CHAPTER 6: RESEARCH FINDINGS

6.1 QUANTITATIVE DATA ANALYSIS

In this chapter on the research findings, an analysis of good quality items and poor quality items, both PRQs and CRQs, in terms of the Quality Index developed in section 5.3.2, within each of the seven mathematics assessment components, will be presented.

6.1.1 Methodology

Stage 1

The traditional statistical analysis of data, supplied by the Computer Network Services (CNS) Division of the University of the Witwatersrand, include the Performance Index, Discrimination Index and Easiness/Difficulty factor per question for all tests (PRQ and CRQ) during the period of study, July 2004 to July 2006.

Raw data, including students' responses to test items and confidence of responses, was obtained from the Computer Network Services (CNS) Division of the University of the Witwatersrand. Spreadsheets were constructed using a 'Mathematica' programme developed by a statistician from the School of Statistics at the University of the Witwatersrand. The following information was captured on every spreadsheet per test:

- students' responses to all test items, both PRQ and CRQ
- students' confidence of responses per test item, both PRQ and CRQ.

The correct answers and mathematics assessment components per test item were also recorded for reference purposes. Student numbers were not recorded on every spreadsheet. In constructing these spreadsheets, records were excluded if:

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- (i) the student had failed to provide an answer; or
- (ii) the student had failed to provide a confidence of response; or
- (iii) the student had filled in the MCQ card incorrectly.

It should be noted that in most cases the excluded records were due to (ii) above. The proportion of all the records excluded in this manner ranged between 7,2% and 8,9% across the tests. All subsequent calculations were performed on this filtered data.

For PRQs and CRQs, the Performance Index (PI) per question was equal to the proportion of (filtered) respondents who obtained the correct answer. It should be noted that the "easiness/difficulty" statistic provided on the CNS printouts is equal to the Performance Index i.e. Performance Index = Difficulty Index. An overall Confidence Index (per assessment component) was calculated by averaging the CIs per question for all questions in that assessment component. An overall Performance Index or Difficulty Index (per assessment component) was calculated in a similar manner by averaging the PIs per question for all questions in that assessment component.

Stability (test- retest) was achieved by administering the same tutorial tests in March and August over the period 2004-2006. Equivalence was achieved over the period of study by administering different tests to the same cohort of students (Mathematics I Major) in each of the 3 years, 2004, 2005 and 2006 respectively. Internal consistency was achieved by correlating and equating the items in each test to each other, as described under test item calibration in section 6.2.1.

Stage 2

The Rasch model (Rasch, 1960), as discussed in section 3.4.1, was used to evaluate both the attitudinal data (confidence levels) as well as test data. The Winsteps (Linacre & Wright, 1999) Rasch analysis programme was utilised by a data analyst from the University of Pretoria for the quantitative data analysis in this research study. In particular the WINSTEPS® Version 3.55.0 was used to



analyse the data in this study. SAS Version 9 and Microsoft EXCEL 2003 were also used in calculating totals and means.

The Winsteps software, developed by John M Linacre in 2005, constructs Rasch measures from simple rectangular data sets, usually of persons and items. Item types that can be combined in one analysis include dichotomous, multiple choice and partial credit items. Paired comparisons and rank-order data can also be analysed. Missing data is no problem. Winsteps is designed as a tool that facilitates exploration and communication. The structure of the items and persons can be examined in depth. Unexpected data points are identified and reported in numerous ways. Powerful diagnosis of multidimensionality through principal components analysis of residuals detects and quantified substructures in the data. The working of rating scales can be examined thoroughly, and rating scales can be recoded and items regrouped to share rating scales as desired (Linacre, 2002). Measures can be fixed (anchored) at pre-set values (Linacre, 2005).

In order to prepare the data in an ASCII format to import into Winsteps, SAS was used to create ASCII files with a specific layout. Control files were prepared in Winsteps for each part of each test, i.e. the PRQ part, the CRQ part as well as the confidence index part. This was done as the different Rasch models, discussed in section 3.4.1.3, were applicable to the different types of data. These parts of the tests were first analysed separately to check for model "fit". Such "fit" statistics help detect possible idiosyncratic behaviour on the part of respondents and test items. Those respondents who exhibited "misfit" were first investigated for coding errors, and then their raw hard-copy responses were reviewed for evidence of non-attention to the test. Such individuals might be ones who are haphazardly circling responses or those who are guessing and/or miscoding.

Winsteps provides ways of diagnosing problems in the analysis. In the first place the point measure values were considered. Where items exhibited negative point measure values, these items were scrutinised for errors such as an



incorrect key and corrected. If the point measure stayed negative, the item was removed from the analysis. Subsequently, the output tables for person ability and item difficulty were checked for misfitting entries. Person ability tables were considered first.

Misfit

Some explanation in terms of misfitting items or students is in order. One would expect that a student of medium mathematical ability would be able to respond correctly to easier items in the test and incorrectly to the difficult items in a test. Where the item difficulty matches the ability of the student, one would expect the student to answer some of these items correct and some incorrectly. If an item's difficulty corresponds exactly to the student's ability, the probability of success of the student on that item is 0.5, in other words, success or failure is expected equally. The Rasch model assumes this pattern of responses, and the Infit and Outfit mean-square statistics are 1.0. If for example, a student would guess the answer to a difficult item correctly (one that the student should really get wrong) the Outfit statistic would be much larger than 1.0 because it is sensitive to outliers.

The approach used in the analyses of this study's data was that items and persons were accepted as not misfitting when Infit mean-square statistics was from 0.5 to 1.5. Where the values were less than 0.5, too much predictability or overfit was experienced and when the value exceeded 1.5, too much noise was present in the data or a situation of underfit existed. The Infit statistics were considered first, and then the Outfit statistics.

