Chapter 3

3 Measuring the welfare cost of inflation in South Africa: A reconsideration*

3.1 Introduction

In chapter 2, we measured the welfare cost of inflation in South Africa, based on estimates of the interest elasticity and semi-elasticity of money demand functions, which were obtained using the Johansen (1991, 1995) methodology on quarterly data for M3, GDP and the Treasury bill rate. Given the estimates for the elasticities, we then calculated the welfare cost of inflation using Bailey's (1956) consumer surplus approach. Relying more on results obtained from the log-log specification of money demand, rather than the semi-log model for the same,[15] they indicated that the welfare cost in South Africa ranged between 0.34% and 0.67% of GDP, for a band of 3-6% of inflation, over the period of 1965:02 to 2007:01.

In this chapter, we re-estimate the long-run relationship between money balance and interest rate for South Africa, using the same data set and over the same period as that used in chapter 2, but applying an alternative approach, namely the long run horizon regression proposed by Fisher and Seater (1993). One of the advantages of using the long-horizon regression approach is that cointegration is neither necessary nor sufficient for tests on the interest rate elasticity of money demand. As in chapter 2, the coefficients obtained in regression for both alternative money demand specifications, a double-log version

[15] The decision to place more confidence on the log-log model of money demand was due to two reasons: First, the R^2 and the Adjusted R^2 values of the inverse money demand relationship captured by the log-log specification was higher than the corresponding values of the semi-log model, and; Second, although there existed overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen test with two lags. We had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship.

*Published in South African Journal of Economic and Management Sciences, (2009), 12(2):137-146.

originated by Meltzer (1963) with constant elasticity and a semi-log version originated by Cagan (1956) with constant semi-elasticity of money, are then used to calculate welfare cost of inflation. In addition, the welfare cost of inflation is then estimated using both Bailey's (1956) consumer's surplus approach and Lucas's (2000) compensating variation approach.

The necessity to compare the welfare cost estimates with that obtained in chapter 2, based on the Johansen (1991, 1995) methodology, arises from the issue of sensitiveness of the estimates of the interest elasticity of alternative forms of money demand, based on alternative econometric techniques adopted to estimate the long-run relationship between money balance and the nominal interest rate. Given that welfare cost estimates hinge critically on the estimate of the interest elasticity and semi-elasticity, it is important to check for the robustness of the results obtained using alternative econometric methodologies.

The above claim regarding the need to use alternative estimation techniques to obtain values for interest elasticity and semi-elasticity is not without empirical basis. Basing their study on the long-run horizon regression approach proposed by Fisher and Seater (1993), the researchers Serletis and Yavari (2004), in their study dealing with the welfare cost of inflation for Canada and the United States, came up with much smaller figures than those of Lucas (2000), who had indicated that a reduction in the nominal rate from 0-3% would yield a benefit equivalent 0.90 % of real income. However, Serletis and Yavari (2005), while repeating the above study for Italy, came up with very similar numbers for the welfare cost they had obtained earlier for Canada and the United States.[16] The authors indicated that reducing the interest rate, in Italy, from 14% to 3% would yield a benefit equivalent to an increase in real income of 0.40%. This, in turn, was fairly comparable to their estimates for Canada (0.35%) and the United States (0.45%) for the same percentage point reduction in the nominal interest rate. More recently, Serletis and Yavari (2007) estimated the welfare cost of inflation using the Fisher and Seater (1993) approach for seven European economies. The results indicated that in big countries, like France and Germany, the welfare cost of inflation is much lower than in small countries, like Austria, Belgium, Ireland, Italy and The Netherlands. But importantly, the numbers were quite comparable to their earlier studies. On the other hand, based on the Phillips-Ouliaris (1990) test for cointegration, Ireland (2009) found that a 10 percent rate of inflation when compared to

[16] See Serletis and Virk (2006) for the sensitiveness of the welfare cost estimates to the choice of monetary aggregation procedure.

price stability, in the United States, would imply a welfare cost of 0.21 percent of income. This figure, though lower than that of Lucas (1981, 2000) and Serletis and Yavari (2004), was in line with Fischer's (1981) findings of 0.30%. Clearly then, apart from sample period and the country under investigation and alternative money demand specifications, welfare cost estimates are sensitive to alternative estimation methodologies. Our need to reconsider the welfare cost estimates obtained in the previous chapter therefore cannot be overlooked. Table 3-1 summarizes the studies discussed above and includes the methodology, country and the size of the welfare cost.

Table 3-1: Summarizing the Literature.

Study	Country	Methodology (Functional Form)	Inflation comparisons (Nominal interest rate)	Welfare costs(percent of GDP)
Fischer(1981)	USA	Calibration[a] (Log-Log)	0 to 10%	0.30%
Lucas(1981)	USA	Calibration[b] (Semi- Log)	0 to 10%	0.45%
Lucas(2000)	USA	Calibration[c] (Log-Log)	0 to 3%	0.9%
Serletis and Yavari(2004)	Canada and USA	Fisher and Seater (1993) Long-Horizon (Log-Log)	0 to 3% 3 to 14%	USA:0.18% Canada:0.15% USA: 0.45% Canada: 0.35%
Serletis and Yavari(2005)	Italy	Fisher and Seater (1993) Long-Horizon (Log-Log)	3 to 14%	0.4%
Serletis and Yavari(2007)	Europe	Fisher and Seater (1993) Long-Horizon (Log-Log)	5 to 10%	Belgium: 0.3% Austria:0.45% France:0.1% Germany:0.2% Netherlands:0.4% Ireland:0.4% Italy:0.4%
Ireland(2009)	USA	Phillips- Ouliaris (1990) Cointegration (Semi-Log)	0 to 10%	0.21%
Gupta and Uwilingiye (2008)	South Africa	Johansen (1991,1995) Cointegration (Log-Log)	0 to 3% 3 to 6%	0.34% 0.67%

Notes: a: Interest elasticity used 0.25 based on Goldfeld (1971); b: Lucas (1981) uses a value of 5.0 for the interest semi-elasticity; c: Lucas (2000) uses a value of 0.50 for the interest elasticity.

Given that, inflation has an effect on economic activity, and ultimately on people's well-being as it reduces the purchasing power of money balances when inflation rises, a correct and fair evaluation of welfare cost of inflation is crucial. This is because inflation creates and amplifies distortions in many areas of economic activity and it has also an influence on all decisions of economic agents. Besides, in a country like South Africa, where the central bank targets inflation, it is of paramount importance to investigate how substantial the welfare costs of inflation are under the current inflation target zone of 3-6% pursued by the South African Reserve Bank. This would help us to decide if there is a need to rethink the band of the target in terms of the welfare cost of inflation. To the best of our knowledge, this is the first attempt to measure the welfare cost of inflation for the South African economy, based on the long-run regression approach proposed by Fisher and Seater (1993).

