1 INTRODUCTION

1.1 Background

The calculation of two-way slab deflections poses the dual difficulties of solving complex governing differential equations and taking into account the effects of material non-linearity, i.e. cracking and creep.

Various simplified methods of elastic analysis for two-way slab systems, suitable for hand calculation, have been proposed and adopted in national building codes. Unfortunately these methods have limited applicability when the slab supports are not rectangular in plan. In contrast, the finite element approach provides a convenient method for analysis where support layouts are irregular. The discretisation of the slab allows the analyst to model almost any shape of slab and the support layout is limited only by node positions. Although the finite element method is a numerical approximation of the 'exact' solution, careful modelling and choice of formulation yields results in good agreement with classical methods.

Load induced cracking, which for the purposes of this dissertation occurs when the stress at the tension face exceeds the modulus of rupture of the concrete, influences both the magnitude of deflection and the distribution of bending moments and shear forces in the slab. At the cracked section, it is assumed that the concrete is free of tensile stress, but this does not hold true for the uncracked zones between fully cracked sections where tensile stresses are transferred to the concrete by bonded reinforcement. The ability of uncracked concrete to contribute to the overall stiffness of a member is referred to as tension stiffening. The simplest method of modelling this effect involves the use of an average cracked section. Numerically, this implies a modification of the second moment of area.

Concrete response to load comprises instantaneous and time-dependent components. The time-dependent component can be attributed to the related effects of creep and shrinkage.

The study of the creep deflection consists of two aspects:

- Prediction of the creep and shrinkage behaviour of a concrete element which includes material and environmental factors. This prediction usually takes the form of a creep factor or function and a shrinkage strain.
- Incorporating the predictive parameters in the analysis of a reinforced concrete member.

This dissertation focuses on the latter aspect and no attempt is made to investigate the various creep and shrinkage models that are currently in use. Numerous methods are available for time-dependent analyses, ranging from complex rheological models to simple effective modulus methods. Faber's Effective Modulus method (Gilbert, 1988), where the elastic modulus of the concrete is adjusted with a creep factor, is probably the best known method.

The method proposed here employs the finite element method in a semi-iterative approach to the deflection problem. The instantaneous deflection is calculated using Branson's Effective Moment of Inertia (Branson, 1968) or the bilinear method (Favre *et al*, 1985). The resulting member actions are then used in a time analysis to establish the creep and shrinkage contribution to the final deflection.

The method derives from a simplified approach to slab tension stiffening suggested by Polak (1996). Polak applied Branson's effective moment of inertia to the finite element method using reduction factors to account for cracking and tension stiffening. Reinforcement has the effect of reducing the magnitude of creep deflections in concrete slabs. This effect is also accounted for using reduction factors.

Rigorous approaches to the long term deflection of concrete slabs, as applied to the finite element method, abound in journals such as *Advances in Engineering Software* published by *Elsevier* and are even commercially available in structural simulation software such as *DIANA,* which is developed by TNO Diana, a company based in the Netherlands. This dissertation proposes a simple method for use and elaboration by practicing engineers.

1.2 Objectives of the Study

The study has the broad objective of establishing a simple method of analysis, based on the finite element method, for the calculation of the long-term deflections of reinforced concrete flat slabs.

This objective is pursued under four components of deflection:

- Elastic deflection:
  The Mindlin Serendipity plate element, as applied to slab problems, is investigated. Convergence characteristics and the influence of aspect ratios are studied to verify the applicability of the element in the current context.

- Cracked deflection:
  The results of previous studies of tension stiffening are reproduced and the bilinear method is investigated as an alternative to Branson's method. The alternative is attractive due to the simple manner in which the influence of creep on the behaviour of a cracked section is modelled.
- Creep deflection:
  The objective of this section is to find a technique of incorporating the method of section curvatures in the finite element formulation.
- Shrinkage deflection:
  The resulting deformation of shrinkage is transformed into applied loading and the results are compared to experimental results to verify the accuracy of the procedure.

1.3 Scope of the Study

1.3.1 Geometry and Supports

Although nothing in the method prohibits non-rectangular layouts, only this type of layout was studied, mainly to avoid the use of distorted rectangular or triangular elements since distortion impacts negatively on the accuracy of the finite element analysis.

1.3.2 Finite Element Formulation

An eight-noded, Serendipity plate element based on the Mindlin formulation was chosen for the dissertation and no other element or plate formulation types were considered, although consideration is given to the numerical issues pertaining to this element. Membrane effects are not considered.

An important aspect of the finite element formulation is the accurate modelling of supports or boundary conditions. However, discontinuous boundary conditions such as those encountered with the column supports of flat slabs pose some problems. The simplest approach is a support, fixed in both translation and rotation, at a node placed at the column centre. Theoretically this implies infinite stress resultants at the point of support, although with the finite element approximation this value is finite but large.

This dissertation follows this simple approach and no consideration is given to alternative approaches such as elastic supports. Admittedly the simple approach will tend to overestimate the actual long term deflection, due to the large numerical values of the support moments and the associated higher degree of cracking.

1.3.3 Reinforcement Layout

Reinforcement directions are assumed to be parallel with the global x and y-axis. With the rectangular layout constraint introduced in paragraph 1.3.1, this will almost always be the case.

1.3.4 Constitutive model

Concrete and reinforcement behaviour are assumed to be linear elastic, although modifications are made for concrete creep and cracking. Reference is made to rigorous models, but no comparison is done with the results of these methods.

1.4 Methodology

A finite element program was developed specifically for this dissertation. The program follows the Object Orientated Programming (OOP) paradigm, which allows for easy extension and modular problem solution. The finite element is encapsulated in an object and the various effects of cracking, creep and shrinkage were added to this element object.

The results of the program are compared with the following analytical, experimental and hand-calculation results:

- Linear elastic analysis: Classical thin plate and moderately thick plate solutions for simply supported and clamped plates.
- Short term crack analysis: Experimental slab deflection results from two sources.
- Long term creep, crack and shrinkage analysis: Experimental results for a beam and hand calculation results for a flat slab panel.

1.5 Software Development

The source provided by Hinton and Owen (1983) was used as the basis for the implementation. The source was originally written in FORTRAN which was translated by the author to Object Pascal. The source was compiled for the Microsoft Windows environment using Borland Delphi 6.0, Borland (2001).

The user interface components used in the program were donated by Prokon Software Consultants which facilitated the process of input as well as the output of analysis results.

The OOP paradigm mentioned in section 1.4, adds a level of abstraction to software which allows for modular programming and extensibility. The main program remains unaware of the implementation details of the element object and the element formulation can thus be changed without influencing the logic of the main program.

1.6 Organisation of the Report

The report is split into the following chapters:

Chapter 2 provides theoretical background to the topics introduced above under the following headings:

- Analysis: The finite element and a hand-calculation method are considered.
- Cracked Sections: The bilinear method and the effective moment of inertia are compared.
- Creep: A single approach, based on the effective modulus method is presented.
- Shrinkage: A single approach based on basic theory is presented.

Chapter 3 presents the proposed method under the same headings as above and discusses the implementation of the method in computer code. Selected parts of the program are included in the Appendix for reference. Section 7.2 contains the source of the entire, self contained, finite element object. Sections 7.3 to 7.6 contain the various analysis procedures.

Chapter 4 documents the results of various analysis runs compared to alternative methods and published experimental data.

Conclusions regarding the accuracy and applicability of the proposed method are presented in Chapter 5. Chapter 6 lists the references and chapter 7 contains the appendix.