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HOW COMPETENT MATHEMATICS TEACHERS DEVELOP PEDAGOGICAL CONTENT KNOWLEDGE IN STATISTICS TEACHING

BY

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CERTIFICATION

This thesis has been examined and approved as meeting the required standard of scholarship for the fulfilment of the Degree of Doctor of Philosophy in Mathematics Education.

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.. Date................................

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DECLARATION OF ORIGINALITY

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Declaration

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ABSTRACT

This study is concerned with how competent mathematics teachers develop pedagogical content knowledge (PCK) in statistics teaching. Pedagogical content knowledge was used as the theoretical framework that guided the research and data collection.

The study's methodology consisted of two phases. In the first phase, the six identified mathematics teachers undertook a conceptual knowledge written exercise. The result of this exercise was used to select the best four performing teachers for the second phase of the study. The second phase consisted mainly of lesson observations, interviews, written documents in the form of completed questionnaires, written diaries or reports, document analysis designed to produce rich detailed descriptions of participating teachers' PCK in the context of teaching statistics concepts at school level. The concept mapping exercise was used to indirectly assess participating teachers' content knowledge and their conceptions of the nature of school statistics and how it is to be taught. The qualitative data obtained were analysed to try to determine individual teachers' content knowledge of school statistics, related pedagogical knowledge, knowledge of learners' conceptions in statistics teaching, knowledge of learners' learning difficulties as well as how they developed their PCK in statistics teaching. The analysis was done based on iterative coding and categorisation of responses and observations made to identify themes, patterns, and gaps, in school statistics teaching. Commonalities and differences if any, in the PCK profiles of the four participating teachers were also analysed and determined.

The results of the study showed that overall, individual teachers develop their PCK in school statistics teaching by:

(a) formally developing their knowledge of the subject matter in a formal undergraduate educational programme, as well as subject matter content knowledge during classroom practice;

(b) using varied topic-specific instructional skills such as graphical construction skills in teaching statistical graphs;

(c) using diagnostic techniques (oral questioning and pre-activity, class discussions and questioning) and a review of previous lessons to introduce lessons, and to determine learners' preconceptions in statistics teaching ;

(d) Using teaching strategies that can help to identify learners' learning difficulties as well as intervention to address the difficulties;

(e) continually updating their knowledge of school statistics by attending content knowledge workshops and other teacher development programmes designed to improve content knowledge and practice.

Keywords: pedagogical content knowledge (PCK), subject matter content knowledge, pedagogical knowledge, instructional strategies, conceptions, learning difficulties, competent teachers, data handling, procedural knowledge, conceptual knowledge.

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the study

This study focused on how competent secondary school mathematics teachers develop pedagogical content knowledge (PCK) for teaching statistics in high school. While some researchers perceive statistics as a subject on its own (Moore & Cobb, 2001; Gordon, Petocz & Reid, 2007), others believe it should be taught as part of the mathematics curriculum and consequently view it as a mathematical concept (Franklin, Kader, Mewborn, Moreno, Peck, Perry & Schaeffer, 2005; Gattuso, 2006).

According to the National Curriculum Statements (NCS) of South Africa (DoE, 2009), the country in which this study was conducted, statistics is taught as part of the mathematics curriculum under the rubric of 'data handling'. In accordance with the new curriculum, the learning outcomes of mathematics require that learners should be able to use appropriate measures of central tendency and spread to collect, organise, analyse, and interpret data in order to establish statistical and probability models for solving related problems (DoE, 2007).

According to the NCS, instructional guides and other publications, teachers need to be given in-service professional support by mathematics experts or professionals with the statistics knowledge required to implement the new mathematics curriculum. This is because the topic of statistics has been included in the national curriculum for the first time, and it is assumed that most teachers will not have the requisite knowledge for teaching it. Thompson (2005) indicated that in order to implement the new curriculum effectively, teachers need subject matter knowledge, pedagogical knowledge, and pedagogical content knowledge (PCK).

Subject matter knowledge refers to the disciplinary knowledge obtained through formal training in colleges and universities, while pedagogical knowledge pertains to the knowledge of instruction and learning that the teacher needs in order to deal with everyday classroom educational tasks (Vistro-Yu, 2003). Such tasks involve the use of various teaching styles and strategies and the management of learning processes in the classroom (Vistro-Yu, 2003). These skills and competencies are normally acquired through formal training and teaching practice. Simply described, PCK is about the overall knowledge the educator has of the subject matter content that learners should master in a particular topic or subject, and how it should be taught, so that effective and efficient learning can take place (Mitchell & Mueller, 2006). In short, PCK is an amalgam of subject matter content and pedagogy, which is uniquely the province of teachers and involves their own special form of professional understanding for good teaching (Jong, 2003).

PCK is specific to teaching and differentiates between expert teachers in a particular subject area and subject area experts (Griffin, Dodds & Rovengno, 1996). To illustrate, mathematics teachers differ from mathematicians, not necessarily in the quantity and quality of their subject matter knowledge, but more specifically in how that knowledge is organised and used (Cochram, De Ruiter & King, 1993). An experienced mathematics teacher's knowledge of the subject is organised from a teaching perspective and is used as a basis for helping learners to understand specific concepts. A mathematician's knowledge, on the other hand, is normally organised from a research perspective and is used mainly as a basis for developing new knowledge in the field. This implies that PCK may be something beginner or inexperienced teachers may not necessarily learn only from textbooks or from short courses. From the literature reviewed, little is known as to how PCK is developed, or even facilitated, in the context of teaching statistics (Godino, Batenero, Roa & Wilhelmi, 2011; DoE 2008; Jong, 2003). Therefore, further research is needed in order to identify and define the skills and practices necessary for PCK development in statistics education more carefully (DoE, 2008).

To develop PCK, Jong (2003) argues that teachers need to explore instructional strategies for specific topics and their learners in practice. Various studies – such as those by Dooren, Verschattel and Oghenna (2005), Boerst (2003), Halim and Meerah (2002) and Van Driel, Verloop and De Vos (1998) have shown that inadequate PCK is one of the areas that require most attention in teacher education, as many teachers are unable to enhance learner performance because of lack of subject matter content knowledge and PCK. Many beginner teachers, including inexperienced mathematics teachers, do not know how to develop and use PCK in their teaching (Van Driel et al., 1998; Halim & Meerah, 2002). In consequence, they become uncomfortable with teaching certain topics, and, for that reason, may omit them altogether (ICM/IASE, 2007).

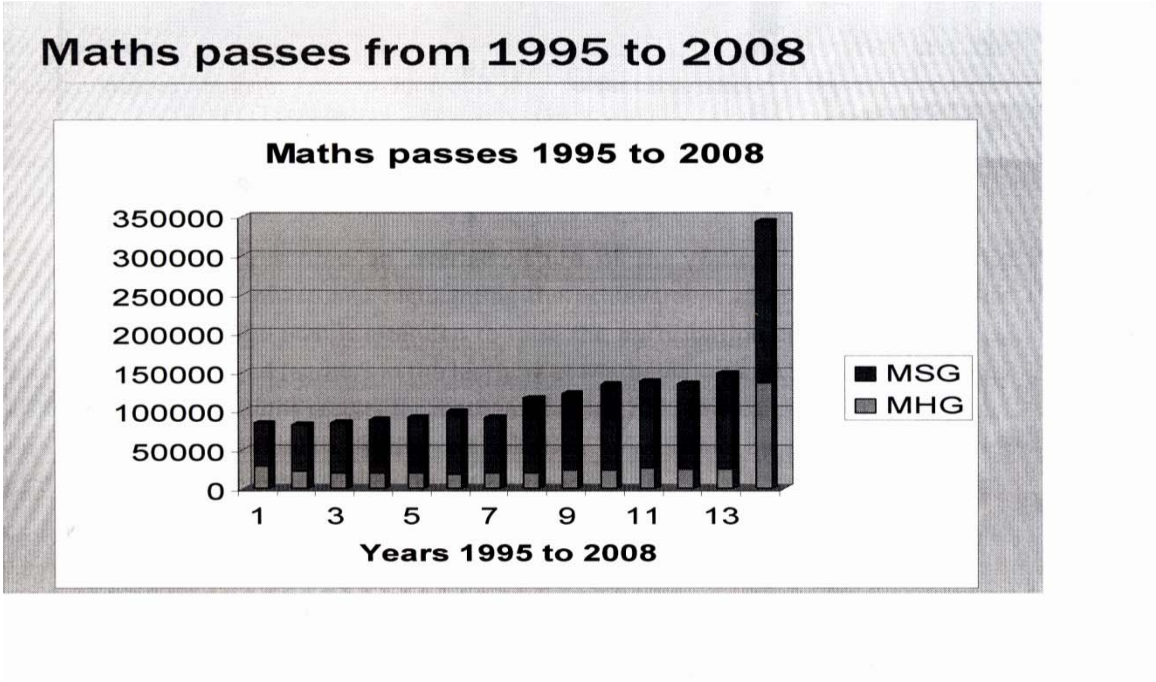
Data on mathematics enrolment and learner performance over a period of five years in the South African Senior Certificate (SC) examination, as displayed in Table 1.1 and Figure 1.1

below, show that learners generally underachieve in mathematics. Mathematics failure rates in the SC examination remain unacceptably high, and the number or percentage of learners that leave Grade 12 with a higher-grade pass in mathematics is unacceptably low. While the percentage of candidates that wrote the mathematics examination over the period of six years increased, the percentage of learners that passed mathematics for standard grade (MSG) was below 30%, and below 10% in mathematics for higher grade (MHG) (Figure 1.1). This suggests a crisis of mathematics underachievement at secondary school level.

Table 1.1: Learners' performance in mathematics from 1999 to 2004 in the South African Senior Certificate Examination

	1999		2000		2001		2002		2004	
No. of candidates	**511,225**		**489,941**		**449,371**		**442,590**		**467,985**	
% of learners that wrote mathematics	**55%**		**58%**		**59%**		**59%**		**81%**	
% of learners that passed mathematics	**SG**	**HG**	**SG**	**HG**	**SG**	**HG**	**SG**	**HG**	**SG**	**HG**
	20%	**4%**	**21%**	**5%**	**24%**	**4%**	**27%**	**5%**	**29%**	**5%**

Source: DoE (2006); CDE (2007)



NB: This is the period in which the standard and higher grades examinations are used to assess mathematics learners in the Senior Certificate Examination.

In addition, the chief examiner's report on learners' performance in both mathematics and mathematical literacy in the 2008 and 2009 SC examinations shows that learners generally underperform in statistics (DoE, 2009). According to this report, there was a steady increase in learners' enrolment and performance in mathematics, compared with previous years, but about 60% of those who passed scored between 30% and 40% (the pass mark for mathematics, according to the NCS, is 30%) (DoE, 2009). Furthermore, the learners' performance in questions relating to statistics in paper 2 was below 35%. As a result of the poor performance in statistics, teachers' ability to teach this topic, the quality of senior certificate products, and university enrolment in mathematics and statistics-related subjects have been subject to review (Keeton, 2009).

Many studies, such as those by Howie (2002) and DoBE (2012), on the causes of poor performance in mathematics in South Africa, show that one of the main factors that is attributable to learners' poor performance is the teacher. Others include language and classroom environment (CDE, 2004). The interest in this study is with the teacher factor. The study is aimed at investigating specifically how competent teachers develop and use PCK to improve the quality of instruction and learning in statistics. The competent mathematics teachers were identified from their learners' final results in mathematics in the public senior certificate exam and on recommendations by principals, peer teachers and subject experts in the Department of Education. Although being competent may not necessarily mean that they are expert in statistics, their selection as competent teachers depends on their final Senior Certificate Examination results in mathematics over time. The research seeks to determine what it is that these teachers who have been classified as competent teachers have and do when using their PCK to teach particular subject matter content in statistics. The assumption here is that PCK can be measured. PCK has been used as a theoretical framework for this study.

The topic of statistics has been chosen because it is completely new in the mathematics curriculum, and many teachers may not have adequate experience in teaching it, let alone in handling the difficulties learners experience with it. Until the introduction of the topic of data handling in mathematics and mathematical literacy in 2006, statistics was not taught in high schools (DoE, 2006). Many, if not all, teachers of mathematics would not have formal knowledge of statistics, let alone knowledge of learners' preconceptions, which need to be addressed in teaching and learning statistics. The assumption is that few in-service teachers would have developed the PCK needed to teach the topic effectively. Therefore, it would be useful to study how teachers who are considered competent go about teaching a new topic in statistics, and to document what it is that they have and do as they go about preparing their lessons, and how they teach those data-handling lessons.

In 2007, this lack of familiarity with statistics content and teaching on the part of secondary school mathematics teachers worldwide was given added support by papers presented at the joint conference of the International Commission for Mathematics Instruction and the International Association for Statistics Educators (ICMI/IASE, 2007). The conference highlighted that mathematics teachers are likely to face challenges in terms of teaching a topic such as statistics in which they do not necessarily have an understanding of learners' learning difficulties, and may not know how to present the content in a way that learners can understand.

Recent studies, such as those by Jong (2003), Jong, Van Driel and Verloop (2005), Capraro, Capraro, Parker, Kulm and Raulerson (2005), Wu (2005), and Godino et al. (2011), showed that most mathematics teachers at high-school level have limited PCK. A clear understanding of how teachers develop PCK and use it to enhance learner achievement in mathematics is useful knowledge for any pre-service and in-service teacher education programme. This study is an attempt to provide a comprehensive description and analysis of how the mathematics teachers selected for the study developed their PCK in teaching statistics.

1.2 The research problem

The NCS Curriculum for Mathematics was introduced in Grade 10 in all high schools in the Republic of South Africa in 2006. Mathematics teachers were charged with the responsibility of delivering the curriculum in the classroom in line with the NCS recommendations and ensuring effective teaching, so that learner achievement could be enhanced (DoE, 2006). However, since the introduction of this curriculum, learners have not been performing as they should, because of internal and external classroom factors that result in underachievement (Howie, 2002; CDE, 2004; DoE, 2008).

Reddy (2006) identifies PCK as one of the limiting factors in enhancing learner achievement in mathematics in the South African context. Other researchers elsewhere in the world, such as Wu (2005), Capraro et al. (2005), Halim and Meerah (2002), and Van Driel et al. (1998), have come to the same conclusion, especially with regard to statistics (Cazorla, 2006), which has only recently been included in the curriculum as a formal aspect of mathematics. The lack of familiarity with statistics has placed teachers' confidence in their ability to teach it in doubt (ICMI/IASE, 2007). Poor learner performance in statistics was also noted at the joint conference of the ICMI and the IASE (ICMI/IASE, 2007), at which conference delegates attributed learners' poor performance to the rudimentary state of mathematics teachers' PCK in statistics. In addition, the chief examiner's report for the Senior Certificate Examination in Mathematics shows that learners underperform in statistics (DoE, 2008). The report suggests that poor PCK background may have contributed to learners' underperformance in statistics, and that this background therefore needs to be investigated (DoE, 2010).

Given the instructional demands of the new mathematics curriculum and the poor performance of learners in statistics, this study was concerned with investigating how competent mathematics teachers at high-school level in South Africa develop PCK in statistics teaching in order to enhance learners' achievement in mathematics.

1.3 Aims of the study

The aims of the study were:

a) To determine how competent secondary school mathematics teachers develop PCK for teaching statistics
b) To determine the implications that PCK has for mathematics education programmes

1.4 Statement of the problem

The problem identified for this study was to determine how secondary school mathematics teachers who are assumed to be competent develop the PCK they use in teaching statistics in school mathematics. In addition, the implications of these findings for mathematics teacher education programmes were determined and discussed.

1.5 Research questions

The problem statement gave rise to the following research questions:

1) What subject matter content knowledge of statistics do mathematics teachers who are considered competent have and demonstrate during classroom practice?
2) What instructional skills and strategies do these teachers use in teaching statistics?
3) What knowledge of learners' preconceptions and learning difficulties, if any, do these teachers have and demonstrate during classroom practice?
(4) How do these teachers develop PCK in statistics teaching?

1.6 Significance of the study

The significance of this study is that it is hoped its findings will provide a knowledge base and process employed by mathematics teachers to develop pedagogical content knowledge in statistics teaching for the improvement of learners' performance; and ideas and knowledge that can be incorporated into a mathematics education programme for in-service and pre-service mathematics teachers.

Besides, PCK development is a complex process and it is not clear how it is developed in statistics teaching for mathematics classroom practices. 'PCK is distinct from a general knowledge of pedagogy, educational purpose and learners' characteristics' (Jong, Van Driel & Verloop, 2005: 948). 'Moreover, because PCK is concerned with the teaching of a particular topic for example statistics, it may turn out to differ considerably from the subject matter itself' (Jong, Van Driel & Verloop, 2005: 948). PCK is said to develop by an iterative process that is rooted in classroom practice (Miller, 2006). The implication is that beginning teachers have little or no PCK at their disposal, particularly if they are new to statistics teaching. A clear understanding of how PCK is developed in statistics teaching will be a requisite for designing effective statistics education programme for in-service and pre-service statistics educators.

A great deal of research has been conducted in an attempt to identify and characterise PCK during classroom practice, but research communities continue to call for studies to devise methods of measuring PCK (Miller, 2006). According to Miller (2006), PCK represents much more than a category of teacher knowledge; it provides a starting point for research involving teacher education. As a theoretical framework of this study, PCK provides a process for organising teacher education research.

1.7 Theoretical framework

Several researchers (Shulman, 1986; Van Driel, 1998; Jong, 2003; Abell, 2008; Hill, 2008; Watson, Callingham & Donne, 2008; and Toerien, 2011) have made serious attempts to develop models to measure teachers' PCK in mathematics and the sciences. These researchers have largely been challenged by the difficulties the models present in distinguishing the boundaries that make up the various constructs (Graham, 2011). These difficulties include the changeable nature of PCK, which makes it difficult to pinpoint specific constructs of this category of teacher knowledge (Miller, 2006). In addition, because of the numerous categories of knowledge that could be integrated into PCK, differences may exist in the boundaries of a PCK construct (Hill *et al.*, 2008); and indeed because teachers, like learners, construct their own knowledge, there is every likelihood that there will be individual examples of teacher PCK. It is precisely because of these constraints that research on PCK development has not always been as straightforward as researchers might have hoped. A review of the literature indicates that the use of PCK in research and for methods of data collection and analysis has mostly taken two forms (Shulman, 1986; Van Driel, 1998; Penso, 2002; Jong, 2003; Cazorla, 2006; Abell, 2008; Hill, 2008; Watson, Callingham & Donne, 2008; and Toerien, 2011). The first form has to do with research on PCK as a category of teachers' knowledge, that is, knowledge specifically constructed by teachers and yet distinctly different for each subject matter content area. The second form involves research using PCK as a theoretical framework, which is based on a number of assumptions, as we shall see later. The fundamental difference between these two forms of using PCK in research is that while the first entails trying to identify or measure PCK, the second utilises the assumption that PCK exists, in order to examine other aspects of teacher knowledge (Miller, 2006). In this study, the interest was in first determining teacher PCK in the context of teaching school statistics, which is assumed to exist, and second in determining the way in which it (PCK) is developed and used in teaching school statistics topics. To this end, the study used PCK as a theoretical framework, consisting of teacher subject matter content knowledge, pedagogical knowledge, and knowledge of learners' conceptions and learning difficulties to explore the main research questions based on a number of assumptions.

The initial model of PCK, which was supported by several studies (eg Shulman 1987), tagged PCK as the specific teacher knowledge that allowed a teacher to more thoroughly understand how to transform content knowledge into a more conceptually accessible version for students or learners. As explained by Shulman (1987) PCK results from the blending of content knowledge and pedagogical methods. Thus, it is a widely accepted belief that PCK represents the category of knowledge that is needed for a novice teacher to mature into an expert (Bodner & Orgill, 2007). Shulman's (1987) vision and Ball *et al.'s* (2008) description of teacher knowledge as an amalgam of categories of knowledge, including content, curricular, pedagogical, and student knowledge, and PCK, has virtually compelled many teacher education programmes to create new instructional activities for improving classroom practice. This same vision of enriching classroom practice has provided a focus on education research. Unfortunately, PCK, because of its nebulous nature, remains a category of knowledge that is difficult to isolate and research (Miller, 2006). Nevertheless, it provides a starting point for researchers who wish to collect and analyse data on other aspects of teacher knowledge. In this study the teachers' classroom practice in statistics was therefore investigated in a series of lesson observations, in order to explore what PCK exists and how the participating teachers demonstrated their PCK in the context of teaching statistics in school mathematics. The first consideration was that identifying the category(ies) of knowledge that the teacher has, as defined, in the teaching of statistics would yield information about teacher's PCK and how it is developed and used during classroom practice.

It was mentioned earlier that the use of PCK as a theoretical framework has provided researchers with a new perspective for collecting and analysing data about teacher knowledge or cognition (Jong, 2003; Rollnick *et al.*, 2008; Toerien, 2011). The use of PCK as a theoretical framework allows researchers to focus on specific questions about a teacher's knowledge base and is founded on a series of assumptions. Miller (2006) has indicated that PCK embodies an epistemological approach to understanding teacher knowledge. Precisely for this reason, in this study, the teachers' PCK in statistics teaching, and the way in which they developed it, was conceptualised as comprising content knowledge, pedagogical knowledge, and knowledge of learners' preconceptions and learning difficulties in the context of teaching school statistics. These central categories of teacher knowledge were used as the theoretical framework that provided a guide for data collection, analysis and discussion of what and how PCK in statistics teaching was developed.

Assumptions of the study

Based on the above considerations, in this study the use of PCK as a theoretical framework was built on the following assumptions, as summarised (Miller 2006).

- PCK represents a category of teacher knowledge that is the essence of an expert teacher in a specific topic (Miller, 2006), in this case in school statistics teaching. In this study, the blending of subject matter content knowledge, pedagogical knowledge, and knowledge of learners' preconceptions and learning difficulties was used to describe the PCK of the participating teachers.

- PCK provides a framework that can be used to describe the origin of this critical teachers' knowledge (Miller, 2006). In other words, PCK represents an epistemological approach, to constructing teaching knowledge.

- PCK is a constructivist process and therefore a continually changing body of knowledge. Teachers, like learners, construct their own knowledge and in this study it is assumed that the development of PCK is a continuously modifying unit, beginning with teacher preparation programmes, evolving through teaching experience and assimilating and accommodating professional development opportunities.
    - Identifying and measuring PCK constructs can be achieved by using instruments designed for that purpose. In this study, the components of PCK were assessed using multiple assessment strategies, which include concept mapping, teacher interviews, teacher questionnaires, lesson observation, written classroom activity reports and document analysis.

According to Shulman (1986), PCK is a specific category of knowledge that goes beyond the knowledge of subject matter per se to include the dimension of subject matter knowledge for teaching. It refers to teachers' interpretations of subject matter in the context of facilitating learning. In consequence, it has been argued that PCK is one of the seven categories in Shulman's (1986) categorisation of a knowledge base for teaching. The key elements of

Shulman's conception of PCK are:

i) Knowledge of the representation of the subject matter for teaching
ii) Knowledge of relevant instructional strategies
iii) Knowledge of learners' conceptions (preconceptions and misconceptions)
iv) Knowledge of learners' learning difficulties

For the purpose of this study, these four elements appear to be most appropriate in defining the PCK that may be used for teaching statistics in school mathematics, namely subject matter content knowledge; knowledge of teaching (pedagogical knowledge); knowledge of learners' conceptions (preconceptions and misconceptions); and knowledge of learners' learning difficulties. These four elements cover the views and constructs of PCK used by various researchers in this domain, such as Jong (2003), Shulman (1986), Jong et al. (2005), Halim and Meerah (2002), Rollnick et al. *(*2008), Hill (2008) and Toerien (2011).

For the construct of PCK, the working definition is that PCK is an amalgam of subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), knowledge of learners' conceptions and knowledge of learners' learning difficulties. In this study, the researcher's intention was to determine the PCK that competent teachers use in teaching statistics by observing the PCK that such teachers demonstrate in the classroom. It is assumed that because such teachers are considered competent and have experience in teaching mathematics, they will be able or will be likely to integrate content knowledge and pedagogical knowledge in ways that contribute to the development of the PCK used for teaching statistics (Jong, 2003). To this end, the development of PCK was inferred from the teacher interviews, questionnaires, written reports, document analysis and lesson observation.

1.7.1 Subject matter content knowledge

According to Manouchehri (1976), subject matter content knowledge consists of an explanatory framework and the rules of evidence within a discipline. The subject matter content knowledge of prospective mathematics teachers is acquired primarily during disciplinary education (Jong, 2003). This knowledge consists of substantive content knowledge and syntactic content knowledge (Barnes, 2007). Substantive content knowledge refers 'to the concepts, principles, laws, and models in a particular content area of a discipline'. Syntactic content knowledge, by contrast, is the 'set of ways in which truth or falsehood, validity or invalidity are established' (Schwab, 1978, cited in Shulman, 1986). In practice, teachers should be able not only to define the acceptable truths for learners in a domain, but to explain, in theory and in practice, why these truths are worth knowing and how they relate to other propositions, within the discipline and outside it.

Both types of subject matter knowledge (substantive and syntactic) are needed for teachers' development of PCK, because they help to create an adequate understanding of the nature of the subject matter and beliefs about how it should be taught (Jong, 2003). It is therefore assumed that mathematics teachers with good PCK have both types of subject matter content knowledge and are able to apply this knowledge in making the topic understandable to learners. This assumption is given empirical support by Wu (2005), who indicated that teachers with good PCK have a firm command of subject matter knowledge and are able to design mathematics instructional material that allows learners to grasp what they teach. Muijs and Reynolds (2000) referred to these teachers as effective teachers.

Other scholars, such as Carpenter, Fennema, Petterson and Carey (1988), Even (1993), Manouchehri (1997), Van Driel et al. (1998), Halim and Meerah (2002), Tsangaridou (2002), Viri (2003) and Hill (2008), have studied the influence of subject matter knowledge on the PCK of pre-service, novice and expert teachers. These studies revealed that teachers' content knowledge goes a long way towards determining the level of teachers' PCK. The subject matter content knowledge is one of the components of PCK that will be assessed in this study.

1.7.2 Pedagogical knowledge

Cochram et al. (1993) define pedagogical knowledge as knowledge about teaching. Vistro-Yu (2003) defines it as the knowledge used for teaching, particularly expertise in teaching techniques, psychological principles, classroom management, and teaching and learning processes. Following these definitions, pedagogical knowledge is believed to be the kind of information that a teacher needs and uses to perform everyday teaching tasks, involving teaching styles and strategies, classroom management and teaching and learning processes relating to learners in the classroom. Research findings by Rollnick et al. (2008), Jong et al. (2005) and Vistro-Yu (2003) show that a mathematics teacher with adequate pedagogical knowledge is able to design good teaching and learning strategies and manage the classroom and other instruction and learning processes. This constituent framework seems appropriate for defining the construct of pedagogical knowledge, as it describes in operational terms what the teacher needs to do to create an environment that is conducive to learning. In this study, the focus was on the instructional skills and strategies used for teaching statistics in school mathematics.

To this end, the pedagogical knowledge of mathematics teachers was assessed by examining their lesson planning and implementation, questionnaires, written reports, and interviews, in order to probe the way in which competent teachers develop their pedagogical knowledge and use it in the instruction and learning process.

1.7.3 Knowledge of learners' conceptions in statistics teaching in school mathematics

Learners' conceptions in statistics in school mathematics consist of preconceptions and misconceptions. A mathematical misconception is a belief or idea that is based on incorrect or erroneous information about a given mathematical concept (Olivier, 1989). According to Olivier (1989), most mathematical misconceptions arise because of pre-existing concepts or preconceptions in the mind of the learner or the teacher. Misconceptions can occur when an attempt is made to link preconceptions and new knowledge to be learned. Olivier (1989) argues that misconceptions play a key role in understanding a new concept. The role of the mathematics teacher in resolving mathematical misconceptions is usually to develop some form of teaching and learning approach, such as teacher-learner or learner-learner discussion, communication, reflection, and negotiation of meaning, that addresses the missing concept (Penso, 2002; Cazorla, 2006). Through these approaches, the mathematics teacher may be able to get to the root of the misconception.

Cazorla (2006) for example reported that misconceptions and the way in which mathematics lessons are taught are among the factors that cause learning difficulties. According to her, most statistics teachers do not have adequate knowledge of the school curriculum and the approaches needed to teach and learn statistics, which can result in poor content delivery in the classroom situation. Jong (2003) noted that in order to identify and resolve misconceptions and learners' learning difficulties during classroom practice, the teacher could use convergent and inferential techniques. Convergent and inferential techniques are data-collection systems that entail developing questions for a topic in short-answer and multiple-choice formats to probe the preconceptions and misconceptions of learners (Jong, 2003). The teachers' written reports and the learners' notebooks may help to identify where the learners' learning difficulties lie (Jong, 2003; Jong et al., 2005, Penso, 2002; Van Driel et al., 1998).

The participating teachers in this study will be examined to determine whether they have prior knowledge of statistics as they teach the assigned topic through lesson planning and implementation.

1.7.4 Knowledge of learners' learning difficulties

Penso (2002) reports that learning difficulties may stem from the way lessons are taught. For example, learning difficulties may arise from the content of the lesson, lesson preparation and implementation and the learning atmosphere (Penso, 2002). Other factors include misconceptions that learners and teachers have about a topic, as well as cognitive and affective characteristics of learners. According to Penso (2002), 'learners consider their learning difficulties to be the result of conditions that existed prior to the process of teaching, as well as those existing in the course of teaching'.

In this study, the ways in which the teachers identified and addressed the learning difficulties that learners encountered during classroom practice were determined in lesson observation.

From the above discussion, subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), learners' conceptions (preconceptions and misconceptions) and learners' learning difficulties were used to conceptualise the construct of PCK for teaching school statistics. These frameworks were derived from the model proposed by researchers such as Shulman (1986), Van Driel (1998), Jong (2003), Cazorla (2006), Penso (2002), Abell (2008), Hill (2008) and Toerien (2011), as discussed in sections 1.7.1□1.7.4. The selection of these components of PCK was based on the assumption that PCK is dynamic, topic specific, and transformative, and can be measured using these frameworks (Corrigan, 2008). While the subject matter content of the participating teachers was measured using a conceptual knowledge exercise, concept mapping, interviews and lesson observation, instructional skills and strategies were assessed using lesson observation, questionnaires, interviews, written reports and document analysis. Lesson observation, questionnaires, written reports and reviews of teachers and learners' portfolios, as well as lesson plans and learners' workbooks, were used to assess the teachers' knowledge of learners' preconceptions and learning difficulties in statistics teaching. The roles of each instrument in measuring the individual component are described in Section 3.5.1.

To summarise, subject matter content knowledge, pedagogical knowledge, and learners' conceptions and learning difficulties were used to conceptualise the PCK needed for teaching statistics in school mathematics.

1.8 Scope of the study

This study explored how selected mathematics teachers at high-school level develop PCK in statistics teaching. The focus was on teachers who were teaching mathematics in accordance with the NCS Curriculum (now called Curriculum and Assessment policy Statement (CAPS)) for Mathematics at high schools in Tshwane North Education District in South Africa. These teachers were selected as participants for this study based on the performance of their learners in the public Senior Certificate Examination and on being recommended as competent teachers by principals, peers, and mathematics specialists at the Department of Basic Education (DoBE). Since PCK is topic specific (Corrigan, 2008), data were collected during statistics lessons by means of lesson observation. The participants in this study were few, because of the criterion used, namely a pass rate of 70%, and because participation was voluntary.

1.9 Criteria for selecting the topic

The concept of statistics is defined by Otumudia (2006) as the science of collecting, organising, and analysing data for any given purpose. Statistics helps us to reduce large and scattered data to an understandable level, thereby enabling us to make decisions in the face of uncertainty (Otumudia, 2006).

Statistics is taught as part of the mathematics curriculum under the rubric of 'data handling'. Data handling is one of the four major topics in the Curriculum and Assessment Policy Statements (CAPS) (DoBE, 2011). The reasons for including data handling in the new curriculum are:

i) Basic statistical knowledge is necessary for all kinds of data interpretation, as people encounter a great deal of categorical and numerical observations that should be used to guide decisions (DoE, 2006).

ii) Data handling helps one to build critical thinking, to understand reality, and to be able to participate in social actions.

iii) Statistics and probability are useful in daily life and play an instrumental role in other disciplines, such as economics, engineering, and medicine (Franklin & Mewborn, 2006).

iv) There is a need for basic stochastic knowledge in many professions, and statistics plays an important role in developing critical thinking that help in the development of this type of knowledge (Innabi, 2002).

For these reasons, the learning outcomes require learners studying statistics to be able to collect, organise, analyse, and interpret data to establish statistical and probability models for solving related problems (DoBE, 2011).

1.10 Definition of terms

In this section, some of the terms that are used to describe how mathematics teachers develop PCK for statistics teaching are defined operationally.

- **National Curriculum Statements (NCS)**

The National Curriculum Statements (NCS) are guidelines that state what each learner should achieve in terms of learning outcomes and assessment standards by the end of each grade. In this study, the NCS for Mathematics is used to describe the curriculum for mathematics as the subject that is taught in Grades 10 to 12.

- **Curriculum and Assessment Policy Statement (CAPS)**

The National Curriculum and Assessment Policy Statement is a ‘single, comprehensive, and concise policy document, which replaced the Subject and Learning Area Statements, Learning Programme Guidelines and Subject Assessment Guidelines for all the subjects listed in the National Curriculum Statement Grades R – 12’ (DoBE, 2012).

- **Pedagogical content knowledge (PCK)**

The construct of PCK constitutes an amalgam of subject matter content knowledge, pedagogical knowledge (instructional skills and strategies), knowledge of learners' conceptions and knowledge of learners' learning difficulties. In this study, PCK is used to describe and measure the way mathematics teachers combine subject matter content knowledge and pedagogical knowledge, as well as use their knowledge of learners' preconceptions and learning difficulties to carry out effective teaching during classroom practice.

- **Pedagogical knowledge**

Pedagogical knowledge is that knowledge that a teacher needs and uses to perform everyday teaching tasks, involving instructional skills and strategies, and classroom management and teaching and learning processes relating to learners in the classroom (Vistro-Yu, 2003). Pedagogical knowledge is used to define the construct of PCK in statistics teaching in this study. Specifically, the instructional skills and strategies will be used to describe the pedagogical knowledge in statistics teaching in this study.

- **Conceptions in the teaching and learning of statistics**

Conceptions in teaching and learning statistics consist of preconceptions and misconceptions. A preconception is regarded as the prior knowledge of a given topic with which learners come to the class (Olivier, 1989) and is used as such in this study. It is manifested during lesson observation. A misconception can occur as a result of a pre-existing concept. Both preconceptions and misconceptions can contribute to learners' learning difficulties in classroom practice. The term 'misconception' was used to describe the learners' beliefs or notions that were based on incorrect or erroneous information about a given statistical concept demonstrated during classroom practice. Teachers' knowledge of learners' conceptions in learning statistics was used to describe the PCK that was likely to be used for teaching statistics in school mathematics.

- **Competent mathematics teachers**

In this study, competent mathematics teachers were identified based on their learners' final results in mathematics in the public senior certificate exam and recommendations made by principals, peer teachers and subject experts at the Department of Education. Although being competent may not necessarily mean that the teachers are knowledgeable or expert in statistics, they were able to help their learners to do well in their final Senior Certificate Examination in Mathematics. The teachers were observed while teaching school statistics in order to determine how they develop their PCK.

- **Conceptual knowledge**

  Conceptual knowledge involves an understanding of mathematical ideas and concepts, as well as the interrelationships among these concepts. It consists of the ability to identify and apply principles, facts and definitions, and to compare and contrast related concepts (Engelbrecht & Potgieter, 2005). In this study the conceptual knowledge approach involves the use by the teacher of mathematical ideas, principles, facts and definitions to explain mathematical concepts and their relationships during the teaching and learning of a particular topic.
- **Procedural knowledge**

  Procedural knowledge is a formal symbolic representation system of a given mathematical task using algorithms, or rules, to complete the mathematical tasks (Star, 2002). In practice, it means for the teacher the use of particular rules, algorithms or procedures to complete a given task without necessarily providing an explanation underpinning the rules or procedures used. For example, the construction of statistical such as bar graph, histogram, ogive and scatter diagrams requires that one should first draw the axes, choose the scale, label the axes, plot the points and join the line of best fit (Leinhardt *et al*, 1990). The four participating teachers followed this procedure during their lessons on bar graphs, histograms, ogives and scatter diagrams. This teaching approach essentially uses what is referred to in this study as a **procedural knowledge approach.**
- **Document analysis**

Document analysis is a technique used in this study to gather information. It describes the act of reviewing the documentation of comparable school systems in order to extract pieces of information that are relevant to the current research project. Hence it is sometimes regarded as a research project requirement. In this study, document analysis was used to extract information about teaching and learning of statistics from the NCS for Mathematics, teacher and learner portfolios, and learners' class workbooks.

1.11 The chapter structure of the thesis

The study is divided into six chapters. Chapter 1 presents the introduction and background of the study and the way in which the background relates to the problem under investigation. The context in which the study took place is described.

PCK, as one of the forms of knowledge needed to implement the curriculum, was defined from four perspectives, namely content-specific knowledge; content-specific instructional strategies; knowledge of learners' conceptions of statistics teaching and learning; and knowledge of learners' learning difficulties. The chapter then presents the guiding research questions and theoretical framework based on the purpose of the study and the statement of the problem. The key concepts in this study are highlighted and discussed. The chapter concludes with a brief discussion of the structure of the thesis.

Chapter 2 focuses on the literature review, which captures the empirical and theoretical aspects related to the process of PCK development and how it is used in classroom practice to teach mathematics and science. The literature review derives its focus from the National Curriculum Statement for Mathematics, theoretical framework and the research questions, which seek to describe the way in which competent mathematics teachers develop PCK in statistics teaching. Chapter 2 is divided into two sections. The first section discusses literature about the content of statistics according to NCS and research on the teaching of statistics in school mathematics. The second section discusses the models of capturing PCK, conceptualisation of PCK and techniques for measuring PCK.

Chapter 3 discusses the methodology of the study. It is argued that a rich description of data comes from using several strategies of investigation, data collection, and data analysis. The chapter describes the methodological plans for the study, the pilot study, the participants, the research activities, and the various instruments used in the collection of data. The validity and reliability of the instruments are also discussed in this chapter.

Chapter 4 presents the results of the data collection discussed in chapter 3. The first presentation concerns the quantitatively analysed data, and the second concerns the qualitatively analysed data. The latter relies on the quantitative data that has been analysed. While the quantitative data are derived from the conceptual knowledge exercises, the concept mapping exercises, and the results of the schools from which the participants were selected, the qualitative data are derived from the interviews, lesson observations, free-response questionnaires, teachers' written reports and documents related to teachers' guides, and learners' portfolios, mathematics workbooks, and textbooks. In this chapter, the guiding research questions and the theoretical framework are revisited in order to determine how competent mathematics teachers develop their PCK in statistics teaching.

Chapter 5 contains a discussion of the results, based on the results of the previous chapter. The guiding research questions are revisited. In line with the theoretical framework, the chapter presents a discussion focusing on the teachers' PCK profiles and how the data help to answer the research questions in order to determine how the mathematics teachers developed their pedagogical content knowledge in statistics teaching.

Chapter 6 presents a summary, the conclusions of the study, and recommendations and suggestions for further research.

1.12 Summary of chapter

This chapter provided insight into the research orientation used in this study, in an attempt to make the reader conversant with the research project. The chapter began with an introduction to the NCS and the learning outcomes for statistics in school mathematics. The knowledge that the teacher needs to implement the curriculum effectively was highlighted from three perspectives, namely subject matter content knowledge, pedagogical knowledge, and PCK. The introduction was followed by an elucidation of the problem of the study, a statement of the research problem, the aims of the research, the research questions, the significance of the study, the scope of the study, and the theoretical framework that guided the study. The key concepts used in this study were then defined and discussed, and the chapter concluded with a discussion of the criteria for selecting the topic, as well as the chapter structure of the thesis.

CHAPTER 2

2.0 LITERATURE REVIEW

2.1 Introduction

This chapter focuses on the literature review, which tries to address the empirical and theoretical issues related to teachers' PCK development and its use in mathematics and statistics teaching. The discussion about the process of PCK is derived from a review of the NCS for mathematics and statistics teaching and the research questions guiding the study. Studies on teaching statistics in school mathematics and the models for capturing PCK are discussed. The techniques of studying PCK are highlighted and studied in order to justify the validity of the instruments used to investigate PCK. The chapter concludes with a summary of the theoretical framework that allows for the development of the research instruments, data analysis and results.

2.2 National Curriculum Statements for Mathematics and Statistics

The National Curriculum Statement (NCS) for Mathematics is based on the nature of the discipline and societal expectations of learners of mathematics (DoE, 2009). Mathematics is a subject that enables creative and logical reasoning about problems in the physical and social world, and in the context of mathematics itself (DoE, 2009:9). From this, mathematics is seen as a human activity that deals with patterns, problem solving, and logical thinking, in an attempt to understand the world and to make use of that understanding (Lebeta, 2006).

According to the views of the Department of Education (2009) and Lebeta (2006), it may be concluded that 'mathematics is part of day-to-day human experiences and relates to human activities that use features of one natural object as a tool for acting on other objects. This means that mathematics is an organic activity'. According to Davydov (1999), human activity is linked to conceptual activity. The purpose of mathematics is to demonstrate how human activity is linked to conceptual activity. Therefore, 'knowledge in mathematical science is constructed by establishing descriptive, numerical and symbolic relationships that are based on observing patterns, using rigorous logical thinking that can lead to theories of abstract relations' (DoE, 2009). By implication, mathematical knowledge can help learners to engage in problem solving to understand the world, and they can use that understanding in their daily lives. Hence, the subject statement for mathematics for Grades 10 to 12 expects learners to expand on their understanding of Learning Outcome 4 (LO4) of the NCS under the category 'Data handling and probability' (DoE, 2007:22), through appropriate teaching and learning of the topic in the classroom context. This learning outcome 'requires learners to be able to collect, organise, analyse, and interpret data, in order to establish statistical and probability models to solve related problems with a focus on human rights issues, inclusivity, and current matters involving environmental and health issues' (DoE, 2009:10). What, then, is the purpose of mathematics, according to the NCS?

According to the NCS (2009:11), the purpose of mathematics is to provide powerful tools:

- To analyse situations and arguments, make and justify critical decisions, and take transformative action, thereby empowering people to work towards the reconstruction and development of society
- To develop equal opportunities and choices
- To contribute towards the widest development of society's cultures, in a rapidly changing, technological, global context
- To derive pleasure and satisfaction through the pursuit of rigour, elegance, and the analysis of patterns and relationships
- To engage with political, organisational and socio-economic relations (DoE, 2009:11)

However, the focus of this study is on statistics, which is part of the mathematics curriculum. Research reports by Gattuso (2006) show that there is a link or relationship between mathematics and statistics. For example, linear function is used in describing the relationship between two variables in scatter plots. Using the stem-and-leaf diagram, one can distinguish between units and tens in mathematics. And in the workplace, statistics is used in representing the records of employees' weekly, monthly and yearly attendance at work on a frequency table and statistical graphs. That is why it is important that mathematics teachers understand this relationship, so that it can be addressed in the teaching and learning situation (DoE, 2009).

For many teachers, the relationships are not clear. They face difficulties in teaching statistics and addressing the relationships between mathematics and statistics in classroom practice (DoE, 2010). As early as 1988, Garfield and Ahlgren (1988) reported that although statistics is related to the learning of mathematics and other disciplines, a large proportion of learners do not understand many of the basic statistical concepts they have studied. The authors reported that 'inadequacies in prerequisite mathematics skills and abstract reasoning' are part of the difficulties encountered by the learners of statistics. Poor learner performance in statistics was also noted at the joint conference of the International Commission for Mathematics Instruction and the International Association for Statistics Educators (ICMI/IASE, 2007).

2.3 Research on teaching statistics in school mathematics

The important role of statistics in mathematics education and other disciplines has now been recognised worldwide. This was confirmed by the introduction of statistics in school mathematics in the school curricula at all levels in South Africa and elsewhere (DoE, 2009). However, recent research on teaching of statistics in school mathematics shows that learners encounter difficulties in learning the subject (Godino et al., 2011).

Baker, Corbett and Koedinger (2001) observed that learners are often confused about the construction of bar graph and histogram. According to these authors, most learners construct a histogram in the same way as a bar graph. The authors noted that in the stage of learning how to construct a histogram, learners transferred their existing knowledge about a bar graph to the construction of a histogram, instead of using knowledge specific to the target representation. And because learners were already familiar with bar graph construction, they found it easy to construct a bar graph instead of histogram (Baker et al, 2001).

Meletiou-Mavrotheris and Lee (2002) note that learners perceive histograms as two-dimensional graphs that must have two variables and thus tend to interpret a histogram as two-variable scatter plots. In addition, learners tend to perceive histograms as displays of raw data on the Y-axis with each bar standing for individual observation and with individual cases on the X-axis. These authors reported that when comparing two histograms with regard to their variability, learners used the vertical axes of the histogram instead of the horizontal axes to compare their variability or spread (Meletiou-Mavrotheris & Lee, 2002).

Baker et al. (2001) extended this research to include the construction and interpretation of statistical graphs with emphasis on scatter plots and stem-and-leaf. Their reports show that the axes of a scatter plots were drawn by the learners as if a bar graph was to be represented and plotted the points on the wrong axes. Consequently, a misinterpretation was obtained from a wrongly constructed scatter plot.

Other research studies (NCTM, 2007; Baker et al., 2001, Cazorla, 2006 and DoBE, 2012) attributed learners' learning difficulties to the way teachers taught the construction and interpretation of stem-and-leaf diagrams. The authors noted that although learners can read and represent stem-and-leaf diagrams, they were unable to interpret them because they had not been exposed to the types (varieties of ways) of stem-and-leaf representation.

Nicholson and Darnton (2005) researched the challenges for the classroom teacher in teaching statistics. In an analysis of questions used in statutory national tests, learners' scripts were used to collect data on their reasoning processes and learning difficulties. The results of an analysis of the questions and scripts at the early stage in the primary school were compared with the difficulties seen at the later stage of secondary statistics. The findings of this study show that pupils at the early stage struggle to articulate their reasoning processes explicitly. Furthermore, teaching and learning at the later stage of their secondary examination were based on computational accuracy and procedural competence in statistics, and less time was spent on interpretational skills. The implication of these findings is that mathematics teachers who are not familiar with the common difficulties and misconceptions may not be able to help learners to overcome their learning difficulties in statistics and achieve a deeper understanding of core concepts (Nicholson & Darnton, 2005).

Mavrotheris and Stylianou (2003) observed that one of the sources of learning difficulties in a statistics classroom is that most mathematics teachers are too formalistic in their approach to the subject. The authors noted that statistics lessons are presented in rigidly established bodies of mathematical knowledge without any reference to the real-world context (Mavrotheris & Stylianou, 2003). Formalist ways of teaching have led to educators failing to convey to the learners the relationship between knowledge they acquire in the statistics classroom and its uses in everyday life (Mavrotheris & Stylianou, 2003). For example, learners were taught first to build a cumulative frequency table, and construct an ogive by drawing the axes, labelling the axes, plotting the points and joining the line of best fit. During interpretation and analysis, values were read off from the vertical and horizontal axes without being linked to the learners' real world (Libman, 2010). Hence, learners had difficulties in understanding what the teacher had taught using the formalistic approach.

Watson, Callingham and Donne (2008) carried out research on establishing PCK for teaching statistics from Grades 1 to 12. The PCK of 42 teachers selected as part of a professional learning programme in statistics was examined. The results of the Rasch analysis to obtain a measure of teacher ability levels in relation to PCK indicate that teachers who did not respond appropriately to the survey items often missed or left out those items that required a response to a specific student misunderstanding (Watson, Callingham & Donne 2008). The inability of the teachers to respond to specific student misunderstanding could mean either that they were not able to move students towards a higher level of statistics understanding or to design instructional interventions to address students' learning difficulties. This study represents an initial attempt to establish the nature of teachers' demonstrated PCK in teaching school statistics.

The intention of the researcher through this study is to determine whether the participating teachers are aware of their learners' difficulties with statistical graphs and the means used by them to elicit these difficulties. PCK is seen as a relevant construct for this study as teachers' topic-specific content knowledge influences what is taught in the classroom context. It therefore becomes necessary to explore the PCK of a mathematics teacher who demonstrates good content-specific knowledge (Godino, Batanero & Font, 2011) to see how this teacher's PCK is enacted while teaching these difficult topics.

2.4 Assessing teachers' PCK

2.4.1 Description of PCK

The concept of pedagogical content knowledge (PCK) was introduced by Shulman (1986) in a paper in which he argued that research on teaching and teacher education ignored questions dealing with the contents of lessons, the questions asked, and the explanations offered. As indicated in the theoretical framework of this study, PCK goes beyond knowledge of the subject per se to encompass the dimension of subject matter knowledge for teaching. It refers to how the teacher interprets the subject matter knowledge in the context of facilitating learning.

Shulman (1986), while categorising a knowledge base for teaching, noted that the way in which the subject matter is presented and formulated is a key element in the conceptualisation of PCK. According to him, this knowledge could originate from research or teaching practice. Other elements in Shulman's categorisation of the knowledge base for teaching are awareness of strategies that may be fruitful in reorganising the understanding of learners, and learners' preconceptions and misconceptions about a particular topic.

In the two decades since Shulman introduced the concept of PCK there have been a number of studies on the subject. Various scholars across the discipline have elaborated on Shulman's work and proposed different conceptualisations of PCK (Grossman, 1990; Marks, 1990; Cochram et al., 1993; Van Driel et al., 1998; Magnusson, Krajcik & Borko, 1999; Gess-Newsome and Lederman, 2001; Barnett & Hodson, 2001; Jong, 2003; Halim & Meerah, 2002). This amplification is in terms of what they include or do not include in their conceptualisations of PCK.

Grossman (1988) developed and expanded the definition of PCK. Her definition is based on four central components: knowledge of learners' understanding; the curriculum; instructional strategies; and the purpose of teaching. Knowledge of learners' understanding refers to how the learners comprehend what is taught. In other words, how do learners understand the subject matter being presented to them? The curriculum pertains to the content of the subject matter, as contained in it. Knowledge of instructional strategies constitutes understanding of the stratagems employed in teaching the subject. The purpose of teaching is to achieve the learning outcomes, as outlined in the curriculum. Using these components, Grossman (1988) examined the influence of teacher education on knowledge growth. The findings regarding the impact of teacher education on knowledge growth demonstrate that teacher education can influence knowledge growth by teachers.

Teacher education involves the disciplinary tutoring through which the subject matter knowledge and pedagogical knowledge can be acquired. This education can provide an opportunity to acquire more knowledge and growth if the teacher continues to practise in the particular discipline (Grossman, 1988). The influence of teacher education on knowledge growth is related to this study in the sense that one can speculate that the disciplinary education acquired by teachers could influence the way in which their PCK is developed and used for teaching statistics in school mathematics, hence the need to examine and assess the level of teachers' subject matter content knowledge, as already indicated. However, in the context of delivering a particular curriculum (DoE, 2007), the model fails to indicate any specific programme and how it influences the teachers' knowledge and its uses during classroom practice (Ibeawuchi, 2010).

Based on an explicit constructivist view of teaching, Cochram et al. (1993), in their research on PCK as an integrative model for teacher preparation, renamed PCK 'pedagogical content knowing' (PCKg), to acknowledge the dynamic nature of knowledge development. In their model, PCKg is conceptualised far more broadly than in Shulman's view. They define PCKg as 'a teacher's integrated understanding of four components of pedagogy, subject matter content knowledge, learner characteristics and the environmental context of teaching' (Cochram et al., 1993). According to these authors, PCKg is generated as a synthesis of the simultaneous development of these four aspects in the context of the integrative model of teaching. Following this argument, it means that the components of PCK, as highlighted above, do not exist independently of one another. In this study, however, the components of PCK were captured individually during classroom practice. Even though the elements of PCK do not exist independently of one another as conceptualised, it is still seen as an amalgam of these components during classroom practice. PCK is individualistic, tacit, and ever changing with time and experience (Miller, 2007).

But according to Lee and Luft (2008), there are two models of PCK, integrative and transformative. In the integrative model, the PCK components exist separately, and at the beginning of teachers' careers they enable teachers to rely on only one of the PCK components to cope with teaching (subject matter content) (Lee & Luft, 2008). Transformative PCK is held by experienced teachers who combine all the components of PCK and convert it into classroom practice. Lee and Luft (2008) claimed that during teaching it is difficult to distinguish subject matter knowledge or general pedagogical knowledge from PCK, which means the components do not exist independently of one another. In this study, based on notion of amalgam, the components of PCK can exist independent of one another or together. The ways the teachers used them were established by attempting to describe the PCK profiles of the participating teachers as evidence in their practice.

Van Driel, Verloop and De Vos (1998) conducted research on developing science teachers' PCK, using classroom observation and interviews. According to them, the idea of integration of knowledge components is also central to the way PCK is conceptualised by Fernandez-Balboa and Steel (1995). These authors identify five knowledge components of PCK: subject matter, the learners, instructional strategies, the teaching context, and the teaching purpose.

Magnusson et al. (1999) presented a model of the relationship between the constituent domains of PCK. According to them, subject matter knowledge (e.g. substantive knowledge, and syntactic knowledge), pedagogical knowledge, knowledge of educational aims, knowledge of the classroom, and context knowledge (e.g. knowledge of specific learners and school characteristics) could be used to interpret PCK. In the teaching process, these domains could be combined (Rollnick et al., 2008) to provide effective teaching and promote learners' understanding of the lesson.

Barnett and Hodson (2001), in their research on how to understand what science teachers know, considered PCK a constituent of pedagogical context knowledge, together with other components. These other components were academic knowledge, classroom knowledge, and professional knowledge. But the components of PCK are not always clear and consistent; rather they look blurry; and the development of a teacher's PCK is not linear, but advances from different angles (Loughran et al., 2004).

Although different researchers have varying opinions about the conceptualisation of PCK, Jong (2003) and Van Driel et al. (1998) stated that these elements seem to be germane to any conceptualisation of PCK with respect to a chosen content area

- Knowledge of learners' learning difficulties, conceptions, and misconceptions concerning the topic
- Knowledge of how to represent specific topics

Several scholars have researched PCK development, and their studies are concerned with how a teacher uses his/her knowledge of the content that the learners are expected to learn and the best approaches to employ to access that content; hence it is called the knowledge base for teaching. A teacher's PCK is therefore unique (Bucat, 2004) as it depends on how he or she interprets learners' preconceptions and learning difficulties and what the learners need in order to understand the content being taught (Mitchell & Mueller, 2006). The development of PCK is mutual and hence the development of one component influences the development of another (Henze, Van Driel & Verloop, 2008). Hill et al. (2008) argued that the impact of teachers' PCK on learners' learning was still to be proven, since there seemed to be a relationship between the teacher's PCK and what the teacher does in the classroom. So far, these authors have agreed that the development of a teacher's PCK is rooted in the classroom and this could contribute to effective teaching and learning of statistics in school mathematics.

The first component of PCK, namely knowledge of learners' understanding and their conceptions of a specific topic, helps teachers to interpret learners' actions and ideas, as well as plan effective instruction (Loughran, Mulhall & Berry, 2004; Halim & Meerah, 2002). These authors argued that ignorance of learners' misconceptions may be due to teachers' lack of content knowledge. The second component, knowledge of how to teach a particular topic, refers to awareness of specific areas that are useful in helping learners understand specific concepts. This involves knowledge of ways of representing specific concepts, in order to facilitate learning (Halim & Meerah, 2002). This component of PCK, which aims to develop learners' conceptual understanding, seems necessarily dependent on having subject matter knowledge relative to the concept being taught. Furthermore, 'the PCK for representing specific topics is a product of previous planning, teaching and reflecting' (Halim & Meerah, 2002).

2.4.2 *Teacher knowledge and PCK*

According to Gess-Newsome (in Jong, 2003), all the various views of PCK can be categorised as integrative or transformative. Where PCK is categorised as integrative, knowledge of teaching is merely the integration of forms of teacher knowledge, such as subject matter content knowledge, knowledge of learners' learning difficulties, and knowledge of learners' preconceptions concerning a topic. In this integrative view, PCK is seen as a mixture. In other words, 'PCK does not really exist in its own domain, and teaching is seen as an act of integrating knowledge of subjects, pedagogy and context' (Gess-Newsome & Lederman, 2001). In classroom practice, knowledge of all these domains is integrated by the teacher to create effective teaching and learning opportunities. Most teacher education programmes that are organised in separate courses of subject matter, pedagogy, and practice follow this model of teacher knowledge (Ibeawuchi, 2010).

In the transformative view (Jong, 2003), forms of teacher knowledge, such as subject matter knowledge, pedagogical knowledge, and contextual knowledge are transformed into a new form of knowledge such as understanding of a concept. In this view, PCK is seen as a compound. This model supports teacher education programmes that contain integrated courses and allow prospective teachers to quickly develop the required skills and knowledge. The integrative view and the transformative view can be considered opposite ends of the PCK spectrum (Jong, 2003). In this study, it is assumed that the transformative view was used by the participating teachers during classroom practice for teaching statistical graphs because the teachers uses the conceptual knowledge approach to describe the concept of histogram, ogive and bar graph which the learners appear to have understood.

Recently the statistics education community's attention has been drawn to the statistical knowledge for teaching (SKT) measures by scholars such as Hill, Blunk, Charalambous, Lewis, Phelps, Sleep and Ball (2008). According to these authors, statistical knowledge for teaching included statistical information that is common to individuals working in diverse professions and the subject matter knowledge that supports such teaching, for example why and how a statistical procedure works, how best to define a statistical term for a particular grade level, and the particular content (Hill et al., 2008). To these authors, the impact of teachers' PCK on learners' learning had yet to be proven, but there seemed to be a relationship between a teacher's PCK and what the teacher did during classroom practice. Following these arguments, the development of PCK is explored in the classroom, and this can contribute to effective teaching and learning.

Toerien (2011) conducted preliminary research on the development of PCK of in-service science teachers and conceptualised PCK as including subject matter content knowledge, the context of the school, knowledge of the curriculum, and teachers' pedagogical knowledge. Using semi-structured interviews and lesson observation, Toerien (2011) noted that these four components could be used to investigate the development of PCK of in-service science teachers in the classroom context.

In looking at how various researchers have conceptualised PCK, it appears that investigating PCK may not always be a straightforward matter, because of its unarticulated and tacit nature. Jong et al. (2005) argued that investigating PCK development is a complex process, because PCK is determined, among other things, by the nature of the topic, the context in which the topic is taught, and the way in which a teacher reflects on the teaching experience (Park & Oliver, 2008). This is because different topics may require different teaching approaches, depending on the learning outcomes. This study sought to determine how PCK is developed by investigating participating teachers through the use of multiple sources for data triangulation.

In summary, Figure 2.1 describes the components of PCK that are likely to be used for teaching statistics in school mathematics. They include subject matter content knowledge, pedagogical knowledge, knowledge of learners' conceptions and knowledge of learners' learning difficulties. In the context of this study, the pedagogical knowledge in statistics teaching will be assessed using multi-evaluation comprising of the lesson observation, written reports, and questionnaire and documents analysis.