The remainder of the chapter is organized as follows: Section 3.2 provides a brief summary of the theoretical issues regarding the estimation of the welfare cost of inflation, while, Section 3.3 and 3.4, respectively, discusses the data and the long-horizon empirical methodology for the estimation of the log-log and the semi-log money demand specifications; Section 3.4 also presents the empirical estimates for the interest rate elasticity and the semi-elasticity, as well as the welfare cost estimates for the South African economy. Finally, Section 3.5 concludes.

3.2 The theoretical foundations

As indicated by Lucas (2000), money demand specification is vital in determining the appropriate size of the welfare cost of inflation. Lucas (2000) contrasts between two competing specifications for money demand. One, inspired by Meltzer (1963), relates the natural logarithm of m, a ratio of money balances to nominal income, and the natural logarithm of a short-term nominal interest rate r. Formally, this can be expressed as follows:

$$\ln(m) = \ln(A) - \eta \ln(r) \tag{3.1}$$

where A>0 is a constant and η>0 measures the absolute value of the interest elasticity of money demand. Another specification, adapted from Cagan (1956), links the log of m to the level of r via the following equation:

$$\ln(m) = \ln(B) - \xi r \tag{3.2}$$

where B>0 is a constant and ξ>0 measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the "consumers's surplus", that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_0^r m(x)dx - rm(r) \tag{3.3}$$

As seen from Equation (3.3), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss. From, Figure 3-1 below, this essentially implies that the welfare cost of inflation is measured by the area A.

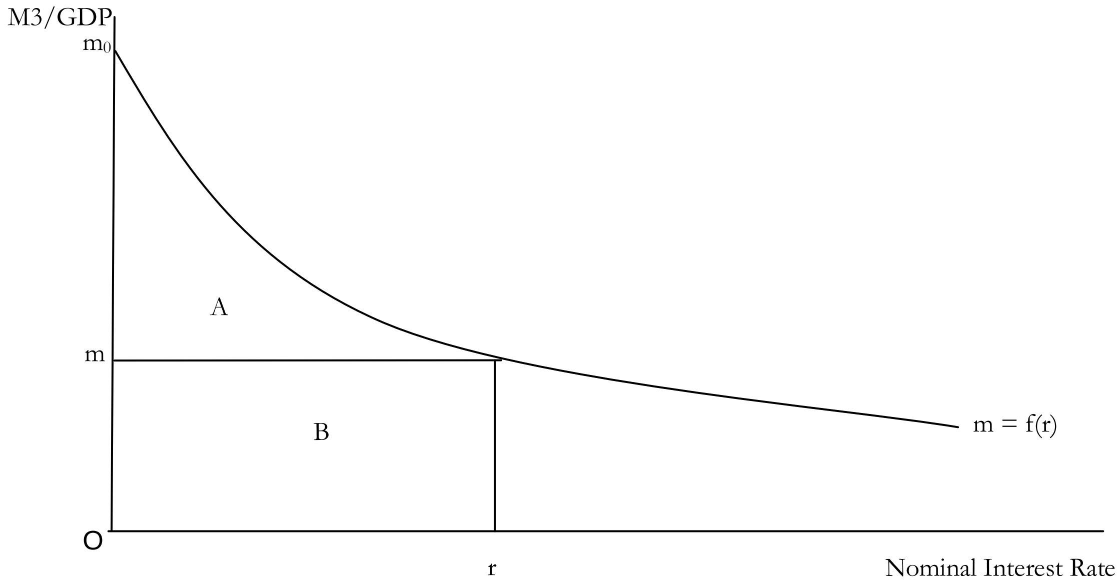

Figure 3-1: Welfare Cost Calculation Using Bailey's Consumer Surplus Approach.

Just as the function m has the dimensions of a ratio to income, so does the function w. The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by (3.1) or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{3.4}$$

While, for a semi-log money demand specification i.e., $m(r) = Be^{-\xi r}$, w(r) is obtained by the following formula:

$$w(r) = \frac{B}{\xi}\left[1-(1+\xi r)e^{-\xi r}\right] \tag{3.5}$$

As demonstrated in (3.4) and (3.5), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, so we first have to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. Lucas (2000) starts by using Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate r, $w(r)$, to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at r and an otherwise identical steady-state with the interest rate of zero with $w(r)$ being obtained from the solution to the following equation:

$$u\left[1+w(r))y, \phi(r)y\right] = u\left[y, \phi(0)y\right] \tag{3.6}$$

Realizing that u is also negatively related to the nominal rate of interest, r, Figure 3.2 presents a diagrammatic illustration of what equation (3.6) essentially implies.

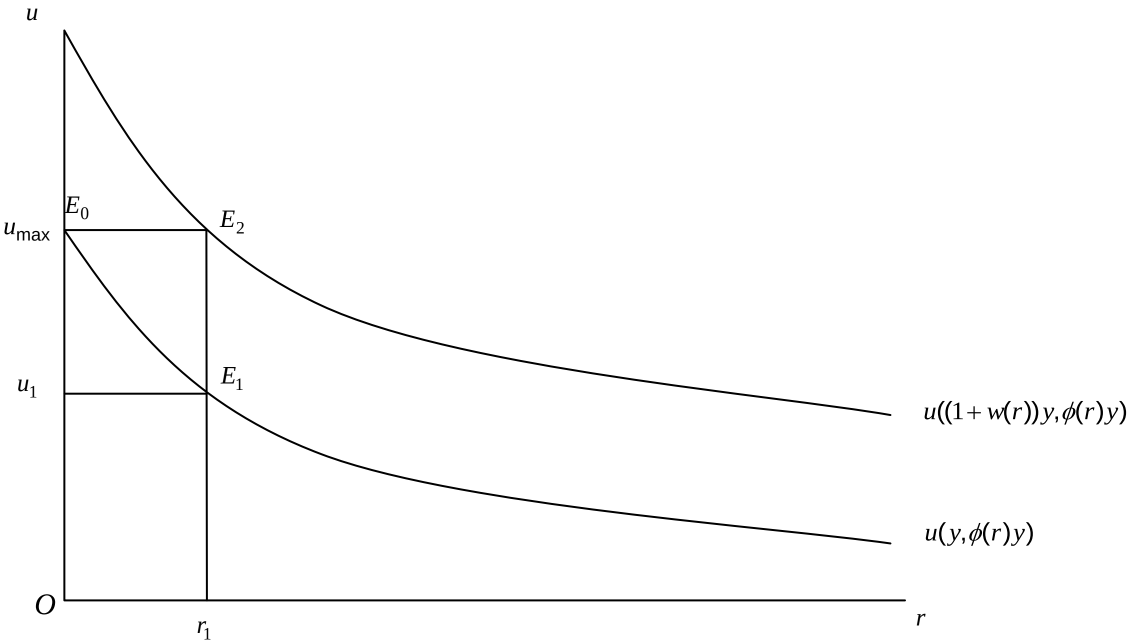

Figure 3-2: Welfare Cost calculation Using the Compensating Variation Approach.

