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In this chapter on the research findings，an analysis of good quality items and poor quality items，both PRQs and CRQs，in terms of the Quality Index developed in section 5．3．2，within each of the seven mathematics assessment components，will be presented．

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The traditional statistical analysis of data，supplied by the Computer Network Services（CNS）Division of the University of the Witwatersrand，include the Performance Index，Discrimination Index and Easiness／Difficulty factor per question for all tests（PRQ and CRQ）during the period of study，July 2004 to July 2006.

Raw data，including students＇responses to test items and confidence of responses，was obtained from the Computer Network Services（CNS）Division of the University of the Witwatersrand．Spreadsheets were constructed using a ＇Mathematica＇programme developed by a statistician from the School of Statistics at the University of the Witwatersrand．The following information was captured on every spreadsheet per test：
－students＇responses to all test items，both PRQ and CRQ
－students＇confidence of responses per test item，both PRQ and CRQ．

The correct answers and mathematics assessment components per test item were also recorded for reference purposes．Student numbers were not recorded on every spreadsheet．In constructing these spreadsheets，records were excluded if：
(i) the student had failed to provide an answer; or
(ii) the student had failed to provide a confidence of response; or
(iii) the student had filled in the MCQ card incorrectly.

It should be noted that in most cases the excluded records were due to (ii) above. The proportion of all the records excluded in this manner ranged between $7,2 \%$ and $8,9 \%$ across the tests. All subsequent calculations were performed on this filtered data.

For PRQs and CRQs, the Performance Index (PI) per question was equal to the proportion of (filtered) respondents who obtained the correct answer. It should be noted that the "easiness/difficulty" statistic provided on the CNS printouts is equal to the Performance Index i.e. Performance Index = Difficulty Index. An overall Confidence Index (per assessment component) was calculated by averaging the Cls per question for all questions in that assessment component. An overall Performance Index or Difficulty Index (per assessment component) was calculated in a similar manner by averaging the Pls per question for all questions in that assessment component.

Stability (test- retest) was achieved by administering the same tutorial tests in March and August over the period 2004-2006. Equivalence was achieved over the period of study by administering different tests to the same cohort of students (Mathematics I Major) in each of the 3 years, 2004, 2005 and 2006 respectively. Internal consistency was achieved by correlating and equating the items in each test to each other, as described under test item calibration in section 6.2.1.

## * 䊾楼

The Rasch model (Rasch, 1960), as discussed in section 3.4.1, was used to evaluate both the attitudinal data (confidence levels) as well as test data. The Winsteps (Linacre \& Wright, 1999) Rasch analysis programme was utilised by a data analyst from the University of Pretoria for the quantitative data analysis in this research study. In particular the WINSTEPS® Version 3.55 .0 was used to
analyse the data in this study. SAS Version 9 and Microsoft EXCEL 2003 were also used in calculating totals and means.

The Winsteps software, developed by John M Linacre in 2005, constructs Rasch measures from simple rectangular data sets, usually of persons and items. Item types that can be combined in one analysis include dichotomous, multiple choice and partial credit items. Paired comparisons and rank-order data can also be analysed. Missing data is no problem. Winsteps is designed as a tool that facilitates exploration and communication. The structure of the items and persons can be examined in depth. Unexpected data points are identified and reported in numerous ways. Powerful diagnosis of multidimensionality through principal components analysis of residuals detects and quantified substructures in the data. The working of rating scales can be examined thoroughly, and rating scales can be recoded and items regrouped to share rating scales as desired (Linacre, 2002). Measures can be fixed (anchored) at pre-set values (Linacre, 2005).

In order to prepare the data in an ASCII format to import into Winsteps, SAS was used to create ASCII files with a specific layout. Control files were prepared in Winsteps for each part of each test, i.e. the PRQ part, the CRQ part as well as the confidence index part. This was done as the different Rasch models, discussed in section 3.4.1.3, were applicable to the different types of data. These parts of the tests were first analysed separately to check for model "fit". Such "fit" statistics help detect possible idiosyncratic behaviour on the part of respondents and test items. Those respondents who exhibited "misfit" were first investigated for coding errors, and then their raw hard-copy responses were reviewed for evidence of non-attention to the test. Such individuals might be ones who are haphazardly circling responses or those who are guessing and/or miscoding.

Winsteps provides ways of diagnosing problems in the analysis. In the first place the point measure values were considered. Where items exhibited negative point measure values, these items were scrutinised for errors such as an
incorrect key and corrected．If the point measure stayed negative，the item was removed from the analysis．Subsequently，the output tables for person ability and item difficulty were checked for misfitting entries．Person ability tables were considered first．

## 大法炇

Some explanation in terms of misfitting items or students is in order．One would expect that a student of medium mathematical ability would be able to respond correctly to easier items in the test and incorrectly to the difficult items in a test． Where the item difficulty matches the ability of the student，one would expect the student to answer some of these items correct and some incorrectly．If an item＇s difficulty corresponds exactly to the student＇s ability，the probability of success of the student on that item is 0.5 ，in other words，success or failure is expected equally．The Rasch model assumes this pattern of responses，and the Infit and Outfit mean－square statistics are 1．0．If for example，a student would guess the answer to a difficult item correctly（one that the student should really get wrong） the Outfit statistic would be much larger than 1.0 because it is sensitive to outliers．

The approach used in the analyses of this study＇s data was that items and persons were accepted as not misfitting when Infit mean－square statistics was from 0.5 to 1.5 ．Where the values were less than 0.5 ，too much predictability or overfit was experienced and when the value exceeded 1.5 ，too much noise was present in the data or a situation of underfit existed．The Infit statistics were considered first，and then the Outfit statistics．

Mean－square statistics indicate the size of the misfit，but the＂significance＂of the improbability of the misfit is important．

Misfitting persons were deleted，and the analysis was repeated．Another round of misfitting persons were removed from the analysis．Only then were the fit
statistics of the items considered. If an item proved to be problematic in terms of the fit statistics, the item was also removed from any subsequent analysis.

The same procedure was followed to explore the misfitting persons and items in terms of the CRQs and the confidence index.

For the PRQs, the dichotomous Rasch model applies:

$$
P_{v i}=\frac{e^{(\beta v-\delta i)}}{1+e^{(\beta v-\delta i)}}
$$

In the confidence index, the same categories were available throughout and were thus analysed according to the Rasch-Andrich rating scale model:

$$
\ln \left(\frac{P_{v i x}}{P_{v i(x-1)}}\right)=\beta_{v}-\delta_{i}-F_{x}
$$

CRQs were analysed through the application of the Partial Credit model:

$$
\ln \left(\frac{P_{v i x}}{P_{v i(x-1)}}\right)=\beta_{v}-\delta_{i}-F_{i x}
$$

These various Rasch models have already been discussed in more detail in section 3.4.1.