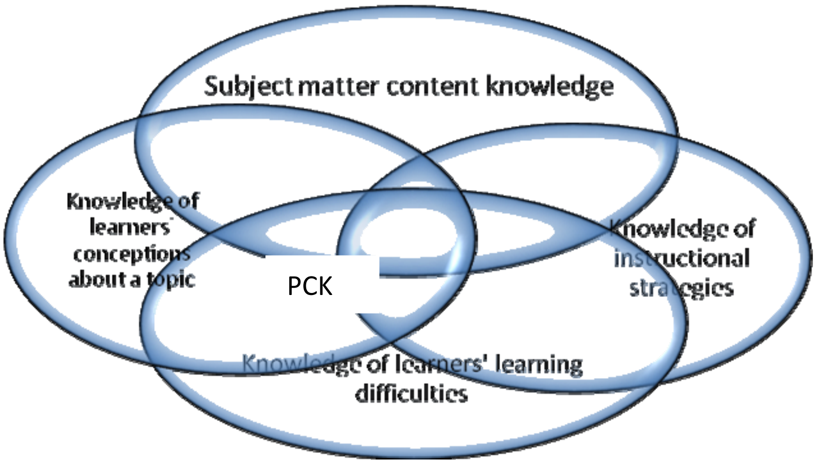


Figure 2.1: Components of PCK used in this study

2.4.3 Pedagogical content knowledge and subject matter for teaching

Several researchers have used the terms 'subject matter knowledge' and 'subject matter content' to describe the kind of knowledge that teachers need for teaching (Shulman, 1986; Ma, 1999; Vistro-Yu, 2003; Jong, 2003; Jong et al., 2005; Halim et al., 2002; Rollnick et al., 2008). In terms of mathematics teaching, Plotz (2007) referred to subject matter content knowledge as 'mathematical content knowledge'. With regard to PCK development in statistics teaching it is necessary to define what each of the concepts means, so that they can be used to define the construct of PCK that was used in statistics teaching. Plotz (2007) argued that mathematical content knowledge is acquired mostly by studying mathematics in school, and this may be described as ‘in-school acquired knowledge’. Van Driel et al. (1998), Jong (2003) and Jong et al. (2005) described subject matter knowledge as the knowledge obtained through formal training at universities and colleges, which may be regarded as disciplinary education. From these assertions, it would seem that subject matter knowledge is acquired through formal training in a subject area.

Ball and Bass (2000) researched the interweaving of content and pedagogy in the teaching and learning of mathematics. The findings of their study indicated that the subject matter knowledge needed by teachers is found not only in the list of topics of the subject matter to be learned, but in the practice of teaching itself (Ball and Bass, 2000; Plotz, 2007). In other words, knowing the content of a subject is not enough to justify the capacity of a teacher to teach; what makes a teacher capable of teaching is also how well the teacher facilitates the learning. According to these authors, little is known about the way in which ‘knowing’ a topic from a list of topics affects teachers’ capabilities. And if one depends on analysing the curriculum to identify the subject matter content knowledge needed for teaching the topics without focusing on practice as well, not much will be gained (Ball and Bass, 2000; Plotz, 2007). Plotz’s (2007) study also reveals that mathematical content knowledge and pedagogical knowledge are both needed for effective teaching and can motivate the development of the PCK used for teaching. He stressed that teachers’ prior knowledge needs to be exposed for effective content knowledge transformation and understanding as the prior knowledge aided the teachers in the written problem-solving activities to design to assess their mathematical content knowledge state.

Capraro, Capraro, Parker, Kulm and Raulerson (2005) researched the role of mathematics content knowledge in developing pre-service teachers’ PCK, using performance in a previous mathematics course, a pre- and post-test assessment instrument, success in the state-level teacher certification examination, and journals. Their study outlined the connection between mathematics content knowledge and pedagogical knowledge in developing PCK, in order to address the increasing expectations of what learners should know and be able to do, and knowledge that the teachers must have in order to meet the educational goals during instruction and learning. A total of 193 undergraduate students who enrolled in integrated method block courses prior to the teaching practice programme were involved in the research project on teaching practice in mathematics. The findings of Capraro et al. (2005) indicated that the teachers’ previous mathematics abilities are valuable predictors of students’ success in teacher certificate examinations. Secondly, the mathematically competent pre-service teachers exhibited progressively more PCK, as they had been exposed to mathematical pedagogy comprising subject matter content and teaching practice during their mathematics method course. Therefore, for one to have pedagogically powerful representations of a topic, one should first have a comprehensive understanding of the topic.

Following this argument, subject matter knowledge in the context of PCK development becomes a product of the interaction between mathematical competence and concern for the instruction and learning of mathematics (Plotz, 2007). In other words, the concern for instruction and learning shown by a competent mathematics teacher must demonstrate that he or she has adequate knowledge of the subject matter, and this may be necessary for PCK development. In this study, it is assumed that during their university preparation programmes, the participating teachers acquired the subject matter knowledge of mathematics and the pedagogical knowledge necessary for PCK development in statistics.

However, the South African mathematics (Grades 10–12) teaching force is made up mainly of practitioners who have three-year teaching diplomas obtained from the old (pre-1994) colleges of education (Rollnick et al., 2008). Less than 40% of these teachers hold a junior degree on the subject they teach. The mathematics content measures only up to that of first year at a university (Rollnick et al., 2008). In this study, the key question is, given that the teachers show competence or understanding of these concepts in mathematics, irrespective of their training, how does this influence their teaching and therefore their PCK for teaching statistics in school mathematics?

Vistro-Yu (2003) conducted a study on how secondary school mathematics teachers faced the challenges of teaching mathematics (in terms of the pedagogical knowledge requirements of PCK in mathematics) in a new mathematics class in college algebra. Thirty-three secondary school mathematics teachers were initially involved in the research project. They were made to write a standardised test in high-school mathematics to determine the level of their subject matter content knowledge. Based on this performance, six teachers were selected for the research project. These six teachers were asked to prepare and teach an assigned topic in a college algebra module, while the researcher conducted classroom observations of the lessons presented by them. They were interviewed before teaching commenced, and after the lessons, the six teachers were given a questionnaire to complete by reflecting on their teaching performance. The findings of the study showed that the teachers were limited in the ways they prepared their lessons. According to Vistro-Yu (2003), they were not able to teach in an organised manner and lacked in-depth subject matter knowledge. The results of the interview showed that some of the participants were dissatisfied with their teacher education preparatory programmes because they lacked thorough content knowledge of the subject matter. In this study, the methods adopted by Vistro-Yu (2003), namely teachers' content knowledge exercise, lesson observation, and interviews, were used to determine the subject matter content knowledge and pedagogical content knowledge of these mathematics teachers (the participants in the study).

Jong et al. (2005) conducted a study of the PCK of pre-service teachers using particle models to teach chemistry at secondary-school level. Responses to written assignments, transcripts of workshop discussions, and reflective reports by the participants were used to collect data. The findings of this study indicated that the pre-service teachers were able to understand and describe the learning difficulties of their learners during teaching with particle models. In addition, they developed PCK using particle models, although development varied among the participants (Jong et al, 2005).

The research methods of Ball and Bass (2000), Vistro-Yu (2003), Capraro et al. (2005) and Jong et al. (2005) provided the rationale for the assessment of subject matter content, pedagogical knowledge, knowledge of learners' conceptions and learning difficulties as constituent elements needed to develop PCK for teaching. However, there were deficiencies in their studies. One of these was that their research was conducted within a relatively short time (Vistro-Yu, 2003; Capraro et al., 2005). For instance, using one, two or four lesson periods to conduct an investigation on the challenges in the instruction and learning of mathematics (Vistro-Yu, 2003; Capraro et al., 2005) may not be adequate, since most topics in mathematics take more than one period to teach.

Second, some of the researchers (Capraro et al., 2005; Ball & Bass, 2000) used grades obtained in their university courses to justify the competency of a teacher in instructing a subject. This may not be adequate, as the number of mathematics courses that a teacher has studied at university or college does not necessarily ensure effective or quality teaching in a classroom situation (Plotz, 2007; Capraro et al., 2005; Geddis, 1993). Rather, what makes him or her an effective teacher is how well he or she understands what learners have to learn, and the way he or she presents the subject matter content (Muijs & Reynolds, 2000; Graffin et al., 1996). Therefore, more precise measures are needed to specify in greater detail the relationships between the various components of PCK and how they are developed in order to improve learner performance in mathematics (DoE, 2008). A third deficiency is lack of lesson observation in conducting some of the investigations (Capraro et al., 2005; Ball & Bass, 2000). The use of lesson observation would have afforded the researchers the opportunity to determine how mathematics teachers use their PCK, for example preparation and presentation of the lesson based on adequate knowledge of the subject matter; and identification of learners' preconceptions and learning difficulties, conceptions and misconceptions concerning the topic (Jong, 2003).

In order to avoid these deficiencies, the study was carried out with the following features:

1) The PCK of teachers were investigated over a relatively long period (between four and six weeks).
2) The study was carried out with experienced secondary school mathematics teachers.
3) Lesson observation was undertaken to determine how the teachers demonstrated their PCK and subject matter knowledge during the teaching process and how they identified learners' preconceptions and misconceptions of the topic.
4) Teachers' and learners' portfolios and workbooks were examined to determine what had made the instruction and learning of the topic easy or difficult.
5) These features were adapted to investigate the way competent mathematics teachers developed their PCK for teaching statistics in school mathematics, in the hope of discovering a further directive for the continuous improvement of the mathematics teachers' PCK in statistics teaching as well as of educational programmes for in-service and pre-service teachers of statistics.

In terms of measuring teachers' subject matter content knowledge in a topic, several techniques and methods have been used by several researchers in the field of mathematics and science education. For instance, Gess-Newsome and Lederman (2001) and Jong (2003) reported that a teachers' subject matter content knowledge can be measured using concept mapping, card sorting and pictorial representation. In this study, the subject matter content knowledge of the participating teachers was assessed with the conceptual knowledge exercise, concept mapping, interview and lesson observation.

The conceptual knowledge exercise in statistics was designed in multiple-choice formats. The multiple-choice questions in statistics consist of a series of question, each with five possible options from which the participating teachers have to choose the best to answer the questions. Critics say that the multiple-choice format may not accurately depict the respondent's personal views about teaching because there is no provision for the reasons for the selection of a particular option. But researchers continue to use multiple-choice questions with success, because the many advantages of this type of question offset their demerits (Gess-Newsome and Lederman, 2001; Kazeni, 2006). For example, multiple-choice questions can be set at different cognitive levels. They are versatile if designed and used appropriately (Miller, 2006). Multiple-choice question assessments can be completed in a short time, and they ensure better coverage of content. In this study, multiple-choice questions were used to assess the changes in statistics content knowledge of the participants (since they have been teaching the topic) as they may have covered enough content area of statistics and to select them for the second phase of the qualitative research.

Considering the role of concept mapping in teaching and learning, Ochonogor and Awaji (2005) and Novak and Cannas (2006) described concept mapping as a learning strategy that aids understanding of complex ideas and clarifies ambiguous relationships between ideas. According to these authors, concept maps may be seen as graphical tools for representing topics, by depicting key concepts and organising knowledge clearly. Following this argument, organising and representing the knowledge of a particular topic can take the form of connecting the concepts by means of arrows, boxes, words or phrases in order to elicit the meaning of the relationships between the concepts. In this connection, concept maps are seen as a special form of web diagram for exploring knowledge and gathering and sharing information visually (Novak & Cannas, 2006). Concept maps can depict how we think, which influences how and what we teach (Miller, 2006). Hence, concept maps can provide opportunities to see relationships between types of knowledge.

Novak and Gowin (1994:96) argued 'that concept maps provide visual representations of knowledge'. According to these authors, concept maps allow researchers to create concrete representations of knowledge that can be used to determine knowledge changes in a teacher. Since concept maps create physical representation of knowledge, changes in this representation are assumed to provide evidence of teacher knowledge change (Miller, 2006:96).

Miller (2006) used concept maps to analyse the construction of pre-service teachers' PCK during a science method course. The participants of the study were asked to construct a concept map of important concepts in a specific chemistry unit that focuses on numerous teaching activities. The findings of this study show that the changes in the structure of the concept map were related to the changes in the personal knowledge of the learner.

Ferry, Hedberg and Harper (1997) investigated how pre-service teachers used a concept map to organise curriculum content knowledge. Participants of the study were asked to use a concept map to plan science-based instruction that could be delivered to an elementary science class. The results of the study showed that pre-service teachers had different perceptions of the connections between the basic statistical concepts, which enhanced their conceptual understanding of the concepts and aided the sequential planning of the sequence of the concepts for teaching (Ferry, Hedberg & Harper, 1997).

Concept mapping may lack reliability in terms of representing all that an individual knows about the content knowledge being assessed (Miller, 2006). Furthermore, if a teacher does not continue with classroom practice, the changes in knowledge of the topic may be short lived.

However, concept maps have been credited with many advantages. For instance, a concept map allows teachers to organise their knowledge of teaching their primary content area much better with high cognitive demand. In this study, a concept mapping exercise was used to indirectly assess teachers' content knowledge of statistics in school mathematics by arranging statistics topics in logical sequence according to the way in which the teachers would present them in their classroom practice.

The interview was used to triangulate the data gathered with the concept mapping. The interview consists of open-ended questions that the interviewer asked the interviewees to respond to. The interview allows the respondent the opportunities to create options for responding and to voice their experiences unconstrained by any perspective of the researcher or past research that may not directly be observed in the respondent action (Cresswell, 2008:225). Some researchers argued that an interview is deceptive and provides the perspective the interviewees want the interviewer to hear, which renders the information inarticulate, perceptive and unclear (Cresswell, 2008). Several researchers (Vistro-Yu, 2003;

Loughran et al, 2004; Hill, 2008) have used the interview to assess teachers' educational background that must have assisted them to develop their topic-specific content knowledge and PCK. In this study, an interview schedule was used to gather data to assess the teachers' educational background that had enabled them to develop their topic-specific content and PCK in statistics teaching. The use of lesson observations in assessing teachers' content and pedagogical knowledge will be discussed in Section 2.4.4.

The research procedures used by researchers such as Jong et al. (2005), Capraro et al. (2005), Vistro-Yu (2003), Jong (2003) and Van Driel et al. (1998) share the same research procedure as this study in terms of the use of these instruments: a conceptual knowledge exercise, interview schedules, concept mapping, to assess subject matter content knowledge and PCK.

2.4.4 PCK and pedagogical knowledge (instructional skills and strategies)

Pedagogical knowledge is believed to be the kind of information that a teacher needs and uses to perform everyday teaching tasks. It involves teaching styles and strategies, classroom management and teaching and learning processes relating to learners in the classroom (Cochram et al., 1993; Vistro-Yu, 2003). Pedagogical knowledge includes knowing and understanding the content to be taught and the specific demands of that content, such as instructional skill and strategies (Kreber, 2004; Loughran et al., 2004; Ball, Thames & Phelps, 2008). Instructional knowledge entails knowing how to sequence the learning outcomes, prepare the lessons, facilitate discussion and group work, construct tests and evaluate learners' understanding through the use of examinations, among others (Kreber, 2004).

In general, different kinds of instructional strategies, representations and activities are used in teaching mathematics. Knowledge of instructional strategies entails understanding ways of representing specific concepts, in order to facilitate student learning. Representations include illustrations, examples, models, and analogies. Each representation has a conceptual advantage and disadvantage over other representations (Ibeawuchi, 2010). PCK in this area includes awareness of the relative strengths and weaknesses of a particular representation. Activities can be used to help learners understand specific concepts or relationships, for example demonstrations, simulations, investigations and even experimentations. PCK of this type incorporates teachers' knowledge of the conceptual power of a particular activity

(Magnusson et al., 1999). For a representation to be powerful or comprehensible, the teacher must know the learners' conceptions about a particular topic, and the possible difficulties they will experience during the teaching and learning of the topic. Representations during teaching must be clearly linked, and the relationships between concepts must be comprehensible (Ibeawuchi, 2010). However, most mathematics teachers are not able to identify learner misconceptions and to teach for conceptual change since most of them have not yet dealt with their own alternative conceptions, and are working with very limited resources, time, and necessary skills (Van Driel, 1998).

Several studies have highlighted certain instructional strategies as a component of PCK. Hashweh (1987) for example emphasises that incorrect and misleading representations, such as analogies and examples that depict the teachers' misconceptions, could result from teaching outside one's own field of expertise. Tobin, Tippins and Gallard (1994) also state that when teachers teach outside their areas of specialisation, they give explanations and analogies that reinforce the misconceptions that learners already have.

Magnusson et al. (1999) argue that pedagogical knowledge as a component of PCK is dependent on teachers' subject matter knowledge about a particular concept. This may not always be true, as subject matter knowledge does not guarantee that PCK will be transformed into representations that will help learners understand targeted concepts, or that teachers will be able to decide when it is most appropriate pedagogically to use a particular representation. Anderson and Mitchener (1994), in their research on science education, support this view and are of the opinion that teachers' knowledge of science teaching may be limited, even if the teachers have knowledge of the subject matter. In a particular topic, pedagogical knowledge, or the way concepts are represented as a component of PCK, seems to depend on previous planning, teaching, and reflection (Halim & Meerah, 2002).

Vistro-Yu (2003) researched pedagogical knowledge in mathematics and focused his study on how the mathematics teacher faces the challenge of teaching algebra in a new class. As explained earlier, pedagogical knowledge is knowledge used for teaching, particularly awareness of instructional techniques, psychological principles, classroom management, and the teaching and learning process. Similar PCK-related studies by Jong et al. (2005) and Rollnick et al. (2008) show that science teachers with adequate pedagogical knowledge should be able to design good teaching and learning strategies that allow them to teach the concepts and manage the classroom and other instruction and learning processes. Hence, the instructional strategies used by the participants in the study for teaching school statistics were investigated in classroom practice. The question that one would ask at this stage is how do we measure the knowledge of instructional skills and strategies demonstrated by the teachers in their statistics lesson.

Current researches on PCK have suggested that the multi-method approach may be appropriate in exploring knowledge of the relevant instructional strategies (Jong, 2003; Miller, 2006; Rollnick et al., 2008; Ibeawuchi, 2010; Toerien, 2011) during classroom practice. Multi-method evaluation involves collecting multiple sources of data. Multi-method analysis tends to create increasing impact on changing knowledge, with each data source adding more dimensions to the findings from another source, thereby biasing the findings of the study (Gess-Newsome & Lederman, 2001). Nevertheless, researchers are using this method with increasing success. Multi-method evaluation is useful for triangulation of data and improving the validity of the data (Gess-Newsome & Lederman, 2001). In this study, multiple sources were used to collect data to assess the instructional skills and strategies that the participating teachers used in teaching statistics.

One of the multiple sources is the lesson observation of the participating teachers. Lesson observation is a process of gathering open-ended, firsthand information by observing the participant physically and gathering the information as it occurs at the research site (Cresswell, 2008:221). Lesson observation has the advantage of studying the actual behaviour of the participants and the difficulties they may have in demonstrating their ideas during research activities. The disadvantages of using lesson observation for data collection are that the researcher will be limited to the site and situations of the research and may have difficulty in establishing rapport with individuals. But despite the disadvantages, researchers continue to use lesson observation with success because of the firsthand information and recording the actual behaviour of the participants at the research site. The lesson observation was also used to triangulate data gathered with the concept mapping exercise (ref Section 2.4.3).

In this study, the teachers' written reports were triangulated with learners' lesson observations which form part of the multiple sources for evaluating teachers pedagogical knowledge in statistics teaching. Several researchers, including Gess-Newsome & Lederman (2001), Penso

(2002) and Jong (2003), Capraro et al (2005), have used the teacher' written report to evaluate teachers' PCK during classroom practices in science and mathematics. It has the advantage of making teachers reflect on their teaching, thereby providing opportunities for the teachers to evaluate it. In this study, the teachers' written reports were used to assess the teachers' pedagogical and triangulate the data collected with lesson observation in terms of reflecting on what transpired during the lesson.

Researchers such as Gess-Newsome and Lederman (2001) and Vistro-Yu (2003) have used questionnaire to determine teachers' pedagogical knowledge in the context of PCK development. According to them, they were able to capture what the teachers did while teaching a specific topic in science and mathematics. In this study, part of the teacher questionnaire responses was used to assess what the teachers did while teaching the assigned topic in statistics. Free-response questionnaire allows the researcher to obtain the teachers' feelings about their actions during the lesson, which they might not have displayed or expressed during the lesson and interview.

The documents analysis and video records were also used to triangulate the data from the lesson observation. Capraro et al (2005), Jong et al (2005) and Ogbonnaya (2011) have used document analysis such as journal and certification to gather data to assess the teachers' content and pedagogical knowledge in mathematics and they were successful in gathering data related to the teachers' content and pedagogical knowledge. In this study, the documents analyse included the teacher portfolios, learners' workbook and portfolios, textbooks as well as school policy guidelines for teaching and learning. They have the advantage of being readily available for reading, analysis and interpretation to the researcher.
Based on these advantages, the documents (learners class workbooks and portfolios, teacher portfolios, lesson plans, and NCS subject assessment guidelines) were considered as a source for gathering data to assess the teachers' pedagogical knowledge in terms of what has made the lesson easy or difficult

Jong (2003:375) explained that teachers are able to explain their cognition in detail while they look at a video record of a lesson that has been taught. Because of the distracting effect of a video recording being made in the classroom, an interview can be considered a replacement for it. The video recording is used as a tool for teachers to remember what they taught during the lesson, and they can experience how the lesson was delivered, unlike the interview, which only allows the respondents to verbalise their actions during the lesson. Jong (2003) noted that the stimulated-recall interview (video records) might be more appropriate in explaining teachers' actions during classroom practice. In this study, the video recorder was used to record the lessons in which the participating teachers demonstrated their pedagogical knowledge in statistics teaching and to triangulate the lesson observations in statistical graphs.

2.4.5 PCK and knowledge of learners' preconceptions and learning difficulties

Instructional strategies, learning difficulties and misconceptions are some of the components of pedagogical content knowledge that are used in teaching a particular topic in a specific subject area (Penso, 2002). Penso (2002) conducted a study on the PCK of pre-service biology teachers, with the emphasis on how student teachers identify and describe learners' learning difficulties. The teacher used classroom observation and learners' diaries to collect data from the participants. Penso's (2002) findings showed that learning difficulties could be identified and described during teaching and by observing lessons. Penso (2002) claimed that these difficulties might originate from the way the lessons were taught, which involves the content of the lesson, lesson preparation and implementation, and the learning atmosphere. Other factors include the misconceptions that the learners and the teachers have about the topic, and the cognitive and affective characteristics of the learners.

According to Penso (2002), learners regard their learning difficulties as being caused by conditions prior to the process of teaching and to those existing in the course of teaching. While the aspect of lesson content relates to the level of difficulty and abstraction of the topic, the teaching, lesson preparation and implementation aspects are concerned with the structure and presentation of the lesson (Cazorla, 2006). Negative lesson structure conditions include overloading content and unsatisfactory sequences in the lesson. Negative lesson presentation conditions include inappropriate instructional strategies for presentation, and not contributing to the process of learning. Negative cognitive and affective characteristics entail lack of prior knowledge about a topic that would enable learners to cope with the lesson in a meaningful way, preconceptions developed by the learners because of previous experiences, partial and inconsistent thinking, and lack of motivation and concentration. These negative cognitive and affective characteristics may result in learning difficulties in a teaching and learning situation if the teacher does not have adequate prior content knowledge of the topic.

Cazorla (2006) researched the ways in which mathematics teachers teach statistics in elementary and secondary schools and teacher training colleges, and reported that mathematics teachers seemed to encounter teaching and learning difficulties during teaching. According to this author, misconceptions and the ways in which mathematics lessons are taught are among the factors that contribute to learners' learning difficulties in statistics teaching. In addition, most statistics teachers do not have adequate knowledge of the curriculum and the necessary approaches to the teaching and learning of statistics. This leads to poor content delivery in the classroom, and consequently affects learners' performance.

Jong (2003), in his research on exploring science teachers' pedagogical content knowledge, used a teacher's log, concept mapping, interviews, and convergent and inferential investigation techniques and notes in order to identify and resolve misconceptions and learning difficulties. Convergent and inferential techniques may be used by the teachers during classroom practice. These refer to data collection techniques in which questions are developed in short-answer and multiple-choice formats to probe the preconceptions and misconceptions of learners in a topic (Jong, 2003). The gap in this study is that lesson observation could have been used to determine how teachers use their PCK to identify learning difficulties during the lesson.

It is thus conclusive that inadequate subject matter knowledge and inappropriate instructional strategies employed in classroom practice can bring about misconceptions and learning difficulties among learners in statistics teaching. However, learning difficulties can be resolved if practising teachers have developed adequate PCK to solve them, which, in turn, can lead to improved learner achievement. In this study, the teachers' knowledge of learners' learning difficulties was assessed through lesson observation, questionnaires, teachers' written reports and document analysis.

In the literature review, the studies by Penso (2002) and researchers such as Jong et al. (2005), Jong (2003), Van Driel et al. (1998), Capraro et al. (2005) and Cazorla (2006) justify the need for this study to investigate how competent secondary school mathematics teachers develop PCK in statistics teaching.

Research reports by Jong (2003) and Gess-Newsome and Lederman (2001) indicated that convergent and inferential techniques may be appropriate in measuring teachers' knowledge of learners' preconceptions and learning difficulties in science. The convergent and inferential technique involves the use of predetermined verbal descriptions of teacher knowledge comprising multiple choices and short-answer questionnaire. A multiple-choice item test is a series of questions with several possible answers, from which a person has to choose the correct one. The multiple-choice format can be used to rate individual performance and ability in a test, as well as to compare the performance between participants (as in this study) (Bontis, Hardie & Serenko, 2009; Kehoe, 1995).

In this study, the teachers' knowledge of learners' preconceptions and learning difficulties were assessed using the lesson observation, as part of the interview schedule, and in the questionnaire, written reports and documents analysis. Based on the way various researchers used these instruments in assessing teachers' content and pedagogical knowledge, and the many advantages of using them to capture teachers' PCK (ref Sections 2.4.3 and 2.4.4), the lesson observation was adapted to assess the teachers' knowledge of learners' preconceptions and learning difficulties in statistics teaching in order to attest how this knowledge manifests in the teacher during classroom practice. The data gathered with the interview, questionnaire, written reports and documents analysis were used to triangulate the lesson observation and to ascertain how the teachers' knowledge of learners' preconception and learning difficulties manifests during the lesson on statistical graph.

2.5 Summary of the chapter

In this chapter, various categories of relevant literature on PCK were presented. It began with a description of the NCS for Mathematics and Statistics, and explained how these subjects relate to each other. Although the studies of Penso (2001), Gess-Newsome and Lederman (2001), Rollnick et al (2008) and Jong (2003) were in the area of the sciences, their framework for describing the PCK in science teaching seemed relevant to describing how the participating teachers developed their PCK in statistics teaching. The researches on teaching and learning statistics, mathematics and sciences provide the benchmarks and suggestions about the process that the study has to consider in describing how the participating teachers develop PCK in statistics teaching. PCK is an appropriate theoretical framework for the study as it addresses the key issues: subject matter content knowledge, pedagogical knowledge, knowledge of learners' conceptions and knowledge of learners' learning difficulties, and bridging the gap in PCK development in statistics teaching. The chapter concluded with a detailed description of how the components of PCK used for this study were assessed to determine the individual topic-specific PCK in statistics teaching.

CHAPTER 3

3.0 RESEARCH METHODOLOGY AND PROCEDURE

3.1 Introduction

This chapter discusses the research methodology and procedures adopted for collecting data. It starts with a description of the research design, followed by the research method, and ends with an outline of the statistical techniques used to address issues of validity and reliability of the instruments used for the collection of data.

3.2 Assumption of PCK development during classroom practice

It was assumed that competent mathematics teachers would have developed their PCK, which enables them, through classroom teaching, to improve learners' performances at the Senior Certificate Examination over time. Observing the participating teachers prepare and teach a lesson in an assigned topic would enable the researcher to determine how they developed their topic-specific PCK in statistics teaching.

3.3 Research design and method used in this study

3.3.1 Research design

The study adopted a descriptive research design using the case study research method. Descriptive research investigates and describes a case about the current situation of an event or how it has happen in the past (Mayer & Fantz, 2004). It is used to tease out possible antecedents of an event that happened in the past. It is assumed that the competent mathematics teachers have developed adequate PCK, which enables them to improve their learners' performance in the Senior Certificate Examinations over time. A descriptive research design was considered appropriate for the nature of the topic under investigation because this study intends to investigate how the teachers developed their PCK over time.

3.3.2 Research method

This study used a qualitative research approach utilising a case study method. Creswell (2008) defines the case study method as 'an empirical inquiry that investigates a contemporary phenomenon within its real-life context when the boundaries between phenomenon and context are not clearly evident and when multiple sources of evidence are used'. This study sought to investigate how competent mathematics teachers developed their PCK in teaching statistics in their statistics lesson.

There is some criticism of the use of case study research methods. 'Critics believe that a small number of cases cannot offer adequate grounds for establishing reliability or generality of findings' (Yin, 1984). Others feel that intense exposure to the study of a case biases the findings (Yin, 1993 & 1994; Feagin, Orum & Sjoberg, 1991). Some argue that case study research is useful only as an exploratory tool (De Vos, 2000). However, researchers continue to use the method successfully in carefully planned practical studies of real-life situations, issues and problems (Soy, 2006). Soy (2006) argued that successful use of case studies in conducting investigations in scientific studies, despite the criticisms, has many benefits, such as providing a rich and detailed account of the case in a real-life context. The case study was chosen for this research in order to provide a rich and detailed account in a real-life context of how the mathematics teachers develop their PCK in statistics teaching. It is considered adequate and conventional in the field of the author's research interest, as it is used to collect information in order to gain greater insight into and understanding of the way in which PCK may have been developed by competent teachers.

This study is a qualitative one that uses both quantitative and qualitative data. The quantitative data was gathered through the conceptual knowledge exercise for teachers and concept mapping. The participants' performance in these exercises involved their marks (expressed in percentages). Interview schedules, observations of lessons, teacher questionnaires, teachers' written reports, video recordings, and document analysis were used to collect qualitative data. The individual teacher's PCK and its development in data handling teaching/statistics constituted the unit of analysis in this study.

3.4 Population and sample description

3.4.1 Study population

The population of the study comprised Grade 11 mathematics teachers in Tshwane North District, Gauteng, South Africa. There are twelve high schools in Tshwane North District. With a criterion of 70% for learners' performance in the Senior Certificate Examination in Mathematics for a period of two years, seven schools were identified from which the participating teachers were selected. The identification of the schools was followed by interviews with the principals, peers and subject specialists at the Department of Basic Education (DoBE) to identify the willing participating teachers.

3.4.2 Study sample

The teachers in the main study were selected, through a process of elimination, according to certain criteria: learners' performance in mathematics in the Senior Certificate Examination; recommendations by school principals, subject specialists at the Department of Education and peers; and competence in statistics through performance in a statistics test. Tshwane North Education District Cluster 3, Gauteng Province, comprises twelve schools. Of these schools, only seven had scored a minimum of 70% mathematics pass rate for two consecutive years in the Senior Certificate Examination. Mathematics teachers from these schools were invited to volunteer for the project. Six teachers from six separate schools indicated their willingness to participate. The researcher requested recommendations from principals, peers and subject specialists from the Department of Basic Education (DoBE) for these teachers. Based on their recommendations, six teachers were selected. Finally, the six teachers wrote the conceptual knowledge exercise in statistics. The top four scorers were selected for the main study. Table 3.1 summarises their performances, and their demographic profiles are described in section 4.3.

Table 3.1: Schools and teachers that participated in the main study

S/NO	SCHOOL	NSC RESULTS	TEACHER
1	School A	81%	Teacher A
2	School B	94%	Teacher B
3	School D	93%	Teacher D
4	School E	98%	Teacher E

3.5 Research instrument used for collecting data

3.5.1 Development of research instruments

3.5.1.1 Teacher conceptual knowledge exercise in statistics

The conceptual knowledge exercise was adopted to collect data in this study.

The National Curriculum Statement for Mathematics for the Senior Phase of the Further Education Training (FET) bands for Grades 10–12 and the prescribed textbooks were reviewed and analysed. The aim was to ascertain the targeted knowledge, competence and skills for developing the test items based on the mathematics assessment taxonomy. A large number of multiple-choice test items were initially formulated by the researcher from sources such as public examinations, locally prepared past examinations and tests, selection tests, achievement tests and textbooks in mathematics. The items were designed in line with Bloom's Taxonomy and the South African Mathematics Assessment Taxonomy, as indicated in the examination guidelines of the NCS (DoE, 2008) and Table 3.2. The competencies tested according to Bloom's Taxonomy included knowledge, comprehension, analysis, synthesis, application and evaluation (DoE, 2010). The levels of the mathematics assessment taxonomy are knowledge (level 1); applying routine procedures in familiar contexts (level 2); applying multi-step procedures in a variety of contexts (level 3); and reasoning and reflecting (level 4) (DoE, 2010). Comprehension and application of Bloom's Taxonomy were used to design the conceptual knowledge exercise, in line with the mathematics assessment taxonomy. The mark allocation was the total mark allocated to all items that were developed according to levels. For instance, all marks allocated to level 1 questions that test knowledge in any mathematics test or examination must not exceed 20 out of the total mark of 100 for the examination or test.

Table of specification 3.2: Mathematics assessment taxonomy and marks allocation

LEVELS OF ASSESSMENT	ASSESSMENT TAXONOMY	MARKS ALLOCATION
1	Knowledge	20
2	Applying routine procedures in familiar contexts	25
3	Applying multi-step procedures in a variety of contexts	30
4	Reasoning and reflecting	25

(DoE, 2010)

Table of specification 3.3: Showing competency and skills and marks allocated

COMPETENCE	ABILITIES	SKILLS DEMONSTRATED	QUESTION	MARKS ALLOCATED	TOTAL
Comprehension (understanding)	Applying routine procedures in familiar contexts	Grasping (understanding) the meaning of informational concept/materials	1, 2, 3, 6, 11, 13, 15, 20	5 for each item	40
Applications	Applying what was learnt in the classroom in solving problems in familiar or other situations by using routine, multi-step procedures	Solving problems using required skills or knowledge	4, 5, 7, 8. 9, 10, 12, 14 16, 17, 18, 19	5 for each item	60
				TOTAL	**100**

(DoE, 2010)

The conceptual knowledge exercise included 40% of the questions designed to test comprehension and consisted of level 2 and 3 questions in statistics (ref Table of specification 3.2). Examples of items measuring comprehension knowledge are 1, 2, 3, 6, 11, 13, 15 and 20 (ref Table of specification 3.3). Below is an example of the levels 2 and 3 questions.

Use the frequency distribution table below to answer question 2

Interval	*0-4*	*5-9*	*10-14*	*15-19*	*20-24*
Frequency	*3*	*5*	*7*	*4*	*1*

2 *Estimate the mode of the distribution*

The remaining 60% tested application knowledge at levels 3 and 4, where participants had to apply higher-order thinking to solve problems in statistics (ref Table of specification 3.2). Examples of items measuring application of knowledge are 4, 5, 7, 8, 9, 10, 12, 14, 16, 17, 18 and 19. The question below is an example of levels 3 and 4 questions.

4 *The mean height of three groups of students consisting of 20, 16 and 14 students is 1.67m, 1.50m and 1.40m respectively. Find the mean height of all the students.*

The conceptual knowledge test was designed to determine how well the teachers could demonstrate that they had adequate content knowledge of the topic by applying routine and multi-step procedures, as well as reasoning and reflection. Initially 30 multiple-choice test items were developed in statistics from the sources indicated, each with five possible responses. Only one of the five options was correct. These items were scrutinised by mathematics experts at the DoBE, and national examiners in NCS mathematics (ref Appendix XXII). The responses from the reviewers were used to modify the test items that formed the first draft of the instrument. For example, item 4 asked, 'The mean heights of three groups of students consisting of 20, 16 and 14 students are 1.67 m, 1.50 m and 1.40 m respectively. What is the mean height of all the students?' The item was modified to 'Find' instead of 'What', as previously used in the question.

- **Scoring the test items**

One mark was allocated to each item. The total mark for the 20 items was therefore 20 marks. While the comprehension part of the question was 8 marks, the application part was 12 marks. For the correct answer to each question, one mark was awarded in both the comprehension and application parts of the question. The marks were later converted to 100 marks. Selection of participants for the concept map and qualitative aspect of the research was based on performance in the conceptual knowledge exercise. A teacher had to score a minimum of 70% to be adjudged to have adequate subject matter content knowledge of statistics in school mathematics.

3.5.1.2 Concept mapping for teachers

The NCS was used to compile the list of contents of statistics in school mathematics. The topics according to the NCS for Grades 10 to 12 are stem-and-leaf; mode, median and mean of ungrouped data; frequency table of grouped data; range, percentiles, quartiles; inter-quartiles and semi-quartile range; bar and compound bar graphs; histograms; frequency polygons; pie charts; line and broken line graphs; box-and-whisker plots; variance, mean deviation; standard deviation; ogives; five number summaries; scatter plots; lines of best fit (DoE, 2010) (ref Appendix XXIV).

The participating teachers were required to use the topics listed above to construct a concept map. The question states:

(a) *Arrange the topics in each grade on how you think they should be taught in grades 10, 11 and 12.*

(*b*) *With an arrow, show how you can teach these topics sequentially in each grade. For example, you observe morning before afternoon and before evening. Therefore;*

Morning → *afternoon* → *evening*

For example, in measures of central tendency, the mode is taught first, followed by the median and the mean. Therefore, the memorandum for question (a) should be:

Table 3.4: Table showing the list of statistics taught in grades 10, 11 and 12 (if any)

GRADE 10	***GRADE 11***	***GRADE 12***
Mode, median, mean, ranges, (ungrouped data), frequency table, bar and compound bar graphs, histogram, frequency polygons, pie charts, line and broken line graphs. mode, median and mean (grouped data), quartiles, inter-quartiles and semi-inter-quartile range	*Five number summary, box and whisker diagrams, ogives, variance and standard deviation, scatter diagrams, lines of best fit*	*N/A*

An example of how question (b) should be answered for grade 10 is:

Mode → *Median* → *Mean Ranges* → *(Ungrouped data)*
Frequency table Bar → *and Compound bar graphs* → *Histogram* → *Frequency* →
Polygon → *Pie Charts Line and broken line graphs.*
Mode → *Median* → *Mean (Grouped data)*
Quartiles → *Inter-quartile and semi-inter-quartile ranges*

- **Scoring of concept mapping**

A rubric was designed by the researcher to indicate how to evaluate the concept map drawn by the participants. It allocated marks to the number of topics that were correctly arranged, and deducted marks for incorrect arrangement of topics (ref Appendix XXV).

As indicated in Appendix XXV, marks were allocated for the number of topics that were correctly arranged, and deducted for incorrect arrangement of topics in each grade. The mark allocation for the concept mapping exercise was 25 marks in each grade for question a). The combined mark for Grades 10 and 11 for question a) was 50. Question a) requested the participating teachers to 'Arrange the topics in each grade on how best they can be taught in Grades 10, 11 and 12'. No mark was allocated for Grade 12, as the topic is not taught in that grade. The same scoring system was applied to question b), in which the participants were requested to 'With an arrow, show how you can teach these topics sequentially in each grade. For example, you observe morning before afternoon and before evening. Therefore, in a sequential order, it is

Morning → Afternoon → Evening. A teacher who scored less than 60 marks could be regarded as not having the knowledge of the curriculum that would inform his or her insight into the topic. The reason for allocating the same mark is that each question required approximately the same time to solve.

3.5.1.3 Interview schedule for teachers

The purpose of the semi-structured interview was to gain some insight into mathematics teachers' content knowledge and educational background that may have enabled them to develop their topic-specific PCK in statistics. The semi-structured interview schedule was based on several literature sources on PCK (e.g. Jong, 2003; Jong et al., 2005; Van Driel et al., 1998; Rollnick et al., 2008)). To this end, questions were developed to address the teachers' teaching experience, qualifications, educational background and professional development, knowledge of instructional strategies, and preconceptions in teaching and learning statistics. The questions were grouped according to the components of PCK being assessed in this study. This approach has been used by several researchers (Jong, 2003; Jong et al., 2005; Van Driel et al., 1998; Rollnick et al., 2008) in the fields of mathematics and science education. The distribution of the questions is shown in Table 3.5. The questions are indicated in Appendix XXVI.

Table of specification 3.5: Item specification table for the interview

PCK components	Subject matter content knowledge	Instructional skills and strategies	Learning difficulties	Workshop
Number of items	1–9	10–13	14	15–20

Questions 1 to 9 were used to assess the teachers' subject matter content knowledge and demographic profile in statistics teaching. For example, question 1 asked:

> '*What university/college did you attend?*'

They were then asked to indicate the course they had studied in their disciplinary education programme and their understanding of the nature of statistics in school mathematics.

Questions 10 to 13 probed the instructional strategies that they used for teaching statistics and why they employed these strategies. For example, in question 12, participants were asked:

> *'If the learners have any problem in understanding the topic based on the instructional approach, what do you do to help them to understand?'*

Question 14 was used to determine the learning difficulties that teachers themselves think learners have about the topic. For example, the teachers were asked:

> *'What learning difficulty do you remember experiencing as a pupil and as a university student or from teaching experience in statistics?'*

Questions 15 to 20 focused on workshops that the teachers had attended. For instance, the teachers were asked:

> *'Have you ever been to a mathematics workshop or teacher development programme?'*

The data related to workshops were used to triangulate data on teachers' content knowledge.

Prior to the validation of the teacher structured interview schedule, it was given to three secondary school Grade 11–12 mathematics teachers for comments about the categories and educational background for developing PCK. Their comments were used to review the questions before the pilot study.

3.5.1.4 Lesson observation schedules

The lesson observation schedules (ref Appendix XXIX) were standard ones recommended by the Provincial Department of Education for normal classroom practice (DoE, 2010). The schedule was therefore adopted for gathering data for assessing instructional knowledge used in teaching statistics, which is the major focus of this study. The purpose of using the standard lesson observation schedule was to collect data from real-life situations and to assess how well the teachers prepared for lessons, as well as to check for consistency in their implementation of plans (Vistro-Yu, 2003 & DoE, 2010) (ref Appendix XXIX).

3.5.1.5 Teacher questionnaire

The teacher questionnaire was designed to assess teachers' PCK in terms of their knowledge of instructional skills and strategies, learners' conceptions in teaching and learning statistics, and learning difficulties. The teacher questionnaire (ref Appendix XXVIII) consisted of 16 questions designed to triangulate data collected during lesson observation. Questions 1 to 9, 12, 13, 15, and 16 were used to assess the instructional strategies that the teachers used in classroom practices in statistics teaching. An example of the questions focusing on instructional skills and strategies is:

> *How did you identify the prior knowledge (preconceptions) which the learners bring to the class about statistical graphs?*

Questions 10, 11 and 14 were used to determine the learning difficulties that learners have with the topics in statistics teaching (ref Table 3.6) (ref Appendix XXVIII). An example of the questions is:

> *What is it about statistics that makes the learning easy or difficult?*

Table of specification 3.6: Item specification table for the questionnaire

PCK components	Instructional skills and strategies	Learning difficulties
Number of items	1–9, 12, 13, 15, 16	10, 11, 14

The questionnaire focused on what the teachers actually does while teaching, namely their strategies or approach and methods (items 7–11, 15–16) and contents of the lessons (item 2). Other information related to how the teacher identified learners' preconceptions and learning difficulties (items 4–6, 10, 17), how these difficulties were resolved (items11, 12, 14), and how the lessons were evaluated (items 13, 15 and 16) (ref Appendix XXVIII). As regards teachers' instructional strategies and skills, participants were requested to indicate the duration of the lesson, topic, and essential prior knowledge (ref Appendix XXVIII). In addition, participants were requested to indicate how learners responded to the class activities, homework and assignments (ref Appendix XXVIII). For instance, the teachers were asked, 'How did learners respond to class activities, homework and assignments?' Knowledge of learners' conceptions and learning difficulties was assessed by asking the teachers to indicate how they identified learners' preconceptions and misconceptions, if any, as well as learning difficulties in the context of teaching (ref Appendix XXVIII). For example, the participating teachers were asked, 'How did you identify the prior knowledge (preconceptions) that the learners bring to the class about statistical graphs?' Table 3.6 displays how the questions were distributed according to the various components of PCK, namely instructional strategies and learning difficulties, and how the components were assessed. The questionnaire was administered to the participants immediately after the last lesson had been observed.

3.5.1.6 Teacher written reports

The teachers' structured written reports (ref Appendix XXVII), in which they recorded what made the lessons easy or difficult, were used to assess instructional strategies and learners' learning difficulties after a four-week period of teaching statistics. The purpose of the teachers' written reports was to determine what (for the teacher) made the lessons easy or difficult, and to triangulate other data related to how the teachers developed their PCK over time. The written reports were compiled from teachers' and learners' portfolios, as well as learners' workbooks. For instance, the participating teachers were asked, 'How did learners respond to classroom activities as well as homework or assignments?'

The teachers' portfolios contained information such as a formal programme of assessment in mathematics for Grade 11, mathematics assessment tasks (standardised tests, assignments, investigations or projects and examination papers), tools for assessments (memoranda, checklists, rubrics, etc), and model answers for all assessment tasks. The learners' portfolios contained continuous moderation reports, a summary of marks, tests, examinations, and assessments (DoE, 2010).

Table of specification 3.7: Item specification table for the written reports

PCK components	Instructional skills and strategies	Learning difficulties
Number of items	5 and 6	1–4 and 7-9

Nine questions were formulated as guidelines for the teachers in compiling the report. Questions 1 to 4 and 7 to 9 were used to examine learning difficulties, and questions 5 to 6 were used to determine instructional skills and strategies (ref Appendix XXVII). An example of an item focusing on learning difficulties is:

> *What learning difficulties do you identify in learners when teaching statistical graphs?*

An example of questions focusing on instructional skills and strategies is:

> *How did the learners respond to classroom activities as well as homework or assignments?*

The reports were given to experienced mathematics teachers in Grades 11 and 12, who were asked to comment on the questions guiding the report for normal classroom practice (ref Appendix XXVII). Their comments were used to review the report guidelines before use in the pilot study. For example, comment on every task in statistics was checked, marked, had comments and suggestions for motivation and improvement to any learning difficulty that learners might have encountered.

3.5.1.7 Document analysis

In this study the documents analysed in terms of teachers' compliance with curricular recommendations for teaching and learning school statistics were the learners' class workbooks, learners' and teachers' portfolios, and the NCS for mathematics. The purpose of the analysis was to triangulate the data, using the teacher interviews, questionnaires, lesson observation and written reports on how teachers developed their PCK in statistics teaching. At the end of the four weeks' teaching, these documents were made available to the researcher.

The learners' workbooks contained completed, written classwork, homework, and remedial work. Teachers' portfolios for example included work samples and reflective commentary by the teachers as to what had made the lesson easy or difficult, and intervention strategies adopted to address learners' learning difficulties, if any (ref Appendix XXI).

The NCS policy documents gave an indication of whether the teachers were adhering to policy recommendations for teaching and learning, such as the work schedule to be used for teaching statistics according to grade, resources, and assessment plans. It is assumed that a teacher with adequate knowledge of the curriculum would be able to design good teaching strategies in line with the curricular goals. In practice, this requirement meant checking for consistency in the implementation of lesson plans according to the NCS.

3.5.1.8 Video recording

The purpose of the video recording was to record the teachers' teaching (lessons), which would enable the researcher to triangulate the data collected from the lesson observations. The duration of the lessons observed ranged from 40 to 45 minutes for each of the eight lessons. The transcribed protocols (ref Appendix V-XII) were used to gain insight into teachers' content knowledge and how it was used, including the instructional strategies demonstrated in the lessons on statistical graphs.

3.6 Validation of the research instruments

Validity tells us whether an instrument measures or describes what it is supposed to measure or describe. It means that whatever scores were obtained from the instrument should make sense, be meaningful, and enable the researcher to draw conclusions from the sample of the population under investigation (Creswell, 2008). The test validity of an instrument could involve construct validity, content validity, and criterion validity (Creswell, 2008). In this study, content validity was chosen to validate the test instrument (conceptual knowledge exercise). The purpose was to determine whether the test covered the content of the domain that it was supposed to measure. The instrument was meant to assess the subject matter content knowledge in statistics (the domain) that the selected mathematics teachers possessed, which, it was assumed, enabled them to develop PCK. The other instruments such as the concept map exercise and semi-structured interview schedule were validated as follow.

3.6.1 Validity and reliability of the concept map

The purpose of the concept mapping exercise (ref Appendix XXIV) was to assess the participating teachers' knowledge of the school statistics curriculum. In this study, although the major instruments used for assessing teachers' school statistics content knowledge were the statistics conceptual knowledge exercise and teacher lesson observation, the concept map exercise was further used as an addendum to that assessment. A concept map is a viable means of gathering information on a person’s conceptual knowledge of a topic (Novak & Canas, 2006). The concept mapping exercise required the participating teachers first to list the given school statistics topics according to the grades for which those topics are taught, namely Grades 10, 11 or 12; and second to arrange them in the order in which they should be taught in a conceptually logical and sequential fashion. The assumption was that ability to arrange the topics for teaching in a hierarchical manner for each grade level provided an indirect indication that the teachers had adequate knowledge of the statistics topics in the mathematics curriculum and the conceptual relationships among them.

A given set of criteria was used by a mathematics specialist from the Department of Education and two university lecturers in the Mathematics Education Department (ref Appendices XXIV and XXV) to content validate the concept map exercise and memorandum. First, the experts had to ascertain whether the concept map exercise would allow the mathematics teachers to list the topics according to Grades 10, 11 and 12 and arrange them in logical order, such that one topic formed the basal knowledge of the next for each of those grades. Second, they were required to ascertain whether the memorandum (expected answers to the concept mapping exercise) was appropriate for answering the concept mapping exercise. The experts’ responses (mathematics specialist from the Department of Education and two university lecturers in the Mathematics Education

Department) showed unanimous agreement that the concept map exercise contained adequate information for assessing teachers' content knowledge of the statistics topics in the various grades and the ways in which they should be taught in a logical and sequential order. In addition, all the raters agreed that the memorandum was adequate and appropriate for assessing the concept mapping exercise.

The reliability of the concept map was determined as follows. The concept map exercise and memorandum were given to four school mathematics teachers that did not participate in the study and who were physically located outside the study site to avoid contamination. There were consistencies in the responses of the mathematics teachers with the anticipated answers (memorandum) of the concept mapping exercise. In other words, the responses of the respondents (mathematics teachers) were consistent with the idea of listing the statistics topics according to grade and the way in which they should be taught in a logical hierarchical and sequential order. The consistency in the responses of the teachers indicated that the concept mapping exercise is reliable enough for assessing the teachers' knowledge of statistics in the school mathematics curriculum (Bush, 2002; Barriball & White, 2006). Where necessary, their responses were used to review the concept mapping exercise and memorandum before they were used for the main study.

3.6.2 Validity and reliability of the interview schedule

The purpose of the semi-structured interview (ref Appendix XXVI) was to assess the educational backgrounds that may have enabled the mathematics teachers to develop their assumed topic-specific PCK in statistics (Jong, 2003; Jong *et al.,* 2005; Van Driel *et al.,* 1998; and Rollnick *et al.,* 2008).

The schedule was validated by a mathematics expert in the Department of Education and two mathematics education specialists from a university, using a specific set of criteria. The raters were requested to establish whether the interview schedule contained appropriate information to determine teachers’ mathematics educational background for developing PCK as defined in statistics teaching (ref Appendix IV). Their responses showed unanimous agreement that the schedule contained the necessary information for assessing how the participating teachers developed their topic-specific PCK (Bush, 2002; Barriball & White, 2006).

To ascertain the reliability of the interview schedule, it was used with some school mathematics teachers who were not participating or involved in the study. The interest was in determining the extent to which the schedule was likely to yield consistent responses from them (Bush, 2002) in terms of assessing the mathematics teachers' educational background that may have enabled them to develop their topic-specific PCK in statistics teaching (ref Appendix XXVI). The responses of the pilot teachers were identical and consistent in terms of the items selected for the interview schedule. The reliability of the instrument was thus generally assured and, where necessary, the respondents' comments were used to review the schedule.

3.7 Pilot study

Purpose of the pilot study

The purposes of the study were:

- To test the validity and reliability of the test instruments
- To test the logistics feasibility for administration of the instruments
- To improve the design of the research instruments and methodology for the administration of the main study
- To check that the instructions given to investigators were comprehensible
- To check the timing for the administration of the instruments

3.7.1 Subjects used in the pilot study

The subjects used in the pilot study were two willing mathematics teachers at high school level who did not participate in the main study. They were selected from their schools' performance in mathematics in the Senior Certificate Examination for at least two years. The participating teachers had taught higher grade or optional mathematics for a minimum period of three years. One of the participants had a BSc (Hons) in mathematics and the others had BEd degrees in mathematics education. All the participants had taught mathematics at high school for a minimum of five years. The schools from which the participants were selected had shown consistent pass rates of 70% and above in mathematics for at least two years.

3.7.2 Administration of the pilot study

The researcher applied for permission to administer the test to the teachers from the Provincial Department of Education (ref Appendix III). Permission was granted and the teachers participated voluntarily in the exercise (ref Appendix I).

The conceptual knowledge exercise, concept mapping, lesson observation schedule, lesson plan schedule, questionnaire, interview schedule, teachers' written report guide, video recording and document analysis schedule were administered to the participants during the pilot study. The conceptual knowledge exercise was administered to the participants in a classroom at the centre where the cluster meeting took place. Before the conceptual knowledge exercise was administered, participants were informed of their right to participate voluntarily or withdraw from the research process if they wished to, and were informed of their role, the aims and objectives of the research, and how their privacy would be maintained. The time for the completion of the conceptual knowledge exercise ranged from 45 to 55 minutes.

3.7.3 Result of the pilot study

3.7.3.1 Conceptual knowledge exercise

As indicated in Section 3.5.1.1, three mathematics lecturers (raters) from the university assessed the first draft of the conceptual knowledge exercise for content validity. Content validity was obtained by determining the extent to which the raters agreed with the researcher (test developer) and whether the test covered the entire content of statistics in school mathematics adequately according to the NCS (ref Appendix XXIX). The raters were asked to rate each question in terms of *sureness* (with rating levels of; 1 = not very sure; 2 = fairly sure; and 3 = very sure) and *relevance* (with rating levels of; 1 = low/not relevant; 2 = fairly relevant; 3 = highly relevant), with a maximum of three marks for each question. By indicating 'sureness', one had no doubt that the instrument measured the content knowledge of the chosen topic. By indicating 'relevance', one had no doubt that the item was a measure or determinant of content knowledge of the chosen topic (ref Appendix XXIX). The raters' responses demonstrated an overall average of 97% agreement (for the first draft) on the extent to which the test items covered the curriculum. Furthermore, based on their comments, the final items agreed upon totalled 20.

Additionally, the instrument was given to some Grade 11 and 12 mathematics teachers who would not participate in the conceptual knowledge exercise in order to identify difficult and confusing terms or phrases and these were modified or rephrased.

- **Scoring the conceptual knowledge exercise**

Marks were allocated for correct responses or correct choice of options, and no mark was allocated for a wrong or omitted choice, or a choice of more than one response per item. The total correct score was determined out of 20 and the percentage of the score was calculated. Both the raw score and percentage score were analysed to determine the reliability of the exercise. Other test characteristics, including the item response pattern, discrimination, and difficulty indices, were determined and are discussed below.

a) Item response pattern

The analysis of the conceptual knowledge test showed that in some of the items, only 1 or 2 or 3 or 4 (both) participants chose the items, as shown in Table 3.6. For instance, all the participants chose option E of item 1, which is the correct answer to the item. In item 3, three participants chose option B and only one participant chose option D. In items 12 and 13, only one participant chose option D in each case. One of the participants wrote 'no answer' and the other two left the question unanswered. This may be due to bad distracters. Such ambiguous items were discarded. All the participants answered items 1, 2, 5, 8, 9, 19, 11, 14, 15, 16, 17, 18, 18 and 20 correctly. These items tested participants' knowledge in statistics in school mathematics according to the NCS. While items 1, 2, 11, 15, and 20 tested comprehension, items 5, 8, 9, 10, 14, 16, 17 and 18 tested application**.** These items seemed to be easy for the teachers**.** The items were modified by replacing and rephrasing the questions and were therefore considered for inclusion in the main study. Few participants answered items 2, 3, 4, 6, 7, and 19 correctly. These items tested teachers' knowledge of measures of central tendency in statistical graphs of grouped data. While items 2, 3 and 6 tested comprehension, in which the teacher applied routine procedures to solve graphing problem in familiar context, items 7 and 19 tested application, in which the teacher applied his knowledge of statistics to solve familiar or other situations by using routine or multi-step procedures**.** These items were considered difficult. At the end of the review exercise, based on test characteristics (item response pattern), 20 items testing statistics in school mathematics (measures of central tendency and spread) were selected for the main study.

Table 3.8: Item response pattern of the conceptual knowledge exercise from the pilot study test items

OPTION	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	0	0	0	0	0	3	3	4	0	0	4	0	0	0	4	4	0	4	1	4
B	0	0	3	0	0	0	0	0	0	4	0	0	0	0	0	0	0	0	2	0
C	0	0	1	3	4	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
D	0	4	0	0	0	0	1	0	0	0	0	1	1	4	0	0	0	0	1	0
E	4	0	0	1	0	1	0	0	4	0	0	0	0	0	0	0	0	0	0	0

b) Reliability of the conceptual knowledge exercise

Reliability is the extent to which a test produces similar results when administered under constant conditions on all occasions (Cohen, Manion & Morrison, 2007). It refers to the ability of a researcher to obtain the same response each time a test is administered. Principally, there are three types of reliability: stability, equivalence and internal consistency reliability (Creswell, 2007). While stability and equivalence can be examined by test-retest procedures (to give the same test to the same group on different occasions), internal consistency can be examined using the Kuder-Richardson split half procedure (KR-20, KR-21) or coefficient alpha (Creswell, 2007). Reliability in terms of stability and internal consistency of the conceptual knowledge exercise was established in this study using the Kuder-Richardson split half procedure (KR-20, KR-21).

In measuring the stability using the test-retest method, the scores of two tests from two similar groups were correlated. The correlation coefficient must be significant at 95% or a higher confidence interval (Cohen, Manion and Morrison, 2007). In this study, similar groups of teachers were used to pilot test the instrument in order to establish the reliability of the instruments.

The correlation (r) of the two equivalent groups was determined with Window SPSS Version 17.0 as shown in Table 3.11.

$$r = \frac{N\sum XY - \left(\sum X\right)\left(\sum Y\right)}{\sqrt{\left[N\sum X^2 - \left(\sum X\right)^2\right]\left[N\sum Y^2 - \left(\sum Y\right)^2\right]}}$$

Where:

r = the correlation between the two half (even numbered and odd-numbered) items
N = total number of scores
$\sum X$ = sum of scores from the first half test (even-numbered items)
$\sum Y$ = sum of scores from the second half test (odd numbered items)
$\sum X^2$ = sum of the squared scores from the first half test
$\sum Y^2$ = sum of the squared scores from the second scores from the second half test
$\sum XY$ = sum of the product of the scores from the first and second half test

Applying the Spearman Brown prophecy formula to adjust the correlation coefficient, (R) was obtained to reflect the full-length exercise (Creswell, 2007; Gay, 1987; Gall and Borg, 1996);

$$R = \frac{2r}{1+r}$$

Where:

R = estimated reliability coefficient of the full length exercise
r = the correlation between the two half length exercises

The actual correlation (r) between the two half-length exercises was found to be 0.70. Hence, the reliability coefficient (R) of the test is 0.81. The reliability coefficient is within the limit of the acceptable range of reliability 0.70–1.00 (Adkins 1974; Hinkle, 1998). The exercise that was developed can therefore be considered reliable for use in the main study.

c) Discrimination index

The discrimination index is a measure of the effectiveness of an item in discriminating between high and low scorers on the whole test (Tristan, 1998). Once a discrimination index of an item has been computed, the value can be interpreted as an indication of the extent to which overall knowledge of the content area is related to the responses on an item. Therefore it is considered that the ability of a test taker to answer an item correctly depends on the level of knowledge that the test taker has about a subject or topic.