Mean-square statistics indicate the size of the misfit, but the "significance" of the improbability of the misfit is important.

Misfitting persons were deleted, and the analysis was repeated. Another round of misfitting persons were removed from the analysis. Only then were the fit



statistics of the items considered. If an item proved to be problematic in terms of the fit statistics, the item was also removed from any subsequent analysis.

The same procedure was followed to explore the misfitting persons and items in terms of the CRQs and the confidence index.

For the PRQs, the dichotomous Rasch model applies:

$$P_{vi} = \frac{e^{(\beta v - \delta i)}}{1 + e^{(\beta v - \delta i)}}$$

In the confidence index, the same categories were available throughout and were thus analysed according to the Rasch-Andrich rating scale model:

$$\ln\left(\frac{P_{vix}}{P_{vi(x-1)}}\right) = \beta_v - \delta_i - F_x$$

CRQs were analysed through the application of the Partial Credit model:

$$\ln\left(\frac{P_{vix}}{P_{vi(x-1)}}\right) = \beta_v - \delta_i - F_{ix}$$

These various Rasch models have already been discussed in more detail in section 3.4.1.

Test item calibration

Through the application of the Rasch family of models it is also possible to put the measures of different tests onto the same scale if certain assumptions are made. The tests can be linked either through common items on the tests or through common students writing the tests. A challenge in terms of the data faced the researcher. Although, as mentioned previously, it was known that the same cohort of students wrote the same tests in a calendar year, the student identification numbers were not available on all the data sets and therefore no linking could take place on a one-to-one basis. The strong assumption was then made that the subject matter of the different tests were distinct and that the tests



could therefore be regarded as independent. In other words, it was assumed that because the subject matter was distinct, students' ability did not improve progressively throughout the year. This assumption led to the decision that all the data could be calibrated together, anchoring the items that were common over the three years. In this way, the item difficulties and the student measures were on the same scale and were deemed directly comparable.

Fit statistics were again considered and if in the combined calibration of items any misfitting items were identified, they were excluded from the analysis. A small number of items misfitted, and this is not to be unexpected in such a large data set.

The same procedure was followed in terms of the CRQs. In order to place the measures of the PRQs and the CRQs on the same scale, a combined calibration of these items was also executed. Another challenge presented itself. At first, when the PRQs and the CRQs were calibrated together, the whole set of CRQs misfitted. It was then decided to recode the partial credit items into dichotomous items in the following way: If a student scored less than half the marks, the student was awarded a 0 for that specific item; if the student scored half or more of the marks on an item, the student was awarded a 1 for the item. The CRQs were therefore eventually analysed through the same model as the PRQs i.e. the dichotomous Rasch model, and the combined calibration of items then produced a set of items that mostly fitted the Rasch model.

Confidence level item calibration

A similar process was followed to determine the item difficulties of the confidence levels. The item difficulty for a rating scale is defined as the point where the top and bottom categories are equally probable (Linacre, 2005).



6.2 DATA DESCRIPTION

Response data from 14 different mathematics tests written between August 2004 and June 2006 were available. Table 6.1 is a representation of the tests written, the number of provided response items (PRQs) per test, the number of constructed response items (CRQs) and the number of students per test. The same cohort of students (Mathematics I Major) wrote the tests in each of the three years, 2004, 2005 and 2006 respectively.

Year	Month	Number of PRQs	Number of CRQs	Number of students
2004	August	10	0	457
2005	March	8	0	410
2005	April Tutorial A	8	0	263
2005	April Tutorial B	8	0	126
2005	Мау	8	0	403
2005	June	12	17	414
2005	August	10	0	389
2005	September	8	17	387
2005	November	15	18	385
2006	March	8	15	352
2006	April Tutorial A	8	0	245
2006	April Tutorial B	8	0	105
2006	Мау	8	14	359
2006	June	12	24	348

Table 6.1: Characteristics of tests written.

Out of a total of 221 PRQ and CRQ items, seven items were discarded because their fit statistics indicated that they did not fit the model. Table 6.2 included in the Appendix A5, presents these items with their fit statistics. Another seven items (I115M09 – I115M15) were discarded because the actual items were not available. Finally, 207 items were included in the analyses. The Rasch statistics



for all 207 test items analysed are included in Appendix A6. Confidence level items Rasch statistics are included in Appendix A7.

6.3 COMPONENT ANALYSIS

Examples of questions in the different mathematics assessment components are now presented. Within each of the seven assessment components, both PRQs and CRQs, ranging from easy to difficult, and of good and poor quality are presented. For each item, the question is followed by a radar plot and a table summarising the quality parameters of the test item i.e. item difficulty; discrimination; confidence index; expert opinion and the final quality index, as discussed in the theoretical framework in Chapter 5. Each of the axes of the radar plots are labelled with the corresponding values for discrimination, confidence index and expert opinion. The Quality Index (QI) is displayed alongside the radar plot. The shading of the radar plot corresponds to one of the six item difficulty levels as classified in Table 5.4. The comments briefly summarise the difficulty level, the three measuring criteria as developed in the theoretical framework and the overall quality of the item.