## 

Through the application of the Rasch family of models it is also possible to put the measures of different tests onto the same scale if certain assumptions are made. The tests can be linked either through common items on the tests or through common students writing the tests. A challenge in terms of the data faced the researcher. Although, as mentioned previously, it was known that the same cohort of students wrote the same tests in a calendar year, the student identification numbers were not available on all the data sets and therefore no linking could take place on a one-to-one basis. The strong assumption was then made that the subject matter of the different tests were distinct and that the tests
could therefore be regarded as independent. In other words, it was assumed that because the subject matter was distinct, students' ability did not improve progressively throughout the year. This assumption led to the decision that all the data could be calibrated together, anchoring the items that were common over the three years. In this way, the item difficulties and the student measures were on the same scale and were deemed directly comparable.

Fit statistics were again considered and if in the combined calibration of items any misfitting items were identified, they were excluded from the analysis. A small number of items misfitted, and this is not to be unexpected in such a large data set.

The same procedure was followed in terms of the CRQs. In order to place the measures of the PRQs and the CRQs on the same scale, a combined calibration of these items was also executed. Another challenge presented itself. At first, when the PRQs and the CRQs were calibrated together, the whole set of CRQs misfitted. It was then decided to recode the partial credit items into dichotomous items in the following way: If a student scored less than half the marks, the student was awarded a 0 for that specific item; if the student scored half or more of the marks on an item, the student was awarded a 1 for the item. The CRQs were therefore eventually analysed through the same model as the PRQs i.e. the dichotomous Rasch model, and the combined calibration of items then produced a set of items that mostly fitted the Rasch model.

## 

A similar process was followed to determine the item difficulties of the confidence levels. The item difficulty for a rating scale is defined as the point where the top and bottom categories are equally probable (Linacre, 2005).

## 

Response data from 14 different mathematics tests written between August 2004 and June 2006 were available. Table 6.1 is a representation of the tests written, the number of provided response items (PRQs) per test, the number of constructed response items (CRQs) and the number of students per test. The same cohort of students (Mathematics I Major) wrote the tests in each of the three years, 2004, 2005 and 2006 respectively.

Table 6.1: Characteristics of tests written.

| Year | Month | Number of PRQs | Number of CRQs | Number of students |
| :---: | :---: | :---: | :---: | :---: |
| 2004 | August | 10 | 0 | 457 |
| 2005 | March | 8 | 0 | 410 |
| 2005 | April Tutorial A | 8 | 0 | 263 |
| 2005 | April Tutorial B | 8 | 0 | 126 |
| 2005 | May | 8 | 0 | 403 |
| 2005 | June | 12 | 17 | 414 |
| 2005 | August | 10 | 0 | 389 |
| 2005 | September | 8 | 17 | 387 |
| 2005 | November | 15 | 18 | 385 |
| 2006 | March | 8 | 15 | 352 |
| 2006 | April Tutorial A | 8 | 0 | 245 |
| 2006 | April Tutorial B | 8 | 0 | 105 |
| 2006 | May | 8 | 14 | 359 |
| 2006 | June | 12 | 24 | 348 |

Out of a total of 221 PRQ and CRQ items, seven items were discarded because their fit statistics indicated that they did not fit the model. Table 6.2 included in the Appendix A5, presents these items with their fit statistics. Another seven items (I115M09- I115M15) were discarded because the actual items were not available. Finally, 207 items were included in the analyses. The Rasch statistics
for all 207 test items analysed are included in Appendix A6. Confidence level items Rasch statistics are included in Appendix A7.

## $x$



Examples of questions in the different mathematics assessment components are now presented. Within each of the seven assessment components, both PRQs and CRQs, ranging from easy to difficult, and of good and poor quality are presented. For each item, the question is followed by a radar plot and a table summarising the quality parameters of the test item i.e. item difficulty; discrimination; confidence index; expert opinion and the final quality index, as discussed in the theoretical framework in Chapter 5. Each of the axes of the radar plots are labelled with the corresponding values for discrimination, confidence index and expert opinion. The Quality Index (QI) is displayed alongside the radar plot. The shading of the radar plot corresponds to one of the six item difficulty levels as classified in Table 5.4. The comments briefly summarise the difficulty level, the three measuring criteria as developed in the theoretical framework and the overall quality of the item.

## A651(a)

Find the constant term in $\left(-x^{2}+\frac{1}{x}\right)^{12}$
CRQ, Algebra, June 2005, Q1a


| A651a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 1.10 | Moderately difficult |
| Discrimination | 0.295 | Discriminates well |
| Confidence Index | 0.385 | Small deviation from expected confidence level |
| Expert Opinion | 0.236 | Small deviation from expected performance |
| Quality Index | 0.119 | Good quality CRQ (excellent) |

Write $-2 \cos x+2 \sqrt{3} \sin x$ in the form $R \cos (x-\theta)$


| A652a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -0.33 | Moderately easy |
| Discrimination | 0.501 | Discriminates fairly well |
| Confidence Index | 0.318 | Small deviation from expected confidence level |
| Expert Opinion | 0.574 | Large deviation from expected performance |
| Quality Index | 0.273 | Good quality CRQ (moderate) |

C115MO7
The limit of the sequence $\left\{\frac{1}{n!}\left(-5+(-1)^{n}\right)\right\}$ is
A. -5
B. 1
C. 0
D. the sequence diverges

PRQ, Calculus, November 2005, Q7

## C115M07



| C115M07 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -1.12 | Moderately easy |
| Discrimination | 0.666 | Does not discriminate well |
| Confidence Index | 0.343 | Small deviation from expected confidence level |
| Expert Opinion | 0.416 | Small deviation from expected performance |
| Quality Index | 0.281 | Good quality PRQ (moderate) |

A1155bii
Let $A=\left(\begin{array}{ccc}1 & 1 & 1 \\ a b & b c & c a \\ a+b & b+c & c+a\end{array}\right)$
For what value(s) of $a, b, c$ does $A^{-1}$ exist?
CRQ, Algebra, November 2005, Q5bii


| A1155bii |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 2.23 | Difficult |
| Discrimination | 0.522 | Discriminates poorly |
| Confidence Index | 0.347 | Small deviation from expected confidence level |
| Expert Opinion | 0.736 | Large deviation from expected performance |
| Quality Index | 0.356 | Poor quality CRQ |