The following statistical formula was used to determine the (DI) of the conceptual knowledge exercise.

$$D = \frac{R_H}{n_H} - \frac{R_L}{n_L} \text{ OR } D = \frac{R_H - R_L}{N} \text{ (if } n_H = n_L\text{)}$$

where:

D = item discrimination index

R_H = number of teachers from the high scoring group who answered the item correctly

R_L = number of teachers from the low scoring group who answered the item correctly

n_H = total number of high scorers

n_L = total number of low scorers

The discrimination index of each item was obtained by subtracting the proportion of low scorers who answered the question correctly, from the proportion of high scorers who answered the question correctly (Trochium, 2001). The discrimination index is a measure of the quality of the items in the exercise and identifies the teachers who possess the desired competency as well as those who do not. The discrimination index ranges from -1.0 to +1.0. If the discrimination index is positive, it means that more test takers in the higher group answered the item correctly than the test takers in the lower group.

Table 3.9: Summary of discrimination indices of the test items

Item no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Discr index	1.0	0.5	0.5	1.0	1.0	1.0	0.5	0.0	1.0	1.0	1.0	0.0o	0.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0

In this study, a discrimination index range of 0.5 to 1.0 was considered appropriate for the inclusion of items in the test instrument (Haladyna, Downing & Rodriguez, 2002). It was therefore necessary to choose more difficult items since the researcher was interested in assessing the content and competency of the teachers in the topic. All the items (e.g. questions 8, 12, 13, and 19) outside the range 0.5 to 1.0 were modified, replaced, or discarded. The overall discrimination index was 0.7, which was within the acceptable range of values for the test characteristics.

(d) Index of difficulty

Another statistical technique that was applied to determine the quality of the test was the index of difficulty. The index of difficulty is given by:

$P = \frac{R*100}{n}$ where; P = index of difficulty

n = total number of teachers in the high and low scoring groups

R = number of high and low scoring teachers who answer the item correctly

The index of difficulty was determined by calculating the proportion of the participants taking the test who answered the item correctly (Nitko, 1996). The larger the proportion, the more students who have learned the content measured by the item (Haladyna, et al., 2002). A test with an overall index of difficulty of more than 0.8 is considered too easy (Nitko 1996). In this study, a difficulty index range of 0.4 to 1.0 was considered appropriate for the inclusion of an item in the test instrument, since participants were assumed to be competent in this topic and were currently teaching it. It was therefore necessary to modify, replace, simplify, or discard items that were outside this difficulty index range. Table 3.10 below summarises the difficulty indices of the tests items.

Table 3.10: Summary of difficulty indices of the test items

Item no	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Diff. index	1.0	0.3	0.5	0.8	1.0	0.8	0.5	0.0	1.0	1.0	1.0	0.0	0.0	1.0	1.0	1.0	1.0	1.0	0.0	1.0

The above analysis shows that about 80% of the items had difficulty indices of between 0.5 and 1.0. This is within the acceptable range. The items cover mode, median, mean, pie charts, histograms, double bar graphs, ogives, variance, standard deviation and scatter diagrams. Therefore, most of the items were retained for use in the main study. Items 2, 12, and 13, with difficult indices of less than 0.4, cover some aspects of grouped data and double bar graphs. The overall difficulty index was 0.7, which was within the acceptable range of values for the test characteristics. Table 3.11 shows the summary of the test characteristics for the conceptual knowledge exercise.

Table 3.11: Summary of test characteristics

Test characteristics	Range of values for test characteristics	Results from pilot study
Reliability	0.70 to 1.00	0.81
Discrimination index	0.3 to 1.0	0,7
Index of difficulties	0,4 to 1,0	0,7
Content validity	0,97	0,7

3.7.3.2 Concept mapping

With reference to Section 3.5.1.2, the concept map and method of assessing (memorandum) the responses of the participating teachers were validated by mathematics specialists from the Department of Basic Education and two lecturers from the university (ref Appendix XXIV) on the content of statistics in school mathematics according to the NCS.

As explained in Section 3.5.1.2, it is assumed that if participants are able to group the statistics topics according to how they should be taught in each grade and in a sequential fashion, then they possess sufficient knowledge of the curriculum and how it should be organised for effective teaching. The raters' responses showed that, first, the list of contents of NCS statistics adequately covered the contents of statistics in school mathematics in accordance with the National Curriculum Statement for Mathematics (DoE, 2010). Second, the memorandum developed for scoring was said to be adequate to assess the performance of the participating teachers in the concept map exercise. Therefore, the concept map instrument was accepted for the pilot study. Using the rubric designed to evaluate the concept map drawn by the participants, the first participant scored 62% and the second participant 64% in the pilot study (see method of scoring in Section 3.5.1.2).

3.7.3.3 Lesson observation schedule

As discussed in Section 3.5.1.4, two mathematics lecturers at the university validated the lesson plan and lesson observation schedules adopted from the Provincial Department of Education. Criteria (ref Appendix XVI) were developed by the researcher by taking into consideration how a standard lesson in normal classroom practice is supposed to proceed (Ofsted, 2010). The two lecturers were questioned, via these criteria, to determine whether the lesson plan and observation schedule contained adequate information to assess normal classroom practice in compliance with the NCS. The validation confirmed that the schedules were the current ones used by mathematics teachers according to the NCS and contained the necessary information to assess normal classroom practice. The classroom observation schedule was used by the researcher during the observation of lessons (ref Appendix XVI).

The lesson observation schedule contained information such as planning, which involves the lesson topic, learning outcomes, assessment standard and resources. The second part described pedagogical issues, such as the introduction of the lesson, general class handling involving class organisation, discipline, interaction, movement, learning climate and involvement of learners in the lesson. Other pedagogical issues contained in the lesson observation schedule were the lesson development, consolidation of the lesson and description of actual teaching and learning. In the actual teaching and learning, the language used for teaching, questioning techniques, assessments, use of resources, knowledge of the teacher, and errors and misconceptions identified were included in the schedule. The teachers and learners' activities, as well as how the lesson was evaluated before the conclusion, were also contained in the lesson observation schedule (ref Appendix XXXII).

3.7. 3.4 Interview schedule

The mathematics expert and the two lecturers were requested to assess the interview schedule to determine if it contained adequate information that will enable the researcher to gather data to gain an insight into the mathematics teachers' content knowledge and educational background for developing PCK in statistics teaching. Their responses to the items in the interview questions showed that the schedule contained adequate information needed to assess teachers' PCK. The items that were not well phrased were modified before they were used in the pilot study (ref Appendix XXVI).

3.7.3.5 Questionnaire for teachers

As explained in Section 3.1.5.2, the questionnaire for the teachers focused on what the teachers did while teaching, namely the strategies used or approach/methods applied, the content of the lessons, the nature of the topic, how the teachers identified the learners' preconceptions and learning difficulties, how the difficulties were resolved if they were, and how the lessons were evaluated. The two lecturers and the mathematics expert validated the designed questionnaire by the researcher with the aid of several sources on classroom practice (Leinhardt et al, 1990; Muijs & Reynolds, 2000; Cangelosi, 1996; Erickson, 1999; DoE, 2010). The two lecturers and the specialists were requested to assess if the questionnaire adequately covered what the teachers are supposed to do while carrying out effective teaching in classrooms with a set of criteria (ref Appendix XIII). Their reports showed that the questionnaire contained questions that are able to elicit from the teachers' information regarding the instructional strategies that mathematics teachers use during classroom practice. The comments from the mathematics specialists and English specialists were used to review the questionnaire before it was used in the pilot study (ref Appendix XXVIII).

3.7.3.6 Written report guide

The written report guide (ref Section 3.5.1.6) was validated by a mathematics specialist in the Department of Basic Education and two lecturers from the university. The three of them were requested to determine whether the written report guides could be used to collect data about what has made the lesson easy or difficult with a set of criteria (ref Appendix XIV). Their responses confirmed that the guide contained adequate questions to guide a mathematics teacher to write such a report. Their comments were used to revise the written report guide before it was considered for the pilot study (ref Appendix XXVII).

3.8 Main study

3.8.1 Subjects used in the main study

The selected four mathematics teachers at high school level in Tshwane North District were involved in the main study.

3.8.2 Administration of the main study

The procedure used in administering the pilot study was also used for the main study. The validated test instruments consisting of i) conceptual knowledge exercises; ii) concept mapping; iii) interview schedule; iv) lesson plan and observation schedule; v) questionnaires; vi) teacher written report guides, vii) and document analysis schedule were administered to the participants. The teachers taught for four weeks and eight periods of lessons were observed on scheduled dates by the researcher.

3.9 Data analysis and results of the main study

3.9.1 Quantitative data analysis

The scores obtained by the four teachers who wrote the conceptual knowledge and concept mapping exercises in the main test were scored as described in sections 4.2 and 4.4 of this study.

3.9.2 Qualitative data analysis

The qualitative data gathered from teachers using the teacher interview, questionnaire, written report and document analysis were analysed by coding and categorising their responses according to the theme in order to determine how the participating teachers developed their PCK in statistics teaching. The analyses were described in Section 4.7.

For the lesson observation, the duration of the observed lessons ranged from 40 to 45 minutes with each of the four participants, and the observation was conducted over four weeks of teaching statistical graphs. The purpose of lesson observation was to determine the subject matter content knowledge, knowledge of instructional skills and strategies as well as insight into learners' conceptions and learning difficulties that the teachers demonstrated in classroom practice over the period. The lesson observations were analysed using the format and content of the lesson observation schedule designed by the Department of Basic Education for normal classroom practice, as was done in the pilot study. The lesson observation reports were coded and categorised in order to determine the similarities and differences between the teachers' teaching methods in the assigned topic (statistical graphs).

The reports of the lesson observations for each of the participants allowed for individual lesson observation analysis and comparison of the instructional skills and strategies used for teaching school statistics. The similarities and differences in content knowledge, knowledge of learners' conceptions in the learning of statistics and learning difficulties that the participating teachers demonstrated and enabled them to develop topic-specific PCK in statistics teaching.

3.10 Ethical issues

Before the commencement of data collection for this study, the researcher applied for ethical clearance. The application was approved and the researcher was issued with a clearance letter (ref Appendices 1, 2, 3A & 3B).

The participants in this study were duly informed of the objectives of the study in writing and oral explanation before the tests were administered to them (ref Appendix I). All the procedures that involved the participants were explained to them. They were informed of their right to decline participation in the study if they so wished. The schools and participants were given codes to ensure that they remained anonymous to the public. The test scripts, interview schedule, responses to questionnaires, the CD for the video recording, and the written reports were kept in a safe place after the information was used for this study. The performance of the participants in the conceptual knowledge exercise was highly confidential. Participants and participating schools were promised access to the result on request. The study report will be submitted to the supervisor of the study, the Gauteng Department of Education, and the University of Pretoria.

3.11 Summary of the chapter

The piloting process of this study was conducted in two phases. The first phase consisted of development, administration and writing of the conceptual knowledge exercise with willing participants. The second phase was to administer and validate the research instruments. The feedback from the pilot study showed that the conceptual knowledge exercise and concept mapping needed to be modified before the main study was undertaken. Other instruments such as the interview schedule, the lesson observation schedule, the lesson plan schedule, the questionnaire and written report were found to contain adequate information that could be used to assess subject matter content knowledge, educational background, instructional skills and strategies as well as knowledge of learners' learning difficulties that teachers use in teaching statistics in school mathematics. The administration of the main study followed the procedure used in the pilot study.

CHAPTER FOUR

4.0 DATA ANALYSIS AND RESULTS

4.1 Introduction

This chapter contains an analysis of the data and presents the results of the main study. Statistical procedures (outlined in Sections 3.7.3.1 and 3.7.3.2) were used to analyse the quantitative data by categorising the responses and lesson observations of the participating teachers according to the components of PCK (pedagogical content knowledge) in order to answer the research questions. The results are presented in the following order:

- Conceptual knowledge exercise
- Concept mapping exercise
- Classroom practice (lesson observation)
- Teacher interview
- Teacher questionnaire
- Teacher written report
- Classroom observations and video recordings
- Document analysis

4.2 Conceptual knowledge exercise

The main purpose of the conceptual knowledge exercise was to make a performance-based selection of teachers for the second phase of the study. The second phase consisted of a concept mapping exercise, an interview, lesson observations, questionnaires, written reports, and document analyses.

The percentage scores of the top four teachers, designated A, B, C, and D, in the conceptual knowledge exercise were 85, 90, 90, and 75 respectively.

4.3 Teacher demographic profiles

The profiles of the four selected teachers are presented below.

Table 4.2: Teacher A, B, C, and D profiles

Name of teacher	Qualification	Subject taught	Teaching experience (in years)	Grade taught
Teacher A	BEd (Mathematics Education), BA (Psychology), Diploma (Mathematics and Science)	Mathematics	21 years	11 and 12
Teacher B	BSc (Mathematics and Statistics)	Mathematics & Mathematical Literacy	10 years	11 and 12
Teacher C	BSc (Mathematics)	Mathematics & Mathematical Literacy	5 years	11 and 12
Teacher D	BEd (Mathematics Education), SED (Mathematics and Biology)	Mathematics	15 years	11 and 12

It is clear that the participants are qualified and experienced mathematics teachers and it was assumed that they have sufficient subject matter content knowledge to competently teach statistics in school mathematics.

4.4 Concept mapping

The four teachers drew a concept map (ref Section 3.5.1.2) on statistics. The results of this exercise, assessed according to the guidelines used to evaluate their responses (ref Section 3.5.1.2), showed that teachers A and C scored 100% each; Teacher B scored 92%; and Teacher D scored 80%. Teachers A and C arranged their topics according to the scheme used, so no marks were deducted. Teachers A and C had greater knowledge than teachers B and D of the school statistics curriculum content and how it should be taught logically so that one topic formed the basal knowledge for the next topic.

4.5 Classroom practice (lesson observation)

The purpose of the lesson observation was to examine interaction patterns in the classroom for each of the teachers, in other words how they used their content knowledge in teaching particular statistics topics. The instructional skills and strategies used by the teachers, the ways in which they tried to identify learners' preconceptions and learning difficulties, and what they did to rectify these misconceptions and learning difficulties, if any, were also examined. The topic in which most lessons were observed was graphing in statistics (line graphs, bar graphs, histograms, pie charts, frequency polygons, ogives, box-and-whisker plots, and scatter plots) since this topic is one of the most challenging in school statistics (DoE, 2010). Two periods of lessons were observed at a time, during site visits to each of the teachers. The observations focused on what the teacher did before (e.g. lesson planning), during (e.g. asking oral probing questions to determine learners' prior knowledge), and after the lesson (e.g. post-teaching discussions and other interventions to address identified learning difficulties).

The same format of analysis was used for all the teachers to identify the components of PCK used in teaching the lessons. The next section presents an analysis of the lesson observation of Teacher A. While observing the teachers, the focus was on how the teachers demonstrated their content knowledge, pedagogical knowledge, knowledge of learners' preconceptions and learning difficulties such as indicating how their assumed PCK manifested during classroom practice. The analysis of the lesson observation will also take into account the coding and categorisation of the themes as shown on the table.

4.5.1 Lesson observation of Teacher A

This section briefly describes Teacher A's lesson observations on teaching statistical graphs. The lesson focused on the construction, analysis, and interpretation of histograms and box-and-whiskers plot respectively. The condition of the classroom is first described, followed by the teacher's classroom practice.

Table 4.5.1 Description of classroom condition and lesson observation of Teacher A

DESCRIPTION OF LESSON	CATEGORISATION/THEMES
Condition of the classroom There were 15 male and 20 female learners of mixed ability. Learners were comfortably seated in six columns of single chairs and desks, with sufficient space to move between the desks. The teacher had a full view of the entire class during the lessons. The classroom walls were decorated with science wall charts. The furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The mathematics class was resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). The classroom had locks, and burglar bars for supervised entry	1) The classroom presented a safe learning environment for both boys and girls. 2) Learners were well resourced with textbooks and other learning materials including workbooks.
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of a histogram. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: After Teacher A had greeted the class, he introduced the lesson on histograms with oral questioning, distributed evenly to different learners, and requested them to define the mode, the median, and the mean in a distribution of ungrouped data	Oral probing questioning was used as an instructional strategy (**pedagogic knowledge**) to introduce the lesson on histograms and determine learners' conceptions and definitions of basic concepts linked to the grouping of data in histogram construction (line 1)
Line 2: One of the learners defined mode as: *'..... The number that appears most often in a distribution,'* and gave an example of mode by verbally listing some numbers and locating the mode within the listed numbers. A second learner defined the median as: '*... The middle number when a distribution of numbers is arranged according to size.'* A third learner defined the mean as: '*... The average of the distribution.'* The last answer was followed by an example from the same learner, who listed some numbers, added them all together, and divided the sum by the number of numbers on the list, to determine the mean. All three learners identified or pointed out by the teacher provided correct definitions for the terms 'mode', 'median', and 'mean'.	Learners correctly defined mode, median and mean (line 2), attesting to **teacher A's content knowledge.** Using a questioning strategy, Teacher A was able to identify learners' previous knowledge about the statistics lesson topic.

Line 3: After the introduction, Teacher A gave the class an example of how to construct and interpret a histogram. He said, '*Write down this example*.' : (i) Construct a frequency table of five classes, starting from 16, (ii) calculate the mean, (iii) draw a histogram, and (iv) use the histogram to calculate the mode of the ages, correct to the nearest year, of 27 members of a netball club. The ages are as follows: 17, 21, 23, 19, 27, 38, 20, 21, 28, 31, 18, 21, 24, 30, 25, 19, 22, 27, 35, 18, 27, 22, 20, 30, 27, 21, and 23. The solution to these questions was presented as follows by Teacher A and the learners, working together:	**Teacher content knowledge** was used to work through an example of how to construct and interpret a histogram (line 3)
(1) Construction of frequency table Line 4a: Teacher A drew a frequency table with the given class intervals, as shown in table 4.5.1a. The table contained the ages of the members of a netball club, the frequencies of the age groups, the mid-values (x) of the age groups, the class boundaries and fx. The teacher did not explain the meaning of the terms. It may be assumed that the learners had come across terms such as class interval (ages), frequencies, mid-values, and the product of frequency and mid-values before because preparing a frequency table of ungrouped data is taught before grouped data according to the curriculum. Teacher A showed the learners how the class intervals (ages) are calculated, using a class of five: for example, he said, '*Beginning from 16 and with five classes, the next class is 20. Therefore, 16–20 is a class interval*.' The teacher continued, '*The next class is 21–25, the other class intervals are: 26-30, 31-35, and 36-40'* (see Table 4.5.1a). Line 4b: Teacher A listed the frequencies of the frequency (f) column on the chalkboard as learners individually counted the ages within the intervals (see Table 4.5.1a) under his instruction. For instance, he asked, '*How many persons are within the ages 16-20?'* The learners counted individually and indicated the frequencies to the teacher who wrote them in the frequency column.	**Teacher content knowledge** was used to describe and complete a frequency table from raw data (lines 4a and 4b) He engages learners by asking them to determine the frequencies within the class intervals row by row (line 4b).

Table 4.5.1a. A frequency table showing the age distribution of members of a netball club

Ages	Freq. (f)	Mid-values (x)	fx	Class boundaries
16–20	6	18	108	15.5–20.5
21–25	10	23	230	20.5–25.5
26–30	8	28	224	25.5–30.5
31–35	2	33	66	30.5–35.5
35–40	1	38	38	35.5–40.5
	27		666	

Line 5a: Teacher A showed the learners how to calculate the mid-values: e.g. he said, '*Mid-value* = $\frac{16+20}{2}$ = 18 (*for the first row*).' Teacher A continued, '*For the second row: mid-value* = $\frac{21+25}{2}$ = *23 (for the second row) .Now continues with row 3, 4 and 5.'* The learners continued with the calculation of mid-values while the teacher wrote the acceptable values on the chalk board.

Line 5b: The next step was to calculate fx, meaning frequency multiplied by mid-values (x). Teacher A demonstrated: '*To calculate fx, you multiply the value of frequency and mid-values, i.e. fx = 6 x 18 = 108 for the first row; for the second row, fx= 10 x 23 = 230; for the third row, fx = 8 x 28 = 224; for the fourth row, fx = 2 x 33=66; and the fifth row, fx = 1x38= 38.'*

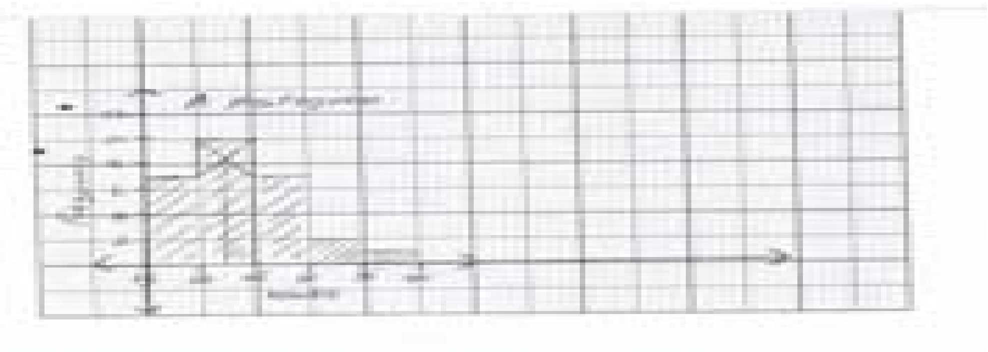
Teacher content knowledge was used to describe how to calculate mid-values and fx (lines 5a and 5b).

Learner content knowledge was used to complete mid-values (line 5b).

Line 6: Teacher A began by describing how to find the class boundaries, beginning with the first row. He then selected an example from table 4.5.1 and calculated the lower class boundary = $\frac{15+16}{2} = 15{,}5$ (for the first row). In the learners' mother tongue, he said, *'15 tlhakanya le 16 arola ka 2, e lekana le 15.5; Meaning add 15 to 16 and divide by 2, equal to* 15.5.' He continued: '*The upper class boundary* = $\frac{20+21}{2}$ *= 20,5.'* (see table 4.5.1a.). Teacher A Further requested the learners to complete the class boundaries for other rows.	**Teacher content knowledge** was used to describe how to complete the frequency table by calculating mid-values and class boundaries to construct the histogram (lines 5a and 5b). The learners' mother tongue (**instructional strategy**) was used to further reinforce a point on how to calculate class boundaries (line 6).
Line 7: The learners completed the table after the teacher had shown them how to calculate the frequencies, mid-values (x), fx, and class boundaries.	**Teacher content knowledge** was used to demonstrate how to complete the frequency table by calculating the frequencies, mid-values, class boundaries, and fx (Line 7).
Teacher A indicated that the next exercise would comprise (II) **Calculating the mean from frequency table:** to begin the demonstration on how to calculate mean from the frequency table. Line 8: Teacher A wrote on the chalkboard: '*Mean is calculated by using the formula,* $\frac{\sum fx}{\sum f}$ *, where* $\sum fx$ *means the sum of frequencies(f) multiplied by the mid-values (x) and* $\sum f$*, means summation of frequencies only, as shown in Table 4.51a.*'Using the formula, he showed the learners how to calculate the mean as follows: Mean = $\frac{\sum fx}{\sum f} = \frac{666}{27} = 24{,}67$	**Teacher content knowledge** was used to calculate mean from the frequency table (line 8) using a **procedural knowledge** approach. *Procedural knowledge approach* is the skill in carrying out procedures flexibly, accurately, efficiently, and appropriately to accomplish a given mathematical task. It includes, but is not limited to, algorithms (the step-by-step routines needed to perform arithmetic operations). An algorithmic approach was used to calculate the mean from the frequency table (line 8).

(iii) **Constructing the histogram** Line 9: Teacher A defined a histogram as: '... *a statistical graph which is used to represent grouped data; a histogram helps to understand complex data in a simpler manner through visualisation*.' He then described how to construct a histogram without explaining what the term grouped data meant. He began by drawing the horizontal and vertical axes on the chalkboard and reinforced the terms using the learners' mother tongue. He said, '*Thala mola o o horizontal le o o vertical',* meaning, draw the horizontal and vertical axis. This was followed by stating the chosen scale. He indicated that the scale was chosen by considering the highest and lowest values of the frequencies and data values as well as the dimension of the graph paper provided, but without demonstrating it mathematically to the learners. He continued with the labelling of the axes and said: '*O be o tsenya di nomore mo meleng',* meaning, label the axes. He drew the first two bars of the histogram. He instructed the class to complete the graph and stated the chosen scale again with no mathematical explanation of how the scale was chosen. To do so would have required a more detailed conceptual explanation.	Teacher A defined a histogram and described how to construct a histogram using a **procedural** as opposed to **conceptual knowledge approach**. A *Procedural knowledge* is a formal symbolic representation system of a given mathematical task using algorithms, or rules, to complete the mathematical tasks (Star, 2002). As indicated above, the participating teachers used more of a procedural knowledge approach than a conceptual knowledge approach because the topic required a particular procedure. It is the common way in which the teachers used algorithms or rules to complete statistics task.He did not explain what was meant by grouped data. Once again the mother tongue equivalent of the technical terms was used to enhance comprehension. Topic specific graph construction skills of drawing horizontal and vertical axes, choosing a scale' and labelling the axes were used to teach the the learners histogram construction (line 9). Teacher A stated and used a chosen scale for constructing the histogram without a conceptual explanation of how it was done (line 9).
Line 10: The learners completed the histogram individually in their workbooks after the teacher had demonstrated on the chalkboard (ref Figure 4.5.1a) with the assistance of another learner how to construct a histogram from the grouped data given.	Learners completed the histogram based on the teacher's demonstration of histogram construction on the chalk board (line10)
Line 11: Some learners seemed to have understood how to construct a histogram for they completed the exercise in	Some learners experienced difficulty in selecting the

their workbooks correctly. Others had difficulties in choosing an appropriate scale so that the histogram could not be accommodated on the graph paper provided. The teacher identified those who were experiencing difficulties because these learners were erasing and correcting their mistakes. He intervened by asking one of the learners, '*Why are you erasing your work*?' The learner answered '*My work is not correct compared to the one on the chalkboard,*' Teacher A then asked, '*Do you understand why your diagram is wrong?'* The learner answered '*Yes, I have seen it on the chalkboard*' and the teacher directed the same question to the other learners who were also erasing their work. They all agreed that they had detected their mistakes from the correction on the chalkboard. The teacher had to allow the learners to write the corrections from the chalkboard into their exercise books for a few minutes before proceeding to calculate the mode from the histogram. The intention of allowing the learners to complete the diagram was to ensure that all participated in using the same diagram to calculate the mode. The next part of the lesson was on how to calculate mode from the histogram.  **Figure 4.5.1a: Histogram of the age distribution of members of a netball club (with a continuation line from the vertical axis)**	appropriate scale (line 11) for constructing a histogram. Insufficient explanation was provided by Teacher A about how to choose scale for constructing a histogram (line 11). Learners who were experiencing some difficulties corrected them with the histogram constructed by the teacher and the learners on the chalkboard (line 11). The learners grasped the rule for the construction of a histogram (line 11).
Line 12a: After the histogram was constructed by Teacher A and the learners, the teacher described another method of constructing histograms. This method allows the histogram to be constructed without a continuation line from origin of the data axis even if the data does not start from o, to reinforce the learners' understanding of histogram construction (ref Figure 4.5.1c). He used the same rule-oriented procedural approach.	**Teacher content knowledge** was used to explain another method of constructing a histogram. It involved creating a continuation line beginning from the vertical axis. The second method helped to reinforce learners'

 Figure 4.5.1b: **A Histogram showing the age distribution of members of a netball club with labelling of the data axis without a continuation line starting from the vertical axis** Line 12b: Teacher A demonstrated the construction of a histogram by beginning the labelling of the data values from the vertical axis, plotting the points, and joining the line of best fit using the same table of values and histogram that had just been constructed. Having constructed the histogram the, next step was to show how to calculate the mode from it..	knowledge of histogram construction and interpretation (line 12a). **Teacher content knowledge** was used to describe the procedure (procedural knowledge approach) of constructing a histogram (line 12b).
(iv) **Calculating the mode from the histogram** Line 13: Teacher A demonstrated how to calculate the mode (using Figure 4.5.1a as presented on the chalkboard). He first drew a diagonal line from the top right-hand corner of the highest bar of the histogram to the top right-hand corner of the bar to the left of it. The next step was to draw another diagonal from the top left-hand corner of the highest bar to the top left-hand corner of the next bar to the right of it. He then drew a line from the meeting point of the two diagonal lines to the horizontal axis and read out the mode at that point (ref Figure 4.5.1a). No explanation was given as to how the drawing of diagonal lines leads to the determination of the mode.	Teacher A used a **procedural knowledge approach** to determine the mode of a histogram (line 13) without explaining the conceptual reasoning behind the drawing of the diagonal lines. *Conceptual understanding* consists of those relationships constructed internally and connected to already existing ideas. It involves the understanding of mathematical ideas and procedures and includes the knowledge of basic arithmetic facts.

	Students use conceptual understanding of mathematics when they identify and apply principles, know and apply facts and definitions, and compare and contrast related concepts. It is called a conceptual knowledge approach when applied in teaching.
Line 14: A learner asked, '*Why do you have to draw a diagonal? Why don't you simply add 20 and 25 and divide by 2 to get the mode?'* This question was posed by the learners because some of them had done it in that way. Many learners nodded their heads in agreement with the question.	Some learners wondered why they should draw diagonals to locate the mode because they calculated the average of the interval of the highest bar instead of locating the mode within the interval of the highest bar identified (line 14). They might have experienced this difficulty of understanding why diagonals should be drawn before locating the mode because the teacher had not explained the term grouped data from the beginning.
Line 15: Teacher A tried unsuccessfully to explain why diagonals should be drawn from both bars on either side of the tallest bar in the histogram to calculate the mode. He said, '*Drawing the diagonals is a procedure for calculating the mode of the grouped data, and the diagonals help to locate the mode within the intervals*.' A conceptual knowledge approach of explaining the relationships among the concepts in histogram construction such as the class boundaries, class intervals, frequency and drawing the line of best fit of a histogram should have been used to answer the question, so as to provide clarity and the answer to the question the learners asked.	The teacher used **procedural knowledge** to answer the learner's question, but the question demanded a conceptual knowledge (explaining the relationship and mathematical connections among the concepts in histogram construction explanation), which the teacher did not provide at this stage (line 15).
Line 16: Teacher A continued with the learner's question on why the diagonal should be drawn and the average of 20 and 25 cannot be used to calculate the mode from the histogram (line 14) when he answered, '*You cannot find the average of 20 and 25 to give you the mode, because the intervals do not contain only the numbers 20 and 25'There are other numbers within the intervals*. 'He referred them to stem-and-leaf diagrams (drawn previously)	A **conceptual knowledge** approach was used to explain why it is not correct to add 20 and 25 in order to determine mode. Comparing the answers obtained from a stem-and-leave with the histogram (line 16) showed

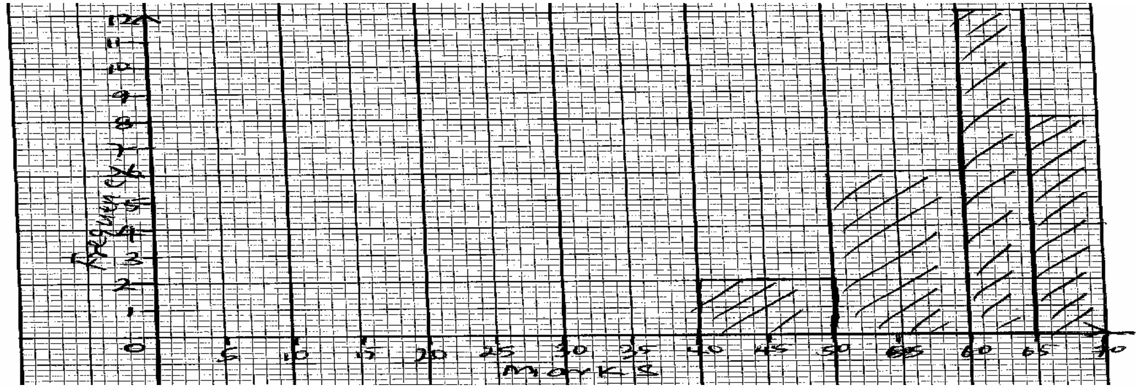
to show how the mode was located and said '*Open to the stem-and-leaf you drew last time and let somebody tell us how we can locate the mode.*' One of the learners raised his hand and explained, *'23 is the most occurring number in the stem-and-leaf diagram and that is the mode.'' Now compare the answer we got from the stem-and-leaf and the one from the histogram, are they the same*?' the teacher asked to elicit an answer as to whether they could link the relationship between the two methods for calculating the mode in grouped data. The learners answered in a chorus, *'Yees sir*.'	that the teacher possesses the content knowledge required to teach histogram construction.
Line 17: Teacher A continues '*Now that you have understood the procedure I have described, write it down in your notebook'*.	Teacher A instructed learners to copy the procedure for calculating the mode on the chalkboard (line 17).
Line 18: The learners wrote the procedure for calculating a mode from a histogram in their exercise books, as provided by Teacher A (see Figure 4.5.1a and 4.5.1b) and shown on the chalkboard. Using photocopied materials, Teacher A provided examples of the useful application of histograms to everyday life situations. For example, '*They can be used to represent the age distribution of teachers in the school and the performance of groups or cohorts of learners in an examination'* he said.	Teacher A related the application of histograms to everyday life familiar situation (line 18) (**instructional strategy).** Learners copy the procedure as written on the chalkboard (line 18).
Line 19a: As the lesson progressedTeacher A asked one of the learners, '*What is the difference between a histogram and a bar graph?'* Line 19b: A learner answered, '*There are constant spaces between the bars in the bar graph, but there is no space in the histogram between the bars. Second, the bar graph is used to represent simple data and histogram is used to represent large groups of data. Because the data that histogram represent are large, they are grouped as class intervals or boundaries in the frequency table. Bar graph do not contain class interval or boundaries'* This answer was satisfactory to the teacher, who asked a second question.	A higher level of questioning (explanation, not recall) was used as an **instructional strategy** to assess how well learners had understood the lesson (line 19a). Learners showed evidence of comprehension in the answer provided about the differences between a bar graph and a histogram (line 19b).
Line 20a: Teacher A asked: *'How can you calculate the percentage of players within the age group of 26–40 in the histogram?'* (ref Figure 4.5.1a). A few learners indicated an interest in answering the question; one was asked to give an answer and she said. '*You add 7 + 2 + 1 = 10* '(from the frequency table), *'then divide 10 by 27 and*	Oral questioning based on **application of knowledge** was used to assess learners' content knowledge about histogram construction (line 20a).

<table>
<tr>
<td><i>multiply by 100; i.e. the percentage of players between 26 and 40 =</i> $\frac{10}{27}$ <i>X 100 = 37%. Therefore, 37% of the players are between the ages of 26 and 40.'</i></td>
<td>Learner content knowledge: an algorithmic approach was used to answer the teacher's oral question on how to calculate the percentage of players within an age group (line 20a).</td>
</tr>
<tr>
<td>Line 21: Teacher A assigned classwork in which the learners were asked a similar question on histogram construction and interpretation to the one they had already done. The classwork required learners to construct a histogram and use it to determine the mode and the percentage of learners who had completed a test. Table 4.5.1b (below) shows the mark distribution of the test. The teacher walked around the class to monitor the learners.

Table 4.5.1b: Frequency table showing learners' performance in a test

<table>
<tr><th>Class interval (%)</th><th>Frequency</th></tr>
<tr><td>40–49</td><td>2</td></tr>
<tr><td>50–59</td><td>6</td></tr>
<tr><td>60–69</td><td>12</td></tr>
<tr><td>70–79</td><td>8</td></tr>
<tr><td>80–89</td><td>4</td></tr>
</table>
a) Draw a histogram to illustrate learners' performance in the test.
b) From your diagram, calculate the mode.
c) If the pass mark is 60%, calculate the percentage of learners who failed the test.</td>
<td>Classwork was used to spontaneously assess how well learners had grasped the content of the lesson (instructional strategy to provide immediate feedback) (line 20b).

Teacher A monitored and analysed learners' responses to classwork on construction and interpretation of histograms (line 21) to ascertain how well the learners were responding to the classwork and to detect learning difficulties and misconceptions, if any.</td>
</tr>
</table>

Line 22a: While most learners completed the class work efficiently, some could not finish it in class. The difficulties experienced were in (i) labelling of the axes with the types of grouped data provided (which began at 40 marks and not from 0 as was the case in the example which the teacher worked on), and ii) the construction (scaling and labelling of the axes) of the histogram. Figure 4.5.1c (below) shows an example of a graph drawn by a learner who experienced difficulties in histogram construction. The histogram could not be accommodated on the graph paper provided due to incorrect scaling.  **Figure 4.5.1c: An example of an incomplete classwork exercise on histogram construction** In this graph, the scale chosen by the learner(s) could not accommodate the histogram on the graph paper; hence part of the histogram was not represented. This made it difficult to calculate the mode and determine the percentage of learners who failed the test. Line 22b: A learner said, '*My graph is not like the one you constructed on the chalkboard.*'	Learning difficulties experienced by learners were labelling and scaling of data axes of grouped data (ref Figure 4.5.1c) (lines 22a and 22b). Lack of comprehension was evident in a learner's statement-(line 22b).
Line 23a: Teacher A analysed what the learner had drawn and said, '*You constructed a bar graph instead of a histogram. It is a wrong histogram.*' He continued, *'I shall organise an extra lesson to rectify the difficulties you are having and explain why your answer is wrong after the lesson.'* (The lesson period had expired.) After the	A learning difficulty of constructing a bar graph instead of histogram was detected by Teacher A from the classwork that the learners were doing (line 23a). A

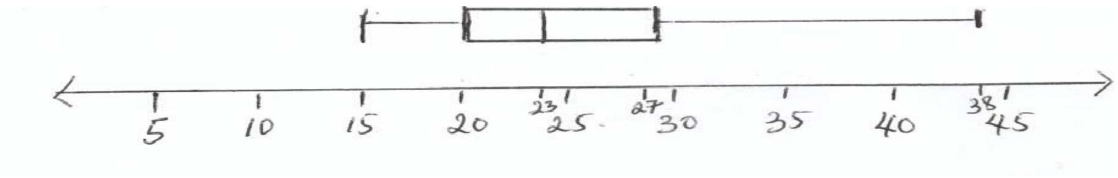
lesson, some learners asked the teacher to explain aspects of the lesson where they lacked clarity (post-teaching discussion). Line 23b: Teacher A gave the learners homework on construction and interpretation of histograms using their textbook in mathematics, to be submitted the following day. The entire lesson was based on the learners' mathematics textbooks, photocopies of mathematics-related materials, and study guides.	post-teaching discussion took place after the lesson to help them (line 23a). Learners' learning difficulties were discovered through an analysis of classwork (**instructional strategy**) (line 21 and 23a). Homework was used as an opportunity for learners to demonstrate their understanding of histogram construction, and later to assess how well the learners had understood the lesson (**instructional strategies** for teaching) (line 23b).

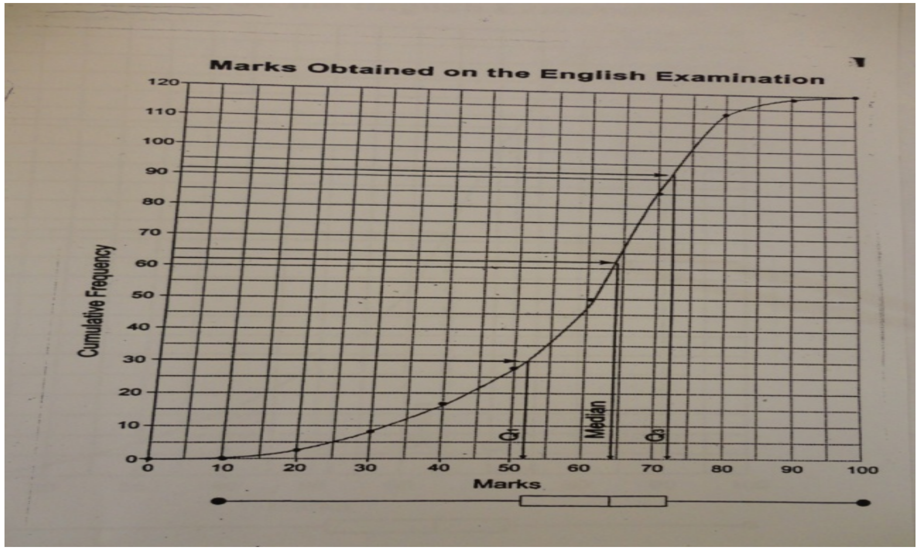
CLASSROOM PRACTICE (SECOND LESSON)

Topic: Construction, analysis, and interpretation of ogives and box-and-whisker plots. Class: Grade 11

DESCRIPTION	**CATEGORISATION/THEMES**
Line 1: Teacher A began the lesson on box-and-whisker plots by checking and marking the homework on cumulative frequency tables and ogives (a distribution curve in which the frequencies are cumulative).	The checking and marking of homework was used to try to determine learners' conceptions (preconceptions) (line 1) (**instructional strategy**) in box-and-whisker plot construction.
Line 2: Teacher A and the learners provided the correct answers to the homework on the construction, analysis, and interpretation of a cumulative frequency table and ogive by calculating the cumulative frequencies and further explaining how it was used to construct an ogive.	The teacher and learners together consolidated the concept previously taught by providing corrections to the difficulties the latter must have experienced while doing the homework (**instructional strategy**) (line 2) on ogive construction.

Line 3a: Teacher A wrote the topic (box-and whisker plots) on the board and referred the learners to photocopied material on ogives, from which they could interpret an ogive using quartiles obtained from an ogive. They were to work in groups of 4 to 5 learners and calculate the quartiles as a way of demonstrating their knowledge of how to construct an ogive. Line 3b: Teacher A said '*Look at the photocopied paper I have given you, question 2.'* He continued and read, '*Find out the percentage of learners who obtained (i) less than the lower quartile; (ii) less than the median; and (iii) less than the upper quartile; and (iv) Minimum and maximum values of the ogive.'*	**Instructional strategies** such as group work were used to interpret ogives and to demonstrate learners' content knowledge and understanding of how to construct an ogive (line 3a). **Teacher content knowledge** and **instructional strategies** were used to design the task to be used to demonstrate box-and-whisker plot construction (line 3b).
Calculation of quartiles from an ogive after the learners had completed the exercise Line 4: The learners interpreted the ogive (it was assumed that Teacher A had provided a description of ogive construction in the previous lesson) as the question on the photocopy indicated, with the first quartile (see definition below)(Q_1) = 20, using the formula; $Q_1 = \frac{(n+1)th}{4}$ to calculate the position of Q_1. The next step was to calculate the second quartile (Q_2) = 23, using the formula, $Q_2 = \frac{(n+1)th}{2}$ to calculate Q_3; and the third quartile (Q_3) = 27, using the formula, $\frac{3(n+1)th}{4}$ (where a quartile is a division of the data distribution into four equal parts). '*The minimum is 15 and maximum is 38 (read from the ogive,*' one of the learners said in response to the questions on the photocopied question. Teacher A accepted the answers provided by the learners for Q_1, Q_2, Q_3, minimum and maximum values as correct, and said, '*Now, we are going to use these values to construct a box-and-whisker plot.*' He defined a box-and-whisker plot as *'... a graph showing the distribution of a set of data along a number line.'* With no further explanation, he went on to describe how to construct a box-and-whisker plot.	An algorithmic approach (**procedural knowledge**) was used by the learners to determine the quartiles (line 4). **Teacher content knowledge** was used to provide the definition of a box-and-whisker plot with no further explanation regarding the basic knowledge or skills to required for the construction of the graph. The teacher did not indicate or anticipate any possible difficulties or misconceptions that the learners might possibly encounter (line 4).
Construction of box-and-whisker plot	Teacher A used a **procedural knowledge** approach to determine the quartiles which were used to construct

Line 5a: Because Teacher A was satisfied with the learners' answers on the quartiles derived from the ogive in line 4, he used the quartile values to show the learners how to construct and interpret box-and-whisker plots. He did this by first drawing a number line with a scale of 1 cm = 5 units (see below). The box was drawn above the number line using the values for Q_1, (23) Q_2 and Q_3 (27) (Fig 4.5.1d). The whisker was then represented by a line, according to the maximum (38) and the minimum value (15) as obtained from the ogive as shown below (Fig 4.5.1d).  **Figure 4.5.1d** **represents a box-and-whisker plot constructed with the values of the quartiles obtained from the ogive** Line 5b: Some learners experienced difficulties making sense of why the minimum and maximum values of Q_1, Q_2 and Q_3, had to be used for constructing a box-and-whisker plot. This was largely because the teacher did not explain the meaning of this term.	a box-and-whisker diagram (line 5a) (**instructional strategy).** **Insufficient teacher content knowledge** and explanation of box-and-whisker plot resulted in learners' learning difficulties (line 5b). Construction skills were used to construct a box-and-whisker plot with the quartiles obtained from the ogive using a **procedural knowledge approach** (line 5a) without explanation.
Line 6: Most of the learners requested clarity on interpreting ogives. For example: how the values of the quartiles were obtained and used to construct the box-and-whisker plot. '*Listen learners*,' the teacher said, '*it appears that some of you do not understand the description I have given about the construction of box-and-whisker plot. Now let me give you another example from the textbook.'*	Learners experienced difficulties as to how the teacher obtained the quartiles. They had no understanding of how the values of the quartiles from an ogive were obtained and used to construct a box-and-whisker plot (line 6). The teacher resorted to the use of the textbook to work through a textbook ogive example.
Line 7a: Teacher A referred the learners to their textbook, unit 8 (containing examples of what they had done). Using these textbook examples (while individual learners took note of the example from their textbook), he then tried to	Teacher subject **matter content knowledge** was supplemented by the use of textbooks to provide

explain the mathematical connection between ogives and box-and-whisker plots. He described an ogive as *’... a cumulative line graph and it is best used when you want to display information involving grouped data,’* He continued, *‘To interpret an ogive, quartiles are usually used. The quartile values are used to construct the box-and whisker plot to provide more clarity about what the data tend to convey.’* ‘, Teacher A said. He continued *‘’Now I*  *want you to study this example in unit 8 in your textbook for five minutes.’* **Figure 4.5.1e: Ogive showing the mark distribution of learners in an English examination** This diagram of an ogive from the learners’ textbook was used to provide another example of the way in which to construct ogives and box-and-whisker plots. The box-and-whisker diagram (ref Fig 4.5.1e) was constructed from information derived from the analysis and interpretation of the ogive.	definitions on the concepts of ogive such as the quartiles. There was no attempt to relate the concepts being studied to any context or examples familiar to the learners. When and how are ogives used for example? The teacher does not demonstrate any flexibility (or insufficient flexibility) in the approaches or methods used to present the topic (line 7a). An example from the learners’ mathematics textbook was used (as an **instructional strategy)** to provide some clarity on how quartiles were obtained from the ogive and used in constructing a box-and-whisker plot (line 7a) **Teacher content knowledge** (Figure 4.5.1e) was used to describe the interrelationship between ogives and box-and whisker plots by reading out the quartile values from the ogive in Figure 4.5.1e and used to construct a box-and-whisker plot (line 7c). Class work was used as an **instructional strategy** to reinforce learners’ grasp of how to calculate quartiles from the ogive (line 7c).

Line 7b: Learners studied the example for about five minutes and compared it with their previous homework box-and-whisker plot construction to try and comprehend how the values for constructing the box-and-whisker plot had been obtained. Line 7c: Teacher A described:*’The first quartile is obtained by first locating the quartile position on the frequency axis, draw a line from there to join the curve, and join the line to the horizontal axis to locate first quartile (Q_1)’* using Figure 4.5.1e. The same procedure is used for Q_2 and Q_3. Teacher A asked, ‘*Do you understand?* ’The learners answered, ‘*Yes sir.’* .As a follow up the teacher gave them a task: Using the same Figure 4.5.1d, the learners were asked to (i) find the estimate of a) the lower quartile (Q_1); b) the median (Q_2); c) the upper quartile (Q_3). (ii) Find out what percentage of the learners had obtained marks that were a) less than the lower quartile, b) less than the median, and c) less than the upper quartile. The intent was to find out if the learners had understood how to obtain the quartiles from the ogive, which could then be used to construct the box-and-whisker plot. Line 7d: As the teacher monitored and analysed learners’ classwork assignment, he discovered that the majority of the learners were unable to locate the position of the quartile from the ogive even after applying the correct formula.., This was either because the learners lacked the knowledge and skills of scaling and labelling of data axis, or that the teacher’s oral explanation was not sufficient for them to grasp the concept.. Teacher A said ’*I can see that some of you cannot locate the quartiles even after you have calculated the position of the quartiles. Now, let me do it with you.’*	Learners experienced some difficulties in locating the quartiles from the data axis due to insufficient **learner content knowledg**e about scale and labelling of the data axis (line 7d). The teacher intervened regarding the errors that the learners were making on their classwork and had to work with them using Figure 4.5.1e to clarify the learning difficulties.
Finding the quartiles (Q_1, Q_2 and Q_3) Line 8a: Using the formula for calculating the position of quartiles as in line 4 and Figure 4.5.1e, Teacher A showed the learners how to calculate quartile positions by the use of a ruler to trace the quartiles beginning from the cummulative frequency axis to the curve and down to the data axis to obtain : a) the lower quartile (Q_1) which was 52. He continued in a similar manner to obtain: b) the median (Q_2) which was 63, and c) the upper quartile (Q_3)	**Teacher content knowledge** was used to demonstrate the procedure for calculating the quartiles from the data axis (line 8a) in order to clarify learning difficulties about box-and-whisker

which was 73.	plot construction.
Calculating the percentage of learners that score marks less than the quartiles Line 8b: Teacher A said, ' *Let us solve the remaining questions,'* and continued,, *'you calculate 25% of 120 as:* $\frac{25}{100} \text{ x } \frac{120}{1} = 30$. *With your ruler at 30 on the cummulative frequency axis, trace it to join the curve and down to the data axis. Therefore, a) 25% of the learners obtained marks of less than 52%. In a similar manner, b) 50% of the learners obtained marks of less than 63%, and c) 75% of the learners obtained marks of less than 73%.'* The answers to the two questions (i) and (ii) are the same but the question was asked in two different ways.The teacher probably wanted to demonstrate varieties of ways of asking questions about quartiles and provide various strategies of answering the question, which illustrates the teachers' PCK.	The learners' oral questions indicated that they had some learning difficulties concerning the formula for calculating the median of grouped and ungrouped data (line 8ci) which may have been due to insufficient teacher explanation of the differences between the way the median in ungrouped and group data is calculated (line 8cii).
Line 8ci: A learner raised her hand and asked, '*Why is the method of calculating the median in the ogive different from the one we did last week?'* The learner referred Teacher A to her exercise book and showed him that the method was different from what she had in her book. Some learners nodded their heads in support of the question. But one of learners raised his hand up and he was recognised by by the teacher to answer the question: And he said '*In the previous example, we calculated the median of ungrouped data. But in this case we are calculating the median of grouped data'.* The methods were different, but the learners had misunderstood the ways the median is calculated in ungrouped data and in grouped data. This learning difficulty may have arisen because the teacher did not explain the difference between determining the median of ungrouped and grouped data in line 8b and in any previous ungrouped data lesson.	A **conceptual knowledge approach** was used to address the learners' lack of understanding of the differences between how to calculate the median of group and ungrouped data (line 8cii) by comparing the differences between the way the median is calculated from grouped data in the current lesson and ungrouped data from previous lesson.
Line 8cii: Teacher A explained, pointing at the previous example in one of the learners' exercise books and directing the whole class to the same example in their individual exercise books, '*The previous example used ungrouped data,*	

in which you arrange the data according to size of the numbers, but the current example used grouped data in which some data were grouped together. You cannot arrange them in the same way like the ungrouped data because, the particular number within the groups are not known. Hence, the formula method was appropriate to calculate the median within the class intervals or group.'	
Line 9: Teacher A asked, as a way of concluding the lesson, '*How do you calculate the first, second and upper quartiles of an ogive? How can you use the quartiles to construct a box-and-whisker plot?'*	Oral questioning was used to assess the learners and evaluate the lesson by requesting the learners to explain how quartiles are calculated (line 9) (**instructional strategy**).
Line 10a: Several learners volunteered to answer the question; the teacher selected one who said: '*Using the formula* $\frac{n+1}{4}th$, *you can calculate* Q_1 *position and locate* Q_1. *Using the formula* $\frac{n+1}{2}th$, *you can calculate* Q_2 *position and locate* Q_2. *Using the formula* $\frac{3(n+1)}{4}th$, *you can calculate* Q_3.*and locate* Q_3.' (rote learning regarding the use of an algorithm). Line 10 b: Teacher A called on another learner to demonstrate how the values of Q_1, Q_2 and Q_3 could be used to construct a box-and-whisker plot. Line 10c: The learner used the teacher's *example to answer the question in a procedural manner by indicating: 'Using the formula (pointing on the chalkboard), you calculate* Q_1 *position by substituting the value of n. After that the quartile position is located on the frequency axis and by drawing a line from that position to the curve and down to the horizontal axis, you locate the first quartile* (Q_1).' Q_2 *and* Q_3 *were calculated in the same way*, the learner said.	**Learner content knowledge** mostly of a procedural or algorithmic nature was used to answer the question on the application of a formula (line 10a). The learners continued with their responses to the teacher's question to indicate that they had grasped the lesson (line 10c)
Line 11a: The learners were then referred to their textbooks for homework. This required the learners to calculate the quartiles from a constructed ogive and use the quartiles to construct a box-and-whisker plot. The assessment task tested learners' conceptual understanding of how to construct, analyse, interpret and apply the knowledge of box-	Homework was used as **instructional strategy** to assess and provide feedback on learners' conceptual understanding of the lesson on box-and-whisker plots

and-whisker plots to a familiar situation. The homework showed that the teacher complied with the assessment guidelines and learning outcomes of data handling-but provided no examples in his teaching of the application of those plots in contexts familiar to the learners. Obviously, Teacher A has displayed inadequate PCK in teaching box-and whisker plot construction at this stage.. Line 11b: A post-teaching discussion took place after the lesson. Some learners asked: '*How do you represent the fractions we got from the graphs during the interpretation of the ogive?'* (following the results from their calculations). The teacher replied: '*The fractions can be represented by rounding off to the nearest whole number.'*	(line 11a). A **post-teaching discussion** was used to address learners' questions and to clarify the method of representing fractions on the box-and-whisker plot (lines 11b).
Line 12: The teacher promised to organise extra tutoring after school for the learners who were experiencing difficulties with the construction and interpretation of ogives and box-and-whisker plots as he could not attend to everybody in the post–teaching discussion.	A **post-teaching discussion** was used to address aspects of the topic which the learners did not grasp (confusion over the use of quartile values to construct box-and-whisker plots), and additional tutoring was proposed (lines 12).

Summary of lesson observation of Teacher A

Teacher A demonstrated that he has the required content knowledge to teach statistical graphs such as histograms, ogives and box-and-whisker plots. He described, and demonstrated how to construct, a histogram and tried to elucidate the differences between ogives and box-and-whisker plots, using a mostly rule-oriented procedural approach; but less of a conceptual knowledge explanation. Using his procedural knowledge he followed a stepwise sequential approach to demonstrate the construction of a histogram and box-and-whisker plot: namely drawing of the axes, choosing a scale, labelling the axes, plotting the points, and then drawing the line of best fit. With regard to section 8cii of the second lesson observation, Teacher A also applied a conceptual approach in clarifying learners' misunderstanding of how to construct a box-and-whisker plot using the quartiles calculated from the ogive. The conceptual approach entails explaining in detail the relationship between the quartiles obtained from the ogive (e.g.$Q_{1,}$ median, and Q_3) of a box-and-whisker plot (ref Section 4.5.1, second lesson observation, and line 8cii), the mathematical connections between quartile positions and the quartiles obtained from the ogive. Teacher A used topic-specific construction skills (as earlier defined) in statistics to construct histograms and box-and-whisker plots. Instructional skills of oral questioning, checking and marking of learners' classroom and homework assignments were also used to try to identify learners' preconceptions and learning difficulties in constructing histograms and box-and-whisker plots. But the teacher identified learners' previous knowledge of histogram and box-and-whisker plot construction using the strategy oral questioning and checking and marking of learners' homework. Other instructional strategies which Teacher A applied in his teaching were the use of examples drawn from everyday familiar situations for the histogram, but for the ogive and box-and-whisker plot he applied the mother tongue to reinforce learners' comprehension. There was no evidence that he anticipated the difficulties learners were likely to have in first coming across the topics of histograms and box-and-whisker plots that he taught. For example, when he tried to identify learners' preconceptions using oral probing questioning on measures of central tendency, learners displayed evidence of having a previous knowledge of histogram construction and no preconception was identified, meaning the teacher may well not likely have knowledge of learners' preconceptions, which would have allowed him to address any anticipated learning difficulty.

From the observed lessons, it can be construed that the PCK of Teacher A consists largely of the procedural use of rules to construct histograms and box-and-whisker plots (statistical graphs) and, less frequently, of conceptual knowledge.

4.5.2 Lesson observation of Teacher B

This section briefly describes Teacher B's lessons on teaching statistical graphs. The lessons, which were observed during two periods of site visits, focused on the construction, analysis, and interpretation of the bar graph and the ogive. The condition of the classroom is described first, followed by the teacher's classroom practice in delivering the lesson.

Table 4.5.2a: Description of lesson observation and classroom conditions at School B

DESCRIPTION OF LESSONS	CATEGORISATION/THEMES
<u>Condition of the classroom</u> There were 16 male and 24 female learners of mixed ability. Learners were comfortably seated in the science laboratory in two columns of single chairs surrounding some big desks with sufficient space to move between the desks. The laboratory was safe and conducive to teaching and learning. The wall of the laboratory was decorated with science charts such as the human circulatory system. The learners were individually resourced with learning material such as the mathematics textbooks, exercise books and calculators. The science laboratory is sometimes used when the teacher want to use an overhead projector for demonstration.	1) Forty learners were seated in single chairs surrounding some big desks in two columns. 2) The school was safe and well protected. 3) The science laboratory is not used exclusively for science subjects. 4) The learners were resourced with learning materials
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION). **Topic: construction and interpretation of bar graphs. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: The teacher arrived in the class and greeted the learners '*Good afternoon learners*?' Learners answered '*Good afternoon sir*' A frequency table was used to introduce **Teacher B's** first observed lesson. Learners were expected to prepare a frequency table of the scores of learners in a test. The data presented to the learners by **Teacher B** was based on the scores that learners had obtained in a 10-mark test, and involved arranging these scores on a frequency table: 2, 3, 4, 5, 5, 6, 4, 7, 5, 6.	Teacher B greeted the class and placed a **pre-activity** on the chalk board to gain information about learners' conceptions **(preconceptions)** of the construction and interpretation of bar graphs (line 1) (**instructional strategy**).
Line 2: The **learners** individually prepared a frequency table within five minutes (ref Figure 4.5.2a).	Learners showed that they had assimilated the knowledge of how to construct a frequency table from their previous lesson as they prepared it efficiently (line 2 and table 4.5.2a).