1. Technical component



CRQ, Algebra, June 2005, Q1a





A651a		Comment
Assessment Component	Technical	
PRQ/CRQ	CRQ	
Item Difficulty	1.10	Moderately difficult
Discrimination	0.295	Discriminates well
Confidence Index	0.385	Small deviation from expected confidence level
Expert Opinion	0.236	Small deviation from expected performance
Quality Index	0.119	Good quality CRQ (excellent)

A652(a)



Write $-2\cos x + 2\sqrt{3}\sin x$ in the form $R\cos(x-\theta)$

CRQ, Algebra, June 2005, Q2a

A652a



A652a		Comment
Assessment Component	Technical	
PRQ/CRQ	CRQ	
Item Difficulty	-0.33	Moderately easy
Discrimination	0.501	Discriminates fairly well
Confidence Index	0.318	Small deviation from expected confidence level
Expert Opinion	0.574	Large deviation from expected performance
Quality Index	0.273	Good quality CRQ (moderate)



PRQ, Calculus, November 2005, Q7

C115M07



C115M07		Comment
Assessment Component	Technical	
PRQ/CRQ	PRQ	
Item Difficulty	-1.12	Moderately easy
Discrimination	0.666	Does not discriminate well
Confidence Index	0.343	Small deviation from expected confidence level
Expert Opinion	0.416	Small deviation from expected performance
Quality Index	0.281	Good quality PRQ (moderate)

A1155bii

Let
$$A = \begin{pmatrix} 1 & 1 & 1 \\ ab & bc & ca \\ a+b & b+c & c+a \end{pmatrix}$$

For what value(s) of a, b, c does A^{-1} exist?

CRQ, Algebra, November 2005, Q5bii

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A1155bii



A1155bii		Comment
Assessment Component	Technical	
PRQ/CRQ	CRQ	
Item Difficulty	2.23	Difficult
Discrimination	0.522	Discriminates poorly
Confidence Index	0.347	Small deviation from expected confidence level
Expert Opinion	0.736	Large deviation from expected performance
Quality Index	0.356	Poor quality CRQ

A661.1

 $P(n) = n^{3} + (n+1)^{3} + (n+2)^{3}$ is divisible by 9

Show that the statement is true for n = 2

CRQ, Algebra, June 2006, Q1.1

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A661.1		Comment
Assessment Component	Technical	
PRQ/CRQ	CRQ	
Item Difficulty	-2.35	Easy
Discrimination	0.975	Discriminates weakly
Confidence Index	0.324	Small deviation from expected confidence level
Expert Opinion	0.410	Small deviation from expected performance
Quality Index	0.367	Poor quality CRQ





PRQ, Algebra, May 2006, Q2





A56M02		Comment
Assessment Component	Technical	
PRQ/CRQ	PRQ	
Item Difficulty	0.77	Moderately difficult
Discrimination	0.563	Weak discrimination
Confidence Index	0.643	Large deviation from expected confidence level
Expert Opinion	0.453	Small deviation from expected performance
Quality Index	0.393	Poor quality PRQ



PRQ, Calculus, June 2005, Q8

C65M08



C65M08		Comment
Assessment Component	Technical	
PRQ/CRQ	PRQ	
Item Difficulty	-1.04	Moderately easy
Discrimination	0.749	Weak discrimination
Confidence Index	0.437	Small deviation from expected confidence level
Expert Opinion	0.674	Large deviation from expected performance
Quality Index	0.488	Poor quality PRQ



2. Disciplinary component

A35M08

Let a, b and c be real numbers. Which of the following is the correct statement?

A.
$$a < b \Rightarrow a + b > b + c$$
.
B. $a > b \Rightarrow ac > bc$.
C. $|x| > a \Leftrightarrow -a < x < a$.
D. $\sqrt{c^2} = c$.
E. $0 < a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$.

PRQ, Algebra, March 2005, Q8

A35M08



A35M08		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	PRQ	
Item Difficulty	2.25	Difficult
Discrimination	0.069	Discriminates very well
Confidence Index	0.842	Large deviation from expected confidence level
Expert Opinion	0.355	Small deviation from expected performance
Quality Index	0.165	Good quality PRQ

C363b

Prove, using the Intermediate Value Theorem, that there is a number exactly 1 more than its cube.

CRQ, Calculus, March 2006, Q3b

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C363b		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	CRQ	
Item Difficulty	3.94	Very difficult
Discrimination	0.295	Discriminates well
Confidence Index	0.274	Small deviation from expected confidence level
Expert Opinion	0.574	Large deviation from expected performance
Quality Index	0.177	Good quality CRQ

C561a(i)



A bacterial colony is estimated to have a population of $P(t) = \frac{24t+10}{t^2+1}$ million, t hours after the

introduction of a toxin.

At what rate is the population changing 1 hour after the toxin is introduced?

CRQ, Calculus, May 2006, Q1a(i)





C561ai		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	CRQ	
Item Difficulty	-2.63	Easy
Discrimination	0.543	Discriminates fairly well
Confidence Index	0.460	Small deviation from expected confidence level
Expert Opinion	0.262	Small deviation from expected performance
Quality Index	0.222	Good quality CRQ



PRQ, Algebra, May 2005, Q7

A55M07



A55M07		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	PRQ	
Item Difficulty	-0.76	Moderately easy
Discrimination	0.790	Does not discriminate well
Confidence Index	0.294	Small deviation from expected confidence level
Expert Opinion	0.290	Small deviation from expected performance
Quality Index	0.236	Good quality PRQ (moderate)



C364b(i)

Let $\llbracket x \rrbracket$ be the greatest integer less than or equal to *x*.

Show that $\lim_{x\to 2} f(x)$ exists if $f(x) = \llbracket x \rrbracket + \llbracket -x \rrbracket$.

CRQ, Calculus, March 2006, Q4b(i)





C364bi		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	CRQ	
Item Difficulty	4.19	Very difficult
Discrimination	0.501	Discriminates fairly well
Confidence Index	0.501	Average deviation from expected confidence level
Expert Opinion	0.547	Large deviation from expected performance
Quality Index	0.346	Poor quality CRQ

C563a(i)

Consider the following theorem:

Let f be a function that satisfies the following three conditions:

- (1) f is continuous on the closed interval [a,b].
- (2) f is differentiable on the open interval (a,b).
- (3) f(a) = f(b).