Assuming a homothetic current period utility function ($u(c,m)=\frac{1}{1-\sigma}\left[cf\left(\frac{m}{c}\right)\right]^{(1-\sigma)}$; $\sigma \neq 1$) and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in $w(r)$ of the following form:

$$w'(r)=\psi\left(\frac{\phi(r)}{1+w(r)}\right)\phi'(r) \tag{3.7}$$

For any given money demand function, Equation (3.7) can be solved numerically for an exact welfare cost function $w(r)$. In fact, with equation (3.1), equation (3.7) can be written as:

$$w'(r)=\eta A r^{(-\eta)}(1+w(r))^{\frac{1}{\eta}} \tag{3.8}$$

yielding a solution for log –log specification

$$w(r)=-1+\left(1-Ar^{1-\eta}\right)^{\frac{\eta}{\eta-1}} \tag{3.9}$$

While, for the semi-log model (7) yields

$$w'(r)=\left[\xi B e^{-\xi r}\left(r+\frac{1}{\xi}\log\left(1+w(r)\right)\right)\right]\approx\left[\xi B e^{\xi r}\left(r+\frac{1}{\xi}w(r)\right)\right] \tag{3.10}$$

with a solution

$$w(r)=-e^{\frac{-Be^{-\xi r}}{\varepsilon}}\left\{e^{\frac{Be^{-\xi r}}{\xi}}-Ei\left[\frac{B}{\xi}\right]+Ei\left[\frac{Be^{-\xi r}}{\xi}\right]\right\} \tag{3.11}$$

and where $Ei(x) = -\int_x^{\infty} \frac{e^{-t}}{t} dt$, and one uses the principal value of the integral.

Note to calculate $w(r)$, in equations (3.9) and (3.11),[17] we use the estimates of η and ξ obtained from the long-horizon regression, discussed in Section 4, while, the values for A and B are obtained such that they match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e., A $= \bar{m}/\left(\bar{r}\right)^{-\eta}$, B $= \bar{m}/\left(e^{-\xi \bar{r}}\right)$ with $\bar{m}$ and $\bar{r}$ being respectively the geometric means of m and r respectively.

3.3 Data

In this chapter, we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this chapter are the money balances ratio (rm3), generated by dividing the broad measure of money supply (M3)[18] by the nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate (tbr).[19] All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by lrm3 and ltbr, respectively.

[17] The calculations were done using the DSolve routine in Mathematica, Version 5.

[18] See chapter 2 for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3 was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

[19] We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

3.4 Empirical methodology and results

As it is standard in time series analysis, we start by studying the univariate characteristics of the data. In this regard, we performed tests of stationarity on our variables (lrm3, ltbr and tbr) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips-Perron (PP) test. As seen in chapter 2 , the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests. For the KPSS test, the null of stationarity was rejected. As the variables were found to be non-stationary, it paved the way for the long-horizon regression proposed by Fisher and Seater (1993) to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions. As stated at the onset, cointegration, is neither necessary nor sufficient for this approach, so we do not test specifically for cointegration.[20]

The basics of the long-horizon regression approach can be described as follows, by starting off with the following bivariate autoregressive representation:

$$\alpha_{mm}(L)\Delta^{\langle m\rangle}m_t = \alpha_{mr}(L)\Delta^{\langle r\rangle}r_t + \varepsilon_t^m \tag{3.12}$$

$$\alpha_{rr}(L)\Delta^{\langle r\rangle}r_t = \alpha_{rm}(L)\Delta^{\langle m\rangle}m_t + \varepsilon_t^r \tag{3.13}$$

where $\alpha_{mm}{}^0 = \alpha_{rr}{}^0 = 1$, $\Delta = 1 - L$. L is the lag operator, m is the money-income ratio, r is the nominal interest rate ,and $\langle x\rangle$ represents the order of integration of x, so that if x is integrated of order γ, or $\mathrm{I}(\gamma)$ in the terminology of Engle and Granger (1987), then $\langle x\rangle = \gamma$ and $\langle \Delta x\rangle = \langle x\rangle - 1$. The vector $\left(\varepsilon_t^m, \varepsilon_t^r\right)'$ is assumed to be independently and identically distributed normal with zero mean and covariance $\sum_{\varepsilon}$, the elements of which are $\mathrm{var}\left(\varepsilon_t^m\right)$, $\mathrm{var}\left(\varepsilon_t^r\right)$, $\mathrm{cov}\left(\varepsilon_t^m, \varepsilon_t^r\right)$. A key result in Fisher and Seater (1993) applies to the case where $\langle m\rangle = \langle r\rangle = 1$,

20 The reader is referred to Gupta and Uwilingiye (2008) for the tests on stationality and cointegration on the variables of the model, reported in Tables 1 through 3.

which is the case with our data as money balance as lrm3, ltbr and tbr are all I(1). In this case, the long-run derivative of m with respect to r, LRD_{mr}, is given by:

$$LRD_{m,r} = \frac{\theta_{mr}(1)}{\theta_{rr}(1)} \tag{3.14}$$

with $LRD_{m,r}$ being interpreted as the long-run elasticity of m with respect to r. In fact, under the Fisher and Seater (1993) identification scheme, which assumes that r is exogenous in the long run, $\theta_{mr}(1)/\theta_{rr}(1)$ can be interpreted as $\lim_{k\to\infty} b_k$, where b_k is the coefficient from the regression:

$$\left[\sum_{j=0}^{k} \Delta^{\langle m\rangle} m_{t-j}\right] = a_k + b_k \left[\sum_{j=0}^{k} \Delta^{\langle r\rangle} r_{t-j}\right] + e_{kt} \tag{3.15}$$

and for $\langle m\rangle = \langle r\rangle = 1$, consistent estimate of b_k can be derived by applying ordinary least squares to the regression

$$m_t - m_{t-k-1} = a_k + b_k \left[r_t - r_{t-k-1}\right] + e_{kt}, \tag{3.16}$$

$$k = 1, \ldots\ldots. K$$

Based on Equation (3.16) and for a value of k=30 as used by Serletis and Yavari (2004 and 2005), our estimate of the interest rate elasticity, η, is 0.1073 and interest semi-elasticity ξ is 1.0099,[21] which, in turn, are much lower than the corresponding values of 0.2088 and 2.1991, obtained in chapter 2 based on the Johansen (1991 and 1995) methodology.

Once we obtain the estimated values for η and ξ, using long-horizon regression, we calculate the values of A and B such that the curves obtained pass through the geometric means of the data. This gives us values of A = 0.4255 and B = 0.6035. Note the values for A and B obtained in chapter 2 based on the cointegrating relationships were, respectively, 0.3323 and 0.6862.[22]

---

[21] Both the estimates of η and ξ are significant at the 1 percent level of significance.

[22] Based on the suggestions of one of the anonymous referees, equation (16) was re-estimated without the constant. The corresponding values of the interest rate elasticity, η, were found to be 0.0965 and that of the interest semi-elasticity ξ was 0.9556. Note both these values were found to be significant at the 1

Having obtained the estimates for η and ξ and the values for A and B, we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation method. The results have been reported in Table 3-2. Note for the sake of comparison, we also present the welfare cost estimates, based on the values of η , ξ , A and B, obtained in chapter 2 , based on the Johansen (1991 and 1995) approach.