Table 4.5.2b: Frequency table showing the performance of learners in a test

Scores (x)	Tally	Freq. (f)	Fx
2	/	1	2
3	/	1	3
4	//	2	8
5	///	3	15
6	//	2	12
7	/	1	7
		∑f = 10	∑fx = 47

Construction of a bar graph

Line 3a: **Teacher B** described algorithmically how to construct a single bar graph, using the data from the frequency table (ref Figure 4.5.2a) prepared by the learners as indicated in line 2. '*Now, watch out, you begin by drawing the vertical and horizontal axes*' he said. **Teacher B** drew the horizontal and vertical axes and asked the learners to explain how to choose the scales for the axes. He asked, '*How do we choose the scale for labelling the axes?*'

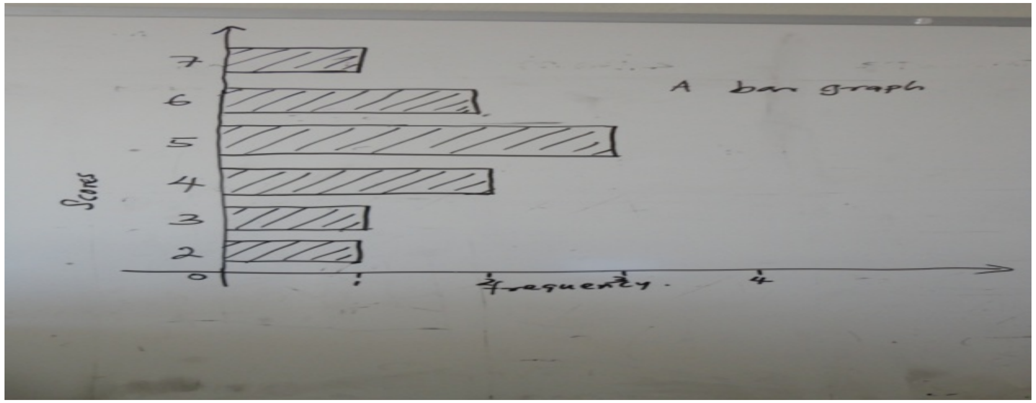
Line 3b: Some learners raised their hands and the teacher pointed at one to explain.

Line 3c: Learners stated how the numbers should be written on both the horizontal and vertical axes, by indicating 1, 2, 3, 4 etc for the horizontal axis and 2, 3, 4, 5, 6, and 7 on the vertical axis, while the teacher wrote the numerals on the chalkboard and elucidated why the scale had been accepted for constructing the bar graph, for such reasons as considering the highest and lowest values on the frequency table and data and the dimension of the graph paper.

Teacher content knowledge was used to describe how a bar graph is constructed (lines 3a and 4).

Teacher B probes learners with a question to find out if they know how to choose a scale for constructing a bar graph (line 3a).

Teacher B merely indicated the scale that was chosen by the learners and why it accepted and wrote them on the chalk board with no mathematical justification of how either the learners or himself had selected the scale (line 3a)

Line 4a: **Teacher B** showed how to label and draw the bars, using the appropriate frequencies on table 4.5.2a: *'Watch and see how to draw the bars; the first score is 2, and the corresponding frequency is 1',* the teacher said. Learners watched as the teacher demonstrated how to draw one of the bars on the axes corresponding to the score (data axis) with a value of 2 and frequency is 1.  **Figure 4.5.2a** **Bar graph of the scores of learners in test on how to construct, analyse, and interpret a bar graph using the scores in column 1** Line 4b: The teacher asked, *'How can I draw the second bar graph?'* The teacher nominated one of the learners, who answered, '*The second score is 3, and the frequency is 1*'. The teacher demonstrated how to draw the second bar (indicating that he was satisfied with how the first bar was constructed) and instructed the learners to copy and complete the bar graph in their exercise books while he monitored them. While monitoring, he discovered that certain learners experienced some difficulties because they had not left a constant space between the bars, which he indicated without explanation. He intervened by helping the learners to complete the bar graph and indicated that there should be constant spacing between the bars.	**Teacher content knowledge** was used to describe how to construct a bar graph using a **procedural approach** (**instructional strategy**) (lines 3a, 3c, 4a, 4c and 5). **Teacher B** analysed learners' classwork as he monitored their work on bar graphs (line 4c). **Graph construction skills** (drawing the axes, choosing scales, labelling axes, plotting the points and drawing the lines of best fit) were used by learners in drawing a bar graph (lines 3, 4a and 4b, and Figure 4.5.2a) (**instructional skill**). **Misconceptions and learning difficulties in constructing a histogram instead of a bar graph** were identified by monitoring and analysing learners' responses to classwork and in the class discussion (lines 4c and 4d). Learners may have experienced such difficulties due to insufficient explanation of why there should be constant spacing between the bars of a bar graph (line 4b).

Line 4c: He asked the learners to watch while he completed the bar graph on the board. Learners who were experiencing learning difficulties (e.g. constructing a bar graph like a histogram) corrected their mistakes as he did so (see Figure 4.5.2a). Line 4d: The **learners** asked, 'Why *they had to leave spaces between the bars?*'	
Line 5: **Teacher B** referred to the graph on the chalkboard and answered: '*The bars represent different scores; the height of the bars represents the number of learners that scored a particular mark, e.g. two learners scored 4 marks, and the constant spacing differentiates one score from another, as the number of learners that score a particular mark is not the same*'	**Teacher B** answered the learners' question by demonstrating how to label the axes and explaining why it is necessary to leave constant spaces between the bars (line 5) (**teacher content knowledge**).
Line 6: **Learners** were given time to correct their misconceptions in their notebooks, as well as learning difficulties. Afterwards, the teacher explained again how to construct the frequency table and bar graph as he did in line 4a to 4c, as some of the learners continued to ask for clarity on why there should be constant spacing between the bars.	**Teacher B used the instructional strategy** of **again explaining** the preparation of a frequency table and bar graph construction to clarify learners' understanding of the need for constant spacing between the bar**s of a bar graph** (line 6).
Line 7: The **learners** asked: '*How do you know that the 10-mark test was easy or difficult?'* This question demanded that the teacher explain the relevance of frequency tables and bar graphs, which he had not done initially.	Learners asked a question that required the teacher to explain the relevance of frequency table and bar graphs (line 7).
Line 8: **Teacher B** explained: '*Other factors could be used to determine whether the test is easy or difficult, but at the moment, the pass mark is considered'*. For example, '*If the pass mark is 4 and the number of learners that scored 4 and above is 8 out of 10 learners, then the test was easy*'. **Teacher B** read out the number of persons who scored 4 and above as 8. '*This means that about 90% of the learners scored between 4 and 10. But if the number of learners that scored between 1 and 3 is 8 (Teacher B read from the graph), and the highest score was 5, the test was difficult, as 80% of the learners scored below 4 marks'*. He continued, *'Thus, with a bar graph, it is easy to show and interpret learners' performance in a test. From Figure 4.5.2a, it is evident that the test was within the level of the learners, as the learners' marks were not too low, and if the pass mark was 4 (40%), then only two of the learners failed'*.	**Teacher content knowledge** was used to explain the criteria and demonstrate how to determine whether the 10-mark test was difficult or easy (line 8) (**teacher content knowledge).**

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<td>Line 9: **Teacher B** gave out photocopies of classwork, in which learners were asked to construct a bar graph individually. The teacher monitored and analysed their responses as they worked. Some **learners** had drawn their diagrams, but had failed to consider the concept of equal spacing (maybe the learners had not understood the teacher's earlier explanation of how and why to leave constant spacing between the bars), causing them to construct bar graphs that resembled histograms. '*The spaces between the bars and the width of each bar should be the same to differentiate one item from the other, although the height of the bars will be different, because of differences in frequency'*, the teacher said as a way of **correcting the learning difficulty** during the lesson.</td>
<td>Iindividual learners did classwork on bar graphs efficiently (**independent instructional strategy**) (line 9).
Learning difficulties occurred from **misconceptions** (constructing a histogram instead of a bar graph) (line 9).
Learning difficulties were identified through **analysis of their classwork** (line 9).</td>
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<td>Line 10: **Teacher B** attempted to correct the misconceptions by explaining again how to construct the bar graph on the chalkboard, while the learners watched. The teacher again demonstrated how the axes were drawn, followed by choosing the scale, labelling the axes and drawing the bars. The problem arose because the teacher had not explained the reasons for the spaces between bars at the beginning.</td>
<td>**Instructional strategy** of **again demonstrating** how to construct a bar graph was used to correct learners' misconceptions (line10). The difficulties that the learners experienced could be traceable to insufficient explanation of how to construct a graph using a **procedural knowledge** approach (line10)</td>
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<td>Line 11: Teacher B provided additional problem-solving activities based on familiar situations (ref table 4.5.2c). For example, learners were provided with a table containing the amount spent on groceries purchased from a supermarket, and were asked to draw a bar graph and to determine what percentage, of the total amount spent, the most expensive item constituted.
Table 4.5.2c: Frequency table showing the distribution of the amount spent on buying some groceries from a supermarket
<table>
<tr><td>**Item**</td><td>Tomatoes</td><td>Rice</td><td>Chicken</td><td>Maize meal</td><td>Onions</td></tr>
<tr><td>**Amount**</td><td>R10</td><td>R70</td><td>R35</td><td>R42</td><td>R3</td></tr>
</table>
</td>
<td>Problems related to a familiar situation were used by Teacher B to try to address the **learning difficulty** of drawing a histogram instead of a bar graph (line 11).</td>
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Table 4.5.2b contains items bought in a supermarket and the amount spent on each. For example, R10 was spent on buying tomatoes, R35 on buying chicken, etc.	
Line 12: Using table 4.5.2b, some of the learners tried to construct the bar graph quickly and efficiently, beginning with the labelling of the axes, choosing the scale for drawing the bar graph, labelling the vertical and horizontal axes, plotting points and drawing the bars, but a few still experienced certain difficulties, as they continued to ask why each bar should be separated from the other. This might indicate that either the learners lack the ability to understand or that the teacher's explanation was not sufficient to elicit an understanding of what had been explained.	**Learners** showed evidence of having understood the lesson on the construction of bar graph (line 12) (**construction skills** of drawing the axes, labelling axes, choosing scale, plotting the points and drawing the line of best fit). Some learners continued to experience difficulties despite the teacher's further explanation of bar graph construction (line 12). The teachers' explanation may not have sufficiently helped the learners to grasp what he had taught or the learners lacked the ability to understand the explanation (line 12).
Line 13: The lesson concluded with oral questioning. For example, the teacher asked, '*Why do we separate one bar from the other with a space?'* Homework on the construction and interpretation of bar graphs from their textbook was also given to the learners to reinforce their understanding of the construction of bar graphs. Teacher B promised to use extra tutoring to help learners who were still experiencing difficulties.	Teacher B asked oral questions and gave homework to learners on construction and interpretation of bar graphs to reinforce their understanding (line 13).
Line 14: A post-teaching discussion took place after the lesson in which some of the learners sought clarity on how to calculate the percentage of the most expensive items bought in the supermarket, which was one of the questions that had not been answered from the classwork. The teacher had to explain orally and asked the learners to complete it at home.	**Post-teaching discussion** was used to address learners' questions (line 14).
CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of ogives. Class: Grade 11**	

Line 1: Teacher B, standing in front of the class, introduces the lesson *'Today's lesson is about the construction and the interpretation of ogives'* Oral questions were directed at individual learners as in line 2.	The lesson was introduced by **oral probing questioning (instructional strategy)** (lines 2 and 4) to **identify learners' conceptions (preconception)**
Line 2: The teacher pointed to individual learners and asked them to mention ways in which data may be represented.	Teacher B identified learners who would answer the question (line 2). **(instructional strategy).**
Line 3: The learners referred to the frequency table, the bar graph, the pie chart, the histogram, the line graph, etc.	The learners' responses to the oral probing showed that they had insight into how to represent data (line 3) (**content knowledge).**
Line 4: **Learners** were referred to page 199 of their textbooks, activity 8.11, question 3, which contains the mark distribution of learners' performance in an English examination. The teacher requested the learners to:' *(a) prepare a cumulative frequency table of the learners' performance; (b) construct an ogive; (c) interpret the ogive by calculating the five-number summary (minimum, first quartile* (Q_1)*, median* (Q_2)*, third quartile* (Q_3) *and maximum value'*. Although question was set for the learners, but the teacher has to use it as an example to demonstrate how to construct and interpret ogive.	**Instructional strategy** of assessing how to construct and interpret an ogive was set for the learners and to be used to demonstrate how to do so (line 4).
a) Preparation of cumulative frequency table **Line 5a: Teacher B** demonstrated how a cumulative frequency table is constructed (see table 4.5.2c), using the first three rows of the table, and said '*add the frequency of the first and second rows to give the cumulative frequency of the second row (0 + 2 = 2 of second row). The cumulative frequency of the second row is added to the frequency of the third row to give the cumulative frequency of third row (2 + 6 = 8 of the third row), and so on'* (table 4.5.2c). He added, '*In groups of eight, complete the table by calculating the cumulative frequencies of the remaining intervals within 10 minutes.*'	**Teacher content knowledge** was used to prepare a cumulative frequency table (line 5). **Instructional strategy** to assess learners' understanding of a cumulative frequency table took the form **of group work activities** in class (**interactive instruction**) (line 4), (line 6b). Teacher's **procedural knowledge** was used to demonstrate how to prepare a frequency table (line 5a).

Table 4.5.2d: Mark distribution of learners in an English examination

Marks	Freq (f)	Cumulative frequency
1–10	0	0
11–20	2	0 + 2 = 2
21–30	6	2 + 6 = 8
31–40	7	8 + 7 = 15
41–50	14	15 + 14 = 29
51–60	20	29 + 20 = 49
61–70	35	49 + 35 = 84
71–80	29	84 + 29 = 113
81–90	6	113 + 6 = 119
91–100	1	119 + 1 = 120

Line 5b: The **learners** completed the cumulative frequency table.

a) Construction of ogive

Line 6a: **Teacher B** explained procedurally '*An ogive is constructed by drawing and labelling the axes with data on the horizontal axis and the cumulative frequencies on the vertical axis. The cumulative frequencies will help in the construction of the ogive' he said.*

Instructional skill mostly used in constructing an ogive was a **topic-specific construction skill** (lines 6a and 6bi).

A procedural knowledge approach was used (**content knowledge and instructional strategy**) to demonstrate how to construct an ogive (line 6a and 6bi).

Teacher content knowledge was utilised to provide

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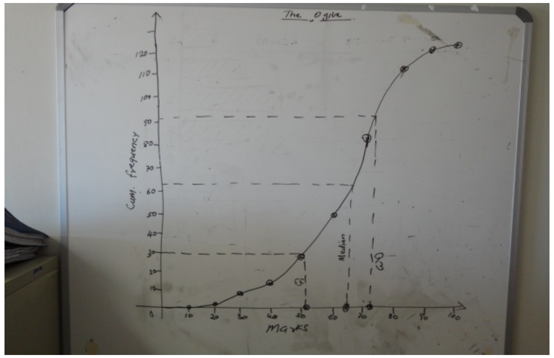

Figure 4.5.2b: Ogive representing learners' performance in an English examination

a) Line 6bi: The teacher demonstrated how to construct an ogive by plotting the cumulative frequency against the marks (e.g. 10, 0; 20, 2; and 30, 8) as indicated on the frequency table and joining the line of best fit. Afterwards the analysis and interpretation were performed using the formula for calculating the quartile positions and the **quartiles.**

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descriptions on how to plot the points from the frequency table on the axes of the ogive (teacher **content and instructional strategy)** (lines 6a and 6bi).

Interpretation of ogive by calculating the five-number-summary (minimum value, Q_1, Q_2, Q_3 and maximum values) (line 6bii) was carried out by Teacher B.

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Line 6bii: The teacher showed the learners how to calculate the position of the quartiles and said '*Using the formula* $\frac{n+1}{4}th$*, you can calculate* Q_1 *position and locate* Q_1*. Using the formula* $\frac{n+1}{2}th$*, you can calculate* Q_2 *position and locate* Q_2*. Using the formula* $\frac{3(n+1)}{4}th$*, you can calculate* Q_3*.and locate* Q_3*.*' All answers were obtained by using the position of the quartile calculated to read out the values of the five-number summary, such as minimum value = 10, Q1 = 52, Q_2 = 63, Q_3 = 73 and maximum value is 120, from the ogive.

Line 6c: A learner asked, '*Must the cumulative frequency always be on the vertical axis? Why don't you put it on the horizontal axis?*' This question showed that the learner did not understand how to label the axes of the ogive because

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Teacher B provided insufficient explanation **(PCK).** He focused on procedure at the expense of conceptual understanding. Hence learners were obliged to request further clarification about the position of the cumulative frequency on the ogive, which the teacher had not previously explained (line 6c) **(teacher content knowledge and instructional strategies).**

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the teacher had not explained this from the beginning, depicting the fact that the teacher displayed insufficient content and pedagogical knowledge to demonstrate how to label the axes of an ogive. Line 6d: **Teacher B** responded, *'You can label it on the horizontal axis, but it is more convenient to label it on the vertical axis, as you are expected to plot the cumulative frequency against the marks'* (see Figure 4.5.2b).	
Line 7a: **Teacher B** referred the learners to a photocopied exercise for classwork with a similar question in which learners were requested to prepare a frequency table, construct an ogive with the table prepared and calculate the five-number- summary (min, Q_1, Q_2, Q_3 and maximum values) from the ogive, but with class intervals starting from 20. He monitored them while they were doing their classwork. Line 7b: Most learners misunderstood the concept of labelling class intervals 0–10, 11–20, 21–30, and 31–40, etc. Instead, they labelled the class intervals on the horizontal axis 20–30, 30–40, 40–50, and 50–60, etc, instead of 10, 20, 30, etc. This approach does not allow the learners to plot the points on the data axis.	**Teacher content knowledge** was used to set classwork on ogive construction (line 7a) to ascertain how well learners have understood the lesson. **Learners' misconceptions** (line 7b) involving how to label the horizontal axis were identified through **analysis of their classwork.** The labelling could result in drawing a histogram instead of an ogive. Lack of understanding stemmed from insufficient elucidation, focusing on the procedural knowledge approach at the expense of conceptual knowledge (line 7b) (**Learning difficulty)**
Line 8a: The lack of understanding of how to label the axes was addressed by **Teacher B** in a class question and answer session (see line 8b). He also explained again how to construct an ogive, as in line 6a, and interpret the ogive, as in line 5a. Line 8b: **Teacher B** referred the learners to the diagram on the chalkboard (Figure 4.5.2b). He again explained how the ogive was interpreted by means of quartiles by using the formula $Q_1 = \frac{1}{4}(n+1)th$ for the first quartile, Q_2 =	**A rule-oriented procedural approach** was used to re-explain ogive construction (line 8b). **Teacher B** explained once more how to construct ogive and position of quartiles to reinforce learners' understanding of ogive construction and interpretation (line 8a)

$(\frac{n+1}{2})th$ for the second quartile, and $Q_3 = \frac{3}{4}(n + 1)$ for the third quartile, to calculate the first, second, and third quartile position and the first, second and third quartile. These quartiles were used to interpret the ogive by deciding the percentage of learners who passed or failed the examination by gaining a given pass mark such as the median.	
Line 9: The other strategy used to address the learning difficulties was the provision of extra-class activities in their textbooks for the learners to solve after normal school hours. Its focus was on drill and practice, using the exercises from their textbooks, in order to make the lesson more accessible and comprehensible to the learners.	**Extra-class activities** on ogive construction were given to the learners from their textbook **(instructional strategy**) (line 9) to deepen their understanding of ogive construction and address learning difficulties.
Line 10a: **Teacher B** concluded the lesson with oral questioning. For instance, **Teacher B** asked, '*What does 'n' represent in the formula for calculating the quartiles? Where can I locate the quartiles using the formula?*' **Teacher B** nominated learners to answer the questions after many of them raised their hands. Line 10b: A **learner** answered the first question by saying '*n = 120 (meaning the sum of the frequencies as in the diagram on the chalkboard)'*. A second learner indicated the answer on the vertical axis but got it wrong. A third learner explained, '*you have to trace it through the vertical axis to meet the curve, and then go down to the horizontal axis, where you have to read off the value for the quartiles, e.g. Q1 = 52'* The teacher and learners accepted the answer.	**Oral questioning** was used in addition to monitoring classwork and homework to assess how well learners had achieved the learning outcomes of the lesson (line 10a). The intention of continuous learner assessment is to ascertain how well learners have understood the teacher's elucidation of ogive construction during the lesson. (**Teacher topic-specific content knowledge and pedagogical knowledge were used to determine learners progress**) (line 10a).
Line 11: **Teacher B** gave the learners homework by referring them to the same exercise in their textbook as mentioned in line 4, as well as to photocopies of past question papers containing questions related to the construction, analysis, and interpretation of ogives.	**Homework** was used as an **instructional strategy** to assess how well learners understood the lesson on ogives and consolidate the lesson (line 11).

Line 12: After the lesson, some learners asked him how to label the horizontal axis if the class boundaries did not start from zero. The teacher explained once more to the learners one by one using the example that had previously been given in class.	**Post-teaching discussion** took place between the teacher and the learners immediately after the lesson to address the learning difficulty (line 12) (**teacher content knowledge and instructional strategy).**

Summary of lesson observation of Teacher B

From the two lessons observed, it is evident that Teacher B demonstrated his knowledge of the content of school statistics which may have been developed through formal education and teaching with the recommended textbooks and work schedule. Teacher B used appropriate topic-specific instructional skills and strategies, such as the use of examples drawn from familiar situations and a formal procedural approach in teaching the construction of the bar graph and ogive. In statistical graph construction and interpretation, measures of central tendency, knowledge of graphing involving drawing axes, choosing scale, etc, are regarded as prior knowledge. In order to identify learners' preconceptions in bar graph and ogive construction, he applied diagnostic techniques of pre-activity that focused on the preparation of a frequency table of ungrouped data and oral questioning on different ways of representing data. The learners displayed evidence of possessing previous knowledge of bar graphs and ogive constructions but with no preconception identified, depicting the fact the teacher may not have had sufficient knowledge of the learners' likely preconceptions of bar graphs and ogives.

The learners' misconceptions in drawing a histogram instead a bar graph, and the learning difficulties that emanated from these, were identified through analysis of their classwork while monitoring, checking and marking their responses to the tasks. Further explanations, extra-class activities and post-teaching discussion were provided to correct their misconceptions and learning difficulties. Teacher B's PCK is largely procedural, focusing on rules and algorithms, and is not always responsive to the needs of the learners, especially when these involve clarification of the construction of grouped data (the ogive). The frequent use of procedural knowledge may stem from the nature of the topic, which requires learners to collect, organise, construct, analyse, interpret statistical and probability model to solve related problems (DoBE, 2010) and demonstrate how graphs should be constructed (Leinhardt et al, 1990). This approach did not appear to accommodate the needs of the learners, because most of them still experienced difficulties with labelling the data axes of graphs of grouped data. Teacher B can be said to have displayed insufficient ability to elucidate concepts of ogive construction **(PCK),** focusing on procedural, at the expense of conceptual understanding.

4.5.3 School C: Lesson observation of Teacher C

In this section, the teacher’s classroom practice on teaching the construction of ogives and scatter plots is described. The condition of the classroom is described first, followed by his classroom practice in the construction, analysis, and interpretation of ogives and scatter plots.

Table 4.5.3a: Description of lesson observation and classroom conditions in Teacher C's mathematics lesson

DESCRIPTION OF LESSONS	CATEGORISATION/THEMES
Condition of the classroom Teacher C's classroom was safe and protected. The teacher had a full view of the entire class during lessons. The classroom walls were decorated with science wall charts; the furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The individual learners were resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). There were 45 learners, consisting of 26 females and 19 males, seated comfortably in twos in four columns of double chairs and desks.	1) The classroom was conducive for learning, safe and well protected. 2) There were 45 learners in the class, who were seated in double chairs in four columns. .3) The individual learners have all the necessary materials for learning statistical graphs.
CLASSROOM PRACTICE (FIRST LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of ogives. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: A histogram had been taught in the previous lesson, and learners had been given homework.	The ogive was taught (line 1). **(Teacher content knowledge).**
Line 2a: **Teacher C** and the learners marked the homework on the construction, analysis, and interpretation of the histogram. Line 2bi: To determine learners' prior knowledge of ogives, **Teacher C** asked, '*What is the difference between a class interval and a class boundary?'* Line 2bii: One of the **learners** voluntarily answered, '*A class interval and a class boundary are the same thing,*	**Oral probing questioning** to identify learners' conceptions **(preconceptions)** (lines 2bi and 2c) was used to introduce the lesson (**Instructional strategy**). **Analysis of homework** (checking if answers were right or wrong) (line 2a) was used to try to identify learners'

because both of them contain a group of numbers between them.' The question was not answered correctly, but none of the other learners volunteered to answer. Other learners, Teacher C indicated, could not provide the answer. Therefore, the **teacher** explained, using an example, '*0–10, 11–20, 21–30, etc., are class intervals. But 0–10, 10–20, 20–30, etc' are class boundaries of a prepared ogive on a photocopied exercise*.' Line 2c: **Teacher C** requested. '*indicate to me how data can be represented based on your experience'* Line 2d: **Learners** referred to the bar chart, the pie chart, scatter plots, the line graph, ogive, etc. This response indicated that learners held some conceptions about ogives, which included data representation, since they had been taught previously.	conceptions in ogive construction (**instructional strategy).** **Teacher content knowledge** was used to explain the differences between class boundaries and intervals (line 2bii). The learners displayed evidence of having previous knowledge about data representation in statistics (line 2c)
Line 3a: **Teacher C** explained the construction of the ogive procedurally, using a frequency table on a photocopied exercise containing the ages of cars, in years, in a sample of 100 car owners. Learners were also asked to interpret the ogive in terms of the five-number-summary. A five-number-summary consists of the minimum value, $Q_{1,}$ $Q_{2,}$ $Q_{3,}$ and Maximum value of the given data. Line 3b: A cumulative frequency distribution table was individually constructed by the learners, based on the teacher's instruction (ref Table 4.5.3b). For instance, Teacher C explained: '*The frequency of the first row (25) should be written under the column for cumulative frequency. The cumulative frequency of the first row is then added to the frequency of the second row (25 + 32 = 57), to get the cumulative frequency of the second row, etc'*.	**Teacher content knowledge** was used to set the example to demonstrate histogram construction (line 3a). **Learners' content knowledge** was used to complete the cumulative frequency table in a procedural manner (**instructional strategy)** following certain algorithms (lines 3a and 3b). **Teacher content knowledge in statistics (**data collection) was used to prepare a frequency table (lines 3b and 3c).

Table 4.5.3b: Table showing the ages of cars in a sample of 100 cars

Age (years)	Freq. (f)	Mid-values (x)	fx	Cum. freq.
$0 < x < 2$	25	1	25	25
$2 < x < 4$	32	3	96	57
$4 < x < 6$	20	5	100	77
$6 < x < 10$	12	8	96	89
$10 < x < 15$	7	12.5	87.5	96
$15 < x < 20$	4	17.5	70	100
	$\sum f = 100$		$\sum fx = 474.$	

Line 3c: '*Continue in the same way to calculate the remaining cumulative frequencies,'* **Teacher C** said. The learners, as shown in table 4.5.3b completed the table.

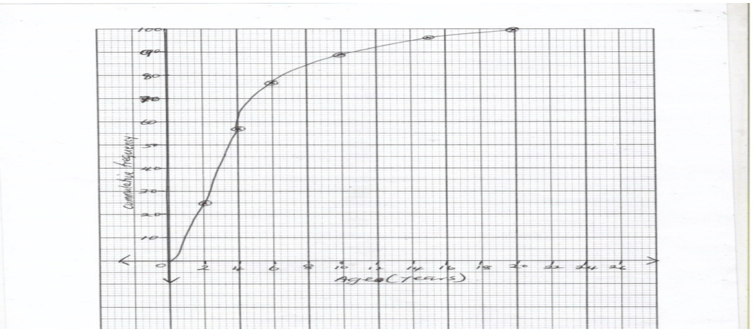
a) Construction of ogive

Line 3d: **Teacher C** used Table 4.5.3b to explain how to construct the ogive, as shown in Figure 4.5.3a. He requested, 'draw the vertical and horizontal axes, choose a scale by considering the highest and lowest value on the cumulative frequency and class boundaries'. The teacher used topic-specific algorithmic knowledge of ogive construction in his demonstration.

Instructional skills such as topic-specific construction skills (drawing of axis, choosing of scale, labelling of axes, plotting the points and joining the line of best fit) (line 3d) were used to construct the ogive.

Topic-specific teacher content knowledge and instructional strategy were used to demonstrate how to construct an ogive (line 3d) using an algorithmic approach. Thus the teacher has content and pedagogical knowledge of histogram construction.

A procedural knowledge approach was used to explain how to construct an ogive (lines 3c and 3d) (**instructional skill and strategy**)

 **Figure 4.5.3a: Ogive of age distribution of sample of 100 cars owners park in a car park**	
Line 4: **Teacher C** explained the ogive construction, using a rule-oriented approach, plotted two points and asked the learners to complete the plotting and join the lines of best fit for the ogive as part of their classwork.	**Teacher C's** use of an algorithmic approach to explain how to construct an ogive (**Teacher content knowledge and instructional strategy**) (line 4)
Line 5: **Learners** completed the ogive by plotting (6; 77), (10; 89), (15; 96) and 20; 100) and joining the line of best fit. (ref Figure 4.5.3a). But some learners were uncertain about the labelling of the data axis.	**Learner content knowledge** was used to complete the ogive (line 5) but some of the learners appeared not to have understood how the ogive was completed especially the labelling of the data axis with data from the frequency table.
Line 6a: **Teacher C** monitored the learners and offered a further explanation of the preparation of the cumulative frequency table to those who were experiencing difficulties, such as being uncertain how to label the data axis with the class boundaries provided on the table of values. He indicated, '*The cumulative frequency was used to label the cumulative frequency axis (vertical axis) and data axes on the horizontal axis*'.	Teacher C **monitored and guided** learners while they were doing their classwork **(instructional skills and strategies)** (line 6a). Insufficient explanation was provided because a procedural approach was used where a conceptual

Line 6b: A **learner** asked, *"Why do we need to add these numbers (frequencies) together?'* Line 6c: **Teacher C** answered, '*Adding the frequencies together to give the next frequency on the cumulative frequency column makes it a cumulative frequency that you are required to calculate for constructing the ogive. Cumulative means adding more numbers each time to get the next number.'* Through non-verbal cues of nodding their heads up and down, learners showed that they had understood the explanation, indicating that the conceptual knowledge approach was sufficient to enable them to comprehend how a cumulative frequency table is prepared. Teacher C demonstrated the required content knowledge of preparing a cumulative frequency table in his explanation regarding the construction of an ogive to the learners.	explanation was more appropriate **(**line 6b). **Teacher content knowledge** was used to explain how the cumulative frequencies were obtained (conceptual knowledge approach) (line 6c) (**instructional strategy**).
Line 7a: **Teacher C** observed a misconception, which resulted in drawing a histogram instead of an ogive with the given data, while he monitored and analysed the learners' responses to classwork. . Line 7b: **Teacher C** told a learner who was experiencing this misconception, '*Look, you were asked to complete the ogive we were plotting on the chalkboard and not to draw something else. Clean it off and continue with the diagram on the chalkboard by plotting the points and joining the line of best fit. For example, when cumulative frequency is 57, age is 4; when cumulative frequency is 77, and age is 6; etc*,' the teacher said.	**Misconception** of drawing a histogram instead of an ogive was identified during monitoring of classwork (line 7a). **Learning difficulties** resulting from this misconception were identified through analysis of learners' responses to classwork (line 7a). Teacher C addressed the misconception through reviewing the learners' work and instructing them to continue with plotting the points and joining the line of best fit (line 7b). (Teacher C displayed **knowledge of the topic content, instructional strategy and learning difficulty.)**

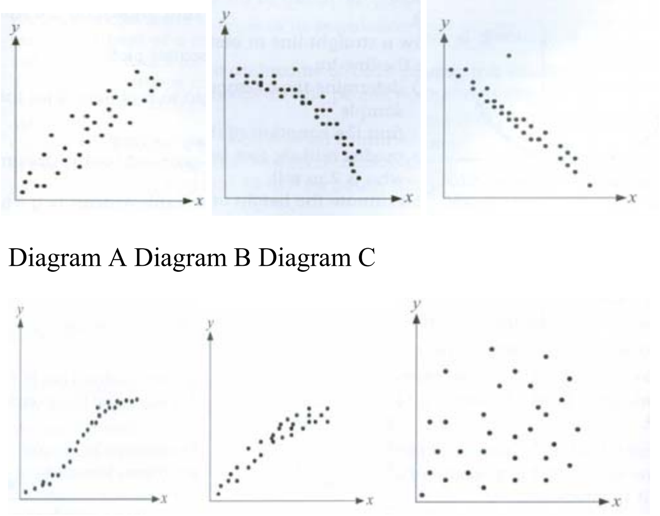
Line 8a: Referring to how the horizontal axis was labelled, a **learner** asked, '*Why do you indicate the numbers that were not on the table?*' The learner displayed a lack of knowledge of selecting a scale of given grouped data, which may not have been addressed through the procedural approach adopted by the teacher.	Teacher C identified lack of knowledge or his insufficient explanation (learning difficulty) of how to choose a scale for constructing an ogive through **oral questioning** from the learners (line 8a).
Line 9a: **Teacher C** re-explained the construction of an ogive by analysing the table of values of the cars and how they were used to construct the ogive, as in line 5. He explained, '*The numbers were not omitted , but grouped together as: $0 < x < 2$; $2 < x < 4$; $4 < x < 6$; etc. And $6 < x < 10$ contains $6 < x < 8$; $8 < x < 10$, In addition, $10 < x < 12$, $12 < x < 14$, $14 < x < 16$, $16 < x < 18$, $18 < x < 20$ is within $10 < x < 15$ and $15 < x < 20$, as indicated in the diagram. Indicating those numbers that were not on the table ensured sequential numbering of the data axis that could help in the construction and interpretation of the ogive*'. Line 9b: After plotting the points, **Teacher C** demonstrated how to join the line of best fit, which gave an S shape. He instructed learners to copy the description from the chalkboard.	**Teacher content-specific knowledge of the construction of an ogive** was used to explain how to label the horizontal axis (line 9a). **A conceptual knowledge approach** based on **teacher's content specific knowledge** of how to label graphs of grouped data was used to explain the construction of an ogive (lines 9a and 9b) (**instructional strategy**).
Line 10: **Learners** listened, and copied notes from the board. One asked, '*Does it mean that the graph of the ogive must be in the form of an S?*'	This question showed lack of understanding of the nature of an ogive. It required further clarification from the teacher from his **content knowledge** of ogive construction using a conceptual knowledge approach (line 10).
Line 11: **Teacher C** answered, '*Yes.*' He explained, *'ogive graphs are typically in an S shaped. If the constructed graph does not display this shape, then it is not an ogive or is constructed wrongly'*.	Teacher C answered learners' oral questions and provided greater clarification to reinforce comprehension of the nature of an ogive **(teacher content knowledge)** (line 11).
b) Interpretation of ogive (calculating the quartiles from an ogive)	The teacher asked how the median is calculated from grouped data as a way of determining learners'

Line 12: **Teacher C** posed this question to the learners, '*How would you calculate the median from the ogive, according to the question?'*	conception in ogive interpretation (line 12).
Line 13a: A **learner** (pointed out by Teacher C) answered, **'***you have to arrange the data in ascending order and locate the middle number. But if they are more than one number at the middle, the average of the two middle numbers is considered as the median.'* The learner quoted the wrong formula for finding the median of ungrouped data, instead of quoting the formula for finding the second quartile of a grouped data showing a lack of understanding of how to calculate median of grouped data. Line 13b: Teacher C explained the formula for calculating all the quartiles and focused on the formula for calculating the median position by indicating, '*Median (second quartile)(* $Q_{2)} = (\frac{n+1}{2})th$ *). The position of the median calculated (second quartile) was used to locate the median on the ogive*. '*Median age = 3 years'*, the teacher said. Line 13c: **Teacher C** and the learners calculated the first and third quartile from the ogive using the formulae ($Q_1 = \frac{1}{4}(n+1)th$ and $Q_3 = \frac{3(n+1)th}{4}$ to locate Q_1 and Q_3. The five number summary was i) minimum age = 1year; Q1 = 2 years; Q_2 = 3 years; Q_3 = 8 years and the maximum age = 20 years. These were all calculated and listed. But some learners appeared to be confused because they regarded the quartile position as the quartile itself. For example, the first quartile position was calculated as 25.5th position. Rather than using this position to find the value of first quartile from the data, the learners simply wrote Q1= 25,5th instead of Q1= 2. Some learners displayed a lack of understanding of how to calculate quartiles from the ogive due to the teachers' procedural knowledge description of how to calculate quartiles.	The learners showed lack of comprehension of how to calculate the median from a graph of grouped data (line 13) **An algorithmic approach** was used, in that the quartiles were calculated according to a particular procedure or formula, without explanation of the use of that algorithm **(insufficient knowledge of learners' conceptions and learning difficulties)** in calculating the median of grouped data, and the difference between calculating the medians of grouped and ungrouped data) (line 13a). **Procedural knowledge** was used to explain how to calculate the quartile's position and locate the quartile itself from the ogive **(lines 13b and 13c).** **Learners experience some difficulties** of using the quartile position to represent the quartile itself (line 13c) which may be linked to the procedural knowledge description adopted by Teacher C during the lesson on ogive construction (line 13c).

Line 14: The teacher provided the following detailed explanation of the mathematical connections between the quartile positions and how they were used to calculate the quartiles from the ogive. The teacher first explained, '*the meaning of 'n' is the number of cars in the park. The value of 'n' was obtained from the table by calculating the frequencies, and substituting the value of 'n' into the formula (in line 13b and c), you can determine the first quartile position* (Q_1)*'*. His next step was to show the mathematical connection between the quartiles position and the value of the quartile from the ogive by using the quartile positions to locate the values of the quartiles from the ogive as indicated in line 13c. Following his explanation in which he substituted 'n' into the formulae as indicated in line 13b, the quartile positions were calculated and used to locate the values Q1 = 2 years; Q_2 = 3 years; Q_3 = 8 years, from the ogive. The learners were able to use the same formula and procedure to calculate the quartile positions and the quartiles in their classwork based on the teachers' conceptual explanation.	**Teacher content knowledge** was used to show the mathematical connections between the quartile position, the quartiles and how they are utilised in interpreting the ogive (line 14) employing a **conceptual knowledge approach.** More learners understood the explanation given via a **conceptual knowledge approach** (line 14).
Line 15: Individualised teaching took the form of post-teaching discussion, so that each learner presented the areas in which he or she was still experiencing problems. The difficulties included labelling data axes and determining the median value of an ogive. The teacher provided more activities applicable to familiar situations using their mathematics textbook as a way of reinforcing learners' competency in ogive construction.	The instructional strategy of using more activities applicable to familiar situations from their mathematics textbook was used to address **learners' learning difficulties** in labelling the data axes of grouped data and determining the median of an ogive (line 15) (**knowledge of learners' learning difficulties and instructional strategy).**
Line 16: The mathematics textbook, as well as examination aids and publications of *Study mate* containing past questions in statistics and mathematics, were used by **Teacher C** to prepare and teach the construction and interpretation of the ogive, as well as to assign homework.	**Textbook and other materials** were used as sources of information for teaching ogive construction **(development of teacher's PCK in respect of content knowledge and instructional strategies) (lines** 4 and 16).

CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction and interpretation of scatter plots. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: Marking and checking homework on the construction and interpretation of scatter plots was used to start the lesson and to identify learners' knowledge or conceptions about scatter plot construction after Teacher C had greeted the class. After the marking and checking were concluded, Teacher C gave the correct answers, while the learners wrote down the corrections in their notebooks.	Teacher C used the **instructional strategy** of checking learners' homework on scatter plot construction and interpretation to try to identify their knowledge and preconceptions of scatter plot construction (line 1).
Line 2: **Teacher C** wrote the topic, '*Construction and interpretation of scatter plots*' on the chalkboard and presented a photocopied exercise containing different types of scatter diagrams to the learners.	**Teacher content knowledge** of scatter plots was utilised to indicate the topic of the lesson and set activities to ascertain learners' knowledge of scatter plot constructions (line 2)
Line 3a: The learners were asked to work in groups and to determine (by analysis and interpretation of the scatter plots) which of the scatter diagrams had a positive correlation, a negative correlation, or no correlation. They had previously been taught how to construct a scatter plot. Line 3b: Learners worked in groups to analyse the scatter plots, to determine the nature of the points plotted and the lines of best fit.	Learners worked in groups (**instructional strategy**) to analyse and interpret scatter plots as a way of identifying how well they had grasped how to construct a scatter plot from their previous lesson (lines 3a and 3b).
Line 4a: After the analysis, learners (in groups) were asked to interpret the graph by indicating their conclusions: whether the diagrams showed a positive correlation, a negative correlation, or no correlation. Line 4bi: Learners through their spokespersons for each group indicated, '*The first diagram displays a positive linear relationship.'* Another group concluded, *'the second diagram displays a graph of negative relationship, but not linear.*' Some of the groups did not seem to be satisfied with the answers presented for two of the graphs B and C.	Learner activity on data handling and interpretation by responding to class activities was undertaken in groups **(Line 4bi).** **Teacher instructional strategy** of giving and monitoring classwork on scatter graph interpretation was used to identify learner knowledge and conceptions

Line 4bii: Teacher C monitored the way in which learners were analysing and interpreting the scatter plots in groups. ‘*In terms of analysis, you were expected to know the values of Y and the corresponding value of X as used in constructing the scatter plots,*’ he said. He continued, ‘*Based on the relationship between X and Y values, one can say whether there is positive correlation, negative correlation, or no correlation* **(interpretation)** *as previously explained***.’** Recognising that some learners appeared to be experiencing difficulties in interpreting a negatively correlated scatter plot as having no correlation in interpreting the diagrams, which could indicate that they lack an understanding or the teachers’ previous lesson explanation on scatter plot construction was not sufficient to enable them to grasp what he had taught them on the topic, Teacher C further handed out another photocopied exercise showing a table of values reflecting the age and mass distribution of players in a rugby game. He asked one of the learners (who appeared to have interpreted the diagram more efficiently), *’Plot the numbers of players against the masses to construct a scatter plot. Can I see you do that on the chalkboard*?’ The learners constructed the scatter plot efficiently. But Teacher C decided again to assess learners’ conceptions in scatter plot construction (using extra-class activity) which would have aided them in interpreting the scatter plot if they had known how to construct these efficiently. Teacher C used his topic-specific content and pedagogical knowledge to assess the learners’ understanding of scatter plots using more activities on their construction in order to improve their grasp of the latter. In this activity, Teacher C plotted some points using the frequency table that he has provided on the activity on the scatter plot and requested learners to complete the remaining points. He said, ‘*let someone complete the scatter plot?’*	of scatter plots (line 4a). Learners misinterpreted a scatter plot owing to insufficient comprehension of scatter plot construction as a result of inadequate teacher explanations regarding how to determine the relationship between X and Y in a scatter plot (**learning difficulty**) (Line 4bii). A negatively correlated scatter plot was interpreted as having no correlation due to an outlier. **Teacher content knowledge** was used to explain (**instructional strategy**) the construction and interpretation of a scatter plot (lines 4di and 4dii).
Line 4c: More learners volunteered and they were requested individually to plot other points on the graph using the table provided by the teacher on the chalkboard, while the other learners watched.	**A procedural approach** of drawing the axes, choosing scale, labelling axes, plotting the points and drawing the line of best fit was used to describe and complete the scatter plot (line 4di).
Line 4di: Teacher C completed the graph that the learners had been plotting, and explained algorithmically how to construct a scatter plot. He then analysed it by reading the value on the vertical axis and the corresponding value on the horizontal or data axis. ‘*From this analysis, the meaning of what the graph intended to convey about the*	**Graph construction skills (**drawing axes, choosing scale, labelling axes, plotting the points and joining the

relationship between the number of players and their masses (correlation or no correlation) was determined', the teacher said. Line 4dii: Some of the learners seemed dissatisfied, as they shook their heads. More explanations were offered by **Teacher C,** who utilised **a conceptual approach** to again demonstrate scatter plot construction and interpretation using the classwork. For instance, Teacher C explained; '*The characteristics (nature of points and shape of line of best fit) of a linear positive correlation with its line of best fit moves from right to left through the origin, and related it to diagrams A and E of Figure 4.5.3b. In a linear negative correlation the line of best fit drops down from the vertical axis to the horizontal axis, as in diagrams B and C, Figure 4.5.3b. And a scatter plot with no correlation has all the points spread through the vertical to the horizontal axis as in diagram F, Figure 4.5.3b*'. '*Diagram D shows a positive correlation, but it is not linear because the points spread through the origin from right to left, but not in a straight line,'* the teacher concluded  Diagram A Diagram B Diagram C Diagram D Diagram E Diagram F **Figure 4.5.3b: Scatter diagrams showing different kinds of correlation between X and Y**	line of best fit) were used to create a scatter plot (line 4di). Teacher C provided further explanation (**using conceptual knowledge**) to address learners' difficulties, showing that he has insight into learners' learning difficulties,; hence the strategy he adopted to provide clarification and reinforce understanding (line 4dii)

Line 5a: A learner asked, '*Do we need to draw the line to show how the two variables X and Y are correlated?'* This question demanded a conceptual explanation which was provided in line 4dii, but the learner may have developed certain misconceptions about drawing the line of best fit in a scatter plot from the earlier procedural explanation which led to a lack of understanding of why such a line has to be drawn based on the nature of the points plotted, to determine the relationship between X and Y'. Another misconception was, '*There were no lines of best fit in Figure 4.5.3b which they had worked on earlier*', the learner indicated. The learner had posed a legitimate question seeking clarification because the teacher simply did not provide a conceptual explanation for the different scatter plots as indicated in the introductory exercise for the lesson and in line 4di. Line 5b: Teacher C answered, '*Yes*' and repeated what he had said in line 4dii by explaining the characteristics of scatter plots, how their correlation can be determine and how they relate to each other as in the diagrams in Figure 4.5.3b.	This learner's question displayed a lack of understanding of how to construct and interpret scatter plots─precisely because of inadequate explanation, using learned rules to explain. The question is how does the teacher makes the leap from the algorithmic to the conceptually meaningful explanation (line 5a).
Line 6: Teacher C observed that in the graphs the learners analysed in groups, they misinterpreted diagram C (Figure 4.5.3b. For example, diagram C was interpreted as a graph with no correlation between X and Y, owing to outliers (the point or points that are farthest from the line of best fit). '*Using one point alone to indicate that diagram C had no correlation may not be adequate as there are other clustered points that would display the correlation between X and Y*,' the teacher explained. This was a misconception of using the nature and shape of a scatter plot with no correlation to interpret a graph of negative linear correlation. In addition, some learners indicated in their exercise book that the line of best fit meant a change in X caused by a change in Y, as in a line graph, which means if Y increases, then X increases by the same percentage. '*Yes, when X increases, Y also increases, which means X and Y are related,*' one of the learners indicated. In a scatter diagram, *'The line of best fit only indicates the association or connection between X and Y, as indicated in diagrams A and B,'* the teacher explained. He continued, '*And depending on how clustered the points are close to the line of best fit, one can say that it is strong, moderate of weak correlation.*' As indicated earlier, *'You were expected to analyse and interpret the scatter plots to determine the relationship between X and Y,'* he emphasised.	**Teacher content knowledge** was used to address learners' misinterpretation of scatter plot (line 6) by explaining why diagram C could not be adjudged to have a negative correlation. A more conceptual explanation was provided of how to describe the relationship between X and Y in a scatter plot and indicate the kind of correlation that the scatter plot is showing (line 6).
Line 7: Teacher C corrected the misconception of using the characteristics of a scatter plot with no correlation to	The **topic-specific content and instructional strategy**

<table>
<tr>
<td>interpret a scatter plot with a negative linear correlation, as well as interpreting a linear scatter plot as if it were a line graph, as in lines 5 and 6, and diagram C of Figure 4.5.3b. He provided more activities on scatter plots and photocopied activities on their construction and interpretation of scatter plots. For example, he said, ‘<i>In this exercise, you were required to construct a scatter plot and indicate the relationship between test 1 and test 2 (see Table 4.5.3c below). The data in the frequency table give the marks (out of 20) that 12 learners attained in the two tests</i>’.

Line 8: Teacher C gave out the classwork as shown below.

Table 4.5.3c: Frequency table showing the distribution of learners’ performance in two tests

<table>
<tr><td>learner</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td><td>L</td></tr>
<tr><td>Test 1</td><td>10</td><td>18</td><td>13</td><td>7</td><td>6</td><td>8</td><td>5</td><td>12</td><td>15</td><td>15</td><td>10</td><td>20</td></tr>
<tr><td>Test 2</td><td>12</td><td>20</td><td>11</td><td>18</td><td>9</td><td>6</td><td>6</td><td>12</td><td>13</td><td>17</td><td>10</td><td>19</td></tr>
</table>
a) Draw a scatter plot and describe by means of two examples whether there is a positive or a negative correlation in the learners’ performance in the tests.
b) How do you account for the outliers, if any?</td>
<td>of providing more examples was used to address the learners’ misconceptions concerning outliers and interpreting a linear correlated scatter plot as if it were a line graph (line 7). Topic-specific content and pedagogical knowledge was utilised to address learners’ misconceptions.</td>
</tr>
<tr>
<td>Line 9: As he monitored the learners’ doing the first classwork, he discovered that some of them did the classwork efficiently. He gave a second classwork activity involving a frequency table of the age distribution of persons infected with HIV/Aids in two towns. They were to work on their own individually to construct a scatter plot showing the relationship between the age distributions of persons infected with HIV/AIDS in the two towns. The objective of using several activities on scatter plots constructions was to identify and correct any difficulties or errors related to the construction and interpretation of scatter plots and reinforce learners’ grasp of scatter plot construction.</td>
<td>Instructional strategy of using real-life context based examples to assess learners’ conceptual understanding of the construction and interpretation of scatter plots and address their learning difficulties (line 9). Several class activities were used to reinforce learners’ grasp of how to construct and interpret scatter plot (line 9)</td>
</tr>
</table>

Line 10: Learners carried out the exercise individually. A few still experienced difficulties in drawing the line of best fit and determining the type of correlation.	An individualised or independent learning strategy/approach was used to evaluate how well learners had learned the construction of a scatter plot (line 10).
Line 11: After the classwork, oral questioning, and homework (as in line 8), were made use of by Teacher C to further assess learning. For instance, he asked a learner, '*What is an outlier?*' '*An outlier is a data value or point that lies apart from the rest of the data'*, the learner replied. Teacher C adjudged the learner to be correct and instructed the learners to answer other questions on the photocopied exercise as homework.	**Oral questioning** and the homework assignment comprised the instructional strategy used to assess how well learners had grasped the concept of constructing scatter plots (line 11).
Line 12: At the end of the lesson, some learners asked more questions about the work that they did, especially the misinterpretation of a negative linear scatter plot and interpreting the line of best fit in scatter plot as if it were a linear algebraic graph. Teacher C held individual discussions with a few learners about diagram C, and asked the others to see him after school the following day.	**Teacher content knowledge and instructional strategy was used to** clarify the misinterpretation of a negative linear scatter plot and interpreting the line of best fit as if it is an algebraic linear graph **in a post-teaching discussion (**responding to learners' oral and written questions after lesson) and various examples (line 12).

Summary of lesson observation of Teacher C

The way in which Teacher C taught his lessons on the ogive and scatter plot showed that he possessed the subject matter content knowledge of school statistics. He utilised recommended statistics and statistics-related textbooks and materials (mathematics study guides) to teach statistical graphs such as the ogive and scatter plot. He demonstrated his subject matter content knowledge by describing how the ogive and scatter plot should be constructed, by adopting an approach that emphasised procedural knowledge and application of formulae, rather than conceptual knowledge. For example, the teacher made greater use of algorithms by slotting values into equations for calculating quartiles without eliciting clear comprehension of the relationships of concepts in the equations. At times he did not provide adequate explanation, and merely repeated the procedures for arriving at an answer when the learner experienced misconceptions and learning difficulties in interpreting an ogive using the calculated quartile positions. Having said that, the teacher used his conceptual knowledge, for instance on how to teach ogive and scatter plot construction, especially when learners encountered misconceptions and learning difficulties such as drawing a histogram instead of an ogive, being unable to label the data axis because of incorrect scaling, and not knowing the distinction between quartile position and quartile value to teach ogive and scatter plots. While the teacher used his procedural knowledge to explain in a step-by-step manner how ogive and scatter plots are constructed, he employed his conceptual knowledge to demonstrate the mathematical connections between quartile positions and to utilise the calculated quartile position to work out the quartile value from the ogive in order to provide the meaning or information that the ogive conveys (interpretation). For example, while the quartile position for Q_1 was calculated to be 25.5^{th}, Q_1 value from ogive was found to be, $Q_1 = 2$.

Concerning the instructional knowledge component of his assumed PCK in data handling, Teacher C used appropriate topic-specific scatter plot construction skills of drawing the axes, choosing the scale, labelling of axes, plotting the points and joining the lines of best fit to make data-handling lessons on ogives and scatter plots accessible to more learners. Post-activity and post-teaching discussions were among the instructional strategies he used to address errors and construction difficulties, etc, in ogives and scatter plots. He applied the required diagnostic techniques of oral probing / questioning, checking and marking of homework at the beginning of the lesson to try to identify learners' prior knowledge about ogive and scatter plot construction. Teacher C identified learners' previous knowledge instead of preconceptions which could indicate that the teacher may not have possessed sufficient knowledge of learners' preconceptions in ogives and scatter plots constructions. The lack of sufficient knowledge of learners' preconception which could have been used to address any anticipated learning difficulties during lesson planning and implementation may have further created room for learners to develop some misconceptions and such learning difficulties as an inability to label data axis, constructing a histogram instead of an ogive and misinterpreting a negative correlated scatter plot as having no correlation. These misconceptions in using content knowledge about algebraic line graph construction to interpret the line of best fit of a scatter plot and learners' inability to label the data axis were identified through analysis of their responses to classwork and homework, and pre- and post-teaching discussions~~.~~ Teacher C provided additional class activities and individualised teaching, post-teaching discussion on the classwork, and further elucidation on scatter plots immediately after the lesson in order to correct any remaining misconceptions and learning difficulties.

From the analysis of the lesson observations of Teacher C, it appears that his PCK was more frequently a procedural approach to teaching, and less often a conceptual approach. The frequent use of procedural knowledge may be a result of the nature of the topic, which requires learners to be able to collect, organise, construct, analyse, and interpret statistical and probability models to solve related problems (DoBE, 2010) and to demonstrate the construction skills of graphs in statistics (Leinhardt et al, 1990). Following this sequence, the teacher may have decided to use his procedural knowledge to teach the construction and interpretation of ogive and scatter plots. On the other hand, the teacher adapted his conceptual knowledge to explain the construction and interpretation of ogives, especially when learners experienced misconceptions and learning difficulties. For example, when some of them misinterpreted a negative linear scatter plot as having no correlation because of an outlier, the teacher explained the meaning and nature of the scatter plot and its line of best fit, which can be used to determine the extent of the correlation (strong, moderate, weak or no correlation) (ref Second lesson observation, line 6). The mathematical connection between calculating the quartile position and using the calculated position to locate the quartile in an ogive was explained conceptually to the learners when they could not distinguish between them during his lesson on ogive construction that involved a procedural approach (line 14).

While the teacher can be said to comprehend learners' learning difficulties by identifying problem areas through the analysis of learners' classwork, homework and from pre-and post-teaching discussion, as well as addressing the difficulties using familiar context-based examples, his knowledge of learners' conceptions may have been developed through the use of oral questioning, checking and marking of learners' homework to assess learners' conceptions in ogive and scatter plot construction.

4.5.4 School D: Lesson observation of Teacher D Grade 11

This section describes briefly the teacher's classroom practice on the teaching of the construction of bar graph and histogram. The condition of the classroom is described first. It is followed by a description of the teachers' classroom practice in the implementation of the planned lesson on the construction and interpretation of bar graph and histogram.

Topic: Construction, analysis, and interpretation of bar graphs

Table 4.5.4a: Description of lesson observation and classroom conditions in School D

<table>
<tr><th>DESCRIPTION OF LESSONS</th><th>CATEGORISATION/THEMES</th></tr>
<tr><td><u>Condition of the classroom</u>

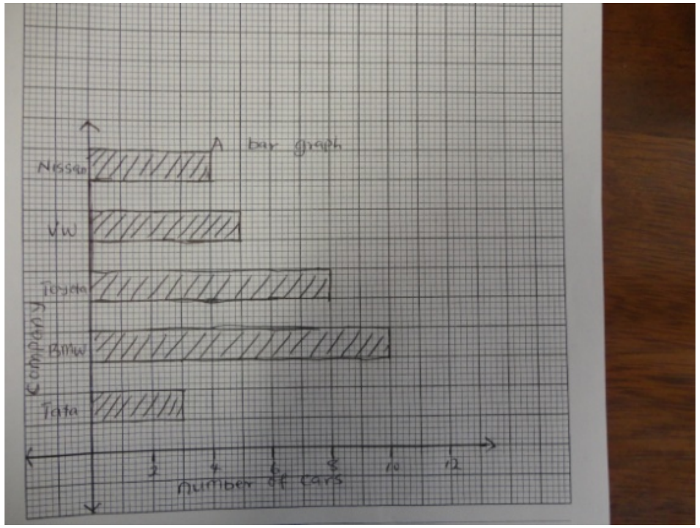
There are 17 male and 23 female learners of mixed ability. Forty learners are seated comfortably in twos in four columns of double chairs and desks. The teacher had a full view of the entire class during lessons. The classroom walls were decorated with science wall charts; the furniture, windows and door were in good condition, with electrical wiring that permitted the use of appliances such as an overhead projector. The individual learners were resourced with textbooks, calculators, exercise books, and graph sheets for each learner, as well as construction instruments for the teacher (ruler, protractor, and pair of dividers). The classroom presented a conducive learning environment, with locks, keys, and burglar bars for supervised entry</td><td>1) The classroom is safe and conducive to teaching and learning.
2) The individual learners were resourced with learning materials.
3) There were forty learners in the class.</td></tr>
<tr><td>CLASSROOM PRACTICE (FIRST LESSON OBSERVATION)

Topic: Construction and interpretation of bar graphs. Class: Grade 11</td><td>CATEGORISATION/THEMES</td></tr>
<tr><td>Line 1: Teacher D introduced the lesson on bar graphs after greeting the class with a pre-activity exercise in which learners were asked to individually prepare a frequency table (shown below) of raw data about the number of cars in a car park manufactured by different companies.

