Chapter 5

5 Some Benefits of Reducing Inflation in South Africa*

5.1 Introduction

South Africa moved to an inflation targeting framework in the February of 2000. Ever since the sole objective of the South African Reserve Bank (SARB) has been to ensure that inflation lies within the target band of 3-6%. In this regard, the measurement of costs and benefits of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink the band in terms of the welfare cost of inflation at least. Among the costs, those that are caused by the interaction of inflation with tax rules needs to be emphasized. Due to the non-indexation of the South African tax system, inflation exacerbates the inefficiencies generated by taxation. The quantitative significance of these efficiencies is likely to be particularly strong in case of taxation of capital – a mobile factor of production. Building on the methodological foundation of Feldstein's (1997, 1999) approaches, this paper examines the welfare implications of the interaction between capital income taxation and inflation. We consider the per-year welfare effects of going from 2 percent[37] inflation to price stability, and compare them with the output costs of disinflation.

Based on the estimate of interest elasticity of money for South Africa, obtained in chapter 4, a two percent inflation rate would translate into a welfare loss of 0.098 percent of Gross Domestic Product (GDP) using Bailey's (1956) consumer surplus approach.[38] However, it must be realized that welfare cost calculations obtained by integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue to deduce the

[37] Note the calculations are symmetric and, approximately, linear. Therefore, it is not difficult to translate the estimates, obtained under an inflation rate of 2 percent, for the width of the target band. The decision to use a 2 percent rate of inflation is dictated by the approach in the original contributions of Feldstein (1997, 1999), and the literature that followed thereof.

[38] See chapter 4 for further details.

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deadweight loss, is merely one-dimensional. This is because, the consumer surplus approach fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects on the intertemporal consumption choice (i.e., saving for old age), housing and the real cost of servicing government debt. Thus, the welfare costs obtained using the money demand approach is likely to provide the lower limit of such estimates, and, hence, a more general approach, like the one adopted here, is desired to obtain the "true" size of the welfare loss caused by inflation. It must, however, be stressed that, recent evidence[39] on the sacrifice ratio of South Africa tends to suggest that disinflation can be achieved at virtually no loss to employment and output. Given this, and Feldstein's (1997, 1999) arguments that costs of disinflation are temporary while the benefits are permanent, i.e., one needs to compare the discounted stream of benefits with one-off output costs, even a small-sized estimated benefit could imply relatively large overall gain from a permanent disinflation of 2%.To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy using Feldstein's (1997, 1999) approaches that accounts for interactions between the tax system and inflation.[40]

The remainder of the chapter is organized as follows: Section 5.2 presents the theoretical background of the analysis, while, Section 5.3 calculates distortion to rates of return and hence the price of retirement consumption, resulting from inflation. Section 5.4 examines the distortion in housing demand, Section 5.5 looks at money demand distortion, and Section 5.6 considers the distortionary effect on government debt servicing. Section 5.7 brings together the several effects of reduced inflation identified in Sections 5.2 through 5.6 and Section 5.8 concludes.

5.2 Theoretical Background

At the moment, most tax systems around the world are not completely indexed to ensure that the price-level changes leave real tax rates and real tax revenue unchanged.

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[39] See for example Akinboade et al. (2004), Woglom (2005), Gonçalves and Carvalho (2008), and Tunali (2008).

[40] For applications of Feldstein's (1997, 1999) approaches on other countries, refer to Bonato (1998) for New Zealand, Bakhshi et al., (1999) for the United Kingdom, Dolado et al., (1999) for Spain, Tödter and Ziebarth (1999) for Germany, O'Reilly and Levac (2000) for Canada and Blaszkiewicz et al., (2003) for Poland and Ukraine.

Inflation-induced distortions generated by the interaction of inflation and the non-indexed tax system have the potential to be much larger than the revenue-related effects on which most of the seigniorage and optimal inflation literature has focused (Walsh, 2003).

One important distortion arises when nominal income and not real interest income is taxed. It must be realized that it is after tax real rates of return that is relevant for individual agents in making saving and portfolio decisions, and if nominal income is subject to a tax rate of τ , the real after –tax return will be

$$r_a = (1-\tau)i - \pi$$
$$= (1-\tau)r - \tau\pi , \qquad (5.1)$$

where $i = r + \pi$ is the nominal return and r is the before-tax real return. Thus for a given pre-tax real return r , the after-tax real return is decreasing in the rate of inflation. Practically speaking, let us consider a two–period overlapping generations model, where individual work and earn income when young and also decides on how much to consume currently and save for their old age. Suppose that savings are invested at the rate of r . Therefore, consumption in old age is related to savings by the following equation:

$$c = s(1+r)^T \qquad (5.2)$$

where T is the length of the period between saving while young and dissaving in the old age. Then price of retirement consumption can be defined as $p = \frac{1}{(1+r)^T}$. (5.3)

Clearly, the relative price of old-age consumption P is affected by both tax system and inflation, since they distort the choice between current consumption and future consumption. Graphically, the scenario is depicted in Figure 5-1.

The figure 5-1 depicts the individual's compensated demand for retirement consumption, labeled as Quantity, as a function of the price of retirement consumption p , denoted as Price, at the time of the decision to save. The different points on the graph represent different scenarios. With combination c_0, p_0 representing consumption decision without tax and inflation, the consumer surplus is A+B+C….+F. Introducing income taxes in an environment of price stability (no inflation) moves the equilibrium point from c_0, p_0 to c_1, p_1 which leads to a lesser retirement consumption at a higher price. Consumer surplus is now reduced to the area: C+E+F and the tax revenues corresponding to that area is B+D. Triangle A, thus, represents the deadweight loss, which, in turn, is the reduction of consumer surplus not compensated by higher tax revenues. When we introduce both taxes and inflation the equilibrium point from c_1, p_1 to c_2, p_2, and again there is a reduction in consumption at high price. The consumer surplus remaining is F and tax revenue is the rectangle D+E. The deadweight loss increases from triangle A to triangle A+B+C.

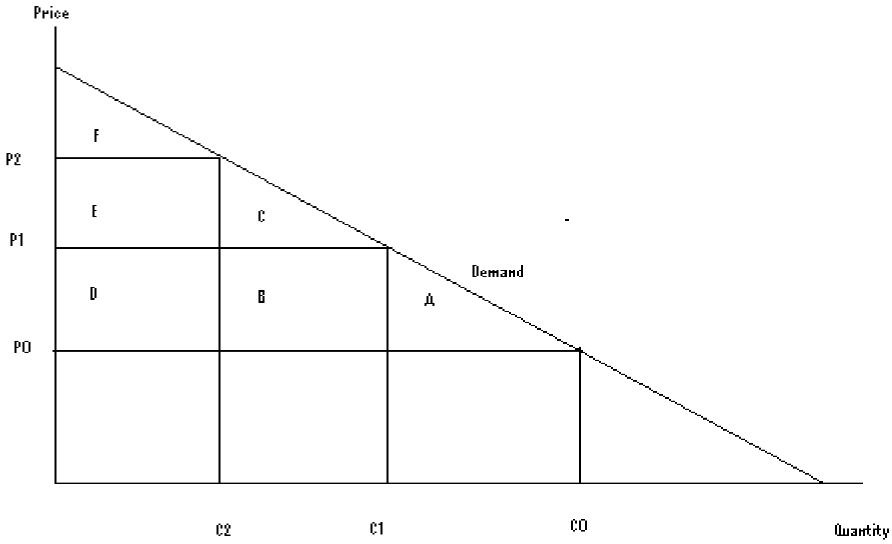

Figure 5-1: Demand for Retirement Consumption.

Thus, moving from equilibrium inflation to price stability increases consumer surplus by the area C+E, the tax revenues change to area B-E(which can be negative or positive) and the welfare gain is B+C which is a trapezoid as shown above. Unlike the traditional welfare analysis where welfare changes were obtained using "Haberger triangles" and, hence, were of second order, i.e., small, in the presence of distortionary taxes, the initial situation is not optimal and welfare changes are of first order, as indicated by a trapezoid rather than triangle.

5.3 Inflation and the Inter-Temporal Allocation of Consumption.

5.3.1 Distortions to Saving Behavior

The household has two main decisions to be make on their expenditure, namely, how much to consume and how much to invest in each period.