Table 3-2: Welfare cost estimates

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Based on the results reported in the Columns 2 and 3, and 4 and 5, the welfare cost estimates obtained under the consumer surplus approach, for 3%, 6%, 10% and 15% of inflation, using the Johansen (1991 and 1995) cointegration method and the long-horizon regression approach respectively, we see that welfare costs are substantially lower in the latter case. In fact they are nearly less by more than half, of the costs obtained using the cointegration approach for both the log-log and the semi-log specifications. When we compare Columns 6 and 7, and 8 and 9, we obtain a similar picture for the welfare cost estimates obtained using the compensating variation approach. Further, the welfare cost estimates within a specific estimation method, but across the consumer surplus approach and the compensating variation approach are quite similar, with the figures being slightly higher under the compensating variation method outlined by Lucas (2000). Specifically, for the log-log (semi-log) specification, estimated using the cointegration approach, under the consumer surplus approach [compensating variation approach], an increase in the inflation rate from 3- 6% would increase the welfare cost from 0.67% of GDP to 1.08% of GDP

percent level. Given, that the values of A and B would stay the same as above, we would obtain even lower estimates of the welfare cost of inflation under the two alternative specifications of money-demand.

[0.72 % of GDP to 1.17% of GDP] (0.76 % of GDP to 1.43% of GDP [0.79% of GDP to 1.4449% of GDP]). While, under the long-horizon approach the welfare cost estimates ranges between 0.18% of GDP to 0.35% of GDP and 0.19% of GDP to 0.37% of GDP with the log-log specification, obtained from the consumer surplus and the compensating variation approaches respectively, for an increase in the inflation rate from 3-6%, the corresponding values under the semi-log specification, for the same increase in the rate of inflation, are 0.15 % of GDP to 0.35% of GDP and 0.16% of GDP to 0.36% of GDP.

The bottom line is that, as in Serletis and Yavari (2004 and 2005) , we find the welfare cost estimates based on the long-horizon approach tends to be much smaller when compared to other standard econometric method of arriving at the long-run equilibrium relationship between ratio of money balance to income and the nominal interest rate. The reason is that, under the long-horizon approach estimates of interest rate elasticity and semi-elasticity tends to be comparatively lower. Given the fact that welfare cost estimates based on money demand estimations critically hinges on the size of interest rate elasticity and semi-elasticity, this brings down the welfare cost of inflation when compared to estimates obtained via econometric methods, such as the Johansen (1991 and 1995) approach.

3.5 Conclusion

In this chapter , using the Fisher and Seater (1993) long-horizon approach, we estimate the long-run equilibrium relationship between money balance as a ratio of income and the Treasury bill rate for South Africa over the period of 1965:02 to 2007:01, and, in turn, use the obtained estimates of the interest elasticity and the semi-elasticity to derive the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach, as well, as Lucas's (2000) compensating variation approach. When, the results are compared to welfare cost estimates obtained in chapter 2 , using the same data set, but based on Johansen's (1991, 1995) cointegration technique, the values are less by more than half of those obtained in chapter 2. This chapter highlights the fact that welfare cost estimates of inflation are sensitive to the methodology used to estimate the long-run equilibrium money demand relationships.

At this stage two aspects of the obtained results needs further emphasis: First, when compared to the literature, the welfare cost estimates obtained for South Africa, whether based on the long-horizon regression or the Johansen (1991 and 1995) cointegration approach, are relatively higher when compared to estimates available in the literature for other economies for similar levels of inflation rates. Second, it must be realised that whatever the estimation methodology used, whether it is a one consumer-surplus approach or a compensating variation method, based on our estimates, we can conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman (1969)-type deflationary rule of zero nominal rate of interest.

However, the following question is undeniably relevant: Given that welfare cost estimates are sensitive estimation methodologies and seem to vary considerably according to econometric approach is undertaken, what is the true size of the welfare cost of inflation in South Africa? The answer to this question is difficult. However, it must be admitted that econometric methodologies deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, a rise in the inflation rates can distort other marginal decisions and can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Given these two additional sources of inflation costs, there is no denying the fact that larger gains can conceivably be achieved by reducing the inflation target below 3%, the lower limit of the current inflation target band.

Chapter 4

4 Time aggregation, long-run money Demand and the welfare cost of inflation*

4.1 Introduction

In chapter 2 and 3 we estimated the long-run money demand relationship for South Africa, and then, in turn, went ahead and used the interest rate elasticity and semi-elasticity to obtain the size of the welfare cost of inflation for the economy. Using the same data set, but two different approaches to estimate the long-run money demand functions, namely the cointegration procedure outlined in Johansen (1991, 1995) and the long-horizon approach proposed by Fisher and Seater (1993) respectively, we ended up with markedly different measures of the welfare cost of inflation. Specifically speaking, in chapter 3, using the long-horizon methodology, found the value to fall by more than half as that obtained in chapter 2, where the estimations were obtained from the cointegration approach. The difference between the results, essentially emanated from the smaller sizes of the interest rate elasticity and semi-elasticity obtained under the long-horizon approach relative to the cointegration procedure.

At this stage, it is important to point out that such a finding is not an exception in the welfare cost literature. Besides, the importance of sample period, the money demand specifications, i.e., double-log (Meltzer, 1963) or semi-log (Cagan, 1956), and the versions of the monetary aggregate, the importance of the estimation procedure, namely cointegration or long-horizon, have been noted by host of authors, with the latter producing the most drastic of differences in the measures of welfare costs within an economy over identical sample periods using same data sets.[23] To the best of our knowledge though, no study thus far has attempted to figure out which of the two methods is more robust and ideally suited in providing the estimates of interest elasticity and semi-elasticity, and, hence, the appropriate measure of the size of the

[23] See for example Fischer (1981), Lucas (1981, 2000), Serletis and Yavari (2004, 2005, 2007), Serletis and Virk (2006) and Ireland (2009), amongst others.

*Published in studies in Economics and Econometrics, (2009), 33(3):95-109.

distortionary effect of inflation in the money market. But, as discussed in both chapter 2 and 3, econometric methodologies, whether based on cointegration or the long-horizon approach, deriving welfare cost measures by estimating money demand relationships provide only the lower bounds to the welfare cost of inflation. Since, such welfare cost estimates merely measures the distortion in the money demand due to positive nominal interest rates, and, hence takes a partial equilibrium approach. Given that, in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output, welfare cost estimates of inflation are likely to be much higher. Hence, the ideal approach to obtaining a welfare cost estimate of inflation would be to use a dynamic general equilibrium model. Nevertheless, this line of argument does not provide an answer to the controversy regarding the true size of the distortionary effect of inflation on the money market or in a partial equilibrium framework. In this chapter, we make an attempt to resolve this issue by delving into the role of time aggregation on the long-run properties of the data, and, hence, the estimates of the welfare cost of inflation based on the money market.