Table 4.5.4bi: Table showing the number of makes of cars in a car park

<table>
<tr><th>Company</th><th>Nissan</th><th>VW</th><th>Toyota</th><th>BMW</th><th>Tata</th></tr>
<tr><td>Number of cars</td><td>4</td><td>5</td><td>8</td><td>10</td><td>3</td></tr>
</table>
</td><td>Teacher D utilised a learner pre-activity exercise of frequency table preparation, which he regards as important for successful bar graph construction, to try to identify learners’ prior knowledge or conceptions (preconception) about bar graphs(line 1) (teacher content specific knowledge and instructional strategy)</td></tr>
</table>

a) Definition of bar graph Line 2a: The learners prepared a frequency table as displayed in table 4.5.4bi. Line 2b: **Teacher D** defined and described a bar graph orally and wrote it on the chalkboard: '*It is a statistical graph used in representing data in the form of a bar. A bar graph is used for representing discrete data. When a bar graph is used to represent information, you can easily see the information physically and understand how one discrete piece of data is different from another. A bar graph can be represented vertically or horizontally.*' The next step was for the teacher to demonstrate how a bar graph is constructed.. **Construction of bar graph** Line 2c: **Teacher D described this on the chalkboard as follows: '***You draw vertical and horizontal axes and labelled them (the horizontal axis represents the frequencies, and the vertica axis represents the companies). The scale of the horizontal axis were determine by considering the lowest and the highest value of the number of cars and appropriately labelling the horizontal axis with names of the companies*. In learners' mother tongue, he said, ***labella ga ke go bontsha**,* meaning '*Watch me as I demonstrate it'*. Teacher D continued**, '** *For the first bar Tata, the frequency is 3; For the second bar, the frequency is 10; for the third bar, the frequency is 8 etc.'*	**Learners** showed evidence of knowing how to prepare a frequency table as they had been taught it previously **(line 2a).** **Teacher content knowledge** was used to define and explain bar graph construction and its uses (line 2b). **Instructional skills** such as **construction skill** involving the drawing of the axes, choosing of scale, labelling of axes, plotting of points, and joining the line of best fit were utilised in constructing a bar graph (line 2c). Teacher D taught a bar graph using a procedural knowledge approach (line 2c) (**content knowledge and instructional strategy).** **Graph construction skills of** drawing the axes, choosing scale, labelling axes, plotting points, and joining the line of best fit were used to construct a bar graph (line 2c). **The learner's mother tongue** was used to direct the learners' attention to the lesson and reinforce their comprehension of the material (line 4b) (**instructional strategy**) (line 2c).

 **Figure 4.5.4a: Bar graph showing the numbers of makes of cars in a car park**	
a) Interpretation of bar graph Line 3: **Teacher D** drew the bar graph, as in Figure 4.5.4a, and interpreted it by indicating that Tata was the least frequent make of car in the car park, while BMW was the most frequent. The second most frequent was Toyota.	**Teacher content knowledge** was made use of to interpret the bar graph (line 3).
Line 4a: **Teacher D** asked, '*Why do you think the most frequent make of car in the car park was BMW?'* Learners answered one by one and gave the following answers: *'BMW produce the most popular cars.' 'BMW produce prestigious cars,' 'BMW produce cars of high quality,' etc.* Line 4b: **Teacher D** further answered the question, *'BMW produced the highest number of cars in the car park.*' In their mother tongue he said*,* '*ke mang a sahlaloganyeng,* meaning *'who does not understand the explanation?*'	**Oral questioning (instructional strategy)** was used to probe learners' views about the most frequent make of car (line 4a). **Open-ended questions that called for reasoning and analytical skills** (line 4a). Reasoning skills were employed to arouse interest and focus the learners' minds on the construction and interpretation of the bar graph.

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b) Classwork

Line 5a: Teacher D set the learners an activity to solve individually. It involved a table of values of the distribution of marks obtained by 50 learners in a class test. Learners were asked to construct the bar graph and calculate the percentage of learners who failed the test if the pass mark was 5 out of 10 or 50%.

Table 4.5.4bii: Frequency table showing the mark distribution of learners in a class test

<table>
<tr><td>Marks</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Frequency</td><td>3</td><td>1</td><td>2</td><td>7</td><td>10</td><td>12</td><td>9</td><td>3</td><td>2</td><td>1</td></tr>
</table>

Line 5b: **Learners** solved the question individually by constructing the bar graph and determining the percentage of learners that had failed the test.

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Instructional strategy was to set an activity on bar graph construction which learners had to solve individually (line 5a)

Learners solve activity of bar graph individually as a way of assessing how well they have understood what the teacher taught them (line 5b).

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Line 6: While the **teacher** monitored how the learners were progressing in their classwork, he offered additional explanations for labelling the axes and drawing the bars. For instance, one of the **learners** asked, '*Why do you leave equal spaces between the bars when the companies produce a different number of cars in the car park?'*

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Learners asked the teacher to explain why there should be constant spaces between the bars, meaning that they did not understand this from the earlier explanation that the teacher had provided using **procedural knowledge (line 6). Learning difficulty** of their lack of understanding of the construction of bar graph was discovered by Teacher D.

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Line 7a: **Teacher D** explained: '*All the companies manufacture cars only, but of different makes, hence they have to be separated by equal spacing by choosing appropriate scale, which differentiates one make of car from another. The difference in height of the bars is because of the difference in the number of cars produced. In terms of your classwork, the differences in the height of the bars are as a result of the number of students which correspond to the marks they scored,' the teacher said.* A conceptual knowledge was used to explain the frequencies, the cars manufactured and while there should be constant spacing between the bars.

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Teacher content knowledge was used to explain why there should be constant spacing between the bars (line 7a).

Teacher used **conceptual knowledge** requiring the drawing of the bars with constant spacing based on the company and the scale that was chosen for constructing the graph and the differences in height resulting from the varying frequencies of

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</table>

Line 7b: Learners showed evidence of a grasp of the lesson as they constructed the bar graph more efficiently, especially after the teacher demonstrated how to construct the bar graph using a conceptual knowledge approach.	the cars manufactured by each company (line 7a). Learners demonstrated evidence of a grasp of the lesson as they constructed the graph more efficiently (line 7b).
Line 8: **Teacher D** identified learners' difficulty in constructing bar graphs during the monitoring of the learners while they are doing classwork, such as unequal spacing between the bars (as most learners think this merely indicates the space and bars without considering the sizes),. For example, while most **learners** used the space between the first bar and the horizontal axis to determine the spaces between the other bars, some did not consider the consistency of the spacing between the bars, irrespective of the size of the space between the first bar and the horizontal axis, as in Figure 4.5.4a. Some **learners** drew the bar graph with different spacing between the bars, and others drew histograms instead of bar graphs.	Teacher D identified **misconceptions,** involving drawing a histogram instead of a bar graph through not considering the spacing between the bars, during the examination of their work on bar graph construction (line 8). Another misconception concerns the inconsistency in spacing and sizes of the bars (line 8).
Line 9: These misconceptions (as stated above) in which learners drew a histogram instead of a bar graph and drew the bars without considering the size of the latter were addressed by **Teacher D** through extra explanations to individual learners-as well as by compulsory additional activities from the textbook which the learners did in class individually.	Extra elucidation on how to construct and interpret a bar graph, especially with respect to the drawing of the histogram instead of a bar graph and inconsistency of spacing between the bars, was offered on a one-on-one basis to correct the misconceptions and learning difficulties (line 9). Extra class activities was given to the learners' to deepen their understanding of bar graph construction (line 9).
Line 10a: During the lesson, **Teacher D** repeated what he said in line 2b and 2c and provided further explanations on the meaning of a bar graph, construction of bar graph with emphasis on the space between the bars drawn according to scale, the size of the bars and the consistency of the space between the bars individually to some learners who were experiencing difficulties. For example, one of the **learners** whose classwork had been marked wrong, because she had constructed a histogram instead of a bar graph, requested clarity as to why her answer was wrong. Line 10b: **Teacher D** stated that the learner had not left spaces between the bars, as explained in the example on the	**Conceptual knowledge** was used to explain the meaning of a bar graph and how it can be constructed by considering the frequency and drawing the bars with appropriate scale. How the scaling affected the consistency of the spaces between the bars and sizes of the bars, and the learners' misconceptions and learning difficulties (inconsistency of spaces between the bars and sizes of the bars) (Line 10b) were addressed. **Teacher used**

chalkboard. The spaces between the bars in a bar graph help to differentiate between categories of data (companies) and must be equal because we are dealing with cars, though of different makes (categories). '*In a bar graph, there should be a constant spacing between the bars and the sizes of the bars must be the same,'* he said. Line 10c: The learner nodded her head in agreement with the teacher's explanation, as explained in line 10b. Teacher D corrected the classwork, and the learners wrote down the corrections in their class workbook.	**content knowledge and instructional strategy** to explain conceptually the construction and interpretation of bar graph (line 10b) to the learners. **Teacher content knowledge** was used to address learners' misconceptions and learning difficulties using teacher's conceptual knowledge (line 10b)
Line 11: At the end of the lesson, the **learners** were given homework from the school supplementary textbook,	**A supplementary mathematics textbook** was used as a source of information for teaching bar graph and assigning homework (line 11).
CLASSROOM PRACTICE (SECOND LESSON OBSERVATION) **Topic: Construction, analysis, and interpretation of histograms. Class: Grade 11**	**CATEGORISATION/THEMES**
Line 1: After greeting the class, **Teacher D** began the lesson on histogram construction by checking and marking homework on the construction and interpretation of stem-and-leaf diagrams. The teacher and learners provided corrections to the homework so that learners who experienced difficulties could correct their mistakes. While providing the corrections, **Teacher D** explained once more how a stem-and-leaf diagram is constructed by arranging the leaves in the right-hand column and the stem in the left-hand column. '*Just as the stem-and-leaf diagram is used to represent group data, the histogram we are about to study now is also used to represent grouped data,'* he added.	Knowledge of stem and leaf diagrams is regarded by the teacher as an important part of learner's prior knowledge before the histogram can be successfully taught to learners, Checking learners' homework on the construction and interpretation of stem-and-leaf diagrams was used as an **instructional strategy** to introduce the lesson and to determine **learners' background knowledge or conceptions** in histogram construction (line 1) (**teacher's PCK**).

Line 2: **Learners** did corrections, which were written on one side of the chalkboard by Teacher D, while he wrote the new topic on the other side of the chalkboard.	Teacher C wrote the new topic while learners corrected their mistakes in their homework (line 2).
Line 3: **Teacher D** presented a photocopy of an activity on the construction of a histogram representing the mass of each player in a 2003 South African rugby squad. The masses of the 30 players were: 115, 122, 110, 110, 105, 112, 80, 98, 90, 93, 85, 87, 99, 84, 112, 76, 96, 128, 110, 108, 118, 105, 108, 118, 90, 89, 90, 88, 103, and 85 kg. The activity requests the learners to: a) prepare a frequency table of the data presented with a class of 10; b) use the frequency table to construct a histogram; and c) determine from the histogram (i) the mean; (ii) interval that has the highest frequency; (iii) percentage of players whose weight fell between 110 and 120 kg and (iv) the mode.	**Instructional strategy** of using photocopied material to provide a source of information for lesson activity was used to set exemplar questions to demonstrate the construction and interpretation of histogram (line 3).
a) Preparation of frequency table Line 4: **Learners** were instructed to prepare a frequency table by calculating the frequencies of each interval. The class boundaries, mid-values, and fx were later calculated to help in answering question (b) and (c), as normally done if the need arises, or based on the questions in the learners' activities (see table 4.5.4c), and to calculate the measures of central tendency (the mean, and the mode) that best describe the masses of the players. The instruction presupposed that learners knew how to prepare a frequency table; hence class boundaries, mid-value and fx were not explained.	**Teacher D** instructed learners to prepare a frequency table from the raw data presented (line 4) (**Instructional strategy**).
Line 5a: The frequency table was constructed by the teacher and the learners. While Teacher D wrote down the frequencies, learners counted the masses within each interval. The mid-values were calculated by finding the average of the upper and lower class of each class interval while fx was calculated by finding the product of mid-value (x) and frequencies (f) of the individual classes, row by row.	**Teacher content knowledge** on the preparation of frequency tables was used to create a frequency table, and to explain how to prepare the frequency table of grouped data by grouping the data according to class; also to determine the frequency as well the class boundaries, mid-values and fx and calculating measures of central tendency, as indicated in questions (b) and (c) (line 5a).

Table 4.5.4c: Frequency table showing the masses of players in the 2003 South African rugby squad

Class intervals	Class boundaries	Freq. (f)	Mid-values (x)	Fx
70-79	70-80	1	75	75
80-89	80-90	6	85	510
90-99	90-100	7	95	665
100-109	100-110	5	105	525
110-119	110-120	9	115	1035
120-129	120-130	2	125	250
		∑f = 30		∑fx = 3060

Line 5b: **Teacher D** defined and described a histogram orally and wrote it down on the chalkboard as indicated in the textbook: '*A histogram is a graphical representation, showing a visual impression of the distribution of grouped data. It consists of tabular frequencies shown as adjacent rectangular bars, erected over discrete intervals, with an area equal to the frequency of the observations in the interval. Unlike the bar graph, a histogram is used to represent a large set of data (e.g. a population census) visually, but with no spaces between the bars,*' the teacher said. After the explanation, he referred to the frequency table and indicated the usefulness of the table in the construction of the histogram beginning with the class boundaries, followed by the frequencies. He thereafter began to demonstrate how to construct the histogram.

Teacher procedural knowledge was used in preparing the frequency table with learners (line 5a).

Procedural knowledge was utilised to describe how a histogram should be constructed, an approach that the teacher felt would make the histogram more accessible to the learners (teacher topic-specific **content knowledge and instructional strategy)** (line 5c).

Teacher content knowledge was used to explain the usefulness of the frequency table in constructing a histogram, beginning with the class boundaries, and followed by the frequencies (line 5b).

Topic-specific **construction skills** of drawing the axes, choosing scale, labelling axes, plotting the points and joining the line of best fit were used to construct a histogram (**instructional skill)** (line 5c).

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a) Construction of a histogram

Line 5c: **Teacher D** illustrated the histogram construction visually using procedural knowledge by drawing the vertical and horizontal axes and labelling them using a scale chosen by the teacher by considering the lowest and highest values of the frequencies, with the vertical axis representing the frequencies, and the horizontal axis representing the masses on the chalkboard. He drew two bars of the histogram and instructed learners to complete it according to the class boundaries and frequencies.

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Line 6a: **Learners** completed the histogram individually in their workbooks while Teacher D monitored and examined their responses. Most of the learners who had correctly completed the table drew a histogram, as shown in Figure 4.5.4b. Other learners who had not drawn their histogram correctly because of incorrect scaling and labelling of the horizontal axis, among other errors, and also because of lack of comprehension, corrected their mistakes by copying the correct diagram presented on the chalkboard. Some learners drew bar graphs instead of histograms by leaving spaces between the bars. The difficulties experienced in scaling could have arisen because at the beginning of the activity the teacher did not describe and explain how to choose a scale for constructing a graph of grouped data.

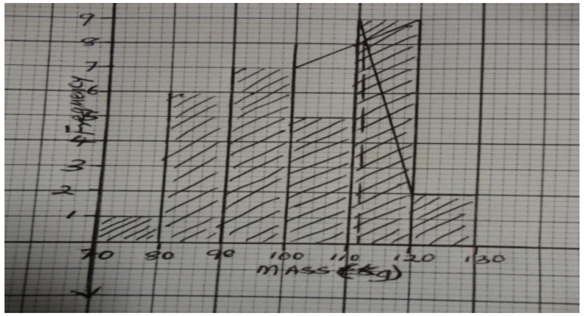

Figure 4.5.4b: Histogram showing the distribution of the masses of players in a 2003 South African rugby squad

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Learning difficulties of drawing a bar graph instead of a histogram were identified through analysis of learners' responses to classwork (line 6a).

Insufficient teacher explanation (pedagogical knowledge) of choosing the scale for constructing a histogram with a procedural approach led to learners constructing a bar graph instead of histogram (line 6a).

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Line 6b: After completing the histogram, **Teacher D** answered follow-up questions (see line 3) such as question (c) which requested the learners to determine (i) the mean; (ii) the interval that had the highest frequency; (iii) the percentage of players whose weight fell between 110 kg and 120 kg and (iv) the mode from the histogram. **c) (i) Calculating the mean** The mean was calculated: '$Mean = \frac{\sum fx}{\sum f} = \frac{3060}{30}$ = *102 kg*'; the teacher said	Learners drew a bar graph instead of a histogram (line 6a) **(misconception).** **Teacher's procedural knowledge was** used in calculating the mean (line 6b). This was done by substituting the values in the equation: $\frac{\sum fx}{\sum f} = \frac{3060}{30}$ **= 102kg**
ii) **Identifying the interval with highest frequency** Line 7a: **Teacher D** analysed the histogram (in which learners determined which interval (110–119) kg had the highest frequency, and which intervals had the next highest frequencies (90–100) kg). He then wrote the answer, 'The class with the highest frequency is 110–119 kg'. '*Any question about how we determine the class that has the highest frequency*?' he asked. As there was no question from the learners, he answered the next question about the percentage of learners that fail the test. iii) The % of players that fall within (110–120) kg = $\frac{9}{30}$ x $\frac{100}{1}$ = 30% From Figure 4.5.4b, it was determined that the individual mass of most of the players (9 out of 30) in the squad fell between 110 kg and 120 kg, which formed 30% of the players in the squad.	**Teacher content knowledge** was used to analyse and interpret the histogram (line 7a), demonstrating the application of **analytical and interpretational skills** by calculating the class with the highest frequency. **Procedural knowledge** was made use of to demonstrate how to determine mode from a histogram (line7b) (**instructional strategy**).

iv) Calculating mode from a histogram Line 7b**: Teacher D** then determined the mode using procedural knowledge by drawing a diagonal line from the top-right corner of the highest bar to the top-right corner of the bar next to it on the left-hand side, and drawing a diagonal line from the top-left corner of the highest bar to the top-left corner of the bar next to it on the right-hand side (as in case A). A line was drawn from the meeting point of the two diagonals down to the horizontal axis to locate the mode. '*After the identification of the interval where the mode will be located, the diagonal lines help to locate the mode within the class interval*,' the teacher said. '*By drawing a line from the point of intersection of the diagonals, the mode was located as 113kg (see Figure 4.5.4b),*' he added.	**Teacher content knowledge and instructional knowledge** were employed to demonstrate how to determine the mode from the histogram by drawing intersecting diagonals and using the point of intersection to locate the mode (line 7b).
Line 8: After rule-oriented procedural knowledge was used to demonstrate how to calculate the mode from the histogram, learners were given time to write the explanation of how the mode was calculated from the histogram that Teacher D had written on the chalkboard into their workbooks. "*Now you can write down the explanation I have given on the chalkboard into your workbooks*," the teacher said.	**Learners** wrote down in their notebooks what the teacher had explained as he instructed them. Teacher's **instructional knowledge** was used to provide time for the learners to write down the explanation given by him on how to calculate the mode.
Line 9: Classwork based on construction and interpretation of bar graphwas then given to the learners to solve individually from their supplementary textbook. Learners had to complete their classwork in their workbooks at home, as they were not able to complete it by the end of the lesson period.	**A supplementary recommended mathematics textbook** was employed as a source of information for teaching histograms (line 9). Using a classwork (line 9) assignment for feedback was part of the teacher's instructional **strategy** during the lesson.
Line 10: When the lesson was about to end and learners were still busy doing the classwork; a **learner** enquired (referring to Figure 4.5.4b), *'why it was necessary to label the horizontal axis from 70, and not from 0, as was done on the vertical axis?'* This question demanded a conceptual knowledge approach, which was provided in line 11.	A **misconception** was identified through oral questioning from the learners on the labelling of the data axis (line 10).
Line 11: **Teacher D** replied that, '*One labels the horizontal axis from 70, because 70 is the lowest value on the table. In addition, a scale of 1cm = 10 units was used to label the data axis. Therefore, if you begin from 0, all the values as indicated on the table of values will not be accommodated on the graph paper provided*,' he added. Alternatively, '*One*	**Teacher's conceptual knowledge** was used to clarify the reason that it was necessary to start labelling the horizontal axes

can label from 0 and make a continuation line between the 0 and 70. The continuation line indicates that the intervals below 70 have been omitted so that the graph can be contained on the graph paper,' the teacher said. A related example was drawn from the same supplementary mathematics textbook.	from 70 (line 11). **Teacher content knowledge and instructional strategy** were applied to explain conceptually why it was not necessary to start labelling from zero as a result of the scale of 1cm = 10 units, which was chosen because of the dimensions of the graph paper (line 11).
Line 12: More **learners** seemed to be satisfied with the teacher's explanation by using a conceptual knowledge approach as explained in line 11. They nodded their heads, while a few others were still experiencing difficulties and shook their heads which may be as a result of lack of understanding due to inadequate explanation regarding why the labelling of the data axis has to start with 70 and not 0 .	While some learners indicated that they were satisfied with the teachers' explanation, others felt that the teacher had not cleared up the difficulty (line 12). Insufficient **teacher content knowledge** was made use of to address learners' difficulties in labelling the data axis correctly (line 12).
Line 13: **Teacher D** gave them homework and promised to organise extra tutoring after normal school hours, where he would try to explain once more how to construct, analyse, and interpret a histogram using activities related to everyday life.	**The instructional strategy** of employing homework (line 13) to assess how well learners understood the lesson was adopted during the lesson. **Extra tutoring** was also proposed for helping learners with difficulties.

Summary of lesson observation of Teacher D

Teacher D demonstrated aspects of procedural knowledge of the topics of bar graph and histogram construction. He combined appropriate pedagogical knowledge of teaching bar graphs and histograms with a rule-oriented procedural and conceptual knowledge approach. The content knowledge of bar graph and histogram construction used for teaching the observed lessons was both procedural and conceptual, but mostly procedural. For example, Teacher D demonstrated procedurally how bar graphs and histograms are constructed using the construction skills of drawing the axes, and choosing a scale by considering the lowest and highest values of the data and frequencies as well as the dimension of the graph paper provided. The next step was to plot the points and draw the line of best fit (ref Section 5.5.4, first lesson observation and line 2c; second lesson observation, and line 5c). In terms of his conceptual knowledge, he explained how histograms should be constructed with a scale, even when data values do not start from zero, so that the values can be accommodated on the graph paper provided (refSection 4.5.4, second lesson observation, and line 11) when he discovered that the learners were experiencing some difficulties.

At the beginning of the lesson, Teacher D used his pedagogical knowledge of instructional skills and strategies to try to identify learners' preconceptions by giving them a pre-activity on the preparation of a frequency table, and by checking and marking their homework on stem-and-leaf diagrams. Through the pre-activity, learners demonstrated that they had mastered the concept of preparing a frequency table of ungrouped data and of constructing bar graphs because they had been taught these in the past (ref Section 4.5.4, first lesson observation line 1). But checking and marking learners' homework on stem-and-leaf diagrams revealed that some learners had experienced difficulties that could have been the results of inadequate explanation or of lack of comprehension by the learners (ref Section 4.5.4, second lesson observation, and line 1). These difficulties were corrected before the new lesson began. In the lesson observed, Teacher D knows that stem-and-leaf diagrams are necessary for histogram construction. There is no evidence in his lessons that he knows of the misconceptions his students are likely to have of bar graph and histogram construction. Hence, he can be said to have provided poor and inadequate explanations that resulted in certain learning difficulties. This is possibly understandable because the topic of data handling is a new one. Learners' misconceptions and learning difficulties were identified through marking and analysing the learners' classwork, as well as through oral questioning, where learners could request clarification of what they did not understand about determining the mode from a histogram. These misconceptions and learning difficulties were not adequately addressed through individual problem-solving class activities and further explanations on the construction and interpretation of bar graphs and histograms, because some learners continued to experience difficulties. For example, when the lesson was about to end and learners were doing the classwork, a learner enquired (referring to Figure 4.5.4b), *'Why is it necessary to label the horizontal axis from 70, and not from 0, as was done on the vertical axis?'* (ref Section 4.5.4, second lesson observation, and line 10). **Teacher D** replied that, '*One labels the horizontal axis from 70, because 70 is the lowest value on the table. In addition, a scale of 1cm = 10 units was used to label the data axis. Therefore, if you begin from 0, all the values as indicated on the table of values will not be accommodated on the graph paper provided*,' he added. Alternatively, '*One can label from 0 and make a continuation line between the 0 and 70. The continuation line indicates that the intervals below 70 have been omitted so that the graph can be contained on the graph paper*,' the teacher said. A few learners shook their heads to indicate that they had not understood the explanation. Teacher D probably does not command sufficient content and pedagogical knowledge to address learners' misconceptions and learning difficulties effectively in this respect.

4.6 Video recordings of lesson observation of the four teachers

The video recordings of the four participating teachers confirmed the teaching of the construction and interpretation of bar graphs, histograms, ogives, box-and-whisker plots, and scatter plots during lesson observations (see Section 4.5.1–4.5.4). The video recordings were also used to triangulate the written notes taken during classroom observations.

4.7 Teacher development of PCK

4.7.1 Teacher development of subject matter content knowledge

In the interviews, the teachers claimed that they had studied mathematics and general method courses at university, which helped them to adapt the way they taught school statistics (ref Appendix XVII, items 1, 2 and 3) by employing appropriate instructional skills and strategies to teach statistical graphs. For instance, when they were asked, "If one of the courses you studied at university is mathematics methodology, how did it help you to prepare for your lessons for teaching?” Teacher A indicated that the method course he had studied helped him to vary his instructional strategies (ref Appendix XVII, item 5a). Teacher B asserted, “The mathematics method courses help me to vary formulae and strategies for teaching statistics.” Teacher C averred that the courses had helped him to prepare his lessons in line with the objectives of the lessons. And Teacher D said the courses helped him to plan his lessons in line with the work schedules, assessment and evaluation of his lessons.

The participating teachers were further requested to indicate how they knew that their teaching in statistics was effective, as a way of establishing whether the contents of statistics lessons are adequately delivered by teachers with content knowledge of statistics. Teacher A claimed that through analysis of the learners’ responses to classwork, homework, and assignments, he knew that his lessons were effective (ref Appendix XVII, item 8). Teachers B, C and D said virtually the same thing, which means that the teachers may have demonstrated the content knowledge of school statistics which they possess during their lessons.

To further ascertain how the participating teachers gained their content knowledge for teaching, they were asked, “Have you attended a mathematics workshop or teacher development programme?” and also, "as a mathematics teacher, did you benefit from the workshop?” Teachers A, B and C responded that they had attended workshops on data handling (the new topic in the curriculum) and learnt how to teach challenging topics in this respect. Teacher D responded: “Yes, I attended many workshops on teacher development in content knowledge especially in data handling. I did not benefit much because I was taught what I already know in mathematics”, which could mean that Teacher D became more aware that he already possessed the required content knowledge for the subject he was teaching.

From the above analysis, the teachers can be said to have developed their content knowledge in statistics teaching through formal education, which gave them the opportunity to study mathematics and the methodology of teaching and enabled them to design instructional strategies for carrying out effective teaching. Through classroom practice, lesson planning and preparation, and content knowledge workshops, they gained further content knowledge.
The teacher portfolios and concept mapping exercise confirmed that the teachers possess the content knowledge of school statistics as they listed the subject matter content of school

statistics to be taught in a sequential and logical manner (ref Appendix XXI, teachers' portfolios; Section 4.4).

In addition to listing the content of school statistics, the participating teachers taught statistical graphs using both procedural and conceptual knowledge approaches following the learning outcome of data handling as stipulated in curriculum (DoBE, 2012) and how graphing concepts should be taught (Flockton et al, 2004; Leinhardt et al, 1990) in their lessons on statistical graphs. Using topic-specific content knowledge and instructional skill (construction skill) of drawing the axes, choosing of scale, labelling of the axes, plotting the points and joining of the line of best fit, Teacher A for instance, demonstrated procedurally how to construct a histogram (ref Section 4.5.1, first lesson observation, and line 9). While some learners displayed evidence of grasp of their lesson, a few experienced some learning difficulties (ref Section 4.5.1, first lesson observation, and line 11) which resulted in the teacher adopting a conceptual knowledge approach to assist learners who are experiencing some difficulties (ref Section 4.5.1, first lesson observation, line 16). Thus, the participating teachers can be said to have mastered the content of school statistics which they developed through formal education and classroom practice, and demonstrated it by teaching with procedural and conceptual knowledge approaches, using recommended textbooks, a work schedule and by attending content-driven knowledge workshops.

4.7.2 Teacher development of pedagogical knowledge (instructional skills and strategies)

The focus of this section was to determine the instructional skills and strategies that the participating teachers utilised in teaching school statistics. The teacher questionnaire, lesson observation, written reports and documents analysis were used to collect data to ascertain the teachers' pedagogical knowledge in statistics teaching. The purpose of the questionnaire was to establish what the teachers actually did while teaching assigned topics in school statistics and to determine the pedagogical knowledge (instructional skills and strategies) they possess and use in teaching school statistics.

In their responses to the questionnaire (ref Appendix XVIII), the teachers claimed they had achieved the objectives of their lessons, in which learners are expected to construct, analyse and interpret statistical graphs, and apply the knowledge to everyday real life situations according to the learning outcomes of data handling (DoBE, 2010). This means that the teachers applied content and pedagogical knowledge that was adequate to elicit understanding of school statistics. For example, they were asked, “Do you think that the learners achieved the objective of the lesson and if not, what do you do to improve their understanding?” to establish what strategies they adopted and how good these strategies were (ref Appendix XVIII, item 7). All four teachers claimed they knew that the objectives of their lessons had been achieved through active participation of learners in their lessons, and responses to classwork, homework, assignments, tests, and examinations in statistics (ref Appendix XVIII, item 7). Teacher A tried to engage the learners in extensive class discussions to improve their understanding of statistical graphs, while Teacher B used teaching aids such as statistical charts and an overhead projector to display statistical diagrams. Teacher C indicated that he made use of extra class activities related to real life to improve learners’ understanding of the lessons, whereas Teacher D claimed that he used additional examples and past questions in tests to improve learners’ understanding of statistical graphs.

From the responses of the four teachers to the questionnaire, it can be understood that they gained their pedagogical knowledge through classroom practice, which involved planning and presentation of lessons, as well as using classwork, homework, exams and assignments, to assess how well learners understood the lessons on statistical graphs. The participating teachers taught statistical graphs with instructional strategies which they felt could help learners to understand the topics and learners responded positively to classwork, homework and assignments. They also claimed to have used class activities related to familiar real life and problem solving on past test questions in statistics to help learners improve their understanding of statistical graphs. The lesson observation, teacher written reports, and document analysis confirmed that the teachers used class activities related to familiar real life situations, problem solving in the form of drill and practice, as well as employing classwork, homework and assignments to assess how well learners had understood the lessons on statistical graphs. For example, during the lesson observation on scatter plot construction, Teacher C made use of the age distribution of persons infected with HIV/AIDS in two towns (familiar real life situation) as classwork to assess how well the learners understood his lesson on the construction and interpretation of scatter plot (ref Section 4.5.3, second lesson observations, and line 9). The teachers also utilised both procedural and conceptual knowledge approaches in teaching statistical graphs (ref Section 4.5.4, first lesson observation, line 2c and 7a). In the teacher's written report, Teacher D indicated that he tackled learners' learning difficulties by adopting different teaching approaches and providing additional class activities related to real life (ref Appendix XX, item 6).

In the learners' notebooks (ref Appendix XXI, learner workbooks) there are examples of statistical graphs, calculations and exercises related to the concepts they were taught according to the procedures for constructing statistical graphs, indicating also that the teachers may have used a procedural knowledge approach. For example, the workbooks of learners in Teacher A's class displayed diagrams of histograms constructed as examples by the teacher and others done as classwork by drawing the axes, labelling the axes based on a given scale, plotting points, and drawing lines of best fit (ref Appendix XXI, learner workbooks). Teachers B, C and D's learner workbooks (ref Appendix XXI, learner workbooks) contained similar records of examples in which a procedural knowledge approach may have been used for teaching statistical graphs. The conceptual knowledge was used less frequently to assist learners that were experiencing some learning difficulties (ref Section 4.5.3, second lesson observation, and line 4dii). All four teachers made use of classwork, homework and assignments as well as the SBA to assess how well learners understood the lessons on statistical graphs. The assessment tasks appeared to be similar because the four participating teachers used the same assessment guidelines, work schedules and textbooks as recommended by the Department of Basic Education (ref Appendix XXI, teacher and learners' portfolios) for teaching Grade 11 mathematics. Learners' recorded examples from extra lessons (ref Appendix XXI, learner workbooks) indicating that the teachers must have individually conducted extra tutoring to help learners who experience learning difficulties (inability to choose scale of grouped data) in order to deepen their understanding of data handling.

From the above discussion, it is evident that the participating teachers used predominantly a procedural knowledge approach and to some extent a conceptual knowledge approach, construction skills, extra tutoring, examples drawn from familiar real life situation, additional class exercises in the form of drill and practice in the teaching of statistical graphs. By doing so, the teachers may have developed more knowledge of the instructional skills and strategies for teaching school statistics.

4.7.3 Teacher development of knowledge of learners' preconception and learning difficulties

A teacher questionnaire, lesson observation, written reports and documents analysis were used to investigate whether the teachers had knowledge of learners' preconceptions and misconceptions, if any, as well as of learning difficulties about statistical graphs such as bar graphs, histograms, ogives, and scatter plots. The investigation revealed that despite many years of teaching experience held by the participating teachers, they possessed no knowledge of learners' preconceptions in statistical graphs. For instance, in the questionnaire, they were asked, "What prior knowledge does your lesson require?" Teachers A and D claimed that learners need measures of central tendency as prior knowledge for bar graphs, histograms and ogives construction (ref Appendix XIX, item 4). Teacher B said that learners need simple addition and subtraction skills, as well as measures of central tendency as prior knowledge for bar graph and ogive construction. Teacher C asserted that learners need to understand measures of central tendency and know how to interpret information from straight-line graphs as prior knowledge for scatter plot and ogive construction. All their responses indicated that they had acquired previous knowledge about the topics they were teaching. But what was needed was the knowledge the learners had before they were taught the concept of statistical graph (preconception). It means that the instructional strategies adopted by the teachers could not elicit learners' preconceptions of the various topics they taught depicting the fact that the teachers have no knowledge of learners' preconceptions in statistics teaching.

The teachers were also asked, "How did you identify the prior knowledge (preconceptions) about statistical graphs with which the learners came to the class?" Teachers A and C claimed that they used probing questioning to establish if learners had gained prior knowledge of measures of central tendency linked to histograms, ogives and scatter plot construction (ref Appendix XIX, item 4–6). This was confirmed in the lesson observation of Teacher A (ref Section 4.5.1, of the first lesson observation, and line 1) in which learners mentioned mode, median and mean when the teacher attempted to probe their preconceptions of histogram construction. Teacher B claimed that he determined their prior knowledge in statistical graphs constructions while correcting their responses to homework and using pre-activities related to the topic he was going to teach (ref Appendix XIX, item 6). This was confirmed in the observation of a bar graph construction lesson given by Teacher B (ref Section 4.5.2), of the first lesson observation, and line 1) in which learners used knowledge of simple addition to prepare a frequency table in a pre-activity in ungrouped data. Learners also mentioned different ways of representing data as prior knowledge for ogive construction. Teacher D claimed that he made use of pre-activities and probing questions to determine prior knowledge in statistical graph constructions such as bar graphs and histograms (ref Appendix XIX, item 6).

This employment of pre-activities and oral probing questions was confirmed in the lesson observation of Teacher D (ref Section 4.5.4, of the first lesson observation, and line 1) who used pre-activities, and checking and marking learners' homework, to attempt to identify learners' preconceptions of bar graphs and histogram construction.

From the responses of the participating teachers to the questionnaire, it appears that they have used topic-specific instructional strategies such as asking oral probing questions, checking and marking learners' homework, and utilising pre-activities at the beginning of the lessons to try to identify learners' prior knowledge in the topics taught in statistical graphs. By employing these strategies, all four teachers could have been adjudged to have demonstrated that they knew about the learners' possible preconceptions and were therefore able to decide which instructional strategy was best to elicit the prior knowledge that was essential for the learning of the new concepts. But the strategies only elicited learners' previous knowledge and not the preconceptions, which means the teachers possess no knowledge of the learners' preconceptions. The teachers' written reports and documents analysis confirmed that the participating teachers tried to identify learners' prior knowledge in statistical graphs using diagnostics techniques such as oral probing questioning, pre-activities as well as checking and marking of learners' homework (ref Appendix XIX, items 8 and 9; Appendix XXI, teacher portfolios).

Regarding the learners' misconceptions and learning difficulties, all the participating teachers adopted monitoring and analysis of learners' responses to classwork to identify any misconception and learning difficulty that the latter may experience during their lessons on statistical graphs. As noted in their responses to the interview (ref Appendix XX, item 14), the learners' learning difficulties range from basic computations of mode, median and mean of grouped data (as in the case of teacher A), to choosing of the scale for constructing graphs of grouped data (for Teachers B and C), and determining the mid-points of graphs of grouped data. From the teachers' responses to the questionnaire, while Teachers A and C addressed the difficulties by giving learners additional exercises in graphs of grouped data, Teacher B did so by specifically teaching the learners how to choose different scales for different data for the sake of uniformity in graph construction. Teacher B tackled the learners' difficulties in graphs of grouped data by giving them additional examples and possibly repeating the lesson in order to reinforce learners' understanding of statistical graphs. The teachers were further asked, "What is it about statistics that makes it easy or difficult?" Teachers A and B said that measures of central tendency represent an easy concept to learn. Teacher C commented that relating statistics to real life makes it lively, interesting, and easy to learn. Teacher D said that statistics is easy to learn if someone who is knowledgeable presents the topic. Therefore, teacher content knowledge of a topic should be adequate in order to make the teaching of statistics comprehensible and accessible to the learners.

In the document analysis, misconceptions such as drawing a histogram instead of a bar graph, as in the case of Teacher B, and drawing a bar graph instead of histogram, as in the cases of Teacher A, C and D, were addressed individually through extra tutoring, extra class activities and post-teaching discussions in statistical graphs (ref Appendix XX1, teacher portfolios) during and after school hours.

The lesson observations and the teacher written reports confirmed that the teachers identified learners' misconceptions and learning difficulties by monitoring and analysis of learners' responses to classwork, homework and assignments in statistical graphs and addressing the misconceptions and learning difficulties by extra tutoring, teaching learners how to choose scale, re-demonstrating or repeating the lessons, extra class activities and post-teaching discussions in statistical graphs. For example, the learners' misconception of drawing a histogram instead of an ogive (ref Section 4.5.2, second lesson observation, and line 7a) and the learning difficulty emanating from the misconceptions of interpreting a negatively correlated scatter plot as having no correlation due to an outlier (ref Section 4.5.3, second lesson observation, and line 4bii) were identified during the monitoring and analysis of learners' responses to classwork by Teachers B and C on ogive and scatter plots respectively (ref Appendix XX, items 1 and 2). The misconceptions and learning difficulties were addressed by post-teaching discussion (ref Section 4.5.3, second lesson observation, and line 12) and extra class activities in the form of drill and practice (ref 4.5.2, first lesson observation, and line 15).

From the above analysis, it can be concluded that the individual participating teachers developed their knowledge of learning difficulties through analysing and monitoring learners' responses to classwork, homework and assignments to identify learners' learning difficulties in statistical graphs. The teachers also extended their knowledge of these difficulties by addressing the difficulties using additional tutoring, extra class activities, post-teaching discussions, re-teaching, and further explanation of the lessons they taught, individually to learners during and after the lessons.

4.7.4 Teacher development of PCK in statistics teaching

By summing the ways through which the participating teachers developed the subject matter content knowledge, pedagogical knowledge and knowledge of learners' preconceptions and learning difficulties, one would be able to determine how the participating teachers developed their PCK in statistics teaching. In section 4.7.1, it was deduced that the participating teachers possess the content of school statistics which they acquired through formal education, and demonstrated it by employing procedural and conceptual knowledge approaches, using recommended textbooks, devising a work schedule and by attending content-driven knowledge workshops. In section 4.7.2, it was discovered that the participating teachers utilised both procedural and conceptual knowledge approaches, construction skills, extra tutoring, examples drawn from familiar real life situation, and additional class exercises in the form of drill and practice in the teaching of statistical graphs. By employing these instructional skills and strategies for teaching statistical graphs, the teachers may have developed more knowledge of the instructional skills and strategies for teaching school statistics. And in section 4.7.3, the individual participating teachers developed their knowledge of learning difficulties through analysing and monitoring learners' responses to classwork, homework and assignments to identify such difficulties in statistical graphs. The teachers may have also developed further knowledge of these difficulties by tackling these using additional tutoring, extra class activities, post-teaching discussions, re-teaching, and further explanation of the lessons they taught, individually to learners during and after the lessons.

4.8 Summary of chapter

In this chapter, the data collected with the instruments mentioned in section 4.1 were presented and analyse in order to determine how the participating teachers developed their assumed PCK in statistics teaching. The results of the qualitative data collected with the conceptual knowledge exercise and concept mapping were analyse in order to select the participants for the second phase of the research and determine the teachers' content knowledge of the statistics curriculum respectively. The lesson observations of the four participating teachers were analysed and discussed in detail in order to tease out how they demonstrate the PCK they have during classroom practice. The video records were used to triangulate the data collected during the lesson observations. The teacher interview, questionnaire, written reports and documents analyses were analysed by categorising the responses of the participating teachers according to the theme of the study. The chapter concluded with a highlight of how the teachers developed their assumed PCK were determined with a summation of their subject matter content knowledge, pedagogical knowledge and knowledge of learners' preconceptions and learning difficulties.

CHAPTER 5

5.0 DISCUSSION OF RESULTS

5.1 Introduction

The results of the study are discussed in this chapter. The similarities and differences in the ways in which the participating teachers develop their pedagogical content knowledge (PCK) in teaching statistics are examined.

The discussion begins by highlighting the research questions about teaching school statistics. The following four components of PCK were used as the theoretical framework: (1) subject matter content knowledge, (2) pedagogical knowledge (instructional skills and strategies), (3) learners' conceptions (preconceptions and misconceptions), and (4) individual learning difficulties in the topics investigated. Pedagogical content knowledge in statistics teaching represents a category of knowledge that teachers need to have assimilated in order to teach the subject effectively.

These research questions were:

1 What subject matter content knowledge of statistics do mathematics teachers who are considered to be competent have and demonstrate during classroom practice?
2 What instructional skills and strategies do these teachers use in teaching statistics?
3 What knowledge of learners' preconceptions and learning difficulties, if any, do they have and demonstrate during classroom practice?
4 How do these teachers develop their PCK in statistics teaching?

Components (1) and (2) above were used to answer research questions 1 and 2. In the third component, the learners' preconceptions and learning difficulties were identified and discussed in order to understand how the teachers acquired their knowledge in teaching statistics. The fourth research question was discussed as an amalgam of the key findings for the other PCK components.

The assumed PCK profiles of the participating teachers were examined in order to determine the similarities and differences, if any, in the ways in which the teachers develop their PCK in school statistics teaching.

The chapter concludes with a detailed discussion of how the results of the study provide insight into the way in which teachers who are reputed to be competent in teaching school mathematics develop their PCK in school statistics and evaluation of the theoretical framework.

5.2 Teacher development of PCK

5.2.1 Teacher A

Teacher A was observed teaching histogram construction and box-and-whisker plots in a step-wise fashion (ref Section 4.5.1: first lesson observation, and line 9; second lesson observation, and line 5a), using the recommended mathematics textbooks and work schedule. He started the lesson by asking the learners to name orally components of measures of central tendency such as modes, medians and means of ungrouped data (ref Section 4.5.1, first lesson observation, and line 1) in an attempt to determine their prior knowledge of histogram construction. The components of measures of central tendency having been identified, the teacher and learners prepared a frequency table from the raw data (ref Section 4.5.1, first lesson observation and line 4a). Using this table, the histogram was constructed by first drawing its horizontal and vertical axes. The axes were labelled with data values on the horizontal axis, and frequencies on the vertical axis. A scale was chosen by the teacher, who stated that the highest and lowest values of the frequencies and data values, as well as the dimensions of the graph paper provided, had been considered (ref Table 4.5.1a). Next, the bars of the histogram were drawn by joining the line of best fit (ref Figure 4.5.1a). Teacher A’s lesson showed that he had adopted a rule-oriented procedural approach to teaching histogram construction.

In teaching the construction of histograms, he gave further evidence of using more procedural knowledge, focusing primarily on rules and algorithms, than conceptual knowledge. The procedural approach requires simply plugging the data into the appropriate formulae, and then working out the correct values of the quartiles for the box-and-whisker plots (ref Section 4.5.1, second lesson observation, and line 4). The most challenging aspect for this teacher was knowing how to move from an algorithmic stage to a conceptually meaningful one as far as the students’ learning was concerned.

However, he used a conceptual teaching approach during the lesson and demonstrated the mathematical connections and relationships between ogives and box-and-whisker plots by describing how quartiles were obtained from the ogive and used in the construction of the box-and-whisker plot (ref Section 4.5.1, second lesson observation, and line 8cii). The relationships between the ogive and box-and-whisker plot, the calculation of the first, second, and third quartiles, and the description of the number line on which the box-and-whisker was drawn, with its mathematical connections, were elucidated during his lesson. A conceptual-based instructional approach endeavours to provide the reasons that make algorithms and formulae work (Peal, 2010). The emphasis is placed on the learners' understanding of the relationships and connections between important statistical concepts such as the use of quartiles to construct the box-and whisker plots on a number line (ref Figure 4.5.1c). Overall, Teacher A implemented more of a rule-oriented procedural knowledge approach in teaching histogram and box-and-whisker plot construction than a conceptual one. What can be surmised from this is that he did use both knowledge approaches except, of course, that one was dominant.

Interestingly enough, through the non-verbal cue of nodding their heads, the learners seemed to grasp the lesson on histogram construction through the use of conceptual knowledge better than when Teacher A adopted a rule-oriented approach. This observation was illustrated by the fact the learners were able to answer questions involving recall and application of procedures posed by him in order to assess how well they had understood the lesson on histogram construction. In answering the question how do you calculate the percentage of learners in the age group of 26–40?, learners first of all calculated the number of learners, divided by 27 and multiplied the result by 100 to get the percentage of learners within that age group. (ref Section 4.5.1, first lesson observation, and line 20). In the explanation, based on his conceptual knowledge, he demonstrated his PCK in a manner that enhanced learners' comprehension of histogram and box-and-whisker plot construction.

During the lesson, a few of the learners experienced learning difficulties such as being uncertain about choosing a scale for labelling the data axis of the histogram (ref Section 4.5.1, first lesson observation, and line 11). The teacher identified such difficulties as being due to lack of comprehension on the part of the learners (ref Section 4.5.1, first lesson observation, and line 22a).

Teacher A's preference for the use of procedural knowledge in teaching histograms was confirmed in the learners' workbooks (document analysis). It was discovered that the learners had written down the teacher's rules or steps on how to construct histograms and box-and-whisker plots, as well as the diagrams of histogram and box-and-whisker plot (ref appendix 21, learner workbooks). Teacher A might have adopted the use of procedural knowledge because the construction of histograms, which demands that specific procedural rules must be followed, is consistent with a conceptual understanding of the term. In a study conducted by Flockton, Crooks and Gilmore (2004) and Leinhardt et al (1990) on graphing, they stress that the construction of graphs requires the sequence of drawing the axes, choosing the scale, labelling the axes, plotting the points, and joining the lines of best fit. The order of steps, in the case of Teacher A, demonstrated the knowledge and skills required for histogram construction.

As observed, the learners experienced learning difficulties, particularly in labelling the data axis with incorrect scale, which could mean that he possibly presented his lesson in a limited way, that is, solely procedurally, without providing the reasons underlying these procedures and clarifying the relationship between concepts (a conceptual knowledge approach) in histogram construction (ref Section 4.5.1, first lesson observation, and line 12a). The teacher omitted a detailed description of how to choose a scale of given data before labelling the data axis. He merely stated the scale and used it to demonstrate the construction of a histogram. During classwork, the learners tried to draw a histogram, which could not be accommodated on the graph paper provided because they scaled the data axis incorrectly (ref Section 4.5.1, Figure 4.5.1c).

It may be said that Teacher A's PCK in terms of subject matter content knowledge presentation did not always reveal the required variety of ways of presenting the data handling topics to his learners for ease of access. In some instances, he demonstrated the use of both procedural and conceptual knowledge in teaching histograms and box-and-whisker plots, but he predominantly used a set of algorithms to demonstrate graph construction. In the main lesson on histogram and box-and-whisker plots, he displayed factual knowledge, procedural proficiency and conceptual understanding of the data handling topics that were taught.

Gersten and Benjamin (2012) note that the use of different strategies for teaching mathematics helps to anchor the learners behaviourally and mathematically, avoids possible learning difficulties, and achieves effective learning. This finding conforms with the suggestion being made here, based on this study's results, that teachers' flexibility or the ability to use a variety of instructional approaches (both conceptual and procedural knowledge) should make data-handling concepts (which are said to be difficult for learners to grasp) more meaningful and accessible to more learners. Teacher A can thus be said to have possessed and demonstrated the required knowledge of histogram and box-and-whisker plot construction.

Grouping method was also used as an instructional strategy for teaching the construction of ogive by teacher A in order to provide interactive engagement , collaborative learning and to ensure sustainability of interest in learning statistics among the learners. Learners work in groups of four to five to calculate the quartiles of an ogive for constructing box-and-whisker plots. The use of grouping method to sustain learners' interest in learners was given an empirical support by Adodo and Agbayewa (2011) who report that effective classroom lesson is achieved using grouping method for teaching. Adodo and Agbayewa (2011) further noted that grouping method allows the teacher to better tailor the pace and content of instruction to learners' ability level and needs and easy management of the classroom is achieve especially in the homogeneous grouping which teacher A adopted.

With regard to his pedagogy, Teacher A often used examples that are familiar to learners for teaching data handling. Using the mark distribution of learners' performances in an English examination, he described in a step-by-step fashion how ogives are constructed, and how quartiles are obtained and used to construct the box-and-whisker plot (ref Section 4.5.1: second lesson observation, and line 8a). The use of familiar examples and contexts by Teacher A is consistent with the approaches used by other workers to make the topic more meaningful and accessible (Ball & Bass, 2000; Meletiou-Mavrotheris & Stylianou, 2002). For example, Meletiou-Mavrotheris and Stylianou (2002) used familiar situations as examples in the context of teaching statistics in order to improve learner access and comprehension. According to these researchers, the teaching of rules alone (algorithmic teaching) does not always convey meaningful relationships between the mathematics knowledge taught in class and daily life situations (Meletiou-Mavrotheris & Stylianou, 2002).

So this disconnects with and is seen to obscure the relevance of statistics teaching and mathematics education in general.

Teacher A's knowledge of learners' preconceptions of the statistics lessons observed was derived largely from what transpired in the classrooms, notably through his analysis of learners' responses to teacher classroom questions (oral probing questioning and pre-activities) and classwork or assignments. During the lessons, Teacher A was able to identify some of these difficulties or inaccurate conceptions – such as the learners' inability to select appropriate scales for labelling the data axis of the histogram correctly through monitoring learner activity and questioning (ref Section 4.5.1, first lesson observation, and line 21; Figure 4.5.1c).

In the lessons observed, for instance, the teacher did not display evidence of anticipating learners' potential difficulties with any of the topics. The teacher went into the lessons without necessarily having prior knowledge or expectations of the type and nature of learning difficulties that his learners were likely to have in teaching histogram construction. For example, at the beginning of the lesson on histogram construction, Teacher A requested learners to define mode, median and mean. The learners did so efficiently, based on knowledge that they had been taught (ref Section 4.5.1, first lesson observation, and line 2). Thus the teacher detected learners' previous knowledge instead of preconceptions. Since the teacher could not identify their preconceptions of histogram construction, learners were likely to experience misconceptions and learning difficulties such as constructing a bar graph instead of a histogram because of their poor background in scaling. Teacher A can therefore be said to have displayed insufficient PCK in terms of the knowledge of learners' preconceptions of histogram and box-and-whisker plot construction.

Teacher A could have addressed possible learning difficulties before or during the lesson if he had had sufficient knowledge of learners' preconceptions of histogram construction. When asked in the questionnaire about his expectations of learners' difficulties, he said merely that there were no major problems, but he would deal with these when the learners asked him (ref Appendix xx, item 10). The insufficiency or inadequacy of his PCK in terms of his insight into learners' preconceptions was a knowledge deficit that was common to all the four teachers that were studied. The finding justifies further investigation into the reasons that teachers, in spite of many years of teaching experience, do not seem to give much thought to possible misconceptions or alternative frameworks their learners are likely to bring with them when they first come across new topics.

Penso (2002) noted that learners' thinking about and prior knowledge of a topic is an important aspect that should be taken seriously into consideration during teaching as it helps to avoid possible learning difficulties that learners may encounter during the lesson. Penso (2002) suggested that during their lesson planning, practising teachers should be encouraged to explore varieties of instructional strategies that could elicit learners' thinking and prior knowledge of the concept being taught in order to be able to deal with their learning difficulties effectively. Hill *et al* (2008) note that the sequence of teaching and learning may be distorted if learners' preconceptions are not identified in order to address learning difficulties that learners are likely to encounter during teaching.

Teacher A addressed the learning difficulties through individual after-lesson or post-teaching discussions, including additional exercises that were given as homework (ref Section 4.5.1, first lesson observation, lines 23a and 23b). In his interview and written reports (ref Sections 4.7.2) the teacher confirmed the use of oral questioning, classwork and homework assignments as strategies that he purposefully uses to evaluate how well learners have understood the lesson and to gain insight into their pre-existing knowledge of histogram and box-and-whisker plot constructions.

In sum, Teacher A used several instructional strategies of oral questioning, group work, using contexts and examples familiar to learners to introduce a topic, checking and marking learners' classroom and homework assignments, as well as using content-specific rule-oriented graphing skills (drawing axes, choosing scale, labelling axes, plotting points and joining line of best fit) for constructing histograms. By identifying learners' learning difficulties, using diagnostic questioning and monitoring techniques (already indicated), Teacher A can be said to have used effective pedagogical strategies to elicit learners' difficulties. But these monitoring strategies were not usually followed up with probing questions to determine the sources of difficulty or of incorrect preconceptions.

From the discussion so far, the question is how Teacher A developed his PCK. Specifically Teacher A's PCK on the construction of histogram and box-and-whisker plots could be said to have been developed over time through a series of teaching and learning experiences. It would be useful to identify and briefly discuss the sources of such experiences. First, in terms of his formal education, Teacher A received further training in the teaching of mathematics after his initial teacher training programme. He holds a BEd degree, majoring in mathematics education, and has an Advanced Certificate in Education, specialising in teaching mathematics and science. His qualifications may be part of the reason that his content knowledge of the subject matter can be considered adequate. In his teaching he demonstrated a good grasp of the various topics of histograms and box-and-whisker plots related to school statistics.

Teacher A has 21 years' mathematics teaching experience. Over the years his pedagogy or instructional strategies in teaching statistics would have involved lesson planning based on the recommended work schedule and textbooks in school statistics, delivery of lessons based on his teaching philosophy, learned skills and feedback from his learners. Other sources of development would have included reviews of his teaching portfolios and learners' workbooks. All of these activities would have contributed to the development of topic-specific PCK in statistics teaching.

Teacher A attended workshops arranged by his educational district office. Most of these workshops dealt with aspects of how to teach various mathematics topics that are considered difficult to learn, such as data handling, analytical geometry and trigonometry. It would appear, however, that the workshops barely considered facets of teacher knowledge of learners' preconceptions and sources of learning difficulties in data handling. But if they did, the teacher did not demonstrate their potential usefulness in planning his lessons. Teacher A appears to have limited knowledge of learners' preconceptions that could have been used in teaching on learners' behalf.

In summary, Teacher A may have developed his pedagogical content knowledge from the formal initial teacher education programme that he received; the further training obtained at the completion of his tertiary education; attendance at in-service training workshop programmes; periodic reviews of his own lessons and learner workbooks; and feedback over his many years of mathematics teaching.

5.2.2 Teacher B

Teacher B planned and taught his statistics lessons on bar graphs and ogives from the recommended mathematics textbooks and work schedule (ref Section 4.5.2, second lesson observation and line 9). He used a predominantly rule-driven formal procedural approach to statistical graphs (ref Section 4.5.2: second lesson observation, lines 6a and 6b; and section 4.5.2, first lesson observation, lines 3a, 3b, 3c and 4a). As observed, in starting his lessons he tried to identify learners' prior knowledge of the new topic. For instance, he introduced bar graph construction and interpretation with a pre-activity (ref Section 4.5.2: first lesson observation, and line 1) that assessed learners' understanding of the way in which to prepare a frequency table. His use of pre-activities as diagnostic strategies to identify learners' pre-existing knowledge was also attested to in his responses to the teacher questionnaire and written reports (ref Sections 4.7.3).

Teacher B taught graphical constructions of bar graphs and ogives according to the learning outcomes of data handling as stated in the mathematics curriculum (DoBE, 2010) (ref Section 2.2). These outcomes require that learners should be able to use appropriate measures of central tendency and spread to collect, organise, analyse, and interpret data, in order to establish statistical and probability models for solving related problems (DoE, 2007). Teacher B followed precisely the order in which the learning outcomes were stated in teaching his learners how to construct bar graphs and ogives. In practice, this meant, as observed in his lesson, drawing the axes, choosing the scale, labelling the axes, plotting the points, and joining the line of best fit, in that order (ref Section 4.5.2, first lesson observation, lines 3a, 3c, 4a, 4c and 5). Teacher B demonstrated his PCK for drawing bar graphs in line with the sequence described. Flockton *et al* (2004) confirm that for a person to understand a graph, he or she should be able to use the construction skills of drawing the axes, labelling the axes, plotting the points, and joining the line of best fit to construct a graph.

Teacher B's assumed PCK on bar graphs and ogive constructions could be characterised as procedural in terms of his lesson planning and teaching approach. Teacher B's predominant use of a formal procedural approach was also triangulated in the analysis of his learners' workbooks (document analysis). The learners drew the bar graph and wrote down the teacher's steps on how to construct bar graphs and ogives (ref Appendix xxi; learners' workbooks). Teacher B might have been influenced to adopt a formal procedural approach because of the learning outcomes of data handling as laid down in the Curriculum and Assessment Policy Statement (CAPS) (DoBE, 2012). Besides, the construction of bar graphs and ogives demands specific procedural rules (Flockton *et al*, 2004 and Leinhardt *et al,* 1990).

Having said that, when the teacher merely taught them the rules for constructing bar graphs, some learners experienced certain misconceptions, confusing bar graphs with histograms, and histograms with ogives (ref Section 4.5.2: first lesson observation, and line 9; second lesson observation, and line 7a). A histogram is usually used to display continuous data. The horizontal axis shows class intervals, and there are no gaps between the bars. The area of each bar shows the frequency for the class interval. Teacher B can be said to have presented his lesson in a limited way with insufficient explanations of how to choose the scales of grouped data (consisting of histogram, frequency polygon, ogive, scatter plot) that are used to analyse and interpret large data. Further, Teacher B seems not to have the flexibility to present the topics to the learners in different ways because his lessons were presented solely according to the procedural knowledge approach.