Then there exists a number $c \in (a, b)$ such that f'(c) = 0.

What is this theorem called?



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C563ai		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	CRQ	
Item Difficulty	-4.74	Very easy
Discrimination	0.831	Discriminates poorly
Confidence Index	0.545	Large deviation from expected confidence level
Expert Opinion	0.273	Small deviation from expected performance
Quality Index	0.359	Poor quality CRQ





C45MB5



C45MB5		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	PRQ	
Item Difficulty	1.91	Difficult
Discrimination	0.749	Discriminates poorly
Confidence Index	0.521	Large deviation from expected confidence level
Expert Opinion	0.409	Small deviation from expected performance
Quality Index	0.394	Poor quality PRQ



C36M02



C36M02		Comment
Assessment Component	Disciplinary	
PRQ/CRQ	PRQ	
Item Difficulty	-5.05	Very easy
Discrimination	0.872	Discriminates very poorly
Confidence Index	0.822	Very large deviation from expected confidence level
Expert Opinion	0.239	Small deviation from expected performance
Quality Index	0.486	Poor quality PRQ



3. Conceptual component C65M09 Choose the correct statement, given that $\int_0^5 f(x)dx = 9$ and $\int_2^5 f(x)dx = -1$. A. $\int_0^2 f(x)dx = 10$ B. $\int_2^0 f(x)dx = 10$ C. $\int_5^2 f(x)dx = -1$ D. $\int_0^2 f(x)dx = 8$ E. None of the above

PRQ, Calculus, June 2005, Q9





C65M09		Comment
Assessment Component	Conceptual	
PRQ/CRQ	PRQ	
Item Difficulty	1.72	Difficult
Discrimination	0.110	Discriminates well
Confidence Index	0.351	Small deviation from expected confidence level
Expert Opinion	0.608	Large deviation from expected performance
Quality Index	0.138	Good quality PRQ



A1152b

Find the equation of the plane which passes through the point A(2,3,-5) and which contains the line l: (-1,3,-2) + t(-2,1,5)

CRQ, Algebra, November 2005, Q2b

A1152b



A1152b		Comment
Assessment Component	Conceptual	
PRQ/CRQ	CRQ	
Item Difficulty	2.93	Difficult
Discrimination	0.357	Discriminates well
Confidence Index	0.255	Small deviation from expected confidence level
Expert Opinion	0.373	Small deviation from expected performance
Quality Index	0.138	Good quality CRQ (excellent)

C1157a



Find $\int x \cos x dx$

CRQ, Calculus, November 2005, Q7a





C1157a		Comment
Assessment Component	Conceptual	
PRQ/CRQ	CRQ	
Item Difficulty	-1.45	Moderately easy
Discrimination	0.522	Average discrimination
Confidence Index	0.249	Small deviation from expected confidence level
Expert Opinion	0.483	Small deviation from expected performance
Quality Index	0.218	Good quality CRQ





PRQ, Calculus, March 2005, Tut Test 1B, Q8

C45MB8



C45MB8		Comment
Assessment Component	Conceptual	
PRQ/CRQ	PRQ	
Item Difficulty	-1.94	Easy
Discrimination	0.604	Discriminates poorly
Confidence Index	0.410	Small deviation from expected confidence level
Expert Opinion	0.284	Small deviation from expected performance
Quality Index	0.232	Good quality CRQ (moderate)

A95M02 PQR is a triangle with vertices P(3,1), Q(5,2) and R(4,3). $P\hat{Q}R$ equals A. $\arccos \frac{4}{5}$ B. $\arccos \frac{1}{\sqrt{10}}$ C. $\pi - \arccos \frac{4}{5} - \arccos \frac{1}{\sqrt{10}}$ D. $\arccos \frac{-1}{\sqrt{10}}$

PRQ, Algebra, August 2005, Tut Test, Q2

A95M02



A95M02		Comment
Assessment Component	Conceptual	
PRQ/CRQ	PRQ	
Item Difficulty	-3.22	Very easy
Discrimination	0.769	Discriminates poorly
Confidence Index	0.406	Fairly small deviation from expected confidence level
Expert Opinion	0.333	Small deviation from expected performance
Quality Index	0.305	Poor quality PRQ (moderate)



PRQ, Calculus, May 2005, 04

C55M04



C55M04		Comment
Assessment Component	Conceptual	
PRQ/CRQ	PRQ	
Item Difficulty	1.50	Moderately difficult
Discrimination	0.336	Discriminates well
Confidence Index	0.723	Large deviation from expected confidence level
Expert Opinion	0.546	Large deviation from expected performance
Quality Index	0.356	Poor quality PRQ

C953a

Consider the following theorem:

Theorem: If a function f is continuous on the closed interval [a,b] and F is an antiderivative

of f on [a,b], then
$$\int_a^b f(x)dx = F(b) - F(a)$$
.

What is this theorem called?

CRQ, Calculus, August 2005, Q3a

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C953a



C953a		Comment
Assessment Component	Conceptual	
PRQ/CRQ	CRQ	
Item Difficulty	-5.56	Very easy
Discrimination	1.000	Discriminates very poorly
Confidence Index	0.497	Large deviation from expected confidence level
Expert Opinion	0.434	Fairly small deviation from expected performance
Quality Index	0.562	Poor quality CRQ

C953b

Consider the following theorem:

Theorem: If a function f is continuous on the closed interval [a,b] and F is an antiderivative

of
$$f$$
 on $[a,b]$, then $\int_a^b f(x)dx = F(b) - F(a)$.