It must be realized that the data on the three critical variables, required in the estimation of a money demand function, namely, a monetary aggregate and measures of real income and the opportunity cost variable, are generally available at different frequencies. Specifically, the interest rate is available at the highest frequency of weeks, the monetary aggregates in monthly form, while, the real income, generally measured by real GDP, is available only at quarters. Given this, a quarterly money demand estimation would require one to convert the weekly and the monthly variables into their quarterly form. In this regard, two approaches that are generally used are either temporal aggregation or systematic sampling. Temporal aggregation simply means aggregating over the weeks (for the interest rate) or months (for the monetary aggregate) of a quarter and using the average value as the quarterly value. Systematic sampling, on the other hand, involves using a single observation from the sampling interval, such as the end of the interval observation, which in our case would be the last week or month of a specific quarter, depending on whether we are trying to convert the measure of the interest rate or the monetary aggregate,

The motivation to use the effect of time aggregation on the two methods of estimating long-run money demand relationships is derived from the recent work of Marcellino (1999). In this paper, the author indicated, theoretically and via an example, that aggregation, via temporal aggregation or systematic sampling, tends to affect only the short-run properties of the data, leaving the long-run aspects of the data unchanged.[24] Given this then, one would expect that within a specific econometric methodology, i.e., long-horizon or the cointegration approach, the estimates of the parameters in the long-run money demand relationships, log-log or semi-log, should not be significantly affected depending on whether the opportunity cost and the monetary aggregate variables were converted to their respective quarterly values based on temporal aggregation or systematic sampling. In other words, by using time aggregation, we expect to determine which of the estimates of the welfare cost of inflation via the money market, obtained through either the Johansen (1991, 1995) methodology or the Fisher and Seater (1993) approach is more robust, and, hence, should be taken more seriously. It is important to point out that, though Marcellino (1999) indicates that long-run properties of the data are virtually unchanged because of alternative sampling methods, the author does indicate the need to verify the theoretical claims with data relating to the specific question under consideration.

In this respect, we re-evaluate, based on the same data set, the results obtained in chapter 2 and 3 by using systematic sampling, instead of temporal aggregation used in chapter 2, to convert the measures of the monetary aggregate and the interest rate into their respective quarterly values. At this stage, it must be emphasized that we are not really trying to draw overwhelming conclusions regarding the robustness of these two alternative estimation methodologies, but, merely trying to deduce what is the appropriate size of the inflationary distortion on the welfare of the South African economy, via the money market. Given that South Africa has an inflation targeting band of 3-6%, the importance of knowing what is the true size of the welfare cost of inflation due to the distortion caused by the positive nominal interest rate on the money market, is of utmost importance. So, our study should not be evaluated in the light of an attempt to check for the credibility of the two methodologies under alternative sampling techniques, since the possibility of obtaining different conclusions based on a different set of variables, sample sizes and the economy(ies) concerned cannot be ignored. The remainder of the chapter is organized as follows: Section 4.2 outlines the theoretical foundations involved in the estimation of the welfare cost of inflation based on the money market distortion. Section 4.3 discusses the data and the results, which includes the estimates of the parameters in the money demand functions, as well as the, measures of the welfare cost of inflation. Finally, Section 4.4 concludes.

[24] Similar observations has also been made by Gupta and Komen (2008) while analyzing the causal relationship between the repo rate and the CPIX inflation in South Africa.

4.2 The theoretical foundations

By applying the methods outlined in Bailey (1956), Lucas (2000) transformed the evidence on money demand into a welfare cost estimate. Note Bailey (1956) described the welfare cost of inflation as the area under the inverse money demand function, or the "consumer's surplus", that could be gained by reducing the interest rate to zero from an existing (average or steady-state) value. So if $m(r)$ is the estimated function, and $\psi(m)$ is the inverse function, then the welfare cost can be defined as:

$$w(r) = \int_{m(r)}^{m(0)} \psi(x)dx = \int_0^r m(x)dx - rm(r) \qquad (4.1)$$

where m, is the ratio of money balances to nominal income, and r measures the short-term nominal interest rate.

As seen from Equation (4.1), obtaining a measure for the welfare cost amounts to, integrating under the money demand curve as the interest rate rises from zero to a positive value to obtain the lost consumer surplus and then deducting the associated seigniorage revenue rm to deduce the deadweight loss.

Since the function m has the dimensions of a ratio to income, so does the function w. The value of $w(r)$, represents the fraction of income that people needs, as compensation, in order to be indifferent between living in a steady-state with an interest rate constant at r or an identical steady state with an interest of close or equal to zero. Given this, Lucas (2000) shows that when the money demand function is given by: $\ln(m) = \ln(A) - \eta \ln(r)$ or is $m(r) = Ar^{-\eta}$, the welfare cost of inflation as a percentage of GDP is obtained as follows:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right) r^{1-\eta} \qquad (4.2)$$

where A>0 is a constant and η>0 measures the absolute value of the interest elasticity of money demand.

While, for a semi-log money demand specification i.e., $\ln(m) = \ln(B) - \xi r$ or $m(r) = Be^{-\xi r}$, w(r) is obtained by the following formula:

$$w(r) = \frac{B}{\xi}\left[1 - \left(1 + \xi r\right)e^{-\xi r}\right] \tag{4.3}$$

where B>0 is a constant and ξ>0 measures the absolute value of the semi-elasticity of money demand with respect to the interest rate.

As can be seen from (4.2) and (4.3), an estimate of the interest elasticity of money demand is crucial in evaluating the welfare cost of inflation, and, hence, we first need to obtain the long-run relationship between the ratio of money balance to income and a measure of the opportunity cost of holding money, captured by a short-term nominal interest rate.

Besides providing the theoretical general equilibrium justifications for Bailey's consumer surplus approach, Lucas (2000), also takes a compensating variation approach in estimating the welfare cost of inflation. To start off, Lucas (2000) uses Brock's (1974) perfect foresight version of Sidrauski's (1967) Money-in-the-Utility (MIU) model, and defines the welfare cost of a nominal interest rate r, $w(r)$, to be the income compensation needed to leave the household indifferent between living in a steady-state with an interest rate constant at r and an otherwise identical steady-state with the interest rate of zero. With, $w(r)$ being obtained from the solution to the following equation:

$$u\left[1 + w(r))y, \phi(r)y\right] = u\left[y, \phi(0)y\right] \tag{4.4}$$

Assuming a homothetic current period utility function ($u(c,m) = \frac{1}{1-\sigma}\left[cf\left(\frac{m}{c}\right)\right]^{(1-\sigma)}; \sigma \neq 1$) and setting up the dynamic programming problem (see Lucas (2000) for details), Lucas obtains a differential equation in $w\left(r\right)$ of the following form:

$$w'\left(r\right) = \psi\left(\frac{\phi\left(r\right)}{1 + w\left(r\right)}\right)\phi'\left(r\right) \tag{4.5}$$

For any given money demand function, Equation (4.5) can be solved numerically for an exact welfare cost function $w\left(r\right)$. In fact, with $m(r) = Ar^{-\eta}$, equation (4.5) can be written as:

$$w'(r) = \eta A r^{(-\eta)}(1 + w(r))^{\frac{1}{\eta}} \tag{4.6}$$

yielding a solution for log –log specification

$$w(r) = -1 + \left(1 - Ar^{1-\eta}\right)^{\frac{\eta}{\eta-1}} \tag{4.7}$$

While, for the semi-log model (4.5) yields

$$w'(r) = \left[\xi B e^{-\xi r}\left(r + \frac{1}{\xi}\log\left(1 + w(r)\right)\right)\right] \approx \left[\xi B e^{\xi r}\left(r + \frac{1}{\xi} w(r)\right)\right] \tag{4.8}$$

with a solution

$$w(r) = -e^{\frac{-Be^{-\xi r}}{\varepsilon}}\left\{e^{\frac{Be^{-\xi r}}{\xi}} - Ei\left[\frac{B}{\xi}\right] + Ei\left[\frac{Be^{-\xi r}}{\xi}\right]\right\} \tag{4.9}$$

and where $Ei(x) = -\int_x^{\infty} \frac{e^{-t}}{t} dt$, and one uses the principal value of the integral.