A detailed description of the construction of bar graphs and ogives using a conceptual knowledge approach would have been ideal in presenting the lesson and would have avoided possible misconceptions and learning difficulties that the learners might have encountered in the lesson. Conceptual knowledge involves understanding mathematical ideas and procedures and includes basic arithmetic facts (Engelbrecht, Harding & Potgieter, 2005). It is rich in relationships among important mathematical concepts such as calculating the quartile positions and locating the quartile itself on the ogive, class intervals and boundaries, frequencies and cumulative frequencies of an ogive. But Teacher B’s teaching of bar graphs and ogives was dominated by a procedural knowledge approach, which involves following a rule or procedure without a detailed explanation of the relationships and mathematical connections between the concepts being learned, such as calculating a quartile position and locating it in an ogive. Thus, the teacher is probably unable to present his lesson in a variety of ways to ensure better comprehension and understanding. A detailed description of the concepts and their relationships, and the mathematical connections between these concepts and even existing ideas, may help to avoid possible misconceptions and learning difficulties that learners are likely to encounter during and after the lessons.

Baker *et al* (2001) and Bornstein (2011) note that a teacher who is unable to present mathematics content to learners in a variety of ways tends to expose them to learning difficulties, such as constructing a histogram instead of an ogive because of the use of an incorrect scale for labelling the data axis. A combination of procedural and conceptual knowledge approach would have helped to deepen learners' understanding and would have avoided misconceptions and learning difficulties that learners might develop during the lesson, as suggested by Engelbrecht, Harding & Potgieter (2005).

Teacher B often used familiar situations as examples for teaching data handling (ref Section 4.5.2: first lesson observation, lines 1 and 11). For instance, he described how a bar graph is constructed using a frequency table prepared by the learners from the raw scores obtained by learners in a mathematics test (ref Section 4.5.2, first lesson observation, and line 1). In his lesson on bar graphs (as explained earlier) he demonstrated the construction of bar graphs using a procedural knowledge approach. The use of familiar contexts is consistent with the recommendations of Meletiou-Mavrotheris and Stylianou (2002), who employ everyday situations as examples in order to make the topic accessible and meaningful to more learners. Although Engelbrecht *et al* (2005) suggest that a procedural knowledge approach could help learners to understand important demanding rule-oriented concepts, they affirmed that the use of both procedural and conceptual knowledge would be more effective and would create greater opportunities for improving learners' conceptual understanding of mathematics during the lesson (Engelbrecht *et al*, 2005; and Star, 2002).

During the lesson, Teacher B identified the learners' inability to label the data axis of the histogram correctly (ref Section 4.5.2: second lesson observation, and line 7b) by monitoring and analysing their responses to classwork. In one example, the learners chose the scale of grouped data and labelled the axes for data values incorrectly (ref Section 4.5.2: second lesson observation, line 7b). Teacher B addressed such learning difficulties through extra class activities in the form of drills and practice, as well as individual post-teaching discussions after formal classes (ref Section 4.5.2: second lesson observation, lines 9 and 12). The use of classwork and homework to evaluate how well learners had understood the lesson was confirmed in the teacher's responses to the questionnaire and written reports (ref Sections 4.7.3).

In terms of his knowledge of learners' learning difficulties, Teacher B was able to detect the misconception and learning difficulty of drawing a histogram instead of an ogive (ref Section 4.5.2: second lesson observation and line 7b). This misunderstanding could have been because of insufficient explanation of the construction of bar graphs and ogives via the procedural knowledge approach. As explained earlier, these learning difficulties were discovered while monitoring and analysing the learners' responses to classwork on bar graph and ogive construction (ref Section 4.5.2: first lesson observation, and line 9; second lesson observation, and line 10a). These problems were addressed by re-demonstrating the construction of bar graphs and by using extra class activities in the case of the ogive (ref Section 4.5.2: first lesson observation and line 10; second lesson observation, and line 9). The teacher interview, questionnaire, written reports and teacher's portfolios confirmed that learners had difficulties with the construction of graphs of grouped data such as the ogive (ref Appendix xvii, item 14; Appendix xx, item 10; items 1 and 2; and Appendix xxi, teacher portfolios).

In terms of his knowledge of learners' conceptions (preconceptions and misconceptions) in statistics teaching, Teacher B tried to identify them from pre-activities and oral probing questioning. Learners demonstrated previous knowledge of frequency tables and how data is represented by preparing the frequency table efficiently and explaining the way in which data is represented, but the strategy that was adopted failed to elicit learners' preconceptions of bar graph construction. In other words, the teacher therefore displayed insufficient knowledge of the learners' preconceptions of bar graphs and ogives. Learners are likely to experience misconceptions and learning difficulties, such as an inability to label the data axis due to incorrect scaling during the construction of ogive (ref Section 4.5.2, second lesson observation, line 7b) when the procedural knowledge approach was adopted to teach ogive construction. Teacher B would have been able to tackle this learning difficulty had the learners' preconceptions had been detected at the beginning of the lesson. When asked what learning difficulties did the learners experience during the lesson? (ref Appendix xx, item 6), he indicated that learners could not choose a scale of grouped data, revealing that their learning difficulties may have emanated from the teacher's insufficient knowledge of learners' preconceptions.

Inadequacy of knowledge of learners' preconceptions appeared to be common to all four teachers observed during the case study period. This finding points to a further investigation into the reasons that teachers with so many years of experience do not possess the knowledge of learners' possible preconceptions that may be necessary for effective teaching in the topics. Hill *et al* (2008) note that the sequence of teaching and learning may not lead to easy understanding of a concept and may not permit effective teaching if learners' preconceptions are not detected at the beginning of the lesson. Penso (2002) opines that teachers should consider several opportunities to detect learners' prior knowledge of a topic in their planning so that the anticipated learning difficulties can easily be addressed during lesson planning and presentation. This is an important agenda for inclusion in mathematics teachers' education programmes to ensure continuous improvement of PCK in statistics teaching.

How then does Teacher B develop his PCK in statistics teaching? In terms of his formal education, Teacher B received further training in the teaching of mathematics and statistics. He holds a BSc degree, majoring in mathematics and statistics. His qualifications may have contributed to his content knowledge of the subject matter which can be considered adequate. In his teaching he did not demonstrated sufficiently a good grasp of the various topics of bar graph and ogive construction related to school statistics because his teaching was dominated with a procedural knowledge approach that resulted to more questions from the learners during and after the lesson seeking for clarity of the misconceptions and learning difficulties they have encountered.

Teacher B has 10 years' mathematics teaching experience. Within these years of teaching, his pedagogy or instructional strategies in teaching statistics would have involved lesson planning based on the recommended work schedule and textbooks in school statistics, delivery of lessons based on his teaching ideology, learned skills and learners' responses to class activities in statistics. The review of his teaching portfolios and learners' workbooks were other sources for PCK development. All of these activities would have contributed to the development of topic-specific PCK in statistics teaching.

Teacher B attended workshops organised by his educational district office. As in the case of Teacher A, most of these workshops dealt with aspects of how to teach various mathematics topics that are considered difficult to learn, such as data handling, analytical geometry and trigonometry. The workshops sometimes appeared not to consider different aspects of teacher knowledge of learners' preconceptions and sources of learning difficulties in statistics teaching. But if they did, the teacher did not demonstrate their potential usefulness of the workshop in planning his lessons as the participating teachers were unable to demonstrate their knowledge of learners' anticipated learning difficulties of statistical graphs. Teacher B appears to have limited knowledge of learners' preconceptions that could have been used in teaching bar and ogive construction.

In summary, Teacher B may have developed his pedagogical content knowledge from the formal initial teacher education programme that he received, attendance at in-service training workshop programmes, periodic reviews of his own lessons and learner workbooks; and learners' responses to class activities in bar graphs and ogives construction.

5.2.3 Teacher C

During classroom practice, Teacher C taught his planned lessons on ogives and scatter plots as laid out in the work schedule (DoBE, 2010). He used the recommended and supplementary mathematics and statistics-related textbooks as sources of information for planning and teaching his lessons on data handling (statistics) (ref Section 4.5.3, first lesson observation, and line 16). Teacher C also displayed evidence of a procedural rather than a conceptual knowledge approach to teaching the construction of ogives and scatter plots (ref Section 4.5.3, first lesson observation, lines 3a, 3c and 4). Teachers need to possess good understanding of both conceptual knowledge and procedural knowledge of mathematics to be able to provide learners with clear explanations (Engelbrecht *et al*, 2005 and Star, 2002). Schneider and Stern (2010) view conceptual knowledge as mastery of the core concepts and principles and their interrelations in the mathematics domain. It is knowledge that is rich in relationships. On the other hand, procedural knowledge can be viewed as consisting of rules and procedures for solving mathematics problems. Procedural knowledge in mathematics allows learners to solve problems quickly and efficiently because to some extent it is automated through drill work and practice.

Teacher C demonstrated the requisite knowledge of and skills for constructing ogives in a step-by-step manner (ref Section 4.5.3, first lesson observation, and line 4) and scatter plots (see Section 4.5.3, second lesson observation, and line 4di). In his teaching, he moved from the algorithmic to the conceptually meaningful stage. He began his lesson on ogives and scatter plots by identifying the learners' prior knowledge of the concept of ogives through oral questioning, and the accuracy of the homework on histograms that had previously been taught (ref Section 4.5.3, first lesson observation, line 2bi; second lesson observation, and line 1). Subsequently, using a cumulative frequency table prepared by the learners, an ogive was constructed by first drawing its horizontal and vertical axes (ref Section 4.5.3, first lesson observation, and line 4). The data values were labelled on the horizontal axis (the upper class boundaries), and the cumulative frequencies on the vertical axis. A scale was chosen by the teacher, who indicated that he had chosen it by considering the highest and lowest values of the frequency and data values. The points were plotted and the line of best fit was joined to produce the ogive (ref Section 4.5.3, first lesson observation, line 9b).

This process of constructing an ogive from grouped data depicted a rule-oriented procedural approach. His procedural knowledge in teaching ogives (which was understandable to his learners) is believed to have been developed as a result of his five years' mathematics teaching experience, using the recommended lesson plan and work schedule of the Department of Education (DoE, 2010). The same procedural approach was used to teach scatter plots (ref Section 4.5.3, second lesson observation, and line 4di). To demonstrate the construction of a scatter plot, the teacher followed an algorithmic approach with progressively less conceptual knowledge. That is, the teacher's lesson was dominated to a large extent by a procedural knowledge approach rather than by conceptual knowledge. Some of the factors that may have contributed to Teacher C teaching scatter plots in a step-wise manner, following a particular order or sequence, could be attributed to the way in which the learning outcome of data handling is stated in the mathematics curriculum (DoBE, 2010). The document indicates that competency in graphing requires that the learner is able to construct, analyse, interpret statistical and probability models to solve related problem. The construction of graphs, as stated, entails scaling, drawing axes, labelling the axes, plotting points, and joining the line of best fit (Flockton *et al*, 2004; Leinhardt et al, 1990**)**. Teacher C followed this sequence for teaching scatter plots (ref Section 4.5.3, second lesson observation). In the lessons observed, the teacher gave a full explanation of how to construct a scatter plot before demonstrating how to analyse and interpret it. The learners did their classwork in groups. They were presented with exercises on scatter plots, and were requested to analyse and interpret the plots to determine whether there was a correlation between the variables X and Y (ref Section 4.5.3, second lesson observation, lines 3a and 4a).

Teacher C's preferred procedural approach to teaching the topic was confirmed in the learners' workbooks, portfolios and teacher's written reports (see appendices xx and xx1). Owing to the limited use of the conceptual knowledge approach rather than the procedural one – namely knowledge of the core concepts and principles and their interrelations in teaching ogive and scatter plots – it did not come as a surprise that some learners displayed certain misconceptions and learning difficulties in their analysis and interpretation of scatter plots (ref Section 4.5.3, second lesson observation, and line 6). For example, a negatively correlated linear scatter plot was interpreted by the learners as having no correlation because of an outlier that lay far from the line of best fit (ref Section 4.5.3, second lesson observation, lines 6 and 7). This misconception could be attributed to the rule-oriented approach that had been adopted to describe the construction of scatter plots (ref Section 4.5.3, second lesson observation, and line 4di), which did not allow for sufficient explanation of the interrelationships among the data values, frequencies, lines of best fit and outliers. The learning difficulty of interpreting a negatively correlated scatter plot as having no correlation owing to outliers may further indicate that in teaching the construction of scatter plots the teacher did not explain an outlier, line of best fit, type and nature of correlation, and how the presence of an outlier affects the correlation of the X and Y variables of the scatter plot.

What can be gleaned from the discussion so far is that teachers need to possess deep conceptual understanding of the mathematics concept that they are teaching and must be able to illustrate why mathematical algorithms work and how these algorithms could be used to solve problems in real-life situations (Nicholson & Darnton, 2003). The learning difficulties experienced by the learners were subsequently addressed by Teacher C during post-activity discussions (instructional strategy). This strategy was frequently used by Teacher C (ref Section 4.5.3, second lesson observation, line 12) during his lessons on ogives and scatter plots.

An important task of any teacher is to attempt to transform the content to be taught in such a way as to make it comprehensible to the learners (Mohr & Townsend, 2002). Teacher C also displayed evidence of a conceptual approach by providing the reasons that make the algorithm and formula work, and by explaining the relationships between important statistical concepts, as well as the mathematical connection between them during the lessons on ogives (ref Section 4.5.3, first lesson observation, lines 13b and 14). It was significant that more learners seemed to possess a better grasp of the topic in that they were able to construct and interpret ogives by means of this approach rather than the procedural approach (ref Section 4.5.3, first lesson observation, and line 14). In the particular lessons observed, Teacher C explained the mathematical connections and relationships between quartile positions and the quartiles and how quartiles can be used to interpret ogives (ref Section 4.5.3, first lesson observation, lines 13b and 14). In doing so, Teacher C could be regarded as having displayed progressively adequate PCK.

In his pedagogy, Teacher C used activities from everyday-life situations as examples (ref Appendix xxi, learner workbook). For example, he demonstrated how to construct a scatter plot using the frequency distribution table of the ages of persons infected with HIV/AIDS in two towns (ref Section 4.5.3, second lesson observation, line 9). This use of examples drawn from everyday life situation to illustrate scatter plot construction is in accordance with the view held by Shulman (1987) and Krebber (2004) that transformation of the subject matter by the teacher into a form that is more easily understood by the learners involves explanation with examples and instructional selection of teaching methods that are adaptable to the general characteristics of the learners. Teacher C may have decided to use examples drawn from everyday-life situations because the topic is new in the curriculum and may be looking for a more manageable way of presenting it to the learners in order to reinforce their understanding.

Teacher C gained knowledge of learners' preconceptions and learning difficulties mostly during classroom practice. The results of this study show that he had limited knowledge of learners' preconceptions. As observed, learners revealed previous knowledge of ogives and scatter plots from their responses to homework on these topics. For instance, at the beginning of the lesson on scatter plot construction, he checked and marked learners' homework on scatter plots based on their previous knowledge of what they had been taught and corrected some of their errors. While he was doing the corrections, he did not display any indication of having knowledge of other anticipated learning difficulties. Instead, he presented the correction procedurally, with no emphasis on the way in which previous errors that learners had committed could be avoided during the lesson or subsequently. Learners' learning difficulties led to Teacher C having to provide corrections to the homework. This leads one to the conclusion that he may not have considered identifying learners' preconceptions in scatter plot construction. This information should have been used in planning the current lesson and avoiding probable learning difficulties. The learning difficulties (constructing a histogram instead of an ogive and misinterpreting negatively correlated scatter plots as having no correlation due to an outlier) that were identified through monitoring and analysing the learners' responses to classwork (ref Section 4.5.3, second lesson observation, and line 4bii) would have been taking into consideration during lesson planning on scatter plot construction. Their inability to label the data axis due to incorrect scaling was identified by oral questioning from the learners (ref Section 4.5.3, first lesson observation, and line 8a).

Penso (2002) opines that practising teachers should be encouraged to consider learners' thinking and prior knowledge in lesson planning to avoid possible learning difficulties that learners may experience during lesson. Hill *et al* (2008) also reported that the sequence of teaching and learning may be altered if learners' prior knowledge is not considered during lesson planning and presentation. Teacher C addressed the learning difficulties by using a conceptual knowledge approach, and reviewing the learners' homework to reinforce their understanding. He also conducted post-teaching discussions during and after ogive and scatter plot construction lessons (ref Section 4.5.3 second lesson observation, and line 12).
The difficulties in terms of labelling the data axis of grouped data graph incorrectly were confirmed through analysis of the learners' workbooks (ref Appendix xxi, learners' workbooks), as well as the teacher's responses to the questionnaire and written reports (ref Section 4.7.3) in which he indicated that he identified learners' learning difficulties on graphs of grouped data through analysis of their classwork, homework and assignments. The learners, however, still followed the teacher after the lesson on scatter plot construction, demanding clarification about misinterpretation of a negative linear scatter plot that he had re-explained during the lesson. The teacher had evidently not addressed their learning difficulties sufficiently, which means that in teaching the construction of scatter plots his PCK was not comprehensive enough to cater for the learners' learning difficulties (Westwood, 2004). At this stage Teacher C did not exhibit enough PCK because his teaching could not cater for all the learners' learning difficulties in ogive and scatter plot construction. He subsequently addressed the learning difficulties experienced by the learners (such as misinterpreting a scatter plot because of outliers) in post-activity discussions, a strategy that he used frequently in his lessons (ref Section 4.5.3, second lesson observation, line 13). Capraro *et al* (2005) note that a competent mathematics teacher should be able to exhibit progressively more PCK in his or her lessons since he or she has acquired more experience

from formal education programmes and should plan his or her lessons in a way that is designed to avoid any learning difficulty that learners are likely to encounter.

In summary, the PCK profile of Teacher C may be construed as an amalgam of the various components of PCK, as defined earlier. His presumed PCK in teaching data-handling topics lies in his ability to use oral questioning and homework to identify the learners' preconceptions, as well as his use of construction skills and recommended mathematics and statistics-related textbooks, and past Senior Certificate Examination question papers in statistics to plan how to teach the construction of ogives and scatter plots. A combination of procedural and conceptual approaches, as well as the use of everyday situations and examples in teaching the statistics topics, constituted the instructional strategies that Teacher C employed to teach ogives and scatter plots. By identifying learners' learning difficulties through monitoring and analysing learners' responses to classwork, Teacher C can be said to have knowledge of learners' learning difficulties. But these difficulties were not always followed up in terms of taking them into consideration when planning the next lesson in order to identify learners' preconceptions of the new topic.

The question that one would want to ask at this stage is how, then, do the teachers develop their PCK in statistics teaching? Precisely, Teacher C's PCK on the construction of ogive could be said to have been developed through classroom practice and learning experiences over time. In terms of his formal education, Teacher C received further training on the teaching of mathematics. He holds a BSc degree, majoring in mathematics. His qualifications may have informed the reason that his content knowledge of the subject matter can be considered adequate.

Teacher C has five years of mathematics teaching experience. His instructional strategies in teaching statistics would have involved lesson planning, using the recommended work schedule and textbooks in school statistics, delivering lessons, and checking and marking learners' responses to homework. Other sources of PCK included reviews of his teaching portfolios and learners' workbooks. These activities may have contributed to the development of topic-specific PCK in statistics teaching

Teacher C attended workshops arranged by the District office of the Department of Basic Education. Most of these workshops focused on the new topic of data handling and particularly on how to teach it.

5.2.4 Teacher D

In Teacher D's observed lessons, it was noted that he had planned and taught his lessons on bar graphs and histograms using the Department of Basic Education's mathematics work schedule, and the recommended textbooks as sources of information (ref Section 4.5.4 first lesson observation, and line 11). During his teaching of bar graph and histogram construction (ref Section 4.5.4, first lesson observation, and line 2c), he gave more evidence of a procedural approach to teaching bar graphs and histograms than a conceptual one. For example, Teacher D taught the lesson on bar graphs in a step-by-step manner, beginning with pre-activities to identify learners' prior knowledge of bar graph construction, followed by the preparation of a frequency table compiled by the learners using a familiar daily life example (ref Section 4.5.4, first lesson observation, lines 1 and 2c). In this case, a frequency table was prepared of the number of cars in a car park according to their make (ref Section 4.5.4, first lesson observation, line 1). Next, with the help of the frequency table, a bar graph was constructed by first drawing its horizontal and vertical axes and labelling them appropriately. A scale was chosen by the teacher with the explanation that this was done by considering the highest and lowest values of the frequencies and the companies that manufactured the cars. Next, the points were plotted and the line of best fit was joined to produce the bar graph (ref Section 4.5.4, first lesson observation, lines 2c and 3). The teacher's specific strategy for teaching bar graph construction followed a rule-oriented procedural approach using procedural knowledge.

Engelbrecht *et al* (2005) describe the procedural knowledge approach as "following a rule or procedures flexibly, accurately, efficiently and appropriately in completing a given task". For example, in constructing a statistical graph, procedural knowledge approach requires a series of actions such as drawing the axes, choosing the scale, labelling the axes, plotting the points and joining the line of best fit. But what may be sometimes challenging is knowing how to move from the procedural stage to a conceptual meaningful one in terms of the students' learning.

As with the other teachers, Teacher D's procedural knowledge may have been developed over his 15 years of teaching mathematics in high school, using the recommended lesson plan and work schedule for statistics (DoBE, 2010). It could be suggested that although Teacher D possesses adequate ways of presenting bar graph construction to his learners, his PCK may be limited in the sense that he presented his lesson procedurally, an approach that was not always responsive to the learners' needs. Consequently, some of the learners constructed the classwork task without leaving spaces between the bars of the graph. The inability to consider the consistency of spaces between the bars of a graph during lesson presentation resulted in the learning difficulties that the learners experienced during classroom practice.

According to Shulman (1987), representation involves the teacher thinking through the key ideas and identifying alternative ways of presenting them to the learners. It is a stage in which suitable examples, demonstrations and explanations are used to build a bridge between the teacher's comprehension of the subject matter and what is required for the learners (Ibeawuchi, 2010). Multiple forms of representations are highly desirable if one is to be successful in the teaching process (Rollnick *et al*, 2008). Teacher D, however, in certain graphing topics, did display evidence of an alternative conceptual knowledge approach in teaching histograms (ref Section 4.5.4, second lesson observation, lines 11). Engelbrecht *et al* (2005) describe the conceptual knowledge approach as "involving an understanding of mathematical ideas and procedures consisting of the knowledge of basic arithmetic facts". Therefore it is knowledge that is rich in relationships and understanding of important statistical concepts in bar graph and histogram constructions. In the lesson observed, Teacher D explained in detail the meaning of a histogram. According to Teacher D, "a histogram is a graphical representation, showing a visual impression of the distribution of grouped data. It consists of tabular frequencies shown as adjacent rectangular bars, erected over discrete intervals, with an area equal to the frequency of the observations in the interval. Unlike the bar graph, a histogram is used to represent a large set of data (e.g. a population census) visually, but with no spaces between the bars" (ref Section 4.5.4, second lesson observation, lines 5b). His conceptual approach (presumably PCK) to teaching the construction of a histogram enhanced conceptual understanding of the topic as the learners seemed to be satisfied with Teacher D's conceptual explanation (ref Section 4.5.4, second lesson observation, and line 11) of how to construct a histogram after the learners had experienced misconceptions and learning difficulties in labelling the data axis (ref Section 4.5.4,second lesson observation, and line 10). They displayed the non-verbal cue of nodding their heads in agreement with the teacher's explanation (ref Section 4,5,4, second lesson observation, and line 12).

From the lessons observed with Teacher D, he used a procedural knowledge approach more rather than a conceptual knowledge approach. His preferred use of this approach was confirmed in the document analysis conducted in Teacher D's learner workbooks. The learners had completed the diagrams on bar graphs and histograms efficiently, with indications of the procedures that had been adopted in constructing these statistical graphs. Star (2002) argues that it is important for practising teachers to possess both kinds of knowledge in order to impart teaching to the learners in a meaningful way. The use of a rule-oriented procedural approach and a conceptual knowledge approach reveals that teachers are looking for ways of making the teaching of bar graphs and histogram comprehensible and accessible to their learners. Moreover, the construction of graphs demands that a particular order of actions should be followed, consistent with conceptual understanding. Teacher D can therefore be said to possess and demonstrate the required knowledge of bar graph and histogram construction.

Over and above this, Teacher D was able to identify learning difficulties experienced by the learners during the lesson and alternative conceptions from the various graphing exercises that were carried out by the learners. One such learning difficulty was their inability to choose the correct scale for labelling the data axis. This meant that they constructed a bar graph instead of a histogram (ref Section4.5.4, second lesson observation, and line 6a). In this case, Teacher D may be said to have presented his lesson in a limited way. His lessons were dominated by procedural knowledge teaching without providing the reasons underlying such procedures. He may not have accommodated the possibility of anticipating learning difficulties during the lessons on bar graph and histogram construction in his lesson planning and presentation and resolving them. For instance, he indicated the scale for constructing a bar graph and how it was obtained without explaining his reasons for choosing it, which shows that he may have presented his lesson in a limited way. When the learners adopted the same procedure to construct the bar graph during classwork, they did not consider the consistency of spacing in a bar graph, which resulted in a histogram instead of a bar graph. In terms of subject matter content knowledge, Teacher D may not have demonstrated the required variety of ways of presenting bar graphs and histogram construction for easy comprehension by the learners. Gersten and Benjamin (2012) note that the use of several strategies for teaching mathematics helps to deepen learners' understanding behaviourally and mathematically, avoids possible alternative conceptions and learning difficulties, and achieves effective learning. This means that teachers should be flexible (able to use a variety of instructional strategies to make content more accessible to more learners) in the representation of bar graph and histogram construction.

Regarding knowledge of instructional skills and strategies, Teacher D used analysis of learners' responses to classwork on bar graphs and histograms to identify their alternative conceptions and learning difficulties (ref Sections 4.5.4, second lesson observation, and line 6a). The teacher questionnaire, written report and learner workbooks confirmed this use of monitoring and analysing learners' responses to classwork to identify their alternative conceptions and learning difficulties. He addressed these difficulties through the instructional strategies of additional explanations, extra class activities, and examples related to familiar situations. These methods are consistent with the findings of Penso (2002), Westwood (2004), Bucat (2004), Mitchel and Mueller (2006) and Cazorla (2006), who adopted the same strategies for dealing with learners' misconceptions and learning difficulties. In practice, teachers are expected to design good teaching and learning instructions that take into consideration ways of identifying and addressing learners' learning difficulties (Westwood, 2004; Jong *et al*, 2005; and Rollnick *et al*, 2008). The other instructional skills that Teacher D used in teaching bar graphs and histograms were the construction skills involving drawing the axes, choosing the scale, labelling the axes, plotting points and joining line of best fit, which require a procedural knowledge approach.

The greater part of Teacher D's knowledge of learners' preconceptions and learning difficulties was gathered while teaching the assigned topic in statistical graphs. As observed earlier, during classwork the learners' inability to choose an appropriate scale for histogram and bar graph construction was identified through monitoring and analysing their responses (ref Section 4.5.4, second lesson observation, and line 6a). But the teacher did not display any evidence of having anticipated the learners' learning difficulties with bar graph and histogram construction, revealing that he may have gone into class without necessarily having knowledge of learners' possible learning difficulties in these constructions. To this end, Teacher D can therefore be said to have displayed insufficient PCK in terms of awareness of learners' preconceptions of bar graph and histogram constructions. From instance, at the start of the lesson, Teacher D requested the learners to prepare a frequency table of the makes of cars in a car park. The learners prepared the frequency table efficiently using previous knowledge. Thus, the teacher realised that the learners had previous knowledge that could be linked to bar graph construction, and no preconception was identified. When asked about the learning difficulties that learners might have had or were likely to experience during his lesson, he indicated that although that the learners had problems in determining the mid points and constructing graphs of group data, he would deal with difficulties that might arise (ref Appendix xx, item 10).

Inadequacy in teachers' knowledge of learners' preconceptions was common to the entire group of teachers involved in this study. This suggests the need for further investigation into the reasons that such teachers with many years of experience should have such a knowledge deficit in an area that is essential for effective classroom practice. As indicated in the section on Teacher A, Penso (2002) suggested that in their lesson planning, practising teachers should explore a variety of instructional strategies that would elicit learners' thinking and prior knowledge of the concept being taught in order to deal with their learning difficulties effectively. Hill *et al* (2008) note that the sequence of teaching and learning may by interfered with and possibly create opportunities for learning difficulties to occur if learners' preconceptions are not considered when planning and presenting a lesson.

Teacher D tried to address difficulties through extra explanations and homework assignments (ref Section 4.5.4, second lesson observation, and line 13). The teacher questionnaire, written reports and document analysis confirm the use of pre-activities, extra explanations and class activities in the form of classwork and homework to evaluate how well learners have understood the lessons and to gain insight into learners' pre-existing knowledge of bar graph and histogram construction.

It can be gleaned from the above discussion that Teacher D displayed a combination of the components of PCK that were identified earlier (Hill *et al*, 2008). Teacher D's presumed PCK is evidenced in his lesson planning and preparation, and in the use of textbooks in school statistics and mathematics, as well as other learning materials, such as past Senior Certificate Examination question papers. Pre-activities and correction of homework assignments were the instructional strategies he used to identify preconceptions about bar graphs and histograms. A combination of procedural and conceptual approaches to the teaching of statistics, as well as the use of exemplars drawn from familiar situations, was another instructional strategy that Teacher D used to teach the construction of bar graphs and histograms (ref Section 4.5.4, second lesson observation, lines 5c). Teacher D at this stage of using several instructional skills and strategies can be considered to have displayed knowledge of instructional skills and strategies for teaching bar graphs and histograms. Misconceptions such as drawing a histogram instead of a bar graph (ref Section 4.5.4, first lesson observation, and line 8) were addressed through post-teaching discussions, additional explanations during the lessons and homework (ref Section 4.5.4, first lesson observation, lines 9 and 11).

In sum, the sources for the development of Teacher D's PCK can partly be linked to the formal education that he acquired from a teacher training programme. He holds a BEd and SED, majoring in mathematics education. These qualifications and his 15 years of experience may have provided Teacher D with the opportunities to develop his content and pedagogical knowledge in statistics teaching. His instructional strategies over the years would have involved the use of lesson planning in line with the recommended work schedule, textbooks in school statistics and presentation of his lessons based on learning skills and reviewing of learners' classwork, homework and assignments. Other sources of development of his topic-specific PCK would have included reviews of his portfolios and learner workbooks. Teacher D attended content knowledge workshops organised by the Department of Basic Education (DoBE). Most of these workshops dealt with new aspects of the mathematics curriculum, especially the issues around teaching topics such as data handling.

5.3 Evaluation of theoretical framework

To evaluate the theoretical framework of this study is to determine to what extent the theoretical framework has enabled the researcher to answer the research questions.

The conceptual knowledge exercise, concept mapping exercise, teacher interviews, lesson observations and document analysis were the instruments used to examine the subject matter content knowledge of the participating teachers in school statistics in this study. The intention of the researcher in using these instruments for data collection was to determine the subject matter content knowledge that the participating teachers demonstrated in classroom practice. What can be gleaned from the results is that the instruments allowed the researcher to capture the teachers' PCK in terms of the subject matter content knowledge in statistics teaching. The concept map exercise was used as a proxy, but was not sufficient to determine how knowledgeable the teachers were about the contents of the curriculum (ref Section 4.4). The teachers should have been requested to write an examination in order to determine their content knowledge of the topic. But because it might be difficult to get the teachers to write an examination, a concept mapping exercise was considered a good proxy for assessing their content knowledge. Another way in which the teachers' content knowledge could have been examined was through certification. That is by reviewing the certificate obtained from colleges and universities. Considering a certificate in mathematics education without observing how a teacher demonstrates his or her content knowledge in the classroom may not be sufficient to determine whether that teacher possesses content knowledge of a topic. Hence, lesson observations were used to assess the teachers' subject matter content knowledge and how well they demonstrated this knowledge in statistics teaching. Although Mahvunga and Rollnick (2011) suggest that a quantitative research study may be sufficient to assess teachers' content knowledge, their study failed to indicate how to assess the quality of teachers' content knowledge, which can be determined only during classroom practice. This assertion is given wide empirical support by researchers such as Toerien (2011), Ball *et al* (2008), Capraro *et al* (2005), Jong *et al* (2005), Lee and Luft (2008), Jong (2003) and Gess-Newsome and Lederman (2001), who all note that PCK is rooted in classroom practice. Any research into teachers' PCK that does not consider the use of lesson observation may fail to fully convey the required information about how teachers develop topic-specific PCK. Through lesson observation, it was possible to determine how the teachers demonstrated their content knowledge of certain topics. Lesson observation provided opportunities to experience the details, nuances and dimensions that the teachers used in their classroom practice in order to determine the adequacy of their subject matter content knowledge (ref Sections 4.5.1–4.5.4). Through the teacher interviews, it appears that the teachers' educational backgrounds that may have enabled them to develop topic-specific content knowledge in statistics were determined (ref Section 4.7.1).

While researchers such as Shulman (1986), Van Driel *et al* (1998), and Magnusson (1999) use subject matter knowledge consisting of syntactic and substantive knowledge acquired in formal education, in this study the subject matter content knowledge focused on the content to be taught and learned by the students. The use of subject matter content knowledge as the theoretical framework for this study proved useful in determining the procedural and conceptual knowledge (component of the PCK) that a teacher demonstrates in teaching statistical graphs. Other PCK studies (Plotz, 2007; Lee & Luft, 2008; Adela, 2009; Ibeawuchi, 2010; Ogbonnaya, 2011; and Toerien, 2011) share the same view of using subject matter content knowledge as a theoretical framework for examining teachers' PCK development in mathematics. These authors also assess the subject matter by making the teachers write a test on the content of the topic under investigation. The instruments developed with the framework were therefore considered adequate to determine teachers' subject matter content knowledge in statistics teaching and the theoretical framework can be considered adequate and valid.

The teacher questionnaire, which focused on what the teachers did while teaching the assigned topic, and the written reports used to triangulating the data collected with lesson observations were used to determine the pedagogical knowledge (instructional skills and strategies) that the teachers used in teaching school statistics. Other instruments used to assess the teachers' pedagogical knowledge were lesson observation and document analyses.
The questionnaire revealed many aspects of the teachers' PCK, such as knowledge of instructional skills and strategies for teaching statistical graphs. These strategies included oral probing questioning, checking and marking learners' homework and pre-activities to determine learners' pre-existing knowledge (ref Section 4.8). The lesson observations, teacher written reports and document analyses confirmed the use of these instructional strategies. These activities were crucial in determining learners' conceptions about statistical graphs, as suggested by Krebber (2004), Westwood (2004) and Ball *et al* (2008), but did not elicit learners' preconceptions in statistical graphs. From the lesson observations, it was not possible to determine learners' preconceptions because the strategies the teachers adopted to do so did not elicit them. Instead, the learners displayed previous knowledge linked to learning the new topic. In fact, the teachers did not have knowledge of the instructional skills and strategies that might have been necessary to determine the learners' preconceptions in statistical graphs.

As Krebber (2004) and Westwood (2004) suggest, the use of the instructional strategies of oral probing questioning, pre-activities, checking and marking learners' responses to classwork, homework and examining learners' understanding, as well as identifying their misconceptions and learning difficulties in statistical graphs, is critical in learning and could motivate the development of teachers' pedagogical knowledge. Loughran *et al* (2004), Ball *et al* (2008), and Vistro-Yu (2003) regard teachers' pedagogical knowledge as crucial to PCK development. Having ascertained the instructional skills and strategies demonstrated by the teachers through the teacher questionnaire, written reports, document analyses and lesson observation, the researcher believes that the teachers' pedagogical knowledge can be considered a valid theoretical framework for determining the PCK required for teaching school statistics.

However, the framework provided an opportunity to reveal that the teachers had some knowledge of learners' misconceptions, as individually they were able to identify misconceptions through analysis of learners' responses to classwork, homework and assignments in statistical graphs. The activities of identifying and addressing learners' misconceptions are critical aspects of teaching and learning. Penso (2002), Carzola (2006) and Westwood (2004) note that a teacher who lacks the ability to identify and address learners' misconceptions may experience poor content delivery in classroom practice. Practising mathematics teachers are encouraged to learn about the possible instructional skills and strategies for identifying and addressing learners' alternative conceptions in statistical graphs.

Penso (2002), Westwood (2004) and Carzolia (2006) also posit that if learners' alternative conceptions and difficulties are not identified and addressed in the preparation and presentation of lessons, negative lesson presentations can occur. The lesson observations (ref Section 4.5.1–4.5.4), teachers' written reports, and learners' and teachers' portfolios confirmed that the participating teachers know about learners' learning difficulties in statistics (ref Sections 4.7–4.10). Therefore, knowledge of learners' learning difficulties can be considered adequate as a theoretical framework for capturing teachers' PCK in statistics teaching.

5.4 Summary of the chapter

This chapter opened with a brief recapitulation of the research questions and PCK components as a theoretical framework for this study.

The teachers demonstrated that they possess content knowledge of school statistics. However, the predominant approach to putting across statistical ideas to their learners about data handling, particularly the construction of statistical graphs, was procedural. A conceptual approach was used less to some extent and not as the same degree as procedural approach. The individual teachers are presumed to have developed their PCK in statistics teaching by extending their knowledge of the subject matter content through formal education programmes and the use of topic-specific mathematics and statistics textbooks and other publications as sources for lesson planning and teaching.

The instructional skills and strategies used by the participating teachers for teaching specific statistics topics consisted largely of oral questioning, pre-activities, and post-teaching discussions to determine preconceptions. By using these instructional skills and strategies to teaching statistical graphs, the participating teachers may have developed their PCK in statistics teaching. An analysis of the learners' classwork, homework, assignments, and post-teaching discussions was used to determine where the learners' misconceptions and learning difficulties lay. All four teachers, although at different times, used extra tutoring, problem-solving activities involving familiar daily-life contexts, individualised teaching, post-teaching discussions, and repetition of the lessons to address learners' difficulties and misconceptions (ref Section 5.2.1-5.2.4).

This chapter concludes with an evaluation of how the theoretical framework was used to ascertain whether it provided adequate opportunities to develop instruments for collecting data to answer the research questions.

CHAPTER 6

6.0 SUMMARY AND RECOMMENDATIONS OF THE STUDY

6.1 Introduction

This chapter presents a summary of the study, recommendations, and suggestions for further research. The main aim of this study was to investigate how competent mathematics teachers whose learners perform consistently well in the Grade 12 mathematics National Senior Certificate Examination develops pedagogical content knowledge in school statistics teaching. Specifically, it explored what these teachers do in classroom practice when teaching data handling topics to the learners. In addition, the study probed the implications that PCK has for mathematics teacher education programmes (ref Section 6.5).

6.2 Focus of the study

The study investigated how competent mathematics teachers develop PCK in statistics teaching (which has recently been introduced as a formal aspect of mathematics in the National Curriculum Statements (now changed to Curriculum and Assessment Policy Statements (CAPS)) in South Africa. The chief examiner's report (DoE, 2009; DoBE, 2012) states that learners' poor performance in statistics may mean that mathematics teachers have not acquired sufficient PCK for teaching the subject. In addition, delegates at the International Commission for Mathematics Instruction and International Association for Statistics Educators joint conferences (ICMI/IASE, 2007, 2011) attributed learners' poor achievement in statistics to underdevelopment of PCK by practising mathematics teachers. This research is therefore intended to explore the manner in which competent mathematics teachers develop PCK in statistics teaching.

A multi-method approach involving the use of several research instruments such as a conceptual knowledge exercise, concept mapping, lesson observation, teacher questionnaires, interviews, written reports, video records and document analysis for data collection, was adopted to carry out the investigation. Mathematics teachers were identified who were perceived to be competent in teaching mathematics, based on their school performance in the senior certificate examination, together with recommendations from principals, peers and subject facilitators.

The research questions that guided the study are:

1) What subject matter content knowledge of statistics do the mathematics teachers have and demonstrate during classroom practice?

2) What instructional skill and strategies do these teachers use in teaching statistics?

3) What knowledge of learners' preconceptions and learning difficulties, if any, do they have and demonstrate during classroom practice?

4) How do these teachers develop PCK in statistic teaching?

A qualitative research approach involving the case study method was used to collect data. The data were analysed to determine the teachers' assumed PCK and how they might have developed their PCK profile in statistics teaching. PCK, in the context of this study, was used as a theoretical framework to try to determine how they developed their assumed PCK in statistics teaching (ref Section 1.7). It was defined as 'an amalgam of practising teachers' content knowledge in school statistics; their pedagogical knowledge (instructional skills and strategies) and learners' conceptions and learning difficulties in statistics teaching' (Shulman, 1987 and Ball et al, 2008: 391)

In applying the PCK as a theoretical framework, certain assumptions were made, as indicated in chapter 1, to enable the investigator to proceed with the study. These assumptions are as follows:

- PCK represents a category of knowledge that describes the quality of an expert teacher (Miller, 2006).
- PCK provides a framework that can be used to describe the origin of its critical teacher knowledge, but not all the teachers have the same PCK (Miller, 2006).
- PCK is a constructivist process and, therefore, a continually changing body of knowledge (Miller, 2006).
- PCK can be measured by conceptualising the construct and using multiple assessment techniques, including classroom practice (Hill, 2008).

It is currently a widely accepted belief that PCK represents a category of knowledge needed for a novice teacher to mature into an expert (Miller, 2006). Ball *et al* (2008) described teacher knowledge as an amalgamation of subject matter and pedagogy. The blend of different forms of teacher knowledge has forced many teacher education programmes to create new pedagogical activities that engage pre-service teachers in terms of the teachers' classroom practice. 'The same vision of how to improve classroom practice has provided a focus on education research; unfortunately PCK remains a category of knowledge that is difficult to isolate and research' (Miller, 2006). However, the teachers' classroom practice in statistics teaching in the context of this study was investigated in a case study using lesson observation to see how they demonstrated their subject matter content knowledge, pedagogical knowledge, and knowledge of learners' conceptions and learning difficulties. Data gathered from lesson observation were triangulated with data collected from concept mapping, teacher questionnaires, interviews, written reports, video records and document analysis in order to determine how the mathematics teachers develop their PCK in statistics teaching for learner performance and classroom practice improvement.

6.3 Summary of the results according to the theoretical framework

A summary of the results from the investigation is as follows:

6.3.1 Knowledge of the subject matter content

The four participating teachers taught statistical graphs predominantly using procedural knowledge and less frequently as conceptual knowledge. The use of procedural knowledge was to some extent dictated by the nature of the topic, which required learners to be able to collect, organise, analyse and interpret statistical and probability models to solve related problem (DoBE, 2010). A second factor that leads to the use of procedural knowledge is the way in which statistical graphs should be constructed, which involves drawing axes, choosing scales, labelling axes, plotting points and joining the lines of best fit. Other processes in developing subject matter content knowledge included the frequent use of mathematics textbooks, CAPS documents, as well as attendance at workshops (ref Appendix xvii).

6.3.2 Pedagogical knowledge (instructional skills and strategies)

Instructional skills are the most specific category of teaching behaviour. They are necessary for procedural purposes and for structuring appropriate learning experiences for learners. In this study, instructional skills and strategies, involving construction skills such as drawing axes, choosing scale, labelling axes, plotting the points and joining the lines of best fit were used in constructing statistical graphs. Instructional strategies such as oral probing questioning, pre-activities, pre- and post-teaching discussion were used by the individual teachers to determine learners' prior knowledge in statistical graph construction. Checking and marking learners' responses to homework were other assessment strategies that helped to determine learners' prior knowledge in statistical graphs and learning difficulties. Procedural and conceptual approaches were used to describe how to construct statistical graphs such as the bar graphs, box-and-whisker plots, ogives, histograms and scatter plots. Individual and grouped classwork, homework and assignments as well as oral probing questioning were also used to assess how well learners' have understood the lessons on these graphical constructions. An analysis of learners' responses to classwork, homework and assignments was the main assessment strategy that the participating teachers used to identify learners' misconceptions and learning difficulties in statistics teaching. While some learners showed that they had grasped what the teachers taught, a few experienced learning difficulties. Instructional strategies such as the use of extra tutoring, class activities in the form of drill and practice, explanation, examples drawn from familiar situations and post-teaching discussions were used to address learners' misconceptions and learning difficulties in statistical graph construction.

The participating teachers claimed that the instructional skills and strategies used in teaching statistics were developed through formal education and classroom practice (ref Section 4.7.2 and Appendix xvii). The development of instructional skills and strategies varies from teacher to teacher, depending on the topic, feedback from the learners, and the learners' prior knowledge of that topic. The results drawn from lesson observation (ref Sections 4.5.1–4.5.4), interviews and the questionnaires (ref Appendices xvii and xxviii) showed that the participating teachers used topic-specific instructional strategy of providing exercises in statistics in which learners were required to solve problems, while the teachers monitored and guided them (as in classwork), which allowed learners to construct knowledge by themselves, thereby influencing their active participation in the lessons. By using instructional skills such as topic-specific construction skills, and the instructional strategies of oral probing questioning, pre-activities, extra tutoring and class activities and post-teaching discussion, as well as assessment strategies of analysing learners' responses to written works to determine learners' misconceptions and learning difficulties, the participating teachers' may have intensified and broaden their knowledge of the instructional skills and strategies used in teaching school statistics.

6.3.3 Knowledge of learners' preconceptions and learning difficulties

The most notable learning difficulty observed in the lessons of all four teachers was the inability to construct and interpret graphs of grouped data (ref Sections 5.2.1-5.2.4). The main challenge, in part, was owing to learners' inability to choose an appropriate scale (ref Sections 5.2.1- 5.2.4). Second, the learners had difficulty in labelling the axes without proper scaling for constructing the statistical graph on the paper provided (ref Figure 4.5.1c).

The teachers developed knowledge of learners' learning difficulties through analysis of their classwork, homework and assignments, as well as through post-teaching discussions on statistical graphs construction (ref Sections 5.2.1-5.2.4). Constant examination of the learners' workbooks helped to reinforce the teachers' insight into learners' conceptions (preconceptions and misconceptions) of statistics topics (ref Sections 4.5.1–4.5.4).

The teachers addressed these difficulties at various times through extra classes, problem-solving tasks using familiar real-life examples, post-teaching discussions, and teaching on a one-to-one basis after normal school hours (ref Section 5.3.4). The process of identifying and addressing learners' learning difficulties should have provided the teachers with ample knowledge of learners' preconception and learning difficulties in statistics teaching. But it is surprising that after so many years of teaching mathematics, some of the teachers are not aware of these problems. This lack of familiarity with learners' anticipated learning difficulties could be because the topic was recently introduced into the curriculum. Therefore, the teachers may not have developed the required PCK for addressing the difficulties which learners' may experience in learning school statistics. However, by identifying and addressing learners' alternative conceptions and learning difficulties, the participating teachers may have gained more knowledge of the learners' learning difficulties in statistics teaching.

6.4` Concluding remarks

Based on the findings of this study, individual teachers constructed their PCK in statistics teaching by:

- Formally developing their knowledge of the subject matter in an accredited formal education programme in which they had the opportunity to study the subject matter and methodology of school statistics

- Teaching school statistics using procedural and conceptual knowledge to some extent (ref Sections 4.5.1-4.5.4.).

- Using several mathematics and statistics textbooks, past senior certificate examination question papers in statistics and other materials in lesson preparation, consistent with their understanding of the nature of statistics in school mathematics and how it should be taught (ref Section 5.2.1-5.2.4). For example, Teacher A taught his lesson of histogram construction and assigned classwork and homework using learners' mathematics textbook (ref Section 4.5.1, first lesson observation, and line 23b).

- Using varied topic-specific instructional skills such as construction skills (involving the drawing of axes, choosing of scale, labelling of axes, plotting thee points and joining the line of best fit), problem-solving, assessment (in the form of oral probing questioning, classwork, homework and assignments), and interpretation skill (comprising of determining the relationship between X and Y, and based on the relationship between X and Y values, one can say whether there is positive correlation, negative correlation, or no correlation as in, second lesson observation, and line 4bii), in teaching scatter plots (ref Section 4.5.3)

- Using diagnostic techniques (oral questioning, pre-activity and class discussions) and a review of previous lessons to introduce lessons, and to determine learners' preconceptions in statistics teaching (ref Section 4.7.3)

- Using a variety of assessment techniques such as classwork, homework and assignments and grouped work in statistical graphs to assess how well learners understood the lesson on statistical graphs and to identify their difficulties (ref Sections 4.7.3).

- Continually updating their knowledge of school statistics by attending content knowledge workshops and other teacher development programmes designed to improve content awareness and practice (ref Section 5.3)

By knowing how teachers develop PCK for teaching school statistics, teacher educators will be able to develop greater understanding and insight into designing programmes to teach topics that were previously included only at tertiary level.

6.5 Educational implications of the study

Based on the results of this study, the educational implications can be summarised as follows:

The findings of this study can be used to provide a knowledge base and process to be employed by mathematics teachers to develop PCK for the continuous improvement of effective mathematics classroom practice. For instance, the teachers developed knowledge of learners' learning difficulties by analysing their responses to classwork, homework and assignments and during pre- and post-activity discussions. Regular examinations of learners' workbooks helped to reinforce the teachers' familiarity with learners' conceptions and learning difficulties of statistics topics. Learning difficulties were generally addressed by the teacher engaging the learners on a one-to-one basis or collectively during or after school hours.

The development of subject matter content knowledge of statistics renders it an essential component of PCK for teaching it at school level. When teaching statistics, teachers' actions were determined to a large extent by the depth of their PCK, thereby making subject matter content knowledge an essential component of their ongoing learning of school statistics for the improvement of their expertise in statistics and effective classroom practice.

'Pedagogical content knowledge research links knowledge of teaching with knowledge of learning' (Adela, 2009). This is a powerful base on which to build teaching expertise. In this study, formal education in mathematics was found to be a prerequisite in developing teachers' subject matter content and pedagogical knowledge. Several research reports have attempted to establish how PCK is developed in science and mathematics. As PCK is topic-specific, however, little attempt has been made to determine how PCK is developed in the context of teaching statistics by mathematics teachers. The research that is available suggests that this type of information is meagre. This study has therefore furnished insight into how PCK is developed by competent mathematics teachers. A detailed description was given of examples of the PCK of mathematics teachers in terms of improving learners' performance in statistics and for consideration by teacher trainers in designing statistics teacher education programmes for in-service and pre-service teachers.

In this study as indicate in section 1.6, PCK was conceptualised to include content specific knowledge, content specific instructional strategies and learners' preconceptions of specific concept, rules and skills. 'PCK development is a complex process and it is not clear how it is developed in statistics teaching for mathematics classroom practices (Jong, 2003). PCK is distinct from a general knowledge of pedagogy, educational purpose and learners' characteristics. Moreover, because PCK is concerned with the teaching of a particular topic e.g statistics, it may turn out to differ considerably from the subject matter itself' (Jong, Van Driel and Verloop, 2005:948). PCK is said to develop by an iterative process that is rooted in classroom practice. The implication is that many beginning teachers have little or no PCK at their disposal, particularly in statistics teaching (ref Section 1.6).

From the description of the knowledge-base and process employed by competent mathematics teachers in developing PCK in statistics teaching, notions of and insight into PCK could be obtained that can be incorporated into a mathematics education programme for in-service and pre-services mathematics teachers, thereby contributing to the continuous improvement of the mathematics teacher education programme and teachers' PCK.

6.6 Suggestions for further study

The results of this study present several areas for further research opportunities. These areas are suggested:

- Large-scale research needs to be conducted on the kind of subject matter content knowledge that a teacher needs for development of PCK in statistics, especially in the construction and interpretation of graphs of grouped data, which many teachers seem to find difficult to teach.

- More studies need to be conducted to determine the impact of teachers' knowledge of learners' preconceptions as a theoretical framework for investigating teachers PCK in statistics teaching.

- This study found that procedural and conceptual knowledge were both necessary for teaching statistical graphs, especially in addressing learners' misconceptions and learning difficulties. Further studies are needed to determine how well both approaches can be applied to other aspects of school statistics.
- More researches need to be conducted on why teachers with over five years

experience of teaching mathematics lack sufficient knowledge of learners' preconceptions in statistics teaching.

6.7 Limitations of the study

This study may have been influenced by these limitations, which should be taken into consideration when interpreting the results:

- Selection of the participants created a problem that led to having only a few in the study. The number of participants was reduced because of the criteria used in selection. The schools from which the participants were selected had to obtained a pass rate of 70% and above in mathematics in the senior certificate examination for at least two years. This left the researcher with a small number of schools from which to select willing participants.
- Assessment of teaching competencies is usually associated with inherent limitations as they are coloured by personal observer idiosyncratic tendencies. The results of this study during the lesson observations may not necessarily be replicated. The process of interpreting teachers' practice and decisions, and placing them into specific pedagogical categories may not always be 100% correct. The possible errors in the interpretations were reduced by the triangulation of data, using open assessments (questionnaires to confirm the teacher observations and the categories assigned) and negotiations for placing pedagogical actions into appropriate categories of how the teachers developed their PCK in the teaching of statistics. Discourse on classifying pedagogical actions into appropriate categories depended on the negotiations that took place between the researcher and the teachers, and was bound to differ from one teacher to another. The interpretations of the lessons and post-teaching discussions could be viewed as temporal (dependent on time and pairs) and tentative. The possible significant errors could be minimised by using multiple strategies to collect data.
- Another limitation included external validity or the ability to generalise the results. Only four teachers participated in the entire study. The number of cases was limited to making broad generalisations. Not only the number of cases, but also the geographical location and the school types may be too limited to produce a general theory on PCK appropriate for teaching statistics in school mathematics. The number of participants

also provided the possibility of variation or similarity in PCK assessment for mathematics teachers using the same working document such as the mathematics work schedule, the results in the senior certificate examination in mathematics, recommendations from principals, subject specialists and peers.

- Organising lessons outside normal school hours posed its own challenges. Learners were sometimes tired at the end of the school day. Extra-curricular activities at the schools occasionally affected the teaching programme. Therefore adjustments had to be made to assure consistency and uniformity in all the statistics topics.

6.8 The role of the researcher in the non-participatory lesson observation.

In this study, non-participatory classroom observation in statistics lessons was conducted with the four participating teachers. As explained in paragraph 2 of section 6.7, assessment of teaching competencies involving a non-participating observer is usually associated with inherent limitations, owing to the presence of the observer. The teacher and the students might behave differently from the ways in which they would normally comport themselves. (Cresswell, 2008; University and College Union, 2012). In this study there was always the possibility that the participating teachers teaching could have been somewhat influenced, as indicated later, by the presence of the researcher and the research interest. The interest was with determining how mathematics teachers considered competent developed their PCK in statistics teaching by observing them in statistics lessons among other things. Any one of such influences could possibly occur in the planning and presentation of statistics topic lessons. For example, to perhaps try to impress the observer they could select instructional materials and use instructional strategies they think are effective but not necessarily economical that they would not normally use routinely in teaching the assigned statistics lessons.

The presence of the researcher could also have influenced learners' responses or active participation (freely or inhibited) during the lessons. While these are the possibilities that could have arisen during the lesson, I believe that I tried to minimise those instances by first introducing myself to the participating teachers and their learners during negotiations with them on the extent and nature of the lessons to be taught and observed (Creswell, 2008, University and College Union, 2012) and spending some time with the teachers (familiarisation) before embarking on any formal classroom observation. Additionally, during the meetings at which the teachers were briefed about the objectives of the research, they were assured that the observation was not an assessment in any form or shape of their teaching performance, but was designed to gain better understanding of how to help teachers with a new topic, statistics, that has recently been introduced into the mathematics curriculum. The learners were also encouraged by both the researcher and their teachers to feel free and less anxious to participate, just as in a normal lesson, since no assessment was involved. The participating teachers were given access to all the recorded field notes, the video recordings and their transcriptions to comment on, and to approve, before the analysis of data. Furthermore, triangulation of data helped to minimise and/or address any inconsistencies in the participating teachers' questionnaire and interview responses, and classroom behaviour.

6.9 Summary of the chapter

In this chapter, the summary, conclusion and recommendations for further investigations were presented. The results of the study indicate that mathematics teachers may have constructed PCK in teaching statistics through the acquisition of formal subject matter knowledge of the topic in formal education programmes, and they develop their subject matter content knowledge during classroom practice. The teachers taking part in the study possessed the necessary content knowledge, and demonstrated it through procedural and conceptual approaches to teaching statistical graphs, although the rule-oriented procedural approach was dominant in teaching data-handling topics. Mathematics and statistics-related textbooks and other learning materials were other sources used by the teachers to acquire the subject matter content knowledge that was needed to plan and deliver their lessons.

Knowledge of instructional strategies, notably the use of a formal rule-oriented approach and instructional skills such as the construction skills, was developed through formal education and years of experience in classroom practice. Analyses of learners' classwork, homework and assignments were used mostly to gain teacher knowledge of learner misconceptions and topic-specific learning difficulties. Intervention strategies such as the used extra tutoring, class activities in the form of drill and practice, repeating and re-explaining of lessons in which learners are experiencing difficulties as well as post teaching discussions were used to addressed the alternative conceptions and learning difficulties. The chapter concluded with highlights of the educational implications, suggestions and limitations of the study for future researchers to note.

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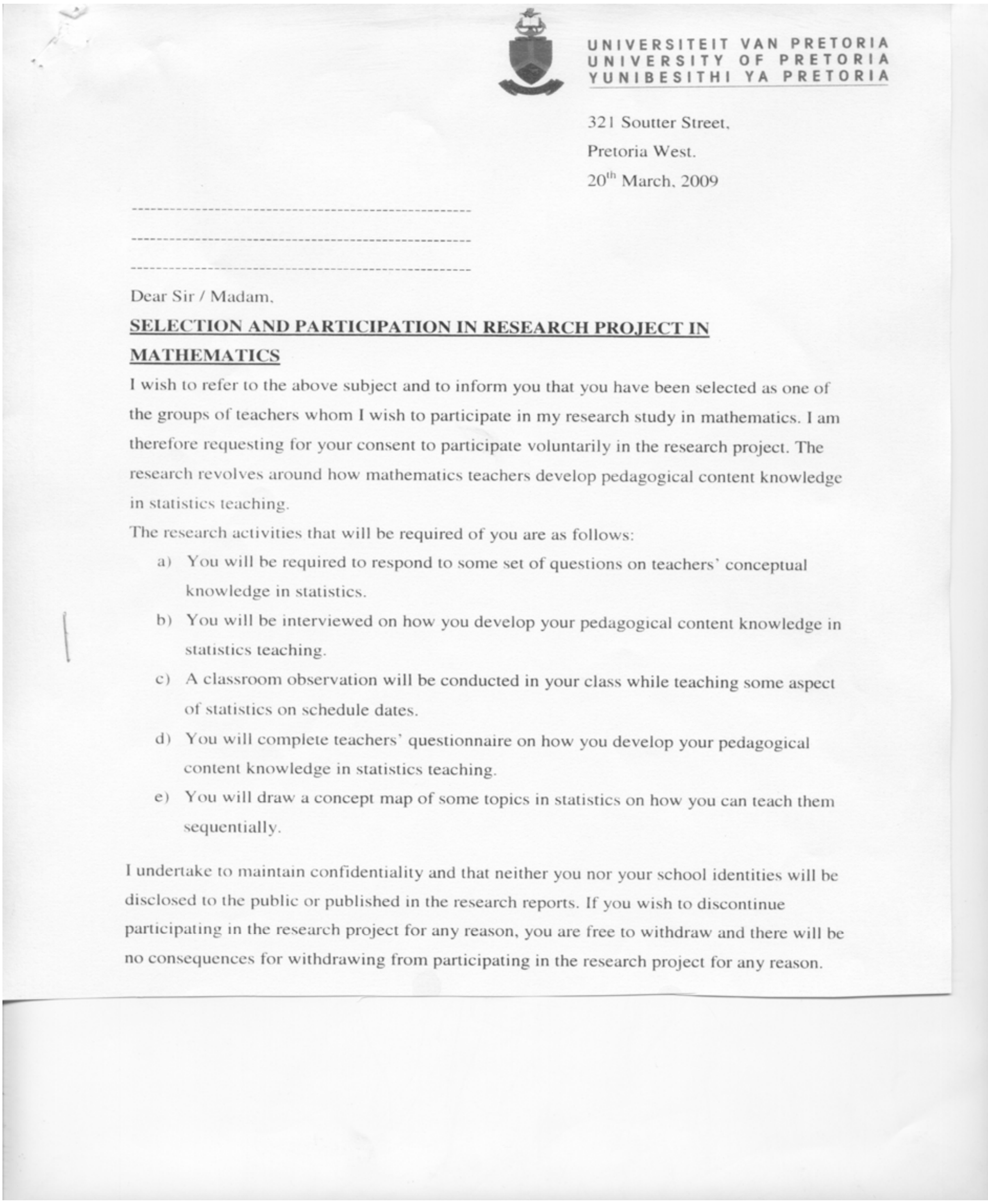
APPENDIX

APPENDIX I



UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

321 Soutter Street,
Pretoria West.
20th March, 2009

Dear Sir / Madam,

SELECTION AND PARTICIPATION IN RESEARCH PROJECT IN MATHEMATICS

I wish to refer to the above subject and to inform you that you have been selected as one of the groups of teachers whom I wish to participate in my research study in mathematics. I am therefore requesting for your consent to participate voluntarily in the research project. The research revolves around how mathematics teachers develop pedagogical content knowledge in statistics teaching.