Feldstein (1997) derives the welfare gain from reducing inflation in a two period consumption model. Individuals are given an initial endowment and then they decide on the portions of their income to consume and save in the first period in order to consume when they are retired in the second period. The agent's first period savings earns a real rate of return, and in period one, the price of retirement consumption (p) is thought to be inversely related to this rate of return, i.e., the higher the rate on saving, the cheaper the effective price of retirement consumption. The rate of return on saving depends on both inflation and the tax system. According to Feldstein et al., (1978), inflation is a source of irregular change on the effective tax rate of capital income, which leads to changes in real net of post- tax return. Taxes drive a wedge between the pre-tax rate of return which is assumed to be invariant to inflation and the post tax return that household earn. Higher inflation raises the tax wedge and reduces the effective real post-tax return to saving and increases the price of retirement consumption. Given this, the welfare gains associated with reduction in inflation with current tax system can be obtained from the following expression:

$$G_1 = B + C = \left[\left(\frac{p_1 - p_0}{p_2}\right) + 0.5\left(\frac{p_2 - p_1}{p_2}\right)\right]\left(\frac{(p_2 - p_1)}{p_2}\right)S_2\left(1 - \eta_{S_p} - \sigma\right) \tag{5.4}$$

where p_0 is the price of retirement consumption at zero inflation with no distortionary taxes; p_1 is the retirement price evaluated under the current tax regime with price stability (zero inflation) and p_2 is the price evaluated under current tax regime with 2 percent inflation[41]; s_2 represents the initial gross saving of individual when young; η_{s_p} is the uncompensated elasticity of saving with respect to the price of retirement consumption, and; σ is the propensity to save out of exogenous income.

Note, to calculate p_0, we need the pre-tax real rate of return on savings. For this purpose, we use the rate of return on equity which on average was equal to 7.06 percent[42] for the period of 1990-2007. Assuming that at this rate of return, both inflation and taxes are non-existent and the time interval between saving and consumption is 30 years; $p_0 = (1+0.0706)^{(-30)}=0.1292$. In order to calculate the real return to savings in a world of taxes and inflation, we need to adjust the

41 Even though the inflation over the period of 1990-2007 was 7.3 percent on average, we decided to use a figure of 2 percent to make our analysis comparable with the literature that uses Feldstein's (1997, 1999) approaches. Note also, we consider the period of 1990-2007 for our calculations, due to data availability on all the relevant variables over this time period.

42 The return on equity is calculated using the percentage change in the All Share Index (ALSI). Source: International Financial Statistics.

above real rate of return on savings for both corporate and personal sector taxes. The average rate of corporate income tax between 1990-2007 was 25.68 percent[43], which, in turn, leaves a net real return r to 5.25 percent, before personal tax deductions. The net of tax rate of return depends not only on the tax at the corporate level but also on the taxes that individuals pay on interest income, dividends and capital gains. The effective marginal tax rate depends on the form of the income and on the tax status of the individual. Feldstein (1997) summarizes these effects by assuming an uniform individual marginal tax rate across all sources of income. Given this, the individual marginal tax rate in South Africa across all source of income averaged to 25 percent[44] over the period of 1990-2007. This reduces the net return further to 3.94 percent. The price of retirement consumption that correspond to this net return of 3.94 percent is therefore $p_2 =$ $(1.0394)^{(-30)} = 0.3137$ where the subscript 2 on the price indicates the price of retirement at an inflation rate of 2 percent.

Reducing the equilibrium inflation rate from two 2 percent to zero lowers the effective tax rate at both corporate and individuals levels. At the corporate level, this has two opposing effects: First, the changes in the equilibrium inflation rate alter the effective tax rate by changing the value of depreciation allowances, and; second, it changes the value of the deduction of interest payments. Because the depreciation schedule that is allowed for calculating taxable profits is defined on the basis of historical nominal terms, a higher rate of inflation reduces the present value of depreciation and thereby increases the effective tax rate. This relation was approximated by Feldstein (1997) using a rule of thumb of 0.57 percent increases in taxable profit for each percentage point of inflation. Due to lack of this estimate in South Africa, we use the same value as Feldstein (1997). With marginal corporate income tax rate at 30 percent[45], a 2 percent reduction in inflation raises the net of tax return and hence decreases effective tax rate by $0.30(0.57)(0.02) = 0.0034$ or 0.34 percentage points. The interaction of the interest deduction and inflation moves the after tax yield in the opposite direction. If each percentage point of inflation raises the nominal corporate borrowing rate by one percentage point[46], the real pre-tax cost of borrowing is unchanged but companies get an addition deduction in calculating their

43 Source: McGregory BFA.

44 The value corresponds to the average of marginal individual tax rate and capital gains tax for individuals (Source: Tax Pocket Guide 2006/7).

45 Source: The value corresponds to the average marginal corporate tax rate (Source: World Bank, World development Indicators).

46 See footnote 24 in Feldstein (1997) for further details.

taxable income. With debt to capital ratio of 59 percent[47] and a corporate tax rate of 30 percent, a 2 percent decline in inflation raises the effective tax rate by 0.30(0.59)(0.02)=0.0035 or 0.35 percentage points. The difference of the two effects at corporate level is almost insignificant.

Beside the impact of inflation at corporate level, the lower inflation rate affects the taxes at the individual level as well. As individual income taxes are levied on nominal interest payments and nominal capital gains, a reduction in the rate of inflation further reduces the effective tax rate and raises the real after-tax of return. The part of this relation that is associated with the taxation of nominal interest at the level of the individual can be approximated in a way that mirrors the effect at the corporate level. If the nominal interest rate increases by one percentage point for every percentage point of inflation, the individual investors' real pretax return on debt is unchanged, but the after tax return falls, and is given by the product of the statutory marginal tax rate and the change in inflation. Assuming the same 59 percent debt share at the individual level, as assumed for the corporate capital stock, and 25 percent average individual marginal tax rate, a 2 percent decline in inflation lowers the effective tax rate by 0.25(0.59)(0.02)=0.003 or 0.3 percentage points.

Next, we consider the effect of inflation on capital gains excluding dividend, as the individual dividend return on capital ownership is unaffected by inflation except at the corporate level. A higher rate of inflation increases the taxation of capital gains. Even though the effective tax rate on capital gains are taxed at the same rate as other investment income, the effective tax rate is lower because the tax is only levied on realization of the gains. Given effective tax rate of 10 percent[48] on nominal capital gains in South Africa, in equilibrium, each percentage point increase in the price level raises the nominal value of the capital stock by one percentage point. Since the nominal value of the liabilities remains unchanged, the nominal value of the equity rises by 1/(1-b) percentage points, where b is the debt to capital ratio. With b=0.59 and an effective tax on nominal capital gain of 10 percent, i.e., $\theta_g = 0.1$, a 2 percent decline in the rate of inflation raises the real after tax rate of return on equity by $\theta_g \left[1/(1-b)\right] d\pi$ =0.0049 or 0.49 percentage points. However, since equity is assumed to represents 35 percent of the individuals 'portfolio'[49], the lower effective capital gains tax raises the overall rate of return by only 35 percent of this 0.49 percentage points or 0.17 percentage points. Combining the debt and capital effects implies that

47 Source: McGregory BFA. The value of debt to capital ratio is obtained by dividing total liabilities with total assets using balance sheets of all companies between 1990-2007.

48 Source: Tax Pocket Guide 2006/7.

49 Source: Financial Services Board.

reducing the inflation rate by 2 percentage points reduce the effective tax rate at the individual investor level by the equivalent of 0.47 percentage points, with the real net return to the individual saver is 4.41 percent. This implies that price of retirement consumption is: p_1=0.2740. Substituting these values for the price of retirement consumption into equation (4) yields:

$$G_1 = 0.066\, S_2 \left(1 - \eta_{sp} - \sigma\right) \tag{5.5}$$

5.3.2 The Saving Rate and the Saving Behavior

The value of S_2 in equation 5.5 represents the saving during pre-retirement years at the existing inflation. To evaluate (5.5), we need an estimate of the saving of the young at an inflation rate of 2 percent (S_2). Feldstein (1997) derives an estimate from the steady-state relationship between savers and dis-savers implied by the two–period model. He shows that the saving of the young is $\left(1+\eta+g\right)^T$ times the saving of the older generation, where n is the rate of population growth and g is the growth in per capita wages. Thus net personal saving (S_N) is related to S_2 according to:

$$S_N = S_2 - (1+n+g)^{-T} S_2 \tag{5.6}$$

Real average wage growth in South Africa over 1990-2007 was 3.73 percent, while, population growth was 1.71 percent. On the other hand, average private saving rate over the same period was 5.4 percent of GDP. Based on these numbers, we have: n+g=0.0218 and with T=30, implies S_2 =2.1 S_N. Further, using private saving to be 5.4 percent of GDP, results in S_2 =0.11GDP. Further, the average share of wage in GDP between 1990 and 2007 was equal to 48 percent. Then, the propensity to save out of exogenous income is: $\sigma = S_2(\alpha * GDP)$ where α is the share of wage in GDP. With α =0.48, $\sigma = 0.23$.

