12(1): 55-93.



- 63. Ludi, K. L. and Ground, M. (2006). Investigating the Bank-Lending channel in South Africa: A VAR Approach, *Working paper No. 2006-04*.Department of Economics, University of Pretoria.
- 64. Marcellino, M. (1999). Some consequences of temporal Aggregation in Empirical Analysis, *Journal of Business and Economic Statistics*, 17(1):129-136.
- 65. Marquis, M.H. and Reffett, K. L. (1994). New technology spillovers into the payment system, *Economic Journal*, 104(426): 1123-1138.
- 66. Meltzer, A.H. 1963. The demand for money: the evidence from the Time series, *Journal of Political Economy*, 71(3):219-246.
- Mincer, J. and Zarnowitz, V. (1969). The evaluation of economic Forecasts. In J. Mincer (ed.), Economic Forecasts and Expectations. New York. NBER
- Mishkin, F.S. (2003). A Comment on "Inflation Targeting in Emerging Market Economies, by Arminio Fraga, Ilan Goldfajn and Andre Minella, NBER Macro Annual, 403-413.
- 69. Mishkin, F.S. and Schmidt-Hebbel, K. (2007). Does Inflation Targeting Make Difference?, *NBER Working Paper No. W12876*, Oxford: Oxford University Press.
- 70. Mishkin, F.S. and Schmidt-Hebbel, K. (2002). One decade of inflation targeting in the world: What do we know and what do we need to know?, *Inflation targeting: Design, Performance, Challenges*, edited by N. Loayza and R. Soto, 2002, pp. 171-219. Central Bank of Chile.
- 71. Naraidoo, R., and Gupta, R. (2009). "Modelling Monetary Policy in South Africa: Focus on Inflation Targeting Era Using a Simple Learning Rule", *Working paper No 200904*. Department of Economics, University of Pretoria.
- 72. Neuman, M.J.M., and Von Hagen, J. (2002) Does Inflation Target Matter?, *Review Federal Reserve Bank of St. Louis*, 84(4):127-148.



- 73. Newey, W.K. and West, K.D. (1987). A simple, positive-semi definite, heteroskedasticity and autocorrelation consistent covariance matrix, *Econometrica*, 55(3): 703-708.
- 74. O'Reilly, B. and Levac, M. (2000). Inflation and the Tax System in Canada: An Exploratory Partial Equilibrium Analysis, *Bank of Canada Working Paper 2000-18*.
- 75. Ortiz, A and Sturzenegger, F (2007). Estimating SARB's Policy Reaction Rule, *South African Journal of Economics*, 75(4): 659-680.
- 76. Petursson, T.G. (2004). The effects of inflation targeting on macroeconomic performance, *Working paper 23*, Central Bank of Iceland.
- 77. Phelps, E.S. (1973). Inflation theory of public finance, *Swedish journal of Economics*, 75:67-82.
- 78. Phillips, P. C. B. and Ouliaris, S. (1990). Asymptotic Properties of Residual Based Tests for Cointegration, *Econometrica*, 58(1):165-193.
- Poisson, S.D. (1823). Mémoire Sur la distribution de la chaleur dans les corps solides, Journal de l'Ecole Polytechnique, 19:249-403.
- 80. Prescott, E.C. (1987). A multiple means of payment model, In W.A. Barnett and K.J, Singleton, eds., *New approaches to monetary economics*. Cambridge. Cambridge University Press.
- Romer, P. M. (1986). Increasing Returns and Long –run Growth, Journal of Political Economy, 94(5):1002-1037.
- Schreft, S.L. (1992). Welfare- Improving Credit Controls, Journal of Monetary Economics, 30(1):57-72.
- Schwarz, H.A. (1872). Zur Integration de partiellen Differentialgleichung, Journal fur die Reine und Angewandte Mathematik, 74: 218-254.



- Serletis, A. and Virk, J. J. (2006). Monetary Aggregation, Inflation and Welfare, *Applied Financial Economics*, 16(7): 499-512.
- 85. Serletis, A. and Yavari, K. (2007). On the welfare cost of inflation in Europe. *Applied Economics Letters*, 14(2): 111–113.
- 86. Serletis, A. and Yavari, K. (2005). The Welfare Cost of Inflation in Italy, *Applied Economics Letters*, 12(3): 165–168.
- Serletis, A. and Yavari, K. (2004). The welfare cost of inflation in Canada and the United States, *Economics Letters*, 84(2): 199–204.
- Sichei, M. M (2005). Bank-Lending Channel in South Africa: Bank-Level Dynamic Panel Data Analysis, *Working Paper No 2005-10*, Department of Economics, University of Pretoria.
- 89. Sidrauski, M. (1967). Rational choice and patterns of growth in a monetary economy, American Economic Review, 57(2):534-544.
- 90. Szëgo, G. (1915). Ein Grenzwertsatz uber die Toeplitzschen Determinanten einer reellen positiven Funktion, *Mathematische Annalen*, 76: 490-503.
- 91. Tödter, K-H. and Ziebarth, G. (1999). Price Stability versus Low Inflation in Germany: An Analysis of Costs and Benefits, in M. Feldstein, ed., The Cost and Benefit of Price stability Chicago, University of Chicago Press.
- 92. Truman, E.M. (2003). Inflation Targeting in the World Economy. Washington, DC: Institute for International Economics.
- Tunali, D. (2008). Inflation Targeting and The sacrifice Ratio: The effect of Monetary Policy Design, Rutgers University.



- 94. Vega, M., and Winkelfried, D. (2005). Inflation Targeting and Inflation Behavior: A Successful Story?, *International Journal of Central Banking*, 1(3): 153-175.
- 95. Walsh, C.E. (2003). *Monetary Theory and Policy*, second Edition. Cambridge, Massachusetts, London, England: The MIT press.
- 96. Wang, P. and Yip, C.K. (1993). Real effects of money and welfare costs of inflation in an endogenously growing economy with transactions costs, *Research Paper 9311*. Research department, Federal Reserve Bank of Dallas.
- 97. Woglom, G. (2005). Forecasting South African Inflation, South African Journal of Economics, 73(2): 302-320.
- 98. Wu, Y. Zhang, J. (1998). Endogenous growth and the welfare cost of inflation: a reconsideration, *Journal of Economic Dynamics and Control*, 22(3):265-482.