## A661.1

$$
P(n)=n^{3}+(n+1)^{3}+(n+2)^{3} \text { is divisible by } 9
$$

Show that the statement is true for $n=2$
CRQ, Algebra, June 2006, Q1.1


| A661.1 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -2.35 | Easy |
| Discrimination | 0.975 | Discriminates weakly |
| Confidence Index | 0.324 | Small deviation from expected confidence level |
| Expert Opinion | 0.410 | Small deviation from expected performance |
| Quality Index | 0.367 | Poor quality CRQ |

## A56MO2

The exact value of $\arctan (\tan (5 \pi / 3))$ is
A. $5 \pi / 3$
B. $-5 \pi / 3$
C. $-\pi / 3$
D. $\pi / 3$
E. $2 \pi / 3$

PRQ, Algebra, May 2006, Q2


| A56M02 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 0.77 | Moderately difficult |
| Discrimination | 0.563 | Weak discrimination |
| Confidence Index | 0.643 | Large deviation from expected confidence level |
| Expert Opinion | 0.453 | Small deviation from expected performance |
| Quality Index | 0.393 | Poor quality PRQ |

C65M08
If $\int_{3}^{5} g(x) d x=5$ and $\int_{3}^{5} h(x) d x=-1$, then $\int_{3}^{5}(2 g(x)-5 h(x)) d x=$
A. 5
B. 15
C. 7
D. 0
E. -27

PRQ, Calculus, June 2005, Q8

## C65M08



| C65M08 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Technical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -1.04 | Moderately easy |
| Discrimination | 0.749 | Weak discrimination |
| Confidence Index | 0.437 | Small deviation from expected confidence level |
| Expert Opinion | 0.674 | Large deviation from expected performance |
| Quality Index | 0.488 | Poor quality PRQ |

## A35M08

Let $a, b$ and $c$ be real numbers. Which of the following is the correct statement?
A. $a<b \Rightarrow a+b>b+c$.
B. $a>b \Rightarrow a c>b c$.
C. $|x|>a \Leftrightarrow-a<x<a$.
D. $\sqrt{c^{2}}=c$.
E. $0<a<b \Rightarrow \frac{1}{b}<\frac{1}{a}$.


| A35M08 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 2.25 | Difficult |
| Discrimination | 0.069 | Discriminates very well |
| Confidence Index | 0.842 | Large deviation from expected confidence level |
| Expert Opinion | 0.355 | Small deviation from expected performance |
| Quality Index | 0.165 | Good quality PRQ |

Prove, using the Intermediate Value Theorem, that there is a number exactly 1 more than its cube.

CRQ, Calculus, March 2006, Q3b

## C363b



| C363b |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 3.94 | Very difficult |
| Discrimination | 0.295 | Discriminates well |
| Confidence Index | 0.274 | Small deviation from expected confidence level |
| Expert Opinion | 0.574 | Large deviation from expected performance |
| Quality Index | 0.177 | Good quality CRQ |

A bacterial colony is estimated to have a population of $P(t)=\frac{24 t+10}{t^{2}+1}$ million, $t$ hours after the introduction of a toxin.

At what rate is the population changing 1 hour after the toxin is introduced?
CRQ, Calculus, May 2006, Q1a(i)

## C561ai



| C561ai |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -2.63 | Easy |
| Discrimination | 0.543 | Discriminates fairly well |
| Confidence Index | 0.460 | Small deviation from expected confidence level |
| Expert Opinion | 0.262 | Small deviation from expected performance |
| Quality Index | 0.222 | Good quality CRQ |

The Cartesian coordinates $(x, y)$ of the point $(r, \theta)=\left(2 \sqrt{3}, \frac{3 \pi}{4}\right)$ are:
A. $(-\sqrt{6},-\sqrt{6})$
B. $(-\sqrt{6}, \sqrt{6})$
C. $(\sqrt{6},-\sqrt{6})$
D. $(-3,2)$

## A55RA07



| A55M07 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -0.76 | Moderately easy |
| Discrimination | 0.790 | Does not discriminate well |
| Confidence Index | 0.294 | Small deviation from expected confidence level |
| Expert Opinion | 0.290 | Small deviation from expected performance |
| Quality Index | 0.236 | Good quality PRQ (moderate) |

C364b(i)

Let $\llbracket x \rrbracket$ be the greatest integer less than or equal to $x$.
Show that $\lim _{x \rightarrow 2} f(x)$ exists if $f(x)=\llbracket x \rrbracket+\llbracket-x \rrbracket$.
CRQ, Calculus, March 2006, Q4b(i)

## C364bi



| C364bi |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 4.19 | Very difficult |
| Discrimination | 0.501 | Discriminates fairly well |
| Confidence Index | 0.501 | Average deviation from expected confidence <br> level |
| Expert Opinion | 0.547 | Large deviation from expected performance |
| Quality Index | 0.346 | Poor quality CRQ |

## C563a(i)

Consider the following theorem:
Let $f$ be a function that satisfies the following three conditions:
(1) $\quad f$ is continuous on the closed interval $[a, b]$.
(2) $f$ is differentiable on the open interval $(a, b)$.
(3) $f(a)=f(b)$.

Then there exists a number $c \in(a, b)$ such that $f^{\prime}(c)=0$.
What is this theorem called?

## C563ai



| C563ai |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -4.74 | Very easy |
| Discrimination | 0.831 | Discriminates poorly |
| Confidence Index | 0.545 | Large deviation from expected confidence level |
| Expert Opinion | 0.273 | Small deviation from expected performance |
| Quality Index | 0.359 | Poor quality CRQ |

## C45MB5

If $\lim _{x \rightarrow 2} f(x)$ exists, then
A. $f(2)$ is undefined
B. $f(2)=3$
C. $f(2)=2$
D. $f(2)$ is unknown

PRQ, Calculus, March 2005, Tut Test 1B, Q5

## C45RBB5



| C45MB5 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 1.91 | Difficult |
| Discrimination | 0.749 | Discriminates poorly |
| Confidence Index | 0.521 | Large deviation from expected confidence level |
| Expert Opinion | 0.409 | Small deviation from expected performance |
| Quality Index | 0.394 | Poor quality PRQ |

C36M02

Find the following limit: $\quad \lim _{x \rightarrow 2} \frac{x^{2}-4}{x-2}$
A. does not exist
B. -2
C. 4
D. 2
E. 1

PRQ, Calculus, March 2006, Q2
C36 ${ }^{\text {H }} 02$


| C36M02 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Disciplinary |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -5.05 | Very easy |
| Discrimination | 0.872 | Discriminates very poorly |
| Confidence Index | 0.822 | Very large deviation from expected confidence <br> level <br> Expert Opinion$\quad 0.239$ |
| Quality Index | 0.486 | Small deviation from expected performance |