The research activities that will be required of you are as follows:

a) You will be required to respond to some set of questions on teachers' conceptual knowledge in statistics.
b) You will be interviewed on how you develop your pedagogical content knowledge in statistics teaching.
c) A classroom observation will be conducted in your class while teaching some aspect of statistics on schedule dates.
d) You will complete teachers' questionnaire on how you develop your pedagogical content knowledge in statistics teaching.
e) You will draw a concept map of some topics in statistics on how you can teach them sequentially.

I undertake to maintain confidentiality and that neither you nor your school identities will be disclosed to the public or published in the research reports. If you wish to discontinue participating in the research project for any reason, you are free to withdraw and there will be no consequences for withdrawing from participating in the research project for any reason.

Kindly indicate your willingness to participate in the research on voluntary basis by signing the space provided below.

Yours sincerely,

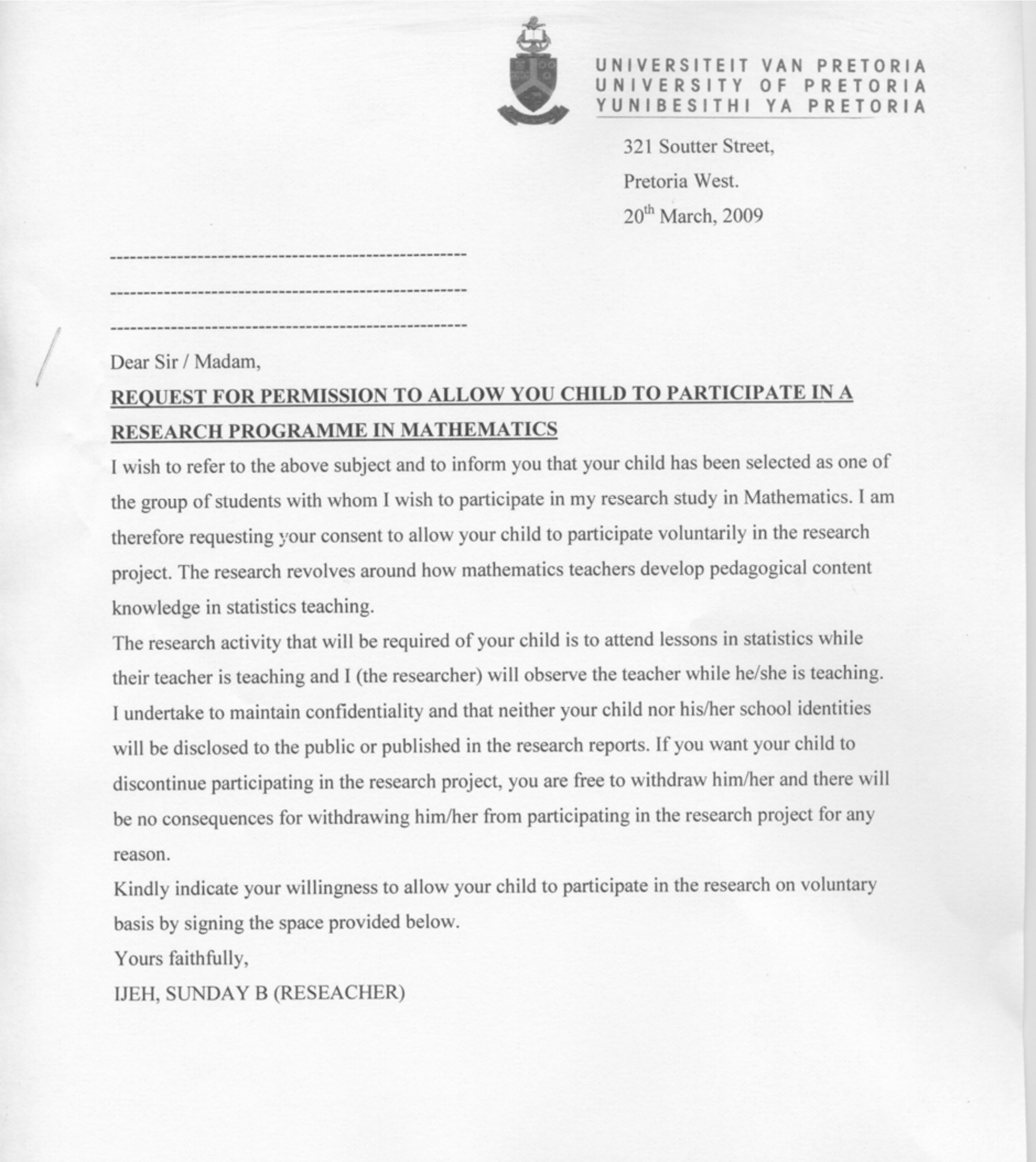
IJEH, SUNDAY B. (Researcher).

I, Mr/Mrs/Miss____________________________of_____________________________ high school have agreed to participate in the research project in mathematics education conducted by Mr Ijeh, Sunday B.

____________________________________ ____________________________

Signature of Participant Date

APPENDIX II



UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

321 Soutter Street,
Pretoria West.
20th March, 2009

Dear Sir / Madam,

<u>REQUEST FOR PERMISSION TO ALLOW YOU CHILD TO PARTICIPATE IN A RESEARCH PROGRAMME IN MATHEMATICS</u>

I wish to refer to the above subject and to inform you that your child has been selected as one of the group of students with whom I wish to participate in my research study in Mathematics. I am therefore requesting your consent to allow your child to participate voluntarily in the research project. The research revolves around how mathematics teachers develop pedagogical content knowledge in statistics teaching.

The research activity that will be required of your child is to attend lessons in statistics while their teacher is teaching and I (the researcher) will observe the teacher while he/she is teaching. I undertake to maintain confidentiality and that neither your child nor his/her school identities will be disclosed to the public or published in the research reports. If you want your child to discontinue participating in the research project, you are free to withdraw him/her and there will be no consequences for withdrawing him/her from participating in the research project for any reason.

Kindly indicate your willingness to allow your child to participate in the research on voluntary basis by signing the space provided below.

Yours faithfully,

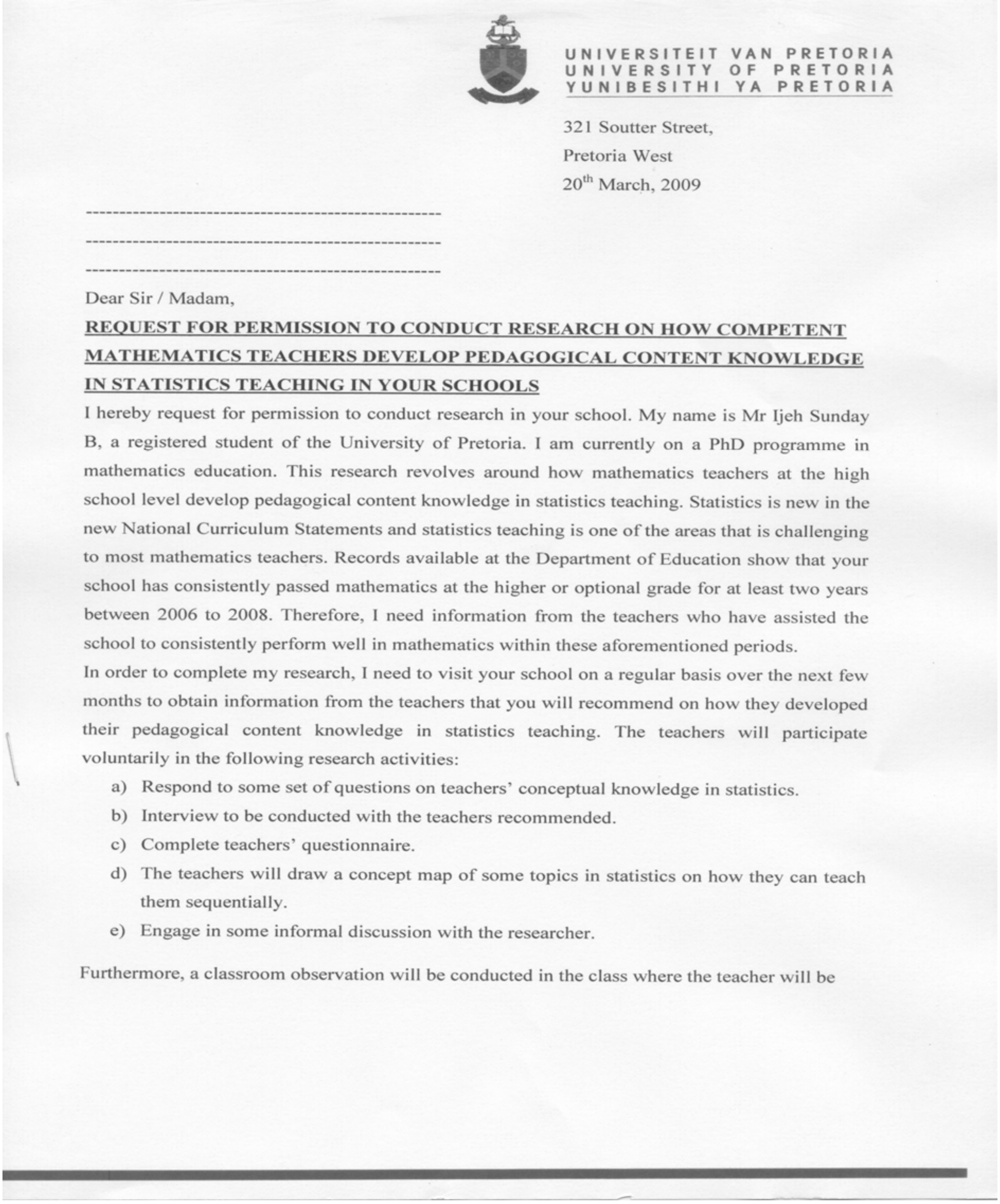
IJEH, SUNDAY B (RESEACHER)

I, /Mr/Mrs/Miss_______________________________ being the father/mother have agreed that my child will attend lessons/ participate in the research project in Mathematics education conducted by Mr. Ijeh, Sunday B (Researcher).

___________________________________ ______________________________

Signature of Parent Date

APPENDIX IIIA



UNIVERSITEIT VAN PRETORIA
UNIVERSITY OF PRETORIA
YUNIBESITHI YA PRETORIA

321 Soutter Street,
Pretoria West

20th March, 2009

Dear Sir / Madam,

REQUEST FOR PERMISSION TO CONDUCT RESEARCH ON HOW COMPETENT MATHEMATICS TEACHERS DEVELOP PEDAGOGICAL CONTENT KNOWLEDGE IN STATISTICS TEACHING IN YOUR SCHOOLS

I hereby request for permission to conduct research in your school. My name is Mr Ijeh Sunday B, a registered student of the University of Pretoria. I am currently on a PhD programme in mathematics education. This research revolves around how mathematics teachers at the high school level develop pedagogical content knowledge in statistics teaching. Statistics is new in the new National Curriculum Statements and statistics teaching is one of the areas that is challenging to most mathematics teachers. Records available at the Department of Education show that your school has consistently passed mathematics at the higher or optional grade for at least two years between 2006 to 2008. Therefore, I need information from the teachers who have assisted the school to consistently perform well in mathematics within these aforementioned periods.

In order to complete my research, I need to visit your school on a regular basis over the next few months to obtain information from the teachers that you will recommend on how they developed their pedagogical content knowledge in statistics teaching. The teachers will participate voluntarily in the following research activities:

a) Respond to some set of questions on teachers' conceptual knowledge in statistics.
b) Interview to be conducted with the teachers recommended.
c) Complete teachers' questionnaire.
d) The teachers will draw a concept map of some topics in statistics on how they can teach them sequentially.
e) Engage in some informal discussion with the researcher.

Furthermore, a classroom observation will be conducted in the class where the teacher will be to teaching some aspects of statistics on scheduled dates.

I undertake to maintain confidentiality and that neither the school nor the mathematics teacher involved in my research will be identified, and, will be free to withdraw at any time.

In line with the department regulations, a letter of consent will be given to the mathematics teacher recommended by you, requesting for his/her voluntary participation.

I will be very appreciative of your assistance in this regard. Kindly indicate your willingness by signing the space provided below.

Yours faithfully

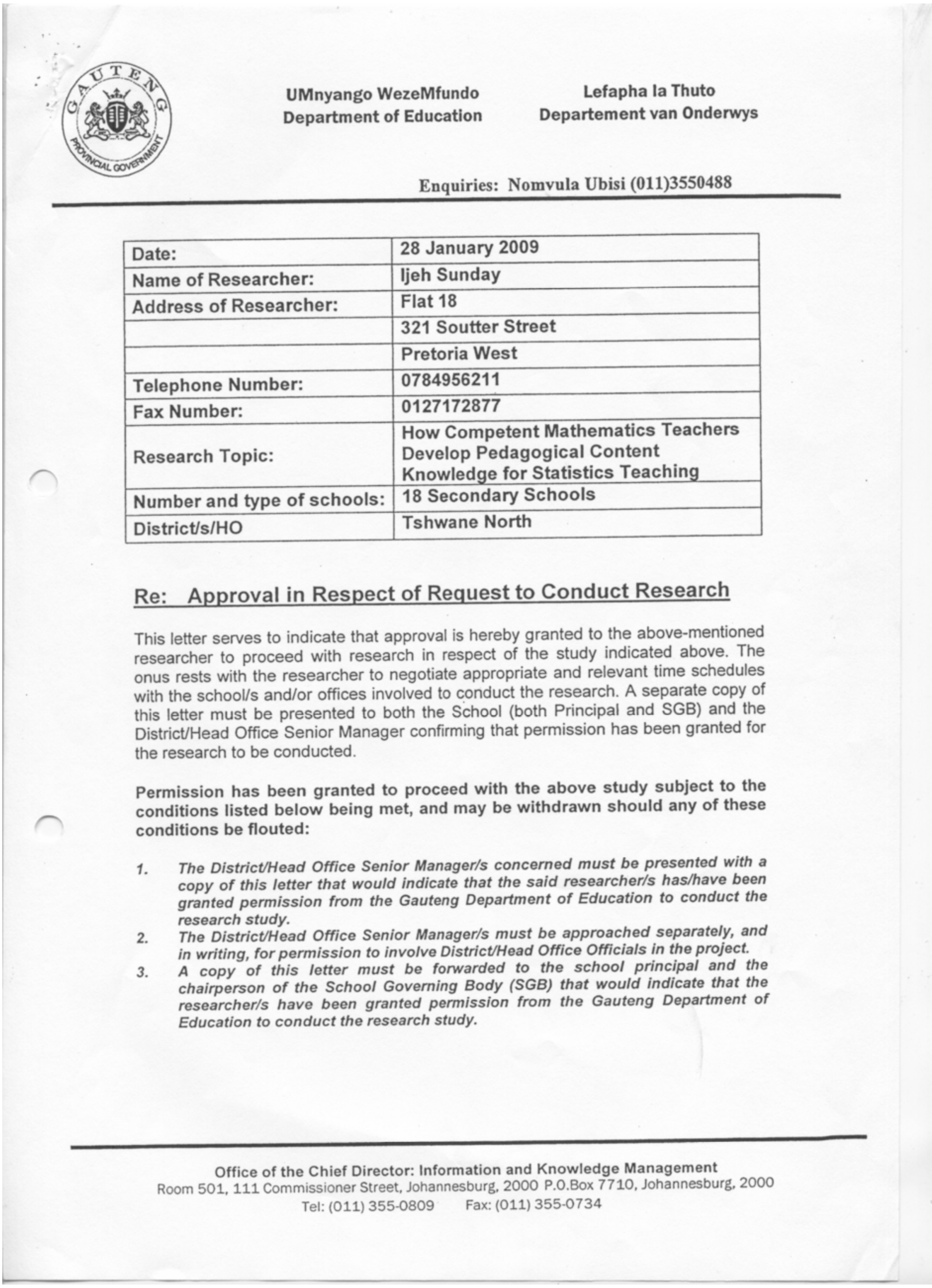
IJEH, SUNDAY B.
(Researcher)

I, Mr/Mrs/Miss ____________________________ the principal of _________________________ hereby grant Mr Ijeh, Sunday B. (Researcher) the permission to conduct a research project on the topic indicated above.

__________________________________ ______________________________________

Signature of the Principal Date

APPENDIX IIIB



UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Enquiries: Nomvula Ubisi (011)3550488

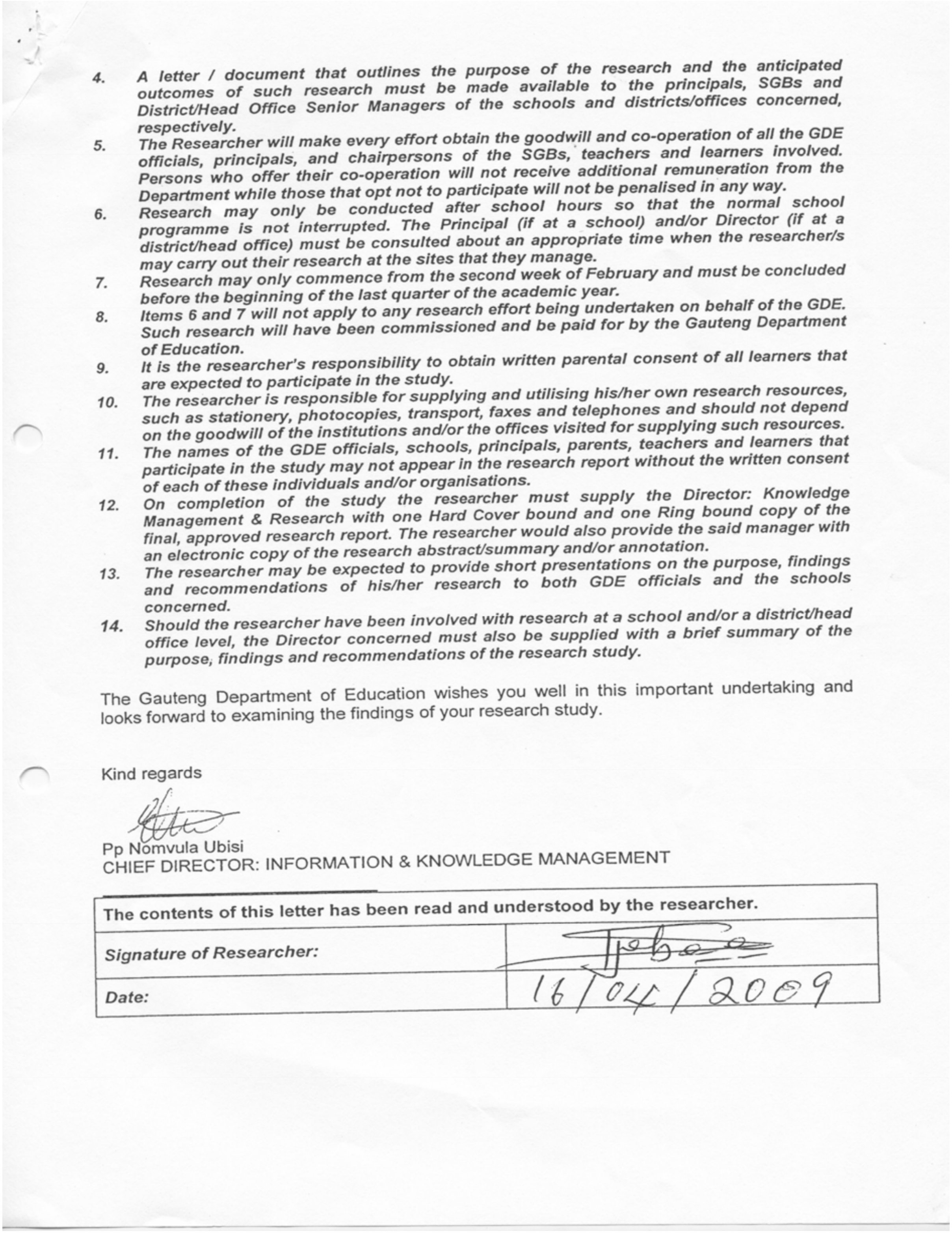
Date:	28 January 2009
Name of Researcher:	Ijeh Sunday
Address of Researcher:	Flat 18
	321 Soutter Street
	Pretoria West
Telephone Number:	0784956211
Fax Number:	0127172877
Research Topic:	How Competent Mathematics Teachers Develop Pedagogical Content Knowledge for Statistics Teaching
Number and type of schools:	18 Secondary Schools
District/s/HO	Tshwane North

Re: <u>Approval in Respect of Request to Conduct Research</u>

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*



4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

Pp Nomvula Ubisi
CHIEF DIRECTOR: INFORMATION & KNOWLEDGE MANAGEMENT

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	16/04/2009

APPENDIX IV

Criteria for Validating Interview Schedule for Teacher on How They Develop PCK in Statistics Teaching.

Preamble

An educational background means where and what school you attended within a particular period. Basically all the schools that one has been to study a given or a particular subject (DoE, 2008). It comprises the university attended, courses/modules studied, qualification obtained and duration of the study. According to Bucat (2004), subject matter content knowledge is the knowledge of the subject matter about what should be taught and how it should be taught for effective learning. The outstanding teacher is not simply a ‘teacher’, but rather a ‘history teacher’, a ‘chemistry teacher’, or an ‘English teacher’. While in some sense there are generic teaching skills, many of the pedagogical skills of the outstanding teacher are content-specific. Beginning teachers need to learn not just ‘how to teach’, but rather ‘how to teach mathematics’, how to teach world history’, or ‘how to teach fractions in mathematics. With these skills, subject content knowledge can be transformed into pedagogical content knowledge (PCK) (Geddis, 1993). In order to be able to transform subject matter content knowledge into a form accessible to students, teachers need to know a multitude of particular aspects about the content that are relevant to its teachability (Bucat, 2004). Those teaching aspects that the teacher needs to know included the topics, method of teaching, effectiveness of the lesson, nature of the topic, how to assess learners’ understanding of the topic and effective participation, instructional strategies used and relevance of the topic to the learners. Others included are how to identify learners’ learning difficulties and the intervention used to address the learning difficulties such as workshops, extra tutoring and more problem solving activities that can enhance learners’ participation in the topic or subject. Kindly indicate in the space provided whether the attached interview covered what it supposes to cover in terms of assessing the mathematics teachers’ content knowledge and educational background that enabled them to develop their PCK in statistics.

1)	**Educational Background and subject matter content knowledge**	**Options**	**Response**
	a) Does the schedule request for the university/college attended?	**Yes/No**	
	b**)** Does the schedule request for the participants' qualifications?	**Yes/No**	
	c) Does the schedule request for the course/module/subject studied in the university/college?	**Yes/No**	
	d) Does the schedule request for how the module/subject help in lesson preparations?	**Yes/No**	
	e) Does the schedule request for how the teacher knows that his teaching was effective?	**Yes/No**	
	f) Does the schedule request if the teacher has interest in teaching mathematics?	**Yes/No**	
	g) Does the schedule request for how the teachers understand the nature of the subject/topic?	**Yes/No**	
	h) Does the schedule request if learners understand the topic?	**Yes/No**	
	i) Does the schedule request if the learners enjoy the topic?	**Yes/No**	
	j) Does the schedule request for how the teachers update their content knowledge for teaching the topic/subject?	**Yes/No**	
	K) If the teachers attend workshop for instance, does the schedule request to know how effective was the workshop?	**Yes/No**	
	l) Does the schedule request to know if the facilitators of the workshop are mathematics teachers or not?	**Yes/No**	
	m) Does the schedule request for the duration of the workshop?	**Yes/No**	
	n) Does the schedule request for what was benefited from the workshop?	**Yes/No**	
	o) Does the schedule request if the workshop participants need similar workshop in subsequent time?	**Yes/No**	
2	**Instructional skills and strategies**		
	a) Does the schedule request if the teachers are adhering to the instructional approach as recommended in the NCS curriculum?	**Yes/No**	
	b) Does the schedule request for how learners can be assisted if they experience some learning difficulties based on the instructional approach used by the teacher?	**Yes/No**	

	c) Does the schedule request for instructional skills and strategies used for teaching statistics?7	**Yes/No**	
	c) Does the schedule request for other instructional approach used by teachers apart from the recommended approach according to NCS?	**Yes/No**	
	d) Does the schedule request for how learners learning difficulties were resolved if any?	**Yes/No**	
3	**Learners' learning difficulties**		
	a) Does the schedule request for the learning difficulties which learners encounter during teaching?	**Yes/No**	

Comments

NAME AND SIGNATURE OF RATTER

APPENDIX V

TRANSCRIPTION OF VIDEO RECORDS OF FIRST LESSON OBSERVATION OF TEACHER A

The teacher came into the classroom and began the lesson as follows:

Teacher A: Good afternoon learners?

Learners: Learners answered, Good afternoon sir.

Teacher A: “Let somebody tell me how to calculate mode, median and mean of ungrouped data?”

Learner: “Mode is the number that appears most often in a distribution. For example; 1, 2,3,2,5. The mode is 2. For the median, the learner continued, you arrange: 1, 2. 2. 3, 5. Therefore, Median is 2, because it is the middle number after arranging the numbers according to size”.

Teacher A: Wrote down the numbers mentioned by the learner as an example of the data and requested another learner to tell him how to calculate mean. In their mother tongue, he said, ***ke bokae?*** Many learners raised their hand to answer the question but the teacher A nominated one of the learners to calculate the mean with the data on the chalkboard.

Learner: Mean is the average of the numbers. i. e. Mean = $\frac{1+2+3+2+5}{5}$,

Mean = $\frac{12}{5}$ = 6.0

Teacher A: Gave an example and explained to the learners how to prepare the frequency table. How many members are their ages within 16-20?

Learner: One of the learners counted and said: it is 7. Another learner said it is 6 (The correct one).

Teacher A explains: The class boundaries are calculated thus, $\frac{15+16}{2}$ = 15.5,

$\frac{20+21}{2}$ = 20.5, etc; Mid-values = $\frac{16+20}{2}$ =18; and fx is calculated as: 6 x 18 = 108.

Teacher A: “I have completed the first three rows.” Then, “complete the remaining rows by calculating the frequencies, class boundaries, mid-values and fx”.

Teacher A: Is it clear? In their mother tongue, he said, ***le a nkutlwa?***

Learner: Yees Sir.

Learner: Completed the frequency table individually. “I have completed mine” (The learners who have finished raise their hands).

Teacher A: Constructed the histogram by drawing the vertical and horizontal axes on the chalkboard, choose scale, label the axes with class boundaries as on the table and draw the bars on the axes (Teacher A drew three bars and asked them to complete the remaining bars).

Learner: Listened and watched how the teacher constructed the histogram using topic specific construction skills (Drawing of axes, choosing of scale and labelling of axes, drawing the line of best fit).

Teacher A: "Now complete the histogram."

Learner: Completed the histogram individually.

Teacher A: Went round to check how learners were constructing the histogram and further to answer the follow-up questions (By explanation).

Teacher A: Calculated the mode from a histogram by drawing a diagonal form the top right corner of the highest bar of the histogram to the top right corner of the next bar on the left hand side and draw a second diagonal from the top left corner of the highest bar to the top left corner of the next bar on the right of the highest bar. He further refers the learners to how mode is calculated in a stem and leaf diagram and to use that method for confirmation of the answer obtained.

Learner: Learners did as teacher A explained with their graph sheet on individual basis.

Teacher A: Analysed and interpreted the histogram i.e. 7 members of a netball club are within the ages of 16-20 years. 67% of the members of the club are within the ages of 21-30 years (e.g. add all numbers within the ages 20 – 30 divide by 27 and multiply by 100).

Learner: Watched how the teacher calculates the percentage of learners within the ages of between 21-30 years and write it on their notebook.

Teacher A: Now, do this as classwork.

Learner: Did classwork by preparing class boundaries, drawing of axes, choose scale for drawing and labelling the axes, draw the line of best fit (bars).

Teacher A: Monitored and guided learners as they did the classwork.

Teacher A commented: "I can see that your (some of them) diagrams are not correct. Make sure that you have chosen the correct scale as in the example, otherwise you diagram cannot be correct. Some of you have constructed the histogram very well, but many have not, because you choose a wrong scale. Please, go back to your example and see how we choose the scale and do the same for this exercise."

Learner: Continued with classwork and were still experiencing some difficulties about the construction and interpretation of histogram especially determining the mode from the histogram.

Teacher A explains: “The lesson is about to end. Please, those of you who have not completed their classwork should do so at home and bring it to school tomorrow. Here is your homework (referring them to the exercise on their mathematics textbook) which you have to submit with the classwork you could not complete. I want to see those of you who could not complete your classwork immediately after closing tomorrow so that I can assist you on those areas where you are experiencing problems.”

APPENDIX VI

RANSCRIPTION OF VIDEO RECORDS OF SECOND LESSON OBSERVATION OF TEACHER A

The teacher came into the classroom and began the lesson as follows:

Teacher A: “Good afternoon learners?”

Learner: Learners answered, “Good afternoon sir”.

Teacher A: “Let me see how you did the homework which I gave you yesterday?”

Learner: Opened to the page in their mathematics notebooks where they did the homework.

Teacher A: Checked and marked the homework from one learner to the other.

Teacher A: Solved homework on chalkboard.

Learner: Wrote correction on notebook.

Teacher: Provided a photocopied exercise on ogive and requested learners to interpret it. That is; calculate first, 2nd, 3rd quartiles, minimum and maximum values.

Learner: Find first quartile, 2nd quartile, third quartile, minimum and maximum value in Groups.

Teacher A: Used the values got from the interpretation of graph to construct a box-and whisker plot while learner watched.

Learner: “Could you please explain again how to calculate the first and third quintiles?”

Teacher A Explains: Using the formula as in previous examples, $Q_1 = \frac{1}{4}(N + 1)^{th}$ position you Can find the position of the first quartiles and Q_3 = ¾ $(n + 1)^{th}$ can be used to find the position of the third quartiles Q_3 can be traced from the cumulative frequency to the curve down to the horizontal axis to determine the value of the first quartile. The same applies to the value of the third quartiles. While $Q_1 = 52$, $Q_2 = 63$ and $Q_3 = 73$ the maximum 100 (using a similar example in their textbook for explanation).Further interpretation: 25% of the learners got less than 52%, 50% the learners got less than 63% and 75% of the learners got less than 73%.

Teacher A: “The formula can also be applied to ungrouped data. You may apply it to the exercise you did previously in ungrouped data.”

Learner: Wrote the reference for the homework and noted it in their textbooks as indicated above.

Teacher A Comment: “It appears that some of you do not understand how to calculate the quartiles in ungrouped and grouped data. Can I see you tomorrow at 15h00 to explain more?”

Learner: “Thank you, see you tomorrow.”

APPENDIX VII

TRANSCRIPTION OF VIDEO RECORDS OF FIRST LESSON OBSERVATION OF TEACHER B

Teacher B: Greeted the learners, “Good afternoon class”

Learner: “Good afternoon sir?”

Teacher B: “Can we move to the science laboratory because we want to use electricity in our lesson today?”

Learner: Moved to the laboratory before the lesson began.

Teacher B Commented: “I want you to solve the exercise on preparation of frequency table on the photocopied paper within 5mins”

Learner: Prepared a frequency table.

Teacher B explained: **“**A bar graph is a pictorial representation of statistical data in the form of rectangle called bar. A bar graph is often needed to compare two or more values that are taken under different conditions or over time.” The frequency table prepared (pre-activity) by the learners was used further used to explain and construct a single bar graph. Teacher B drew the vertical and horizontal axes, label vertical axis as frequency and horizontal as scores. Draw the bar for each score”, teacher B said.

Learner: Watched as the teacher explains how to construct and interpret the bar graph and write explanation in their notebook and asked “How do we know that the test is easy or difficult using these scores?”

Teacher B Explains: “If the number of learners that scored 7 and above was more than six, then the test was easy.” (**Teacher B** read out the number of persons who scored 7 and above as 3 x 1 = 3.) “This means that about 60% of the learners scored between 7 and 10. But if seven learners scored between 1 and 3 (teacher B read from the graph), and the highest score was 5, the test was difficult, as 70% of the learners scored below 4 marks, and the highest score was 5.”

Thus, with a bar graph, it is easy to visualise and interpret learners' performance in a test. From Figure 4.5.2a, it is evident that the test was within the level of the learners, as the learners' marks are not too low, and if the pass mark is 4 (40%), then only four of the learners failed.

Teacher B: After explanations, he gave classwork on the construction and interpretation of bar graph (referring to their textbooks).

Learner: Did their classwork as instructed by the teacher on one to one basis.

Teacher B: Monitored and guided learners while they were doing their classwork.

Learner: Some learners did not consider the concept of spacing which resulted to misconception.

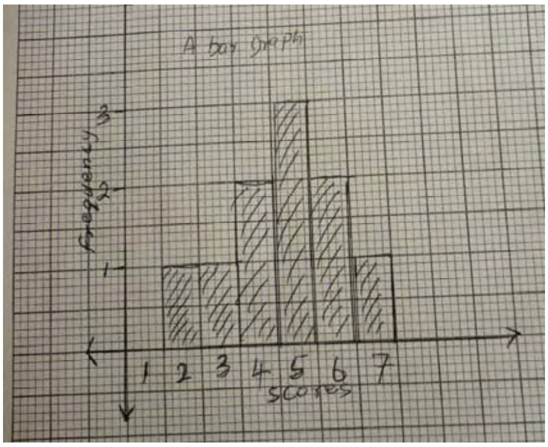


Figure A7: **Learners constructed a histogram instead of bar graph.**

Teacher B: Indicated to some of them that their classwork was wrong

Learner: Demanded clarity why their classwork was wrong.

Teacher B: Explained and re-explained and gave extra activities for learners to solve in class and at home on how to solve the problem in the afternoon next day.

APPENDIX VIII

TRANSCRIPTION OF VIDEO RECORDS OF SECOND LESSON OBSERVATION OF TEACHER B

Teacher B: Greeted the learners “Good afternoon learners?”

Learner: “Good afternoon sir?”

Teacher B: Wrote the topic on the chalk board

Learners: Watched and listened.

Teacher B “Mention two ways of presenting data.”

Learner: Mentioned frequency table, bar graph, pie chart, histogram

Teacher B: Referred the learners to the exercise in their textbook. Explained how to prepare a cumulative frequency table with the first three rows and instructed the learners to complete the preparation of the frequency table with the remaining rows.

Learners: Completed the frequency table.

Teacher B: Wrote all the cumulative frequencies calculated to ensure that every body agreed on common cumulative frequency table.

Teacher B: “Draw the vertical and horizontal axes like this .Label the axes with the vertical as cumulative frequency and the horizontal with the marks value. Plot the point (10, 0); (20, 2); (30, 8) etc. Continue with the remaining points,” the teacher said

Learners: Learners plotted the remaining points e.g. (40, 15); (50, 29); (60, 49), (70, 84); 80,113), (50, 29) ;(90,119); 100,120); in groups

Teacher B “This is how to interpret the graph.”

Learners: Listened and watched as teacher B interprets the graph (Using the follow up questions).

Teacher B: Interprets the ogive by determining the quartiles e.g. 1st, 2nd and 3rd quartiles as 52, 63 and 73 respectively.

Teacher B: “Now do this exercise as classwork but with class internal beginning from 20-30, 30-40, 40-50, 50-60, while he monitored and guided them.

Teacher B Commented: “Most of you appear not to know how to label the horizontal axis with class boundaries beginning from 0-10 , 10-20, 20-30, etc and beginning from 20-30, 30-40 40-50 etc. Look at how you can do it (referring the learner to a graph paper and showing how to mark out the value on the horizontal axis.”

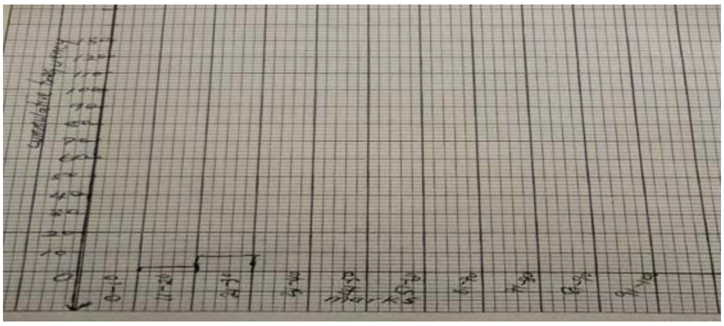


Figure A8. Learners constructed a histogram instead of an ogive.

Learners: Tried to do as the teacher instructed, yet some were still experiencing problems.

Teacher B: Re-explains how to construct and interpret the ogive.

Teacher B: “Now do the exercise at 8.11, 8.12, and 8.13 in your textbook.” Teacher B instructed them to finish and submit the extra activities before going home (The activities were divided into two: one part as homework for those who were not experiencing problems and the other part for learners who were experiencing some difficulties.

Teacher B: Asked oral questioning as a way of concluding the lesson: ‘What does ‘n’ represent in the formula for calculating the quartiles? Where can I locate the quartiles using the formula?’ Learners nominated by Teacher B gave satisfactory answers which were followed by homework and post-teaching discussion.

APPENDIX IX

TRANSCRIPTION OF VIDEO RECORDS OF FIRST LESSON OBSERVATION OF TEACHER C

Teacher C: “Good after noon learners?”

Learners: “Good afternoon sir.”

Teacher C: Specified the outcome of the lesson and said, “We are going to learn how to construct, analyse and interpret the ogive. Before we do that, let us look at the homework on the histogram.”

Teacher C: Marked and checked the homework on histogram on a desk to desk basis.

Learner: Some of the learners wrote corrections from friends who got the homework correct for the questions they got wrong before the teacher could do that.

Teacher C: “What is the difference between a class interval and class boundary?”

Learner: “They are the same. Both of them contain groups of numbers.”

Teacher C: “Mention various ways of representing data?”

Learners: Mentioned bar chart, pie chart, scatter plots, line graph, etc.

Teacher C Comment: “Let us prepare a cumulative frequency table using the table on the paper that I gave you.”

Teacher C: Prepared the cumulative frequency of the first 3 rows and instructs learners to complete the cumulative frequency table of the remaining rows.

Learner: Completed the cumulative frequency of the remaining row.

Teacher C Explains: “Now we can construct the ogive. Draw the two axes, label them using the

Cumulative frequencies for the vertical axis and profit for the horizontal axes. Now I will plot three points and you will plot and connect the remaining points and lines of best fit for the curve.”

Learner: Completed the plotting and connect the curve.

Teacher C: Walked around the class from desk to desk, analysing learners' classwork and monitoring how learners are completing the plotting and connect the curve.

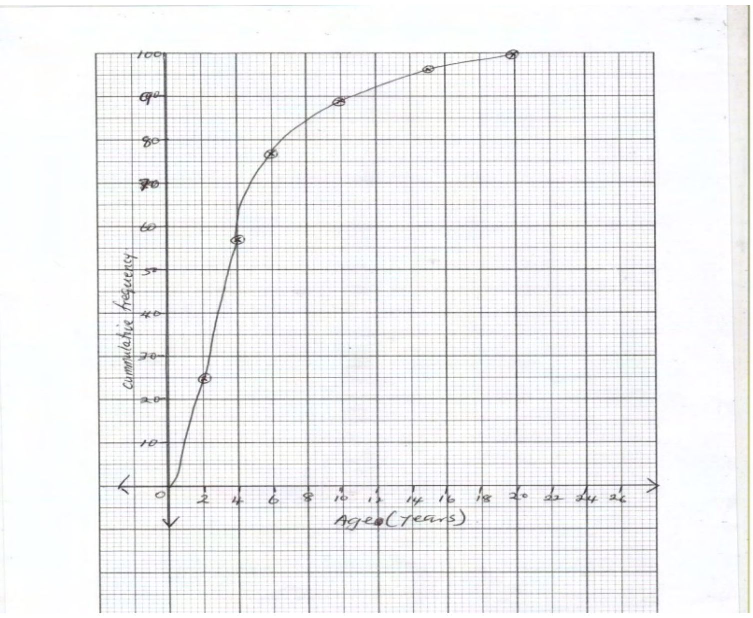


Figure 4.5.3: An ogive showing the ages of cars of a sample of 100 car owners.

Learner: Continued with the completing of the ogive.

Teacher C: "How do we calculate the median?"

Learner: Quoted a formula; $Q_1 = \frac{1}{2}(N + 1)$.

Teacher C: How do we calculate First quartile, second quartile and third quartile?

Learner: Quoted some formulae: $Q_1 = \frac{1}{4}$ (N + 1) and $Q_3 = \frac{3}{4}$ (N + 1). These formulae were used to calculate the quartiles' position as a way of interpreting the graph and further explain how we calculate the quartiles.

Learner: Listened and watched as he described how the quartiles were obtained.

Learners: Some learners drew a histogram instead of an ogive. This is a misconception.

Teacher C: "Now I observed that some of you constructed a histogram instead of ogive the question says construct an ogive and not a histogram"

Learner: Reconstructed the ogive with the help of the teacher.

Teacher C: Identified learners who are experiencing difficulties due to the misconception and requested them to see him after the lesson one by one in order to help them correct the difficulties they had.

Teacher C: Summarises the lesson with oral questioning (How do you calculate the cumulative frequencies? In constructing the ogive, and do you plot the cumulative frequencies against the lower or upper class boundaries? He gave the learners homework by referring them to their textbooks and other statistics related materials.

Some of the learners who got their classwork correct went home at the end of the lesson but those who were experiencing difficulties had to wait and see the teacher one after the other for immediate assistance.

Learner: Noted the homework given to them.

APPENDIX X

TRANSCRIPTION OF VIDEO RECORDS OF SECOND LESSON OBSERVATION OF TEACHER C

Teacher C: **Indicate the Outcomes of the Lesson**: The purpose of the lesson is to learn how to construct, analyse and interpret scatter plots.

Teacher C: Wrote the topic on the chalk board and introduces the lesson by giving the learners some photocopied exercise to analyse as a pre- test (requesting them to indicate how they have constructed the scatter plots (analysis).

Learner: Analysed the scatter plot in groups of two and three.

Teacher C: Wrote a table and requested for a volunteer to plot the point on the chalkboard as a way of explaining more about the construction of scatter plots.

Teacher: Teacher C walked around the class to monitor how learners are analysing the scatter plots. He further asks, "Do you need to draw a line of best fit in order to determine how the variable x and y are connected?"

Learners: "Yes sir."

Teacher: "Some of you interpreted diagram C as having negative correlation. Why?"

Learner: "Because the points are scattered all over (misconception)."

Teacher C Explains: "No; only one point stood out of others as outliers. It has little or no impact on the correlation of the two variables. He presents more photocopies of related examples in real life situation (see table 4.4.4c).

Learners: "How do you account for outliers in a scatter plot?"

Teacher C: “Outliers of two or more can affect the correlation of two variables and it depends on the number of the correlating points.”

Learner: Wrote explanations.

Teacher C: “Why do we say that diagram A has a strong positive correlation?”

Learner: Explains how the points are clustered along a given line indicating that the more learners are taught, the more they perform in the test. This shows a relationship between learner performance and the period they were taught.

Teacher C: Summarises the lesson with oral question and gave homework.

APPENDIX XI

TRANSCRIPTION OF VIDEO RECORDS OF FIRST LESSON OBSERVATION OF TEACHER D

Teacher D: Wrote the topic on the chalk board and gave learners photocopies of statistics exercises. “Do the exercise I have given to you for 5mins”

Learners: Solved the exercise individually involving the preparation of the frequency table of given data.

Teacher D explains: After all learners had agreed on a common answer to frequency table prepare, he showed the learners how to construct and interpret a bar graph: “Draw the axes, label them with number of rows on the vertical axis and accompanied on the horizontal axis constructing the bar graph.”

Learner: Listened and watched the teacher as he constructed the bar graph. In their mother tongue, he said, ***labella ga ke go bontsha.***

Teacher D: After he had finished, he asked the learners, “What was the first thing I did, when constructing a bar graph?”

Learner: “You draw the vertical and the horizontal axis and label it”

Teacher D: Which company manufactures the least number of cars?

Learner: “Tata”

Teacher D: “Why do I have to leave space between the bars?”

Learner: “To show that they are different companies.”

Teacher D: Gave a classwork on construction and interpretation of the bar graph.

Learner: “Do the classwork by constructing and interpreting the bar graph and find out how many learners fail the test if the pass mark is 5.”

Teacher D: Monitored and guided learners as they were doing their classwork.

Learner: "Why do we need to leave space between the bars?"

Teacher D: "All companies manufacture cars but of different types (or different companies)"

Learner: Some learners drew a histogram instead of a bar graph by not leaving a space between the bars. Some did not consider the constancy of equal spacing between the bars.

Teacher D explained: Re -explained the constructed bar graph as some learners are still experiencing difficulties in terms of constructing the bar graph as explained above. And a bar graph is not a histogram as some of you have done. While a bar graph have a common space between the bars, a histogram does not.

Learner: Listened, watched and wrote explanation on their notebooks. In their mother tongue, he said***, labella ga ke go bontsha.***

Teacher D: Gave homework on construction and interpretation of bar graphs.

Learner: Wrote down the homework.

APPENDIX XII

TRANSCRIPTION OF VIDEO RECORDS OF SECOND LESSON OBSERVATION OF TEACHER D

Teacher D: Requested for the homework given to the learners in the previous lesson.

Learner: Presented their completed homework individually.

Teacher D: Checked and marked the learners' completed homework on stem and leaves.

Learner: Wrote corrections on the homework (for those who got some answers wrong).

Teacher D: Wrote the topic on the chalkboard (construction, analysis and interpretation of histogram).

Learner: Listened and watched.

Teacher D: Presented photocopies of exercise which was used to explain the topic.

Learner: Received the photocopy and watched as the teacher demonstrated how to construct, analyse and interpret a histogram.

Teacher D: Prepared the frequency table of the data (To determine preconception).

Learner: Prepared the frequency table using a given class interval.

Teacher D explained: "This is how you draw a histogram. Draw the vertical and horizontal axis. Label the vertical as frequency and horizontal axis as masses of the player. Join the line of best fit in the form of a rectangle. I will draw two rectangles and you will complete the remaining one."

Learner: Completed the histogram as the teacher had instructed.

Teacher D: Interpreted the histogram by determining the measures of the central tendency (Mode, Mean) that best describes the players according to their weight, and gave learners classwork.

Learner: "Why is it necessary to start marking the horizontal axes with 70 and not 0?"

Teacher D: “You may make a zig-zag to indicate that you did not start from 0 or start from the vertical line as shown in the table of values. You will have enough space to construct the histogram.”

Learner: Looked on for some time as a way of showing that they were not satisfied with the explanation. They noted it and used the same method to do their class work.

Teacher D: Summarised and concluded the lesson with more explanation on the examples

and gave them homework on the same topic. Some of the learners who were still experiencing some difficulties about how to construct a histogram of grouped data were given extra compulsory activities to solve after the lesson from their recommended textbook. “All of you who failed this activity have to do this exercise and see me tomorrow after the normal school hours or closing so that I can explain to you more about how to construct histogram.”

APPENDIX XIII

Criteria for validating questionnaire schedule for teachers on how they develop PCK in statistics teaching.

Preamble

The attached questionnaire aims at investigating what the teachers actually did while teaching such as the method applied, content of the lessons, nature of the topic, how the teacher identified the learners preconceptions and learning difficulties, how the difficulties were resolved if any and how the lessons were evaluated. Kindly indicate with the options provided, your opinion about using the schedule to assess what the teacher actually did while he was teaching statistical graph during the case study period.

S/No	Descriptions	Option	Respond
1	**Instructional strategies used for teaching statistical graphs**		
A	Does the questionnaire asked for the duration of the lesson?	Yes/no	
B	Does the questionnaire request for the topic of the lesson?	Yes/no	
C	Does the questionnaire request for the objective of the lesson?	Yes/no	
D	Does the questionnaire request for the prior knowledge the lesson needed?	Yes/no	
E	Does the questionnaire request if learners have prior knowledge of the topic?	Yes/no	
f	Does the questionnaire request for how the teacher identifies the preconception with which learners come to the class about the topic?	Yes/no	
G	Does the questionnaire request whether learners achieved the objective of the lesson or not?	Yes/no	
H	Does the questionnaire request for how learners responded to class activities, homework and assignments?	Yes/no	
I	Does the questionnaire request if the teachers were able to follow the planned lesson from beginning to the end?	Yes/no	
J	Does the questionnaire request for how teachers will improve their lesson if their lesson was not successful?	Yes/no	
K	Does the questionnaire request whether teachers evaluate their lesson or not?	Yes/no	
L	Does the questionnaire request how the teachers evaluate their lessons?	Yes/no	
M	Does the questionnaire request the reason for evaluating a lesson?	Yes/no	

S/No	Descriptions	Option	Respond
2	**Learning difficulties in the teaching of statistical graph**		
a	Does the questionnaire request for information about learning difficulties that learners are experiencing?		
b	Does the questionnaire request for how teachers resolve learners' learning difficulties if any?		
c	Does the questionnaire request for what makes the learning of statistics easy or difficult?		

APPENDIX XIV

Criteria for validating written reports schedule for teacher on how they develop PCK in statistics teaching

The attached is a teacher written report schedule for a period of four weeks for teaching statistical graph. The schedule focuses on what has made the lessons easy or difficult as well as where the learners' learning difficulties lie during the teaching of statistical graphs for a period of 4 weeks. Kindly indicate with the options provided, your opinion about using the schedule to assess what has made the lesson easy or difficult.

S/No	Descriptions	Option	Response
1	**Learners' learning difficulties**		
A	Does the written report schedule request for information about the learning difficulties the teacher identifies when teaching the statistics?	Yes/no	
b	Does the written report schedule request for the difficulties that the teacher experiences when teaching statistical graphs?	Yes/no	
C	Does the written report schedule request for what the teacher finds interesting or difficult when teaching the statistics?	Yes/no	
D	Does the written report schedule request why the teacher finds certain topic interesting when teaching?	Yes/no	
E	Does the written report schedule request what the teacher find less difficult to teach in the topic?	Yes/no	
F	Does the written report schedule request for how the teachers identify the preconceptions and misconceptions which learners have about statistics during teaching?	Yes/no	
G	Does the written report schedule request the preconception identified when teaching statistics?	Yes/no	
H	Does the written report schedule request for the misconceptions identified by the teacher when teaching the topic?	Yes/no	
J	Does the written report schedule request for how the teachers address the misconceptions which they identified when teaching the topic.	Yes/no	
2	**Instructional skills and strategies used for teaching**		
a	Does the written report schedule request for how learners respond to class activities, homework and assignments		
b	Does the written report schedule request for the changes that the teacher will make next time with regards to the difficulties encountered while teaching the topic both on the part of the teacher or the learners'.		

APPENDIX XV

Criteria for validating document analysis schedule for teachers on how they develop PCK in statistics teaching

The teacher and learners' portfolios, learners' workbook and recommended mathematics textbooks are the documents that will be used to examine if mathematics teachers are complying with the National Curriculum Statements (NCS) policy for teaching and learning of mathematics and have sufficient content knowledge of school statistics. Using these criteria listed in the table below, kindly indicate with the options provided if the documents contain adequate information that can be used to determine how well the mathematics teachers are complying with implementation plan according to NCS.

S/No	Descriptions	Option	Response
1	**Learners' workbook**		
a	**Authenticity**	**Yes/No**	
i	Is learners' workbook a genuine instrument for capturing where their learning difficulties lie?	**Yes/No**	
b	**Credibility**	**Yes/No**	
ii	Can the workbook be used to gather enough evidence that is free from error and distortion about learners' learning difficulties in statistics teaching?		
c	**Representativeness**		
iii	Can the evidence obtained from the learners' workbook give a true representation about the learning difficulties they have in statistics teaching.	**Yes/No**	
d	**Meaning.**		
iv	Is the evidence about the learners' learning difficulties, gathered with the learners' workbook, clear and comprehensible?	**Yes/No**	
2	**Learners' portfolio**	**Yes/No**	
a	**Authenticity**	**Yes/No**	
i	Is learners' portfolio a genuine instrument for capturing where their learning difficulties lie?	**Yes/No**	

S/No	Descriptions	Option	Response
b	**Credibility**	**Yes/No**	
ii	Can the Learners' portfolios be used to gather enough evidence that is free from error and distortion about learners' learning difficulties in statistics teaching?	**Yes/No**	
c	**Representativeness**	**Yes/No**	
iii	Can the evidence obtained from the learners' portfolios give a true representation about the learning difficulties they have in statistics teaching.	**Yes/No**	
d	**Meaning.**		
iv	Is the evidence about the learners' learning difficulties gathered with the learners' portfolio clear and comprehensible?	**Yes/No**	
3	**Teachers' portfolio**	**Yes/No**	
a	**Authenticity**	**Yes/No**	
i	Can the teachers' portfolio be used to gather genuine evidence about how they (teachers) are complying with the teaching and learning policy in mathematics such as using the work schedule, instructional strategies used for teaching and learning difficulties that learners encountered?	**Yes/No**	
b	**Credibility**	**Yes/No**	
ii	Can the teachers' portfolios be used to gather enough evidence that is free from error and distortion about work schedules, instructional strategies used for teaching and the learning difficulties encountered by the learners?	**Yes/No**	
c	**Representativeness**		
iii	Can the evidence obtained from the teachers' portfolios give a true representation about the learners' learning difficulties in statistics teaching?		
d	**Meaning.**	**Yes/No**	
iv	Is the evidence about the learners' learning difficulties gathered with the teachers' portfolio clear and comprehensible?		
4	**Textbooks**	**Yes/No**	
a	**Authenticity**		
i	Is/are the textbook(s) the recommended textbook(s) for teaching and learning in mathematics in the school?	**Yes/No**	

S/No	Descriptions	Option	Response
b	**Credibility**		
ii	Is the textbook(s) used error free in terms of the content of statistics in school mathematics	**Yes/No**	
c	**Representativeness**		
iii	Does the textbook(s) contain adequate statistics content in school mathematics according to the National curriculum statements (NCS).	**Yes/No**	
d	**Meaning**		
iv	Is the content of statistics in school mathematics in the textbook(s) clear and comprehensible?	**Yes/No**	

APPENDIX XVI

Criteria for validating the lesson plan and observation schedule.

The attached lesson plan/observation schedule was adopted from the Department of education for classroom practice (DoE, 2009). Please indicate with the options provided, if the schedule contains enough information for assessing a normal classroom practice in terms of lesson planning/observation what the teacher did while teaching an assigned topic.

S/No	Description	Option	Response
1	**PLANNING**		
a	Does the schedule request for lesson topic?	**Yes/No**	
b	Does the scheduled request for learning outcomes?	**Yes/No**	
c	Does the Schedule request for assessment standard?	**Yes/No**	
d	Does the schedule request for resources used during the lesson?	**Yes/No**	
2	**Pedagogical issues**		
a	Does the schedule request for how the lesson was introduction?	**Yes/No**	
b	Does the schedule request for general handling of the class e.g.		
	i) Classroom organisation?	**Yes/No**	
	ii) Discipline?	**Yes/No**	
	iii) Classroom interaction?	**Yes/No**	
	iv) Movement?	**Yes/No**	
	v) Learning climate?	**Yes/No**	
	vi) Involvement of the learners?	**Yes/No**	
c	Does the schedule request for lesson development (progression)?	**Yes/No**	
d	Does the schedule request for how lesson is consolidated?	**Yes/No**	
e	Does the schedule request for the description of the lesson in terms of:		

	i) Language?	**Yes/No**	
	ii) Questioning techniques?	**Yes/No**	
	iii) Assessment?	**Yes/No**	
	iv) The use of resources?	**Yes/No**	
	v) Knowledge of the teacher?	**Yes/No**	
	vi) Errors and misconceptions	**Yes/No**	
3	**Learner related activities**		
a	Does the schedule request learners' related activities?	**Yes/No**	
4	**Teacher related activities**		
a	Does the schedule request teacher related activities?	**Yes/No**	
5	**Evaluation/Conclusion**		
a	Does the schedule request how a lesson is concluded?	**Yes/No**	

Comments

NAME AND SIGNATURE OF RATTER

Appendix XVII

Analysis of participants' responses to interview, questionnaire and teachers' written report

Table 4.7.1: Teachers' responses to interview about teachers' subject matter content knowledge in statistics teaching

Items	Interview Question	Responses				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	**Coding**
1	Which university /college did you attend?	Unisa & University of North-West	University of Zimbabwe	Aerica University, Zimbabwe	Vista University, South Africa	-all attended university
2	What qualification did you obtain?	BEd Maths Ed., BA Psychology, & Dip. in Maths Ed.	BSc Maths & Statistics	BSc Mathematics	BEd Maths Ed., SED Maths and Biology	- 2 had degree in maths education -2 had B.Sc in mats. - one also had diploma
3	What course, subject/module did you study at the university/college?	Maths, Physical Sc, & Ed Psychology. The importance, advantages and disadvantages of different instructional strategies.	Maths, Statistics & Edu. Methods of teaching, advantages and disadvantages of different strategies.	Maths & other courses. Different instructional strategies, advantages and disadvantages of the strategies for teaching various topics..	Maths courses & Maths method course. Advantages and disadvantages of different teaching approaches.	-all 4 study mats and education courses. Advantages and disadvantages of different teaching strategies
4	How long did you study this course/subject?	BEd is 3yrs & Dip is 2yrs.	Four (4) years	Four (4) years	Two (2) **&** Four (4) years	-One for 3yrs, three of them for 4 yrs, and two 0f them for 2 yrs in Dip.
5a	If one of the courses in (3) is mathematics methodology, how did it help you to prepare your lessons for teaching?	Use of varied instructional strategies.	Varied formulae and strategies for teaching the same topic	The courses helped me to prepare the lessons using the required format and knowledge for teaching with the objectives of the lessons in mind.	It helps me for planning in line with the work schedule, assessment and evaluation of my lessons.	- Help to plan lessons and vary instructional strategies.
5b	How do you know that your teaching is effective?	Response from learners or feedback to class works, assignments, homework etc.	Response from the learners to classworks, homework and assignments.	Learner's response to class activities, homework and examinations.	Feedback to class activities, homework and other related tasks on the topic.	-Analysis of learners' responses to classwork, homework and assignments.
6	Do you have interest in the teaching of mathematics? If yes / no why?	Very well. I love teaching. Mathematics helps one to improve one's thinking skills and provides opportunity for problem solving.	Yes, because answers are always there for a particular question.	Yes. It is very challenging but interesting, as it makes one to be precise and accurate.	Yes. It is straight forward. It is either you get it right or wrong.	-All four have interest in teaching statistics as mathematics improves ones thinking skill and opportunity for problem solving.