The final term to be evaluated in order to calculate the welfare gain described in equation 5.5 is the elasticity of saving with respect to real interest rate, since the uncompensated elasticity of savings with respect to the price of retirement consumption is related to elasticity with respect to the real rate of return as: $\eta_{sp} = \frac{-\left(1+r)\eta_{sr}\right)}{rT}$. Following Dolado et al. (1998) and Balshki et al.

(1998), we assume that $\eta_{s_r} = 0.2$.[50] As in Feldstein (1997), we also assume a value of $\eta_{s_r} = 0$ to assess the sensitivity of this estimate to the value of η_{s_r}.

Given this, the annual gain from reduced distortion of consumption is G_1=0.0069 GDP or 0.69 percent of GDP when $\eta_{s_r} = 0.2$, and for $\eta_{s_r} = 0$, we have G_1=0.0056GDP or 0.56 percent of GDP. These calculations suggest that the traditional welfare effect on the timing of consumption of a reduction in inflation rate by 2 percent is bound between 0.56 percentage points of GDP to 0.69 percentage point of GDP.

5.3.3 Indirect Revenue Effects

Next we consider the effect on government revenue of the above experiment. The working assumption here, as in the Feldstein (1997), is that any reduction on government revenue due to a move from 2 percent inflation to price stability cannot be made good by a rise in lump-sum taxes. Instead, distortionary taxes are required to fill in the financial gap, with obvious corresponding welfare implications.

Assume that we start from a situation where the price of retirement income is p_2 and consumption level is c_2 (see figure 5-1), with inflation at 2 % and the current tax system in place. Now consider lowering the inflation rate to zero. There are two offsetting effects on revenue. First, lower inflation raises the real return to saving and hence lowers the price of retirement to p_1. This results in a loss of revenue equal to $(p_2 - p_1)c_2$. Against this, the lower the price of retirement consumption stimulates higher consumption by $(c_1 - c_2)$, which in turn generates revenue by the amount of $(p_1 - p_0)\times(c_1 - c_2)$. The change in revenue can thus be captured by:

$$dREV = S_2\left\{\left[\frac{p1-p0}{p2}\right]\left[\frac{p2-p1}{p2}\right](1-\eta_{sp}-\sigma)-\left[\frac{p2-p1}{p2}\right]\right\} \tag{5.7}$$

[50] The decision to use a value of 0.20 for η_{s_r}, which is, in general, the lower bond of this estimate available in the literature, is in line with the observation of low interest sensitivity of savings in South Africa.

This expression can in principle be either positive or negative. But in our case, substituting the earlier estimates for values for p_0, p_1, p_2 . We get net revenue loss of dRE = -0.0079GDP or -0.79% of GDP for η_{s_r} =0.2 and -0.009GDP or -0.9 % of GDP for η_{s_r} =0.Assuming that λ represents the deadweight loss when each rand of revenue that needs to be raised from other taxes due to loss in revenue, the net loss in revenue of shifting from two percent inflation to price stability is 0.36 percent and 0.32 percent of GDP under η_{sr} =0 and 0.2 respectively. Note, following Feldstein (1997) λ is set at 0.4. Overall, net welfare gain (NG_1) from reducing inflation by 2 percent is then given by: $NG_1 = G_1 + \lambda d\,\mathrm{Re}v$. For λ =0.4 the net welfare gains are respectively, equal to 0.37 with η_{sr} =0.2 and 0.20 with η_{sr} =0. For λ =1.5 the welfare gains are forfeited for both η_{sr} =0 and η_{sr} =0.2.

5.4 The Gain from Reducing Distortion in Housing Demand

In some countries, owner-occupied housing is generally given special treatment on individual income taxation in order to encourage investment in housing and therefore stimulate economic growth. The benefit of owner occupied housing is that the mortgage interest payments and property tax rates are tax deductible. This is not the case for South Africa, since such deductions are not applicable. According to (Bonato, 1998), when mortgage interest payment is not tax deductable, inflation affects demand for housing only indirectly. This leads to reduction in the return on alternative assets. The state of price stability reduces this distortion as well as the loss of tax revenue by moving capital from housing to the business sector.

Given this, welfare effect of inflation on housing demand can be graphically represented as follows:

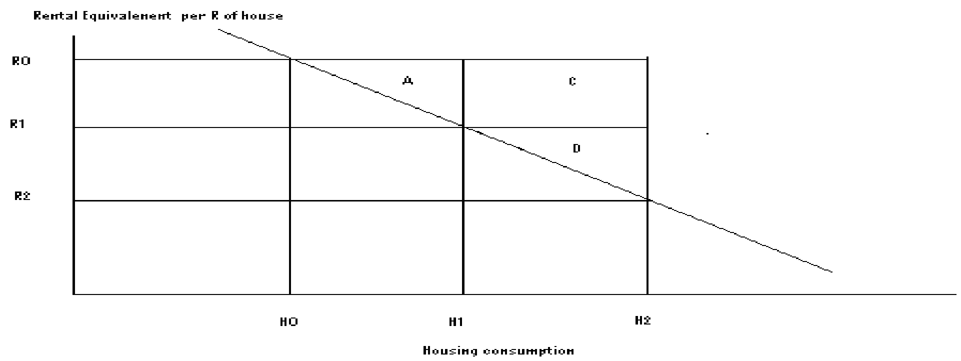

Figure 5-2: Distortion in Housing Demand

Figure 5-2 shows the compensated demand for housing services. The horizontal line at R_0 represents the undistorted cost of housing –the 'true' supply curve. The dead weight loss due to taxation is represented by triangle A, while the deadweight loss due to inflation is represented by the area of the trapezoid C+D. The reduction in the deadweight loss that results from reducing the distortion to housing demand when the inflation decline from two percent to zero is:

$$G_2 = \left[(R_0 - R_1) + 0.5(R_1 - R_2) \right](H_2 - H_1)$$

where $(H_2 - H_1) = (dH/dR)(R_2 - R_1)$

$$= (dH/dR)(R_2/H_2)(H_2/R_2)(R_2 - R_1)$$

$$= \varepsilon_{HR} H_2 \frac{(R_2 - R_1)}{R_2}$$

Then,

$$G_2 = \varepsilon_{HR} R_2 H_2 \left\{ \left[\frac{(R_0 - R_1)}{R_2} \frac{(R_1 - R_2)}{R_2} \right] + 0.5 \left[\frac{(R_1 - R_2)}{R_2} \right]^2 \right\} \qquad (5.8)$$

where ε_{HR} is the compensated elasticity of housing demand with respect to the rental rate.

H_2 is the demand for owner-occupied housing and R represents the rental cost of housing per rand of housing capital. In many countries, effective subsidies to housing demand arising from the combination of inflation and tax system reduces the implied rental cost of housing and leads to overconsumption of housing (H_2), compared to situation of no taxes and no inflation (H_0).

In the absence of tax and inflation, the implicit rental cost is equal to

$$R_0 = r_0 + m + \delta \qquad (5.9)$$

Where r_0 is the return on real rate of return on equity, m is the cost of maintenance per rand of housing capital and δ is the rate of depreciation. With r_0 =0.0706, m =0.074[51] and $\delta = 0.05$[52] implies R_0 =0.1946.

With the current tax regime and inflation, the revised implicit rental cost is

$$R_2 = \mu(r_m + \pi) + (1-\mu)(r_n + \pi) + \tau_p + m + \delta - \pi \qquad (5.10)$$

where μ is the loan to value ratio, r_m is the real mortgage interest rate, r_n is the rate of return on equity with taxes and 2 percent inflation rate, and τ_p is the property tax rate. With μ equal to 0.7[53], $r_m + \pi$ equal to 0.097[54] and τ_p equal to 0.002[55]. R_2 =0.1913. The combination of tax and two percent inflation reduces the rental cost from 19.46 cents per rand of housing capital to 19.13 cents per rand of housing capital.

Next we look at the effect of a decrease in the rate of inflation on this implicit rental cost of owner occupied housing:

$$dR_2 = \mu * dr_m / d\pi + (1-\mu) d(r_n + \pi) / d\pi - 1.$$

With r_1 =0.0441 at $\pi = 0$ and r_n =0.0394 at π =0.02, $dr_n / d\pi$ =-0.235 and $d(r_n + \pi)/d\pi$ =0.765. Therefore,

$$dR_2 / d\pi = \mu + 0.765(1-\mu) - 1. \qquad (5.11)$$

$$=-0.0705.$$

Since R_2 =0.1913 at two percent inflation, this implies R_1 =0.1927 at zero inflation.

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51 Source: Statistics South Africa.