Consider the proof of this theorem:

Proof: Divide the interval [a,b] into *n* sub-intervals by the points

$$a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$$

Show that
$$F(b) - F(a) = \sum_{i=1}^{n} [F(x_i) - F(x_{i-1})].$$

CRQ, Calculus, August 2005, Q3b

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C953b		Comment
Assessment Component	Conceptual	
PRQ/CRQ	CRQ	
Item Difficulty	2.4	Difficult
Discrimination	0.831	Discriminates poorly
Confidence Index	0.839	Large deviation from expected confidence level
Expert Opinion	0.865	Large deviation from expected performance
Quality Index	0.927	Poor quality CRQ

C953b



4. Logical component

A662.2

Use properties of sigma notation and the fact that

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$
 to prove that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

CRQ, Algebra, June 2006, Q2.2





A662.2		Comment
Assessment Component	Logical	
PRQ/CRQ	CRQ	
Item Difficulty	1.52	Difficult
Discrimination	0.048	Discriminates well
Confidence Index	0.495	Average deviation from expected confidence level
Expert Opinion	0.251	Small deviation from expected performance
Quality Index	0.069	Good quality CRQ (excellent)



PRQ, Algebra, May 2005, Q8





A55M08		Comment
Assessment Component	Logical	
PRQ/CRQ	PRQ	
Item Difficulty	0.15	Moderately difficult
Discrimination	0.378	Discriminates well
Confidence Index	0.479	Small deviation from expected confidence level
Expert Opinion	0.504	Average deviation from expected performance
Quality Index	0.265	Good quality PRQ (moderate)



A562a

A polar graph is defined by the equation $r(\theta) = 5\cos 3\theta$ for $\theta \in [0, 2\pi]$

Is the graph symmetric about the x – axis, the y – axis, both or neither? Motivate your answer.





A562a

A562a		Comment
Assessment Component	Logical	
PRQ/CRQ	CRQ	
Item Difficulty	-1.62	Easy
Discrimination	0.295	Discriminates well
Confidence Index	0.620	Large deviation from expected confidence level
Expert Opinion	0.487	Small deviation from expected performance
Quality Index	0.272	Good quality CRQ (moderate)



PRQ, Algebra, August 2005, Tut Test Q5

A85M05



A85M05		Comment
Assessment Component	Logical	
PRQ/CRQ	PRQ	
Item Difficulty	-2.31	Easy
Discrimination	0.687	Discriminates poorly
Confidence Index	0.652	Large deviation from expected confidence level
Expert Opinion	0.249	Small deviation from expected performance
Quality Index	0.338	Poor quality PRQ

C46MA5

- If $\lim_{x \to a} [f(x) + g(x)]$ exists, then
- A. $\lim_{x \to a} f(x) = \lim_{x \to a} g(x).$
- B. neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists.
- C. both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist.
- D. we cannot tell if $\lim_{x \to a} f(x)$ or $\lim_{x \to a} g(x)$ exists.

PRQ, Calculus, March 2006, Tut Test A,Q5

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C46MA5

C46MA5		Comment
Assessment Component	Logical	
PRQ/CRQ	PRQ	
Item Difficulty	2.47	Difficult
Discrimination	0.481	Average discrimination
Confidence Index	0.700	Large deviation from expected confidence level
Expert Opinion	0.470	Small deviation from expected performance
Quality Index	0.386	Poor quality PRQ



CRQ, Algebra, May 2006, Q2d





A562d		Comment
Assessment Component	Logical	
PRQ/CRQ	CRQ	
Item Difficulty	-1.42	Moderately easy
Discrimination	0.625	Discriminates poorly
Confidence Index	0.743	Large deviation from expected confidence level
Expert Opinion	0.424	Small deviation from expected performance
Quality Index	0.452	Poor quality CRQ

C563aii

Consider the following theorem:

Let f be a function that satisfies the following three conditions:

- (1) f is continuous on the closed interval [a,b].
- (2) f is differentiable on the open interval (a,b).
- (3) f(a) = f(b).

Then there exists a number $c \in (a,b)$ such that f'(c) = 0.

Let f(x) > f(a) for some $x \in (a,b)$.

Give a **complete proof** of the theorem in this case.



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C563aii		Comment
Assessment Component	Logical	
PRQ/CRQ	CRQ	
Item Difficulty	-0.46	Moderately easy
Discrimination	0.481	Average discrimination
Confidence Index	0.688	Large deviation from expected confidence level
Expert Opinion	0.466	Small deviation from expected performance
Quality Index	0.379	Poor quality CRQ



5. Modelling component

A652b

Solve
$$-2\cos x + 2\sqrt{3}\sin x = 4\cos^2 x - 4\sin^2 x$$

CRQ, Algebra, June 2005, Q2b

A652b



A652b		Comment
Assessment Component	Modelling	
PRQ/CRQ	CRQ	
Item Difficulty	2.81	Difficult
Discrimination	0.295	Discriminates well
Confidence Index	0.465	Small deviation from expected confidence level
Expert Opinion	0.360	Small deviation from expected performance
Quality Index	0.178	Good quality CRQ (excellent)

A95M03



PRQ, Algebra, August 2005, Tut Test, Q3

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A95M03



A95M03		Comment
Assessment Component	Modelling	
PRQ/CRQ	PRQ	
Item Difficulty	0.84	Moderately difficult
Discrimination	0.357	Discriminates well
Confidence Index	0.443	Small deviation from expected confidence level
Expert Opinion	0.460	Small deviation from expected performance
Quality Index	0.228	Good quality PRQ

C35M01

 $\lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \text{ is equal to}$ A. $\lim_{h\to 0} \frac{1}{\sqrt{9+h}+3}$ B. The slope of the tangent line to $y = \sqrt{x}$ at the point P(9,3)C. The slope of the tangent line to $y = \sqrt{x}$ at the point P(9,-3)D. Both (A) and (B)E. All of (A), (B) and (C)



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C35M01		Comment
Assessment Component	Modelling	
PRQ/CRQ	PRQ	
Item Difficulty	-0.36	Moderately easy
Discrimination	0.460	Discriminates well
Confidence Index	0.587	Large deviation from expected confidence level
Expert Opinion	0.309	Small deviation from expected performance
Quality Index	0.257	Good quality PRQ (moderate)



C1156a

Match each of the differential equations given in Column A with the type listed in Column B.