7	What is your understanding of the nature of the statistics you teach?	Statistics is practical in nature and an aspect of mathematics that one can apply in everyday real life situation especially in summarizing data.	I am quite comfortable with the concepts during teaching because of its practical way of solving problems. It helps to organised and summarized data in a meaningful way.	It helps to simplify complex data into understandable one that can be used for interpretation and analysis.	It is a practical topic that allows learners to participate actively especially during the construction of statistical graphs and preparation of frequency tables. Statistics helps in summarising data in a meaningful and understandable way.	-statistics help to organised and summarise data in an understandable manner for making decisions. The ways statistical graph are constructed make it look practical in nature.
8	Do learners understand the topic?	Yes and I notice it through their response and I make sure that they understand by employing appropriate method of teaching which the topic demanded.	About 90% of the learners were very much comfortable with the topic.	Yes. Learners' level of participation during the lesson is high.	Yes, I understand it through their response to class activities and homework.	-all four claim learners understood their lesson and it is observed from the way they respond to classwork, homework and assignments
9	Do learners enjoy the topic if yes/no why?	Yes, through their involvement in the lessons and participation.	Yes. They really like every aspect and this was evident in the way the learners performed in some of the activities given to them.	Yes. They sometimes make contributions and explain with confidence to their classmates what they do not understand during lessons.	Yes. Through response to oral questions, class work and interaction with fellow learners.	- All four claim that learners enjoy their lesson. -The way they enjoy the lesson was noticeable in the way they interact with their teachers and classmate during class discussion.
15	Have you attended a mathematics workshop or teacher development programme?	Yes. Workshop on NCS for grades 10-12 mathematics, Investec Enrichment Mathematics Programme.	Yes. Workshop was on NCS for Mathematics grades 10-12	Yes. I attended several workshops on mathematics especially data handling which lasted for a week, 3 days, etc.	Yes. I attended many workshops on teachers' development in content knowledge especially in data handling. The duration of the workshop was 7 days.	-all four claim they have attended workshop on professional development programme in maths and data handling in particular.
16	If your answer in (15) is yes, what was the content of the workshop?	Teaching and learning of NCS mathematics and methods for teaching	How to calculate measures of central tendency and spread.	The workshop was on NCS mathematics.	The workshop was on the new topics in mathematics and other challenging topics.	-The workshops were based on: - methods of teaching NCS. -Data handling. New and challenging topics in the new curriculum.
17	What was the duration of the workshop?	Every Saturday and during school holidays for one year.	7 hours workshop on the topic indicated above.	6 hours workshop on the above mentioned topic.	08h00 to 14h00 for four weeks (every Saturday) on data handling.	Durations for the workshops were 6hrs, 7hrs and 8hrs per day for four weeks and 8hrs per day during holidays.

18	Were the workshop facilitators mathematics teachers or mathematics experts?	Yes. Mathematics experts from the universities.	Mathematics teachers.	Yes. Mathematics educators from the university and colleges.	Mathematics educators and experts from the districts.	-The workshop facilitators were mathematics experts from the university, department of education and in-service teachers.
19	As a mathematics teacher, did you benefit from the workshop?	Yes. Very well. How to teach some challenging topics in mathematics such as data handling.	Did not learn new concepts but some changes in the curriculum like data handling that is new.	All about statistics especially lower and upper quartiles ranges etc.	Not as much, I was taught what I already know.	-All four claim that they benefit from the workshops as they gain more confidence in teaching, became abreast with contemporary issues in mathematics education.
20	Would you recommend that similar workshops be held for teachers?	Yes, because the benefits are enormous as it gives confidence in my class practice.	Yes. To provide educators with contemporary issues with regards to teaching and learning but not teaching educators like learners.	Yes. Any time that may be convenient.	Yes. It helps to refresh and reflect on what is already known and be aware of the contemporary issues in the teaching and learning of mathematics.	-all four claim that they will recommend for a similar workshop that can help them to reflect and refreshed their knowledge on the subject.

APPENDIX XVIII

Table 4.7.2: Participants' responses to the interview, questionnaire and written reports about teachers' knowledge of instructional skills and strategies for teaching statistics

Items	**Interview Question**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	**Coding**
10	In your own opinion and based on your experience in the teaching of statistics, how do you see the topic (statistics) in mathematics?	The topic is good to be integrated into mathematics. It lends support to other areas of mathematics because it is very practical in nature and good in summarising information e.g. frequency table	It looks understandable and helps in understanding other topics too because of the way it helps in organising information in a meaningful way.	It is very important to be integrated into mathematics because of its uses in everyday life and other professions in terms of making information to be understandable to the users.	It is a lively topic and very practical in nature. It can also help to understand other subjects.	-Statistics help to oganise data in meaningful way to the users- -it lends support to understand other subjects. -The way data is represented makes it look practical especially statistical graphs.
11	Do your learners understand your lessons based on the instructional approach for teaching as recommended in the curriculum?	Yes, the approach is OBE learner centred for teaching mathematics but not all topics required it. Learners need the basic background because the topic is new.	Yes. OBE approach is the general teaching method. But one can change depending on feedback from learners.	Yes, they do and I ensure that they participate actively or be actively involved in the lessons.	Yes they do and I encourage them to participate actively in the lesson.	-All four claim that learners understood their lessons based on the instructional strategies used in teaching. -OBE is used. -Learners must be actively involved.
12	If learners have problems in understanding the topic based on the instructional approach, what do you do to help them understand?	I will try to explain the topic with familiar examples and situation as well as solving more problems on that topic. I will try to organised extra-tutoring for them after school hours.	I will conduct remedial lessons with the learners concerned after the normal school hours with familiar examples related to real life.	I have to involve them in the discussion after the lesson and provide more class activities and ensure that they understand the lessons by monitoring how well they are doing the activities.	I have to change my methods of teaching, repeat the lesson and organise extra lesson in the topic in which they are experiencing some difficulties to help the learners.	-Explanation with varieties of example, remedial lessons, problem solving, and monitoring strategies were used to resolve any difficulty that may arose during teaching and based of the instructional approach.

13	What other instructional strategies do you use for teaching and why?	Questioning and answers and demonstration methods. The reason for using these methods is that the teaching of statistics is very practical and most material used are within the environment of the learner (real life)	Using teaching aids because it enhanced understanding and encourages the development of manipulative skills such as graph construction.	Definition of basic terms, explanations and using outcome based approach (OBE) for teaching. Extra tutoring is also important in order to improve learners' understanding of the topic and enhance their participation in the study of statistics.	Explanation of basic concepts with examples and more problem solving activities in real life situations.	-Demonstration method, questioning and answer, use of teaching aids, organizing of extra tutoring and definition of basic terms with examples were other instructional strategies used for teaching statistics. -The reason for using these strategies is to improve learner understanding and achievement in the topic.
Items	**Questionnaire**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	
1	How long was the lesson?	Duration of the lesson was 45 minutes.	Duration of the lesson was 40 minutes.	45 minutes.	45 minutes.	-Three teachers taught for 45 minutes each. -only one teacher taught for 40 minutes.
2	What was the topic of the lesson?	Statistical graph (histogram).	Data representation by statistical graphs (communicative frequency and ogives).	Scatter diagrams.	Drawing and interpreting of an ogive.	-All four taught statistical graph comprising of histogram, ogive and scatter plots

3	What was the objective / assessment standard of your lesson?	By the end of the lesson, each learner should be able to construct, analyse and interpret a statistical graph (histogram) as well as using them to solve real life problems.	By the end of the lesson, each learner was expected to be able to draw ogives and answer questions related to constructing analysing and interpreting of ogive using the knowledge of measures of central tendency .	By the end of the lesson, each learner should be able to construct, analyse and interpret statistical graph such as the bar graphs, ogive and scatter diagrams.	By the end of the lesson, each learner should be able to draw, analyse and interpret statistical graphs e.g. cumulative frequency curve.	- All four claims that the objective of their lessons was that learners should, at the end of the lesson, be able to construct, analyse, interpret and apply the statistical knowledge to solve everyday real life problems. -
7	Did you think that the learners achieved the objective of the lesson?	Yes. Through their active participation in the lessons and the reaction to the assessment task given afterwards.	They did because they were able to respond positively to the questions asked by the teacher orally, in classwork, home work and assignments.	They did and I confirm through their response to classwork and homework.	Yes. Based on their response to class activities.	-All four participants claimed that the objectives of their lessons were achieve and it was evidence in their responses to classwork, class discussion, homework and assignments
8	How did the learners respond to the class activities, homework and assignments?	Positively and they showed interests in the topic.	The response was quite positive as the learners wrote the homework activities.	Excellent. Their responses were good to indicate that they understood the lesson.	They performed well in the activities given to them as homework or assignments.	-All our teachers reported that learners' responses to classwork, homework and assignments were positive.
9	Were you able to follow the lesson as planned at the end of the lesson?	Yes and it was done as outline in the lesson plans.	Yes, the lesson was a successful one from the beginning to the end.	Yes and it was based on the lesson plan.	Yes and according to the lesson plan.	- All four participants claimed that they were able to follow the planned lessons from the starting point to the end

12	How would you improve / sustain the lesson?	By ensuring (through oral questioning or pre-test) that the necessary background knowledge does exists before introducing new content.	By using more teaching aids like the charts, overhead projector etc.	By giving extra-class activities related to real life and lessons.	By giving more examples on the topic and solving past questions.	-To ensure that learners has adequate background about the topic. -Explain lesson with enough teaching aids. _give enough and variety of class activities. -Solve past questions on the topics.
13	Do you normally evaluate your teaching?	Yes (using class work, test, exams, homework and assignments).	Quite often. Through class work, test, exams and homework.	Yes. Through class work, oral questions, homework.	Yes, I do. I give classwork, homework and sometimes assignments.	-All four participants claimed that they usually evaluate their teaching using classwork, test and examinations.
15	How do you evaluate your teaching performance?	By asking oral questions, during lessons, and giving classwork, homework and assignments.	By observing the difference in my teaching performance through end of year examination.	As mentioned in question 13.	I use same method as in question 13 to determine performance.	-By oral questioning, classwork, homework, assignments and examinations.
16	For what reason do you evaluate your teaching?	To ensure that learners understand what they need to know and to determine which methods of teaching may be more effective than the other through learners' response to class work and homework.	To improve and adjust to the performance of the learners.	For personal professional development and determine how to give learners the best of my capabilities.	To evaluate my teaching performance based on learners' performance in classworks and homework which consequently determines their progress.	-To ensure that learners understand. -To determine progress. -To select a better method of teaching the topic.

Items	**Teachers' Written Reports**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	
5	How did learners respond to classroom activities as well as homework or assignments?	The learners responded positively and showed much interest in the classroom activities. But a few of them have little problem	Learners were able to write the class activities as well as homework efficiently with exception of few learners..	Their responses to class activities were excellent. But few of them need some help to overcome their little difficulties.	Many of the learners did very well in their classroom activities as well as homework. Only few of them had some problem.	-Learners' responses were positive, excellent and interesting. But a few of them have some difficulties.
6	What changes would you make next with regards to the difficulties you encountered while teaching, either on your part or on the part of the learners?	Try to use different teaching strategies or approach and methods that will best suit the learners.	I shall try to give learners more work on the topic to enhance their understanding of the topic.	No change but more work can be given to the learners for them to understand more.	To adopt a different teaching approach and provide more activities for them to solve.	- Two of them use different teaching methods. - Two of them use more class activities for learners to solve.

APPENDIX XIX

Table 4.7.3: Participants responses to the questionnaire and written reports on teachers' knowledge of learners' preconceptions and misconceptions in statistics teaching

Items	**Questionnaire**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	**Coding**
4	What prior knowledge does your lesson require?	Measures central tendency and common bar graphs.	Calculation of mean, mode, median, quartiles and simple additions and subtractions.	Measures of central tendency and how to interpret information from simple straight line graphs.	Preparation of frequency table, class interval and boundaries, mid-points in the case of grouped data representation.	-Measures of central tendency -How to prepare frequency table from a given data. - Simple addition and subtraction. How to interpret line graphs.
5	Do the learners have prior knowledge (preconceptions) of the topic?	Yes. Measure of central tendency.	Yes. They could even link the previous knowledge with the new concepts such as histogram and frequency polygon.	Yes but not all especially about grouped data.	Yes. They have been taught how to construct and interpret line graph and quadratic graphs.	-All four claimed that learners have prior knowledge of the topic they teach. -Depending on the topic. -As indicated in question 4
6	How did you identify the prior knowledge (preconceptions) which the learners came with to the class about statistical graphs?	By asking diagnostic oral questioning.	Through the correction of answers to previous questions (Homework) that were given to learners and pre-activities related to the topic I want to teach.	Asking refresher questions involving oral questioning.	By making them to participate in solving some short question at the beginning of the lessons; but sometimes you may ask oral questions.	- Using diagnostic techniques (oral questioning, pre-test Through analysis of learners' responses to class activities (problem solving and discussion)
Items	**Teachers' written reports**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	
7	How do you identify the preconceptions and	By continually asking	The preconceptions	Through responses to	Oral questioning at the beginning	- Using diagnostic

	misconceptions of the learners during teaching?	diagnostic oral questions at the beginning of the lesson and during the lesson. Misconceptions were identify through the analysis of learners’ classwork and homework on histogram.	can be identified through oral questioning or pre-test at the beginning of the lessons. Misconceptions can be identified by going through their classwork or homework. The misconception can be resolved by learners in more practice exercise.	classwork and homework. Misconceptions were identify through the analysis of learners’ classwork and homework on scatter plots and ogive.	of the lessons for preconceptions and looking through their responses to classwork and homework on bar graphs in the case of misconceptions.	techniques e.g. Oral questioning, pre-test, classwork and homework. Misconceptions were identify through the analysis of learners’ classwork and homework on statistical graphs.
8	What preconceptions and misconceptions do you identify?	Confusing the concept such as mode, median and mean. The preconceptions identified are mode, median and mean. The misconceptions are inability to draw graphs of grouped data, choosing scale, labelling axes and interpreting the graphs especially for grouped data.	Preconceptions: mode, median and mean. Misconception is drawing a bar graph instead of a histogram.	They are familiar with measures of central tendency but develop some misconceptions such as the construction of bar graph instead of histogram during the lesson.	The drawing of the bar graphs. The misconception is that they tend to draw a histogram instead of bar graph.	- Preconceptions identified were measure of central tendency, construction of simple linear graphs and bar graphs. Misconceptions noticed were construction and interpretation of graphs of grouped data e.g. histogram, ogive and scatter plots
9	How would you address the preconceptions and misconceptions, if any, Identified during the teaching and learning process?	By dealing with clear and practical examples dealing with such concepts until learners understand them.	By providing varieties of examples and activities for learners to solve.	Give learners chance to elaborate on their misconceptions in order to correct the misconceptions.	Re-teach and re-explain using a different approach and give them more activities on same topic.	-Explanations with examples, more class activities and re-teaching of the topic.

APPENDIX XX

Table 4.7.4: Participants responses to the teachers' interview, questionnaire and written reports about teachers' knowledge of learners' learning difficulties

Items	**Interview Question**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	**Coding**
14	What learning difficulties do you remember experiencing as a pupil and as a university / college student or from teaching experience in statistics?	Have problems with plotting of scatter plots, drawing of line of best fit and forming equation from the scatter plot.	Construction of graphs especially choosing an appropriate scale in drawing histograms, frequency polygons, ogives and scatter plots.	Construction and interpreting of graphs especially ogives and scatter diagrams.	Graphical constructions and interpretation especially ogives, scatter plots and regression lines.	-learners' learning difficulties are inability to construct graph of grouped data e.g. ogive, histogram, scatter plots and frequency polygon.
Items	**Questionnaire**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	
10	What difficulties did the learner experience during teaching?	No major problem and but when it arises, I will deal with it Such as some basic calculations such as the median and mean and mode of group data	Some learners could not choose an appropriate scale on the graph paper to construct graphs of grouped data.	Not much except graphs construction and interpretation of grouped data.	None except determination of mid points and construction of graphs. But any problem occur I will be able to deal with it	- Inability to construct graph of grouped data, calculate median, mid values and interpretations. Anticipated learning difficulty to be dealt with if they arose
11	How did you address these difficulties?	By giving them more exercises to solve on problem areas.	I have to guide them on how to choose a suitable scale for a particular data hence we ended up using the same scale in constructing some of the graphs for the sake of uniformity.	By giving them more exercises to solve on problem areas.	Provide more examples and possibly repeat the lessons	-The difficulties identify were resolved by given learners more activities to solve, guiding and monitoring of learners on how to construct graphs of grouped data and repeating the lesson.

14	What is it about statistics that makes the learning easy or difficult?	Measure of central tendency is easy because it is more practical to teach. Construction and interpretation of graphs of group data are difficulty to teach.	The learning of statistics was quite easy because the learners had the background in grade 10 especially for measures of central tendency except for the construction and interpretation of graphs of grouped data such as histograms, frequency polygons and cumulative frequency curves (ogives).	Example of everyday life makes the lesson lively and interesting. The everyday life example makes learners to easily understand the lessons. But graph of grouped data, especially scatter diagrams are difficult to learn.	It is an interesting subject such that if one has a deeper understanding of it, he/she will be able to present it in a manner that learners can understand it. However, grouped data graphs are more difficult to construct and interpret.	-learning of measures of central tendency is simple to teach and learn. -When lessons are taught by an experience teachers. -Graphs of grouped data are difficult to construct.
Items	**Teachers' Written Reports**	**Responses**				
		Teacher A	**Teacher B**	**Teacher C**	**Teacher D**	**Coding**
1	What learning difficulties did you identify in learners when teaching a topic?	Basic knowledge and background information linking to the new content. They were identified by going through the learners' responses to classwork and homework	1. Choosing a suitable scale. 2. Neatness of axes and title of the graphs. 3. Labelling of axes and title of the graphs of grouped data. All of them were identified by analysis of their classwork, homework, assignments, project, etc.	Some learners are slow in learning of the topic especially the drawing of graphs of grouped data. I observe this while they are doing their classwork. I also use homework and assignments to discover their mistakes	Learners' inability to interpret statistical graphs such as the histogram, the ogive and scatter plots. These difficulties were discovered by analysing learners' classwork, homework and assignments.	- Learning difficulties identify in learners are poor background in the topic, construction of grouped data (labelling of axis), interpretation of graph of grouped data and choosing suitable scales for constructing graph of grouped data. They were identified by analysis of learners' classwork, homework, assignments, etc.

2	What difficulties do you experience in the teaching of statistical graphs?	The construction of graphs of grouped data.	As described in (1) above.	None except that learners struggle to construct graphs of grouped data.	Non availability of graph board which make it to inaccurately display the graphs during teaching.	-Construction and interpretation of graph of grouped data.
3	What do you find interesting in this topic and why?	The practical application of statistical concepts and knowledge in everyday life.	The topic has some practical examples that can be used in day to day life situations and the topic can be used in solving related problems in other subjects such as research projects.	Frequency graphs such as the bar graphs because of easy and smart presentation.	Displaying graphs of the same data using different graphs. This reinforces the uses of statistics in everyday life.	-The way the topic is relatable to everyday real life, graphical representation of data and how it encourages manipulative skills.
4	What do you think you find less difficult to teach in the topic?	Mode, median and mean of all ungrouped data	The topic is less difficult to teach and learners enjoy it especially mode, median and mean of ungrouped data.	The stem-and-leaf diagram, mode, median, mean and range.	Mode, median and mean of ungrouped data and bar graphs.	-Mode, median and mean of ungrouped data. -Stem-and-leaf.
5	How did learners respond to classroom activities as well as homework or assignments?	The learners responded positively and showed much interest in the classroom.	Learners were able to write the class activities as well as homework.	Their responses to class activities were excellent.	Many of the learners did very well in their classroom activities as well as homework.	-Learners’ responses to classroom activities such as classwork; homework and other related class activities were positive.
6	What changes would you make next with regards to the difficulties you encountered while teaching, either on your part or on the part of the learners?	Try to use a different approach and methods that will best suit the learners.	I shall try to give learners more work on the graph to enhance their understanding of the topic.	No change but more work can be given to the learners for them to understand more.	To adopt a different approach and provide more activities for them to solve that may relate to real life.	- Changes expected of the teacher are adopting different teaching approach, providing more activities for learners to solve.

APPENDIX XXI

Table 4.8: A Comparison of the documents used by participants in statistics teaching

TEXTS	CASE STUDY 1	CASE STUDY 2	CASE STUDY 3	CASE STUDY 4	CODING
Learner workbooks	The learner workbooks contain written and marked class work and home-work. For example, there was evidence of learners' inability to construct and interpret graphs of grouped data due to wrong scaling of data axis (Learning difficulties) such as histograms, cumulative frequency curves, and the drawing of lines of best fit in a scatter diagram. Learning difficulties were identified by analysis of learners' classwork and homework. Teacher A seem to teach with procedural knowledge (construction of a histogram) and conceptual (Defining and explaining with examples the meaning of mode, median, mean and histogram) approaches However, there were also some instances where learners performed well e.g. construction, analysis and interpretation of bar graphs, stem-and-leaf, histogram and ogives which they did individually and in groups (flexibility in approaches). Detail	The learner workbooks contain written and marked class work and home- work. For example, there was evidence of learners' inability to construct and interpret graphs of grouped data due to wrong scaling of data axis such as histograms (learning difficulties). They were identified by analysis of learners' classwork and homework. In some cases, learners did well in terms of graphical constructions and interpretations in bar (with horizontal and vertical bar graphs) and double bar graphs, histograms and ogives. There was evidence of intervention to resolve learning difficulties. Teacher B seemed to be flexible in his approaches to the teaching of statistics. Learners' were sometimes given extra lessons.	The learner workbooks contain written and marked class work and home- work. There was similar evidence in case study 3 as in case studies 1 and 2. Detail descriptions of statistics concepts and mathematical connections between them were available in which quartiles were described and used to interpret ogive. Learners' misconceptions (drawing a histogram instead of bar graph) and learning difficulties of not being able to label data axis due to wrong scaling were identified by analysis of learners' classwork and homework on graph of grouped data during marking. Scatter plots construction done procedurally and interpretation examples and learners' class activities relatable to real life was available in the learners' workbook. Teacher C seemed to be flexible in his approaches to the teaching of statistics. Learners' were sometimes	The learner workbooks contain written and marked class work and homework. There was similar evidence in case study 4 as in case studies 1, 2, and 3 but the rate at which learners made progress during the lesson was much better. There were details of definition of statistics concepts with examples relatable to real life. Comparative relationship between a bar graph and a histogram. Learners' misconceptions (drawing a histogram instead of bar graph and vice versa) and learning difficulties of not being able to label data axis due to wrong scaling were identified by analysis of learners' classwork and homework of graph of grouped data. Teacher D seemed to be flexible in his approaches to the teaching of statistics where he uses both procedural and conceptual knowledge to teach statistical	-Learners' workbook contains written and marked classwork but the rate at which learners make progress differ in each case. -Analysis of learners' classwork, homework and assignments were used to identify learners' misconceptions and learning difficulties. -There was evidence of learning difficulties of not being able to label data axis due to wrong scaling, and misconception of drawing a histogram instead of a bar graph and vice versa. Teachers taught with both procedural and conceptual knowledge in their statistics lessons.

TEXTS	CASE STUDY 1	CASE STUDY 2	CASE STUDY 3	CASE STUDY 4	CODING
	descriptions of concepts. etc Learners' were sometimes given extra lessons.		given extra lessons.	graphs. Learners' were sometimes given extra lessons.	
Learner portfolios	The learner portfolios contain written and marked tasks such as assignments, projects and investigations, informal and formal tests and examinations done individually and in groups. These assessment instruments with feedback from the learners contain similar evidence of where the learners perform well and where their misconceptions and learning difficulties lie as shown in the learner workbooks.	The learner portfolios contain written and marked tasks such as assignments, projects and investigations, informal and formal tests and examinations. Constructions and interpretations were observed as two of the areas were learners had difficulties. There was also evidence of teaching intervention.	The learner portfolios contain written and marked tasks such as assignments, projects and investigations, informal and formal tests and examinations. There was also evidence of intervention in learners' learning difficulties in constructing and interpreting graphs of grouped data such as scatter plots, histogram, ogive, etc.	The learner portfolios contain written and marked tasks such as assignments, projects and investigations, informal and formal tests and examinations. Similar evidence of learner difficulties in statistical graphs was observed. Evidences of interventions to address the difficulties were also observed.	-Contains written and marked tasks, tests, examinations -Learners' performances in the various tasks in statistical graph given to them were indicated showing whether they perform well or not and what difficulties they may have encountered - learners had difficulties with the construction and interpretation of graph of grouped data.
Teacher portfolios	The teacher's portfolios contain policy documents such as the National Curriculum Statement (NCS), SBA and other related assessment instruments. Other documents are lesson plans showing method used for teaching, how preconception were identified using oral questioning,, checking and marking of learners' homework, work schedules and records of assessment areas, how learners misconceptions (drawing bar graph instead of histogram) and learning difficulties (construction of graphs (labelling of	Teacher Bs' portfolio contains similar documents to that of teacher A. Intervention strategies similar to the ones used in case study 1 were used to address the misconceptions (e.g. drawing a histogram instead of bar graph) and learning difficulties about labelling the horizontal axis, observed by teacher B in case study 2 by analysing learners classwork, homework, and assignment. Oral probing questioning, pre-activities and checking and marking of learners' homework were used to identify learners' prior knowledge in histogram	Teacher Cs' portfolio contains similar documents as those of teachers A and B. Similar intervention strategies used in case studies 1 and 2 were used to address the misconceptions (drawing bar graph instead of histogram) and learning difficulties in graph constructions of graph of grouped data (eg give and scatter plots) as observed by teacher C in case study 3. Oral probing questioning, pre-activities and checking and marking	There were similar documents and records in teacher Ds' portfolio as seen in teacher A, B and C's portfolios. Oral probing questioning, pre-activities and checking and marking of learners' homework were used to identify learners' prior knowledge. Similar intervention strategies used in case studies 1, 2 and 3 were used to address the misconception ((drawing bar graph instead of histogram) and learning difficulties	-Contains policy documents, tasks, tests, examinations with their memoranda in. -Marks of learners in the tasks and test already completed were available. -Intervention strategies used to identify learning difficulties such as more activities days for afternoon lessons were indicated. Learning difficulties and misconceptions were addressed by re-explanation of the concept, extra tutoring and class

TEXTS	CASE STUDY 1	CASE STUDY 2	CASE STUDY 3	CASE STUDY 4	CODING
	data axis)of grouped data such as histogram, ogive, scatter plots, etc.) were identify by analysing learners classwork, homework during marking, and the intervention strategies such as extra tutoring and class activities were put in place to address them. There were also invitation letters to mathematics workshops.	construction. There were also invitation letters to mathematics workshops.	of learners' homework were used to identify learners' prior knowledge. Learning difficulties and misconceptions were addressed by re-explanation of the concept and as Teacher A and C did. There were also invitation letters to mathematics workshops.	of labelling data axis of grouped data observed by teacher D in case study 4. Learning difficulties and misconceptions were addressed by re-explanation of the concept and as Teacher A, B and C did using extra tutoring and class activities in statistics. There were also invitation letters to mathematics workshops.	activities in statistics. The teachers had invitation letters to several workshops in mathematics teaching.
School based assessment (SBA)	The SBA of teacher A contains topics to be taught, assessment tasks, and memoranda for the tasks and formats for grading and recording learners' performance in the tasks.	The SBA of teacher B contains content to taught assessment tasks, memoranda for the tasks and formats for grading and recording of learners' performance in the tasks.	The SBA of teacher C contains contents to be taught, assessment tasks, memoranda for the tasks and formats for grading and recording of learners' performance in the tasks.	The SBA of teacher D contains topics to be taught assessment tasks, memoranda for the tasks and formats for grading and recording of learners' performance in the tasks.	- contains contents of statistics curriculum, assessment tasks, memoranda for the tasks and formats for grading and recording of learners' performance in the tasks.
Textbooks	Recommended	Recommended	Recommended	Recommended	

APPENDIX XXII

An exercise in statistics for mathematics teachers

INSTRUCTION: This exercise is to get an insight into the basic knowledge that you have about statistics teaching in school mathematics. Choose the option that best represent the correct answer to each of the question and write it against the number of the question. Do not write your name or the name of your school.

Duration: 40mins

Example

Find the mode of the following set of data: 4,3,5,4,5,6,4,6,5,4.?

A 4
B 3
C 5
D 6
E 4 and 3

ANSWER: A (The most occurring number in the set of data)

1 What would be the angle of sector representing the interval 10-20 in a pie chart in the following frequency distribution table?

Interval	0-10	10-20	20-30	30-40	40-50
Frequency	10	30	20	8	7

A. 48^{0}
B. 72^{0}
C. 96^{0}
D. 120^{0}
E. 144^{0}

Use the frequency distribution table below to answer question

Interval	0-4	5-9	10-14	15-19	20-24
Frequency	3	5	7	4	1

2 Estimate the mode of the distribution.

A. 11.3

B. 11.5

C. 11.6

D. 12.0

E. 12.1

3 Calculate the mean of the distribution.

A 10.08

B 10.75

C 10.93

D 10.93

E. 11.79

4 The mean height of three groups of students consisting of 20, 16 and 14 students is 1.67m, 1.50m and 1.40m respectively. Find the mean height of all the students

A. 1,52m

B. 1.53m

C. 1.54m

D. 1.55m

E. 1.56m

5 The table below gives the frequency distribution of marks obtained by a group of students in a test

Marks	3	4	5	6	7	8
Frequency	5	x-1	x	9	4	1

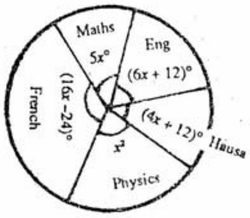
If the mean mark is 5, calculate the value of x.

A. 12
B. 13
C. 11
D. 9
E. 5

6 What is the median mark in question 4

A. 5
B. 4
C. 10
D. 5.5
E. 4.5

In a class, there are 80 students. The statistical distribution of the number of students offering Physics, English, Mathematics, Hausa, and French is shown in a pie chart below.



Use this diagram to answer question 6.

7 How many students offer Mathematics?

A. 12
B. 13
C. 36
D. 10
E. 25

8 Calculate the percentage of student that offer English in the class.

A. 18.7%
B. 18.5%
C. 18.4%
D. 18.2%
E. 18.3%

The number of learners that were absent from a school in a class in a week were 2, 5, 6, 3 and 4. Use this information to answer question 9 and 10.

9 What is the variance of the data?

A. 3.0
B. 2.5
C. 5.0
D. 4.0
E. 2.0

10 Calculate the standard deviation of the distribution.

A. 2.4

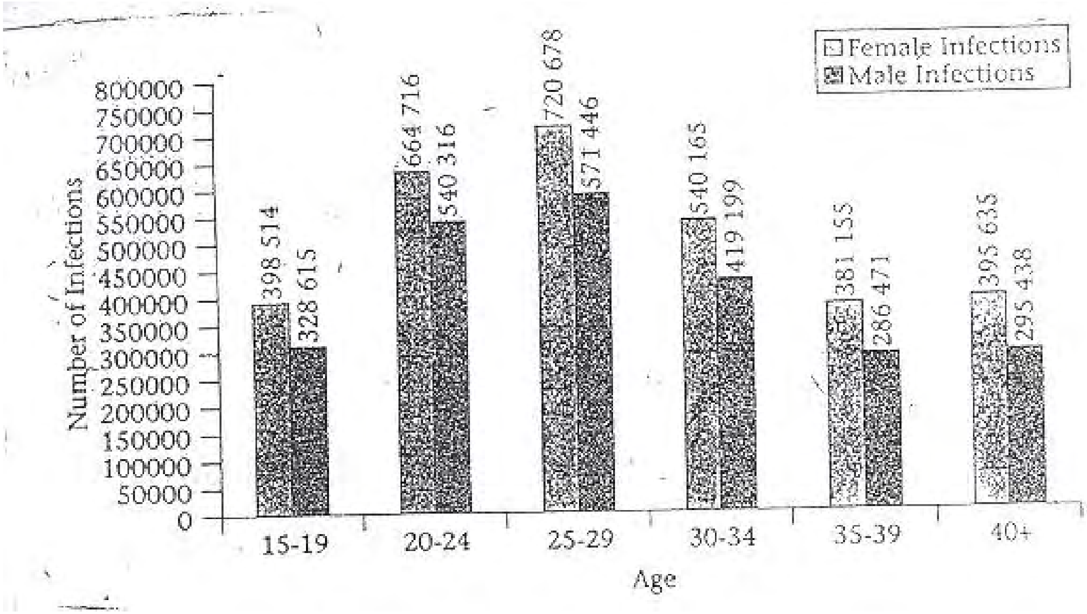
B. 1.4

C. 3.4

D. 4.4

E. 5.0

The diagram below shows the number of HIV+ males and Females per age group in South Africa in 2003. Use this information to answer questions 7 and 8.



11 How many South Africans were HIV+ in 2003?

A. 5542348

B. 5543248

C. 5554238

D. 5542384

E. 5524348

12 What percentage of male and HIV+ South Africans are in all the groups in 2003.

A. 44.50%

B. 4 4.19%

C. 44, 05%

D. 42.40%

E. 4.40%

13 If the population of South Africa is 46560400, how many South Africans are not HIV+?

A. 41018052

B. 441018052

C. 41036052

D. 14018052

E. 5542348

14 Which age group is most infected with HIV/AIDS in 2003?

A. (20-24) years

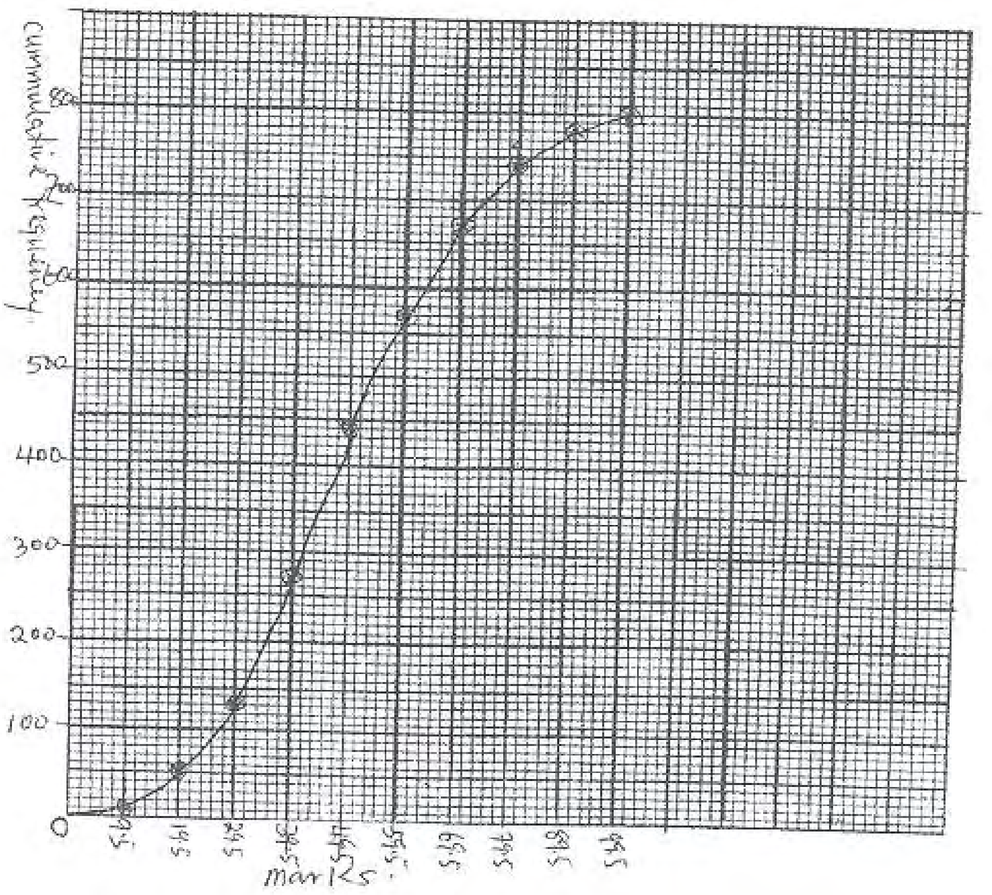
B. 40+ years

C. (30-34) years

D. (25-29) years

E. (15-19) years

The frequency distribution of marks of 800 students in an examination is display in a cumulative frequency curve as shown below. Use the diagram to answer questions9 -11.



15 Use your ogive to determine the 50th percentile.

A. 47.5

B. 43.5

C. 37.5

D. 57.5

E. 67.5

The candidates who score less than 25% are to be withdrawn from the institution, while those that score more than 75% are to be awarded scholarship. Estimate:

16 The number of candidates that will be withdrawn from the institution.

A. 100

B. 80

C. 180

D. 200

E. 70

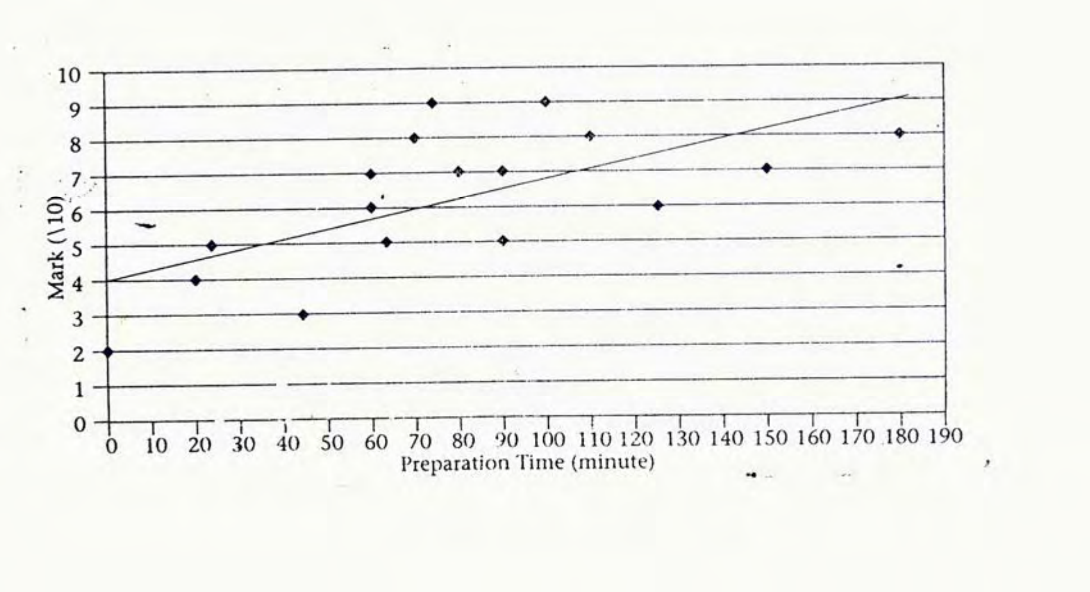
17 How many candidates will be retained in the institution but will not enjoy the scholarship award?

A. 640
B. 560
C. 300
D. 440
E. 540

18 How many candidates will be retained in the institution but will not enjoy the scholarship award?

A. 640
B. 560
C. 300
D. 440
E. 540

The graph below display the amount of time that a group of students spent in preparation for a test and the marks that they scored in the test. The line of best fit has been included on the scatter plot. Use this diagram to answer questions 12.



19 The graph shows a ------------------------------ correlation between the amount of time the students spent in preparing the examinations and the marks scored.

A. Strong

B. No

C. Weak

D. Strong and weak

E. None of the above

20 Determine the equation of the line of best fit for the scatter plot.

A ` Y = 0.22x + 40

B Y = 0.50x + 40

C Y = 0.70x + 40

D Y = 22x + 40

E Y = 2.2x + 40

Total marks: 100

APPENDIX XXIII

Memo for final conceptual knowledge exercise, march 2010

QUESTION	ANSWER
1	E
2	C
3	B
4	C
5	C
6	A
7	D
8	E
9	E
10	B
11	A
12	B
13	A
14	D
15	A
16	B
17	C
18	A
19	A
20	A

APPENDIX XXIV

Examining the content knowledge of mathematics teachers in statistics teaching

Duration: 30mins

The following are the topics to be taught in statistics under data handling in the new National Curriculum Statements for grades 10-12: stem-and-leaf; mode, median and mean of ungroup data; frequency table of group data; range, percentiles, quartiles; inter-quartiles and semi-quartile range; bar and compound bar graphs; histogram; frequency polygons; pie charts; line and broken line graphs; box and whisker plot; variance, mean deviation; standard deviation; Ogives; five number summary; scatter plots; line of best fit.

a) Arrange the topics in each grade on how you think they should be taught in grades 10, 11 and 12.

b) With an arrow, show how you can teach these topics sequentially in each grade. For example, you observe morning before afternoon and before evening. Therefore; Morning ⟶ afternoon ⟶ evening

A)

GRADE 10	GRADE 11	GRADE 12

GRADE 10	GRADE 11	GRADE 12

APPENDIX XXV

Examining the content knowledge of mathematics teachers in statistics teaching

Duration: 30mins

The following are the topics to be taught in statistics under data handling in the new National Curriculum Statements for grades 10-12: stem and leave; mode, median and mean of group data; frequency table of group data; range, percentiles, quartiles; inter-quartiles and semi-quartile range; bar and compound bar graphs; histogram; frequency polygons; pie charts; line and broken line graphs; box and whisker plot; variance, mean deviation; standard deviation; Ogives; five number summary; scatter plots; line of best fit.

c) Arrange the topics in each grade on how you think they should be taught in grades 10, 11 and 12.

d) With an arrow, show how you can teach these topics sequentially in each grade. For example, you observe morning before afternoon and before evening. Therefore; Morning → afternoon → evening

SOLUTION

A)

GRADE 10	GRADE 11	GRADE 12
Mode, Median, Mean, Ranges, (ungrouped data), Frequency table, Bar and Compound bar graphs, Histogram, Frequency polygons, Pie charts, Line and broken line graphs. Mode, median and mean (grouped data), Quartiles, Inter-quartiles and semi-inter-quartile range	Five number summary, Box and whisker diagrams, Ogives, Variance and Standard deviation, Scatter diagrams, Lines of best fit	N/A

	GRADE 10			
	Mode → Median → Mean → Ranges (Ungrouped data) Frequency table → Bar and Compound bar graphs → Histogram → Frequency Polygon → Pie Charts Line and broken line graphs. Mode → Median → Mean (Grouped data) Quartiles → Inter-quartile and semi-inter-quartile ranges			
	GRADE 11			
	Five number summary → Box and whisker diagrams → Box and whisker diagrams → Ogives → Variance and Standard deviation → Scatter diagrams → Scatter diagrams → Lines of best fit			
1	**A(1)**	**Grade 10 topics (25%)**	**Marks deducted**	**Marks obtained**
	a	Missing one topic in the arrangement.	2	23
	b	Missing two topics in the arrangement.	4	21
	c	Missing three topics in the arrangement.	6	19
	d	Missing four topics in the arrangement.	8	17
	e	Missing five topics or more in the arrangement.	10	15
	A2	**Grade 11 topics (25%)**	**Marks deducted**	**Marks obtained**
	a	Missing one topic in the arrangement.	2	23
	b	Missing two topics in the arrangement.	4	21
	c	Missing three topics in the arrangement.	6	19
	d	Missing four topics in the arrangement.	8	17
	e	Missing five or more topics in the arrangement.	10	15

2	**B1**	**Grade 10 with links (25%)**	**Marks Deducted**	**Marks obtain**
	a	Missing one topic in the arrangement.	2	23
	b	Missing two topics in the arrangement.	4	21
	c	Missing three topics in the arrangement.	6	19
	d	Missing four topics in the arrangement.	8	17
	e	Missing five or more topics in the arrangement.	10	15
	B2	**Grade 11 with links (25%)**	**Marks Deducted**	**Marks obtain**
	a	Missing one topic in the arrangement.	2	23
	b	Missing two topics in the arrangement.	4	21
	c	Missing three topics in the arrangement.	6	19
	d	Missing four topics in the arrangement.	8	17
	e	Missing five or more topics in the arrangement.	10	15
	B3	Grade 12 (N/A)	**Marks Deducted**	**Marks obtain**
		N/A	Deduct 10 marks from total marks if any topic is written in grade 1 2. But if the same topic in grade 11 is written in grade 12, no mark should be deducted. It should be regarded as a revision in grade 12.	Balance after deduction

APPENDIX XXVI

The interview schedule for mathematics teachers.

This interview probes the content knowledge in statistics and educational background that may have enabled the teachers to develop their topic-specific PCK in statistics.

Time : 30 minutes

1) Which university / college did you attend?

2) What qualifications did you obtain?

3) What course/subject/module did you study at the university/ college?

4) How long did you study this course/subject?

5a) If one of the courses in (3) is mathematics methodology, how did it help you to prepare lessons for teaching?

5b) How do you know that your teaching is effective?

6) Do you have an interest in the teaching of mathematics? If yes/no, why?

7) What is your understanding of the nature of the statistics you are teaching?

8) Do learners understand the topic?

9) Do learners enjoy the topic? If yes/no, why?

10) In your own opinion and based on your experience in the teaching of statistics, how do you see the topic (statistics) in mathematics?

11) Do your learners understand your lessons based on the instructional approach for teaching as recommended in the curriculum?

12) If the learners have any problems in understanding the topic based on the instructional approach, what do you do to help them to understand?

13) What other instructional strategies do you use for teaching and why?

____

____

____

14) What learning difficulties do you remember experiencing as a pupil and as a university student or from teaching experience in statistics?

____

____

____

15) Have you ever been to a mathematics workshop or teachers' development programme?

____

____

____

16) If your answer in (15) is yes, what was the content of the workshop?

____

____

____

17) What was the duration of the workshop?

____

____

____

18) Were the workshop facilitators mathematics teachers or mathematics expert?

____

____

____

19) As a mathematics teacher, what did you benefit from the workshop?

____

____

____

20) Would you recommend that similar workshops be held for teachers in subsequent time?

____

____

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____

APPENDIX XXVII

Report on the teaching of statistics.

This schedule is to guide the mathematics teachers in written report during the four weeks of teaching statistical graphs in grade 11. Any other relevant information may be added by the teacher during the course of teaching.

Duration: 4 weeks

1) What learning difficulties do you identify in learners when teaching a topic?

____

____

____

2) What difficulties do you experience in the teaching of statistical graphs?

____

____

____

3) What do you find interesting in this topic and why?

____

____

____

4) What do you think you find less difficult to teach in the topic?

____

____

____

5) How did the learners respond to classroom activities as well as homework or assignments?

____

____

____

6) What changes would you make next time with regard to the difficulties you encountered while teaching, either on your part or on the part of the learners?

____

____

____

7) How do you identify the preconception and misconceptions of the learners during teaching?

____

____

____

8) What preconceptions or misconceptions do you identify?

____

____

____

9) How would you address the preconceptions and misconceptions, if any, identified during the teaching and learning process?

____

____

____

APPENDIX XXVIII

The questionnaire for mathematics teachers.

This questionnaire aimed at investigating what the teacher did while teaching statistical graphs in grade 11.

Duration: 15 mins

1. How long was the lesson?

____

____

____

2. What was the topic of your lesson?

____

____

____

3. What were the objectives of your lesson?

____

____

____

4. What prior knowledge does your lesson require?

____

____

____

5. Did the learners have the prior knowledge (preconceptions) of the topic?

____

____

____

6. How did you identify the prior knowledge (preconceptions) which the learners bring to the class about statistical graphs?

____

____

____

7. Did you think the learners achieved the objective of the lesson?

____

____

____

8. How did the learners respond to class activities, homework and assignments?

____

____

____

9. Were you able to follow the lesson as planned to the end of the lesson?

____

____

____

10 What difficulties did the learners experience?

____

____

____

11 How did you address these difficulties?

____

____

____

12 How would you improve the lesson?

____

____

____

13 Do you normally evaluate your teaching?

____

____

____

14 What is it about statistics that makes the learning easy or difficult?

____

____

____

15 How do you evaluate your teaching performance?

____

____

____

16 For what reasons do you evaluate your teaching?

____

____

____

17 Were the students able to use the knowledge acquired to solve other problems?

____

____

____

APPENDIX XXIX

Instrument validation form for the conceptual knowledge exercise

Pedagogical content knowledge which is topic specific is conceptualised to include content specific knowledge, content specific instructional strategies, conceptions and learners' learning difficulties. The study participants (competent mathematics teachers) have developed PCK and used it to assist learners to perform well in mathematics as evidence at the senior certificate examination result for some period of years. This instrument is meant to measure the content of a chosen topic (statistics) according to the National Curriculum Statements (NCS) in mathematics, which the competent mathematics teachers have and demonstrate in statistics teaching.

I therefore solicit few moment of your time to help me to validate the instrument using **SURENESS** and **RELEVANCE**. By indicating **sureness,** one has no doubt that the instrument measures the content of the chosen topic. By indicating relevance, one has no doubt that the instrument is valuable and useful in measuring the content knowledge of the chosen topic.

The rating levels for SURENESS are: 1 = not very sure; 2 = fairly sure; and 3 = very sure.

The rating levels for RELEVANCE are: 1 = low/not relevant; 2 = fairly relevant; and 3 = highly relevant.

QUESTION NUMBER	**SURENESS**	**RELEVANCE**	**DO NOT WRITE ON THIS COLUMN**
1			
2			
3			
4			
5			

6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

Comments:

Signature of Reviewer and qualification Date

APPENDIX XXX

Ethical clearance certificate

UNIVERSITY OF PRETORIA
FACULTY OF EDUCATION
RESEARCH ETHICS COMMITTEE

CLEARANCE CERTIFICATE	**CLEARANCE NUMBER :** SM 09/08/02
DEGREE AND PROJECT	PhD How competent mathematics teachers devleop pedagogical content knowledge in statistics teaching
INVESTIGATOR(S)	Sunday Bomboi Ijeh
DEPARTMENT	Department of Science, Mathematics and Technology Education
DATE CONSIDERED	1 December 2011
DECISION OF THE COMMITTEE	APPROVED

Please note:
For Masters applications, ethical clearance is valid for 2 years
For PhD applications, ethical clearnace is valid for 3 years.

CHAIRPERSON OF ETHICS COMMITTEE	Prof L Ebersohn
DATE	1 December 2011
CC	Jeannie Beukes Prof G.O.M Onwu

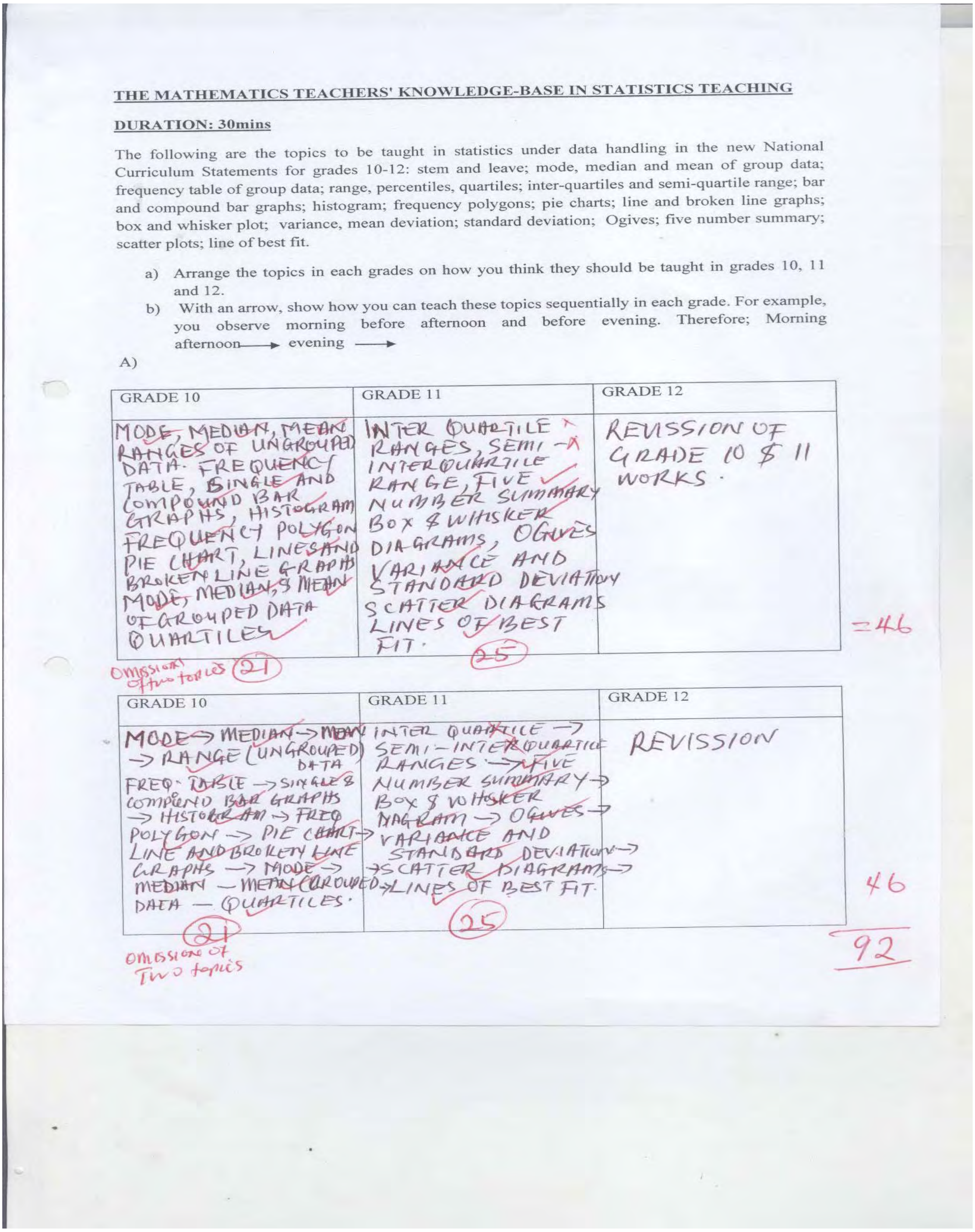
This ethical clearance certificate is issued subject to the following conditions:

1. A signed personal declaration of responsibility
2. If the research question changes significantly so as to alter the nature of the study, a new application for ethical clearance must be submitted
3. It remains the students' responsibility to ensure that all the necessary forms for informed consent are kept for future queries.

Please quote the clearance number in all enquiries.

APPENDIX XXXI

A sample of teachers' response to concept mapping exercise



THE MATHEMATICS TEACHERS' KNOWLEDGE-BASE IN STATISTICS TEACHING

DURATION: 30mins

The following are the topics to be taught in statistics under data handling in the new National Curriculum Statements for grades 10-12: stem and leave; mode, median and mean of group data; frequency table of group data; range, percentiles, quartiles; inter-quartiles and semi-quartile range; bar and compound bar graphs; histogram; frequency polygons; pie charts; line and broken line graphs; box and whisker plot; variance, mean deviation; standard deviation; Ogives; five number summary; scatter plots; line of best fit.

a) Arrange the topics in each grades on how you think they should be taught in grades 10, 11 and 12.

b) With an arrow, show how you can teach these topics sequentially in each grade. For example, you observe morning before afternoon and before evening. Therefore; Morning afternoon → evening →

A)

GRADE 10	GRADE 11	GRADE 12
MODE, MEDIAN, MEAN RANGES OF UNGROUPED DATA. FREQUENCY TABLE, SINGLE AND COMPOUND BAR GRAPHS, HISTOGRAM FREQUENCY POLYGON PIE CHART, LINES AND BROKEN LINE GRAPHS MODE, MEDIAN, & MEAN OF GROUPED DATA QUARTILES	INTER QUARTILE RANGES, SEMI-INTERQUARTILE RANGE, FIVE NUMBER SUMMARY BOX & WHISKER DIAGRAMS, OGIVES VARIANCE AND STANDARD DEVIATION SCATTER DIAGRAMS LINES OF BEST FIT.	REVISSION OF GRADE 10 & 11 WORKS.

OMISSION of two topics ⑳①

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=46

GRADE 10	GRADE 11	GRADE 12
MODE → MEDIAN → MEAN → RANGE (UNGROUPED DATA FREQ. TABLE → SINGLE & COMPOUND BAR GRAPHS → HISTOGRAM → FREQ POLYGON → PIE CHART → LINE AND BROKEN LINE GRAPHS → MODE → MEDIAN — MEAN (GROUPED DATA — QUARTILES.	INTER QUARTILE → SEMI-INTERQUARTILE RANGES → FIVE NUMBER SUMMARY → BOX & WHISKER DIAGRAM → OGIVES → VARIANCE AND STANDARD DEVIATION → → SCATTER DIAGRAMS → LINES OF BEST FIT.	REVISSION

21

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46

92

OMISSION OF TWO topics

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YUNIBESITHI YA PRETORIA

APPENDIX XXXII

LESSON OBSERVATION SHEET

DATE: ....

DURATION OF THE LESSON: ....

The practical investigation lesson will be observed against the following attributes:

1) PLANNING
    1.1 Lesson topic
    1.2 Learning outcomes
    1.3 Assessment Standards
    1.4 Resources used
2) PEDAGOGICAL ISSUES
    2.1 Introduction of the lesson
    2.2 General class handling
    2.2.1 Class organization
    2.2.2 Discipline
    2.2.3 Interactions
    2.2.4 Movement
    2.2.5 Learning climate
    2.2.6 The involvement of the lesson
2,3 Lesson Development (Progression)
2.4 Consolidation of the lesson
2.5 Description of teaching and learning
    2.5.1 Language
    2.5.2 Questioning techniques
    2.5.3 Assessments
    2.5.4 The use of resources
    2.5.5 Knowledge of the teacher
        a) How did the teacher identify learners’ preconceptions, if any, in a topic as indicated in the lesson plan? Did he or she demonstrate knowledge of learners’ anticipated learning difficulties in the topic during the lesson and in the lesson plan?

b) Did the teacher demonstrate his or her subject matter content knowledge of the topic he or she was teaching?

c) What instructional skills and strategies did he or she use in teaching the topic (statistics)?

2.5.6 Errors and misconceptions

d) How did he or she identify the learners' misconceptions and learning difficulties in the topic he or she was teaching?

e) How did he or she address the identified misconceptions and learning difficulties?

3) LEARNER RELATED ACTIVITIES
4) TEACHER RELATED ACTIVITIES
5) EVALUATION / CONCLUSIONS