52 Source: National Department of Housing, South Africa.

53 Source: Standard Bank of South Africa Limited.

54 Source: SARB.

55 Source: SARB.

We now go on and calculate G_2. Due to lack data on housing stock and rental rate in South Africa we use an elasticity of 0.3 as in Bonato (1998).[56] With H_2 equal to 2.08, which is gross fixed capital formation for residential building as percentage of GDP[57], the welfare gain is equal to 0.001 percent of GDP.

5.4.1 Indirect Revenue Effects

In the case of owner occupied housing, zero inflation would result in an increase in tax revenues. Shifting capital from owner occupied housing to business capital will lead to additional revenue equal to

$$d\,\mathrm{Re}v_1 = \varepsilon_{HR} \frac{R_1 - R_2}{R_2} H_2 \left(r_0 - r_1 \right) \tag{5.12}$$

=0.012 percent of GDP

Secondly, this increase in tax revenue is partly offset by a loss in the revenue from property taxes due to reduction of the housing stock. This loss can be estimated form

$$d\,\mathrm{Re}v_2 = \varepsilon_{HR} \frac{R_1 - R_2}{R_2} H_2 \tau_p \tag{5.13}$$

=0.0009

Recalling that λ =0.4 represents the deadweight loss when each rand of revenue that needs to be raised from other taxes due to loss in revenue, the overall effect on tax revenue is about 0.004 percent of GDP, and, hence, the welfare gain from reducing distortion in housing is equal to 0.005 percent of GDP.

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[56] Given the inelastic rental market in South Africa, we believe that the choice of this value is a reasonable one.

[57] Source: SARB.

5.5 Seigniorage and Distortion of Money Demand

5.5.1 Money Demand

An increase in inflation raises the cost of holding non-interest bearing money balances and therefore reduces the demand for such balances below the optimal level. It is this resulting deadweight loss of inflation that has been the primary focus of the literature on the welfare effects of inflation, since Bailey's (1956) pioneering paper.

Assuming that the initial situation is characterized with inflation π_2 and a positive nominal interest rate $i_{n2} = r_{n2} + \pi_2$, reducing inflation entails a welfare gain. Graphically, this can be depicted as follows:

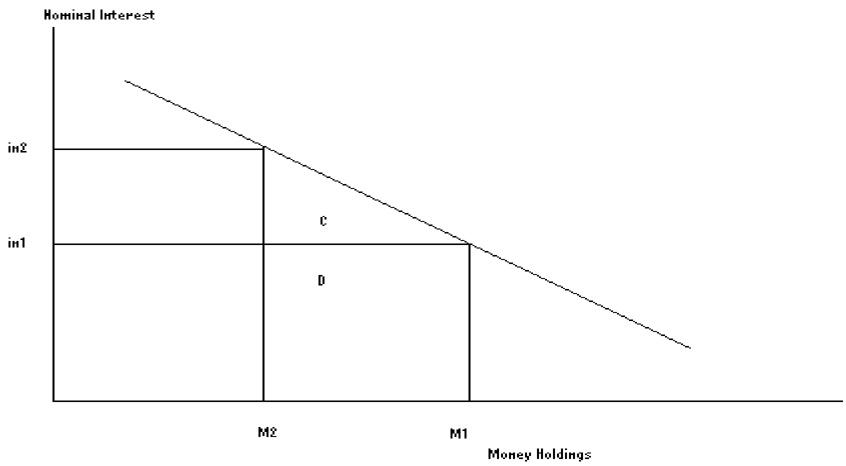

Figure 5-3: Money Market Distortion.

As shown in the Figure 5-3 above, which plots the demand for money as a function of nominal interest rate, a reduction in inflation (from π_2 to π_1) leads to an increase in money demand (from M_1 to M_2) and to a welfare gain presented by the area C plus the area D between the money demand curve and zero opportunity cost line. To compute the welfare gain, it is necessary to estimate the change in nominal interest rates caused by the reduction in inflation and induced increase in money demand $(M_1 - M_2)$. Recall, that a true initial inflation π_2 of 2 percent, the real net of tax return in South Africa is 3.94 percent, this leads to a nominal interest rate i_{n2} of 5.94 percent.

With $dr / d\pi$ =-0.235

$$G_3 = i_{nl}(M_1 - M_2) + \frac{1}{2}\left(i_{n2} - i_{nl}\right)\left(M_1 - M_2\right) \tag{5.14}$$

$$= 0.0441\left(M_1 - M_2\right) + 0.5\left(0.0594 - 0.0441\right)\left(M_1 - M_2\right)$$

$$= 0.05175(M_1 - M_2)$$

$$= -0.05175\varepsilon_M M \frac{1}{r_n + \pi} 0.0153$$

$$= 0.00079\varepsilon_M \frac{M}{GDP}\left(r_n + \pi\right)^{-1} GDP$$

Since the demand deposit component of M1 is now generally interest bearing, non-interest-bearing money in South Africa is represented by M1A. Between 1990 and 2007 the ratio of currency in circulation to GDP was 15.3 percent. Thus, M=0.153GDP. Using Meltzer's (1963) log-log money demand specification, we obtain an elasticity of money demand equal to 0.21, based on the Fischer and Seater (1993) long-horizon approach. Given this:

G_3=0.00079*0.21*0.153*(1/0.0594) GDP =0.00043GDP or 0.043 percent of GDP.

5.5.2 The Revenue Effects of Reduced Money Demand

The reduction in inflation affects government revenue in three ways. First, the reduction in the inflation tax on money balances results in a loss of Seignorage and therefore an associated welfare loss of raising revenue by other distortionary taxes (Phelps, 1973). In equilibrium, inflation at rate π implies revenue equal to πM. Increases in inflation raise the seignorage revenue by:

$$dSeignorage / d\pi = M + \pi\left(dM / d\pi\right) \tag{5.15}$$

$$= M / GDP\left\{1 - \varepsilon_M\left[d\left(r_n + \pi\right) / d\pi\right]\left(\pi / r_n + \pi\right)\right\} GDP$$

M=0.153 GDP, ε_M=0.21, $d\left(r_n + \pi\right) / d\pi$=0.765, $\pi = 0.02$ and $r_n + \pi$=0.0594

A decrease of inflation from $\pi = 0.02$ to $\pi = 0$ leads to a loss of seignorage by 0.0029GDP.

The corresponding welfare loss is 0.29 $\times \lambda$ percent of GDP. With λ=0.4, the welfare cost of lost seignorage is 0.116 percent of GDP. Clearly the benefit of reducing inflation via an increase in money demand is outweighed by the loss of revenue from seigniorage. Specifically, a reduction of inflation by 2 percent would imply a welfare loss of 0.116 percent of GDP which is obviously way lesser than the 0.098 percent gain that could be obtained using the estimation and calculations in chapter 4.

The second revenue effect is the revenue loss that results from shifting capital to money balances from other productive assets. The decrease in business capital is equal to the increase in the money stock, $M_1 - M_2 = \left[dM / d\left(r_n + \pi\right)\right](0.0153) = 0.0153\ \varepsilon_M M\left(r_n + \pi\right)^{-1} = 0.83$ (5.16) percent of GDP. When these assets are invested in business capital, they earn a real pretax return of 7.06 percent but a net of tax return of only 4.41. The difference is the corporate and personal tax payments of 2.65 percent. Applying this to the increment in capital of 0.83 percent of GDP implies a revenue loss of 0.0265×0.83=0.022 percent of GDP. The welfare gain from this revenue loss is 0.022 λ percent of GDP. Again with λ =0.4, the welfare loss from this source is 0.009 percent of GDP.

The final revenue effect of the change in the demand for money is the result of the government's ability to substitute the increases in money balance of $M_1 - M_2$ for interest bearing government debt. Although this a one time substitution, it reduces government debt service permanently by:

$$r_{ng}\left(M1 - M2\right) \tag{5.17}$$

where r_{ng} is the real interest rate paid by the government on its outstanding debt net of the tax that it collects on those payments, given by: r_{ng} =$0.75\left(0.153\right) - 0.075 = 0.04$.

The reduced debt service cost is: $0.04\left(M_1 - M_2\right)$ =0.00033 or 0.033 percent of GDP. For λ =0.4, the corresponding welfare gain is equal to 0.013 percent of GDP.

Combining all three effects, we have total revenue losses equal to 0.112 when $\lambda = 0.4$. The net welfare loss due to decrease revenue is equal to 0.07. Note Phelps' (1973) revenue effects are bigger than Bailey's (1956) money demand effect, which, in turn, means that the welfare loss from reduced seignorage revenue is bigger than the welfare gain from the reduced distortion of money demand following a move from 2 percent inflation to price stability.