C65M09
Choose the correct statement, given that $\int_{0}^{5} f(x) d x=9$ and $\int_{2}^{5} f(x) d x=-1$.
A. $\int_{0}^{2} f(x) d x=10$
B. $\int_{2}^{0} f(x) d x=10$
C. $\int_{5}^{2} f(x) d x=-1$
D. $\int_{0}^{2} f(x) d x=8$
E. None of the above

PRQ, Calculus, June 2005, Q9

## C65MO9



| C65M09 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 1.72 | Difficult |
| Discrimination | 0.110 | Discriminates well |
| Confidence Index | 0.351 | Small deviation from expected confidence level |
| Expert Opinion | 0.608 | Large deviation from expected performance |
| Quality Index | 0.138 | Good quality PRQ |

## A1152b

Find the equation of the plane which passes through the point $A(2,3,-5)$ and which contains the line $l:(-1,3,-2)+t(-2,1,5)$

CRQ, Algebra, November 2005, Q2b

## A1152b



| A1152b |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 2.93 | Difficult |
| Discrimination | 0.357 | Discriminates well |
| Confidence Index | 0.255 | Small deviation from expected confidence level |
| Expert Opinion | 0.373 | Small deviation from expected performance |
| Quality Index | 0.138 | Good quality CRQ (excellent) |

C1157a

Find $\int x \cos x d x$
CRQ, Calculus, November 2005, Q7a

## C1157a



| C1157a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -1.45 | Moderately easy |
| Discrimination | 0.522 | Average discrimination |
| Confidence Index | 0.249 | Small deviation from expected confidence level |
| Expert Opinion | 0.483 | Small deviation from expected performance |
| Quality Index | 0.218 | Good quality CRQ |

## C45MB8

If $\lim _{x \rightarrow a} f(x)=2$ and $\lim _{x \rightarrow a} g(x)=3$ then $\lim _{x \rightarrow a} \frac{3 f(x)-(g(x))^{2}}{g(x)}=$
A. $\frac{13}{3}$
B. -1
C. $-\frac{3}{2}$
D. 1

PRQ, Calculus, March 2005, Tut Test 1B, Q8

## C4519B8



| C45MB8 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -1.94 | Easy |
| Discrimination | 0.604 | Discriminates poorly |
| Confidence Index | 0.410 | Small deviation from expected confidence level |
| Expert Opinion | 0.284 | Small deviation from expected performance |
| Quality Index | 0.232 | Good quality CRQ (moderate) |

## A95M02

$P Q R$ is a triangle with vertices $P(3,1), Q(5,2)$ and $R(4,3) . P \hat{Q} R$ equals
A. $\arccos \frac{4}{5}$
B. $\arccos \frac{1}{\sqrt{10}}$
C. $\pi-\arccos \frac{4}{5}-\arccos \frac{1}{\sqrt{10}}$
D. $\arccos \frac{-1}{\sqrt{10}}$

PRQ, Algebra, August 2005, Tut Test, Q2


| A95M02 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -3.22 | Very easy |
| Discrimination | 0.769 | Discriminates poorly |
| Confidence Index | 0.406 | Fairly small deviation from expected confidence <br> Expert Opinion |
| Quality Index | 0.333 | Small deviation from expected performance |

## C55M04

The graph below is of the derivative of a function $g(x)$, i.e. the graph of $y=g^{\prime}(x)$.


The critical numbers of $g(x)$ are
A. $-2,2$
B. $-3,3$
C. $-2,2,-3,3$
D. $-2,-3,3$

PRQ, Calculus, May 2005, 04


| C55M04 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 1.50 | Moderately difficult |
| Discrimination | 0.336 | Discriminates well |
| Confidence Index | 0.723 | Large deviation from expected confidence level |
| Expert Opinion | 0.546 | Large deviation from expected performance |
| Quality Index | 0.356 | Poor quality PRQ |

Consider the following theorem:
Theorem: If a function $f$ is continuous on the closed interval $[a, b]$ and $F$ is an antiderivative of $f$ on $[a, b]$, then $\int_{a}^{b} f(x) d x=F(b)-F(a)$.

What is this theorem called?

CRQ, Calculus, August 2005, Q3a


| C953a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -5.56 | Very easy |
| Discrimination | 1.000 | Discriminates very poorly |
| Confidence Index | 0.497 | Large deviation from expected confidence level |
| Expert Opinion | 0.434 | Fairly small deviation from expected performance |
| Quality Index | 0.562 | Poor quality CRQ |

C953b

Consider the following theorem:
Theorem: If a function $f$ is continuous on the closed interval $[a, b]$ and $F$ is an antiderivative of $f$ on $[a, b]$, then $\int_{a}^{b} f(x) d x=F(b)-F(a)$.

Consider the proof of this theorem:
Proof: Divide the interval $[a, b]$ into $n$ sub-intervals by the points
$a=x_{0}<x_{1}<\ldots<x_{n-1}<x_{n}=b$.
Show that $F(b)-F(a)=\sum_{i=1}^{n}\left[F\left(x_{i}\right)-F\left(x_{i-1}\right)\right]$.
CRQ, Calculus, August 2005, Q3b

C953b


| C953b |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Conceptual |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 2.4 | Difficult |
| Discrimination | 0.831 | Discriminates poorly |
| Confidence Index | 0.839 | Large deviation from expected confidence level |
| Expert Opinion | 0.865 | Large deviation from expected performance |
| Quality Index | 0.927 | Poor quality CRQ |

## A662.2

Use properties of sigma notation and the fact that
$\sum_{r=1}^{n} r=\frac{n(n+1)}{2}$ to prove that $\sum_{r=1}^{n} r^{2}=\frac{n(n+1)(2 n+1)}{6}$.
CRQ, Algebra, June 2006, Q2.2

A6622


| A662.2 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 1.52 | Difficult |
| Discrimination | 0.048 | Discriminates well |
| Confidence Index | 0.495 | Average deviation from expected confidence <br> level |
| Expert Opinion | 0.251 | Small deviation from expected performance |
| Quality Index | 0.069 | Good quality CRQ (excellent) |

A55M08
You are given the sector $O A B$ of a circle of radius 2 with $A C=p$.