A. Differential Equation	В. Туре
a. $\frac{dy}{dx} - \frac{y}{x} = \ln x$	1. Variable separable
b. $\frac{dy}{dx} = \frac{e^x}{e^y}$	2. Homogeneous
c. $(x^2 + y^2)dx + 2xydy = 0$	3. Exact
d. $2x + y^3 + (3xy^2 + ye^{2y})\frac{dy}{dx} = 0$	4. Linear

CRQ, Calculus, November 2005, Q6a





C1156a		Comment
Assessment Component	Modelling	
PRQ/CRQ	CRQ	
Item Difficulty	-0.22	Moderately easy
Discrimination	0.295	Discriminates well
Confidence Index	0.472	Small deviation from expected confidence level
Expert Opinion	0.617	Large deviation from expected performance
Quality Index	0.265	Good quality CRQ (moderate)





PRQ, Calculus, June 2006, Q6



C66M06

C66M06		Comment
Assessment Component	Modelling	
PRQ/CRQ	PRQ	
Item Difficulty	-1.00	Moderately easy
Discrimination	0.687	Discriminates poorly
Confidence Index	0.452	Small deviation from expected confidence level
Expert Opinion	0.496	Average deviation from expected performance
Quality Index	0.379	Poor quality PRQ



C561aii

A bacterial colony is estimated to have a population of $P(t) = \frac{24t+10}{t^2+1}$ million, t hours after the

introduction of a toxin.

Is the population increasing or decreasing at this time?

CRQ, Calculus, May 2006, Q1aii



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C561aii		Comment
Assessment Component	Modelling	
PRQ/CRQ	CRQ	
Item Difficulty	-4.51	Very easy
Discrimination	0.810	Discriminates poorly
Confidence Index	0.549	Large deviation from expected confidence level
Expert Opinion	0.613	Large deviation from expected performance
Quality Index	0.553	Poor quality CRQ



6. Problem solving component

C1152a
Split
$$\frac{3}{(x-1)(x^2+x+1)}$$
 into partial fractions.
CRQ, Calculus, November 2005, Q2a





C1152a		Comment
Assessment Component	Problem solving	
PRQ/CRQ	CRQ	
Item Difficulty	-1.37	Moderately easy
Discrimination	0.439	Discriminates well
Confidence Index	0.352	Small deviation from expected confidence level
Expert Opinion	0.272	Small deviation from expected performance
Quality Index	0.160	Good quality CRQ (moderate)

C65M10



- A. $(\pi, 8\pi)$ and $(2\pi, 16\pi + 2)$
- B. $(\pi, 2)$ and $(2\pi, 16\pi + 2)$
- C. $(\pi, 8\pi)$ and $(2\pi, 16\pi)$
- D. $(\pi, 8\pi + 2)$ and $(2\pi, 16\pi + 2)$
- E. $(\pi, 8\pi + 2)$ and $(2\pi, 16\pi)$

PRQ, Calculus, June 2005, Q10

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C65M10



C65M10		Comment
Assessment Component	Problem solving	
PRQ/CRQ	PRQ	
Item Difficulty	1.73	Difficult
Discrimination	0.213	Discriminates well
Confidence Index	0.352	Small deviation from expected confidence level
Expert Opinion	0.609	Large deviation from expected performance
Quality Index	0.181	Good quality PRQ









A65M04		Comment
Assessment Component	Problem solving	
PRQ/CRQ	PRQ	
Item Difficulty	0.14	Moderately difficult
Discrimination	0.522	Average discrimination
Confidence Index	0.358	Small deviation from expected confidence level
Expert Opinion	0.280	Small deviation from expected performance
Quality Index	0.188	Good quality PRQ



A951
Evaluate
$$\sum_{r=1}^{100} [(r+1)^{r+1} - r^r].$$

CRQ, Algebra, August 2005, Q1





A951		Comment
Assessment Component	Problem solving	
PRQ/CRQ	CRQ	
Item Difficulty	0.67	Moderately difficult
Discrimination	0.439	Discriminates well
Confidence Index	0.480	Small deviation from expected confidence level
Expert Opinion	0.372	Small deviation from expected performance
Quality Index	0.239	Good quality CRQ (moderate)



A65M02	
$\sum_{k=1}^{k} \pi =$	
$\sum_{i=r+1}^{i=r+1} \Delta \pi (r+1-k)$	
B. $k(r - \pi + 1)$	
C. $\pi(k-r+2)$	
D. $\pi(k-r)$	

PRQ, Algebra, June 2005, Q2





A65M02		Comment
Assessment Component	Problem solving	
PRQ/CRQ	PRQ	
Item Difficulty	0.98	Moderately difficult
Discrimination	0.357	Discriminates well
Confidence Index	0.598	Large deviation from expected confidence level
Expert Opinion	0.475	Small deviation from expected performance
Quality Index	0.289	Poor quality PRQ (moderate)