5.6 Debt Service and the Government Budget Constraint

Finally, we analyze the effect of the higher real cost of servicing the national debt following a reduction in the inflation rate. With inflation, the nominal interest payments are taxed; therefore, lower inflation reduces the nominal interest rate on government debt and reduces the real value of taxes on interest payment to individuals. A lower inflation hence leaves real pre-tax interest rate unchanged which leads to no change on pre-tax cost of debt service, but reduces the tax revenue on the government debt payments which in turn leads to higher level of other distortionary taxes.

Assuming a constant debt to GDP ratio, the increases in the real value of interest payments is equal to the product of the change in inflation times the marginal tax rate on interest payment, θ_i (which we assume to equal to average marginal individual tax rate), times the ratio of debt, B, to GDP as shown in Feldstein (1997) . Hence, the welfare effect of the change in taxes required to offset the change in real government revenue is[58]:

$$d\mathrm{R}EV_4 = -d\pi \times \theta_i \times B / GDP \qquad (5.18)$$

$$= -0.02 \times 0.25 \times 0.41 = 0.00205 \text{ or } 0.21 \text{ percent of GDP.}$$

The reduction of inflation by 2 percentage point will reduce the welfare by 0.21 λ .
With $\lambda = 0.4$, the net welfare revenue is -0.08 percent of GDP.

5.7 The Net Effect of Lower Inflation on Economic Welfare

We can now bring together the several effects of reduced inflation that have been identified and evaluated in Sections 5.2 through 5.6 and compare them with the one-time output losses required to achieve the inflation reduction. As can be seen from Table 5-1, adding up all the four distortion, we obtain a welfare gain of 0.225 percent of GDP welfare by moving form an average inflation of 2 percent to price stability. But, when compared to Feldstein's (1997) estimate, our welfare gain is 4 times lesser than what he obtained.[59] This is mainly due to the fact that the gain due to a move from 2 percent inflation to price stability, resulting from the distortions on intertemporal allocation of consumption and the housing market demand is much higher in case

[58] See Feldstein (1997) for details on derivations to obtain equation (5.18).

[59] In fact, in general, barring Poland, estimated at 0.125 percent of GDP, our estimate of welfare gain is less than all the other estimates, obtained using Feldstein's (1997, 1999) approaches, available in the literature. Specifically, the welfare gains following a permanent reduction of inflation by 2 percent was found to be 1.41 percent of GDP in Germany, 0.39 percent of GDP in New Zealand, 1.88 percent of GDP in Spain and 0.316 percent of GDP in Ukraine. The only estimate that comes close to that of ours is that of the United Kingdom, which is measured at 0.21 percent of GDP.

of the USA when compared to South Africa. This, in turn, results from the facts that the tax structure has a smaller distortionary effect on the choice between current and future consumption in emerging economies like South Africa than in the US, and also because with interest payment and property rates not being tax deductable, inflation affects demand for housing only indirectly. But, at the same time what is more important to us is that this measure of welfare loss is bigger than the value of 0.098 percent of GDP that could be obtained using the consumer surplus approach in chapter 4, which merely measures the distortion in the money demand due to positive nominal interest rates.

Table 5-1: Overall welfare Gain of Moving from 2 percent Inflation to Price Stability.

Welfare effect	Welfare gain as % of GDP	
	South Africa	Feldstein (USA)
Inter-temporal	0.37	0.926
Housing Demand	0.005	0.22
Money Market	-0.07	-0.034
Debt servicing	-0.08	-0.100
Total	0.225	1.012

Moreover, once we take into account Feldstein's (1997, 1999) arguments that benefits of inflation are permanent, and, hence, one should obtain its present value from reducing inflation permanently by 2 percent, even the relatively small-sized welfare gain of 0.225 percent of GDP translates into 15 percent of GDP, realizing that the relevant discount rate is (r_a-χ), since benefits grow at the same rate, χ, as GDP. Recall, r_a is the after tax real return on savings and equals to 3.94 percent, while the average growth rate of GDP (χ) over 1990-2007 was 2.44 percent, yielding a discount rate of 66.67. On the other hand, given that the sacrifice ratio for South Africa is 0.017[60] percent of GDP (Tunali, 2008), the one-time cumulative loss of output is 0.034 percent of GDP following a reduction in the inflation rate from two percent to zero. Clearly, the current benefits, not to say the present value of the same, overwhelmingly outweigh the output loss originating from such a disinflationary policy.

[60] Gonçalves and Carvalho (2008) obtain negative numbers for the sacrifice ratio, implying that disinflation can be achieved without any output costs.

5.8 Conclusion

This chapter makes the first attempt to calculate the benefit of moving from low inflation to price stability in South Africa using a micro partial equilibrium framework, developed by Feldstein (1997, 1999). Looking at interaction between inflation and non indexed tax system, our calculations show that the benefits for moving from an inflation of two percent to zero percent is equal to that 0.225 percent of GDP, which is more than twice the size of the estimates that could be obtained following Bailey's (1956) consumer surplus approach, based on the interest elasticity of money demand obtained in chapter 4..

This chapter emphasizes the distortions caused by the interaction of inflation and capital income taxation, in calculating the gain from moving to a zero rate of inflation. Though the annual deadweight loss of a two percent inflation rate is a relatively small number when compared to the literature, since the real gain from shifting to price stability is permanent, the present value is a substantial multiple of the annual welfare gain and is found to be 15% of GDP. Since the corresponding one-off output cost of moving from two percent inflation to price stability is 0.034% of GDP, the gain outweighs the cost by an overwhelming margin. Further, when one realizes that the calculations are symmetric and, approximately, linear, our results make a strong case for rethinking the width and the upper and lower limits of the target band, at least from the point of view of welfare costs of inflation. Clearly the discounted welfare gains will be quite substantial by moving to a narrower and lower target band and would also come at no cost to employment and output.

Dotsey and Ireland (1996) evaluated the welfare cost of inflation in dynamic general equilibrium endogenous and exogenous growth frameworks. By viewing inflation as a tax on micro-level decisions, the authors were able to identify explicitly, and quantify empirically, sizeable welfare costs of inflation at macroeconomic level, indicating that a partial equilibrium approach, like the one used in this chapter, can significantly underestimate the cost of inflation. Given this, there is no denying the fact that one can achieve, possibly, larger gains by reducing the inflation in a dynamic general equilibrium endogenous growth economic structure, and is an important research question for the future to correctly evaluate the inflation targeting regime in South Africa, based on welfare cost estimates.

Chapter 6

6 Evaluating the Welfare Cost of Inflation in a Monetary Endogenous Growth General Equilibrium Model: The Case of South Africa*

6.1 Introduction

The South African Reserve Bank (SARB) has been in pursuit of low inflation for nearly three decades now. Though not quite successful over the decade of 1980, the SARB made significant progress in reducing the inflation rate during the 1990s. Interestingly, the SARB pursued an implicit inflation target during the latter period. However, since the announcement made by the minister of Finance in the February of 2000, the sole objective of the SARB has been to achieve and maintain price stability. In other words, the SARB has now adopted an explicit inflation targeting regime, whereby it aims to keep the CPIX[61] inflation rate within the target band of 3-6%, using discretionary changes in the Repurchase (Repo) rate as its main policy instrument. In this regard, the measurement of the cost of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink of the level and width of the band in terms of the welfare cost of inflation at least.[62]

Given this, the four previous chapters deserves special mentioning. These four studies used alternative econometric methodologies to obtain estimates for the range of the welfare cost of inflation for the target band pursued by the SARB. While, chapter 2 showed that the welfare cost of inflation ranged between 0.34 % and 0.67 % of Gross Domestic Product (GDP), obtained using the Johansen (1991, 1995) cointegration approach to estimate the long-run money demand function, chapter 3 found the corresponding values to decrease markedly to 0.16 percent to 0.36 percent of GDP, when the long-horizon approach proposed by Fisher and Seater (1993) was used to estimate the long-run money demand function on the same data set.

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[61] CPIX is defined as CPI excluding interest rates on mortgage bonds.

[62] For recent studies that have evaluated the South African inflation targeting regime in terms of average levels of inflation, volatility and a wide array of other macroeconomic variables, refer to Burger and Marinkov (2008), Gupta and Uwilingiye (2010, forthcoming b) and Gupta et al. (forthcoming).

* Forthcoming in International Business and Economics Research Journal.

The difference between the results essentially emanated from the smaller sizes of the interest rate elasticity and semi-elasticity obtained under the long-horizon approach relative to the cointegration procedure. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income and that long-run properties of data are unaffected under alternative methods of time aggregation, chapter 4, tested for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Their results indicated that the long-horizon method is more robust to alternative forms of time aggregation, and given this, the welfare cost of inflation in South Africa for the inflation target band of 3-6 % was found to be between 0.15 % and 0.41% of GDP.