C. $\arctan p / 2$
D. $2 \arctan (p / 2)$

PRQ, Algebra, May 2005, Q8


| A55M08 |  | Comment |
| :--- | :--- | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 0.15 | Moderately difficult |
| Discrimination | 0.378 | Discriminates well |
| Confidence Index | 0.479 | Small deviation from expected confidence level |
| Expert Opinion | 0.504 | Average deviation from expected performance |
| Quality Index | 0.265 | Good quality PRQ (moderate) |

## A562a

A polar graph is defined by the equation $r(\theta)=5 \cos 3 \theta$ for $\theta \in[0,2 \pi]$ Is the graph symmetric about the $x$-axis, the $y$-axis, both or neither? Motivate your answer.

CRQ, Algebra, May 2006, Q2a


| A562a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -1.62 | Easy |
| Discrimination | 0.295 | Discriminates well |
| Confidence Index | 0.620 | Large deviation from expected confidence level |
| Expert Opinion | 0.487 | Small deviation from expected performance |
| Quality Index | 0.272 | Good quality CRQ (moderate) |

If $z=3+2 i$ and $w=1-4 i$, then in real-imaginary form $\frac{z}{w}$ equals:
A. $-\frac{5}{17}+\frac{14}{17} i$
B. $\frac{5}{15}-\frac{14}{\sqrt{15}} i$
C. $3-4 i$
D. $\frac{11}{17}+\frac{14}{17} i$

PRQ, Algebra, August 2005, Tut Test Q5
AB5 $\mathrm{F} \mathbf{4 0 5}$

## Adapted discrimination



| A85M05 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -2.31 | Easy |
| Discrimination | 0.687 | Discriminates poorly |
| Confidence Index | 0.652 | Large deviation from expected confidence level |
| Expert Opinion | 0.249 | Small deviation from expected performance |
| Quality Index | 0.338 | Poor quality PRQ |

If $\lim _{x \rightarrow a}[f(x)+g(x)]$ exists, then
A. $\lim _{x \rightarrow a} f(x)=\lim _{x \rightarrow a} g(x)$.
B. neither $\lim _{x \rightarrow a} f(x)$ nor $\lim _{x \rightarrow a} g(x)$ exists.
C. both $\lim _{x \rightarrow a} f(x)$ and $\lim _{x \rightarrow a} g(x)$ exist.
D. we cannot tell if $\lim _{x \rightarrow a} f(x)$ or $\lim _{x \rightarrow a} g(x)$ exists.

C46MA5


| C46MA5 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 2.47 | Difficult |
| Discrimination | 0.481 | Average discrimination |
| Confidence Index | 0.700 | Large deviation from expected confidence level |
| Expert Opinion | 0.470 | Small deviation from expected performance |
| Quality Index | 0.386 | Poor quality PRQ |

## A562d

A polar graph is defined by the equation $r(\theta)=5 \cos 3 \theta$ for $\theta \in[0,2 \pi]$.
What is the name of this polar graph?

CRQ, Algebra, May 2006, Q2d


| A562d |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -1.42 | Moderately easy |
| Discrimination | 0.625 | Discriminates poorly |
| Confidence Index | 0.743 | Large deviation from expected confidence level |
| Expert Opinion | 0.424 | Small deviation from expected performance |
| Quality Index | 0.452 | Poor quality CRQ |

Consider the following theorem:
Let $f$ be a function that satisfies the following three conditions:
(1) $f$ is continuous on the closed interval $[a, b]$.
(2) $f$ is differentiable on the open interval $(a, b)$.
(3) $f(a)=f(b)$.

Then there exists a number $c \in(a, b)$ such that $f^{\prime}(c)=0$.
Let $f(x)>f(a)$ for some $x \in(a, b)$.
Give a complete proof of the theorem in this case.
CRQ, Calculus, May 2006, Q3aii

## C563aii



| C563aii |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Logical |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -0.46 | Moderately easy |
| Discrimination | 0.481 | Average discrimination |
| Confidence Index | 0.688 | Large deviation from expected confidence level |
| Expert Opinion | 0.466 | Small deviation from expected performance |
| Quality Index | 0.379 | Poor quality CRQ |

```
A652b
Solve -2 cos x+2\sqrt{}{3}\operatorname{sin}x=4\mp@subsup{\operatorname{cos}}{}{2}x-4\mp@subsup{\operatorname{sin}}{}{2}x
```



| A652b |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 2.81 | Difficult |
| Discrimination | 0.295 | Discriminates well |
| Confidence Index | 0.465 | Small deviation from expected confidence level |
| Expert Opinion | 0.360 | Small deviation from expected performance |
| Quality Index | 0.178 | Good quality CRQ (excellent) |

## A95M03

If $\vec{a}=(1,2), \vec{b}=(-1,3), \vec{c}=(4,-2)$ and $\vec{d}=(3,-3)$, then $(\vec{a} \cdot \vec{d}) \vec{b}-(\vec{b} \cdot \vec{c}) \vec{d}$ equals
A. $(-54,12)$
B. -4
C. $3(11,-13)$
D. not possible

PRQ, Algebra, August 2005, Tut Test, Q3
A95月503


| A95M03 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 0.84 | Moderately difficult |
| Discrimination | 0.357 | Discriminates well |
| Confidence Index | 0.443 | Small deviation from expected confidence level |
| Expert Opinion | 0.460 | Small deviation from expected performance |
| Quality Index | 0.228 | Good quality PRQ |

$\lim _{h \rightarrow 0} \frac{\sqrt{9+h}-3}{h}$ is equal to
A. $\lim _{h \rightarrow 0} \frac{1}{\sqrt{9+h}+3}$
B. The slope of the tangent line to $y=\sqrt{x}$ at the point $P(9,3)$
C. The slope of the tangent line to $y=\sqrt{x}$ at the point $P(9,-3)$
D. Both $(A)$ and $(B)$
E. All of $(A),(B)$ and $(C)$

PRQ, Calculus, March 2005, Q1


| C35M01 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -0.36 | Moderately easy |
| Discrimination | 0.460 | Discriminates well |
| Confidence Index | 0.587 | Large deviation from expected confidence level |
| Expert Opinion | 0.309 | Small deviation from expected performance |
| Quality Index | 0.257 | Good quality PRQ (moderate) |

C1156a
Match each of the differential equations given in Column A with the type listed in Column B.

| A. Differential Equation | B. Type |
| :--- | :--- |
| a. $\frac{d y}{d x}-\frac{y}{x}=\ln x$ | 1. Variable separable |
| b. $\frac{d y}{d x}=e^{x} / e^{y}$ | 2. Homogeneous |
| c. $\left(x^{2}+y^{2}\right) d x+2 x y d y=0$ | 3. Exact |
| d. $2 x+y^{3}+\left(3 x y^{2}+y e^{2 y}\right) \frac{d y}{d x}=0$ | 4. Linear |