Note to calculate $w(r)$ under Bailey's (1956) and Lucas'(2000) approaches, we use the estimates of η and ξ obtained from both the cointegration and long-horizon regression. While, A and B are obtained directly from the cointegrating relationships, the values of the same, under the long-horizon regression, is derived to ensure that they match the geometric means of the data for the log-log and the semi-log specifications respectively, i.e., A $= \bar{m}/\left(\bar{r}\right)^{-\eta}$, B $= \bar{m}/\left(e^{-\xi\bar{r}}\right)$ with $\bar{m}$ and $\bar{r}$ being respectively the geometric means of m and r respectively.[25]

4.3 Data and Results

As in chapter 2 and 3 , we use quarterly time series data from the second quarter of 1965 (1965:02) to the first quarter of 2007 (2007:01) for the South African economy, which, in turn, are obtained from the South African Reserve Bank (SARB) Quarterly Bulletin and the International Financial Statistics of the IMF. The variables used in this chapter are the money balances ratio (rm3), generated by dividing the broad measure of money supply (M3)[26] by the

---

[25] For details regarding the estimation methodologies refer to Chapter 2 and 3 .

[26] See chapter 2 for details regarding the reasons behind the choice of M3 as the appropriate monetary aggregate for South Africa, over narrower aggregates generally used in literature. Basically, the ratio of M3 to GDP is less volatile when compared to the corresponding ratios of M1 and M2 to GDP, and also M3

nominal income (nominal GDP), and short term interest rate, in our case, proxied by the 91 days Treasury bill rate (tbr).[27] All series, except for the Treasury bill rate are seasonally adjusted. Further, for the estimation of the log-log specification both the ratio of money balances and the Treasury bill rate are transformed into their logarithmic values, and are denoted by lrm3 and ltbr, respectively. Note, given that weekly values of the 91 days Treasury Bill rate is only available from the beginning of 1981, and to keep our data set identical to the one used in chapter 2 and 3, we use monthly data on both M3 and the interest rate measure to convert them into quarterly figures via systematic sampling, unlike temporal aggregation used in chapter 2 and 3.

After obtaining all the series in their quarterly forms, as is standard in time series analysis, we start off by studying the univariate characteristics of the systematically sampled series. In this regard, we performed tests of stationarity on our variables (lrm3, ltbr and tbr) using the Augmented–Dickey–Fuller (ADF) test, the Dickey-Fuller test with GLS Detrending (DF-GLS), the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test and the Phillips- Perron (PP) test. As in Gupta and Uwilingiye (2008), all the variables were found to follow an autoregressive process with a unit root, as the null hypothesis of a unit root could not be rejected for the variables, expressed in levels for the ADF, the DF-GLS and the PP tests, while for the KPSS test, the null of stationarity was rejected. As all the variables were found to be non-stationary, to avoid obtaining estimates for the interest rate elasticity and semi-elasticity based on spurious regressions, the Johansen (1991, 1995) cointegration method and the long-horizon regression proposed by Fisher and Seater (1993) was used to obtain the long-run relationships.

---

was used to account for the financial innovations that have taken place in the South African economy over the sample period being used of our concern.

[27] We also use the percentage change at seasonally adjusted annualized rates of the CPI to obtain the rate of inflation, and, hence, the real rate of interest. See below, for further details.

Table 4-1: Unit Root Tests (Systematic Sampling).

Series	Model	ADF		PP	KPSS	DF-GLS	Conclusion
		$\tau_\tau \tau_\mu \tau$	$\phi_3 \phi_1$	$\tau_\tau \tau_\mu \tau$	$\tau_t\ \tau_\mu$	$\tau_t\ \tau_\mu$	
LRM3	τ_t	-0.04	2.57	-0.03	0.31	-0.30	Non- stationary
	τ_μ	-0.22	0.05	-0.27	0.32***	-0.62	
	τ	-0.69		-0.69			
D(LRM3)	τ_t	-13.71***	94.04***	-13.71***	0.09***	-13.58***	Stationary
	τ_μ	-13.31***	177.13***	-13.30***	0.56*	-13.35***	
	τ	-13.31		-13.31***			
LTBR	τ_t	-2.61	11.89***	-2.29	0.29	-2.52	Non- stationary
	τ_μ	-2.45	17.21***	-2.28	0.90	-1.35	
	τ	-0.76	-2.28	-0.82			
D(LTBR)	τ_t	-8.60***	37.01	-8.61***	0.03***	-8.52***	Stationary
	τ_μ	-8.60***	73.93	-8.60***	0.09***	-7.84***	
	τ	-8.62***		-8.62***			
TBR	τ_t	-2.50	7.02***	-2.32	0.27	-2.47	Non- stationary
	τ_μ	-2.45	10.25***	-2.30	0.73*	-1.63*	
	τ	-0.86		-0.78			
D(TBR)	τ_t	-9.60***	46.04***	-9.62***	0.03***	-9.65***	Stationary
	τ_μ	-9.60***	92.08***	-9.62***	0.08***	-9.48***	
	τ	-9.62		-9.65***			

*(**) [***] indicates statistical significance at 10(5)[1] percent level.

Before deriving the long-run money demand relationships using the Johansen (1991, 1995) methodology, a test for the stability of the VAR model, including a constant as an exogenous variable was performed. Given that no roots were found to lie outside the unit circle for the estimated VAR based on 2 lags[28] for both specification of money demand, we conclude

[28] The choice of two lags was based on the unanimity of the Schwarz Information Criterion (SC) Hannan-Quinn (HQ) Information Criterion.Note the optimal lag length used by Gupta and Uwilingiye (2008) based on temporally aggregated data was four. However, it must be noted that although there existed

that the VARs are stable and suitable for further analysis. Once the issues of stability and the optimal lag length were settled, we tested for the cointegrating relationship based on the Johansen (1991, 1995) approach. For this purpose, we included two lags in the VAR, and allowed the level data to have linear trends, but the cointegrating equations to have only intercepts. Based on the Pantula Principle, the Maximum Eigen Value tests, showed that there is one stationary relationship in the data (r = 1) at 5 percent level of significance for both the log-log and the semi-log specifications. The results have been reported in Tables 4-2 and 4-3.[29] Interestingly, unlike with the temporally aggregated data used in chapter 2, the trace test failed to detect any cointegrating relationship. Thus immediately, we get to see the differences in the results obtained under the two alternative sampling techniques, even though Marcellino (1999) claims that alternative forms of aggregation do not tend to affect the long-run properties of the data.