PRQ, Calculus, May 2005, Q1





C55M01		Comment
Assessment Component	Problem solving	
PRQ/CRQ	PRQ	
Item Difficulty	-0.50	Moderately easy
Discrimination	0.728	Discriminates poorly
Confidence Index	0.288	Small deviation from expected confidence level
Expert Opinion	0.587	Large deviation from expected performance
Quality Index	0.349	Poor quality PRQ









C663c		Comment
Assessment Component	Problem solving	
PRQ/CRQ	CRQ	
Item Difficulty	-0.13	Moderately easy
Discrimination	0.604	Discriminates poorly
Confidence Index	0.411	Small deviation from expected confidence level
Expert Opinion	0.577	Large deviation from expected performance
Quality Index	0.361	Poor quality CRQ

A1154bii

 $M = \begin{pmatrix} 1 & -2 & -3 & : & 3 \\ -1 & 3 & 5 & : & -4 \\ 4 & -5 & k^2 - 15 & : & k + 12 \end{pmatrix}$

Suppose the system given by M represents three planes, P_1, P_2, P_3 . That is, we have:

 $P_{1}: x - 2y - 3z = 3$ $P_{2}:-x + 3y + 5z = -4$ $P_{3}:4x - 5y + (k^{2} - 15)z = k + 12$

Find the value(s) of k such that the three planes intersect in a single point. Do not calculate the co-ordinates of that point.

CRQ, Algebra, November 2005, Q4biii

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A1154biii		Comment					
Assessment Component	Problem solving						
PRQ/CRQ	CRQ						
Item Difficulty	0.35	Moderately difficult					
Discrimination	0.316	Discriminates well					
Confidence Index	0.717	Large deviation from expected confidence level					
Expert Opinion	0.964	Large deviation from expected performance					
Quality Index	0.529	Poor quality CRQ					



7. Consolidation component

C951

Rewrite the following integral as the sum of integrals such that there are no absolute values. DO NOT solve the integral. Give full reasons for your answer.

$$\int_{-2}^{5} |4x - x^{2}| dx$$

CRQ, Calculus, August 2005, Q1





C951		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	CRQ					
Item Difficulty	0.86	Moderately difficult				
Discrimination	0.419	Discriminates well				
Confidence Index	0.392	Small deviation from expected confidence level				
Expert Opinion	0.323	Small deviation from expected performance				
Quality Index	0.185	Good quality CRQ				



A45MA4

- If f is an odd function and g is an even function then
- A. $f \circ g$ is an even function
- B. $f \circ g$ is an odd function
- C. f is a one-to-one function
- D. g is a one-to-one function

PRQ, Algebra, March 2005, Tut Test A, Q4





A45MA4		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	PRQ					
Item Difficulty	1.11	Moderately difficult				
Discrimination	0.275	Discriminates well				
Confidence Index	0.698	Large deviation from expected confidence level				
Expert Opinion	0.296	Small deviation from expected performance				
Quality Index	0.207	Good quality PRQ				

A661.2

This question deals with the statement $P(n): n^3 + (n+1)^3 + (n+2)^3$ is divisible by 9.

Use Pascal's triangle to expand and then simplify $(k+3)^3$.

CRQ, Algebra, June 2006, Q1.2

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A661.2		Comment					
Assessment Component	Consolidation						
PRQ/CRQ	CRQ						
Item Difficulty	0.02	Moderately difficult					
Discrimination	0.666	Discriminates poorly					
Confidence Index	0.379	Small deviation from expected confidence level					
Expert Opinion	0.301	Small deviation from expected performance					
Quality Index	0.246	Good quality CRQ (moderate)					





PRQ, Calculus, August 2005, Q7





C85M07		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	PRQ					
Item Difficulty	-1.17	Moderately easy				
Discrimination	0.687	Discriminates poorly				
Confidence Index	0.230	Small deviation from expected confidence level				
Expert Opinion	0.514	Average deviation from expected performance				
Quality Index	0.272	Good quality PRQ (moderate)				



C654

State the Fundamental Theorem of Calculus.

CRQ, Calculus, June 2005, Q4



C654

C654		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	CRQ					
Item Difficulty	0.29	Moderately difficult				
Discrimination	0.481	Average discrimination				
Confidence Index	0.248	Small deviation from expected confidence level				
Expert Opinion	0.819	Large deviation from expected performance				
Quality Index	0.310	Poor quality CRQ (moderate)				

A56M01

Let $y = f(x) = \cos(\arcsin x)$. Then the range of f is A. $\{y | 0 \le y \le 1\}$ B. $\{y | -1 \le y \le 1\}$ C. $\{y | -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$ D. $\{y | -\frac{\pi}{2} \le y \le \frac{\pi}{2}\}$ E. None of the above

PRQ, Algebra, May 2006, Q1

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A56M01		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	PRQ					
Item Difficulty	3.07	Very difficult				
Discrimination	0.460	Discriminates fairly well				
Confidence Index	0.655	Large deviation from expected confidence level				
Expert Opinion	0.389	Small deviation from expected performance				
Quality Index	0.318	Poor quality PRQ (moderate)				

C662f

Let
$$f(x) = \frac{x^2}{(x-2)^2}$$
.
You may assume that $f'(x) = \frac{-4x}{(x-2)^3}$ and $f''(x) = \frac{8x+8}{(x-2)^4}$.
Find the points of inflection of f (if any).