## C1156a



| C1156a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -0.22 | Moderately easy |
| Discrimination | 0.295 | Discriminates well |
| Confidence Index | 0.472 | Small deviation from expected confidence level |
| Expert Opinion | 0.617 | Large deviation from expected performance |
| Quality Index | 0.265 | Good quality CRQ (moderate) |

## C66M06

Let $f(x)$ be a function such that $f(4)=-1$ and $f^{\prime}(4)=2$. If $x<4$, then $f^{\prime \prime}(x)<0$ and if $x>4$, then $f^{\prime \prime}(x)>0$. The point $(4,-1)$ is a $\square$ of the graph of $f$.
A. Relative maximum
B. Relative minimum
C. Critical point
D. Point of inflection
E. None of the above

PRQ, Calculus, June 2006, Q6

C66月06


| C66M06 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -1.00 | Moderately easy |
| Discrimination | 0.687 | Discriminates poorly |
| Confidence Index | 0.452 | Small deviation from expected confidence level |
| Expert Opinion | 0.496 | Average deviation from expected performance |
| Quality Index | 0.379 | Poor quality PRQ |

## C561aii

A bacterial colony is estimated to have a population of $P(t)=\frac{24 t+10}{t^{2}+1}$ million, $t$ hours after the introduction of a toxin.

Is the population increasing or decreasing at this time?
CRQ, Calculus, May 2006, Q1aii

## C581aii



| C561aii |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Modelling |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -4.51 | Very easy |
| Discrimination | 0.810 | Discriminates poorly |
| Confidence Index | 0.549 | Large deviation from expected confidence level |
| Expert Opinion | 0.613 | Large deviation from expected performance |
| Quality Index | 0.553 | Poor quality CRQ |

```
C1152a
Split }\frac{3}{(x-1)(\mp@subsup{x}{}{2}+x+1)}\mathrm{ into partial fractions.
```

CRQ, Calculus, November 2005, Q2a

## C1152a



| C1152a |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -1.37 | Moderately easy |
| Discrimination | 0.439 | Discriminates well |
| Confidence Index | 0.352 | Small deviation from expected confidence level |
| Expert Opinion | 0.272 | Small deviation from expected performance |
| Quality Index | 0.160 | Good quality CRQ (moderate) |

C65M10

The points of inflection for the function $f(x)=8 x+2-\sin x$ for $0<x<3 \pi$, are
A. $(\pi, 8 \pi)$ and $(2 \pi, 16 \pi+2)$
B. $(\pi, 2)$ and $(2 \pi, 16 \pi+2)$
C. $(\pi, 8 \pi)$ and $(2 \pi, 16 \pi)$
D. $(\pi, 8 \pi+2)$ and $(2 \pi, 16 \pi+2)$
E. $(\pi, 8 \pi+2)$ and $(2 \pi, 16 \pi)$

PRQ, Calculus, June 2005, Q10

## C65月10



| C65M10 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 1.73 | Difficult |
| Discrimination | 0.213 | Discriminates well |
| Confidence Index | 0.352 | Small deviation from expected confidence level |
| Expert Opinion | 0.609 | Large deviation from expected performance |
| Quality Index | 0.181 | Good quality PRQ |

If $\frac{1}{2} \arccos 2 x=\frac{\pi}{2}$, then $x$ equals
A. 0
B. -1
C. $\frac{1}{2}$
D. $-\frac{1}{2}$

PRQ, Algebra, June 2005, Q4


| A65M04 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 0.14 | Moderately difficult |
| Discrimination | 0.522 | Average discrimination |
| Confidence Index | 0.358 | Small deviation from expected confidence level |
| Expert Opinion | 0.280 | Small deviation from expected performance |
| Quality Index | 0.188 | Good quality PRQ |

```
A951
Evaluate }\mp@subsup{\sum}{r=1}{100}[(r+1\mp@subsup{)}{}{r+1}-\mp@subsup{r}{}{r}]
```

CRQ, Algebra, August 2005, Q1


| A951 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 0.67 | Moderately difficult |
| Discrimination | 0.439 | Discriminates well |
| Confidence Index | 0.480 | Small deviation from expected confidence level |
| Expert Opinion | 0.372 | Small deviation from expected performance |
| Quality Index | 0.239 | Good quality CRQ (moderate) |

## A65M02

$$
\sum_{i=r+1}^{k} \pi=
$$

A. $\pi(r+1-k)$
B. $k(r-\pi+1)$
C. $\pi(k-r+2)$
D. $\pi(k-r)$

A65RA02


| A65M02 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 0.98 | Moderately difficult |
| Discrimination | 0.357 | Discriminates well |
| Confidence Index | 0.598 | Large deviation from expected confidence level |
| Expert Opinion | 0.475 | Small deviation from expected performance |
| Quality Index | 0.289 | Poor quality PRQ (moderate) |

C55M01

Determine from the graph of $y=f(x)$ whether $f$ possesses extrema on the interval $[a, b]$.

A. Maximum at $x=a$; minimum at $x=b$.
B. Maximum at $x=b$; minimum at $x=a$.
C. No extrema.
D. No maximum; minimum at $x=a$.

C55MOT


| C55M01 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -0.50 | Moderately easy |
| Discrimination | 0.728 | Discriminates poorly |
| Confidence Index | 0.288 | Small deviation from expected confidence level |
| Expert Opinion | 0.587 | Large deviation from expected performance |
| Quality Index | 0.349 | Poor quality PRQ |

C663c

In a given semi-circle of radius 2 , a rectangle is inscribed as shown in the figure below.


Find the value of $\theta$ corresponding to the maximum area, and test whether this value for $\theta$ gives a maximum.

CRQ, Calculus, June 2006, Q3c
C663e


| C663c |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | -0.13 | Moderately easy |
| Discrimination | 0.604 | Discriminates poorly |
| Confidence Index | 0.411 | Small deviation from expected confidence level |
| Expert Opinion | 0.577 | Large deviation from expected performance |
| Quality Index | 0.361 | Poor quality CRQ |

A1154bii
$M=\left(\begin{array}{ccccc}1 & -2 & -3 & : & 3 \\ -1 & 3 & 5 & : & -4 \\ 4 & -5 & k^{2}-15 & : & k+12\end{array}\right)$
Suppose the system given by $M$ represents three planes, $P_{1}, P_{2}, P_{3}$. That is, we have:
$P_{1}: x-2 y-3 z=3$
$P_{2}:-x+3 y+5 z=-4$
$P_{3}: 4 x-5 y+\left(k^{2}-15\right) z=k+12$
Find the value(s) of $k$ such that the three planes intersect in a single point. Do not calculate the co-ordinates of that point.