Table 4-2: Estimation and Determination of Rank (Log-Log).

Null hypothesis	Alternative Hypothesis	Test statistic	0.05 critical value	Prob. **
			Trace Statistic	
r=0	r=1	15.15050	15.49471	0.0563
r=1	r=2	0.157021	3.841466	0.6919
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	r=1	14.99348	14.26460	0.0383
r=1	r=2	0.157021	3.841466	0.6919
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

overwhelming evidence that suggested the choice of two lags for the semi-log specification, no cointegration could be detected using the Johansen tests with two lags. We had to use 4 lags, based on the Sequential Modified LR test statistic, to obtain a stable long-run money demand relationship of the semi-log form

[29] As in Ireland (2009), we also used the Phillips-Ouliaris (1990) test for cointegration. However, unlike (2009), the test could not detect any cointegrating relationship between the chosen variables. Hence, the results of the test have been suppressed to save space. They are, however, available upon request.

Table 4-3: Estimation and Determination of Rank (Semi-Log).

Null hypothesis	Alternative Hypothesis	Test statistic	0.05 critical value	Prob. **
			Trace Statistic	
r=0	r=1	14.88209	15.49471	0.0617
r=1	r=2	0.115014	3.841466	0.7345
Trace test indicates no cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				
			Maximum Eigenvalue Statistic	
r=0	r=1	14.76707	14.26460	0.0416
r=1	r=2	0.115014	3.841466	0.7345
Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level * denotes rejection of the hypothesis at the 0.05 level **MacKinnon-Haug-Michelis (1999) p-values				

As we are more interested in the relationship between the money balance ratio and interest rate, for both specifications, lrm3 was restricted to unity. Given that we have only one cointegrating vector, the normalizing restriction on lrm3 is enough to exactly identify the long-run relationship. However, as in chapter 2, we encountered two serious econometric problems with this restriction. First, the restriction was not binding. Secondly, the adjustment coefficient of lrm3 was insignificant under both the specifications. Imposing an additional zero restriction on the adjustment coefficient of lrm3 did ensure binding restrictions, but at the cost of suggesting that the ratio of real balance to income was in fact exogenous and we should not be normalizing on lrm3. Given this, we decided to normalize on the interest rate variable, i.e., ltbr for the log-log specification and tbr for the semi-log specification. Further, with the adjustment coefficients on lrm3 still being insignificant in both the models, we restricted them to zero, and obtained binding restrictions.[30] Note with lrm3 now treated as the right-hand side variable, weak exogeneity of the same is what should be expected. The adjustment coefficients of ltbr and tbr were negative and significant, with them correcting for 7.1 percent and 8.4 percent of the disequilibrium in the next period, respectively.

[30] Note the value of the LR test statistics for binding restrictions, both long- and short-run, for the log-log and the semi-log specifications respectively, were $\chi^2(1)$ = 0.5578 (0.4551) and $\chi^2(1)$ = 0.0587 (0.8085), where the numbers in the parenthesis indicates the probability of committing a Type I error.

Based on the above restrictions, the interest elasticity of money demand is found to be equal to 0.2316, while, 2.4794 was the obtained value for the interest semi-elasticity of money demand.[31], The values of A and B, based on cointegrating relationships are, respectively, 0.3187 and 0.7153.[32] Note in chapter 2, the estimates of the intercept and slope coefficient based on temporally aggregated data implied values of A = 0.3323 and that of η = 0.2088, while for the semi-log specification B = 0.6862 and ξ = 2.1991. So, as can be seen, systematic sampling increases the values of the elasticity and semi-elasticity. However, the value of the intercepts increases for the semi-log model and falls for the log-log model.

After having estimated the money demand relationships via the Johansen (1991, 1995) cointegration approach, we resorted to the long-horizon approach of Fisher and Seater (1993) to obtain the estimates of A and η, and B and ξ. Our estimate of the interest rate elasticity, η, yields a value of 0.1160.and that of interest semi-elasticity, ξ, equal to 1.1027.[33] Once we obtain the estimated values for η and ξ using the long-horizon regression, we then calculate the values of A and B such that the curves obtained pass through the geometric means of the data. This gives us values of A = 0.4221 and B = 0.6166. Note, the corresponding values of A and η, and B and ξ obtained in chapter 3 were 0.4255, 0.1073, 0.6035 and 1.001 respectively. As with the

---

[31] The obtained cointegrating relationships are:

(i) Log-Log: *ltbr = -4.9388 – 4.3186 (lrm3), and;*

[*-3.1490*]

(ii) Semi-Log: *lbr = -0.1352 – 0.4033(lrm3).*

[*-3.0974*]

[32] See chapter 2 for a discussion on how the values for the parameters of the money demand functions was obtained out of the estimated inverse versions of the same. The obtained cointegrating relationships were:

(i) Log-log: *ltbr=-5.2760- 4.7898 (lrm3), and*;

[*-3.8797*]

(ii) Semi-Log: *lbr = -0.1713 – 0.4547(lrm3).*

[*-3.8888*]

[33] Given that *lrm3, ltbr* and *tbr* are all *I(1)*, the interest elasticity and semi-elasticity are obtained from an OLS estimation of the following equation: $m_t - m_{t-k-1} = a_k + b_k\left[r_t - r_{t-k-1}\right] + e_{kt}$, where *m* is the log of the ratio of money balance, while *r* is the log of the nominal interest rate in the log-log specification and is specified in levels for the semi-log version of the money demand. Following Serletis and Yavari (2004 and 2005), *K* is set equal to 30.

Johansen (1991, 1995) approach based on the systematic sampling, the values of the elasticity and semi-elasticity increases, when compared to those obtained in chapter 2 under temporal aggregation. While, as above, the value of the intercepts increases for the semi-log model and falls for the log-log model. Again as with the cointegration approach, under the long-horizon approach, the t-tests on the interest elasticity and semi-elasticity across the two models under alternative sampling techniques reveal that they are statistically different at one percent level of significance. The results have been presented in Table 4-4. So clearly, unlike as suggested by the theoretical results of Marcellino (1999), long-run elasticities of money demand are significantly affected by alternative sampling techniques. Nevertheless, given the theoretical results of Marcellino (1999), the important aspect that needs to be determined here, would be the robustness of the welfare cost estimates based on the alternative values of the interest elasticity and semi-elasticity of the money demand functions. In other words, we want to know, which of the two estimation methods produces the least changes across the alternative sampling methods.