However, Feldstein (1997,1999) point out that welfare cost calculations obtained by integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue to deduce the deadweight loss, is merely one-dimensional. This is because, the consumer surplus approach fails to account for the fact that inflation, operating in conjunction with the tax system, has further distortionary effects on the intertemporal consumption choice (i.e., saving for old age), housing and the real cost of servicing government debt. Thus, the welfare costs obtained using the money demand approaches is likely to provide the lower limit of such estimates, and, hence, a more general approach is desired to obtain the "true" size of the welfare loss caused by inflation. As such , chapter 5) uses a microeconomic partial equilibrium approach, as proposed by Feldstein (1997, 1999), found the annual deadweight loss of a two percent inflation rate to be 0.225 percent of GDP. Realizing that the calculations are symmetric and, approximately, linear, an inflation target band of 3-6%would imply the welfare cost to range between 0.34 percent and 0.68 percent of GDP. Interestingly, the figures are nearly the same as those obtained in chapter 2. Feldstein (1997, 1999) argued that costs of disinflation are temporary while the benefits are permanent, i.e., one needs to compare the discounted stream of benefits with one-off output costs Given this, chapter 5 calculated the present value gain to be 13.33 percent of GDP, while, the corresponding one-off output cost of moving from two percent inflation to price stability was found to be 0.034 percent of GDP. Thus, the gain was found to outweigh the cost by an overwhelming margin.

Dotsey and Ireland (1996) evaluated the welfare cost of inflation in dynamic general equilibrium endogenous and exogenous growth frameworks.

By viewing inflation as a tax on micro-level decisions, the authors were able to identify explicitly, and quantify numerically, sizeable welfare costs of inflation at the macroeconomic level, indicating that Feldstein (1997, 1999)-type partial equilibrium approaches, also used in chapter 5, can significantly underestimate the cost of inflation. Given this, it is important that one revisit the welfare cost estimates for South Africa in a dynamic general equilibrium endogenous growth model to ensure that one correctly evaluate the inflation targeting regime, based on welfare cost estimates.

Against this backdrop, this chapter calibrates the general equilibrium endogenous growth model proposed by Dotsey and Ireland (1996) for South Africa using quarterly data over the period of 1965 to 2008, and obtains the welfare cost of inflation. The decision to use the framework proposed by Dotsey and Ireland (1996) over a large number of other general equilibrium models such as Black et al. (1993), Coleman (1993), De Gregorio (1993), Gomme (1993), Jones and Manuelli (1993), Wang and Yip (1993), Marquis and Reffett (1994) and Whu and Zhang (1998) due to the fact that the transactions technology used here generates a money demand function that has an interest-elasticity similar to those estimated with South African data. Consequently, the model is ideally suited for comparing the welfare cost estimates obtained from the traditional partial equilibrium approaches based on money demand estimations to the full general equilibrium cost of inflationary policy. To the best of our knowledge, this is the first attempt to use a dynamic general equilibrium endogenous growth model to obtain the welfare cost of inflation in South Africa. The remainder of the chapter is organized as follows: Section 6.2 presents the general equilibrium endogenous growth model and Section 6.3 discusses the general equilibrium. Section 6.4 outlines the calibration, while Section 6.5 derives the welfare costs of inflation for alternative values of steady-state inflation. Finally, Section 6.6 concludes.

6.2 The General Equilibrium Model

In this chapter, we use Dotsey and Ireland's (1996) general equilibrium model. We start off by describing the economic environment, followed by the problems of the household, financial intermediary and the goods-producing firm.[63]

[63] The description of the general equilibrium model relies heavily on the discussion available in Dotsey and Ireland (1996) and has been presented here to ensure that the paper is self-contained. As such, we also retain the paper's symbolic representation of the equations.

6.2.1 Economic Environment

The model economy consists of a continuum of markets, indexed by $i \in [0,1]$, arranged on the boundary of a circle with a circumference of one. In each market, a distinct, nonstorable consumption good is produced and traded in each period t = 0, 1, 2,…. Thus, the economy's consumption goods are also indexed by $i \in [0,1]$, implying that good i is sold in market i.

Each market i is populated with a large number of identical households, financial intermediaries, and goods-producing firms. We assume that enough symmetry exists amongst the agents' preferences, endowments and technologies to allow us to consider the behavior of a single representative agent corresponding to each type, i.e., households, intermediaries and firms. The representative agents all live at market 0, so that the index i measures the distance of market i form their home.

At the beginning of period t = 0, the government, which has no other role in the economy, supply non-interest bearing fiat money of m_0^s units to households and augments the initial supply with identical lump-sum transfers h_t to all households at the beginning of each period t. Hence, per-household money supply m_{t+1}^s at the end of period t satisfies:

$$m_{t+1}^s = (1 + g_t) m_t^s \tag{6.1}$$

where the rate of money growth g_t is given by

$$g_t = \frac{h_t}{m_t^s} \tag{6.2}$$

Note, the government pre-announces the complete sequence $\{g_t\}_{t=0}^{\infty}$ of money growth rates at the beginning of period t = 0, which leads all agents to have perfect foresight beyond this point.

6.2.2 Household and Trading

The representative household at market i = 0 has preferences over leisure (J_t) and the entire continuum of consumption goods ($c_t(i)$) as described by the utility function:

$$\sum_{t=0}^{\infty} \beta^{t} \left\{ \int_{0}^{1} \ln\left[c_{t}\left(i\right)\right] di + BJ_{t} \right\}, \qquad \beta \in \left(0,1\right),\ B > 0 \tag{6.3}$$

where, β is the discount rate and B is the substitution elasticity between leisure and household consumption.

The representative household is assumed to be made up of two members: a shopper and a worker (Lucas and Stokey; 1983). The representative worker rents out his capital stock k_t at the real rate r_t and supplies l_t^g units of labor at the real wage w_t to goods-producing firms, at each time point t. He also supplies l_t^f units of labor to financial intermediaries. In each period, the fraction of time allocated to each activity sums to one. The worker makes his labor-supply decisions subject to time constraint:

$$1 \leq J_t + l_t^g + l_t^f \tag{6.4}$$

at each date t.

Meanwhile, the representative shopper travels around the circle to obtain goods for his household's consumption. As in Prescott (1987), Schreft (1992b) and Gillman (1993), the shopper choose two ways of making payment for his purchases in each market i. First, he can make use of government-issued fiat money. Assuming perfect competition, the nominal price p_t of consumption goods is the same across markets, the shopper may acquire $c_t\left(i\right)$ units of good i in exchange for $p_t c_t\left(i\right)$ units of money at time t. Second, he could use the services of financial intermediary to purchase good i on credit that will be paid at the end of the t period from his labor and rental incomes. The credit is obtained at a cost of $\gamma(i)$ units of labor. An intermediary verifies the shopper's identity, proof of income and credit record and guarantees his ability to pay, so that the firm in market i is willing to sell its output on credit at time t. The transaction cost to be paid by the shopper to intermediary does not depend on the quantity of purchase but increases with distance, i.e., the farther the shopper travels from his residence, the more is the cost that he has to incur. Hence, γ is strictly increasing function of i. Under the additional assumption that $\lim_{i \to 1} \gamma(i) = \infty$, there will always be some cash-good, implying a well-defined money demand function.

The intermediary in market i charge the representative household the real price $q_t(i)$ in exchange for its services at time t. Since the intermediary's cost $\gamma(i)$ is independent of the quantity of the transaction but depends on i, competition implies that the representative shopper may acquire

$c_t(i)$ units of good i on credit at time t at a total nominal cost of $p_t\left[c_t(i)+q_t(i)\right]$, where $p_t c_t(i)$ is the amount that the shopper has to pay for goods themselves (which is the same whether cash or credit is used to purchases the goods), and $p_t q_t(i)$ is the amount required to compensate the intermediary.

Let the indicator function $\xi_t(i)=0$ if the representative shopper purchases good i with money at time t, and let $\xi_t(i)=1$ if he uses the services of an intermediary instead. To purchase good with cash in period t, the shopper need to have nominal money balance of m_t, which is augmented at the beginning of the period by the government transfer h_t. Since the shopper must use money whenever he chooses not to hire an intermediary, he faces the following cash-in -advance constraint:

$$\frac{m_t+h_t}{p_t}\geq\left[1-\xi_t(i)\right]c_t(i)di \qquad (6.5)$$

in each period t.

After making its consumption decision at the end of period t, the representative household participates in a centralized assets market, and receives rental payments $r_t k_t$ and wages $w_t(l_t^g+l_t^f)$ and pays for the credit goods purchased earlier during period t. Whatever remains is then used to accumulate cash balances m_{t+1} that he carries in to period $t+1$ and to purchase the unsold output from the representative firm, which it combines with its depreciated capital stock $(1-\delta)k_t$ in order to carry k_{t+1} units of capital into period $t+1$.