CRQ, Algebra, November 2005, Q4biii


| A1154biii |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Problem solving |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 0.35 | Moderately difficult |
| Discrimination | 0.316 | Discriminates well |
| Confidence Index | 0.717 | Large deviation from expected confidence level |
| Expert Opinion | 0.964 | Large deviation from expected performance |
| Quality Index | 0.529 | Poor quality CRQ |

C951

Rewrite the following integral as the sum of integrals such that there are no absolute values. DO NOT solve the integral. Give full reasons for your answer.

$$
\int_{-2}^{5}\left|4 x-x^{2}\right| d x
$$

CRQ, Calculus, August 2005, Q1

## C951



| C951 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 0.86 | Moderately difficult |
| Discrimination | 0.419 | Discriminates well |
| Confidence Index | 0.392 | Small deviation from expected confidence level |
| Expert Opinion | 0.323 | Small deviation from expected performance |
| Quality Index | 0.185 | Good quality CRQ |

```
A45MA4
If f}\mathrm{ is an odd function and g}\mathrm{ is an even function then
A. \(f \circ g\) is an even function
B. \(f \circ g\) is an odd function
C. \(f\) is a one-to-one function
D. \(g\) is a one-to-one function
```


## A45 FiA $^{4}$



| A45MA4 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 1.11 | Moderately difficult |
| Discrimination | 0.275 | Discriminates well |
| Confidence Index | 0.698 | Large deviation from expected confidence level |
| Expert Opinion | 0.296 | Small deviation from expected performance |
| Quality Index | 0.207 | Good quality PRQ |

A661.2

This question deals with the statement $P(n): n^{3}+(n+1)^{3}+(n+2)^{3}$ is divisible by 9 .
Use Pascal's triangle to expand and then simplify $(k+3)^{3}$.

CRQ, Algebra, June 2006, Q1. 2
A661.2


| A661.2 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 0.02 | Moderately difficult |
| Discrimination | 0.666 | Discriminates poorly |
| Confidence Index | 0.379 | Small deviation from expected confidence level |
| Expert Opinion | 0.301 | Small deviation from expected performance |
| Quality Index | 0.246 | Good quality CRQ (moderate) |

## C85M07

On which interval is the function $f(x)=e^{3 x}-e^{x}$ increasing?
A. $(\ln 9, \infty)$
B. $(0, \infty)$
C. $(-\infty, \infty)$
D. $\left(-\frac{1}{2} \ln 3, \infty\right)$
E. None of the above

PRQ, Calculus, August 2005, Q7


| C85M07 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -1.17 | Moderately easy |
| Discrimination | 0.687 | Discriminates poorly |
| Confidence Index | 0.230 | Small deviation from expected confidence level |
| Expert Opinion | 0.514 | Average deviation from expected performance |
| Quality Index | 0.272 | Good quality PRQ (moderate) |

## C654

State the Fundamental Theorem of Calculus.
CRQ, Calculus, June 2005, Q4

## C654



| C654 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 0.29 | Moderately difficult |
| Discrimination | 0.481 | Average discrimination |
| Confidence Index | 0.248 | Small deviation from expected confidence level |
| Expert Opinion | 0.819 | Large deviation from expected performance |
| Quality Index | 0.310 | Poor quality CRQ (moderate) |

Let $y=f(x)=\cos (\arcsin x)$. Then the range of $f$ is
A. $\{y \mid 0 \leq y \leq 1\}$
B. $\{y \mid-1 \leq y \leq 1\}$
C. $\left\{y \left\lvert\,-\frac{\pi}{2}<y<\frac{\pi}{2}\right.\right\}$
D. $\left\{y \left\lvert\,-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right.\right\}$
E. None of the above

PRQ, Algebra, May 2006, Q1


| A56M01 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | 3.07 | Very difficult |
| Discrimination | 0.460 | Discriminates fairly well |
| Confidence Index | 0.655 | Large deviation from expected confidence level |
| Expert Opinion | 0.389 | Small deviation from expected performance |
| Quality Index | 0.318 | Poor quality PRQ (moderate) |

C662f

Let $f(x)=\frac{x^{2}}{(x-2)^{2}}$.
You may assume that $f^{\prime}(x)=\frac{-4 x}{(x-2)^{3}}$ and $f^{\prime \prime}(x)=\frac{8 x+8}{(x-2)^{4}}$.
Find the points of inflection of $f$ (if any).
CRQ, Calculus, June 2006 Q2f

## C662f



| C662f |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | CRQ |  |
| Item Difficulty | 3.75 | Very difficult |
| Discrimination | 0.646 | Discriminates poorly |
| Confidence Index | 0.783 | Large deviation from expected confidence level |
| Expert Opinion | 0.609 | Large deviation from expected performance |
| Quality Index | 0.595 | Poor quality CRQ |

$$
\lim _{x \rightarrow-1} \frac{x^{2}+4 x+3}{x^{2}-1}=
$$

A. -1
B. 0
C. undefined
D. 4

PRQ, Calculus, March 2006, Tut Test B, Q6

## C46 1 AB6



| C46MB6 |  | Comment |
| :--- | :---: | :--- |
| Assessment Component | Consolidation |  |
| PRQ/CRQ | PRQ |  |
| Item Difficulty | -2.24 | Easy |
| Discrimination | 0.996 | Discriminates poorly |
| Confidence Index | 1.000 | Large deviation from expected confidence level |
| Expert Opinion | 0.544 | Large deviation from expected performance |
| Quality Index | 0.933 | Poor quality PRQ |

## 

Table 6.3 summarises the quality of the item，both PRQs and CRQs，within each assessment component．Within each component the number of good and poor quality items are given，both for the PRQ and CRQ formats．The numbers are also given as percentages of the total number of items．