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C662f		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	CRQ					
Item Difficulty	3.75	Very difficult				
Discrimination	0.646	Discriminates poorly				
Confidence Index	0.783	Large deviation from expected confidence level				
Expert Opinion	0.609	Large deviation from expected performance				
Quality Index	0.595	Poor quality CRQ				

CRQ, Calculus, June 2006 Q2f



C46MB6



C46MB6		Comment				
Assessment Component	Consolidation					
PRQ/CRQ	PRQ					
Item Difficulty	-2.24	Easy				
Discrimination	0.996	Discriminates poorly				
Confidence Index	1.000	Large deviation from expected confidence level				
Expert Opinion	0.544	Large deviation from expected performance				
Quality Index	0.933	Poor quality PRQ				



6.4 RESULTS

6.4.1 Comparison of PRQs and CRQs within each assessment component

Table 6.3 summarises the quality of the item, both PRQs and CRQs, within each assessment component. Within each component the number of good and poor quality items are given, both for the PRQ and CRQ formats. The numbers are also given as percentages of the total number of items.

COMPONENT	No. of PRQs	No. of CRQs	Total no. of items	Good quality items	Poor quality items	Good PRQs	Good CRQs	Poor PRQs	Poor CRQs
1.Technical	11	22	33	17 [52%]	16 [48%]	8 [73%]	9 [41%]	3 [27%]	13 [59%]
2.Disciplinary	24	34	58	28 [48%]	30 [52%]	12 [50%]	16 [47%]	12 [50%]	18 [53%]
3.Conceptual	26	30	56	28 [50%]	28 [50%]	14 [54%]	14 [47%]	12 [46%]	16 [53%]
4.Logical	7	6	13	5 [39%]	8 [61%]	1 [14%]	4 [67%]	6 [86%]	2 [33%]
5.Modelling	3	10	13	8 [62%]	5 [38%]	2 [67%]	6 [60%]	1 [33%]	4 [40%]
6.Problem solving	7	4	11	6 [55%]	5 [45%]	4 [57%]	2 [50%]	3 [43%]	2 [50%]
7.Consolidation	16	7	23	12 [52%]	11 [48%]	7 [44%]	5 [71%]	9 [56%]	2 [29%]

 Table 6.3:
 Component analysis – trends.

1. Technical

In the technical assessment component, there is a higher percentage (73%) of good PRQs than good CRQs (41%). 73% good PRQs compared to good 41% CRQs shows us that PRQs are more successful than CRQs as an assessment format in the technical component. There is also a much higher percentage (73%) of good PRQs than poor PRQs (27%). CRQs, however, are not that successful in this component, with the results showing 59% poor CRQs compared to 41% good CRQs. The conclusion is that the technical assessment component lends itself better to PRQs than to CRQs.



2. Disciplinary

In this study, the disciplinary component is the assessment component with the most items (58), of which 34 were CRQs and 24 were PRQs. In this component it is interesting to note that the percentages of good PRQs (50%) and good CRQs (47%) are almost equal. In addition, there is no difference between the good PRQs (50%) and the poor PRQs (50%), with very little difference between the good CRQs (47%) and poor CRQs (53%). PRQs and CRQs can be considered as equally successful assessment formats in the disciplinary component.

3. Conceptual

The conceptual component also contained many items (56), with an almost equal number of PRQs and CRQs (26 PRQs versus 30 CRQs). 50% of the items are of good quality and 50% are of poor quality. In this component, there is no clear trend that PRQs are better than CRQs or vice versa. There is a slight leaning towards good PRQ assessment (47% good CRQs compared to 54% good PRQs). Therefore, in the conceptual assessment component, PRQs could be used as successfully as CRQs as a format of assessment.

4. Logical

In this study, it is interesting to note that the majority of questions within the logical component were of a poor quality mainly due to the large percentage of poor PRQs. There are noticeably more good quality CRQs (67%) than good quality PRQs (14%), and noticeably more poor quality PRQs (86%) than poor quality CRQs (33%). A very high percentage of the PRQs (86%) in the logical component were of a poor quality. The conclusion is that the logical assessment component lends itself better to CRQs than to PRQs.

5. Modelling

In the modelling component, very few PRQs were used as assessment items in comparison to CRQs, 3 PRQs versus 10 CRQs, probably because it is difficult to set PRQs in this component. Despite the small number of PRQs, it was encouraging to note that the good PRQs (67%) far outweighed the poor PRQs



(33%). So in terms of quality, the PRQs were highly successful in the modelling component. There are also more good CRQs (60%) than poor CRQs (40%). It appears that although more difficult to set in the modelling component, PRQs could be used as successfully in the modelling assessment component as CRQs.

6. Problem solving

Although the problem solving component had the least number of items (11), it is interesting to note that there are more PRQs (7) than CRQs (4). There is a slightly higher percentage (57%) of good PRQs than good CRQs (50%). Although the sample is too small to make definite conclusions, there is no reason to disregard the use of PRQs in this assessment component. In fact, PRQs seem to be slightly more successful than CRQs, and the conclusion is that PRQ assessment format can add value to the assessment of the problem solving component.

7. Consolidation

It was somewhat surprising to note that corresponding to the highest level of conceptual difficulty, the consolidation component displayed an unusually higher proportion of PRQs (16) to CRQs (7). This supports the earlier claim that PRQs are not only appropriate for testing lower level cognitive skills (Adkins, 1974; Aiken, 1987; Haladyna, 1999; Isaacs, 1994; Johnson, 1989; Oosterhof, 1994; Thorndike, 1997; Williams, 2006). In the consolidation component there is a significant higher percentage (71%) of good CRQs than good PRQs (44%). In addition, there is a higher percentage of poor PRQs (56%) than good PRQs (44%). The high percentage of good CRQs (71%) in comparison to poor CRQs (29%) indicates that the consolidation assessment component lends itself better to CRQs than to PRQs.