Having obtained the estimates for η and ξ and the values for A and B, both from the Johansen (1991, 1995) approach and the long-horizon regression, we are now in a position to obtain the welfare cost estimates of inflation, using both Bailey's (1956) consumer surplus approach and Lucas' (2000) compensating variation method. The results have been reported in Table 4-4. Note for the sake of comparison, in Table 4-5, we also present the welfare cost estimates, based on the values of η, ξ, A and B, obtained in both chapter 2 and 3 using both of the above mentioned estimation methodologies. Plugging these values into the corresponding formula for the welfare cost measures, given by equations (4.2), (4.3), (4.7) and (4.9), and using the fact that the average real rate of interest[34] over this period was equal to 7.70 percent, so that a zero rate of inflation would also imply a nominal rate of interest equal to 7.70 percent, we obtain the baseline value of w under price stability. Naturally then, a value of r = 10.70 corresponds to a 3% rate of inflation, while, when r = 13.70, the economy experiences a 6% inflation, and so on.
So the welfare costs of inflation are evaluated by subtracting the value of w at an inflation equal to zero from the value of the same at a positive rate of inflation. Based on Tables 4-4, 4-5[35] and 4-6 the following conclusions can be drawn:

[34] Note, as in Ireland (2009), we define the real rate of return to be equal to the difference between the nominal interest rate and the inflation rate, where the inflation rate is obtained as the percentage change in the seasonally adjusted series of the CPI. In addition, the real rate of interest was found to be stationary based on the ADF, the DF-GLS, the KPSS and the PP tests of unit roots.

[35] Note, we have replicated Table 3-1 from chapter 3 as Table 4-5 in this paper.

Except for the welfare costs evaluated under the compensating variation method for the double log model estimated with the Johansen (1991, 1995) cointegration approach,[36] systematic sampling tends to increase the welfare cost of inflation in all the other cases;

Under the long-horizon approach, irrespective of whether we use systematic sampling or temporal aggregation and the compensating variation or the consumer surplus approach, the pattern of movement of the welfare cost of inflation as we increase the interest rate stays the same. In other words, the welfare cost estimates from the semi-log model tends to be higher than the log-log version at higher interest rates across both methods of aggregation. Further, under both systematic sampling and temporal aggregation, the compensating variation approach produces slightly higher welfare cost estimates for both types of money demand functions;

With the cointegration approach, except for the log-log model under compensating variation with systematic sampling, welfare costs are always lower under the consumer surplus method across both sampling technique and econometric models. Again, as with the long-horizon, the semi-log version of the model tends to yield higher costs of welfare at higher interest rate across the sampling techniques;

Over all, when we compare the two methodologies based on the percentage difference in the welfare cost estimates across the two sampling techniques, the long-horizon approach tends to produce more robust estimates of the welfare cost of inflation via the money market. In other words, for 3%, 6%, 10% and the 15% levels of inflation, the percentage change in the welfare cost of inflation for moving from temporal aggregation to systematic sampling is consistently lower under the Fischer and Seater (1993) approach in comparison to the Johansen (1991, 1995) cointegration methodology. Based on this criteria solely, we would want to conclude that the widest range of the welfare cost estimates for a target band of 3-6% rate of inflation, falls between 0.15% (obtained from the semi-log model estimated with temporal aggregation) to 0.41 % (obtained from the log-log model estimated with systematic sampling) These numbers, in turn, are much lower than the range of 0.34% (obtained from the log-log and semi-log model estimated with temporal aggregation) to 0.90% (obtained from the log-log model estimated with systematic sampling) based on the cointegration approach under alternative methods of sampling.

---

[36] The exception arises due to the fact that even though the interest elasticity increases, the size of the fall in A is such that it tends to reduce the welfare cost estimates, based on equation (4.7), under the Johansen (1991, 1995) approach for the log-log model, when compared to the same model estimated with temporally aggregated data.

Table 4-4: Welfare cost estimates (Systematic Sampling).

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0039	0.0039	0.0019	0.0017	0.0023	0.0040	0.0021	0.0017
6	0.0075	0.0087	0.0038	0.0039	0.0046	0.0090	0.0041	0.0040
10	0.0119	0.0162	0.0062	0.0075	0.0077	0.0168	0.0068	0.0077
15	0.0173	0.0270	0.0092	0.0129	0.0116	0.0283	0.0101	0.0135

Table 4-5: Welfare cost estimates (Temporal aggregation).

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	0.0034	0.0034	0.0018	0.0015	0.0037	0.0035	0.0019	0.0016
6	0.0067	0.0076	0.0035	0.0035	0.0072	0.0079	0.0037	0.0036
10	0.0108	0.0143	0.0057	0.0068	0.0117	0.0149	0.0062	0.0070
15	0.0156	0.0241	0.0084	0.0118	0.0172	0.0251	0.0092	0.0123

Table 4-6: Percentage Change in Welfare Cost Estimate Under Temporal Aggregation and Systematic Sampling.

	Consumer Surplus Method				Compensating Variation Method			
	Johansen Approach		Long-Horizon		Johansen Approach		Long-Horizon	
Inflation Rate	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log	Log-log	Semi-log
3	14.71	14.71	5.56	13.33	37.84	14.29	10.53	6.25
6	11.94	14.47	8.57	11.43	36.11	13.92	10.81	11.11
10	10.19	13.29	8.77	10.29	34.19	12.75	9.68	10.00
15	10.90	12.03	9.52	9.32	32.56	12.75	9.78	9.76

Values Computed Using: $\left|\left(\frac{(w^{syst} - w^{temp})}{w^{temp}}\right)\right| \times 100$

4.4 Conclusions

The two previous chapters have found markedly different measures of the welfare cost of inflation in South Africa, obtained through the estimation of long-run money demand relationships using cointegration and long-horizon approaches, respectively. Realizing that the monetary aggregate and the interest rate variables are available at higher frequencies than the measure of income, and that long-run properties of data are unaffected under alternative methods of time aggregation (Marcellino, 1999), in this chapter , we tested for the robustness of the two estimation procedures under temporal aggregation and systematic sampling. Our results indicate that the long-horizon method is more robust, in terms of lower percentage change in the welfare cost measures across the two alternative methods of time aggregation, and, given this the welfare cost of inflation in South Africa for an inflation target band of 3-6% lies between 0.15% and 0.41%. Based on these set of results, we can, thus, conclude that the SARB's current inflation target band of 3-6 percent provides quite a good approximation in terms of welfare, at least when compared to a Friedman (1969)-type deflationary rule of zero nominal rate of interest.

It is, however, important to point out that, in this chapter, we are only looking at welfare cost of inflation using a partial equilibrium approach. But as argued by Dotsey and Ireland (1996), in a general equilibrium framework, rise in the inflation rates can distort other marginal decisions and, hence, can negatively impact both the level and the growth rate of aggregate output. In addition, as pointed out by Feldstein (1997), interactions between inflation and a non-indexed tax code can add immensely to the welfare cost of inflation. Hence, the path ahead should involve obtaining the size of the welfare cost of inflation using a dynamic general equilibrium endogenous growth model. Then only, we will be able to deduce whether there are possibly larger gains of reducing the inflation target below 3%.