The household can also borrow from and lend to other households at the end-of-period asset market by purchasing or issuing one–period, nominally-denominated discounts bonds. In period t+1, bonds pay b_{t+1} units of money and are sold for b_{t+1}/R_t units of money in period t asset market, where R_t is the gross nominal interest rate between period t and $t+1$. Note, $b_{t+1}=0$ must hold as an equilibrium condition in each period t, since these bonds are available in zero net supply at the beginning of each period.

As source of income in period t, the representative household has access to its initial money and bond holdings, its beginning of period government transfer, its rental and wage receipts, and the undepreciated capital stock. On the expenditure side, the representative household purchases consumption goods, pays to the intermediaries, and the capital, money, and bonds that it will carry into period $t+1$. Formally, the representative household faces the following budget constraint:

$$\frac{m_t + b_t + h_t}{P_t} + r_t k_t + w_t\left(l_t^g + l_t^f\right) + (1-\delta)k_t \geq \int_0^1 c_t(i)\,di + \int_0^1 \xi_t(i) q_t(i)\,di + k_{t+1} + \frac{m_{t+1}}{P_t} + \frac{b_{t+1}}{p_t R_t} \qquad (6.6)$$

in each period t. The representative household chooses sequences for $c_t(i)$, $\xi_t(i)$, J_t, l_t^g, l_t^f, k_{t+1}, m_{t+1} and b_{t+1} to maximize the utility function (6.3) subject to time constraint (6.4), cash- in-advance constraint (6.5) and the resource constraint (6.6) by taking the sequences of h_t, $r_t, w_t, p_t, q_t(i)$ and R_t as given . Moreover, the household also takes its initial holdings of capital $k_0 > 0$, money $m_0 = m_0^s$ and bonds $b_0 = 0$ as given.

6.2.3 The representative intermediary's problem

In market i, an intermediary hires $\gamma(i)$ units of labor and charges $q_t(i)$ if the representative shopper purchases good i on credit at time t. Thus, the representative intermediary chooses labor input n_t^f to maximize its profits. Formally at each date t,

$$\pi_t^f = \int_0^1 \xi_t^s(i) q_t(i)\,di - w_t n_t^f \qquad (6.7)$$

is maximized taking w_t, ξ_t^s and $q_t(i)$ as given, subject to intermediaries total demand for labor (the technological constraint):

$$n_t^f \geq \int_0^1 \xi_t^s(i)\gamma(i)\,di \qquad (6.8)$$

6.2.4 The representative goods-producing firm's problem

In market i = 0, the representative goods-producing firm uses k_t units of capital and n_t^g units of labor in each period t and produces consumption good i = 0. Its profits in period t are:

$$\pi_t^g = A(k_t)^\alpha \left(n_t^g\right)^{1-\alpha} (K_t)^\eta - r_t k_t - w_t n_t^g \qquad \alpha \in (0,1),\ \eta > 0. \qquad (6.9)$$

K_t in production function equation (6.9) , represents the aggregate capital stock per household at time t. Following Romer (1986), capital is broadly defined to include human capital and disembodied knowledge, over and above to physical capital. Spillover effects from human capital lead to increasing returns to scale at aggregate level, even though production obeys constant returns to scale at firm level. Increasing returns to scale results in endogenous growth, which is also (possibly) dependent on the inflation rate. When maximizing its profit, given in (6.9), the representative firm takes K_t, r_t and w_t as given.

6.3 Competitive equilibrium

A competitive equilibrium in this economy consists of sequences for prices and quantities which ensures that the optimization problems of households, intermediaries, and firms, outlined above, holds. Given the initial conditions $k = K_0 > 0$, $m_0 = m_0^s$, and $b_0 = 0$, equilibrium prices and quantities must also satisfy the zero profit conditions of the goods producing firms and the financial intermediaries, i.e.,

$$\pi_t^g = \pi_t^f = 0, \tag{6.10}$$

the consistency condition:

$$k_{t+1} = K_{t+1}, \tag{6.11}$$

and the market–clearing conditions in each period t for each market as follows :

Goods market: $A(k_t)^{\alpha+\eta}(n_t^g)^{1-\alpha} + (1-\delta)k_t = k_{t+1} + \int_0^1 c_t(i)di$ (6.12)

Labor market: $n_t^g = l_t^g$ and $n_t^f = l_t^f$, (6.13, 6.14)

Money market: $m_{t+1} = m_{t+1}^s$ (6.15)

Bond market: $b_{t+1} = 0$ (6.16)

Financial intermediaries: $\xi_t(i) = \xi_t^s(i)$ (6.17)

6.4 General Equilibrium effects of inflation tax

Dotsey and Ireland (1996) demonstrates that there exists a borderline index s_t for each date t such that the representative household's all purchases are credit goods with indices $i \le s_t$, and all goods are cash goods when $i > s_t$, with the borderline index determined by the solution to:

$$\gamma(s_t) = \left[\ln(\lambda_t + \mu_t) - \ln(\lambda_t)\right]/(w_t\lambda_t) \tag{6.18}$$

where λ_t is nonnegative multiplier on the resource constraint (6.6) and μ_t is the nonnegative multiplier on the cash-in-advance constraint (6.5) from household's optimization problem. Note, since transaction cost increases with distance, the shopper uses credit close to home and cash far from home, as discussed in Schreft (1992) and Gillman (1993).

The representative household's optimal $c_t(i)$ follows a step-function in each period t:

$$c_t(i) = \begin{cases} c_t^1 = 1/\lambda_t & for \quad i \le s_t \\ c_t^0 = 1/(\lambda_t + \mu_t) & for \quad i > s_t \end{cases} . \tag{6.19}$$

where, since $\mu_t \ge 0$, $c_t^1 \ge c_t^0$, equation (6.19) and cash in advance constraint (6.5) determine equilibrium money demand as:

$$\frac{(m_t + h_t)}{p_t} = (1 - s_t) c_t^0. \tag{6.20}$$

Further, equations (6.8), (6.14) and (6.17) determine the employment in the financial sector as:

$$l_t^f = \int_0^{s_t} \gamma(i) di. \tag{6.21}$$

In this model welfare cost of inflation arise in number of ways: firstly, higher inflation causes the cash-in-advance constraint to bind, implying higher values of μ_t following higher rates of inflation. Since γ is increasing function of i, the larger value of μ_t will lead to higher a value of s_t. Referring to equation (6.18), as s_t increases, the representative household purchases a wider range of goods with the help of intermediaries. Second, based on equation (6.19), inflation tax distorts consumption and production decisions in two ways. (i) with $c_t^1 > c_t^0$, the marginal rate of substitution between cash and credit goods differ from the corresponding marginal rate of transformation, with the representative household buying different consumption goods in different quantities, and; (ii) since c_t^0 is a decreasing function of μ_t, the representative household purchases cash goods in smaller quantities causing a reduction in market activity. Given the production technology in equation (6.9), these allocative effects of inflation changes the level and growth rate of aggregate output. Third, equation (6.20) suggests that as the inflation rate rises, the representative household economizes on its cash balances by not only purchasing a wider range of goods without money, but also by consuming less of cash goods. In other words, s_t increases and c_t^0 decreases respectively, leading the money demand function to be interest-elastic and resulting in a Bailey-Friedman type cost of the inflation tax. Finally, equation (6.21), suggests that as s_t increases following an increase in the inflation tax, the size of the labor force in the financial sector rises, causing a substitution of resources from the production sector and into finance. This also contributes to the welfare cost of inflation, since given the production function in equation (6.9), this allocative effect influences the long-run growth rate.

Clearly then, inflation tax distorts many marginal decisions, however, it is not possible to analytically assess the magnitude of any of these distortions. Given this, one has to resort to numerical methods to measure the effects of the inflation tax in the general equilibrium, which, in turn, requires us to calibrate the model – a process which we discuss in the next section.

6.5 Model calibration

The household's discount rate is set at $\beta = 0.99$ and the depreciation rate at $\delta = 0.019$ (Liu and Gupta), so that the period in the model is one quarter year. To ensure growth in the long-run equilibrium, $\eta = (1-\alpha) = 0.74$, given $\alpha = 0.26$ (Liu and Gupta, 2007), With A=0.3926, the economy grows at a constant annual rate of 3 percent, the average growth rate of the South African economy over the period of 1965-2008, under a constant annual inflation rate of 9.45 percent, again a figure which corresponds to the average of the above period. The households allocate 25 % of their time to labor (Liu and Gupta, 2007), under an inflation rate of 9.48 percent when B =3.5795.