Table 6．3：Component analysis－trends．

| COMPONENT | No． of PRQs |  | Total no． of items | Good quality items | Poor quality items | Good PRQs | Good CRQs | Poor PRQs | Poor CRQs |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1．Technical | 11 | 22 | 33 | $\begin{gathered} 17 \\ {[52 \%]} \end{gathered}$ | $\begin{gathered} 16 \\ {[48 \%]} \end{gathered}$ | $\begin{gathered} 8 \\ {[73 \%]} \end{gathered}$ | $\begin{gathered} 9 \\ {[41 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 3 \\ {[27 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 13 \\ {[59 \%]} \end{gathered}$ |
| 2．Disciplinary | 24 | 34 | 58 | $\begin{gathered} 28 \\ {[48 \%]} \end{gathered}$ | $\begin{gathered} 30 \\ {[52 \%]} \end{gathered}$ | $\begin{gathered} 12 \\ {[50 \%]} \end{gathered}$ | $\begin{gathered} 16 \\ {[47 \%]} \end{gathered}$ | $\begin{gathered} 12 \\ {[50 \%]} \end{gathered}$ | $\begin{gathered} 18 \\ {[53 \%]} \end{gathered}$ |
| 3．Conceptual | 26 | 30 | 56 | $\begin{gathered} 28 \\ {[50 \%]} \end{gathered}$ | $\begin{gathered} 28 \\ {[50 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 14 \\ {[54 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 14 \\ {[47 \%]} \end{gathered}$ | $\begin{gathered} 12 \\ {[46 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 16 \\ {[53 \%]} \\ \hline \end{gathered}$ |
| 4．Logical | 7 | 6 | 13 | $\begin{gathered} 5 \\ {[39 \%]} \end{gathered}$ | $\begin{gathered} 8 \\ {[61 \%]} \end{gathered}$ | $\begin{gathered} 1 \\ {[14 \%]} \end{gathered}$ | $\begin{gathered} 4 \\ {[67 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 6 \\ {[86 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 2 \\ {[33 \%]} \\ \hline \end{gathered}$ |
| 5．Modelling | 3 | 10 | 13 | $\begin{gathered} 8 \\ {[62 \%]} \end{gathered}$ | $\begin{gathered} 5 \\ {[38 \%]} \end{gathered}$ | $\begin{gathered} 2 \\ {[67 \%]} \end{gathered}$ | $\begin{gathered} 6 \\ {[60 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 1 \\ {[33 \%]} \end{gathered}$ | $\begin{gathered} 4 \\ {[40 \%]} \\ \hline \end{gathered}$ |
| 6．Problem solving | 7 | 4 | 11 | $\begin{gathered} 6 \\ {[55 \%]} \end{gathered}$ | $\begin{gathered} 5 \\ {[45 \%]} \end{gathered}$ | $\begin{gathered} 4 \\ {[57 \%]} \end{gathered}$ | $\begin{gathered} 2 \\ {[50 \%]} \end{gathered}$ | $\begin{gathered} 3 \\ {[43 \%]} \end{gathered}$ | $\begin{gathered} 2 \\ {[50 \%]} \end{gathered}$ |
| 7．Consolidation | 16 | 7 | 23 | $\begin{gathered} 12 \\ {[52 \%]} \end{gathered}$ | $\begin{gathered} 11 \\ {[48 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 7 \\ {[44 \%]} \end{gathered}$ | $\begin{gathered} 5 \\ {[71 \%]} \end{gathered}$ | $\begin{gathered} 9 \\ {[56 \%]} \\ \hline \end{gathered}$ | $\begin{gathered} 2 \\ {[29 \%]} \end{gathered}$ |

c）＊＊＊：
In the technical assessment component，there is a higher percentage（73\％）of good PRQs than good CRQs（41\％）．73\％good PRQs compared to good 41\％ CRQs shows us that PRQs are more successful than CRQs as an assessment format in the technical component．There is also a much higher percentage （73\％）of good PRQs than poor PRQs（27\％）．CRQs，however，are not that successful in this component，with the results showing 59\％poor CRQs compared to $41 \%$ good CRQs．The conclusion is that the technical assessment component lends itself better to PRQs than to CRQs．

In this study，the disciplinary component is the assessment component with the most items（58），of which 34 were CRQs and 24 were PRQs．In this component it is interesting to note that the percentages of good PRQs（50\％）and good CRQs（47\％）are almost equal．In addition，there is no difference between the good PRQs（50\％）and the poor PRQs（50\％），with very little difference between the good CRQs（47\％）and poor CRQs（53\％）．PRQs and CRQs can be considered as equally successful assessment formats in the disciplinary component．

## $\sqrt{\otimes} \quad \because \square$＊$\because$ 几

The conceptual component also contained many items（56），with an almost equal number of PRQs and CRQs（26 PRQs versus 30 CRQs）． $50 \%$ of the items are of good quality and $50 \%$ are of poor quality．In this component，there is no clear trend that PRQs are better than CRQs or vice versa．There is a slight leaning towards good PRQ assessment（47\％good CRQs compared to 54\％ good PRQs）．Therefore，in the conceptual assessment component，PRQs could be used as successfully as CRQs as a format of assessment．

## 

In this study，it is interesting to note that the majority of questions within the logical component were of a poor quality mainly due to the large percentage of poor PRQs．There are noticeably more good quality CRQs（67\％）than good quality PRQs（14\％），and noticeably more poor quality PRQs（86\％）than poor quality CRQs（33\％）．A very high percentage of the PRQs（86\％）in the logical component were of a poor quality．The conclusion is that the logical assessment component lends itself better to CRQs than to PRQs．

## X＊转来 光

In the modelling component，very few PRQs were used as assessment items in comparison to CRQs， 3 PRQs versus 10 CRQs，probably because it is difficult to set PRQs in this component．Despite the small number of PRQs，it was encouraging to note that the good PRQs（67\％）far outweighed the poor PRQs
(33\%). So in terms of quality, the PRQs were highly successful in the modelling component. There are also more good CRQs (60\%) than poor CRQs (40\%). It appears that although more difficult to set in the modelling component, PRQs could be used as successfully in the modelling assessment component as CRQs.

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Although the problem solving component had the least number of items (11), it is interesting to note that there are more PRQs (7) than CRQs (4). There is a slightly higher percentage (57\%) of good PRQs than good CRQs (50\%). Although the sample is too small to make definite conclusions, there is no reason to disregard the use of PRQs in this assessment component. In fact, PRQs seem to be slightly more successful than CRQs, and the conclusion is that PRQ assessment format can add value to the assessment of the problem solving component.

## 

It was somewhat surprising to note that corresponding to the highest level of conceptual difficulty, the consolidation component displayed an unusually higher proportion of PRQs (16) to CRQs (7). This supports the earlier claim that PRQs are not only appropriate for testing lower level cognitive skills (Adkins, 1974; Aiken, 1987; Haladyna, 1999; Isaacs, 1994; Johnson, 1989; Oosterhof, 1994; Thorndike, 1997; Williams, 2006). In the consolidation component there is a significant higher percentage (71\%) of good CRQs than good PRQs (44\%). In addition, there is a higher percentage of poor PRQs (56\%) than good PRQs (44\%). The high percentage of good CRQs (71\%) in comparison to poor CRQs (29\%) indicates that the consolidation assessment component lends itself better to CRQs than to PRQs.