The magnitude of the Bailey–Friedman cost of inflation depends on the size of the tax base and the interest elasticity of money demand. When the intermediary's cost function is specialized to:

$$\gamma(i) = \gamma\left[i/(1-i)\right]^{\theta}, \gamma > 0, \theta > 0, \qquad (6.22)$$

One can choose the parameters γ and θ so that the size of the tax base and the interest elasticity of money demand in the model corresponds to figures in the South African economy.

As in Dotsey and Ireland (1996), the size of inflation tax base in the South African economy is measured by the fraction of all purchases that are made using money. Based on our calculations, using data over the period of 1965-2008 from the South African Reserve Bank (SARB), suggests that South African households made 45 percent of their transactions using M1(A), implying a value of 0.45 for 1-s_t under 9.45 percent inflation in the model.

The money demand for M1 is estimated using annual data over the period of 1965-2008, and yields: ln(vM1) =2.29+2.75 R, where vM1 is the income velocity of M1(A) and R is the 91-days Treasury bill rate. The OLS coefficient on R measures the long run interest semi-elasticity of money demand. An analogous statistic in the model economy is:

$$\left[\ln\left(v_{9.45}\right)-\ln\left(v_0\right)\right]/\left(R_{9.45}-R_0\right)$$

where $v_{9.45}$ and v_0 are the constant annual velocities of money and $R_{9.45}$ and R_0 are constant annual nominal interest rates that prevail under constant annual inflation rates of 9.45 percent and zero.

Matching the tax base and the elasticity figures in the data and the model yields γ=0.0078 and θ=1.83. With this combination of γ and θ, the annual velocity of money under 9.45 percent of inflation produced by the model is 11.21, which is very similar to the average velocity of 13.15 found in the South African data over the period of 1965-2008. This justifies our identification of one model period as one quarter year.

6.6 The quantitative effects of inflation in the general equilibrium model

In this section, we analyze the effects of a change in the money growth rate on the critical variables defining the general equilibrium model. Note, the effects of monetary policies, which require constant money growth rates, give rise to steady-state equilibria in which all variables grow at constant rates. In Table 6-1 we compare the steady-state equilibrium under the average inflation rate of 9.45%, with those of the Friedman-rule, 0%, 2%, 3% and 6% of inflation. Recall, under the Friedman (1969)-rule, one must ensure a zero nominal rate of interest, which, in turn, implies that money supply is contracted at the rate of time preference. While, the situation under zero percent corresponds to the case of price stability, and the 3% and 6% of inflation captures the limits of the target band.

The representative shopper uses cash to make a constant fraction of his purchases under a constant inflation rate. The model has been calibrated to ensure that the agent carries out 45% of their transactions using M1 under the steady-state inflation rate of 9.45%. As inflation gets higher, the shopper, understandably, uses money in a smaller range of transaction, implying a positive relationship between the velocity of money and the inflation rate. The model parameterization also ensures that the representative worker devotes 25% of his time to labor. As seen from Table 6-1, household substitute out of market activity as inflation rises, and enjoys more leisure without the use of means of exchange, unlike market activity which requires either money or the costly financial services. Further, besides the substitution effect, there is also a negative wealth effect as the inflation rate increases. As in Cooley and Hansen (1989, 1991), the substitution effect tends to dominate the wealth effect, causing the household's labor supply to fall as the inflation rate rises. The allocation of labor force, besides the total labor supply itself, gets affected with changes in the inflation rate. As shown in Table 6-1, though the fraction of labor force working in financial intermediaries is a small number (always less than 0.6%), it rises with the rate of inflation. The substitution of labor of the production sector into leisure and into the financial intermediaries tends to negatively affect the growth rate of output through the spillover effects of aggregate activity. The effect of inflation on the growth rate, however, is quite small in general, since 9.45% inflation causes the growth rate to fall from 3.11 to 3.01%.

Table 6-1: The welfare cost of inflation.

	Annual inflation rate					
	Friedman Rule	0	2	3	6	9.45
Annual money growth	-0.0394	0.0300	0.0506	0.0609	0.0918	0.1273
Annual inflation	-0.0694	-0.0011	0.0192	0.0293	0.0595	0.0943
Annual growth rate	0.0323	0.0311	0.0309	0.0307	0.0304	0.0301
Fraction of time working	0.2548	0.2527	0.2523	0.2522	0.2515	0.2514
Fraction of labour in finance	0.0000	0.0022	0.0029	0.0033	0.0044	0.0056
Fraction of purchases with money, number	1.0000	0.5639	0.5301	0.5160	0.4814	0.4511
Fraction of purchases with money, value	1.0000	0.5596	0.5245	0.5098	0.4734	0.4412
Annual velocity	4.9306	8.8199	9.4135	9.6869	10.4386	11.2091
Welfare cost (percentage of output)	-2.20	0.00	0.48	0.70	1.33	1.97

Following Cooley and Hansen (1989, 1991) and Dotsey and Ireland (1996), the welfare cost of inflation is captured by the permanent percentage increase in the consumption of all goods that is required to make the representative household as well off under a positive rate of inflation as it is under price stability (under the zero rate of inflation). When we multiply this figure with the consumption output ratio, we are able to express it as percentage of output. Table 6-1 show that the welfare cost is nearly 2 percent of output for a steady-state inflation rate of 9.45 percent. The corresponding values for inflation rates of 2, 3 and 6% of inflation are 0.48, 0.70 and 1.33% of output. Finally, the welfare gain from adopting the Friedman-rule is equivalent to a 2.2 % increase in output.

Importantly, the welfare cost values are way higher compared to those obtained in the four previous chapters. Recall, in chapter 4, based on a money demand approach, concludes that for the inflation target band of 3-6%, the welfare cost ranged between 0.15% and 0.41% of GDP, while in chapter 5 , using Feldstein's (1997, 1999) microeconomic partial equilibrium approach, found the annual deadweight loss of a two percent inflation rate to be 0.225% of GDP. Clearly, when one compares the welfare cost estimates of Table 6-1 with those obtained from Bailey-Friedman-type partial equilibrium analyses used in chapter 2, 3 and 4, one tends to obtain much smaller figures than those under the general equilibrium model since the former approach captures only a fraction of the total cost of inflation – the cost due its effect on the velocity of money. In addition to this effect, inflation causes inefficient allocation of productive labor across its alternative uses. Even though the labor supply effects might seem small quantitatively, they end up contributing to the welfare cost of inflation enough to significantly outweigh the welfare cost estimates under the Bailey-Friedman-type money demand approach. In addition, by viewing inflation as a tax on a host of micro-level decisions, we obtain sizeable welfare costs of inflation at the macroeconomic level, thus, indicating that Feldstein (1997, 1999)-type partial equilibrium approaches used in chapter 5, can also significantly underestimate the cost of inflation.

6.7 Conclusion

Since the February of 2000, the sole objective of the SARB has been to keep the CPIX inflation rate within the target band of 3-6%, using discretionary changes in the Repo rate as its main policy instrument. In this regard, the measurement of the cost of inflation is of paramount importance in determining the legitimacy of the current target band, and, if there is a need to rethink of the level and width of the band in terms of the welfare cost of inflation at least Against this backdrop, this chapter calibrates the general equilibrium endogenous growth model proposed by Dotsey and Ireland (1996), where the inflation tax distorts a variety of marginal decisions, for South Africa using quarterly data over the period of 1965 to 2008, and obtains the welfare cost of inflation.

Higher inflation rate causes the agents to inefficiently economize on their holdings of real cash balances, leads to substitution out of market activity by taking more leisure and diverting productive resources out of goods production and into financial intermediaries. The model shows that individually, none of these distortions is very large, but the various small distortions combine to yield substantial estimates of the total cost of inflation. More importantly, the estimates are way higher than the previous welfare cost values obtained in the four previous chapters based on the Bailey-Friedman-type money demand approach or the Feldstein (1997, 1999)-type partial equilibrium approach. We show that for a target band of 3-6 % the welfare cost of inflation ranges between 0.70 % of GDP and 1.33% of GDP. On the other hand, the

Friedman-rule tends to produce welfare gains of the magnitude of 2.20% of GDP. These higher estimates, thus, tend to strengthen the case for a possibly lower and narrower target band – a proposal made in chapter 7 and 8 based on the findings that the inflation targeting produces higher mean and variance of inflation than it would otherwise be if the SARB had continued to follow its earlier so-called eclectic approach to monetary policy. In general, our findings highlight the usefulness of general equilibrium models for the purposes of evaluating a policy regime. In our case, this amounts to indicating that unless all the distortions induced by a policy is considered, in other words, unless we undertake a general equilibrium approach, reliance on partial equilibrium approaches for measuring the welfare effect of inflation will grossly underestimate the "true" welfare cost.